CERN Summer Student Lecture - 2024

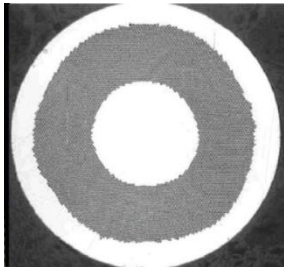
Accelerator Technology Challenges: Superconducting magnets (1/2)

Susana Izquierdo Bermudez
(susana.Izquierdo.Bermudez@cern.ch)
European Organization for Nuclear Research (CERN)

Goal of the course

- Overview of superconducting magnets for particle accelerators (dipoles and quadrupoles)
- Exciting, fancy and dirty mixture of physics, engineering, and chemistry
    - Chemistry and material science: superconducting materials
    - Quantum physics: the key mechanisms of superconductivity
    - Classical electrodynamics: magnet design
    - Mechanical engineering: support structures
    - Electrical engineering: powering of the magnets and their protection
    - Cryogenics: keep them cool …
    - Cost optimization also plays a relevant role

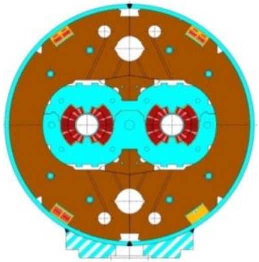
Superconducting strand



Superconducting cable

Superconducting coil

Superconducting magnet



References

Superconducting magnets for particle accelerators are a vast domain. This lecture will be especially focused on magnets for colliders, with a special eye on the CERN high energy infrastructures (LHC and HL-LHC). They are based on:

- P. Ferracin, E. Todesco, S. Prestemon, "*Superconducting accelerator magnets*", US Particle Accelerator School, www.uspas.fnal.gov.
- E. Todesco, "Masterclass -Design of superconducting magnets for particle accelerators", https://indico.cern.ch/category/12408/

Many thanks to Paolo F., Ezio T. and Luca B., for all the material I took from them for this course, and for everything I learnt from them on superconducting magnets!

Outline

- Part I
  - Particle accelerators, magnets and the need of superconductors
  - Magnetic design and coil fabrication

- Part II
  - Mechanical design and assembly
  - Quench, training and protection
  - Outlook, what brings the future

Outline

- Part I
  - Particle accelerators, magnets and the need of superconductors
  - Magnetic design and coil fabrication
- Part II
  - Mechanical design and assembly
  - Quench, training and protection
  - Outlook, what brings the future

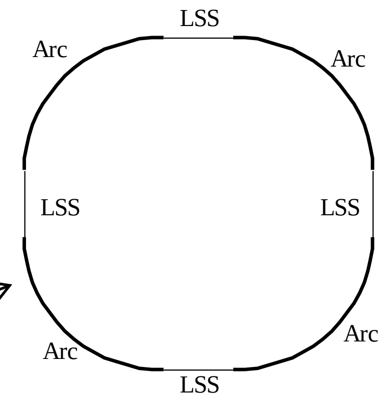
Particle accelerators

Principle of synchrotrons:

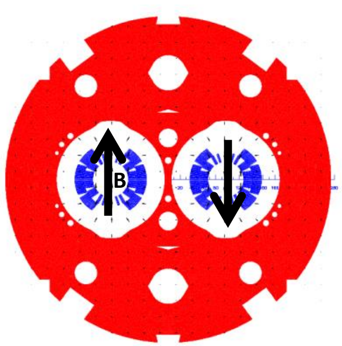
Driving particles in the same accelerating structure several times.

- Electro-magnetic field accelerates particles

$$\vec{F} = e\vec{E}$$



- Magnetic field steers the particles in a ~ circular orbit



$$\vec{F} = e\vec{v} \times \vec{B}$$

$$p = eB\rho$$

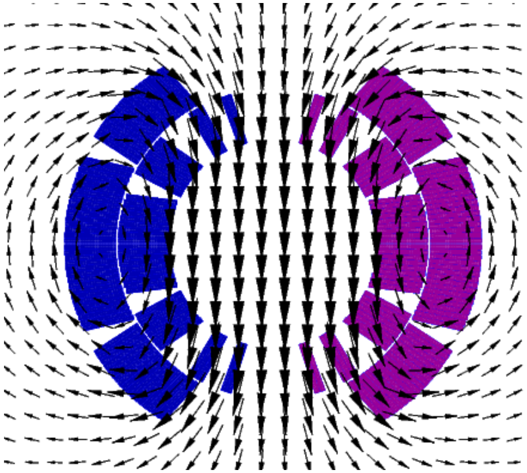
Constant

- Particle accelerated $\rightarrow$ energy increased $\rightarrow$ magnetic field increased ("synchro") to keep the particles on the same orbit of curvature ρ

Particle accelerators and magnets

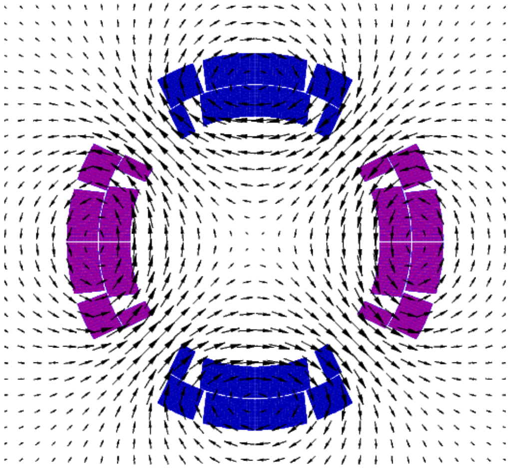
- How do we keep the particles in a cycle? MAGNETS!

  - Dipole magnets provide a constant field, to be increased with time to follow the particle acceleration, steering (bends) the particles in ≈ circular orbit



$$B_y = B_1$$
$$B_x = 0$$

  - Quadrupole magnets keep the particles in the orbit, providing a linear force that keep them focused acting as a spring. They provide a field
    - Equal to zero in the center
    - Increasing linearly with the radius

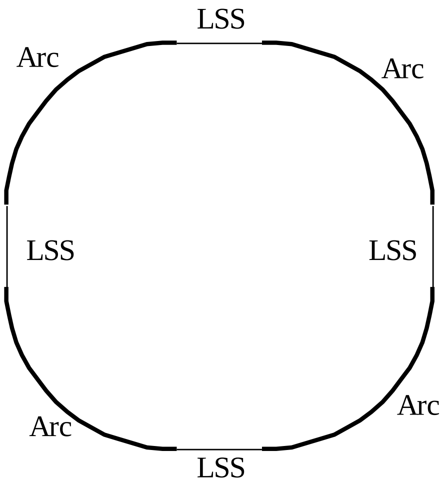


$$B_y = Gx$$
$$B_x = Gy$$

Particle accelerators: the LHC

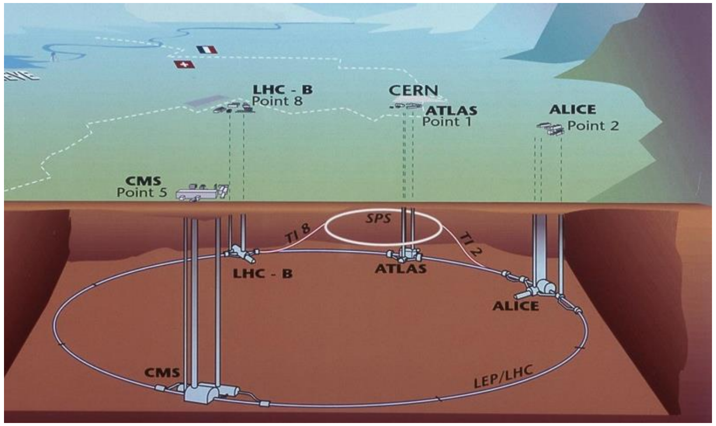
"The Arc (20.7 km)"

- **Dipoles**: magnetic field steers (bends) the particles in a ~circular orbit
- **Quadrupoles**: magnetic field provides the force necessary to stabilize linear motion.
  - They act as a spring: **focus the beam**
  - Prevent protons from **falling** to the bottom of the aperture due to the **gravitational force (**it would happen in less than 60 ms!)
- **Correctors**



"Long straight sections (7.2 km)"

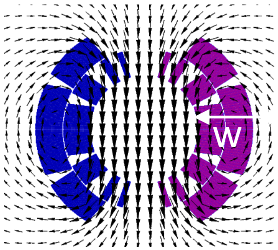
- Interaction regions (IR) where the experiments are housed
  - Quadrupoles for strong focusing in interaction point
  - Dipoles for beam crossing in two-ring machines
- Regions for other services
  - Beam injection (dipole kickers)
  - Accelerating structure (RF cavities)
  - Beam dump (dipole kickers)
  - Beam cleaning (collimators)



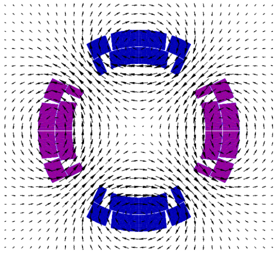
Electromagnets

- Dipoles: the larger B, the larger the energy ($p = eB\rho$)
- Quadrupoles: the larger B, the larger the focusing strength ($G = B/r$)
- For an electro-magnet, the larger B, the larger must be J

$$B_y = -\frac{\mu_0 J_0}{2} w$$



$$G = -\frac{\mu_0 J_0}{2} ln \frac{r_{out}}{r_{in}}$$

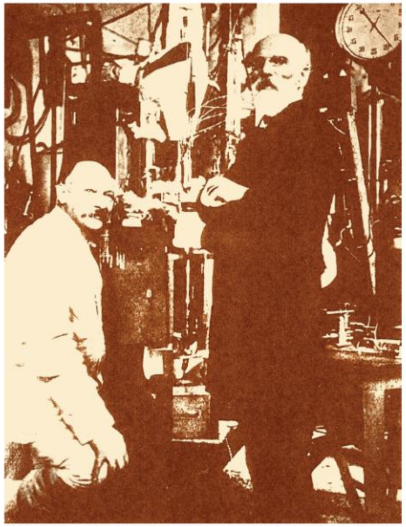


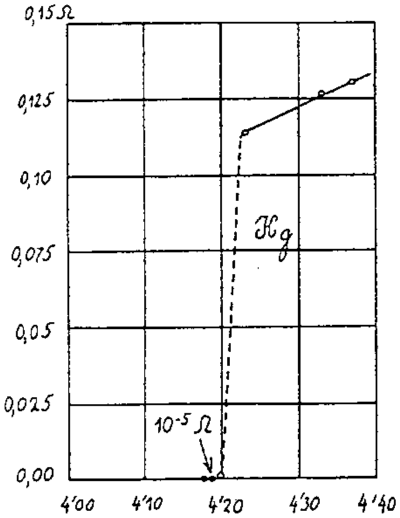
- In normal conducting magnets, $J \sim 5$ A/mm^2
- In superconducting magnets, $J_e \sim$ 600-700 A/mm^2

If we want magnets with B>2T and a reasonable size (and energy consumptions), superconductors are needed

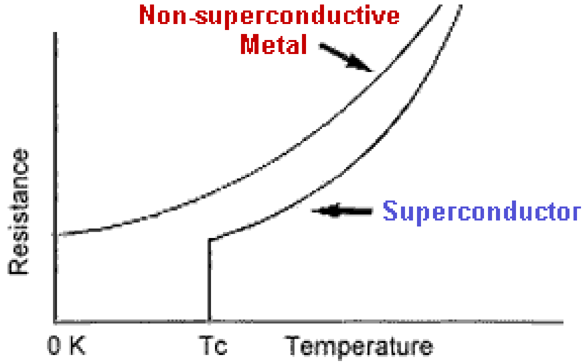
Superconductivity

- In 1911, Kammerling-Onnes, discovered superconductivity (**ZERO resistance** of mercury wire at 4.2 K)





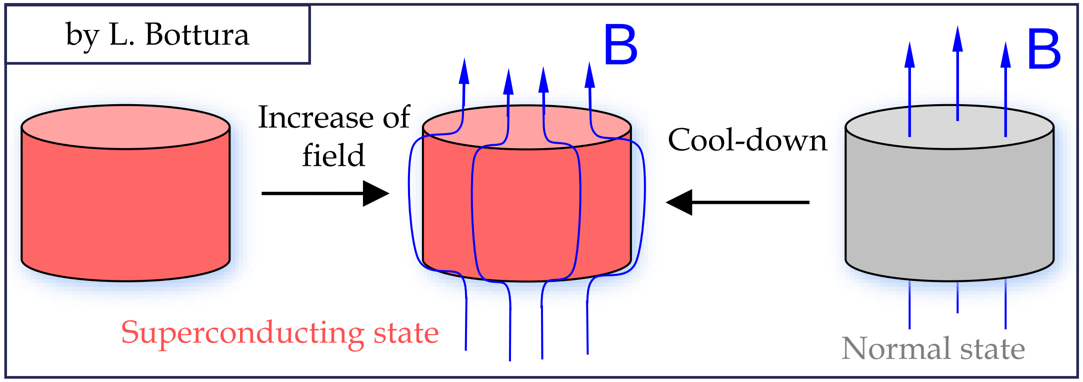
- The temperature at which the transition takes place is called **critical temperature T_c**
- Observed in may materials
  - but not in the typical best conductors (Cu, Ag, Au)
- At $T > T_c$, superconductor very poor conductor



Superconductivity

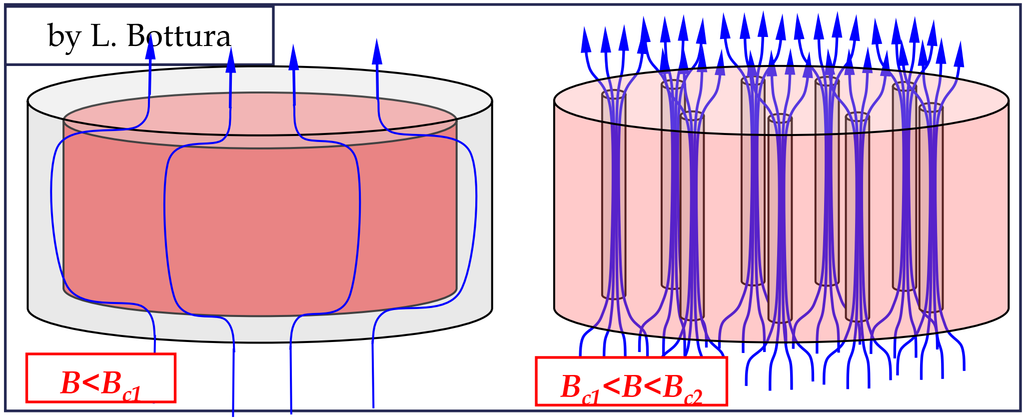
- For 40-50 years, only "**Type I**" superconductors were known.

  - Perfect diamagnetism. With $T<T_c$ magnetic field is expelled
  - But, the B must be < critical field B_c. Otherwise, superconductivity is lost
  - Unfortunately, B_c *very low* (≤ 0.1 T), not practical for electro-magnets

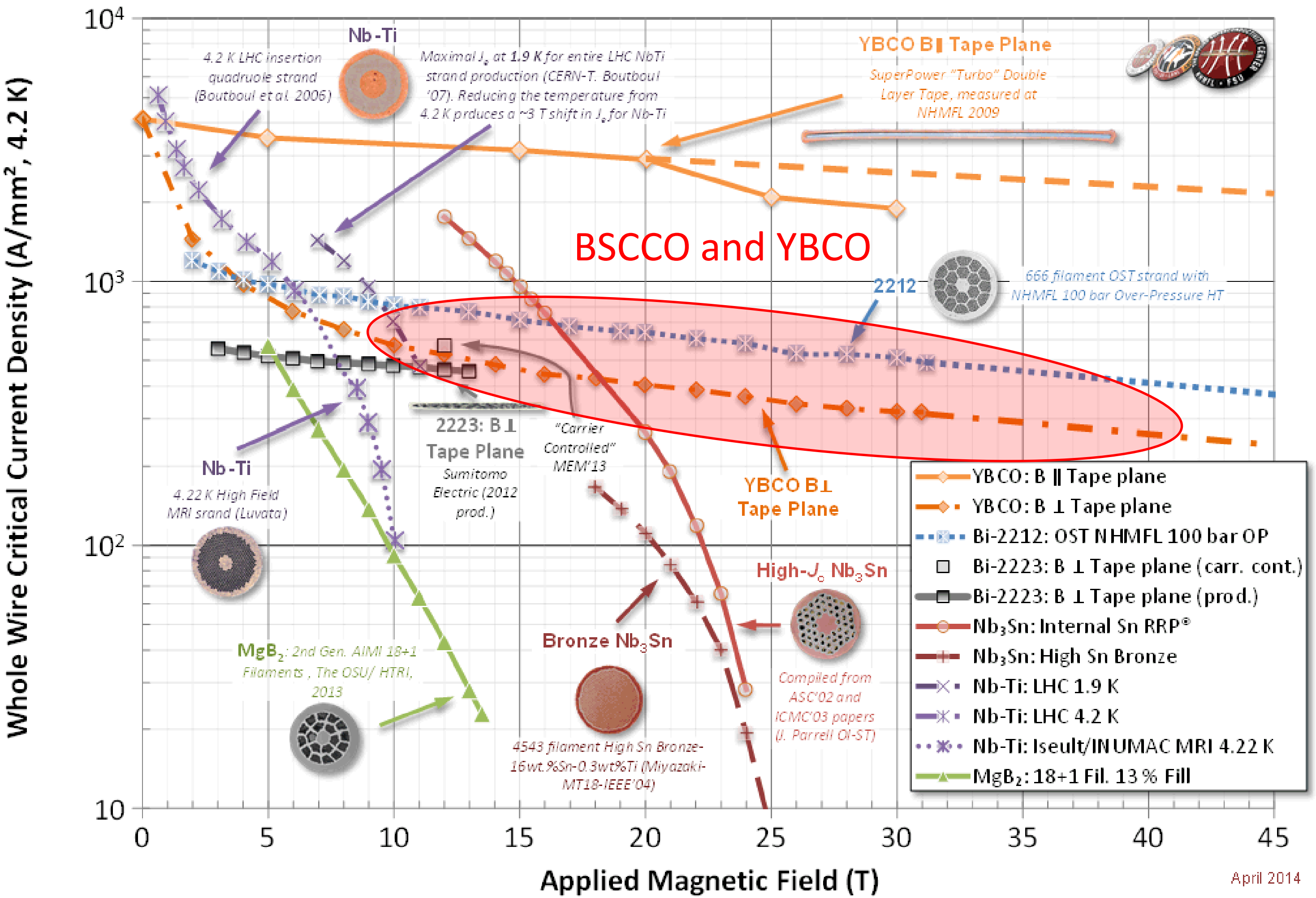


- Then, in the 50's, "**Type II**" superconductors

  - Between B_{c1} and B_{c2} : <u>mixed phase</u>
    - B penetrates as flux tubes: *fluxoids*
  - Much higher fields and link between T_c and B_{c2}



Practical superconductors

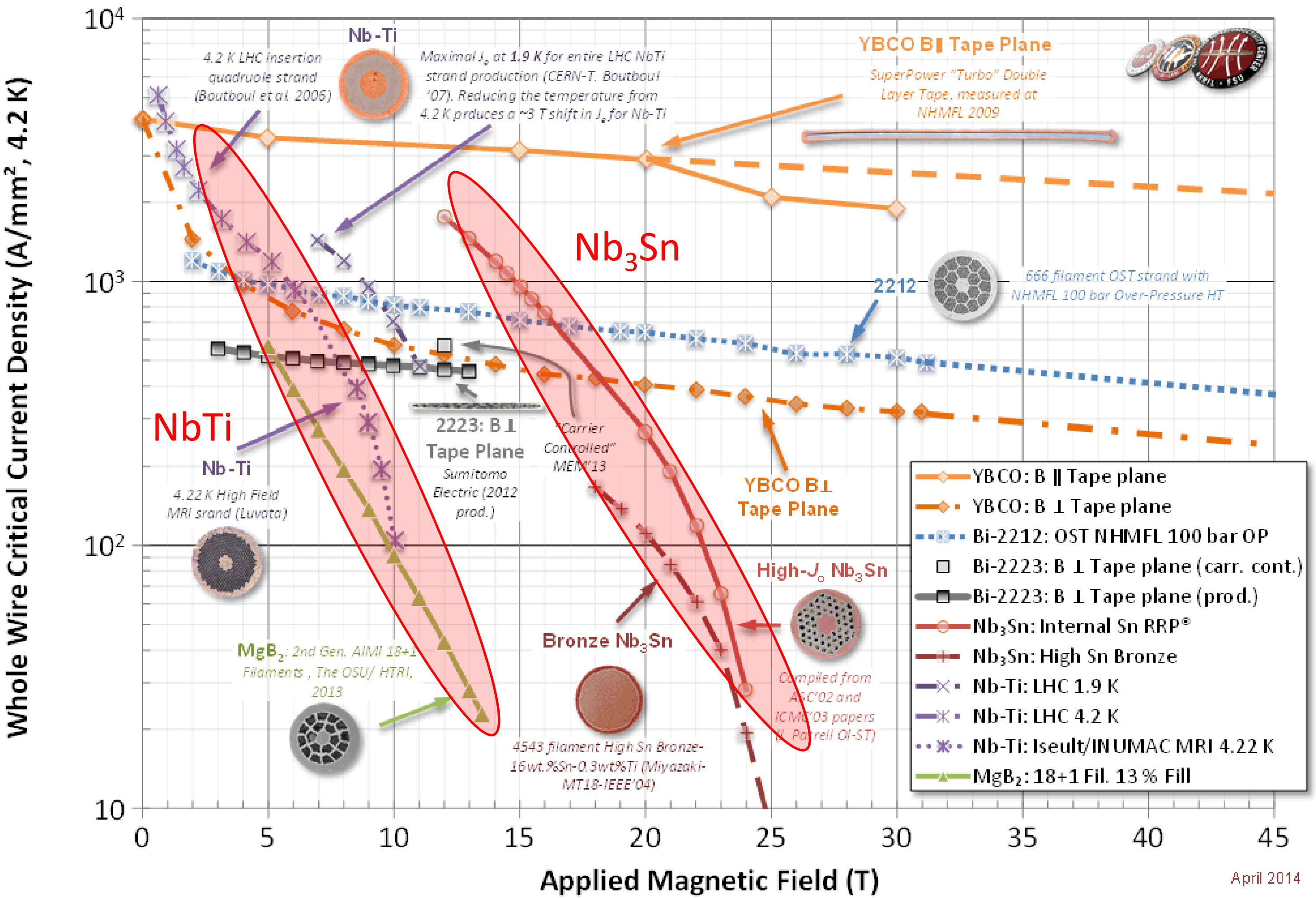


Critical current density in the superconductor versus field for different materials at 4.2 K [P. J. Lee, et al]
https://nationalmaglab.org/images/magnet_development/asc/plots/JeChart041614-1022x741-pal.png

BSCCO and YBCO

- BSSCO and YBCO are the two main HTS (high temperature superconductors)
  - Discovered in 1988/86
  - Large critical temperature ≈100 K
  - Very large critical field above 150 T
  - Flat critical surface (little dependence on field)
  - Large progress in reaching good current density
  - Both expensive (more than 10 times Nb-Ti …)
  - Drawbacks:
    - YBCO round wires are not trivial – most application on tapes
    - BSCCO requires a heat treatment at 800 C , and 100 bar of oxygen to increase *j*
  - NMR/MRI solenoids with HTS tapes have been developed
  - Projects of dipole inserts for accelerator magnets are ongoing in many labs (LBNL, BNL, CERN, CEA, …)

Practical superconductors



Critical current density in the superconductor versus field for different materials at 4.2 K [P. J. Lee, et al]
https://nationalmaglab.org/images/magnet_development/asc/plots/JeChart041614-1022x741-pal.png

NbTi and Nb_3Sn

Nb and Ti (1961) → ductile alloy

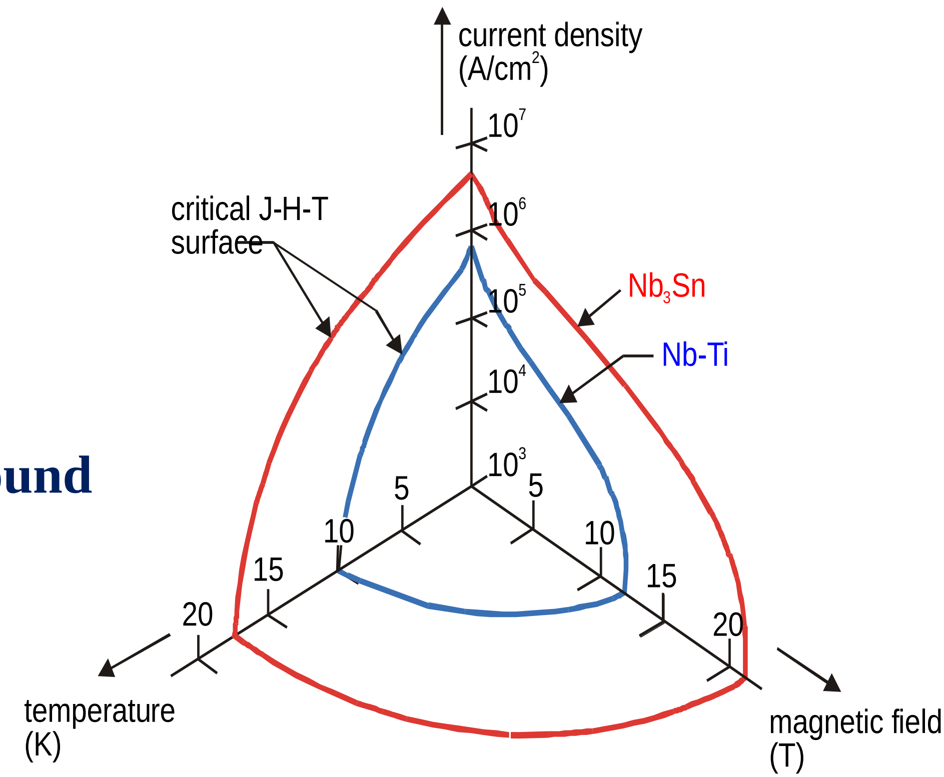
Extrusion + drawing

- T_c is ~ **9.2 K** at 0 T
- B_{C2} is ~ **14.5 T** at 0 K
- Use in **Tevatron** (80s), then all the other
- ~50-200 US$ per kg of wire
  (1 euro per m)

Nb and Sn (1954) → intermetallic compound

Brittle, strain sensitive, formed at ~650-700 ℃

- T_C is ~ **18 K** at 0 T
- B_{C2} is ~ **28 T** at 0 K
- Used in **NMR**, **ITER,** now HL-LHC
- ~700-1500 US$ per kg of wire
  (5 euro per m)



Critical surface for Nb-Ti and Nb_3Sn

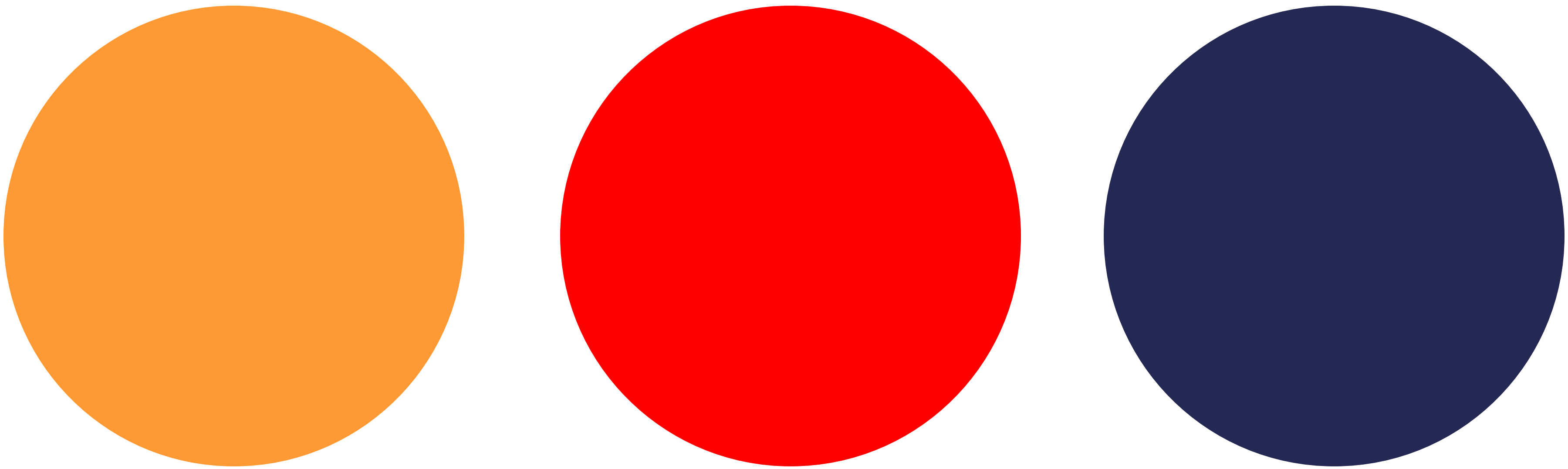
Practical superconductors

Typical operation parameters

(for a 0.85 mm diameter strand)

By P. Ferracin

Cu	Nb-Ti	Nb_3Sn



Cu	Nb-Ti	Nb_3Sn
$\boldsymbol{J_e}$ **~ 5 A/mm^2**	$\boldsymbol{J_e}$ **~ 600-700 A/mm^2**	$\boldsymbol{J_e}$ **~ 600-700 A/mm^2**
I **~ 3 A**	***I*** **~ 300-400 A**	***I*** **~ 300-400 A**
B **= 2 T**	***B*** **= 8-9 T**	***B*** **= 12-13 T**

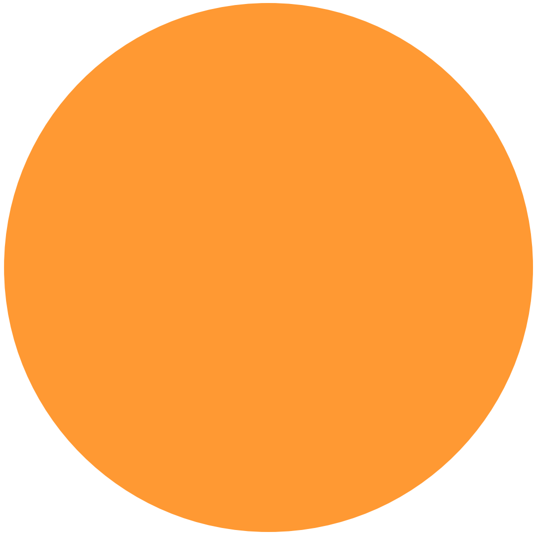
Practical superconductors

Typical operation parameters

(for a 0.85 mm diameter strand)

By P. Ferracin

Cu

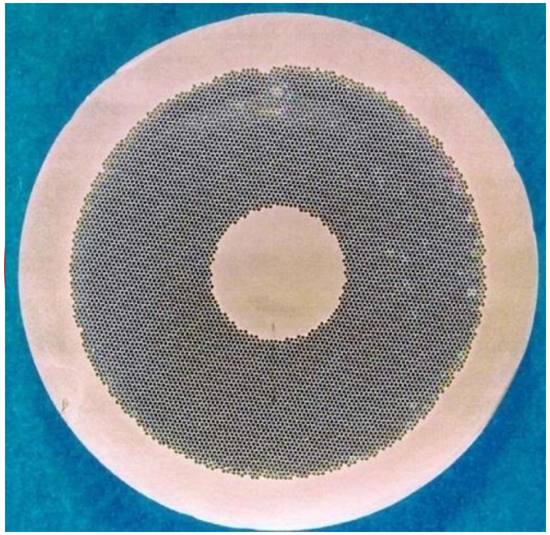


$J_e \sim 5$ A/mm^2

$I \sim 3$ A

$B = 2$ T

Nb-Ti

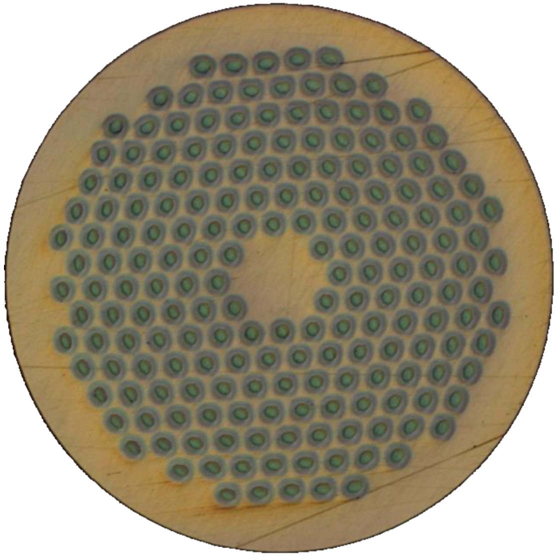


$J_e \sim 600\text{-}700$ A/mm^2

$I \sim 300\text{-}400$ A

$B = 8\text{-}9$ T

Nb_3Sn



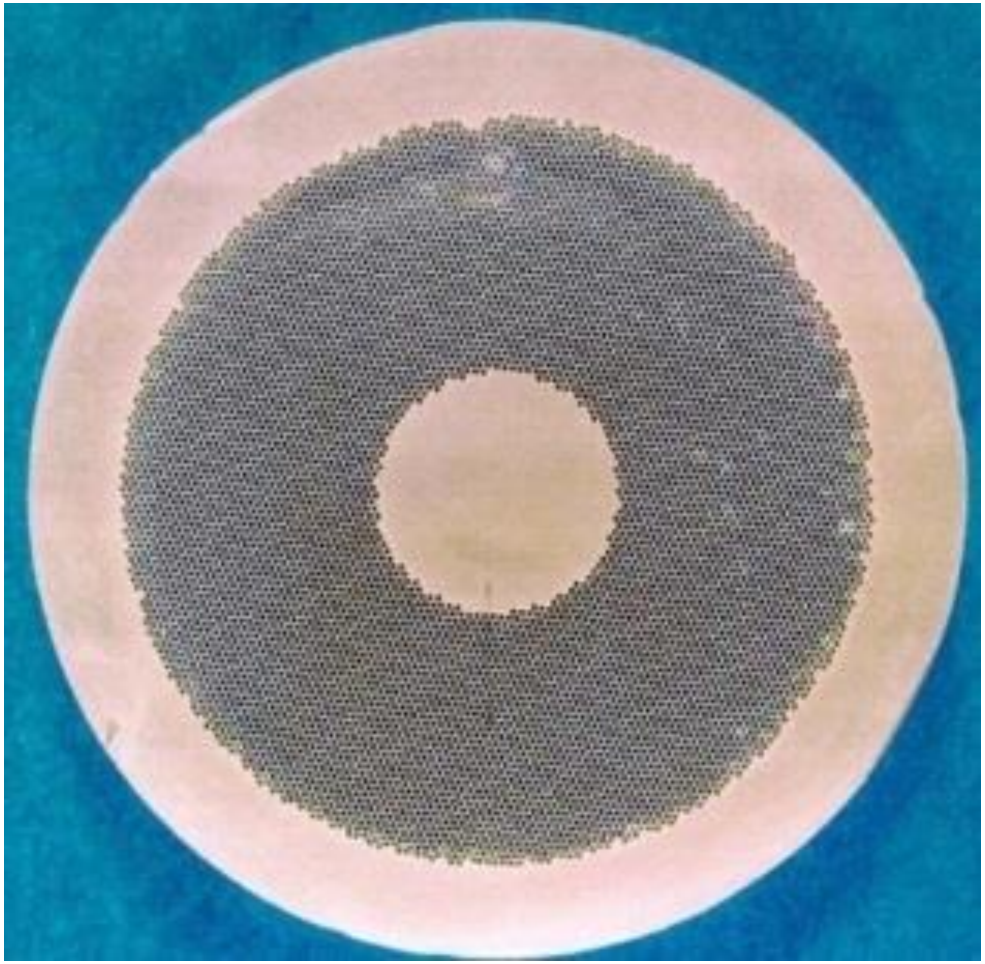
$J_e \sim 600\text{-}700$ A/mm^2

$I \sim 300\text{-}400$ A

$B = 12\text{-}13$ T

Strand: multifilament wire

Superconducting materials are produced in small filaments and surrounded by a stabilizer (typically copper) to form a "*multi-filament wire*" o "*strand*"



18

The strand: multifilament wire

WHY a multi-filament wire in a stabilizing matrix?

1. Flux jumps

Thermal disturbance → the local change in J_c → motion or "**flux jump**" → power dissipation

Stability criteria for a slab (adiabatic condition)

$$\boxed{a \leq \sqrt{\frac{3\gamma C(\theta_c - \theta_0)}{\mu_0 j_c^2}}}$$

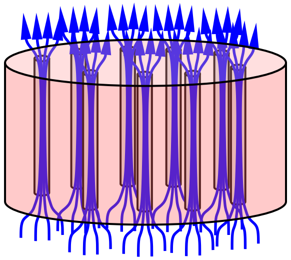
a is the half-thickness of the slab

j_c is the critical current density [A m^{-2}]

γ is the density [kg m^{-3}]

C is the specific heat [J kg^{-1}]

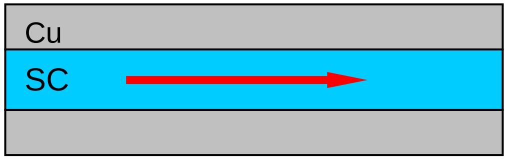
θ_c is the critical temperature.

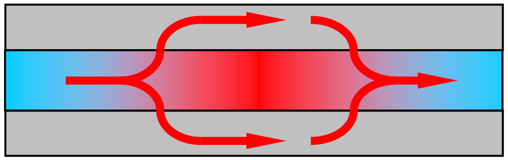


$B_{c1} < B < B_{c2}$

2. Quench protection

- Superconductors have a very high normal state resistivity.
  If quenched, could reach very high temperatures in few ms.
- If embedded in a **copper matrix**, when a quench occurs, current redistributes in the low-resisitivity matrix → **lower peak temperature**





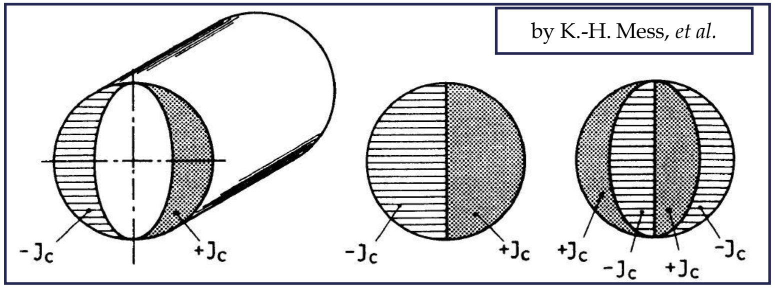
19

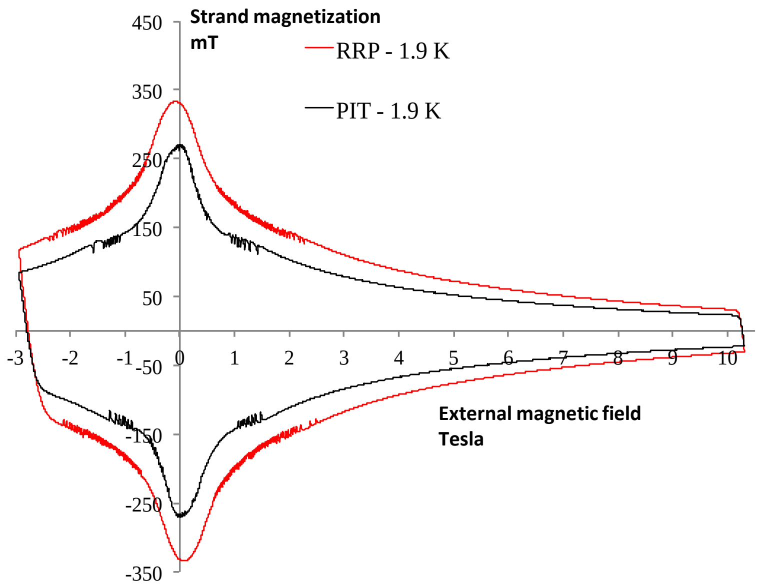
The strand: multifilament wire

3. Persistent currents

When a filament is in a varying B_{ext}, its inner part is shielded by currents distribution in the filament periphery

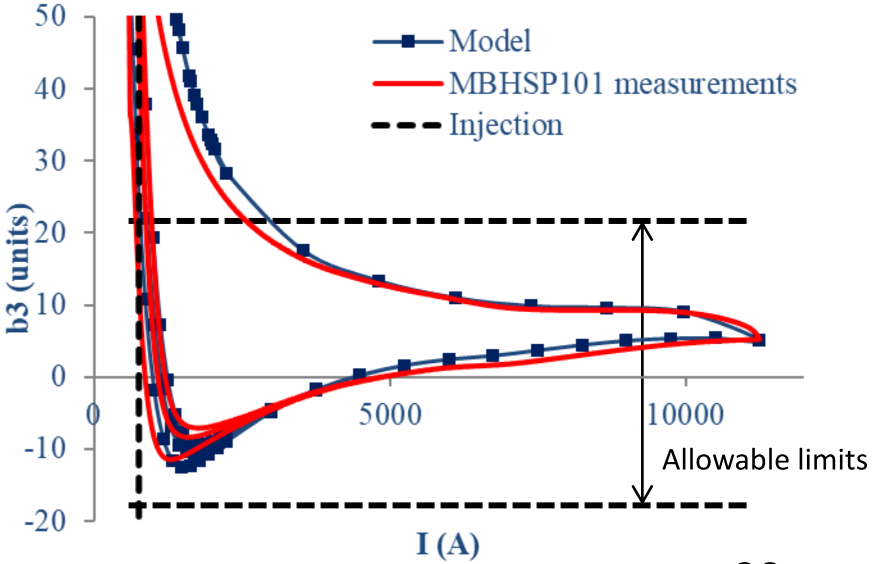
They **do not decay** when B_{ex} is held constant → **persistent currents**





These currents produce **field errors** that are particular important at low energy **(when the beam is injected),** which are proportional to the filament diameter (d_{sub}) and the current density.

$$M(B) \propto d_{sub} \cdot J_c\,(B)$$

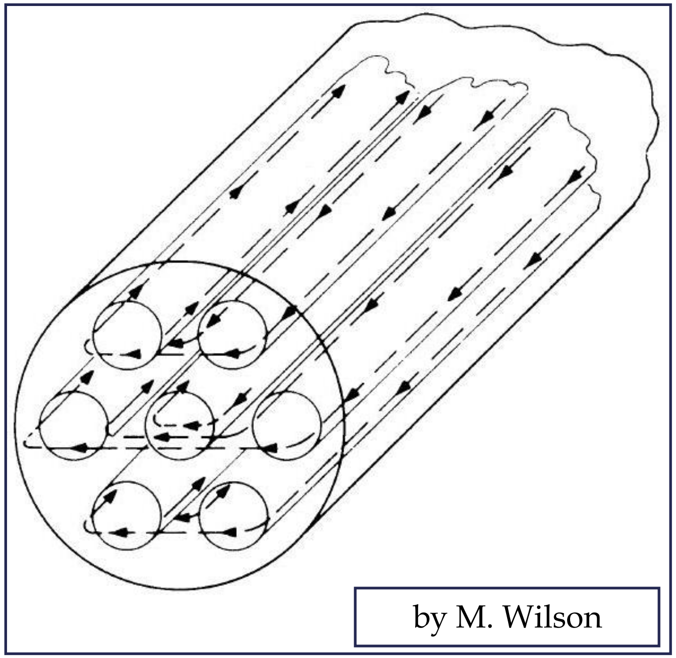


20

The strand: multifilament wire

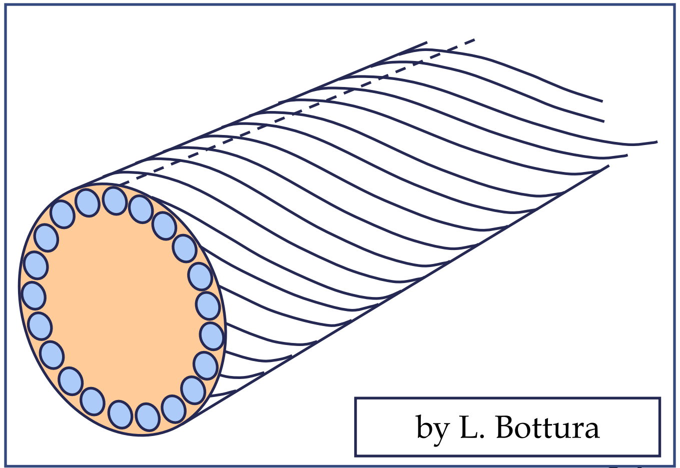
4. Inter-filament coupling

- When a multi-filamentary wire is subjected to a time varying magnetic field, **current loops** are generated between filaments.
- If filaments are straight, large loops with large currents → **ac losses**
- If the strands are magnetically coupled the effective filament size is larger → **flux jumps**



To reduce these effects, filaments are **twisted**

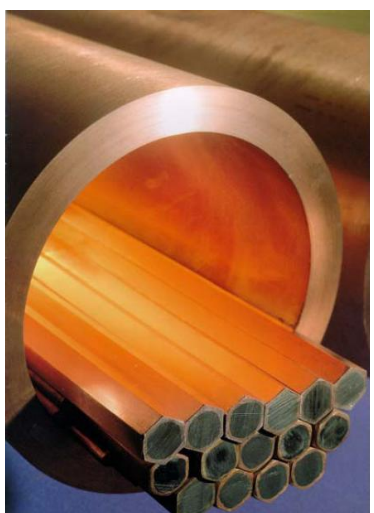
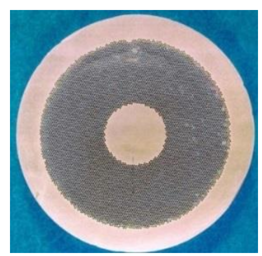
- twist pitch of the order of 20-30 times of the wire diameter.

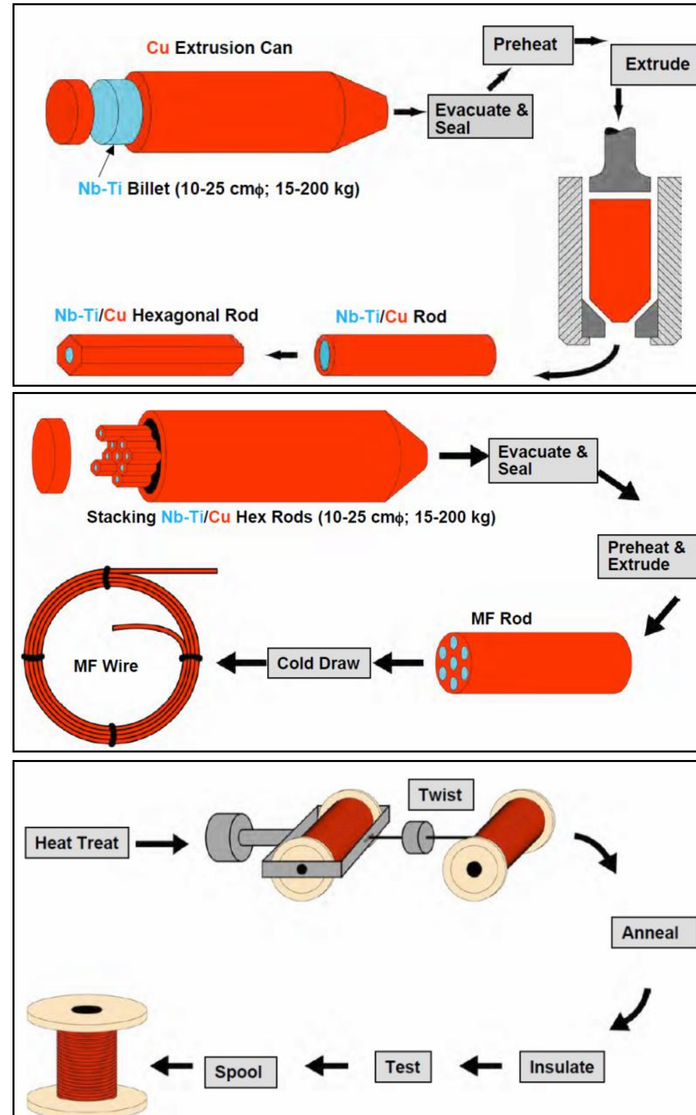


21

Strand: Manufacturing process (NbTi)

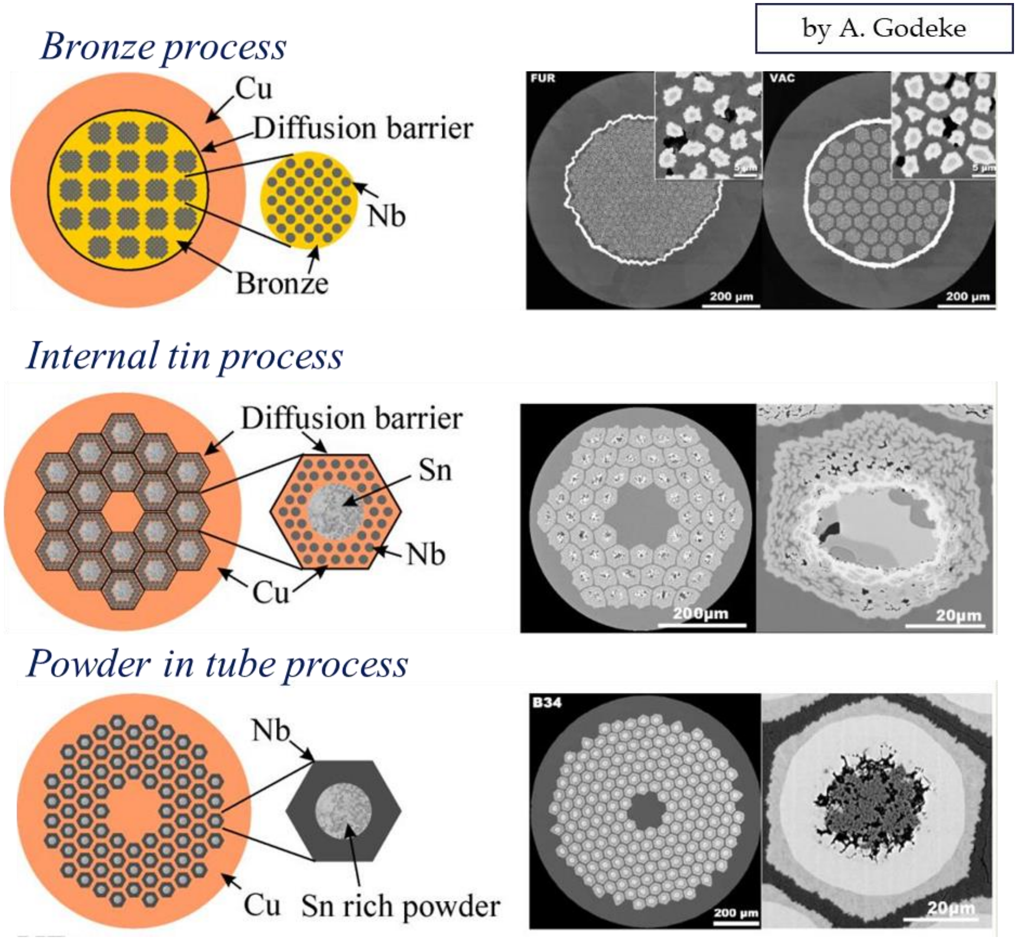
- Nb-Ti **ingots**
  - 200 mm Ø, 750 mm long
- **Monofilament rods** are stacked to form a multifilament billet
  - then extruded and drawn down
  - can be re-stacked: double-stacking process



Strand: Manufacturing process (Nb_3Sn)

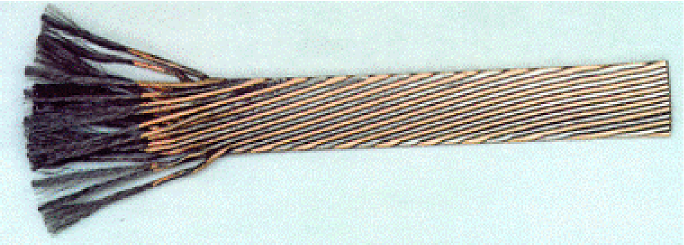
- Since Nb_3Sn is brittle
  - It cannot be extruded and drawn like Nb-Ti. It must be formed at the end of the fabrication of the cable (or the coil).
- Process in several steps
  - Fabrication of the wire, assembling multifilament billets from with **Nb and Sn separated**. Different processes tried in industrial scale (bronze process, internal tin process, powder in tube process)
  - Fabrication of the cable
  - Fabrication of the coil. Two different techniques:
    - **Wind & react”** (more common). First coil winding and then formation of Nb_3Sn
    - **“React & wind”.** First formation of Nb_3Sn and then coil winding
  - **Reaction.** Heating to about 600-700 C in vacuum or inert gas (argon) atmosphere, and the Sn diffuses in Nb and reacts to form Nb_3Sn.

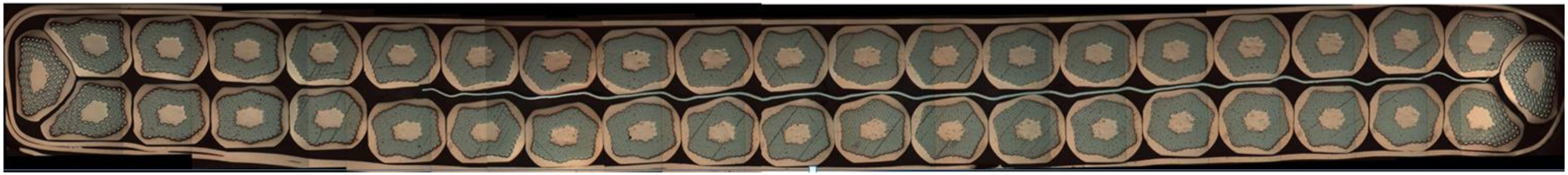


23

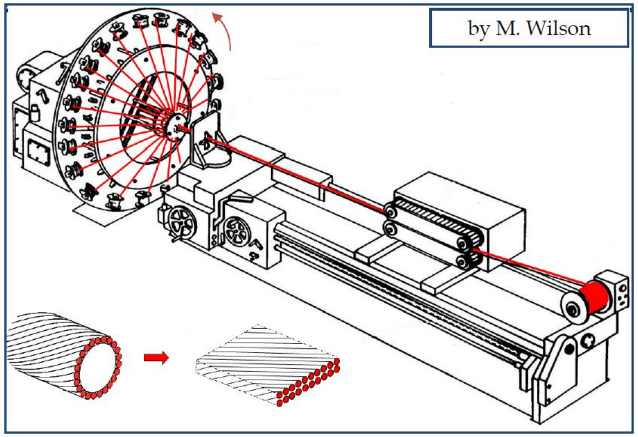
The cable

- Most of the superconducting coils for particle accelerators wound from a multi-strand cable (**Rutherford cable**). The strands are **twisted** to
  - Reduce **inter-strand coupling currents**
    - Losses and field distortions.
  - Provide more **mechanical stability**
  - **Current redistribution** (in case a defect in one strand)
  - Reduction the **number of turns** (easier winding, lower inductance)
  - Reduction strand **piece length**



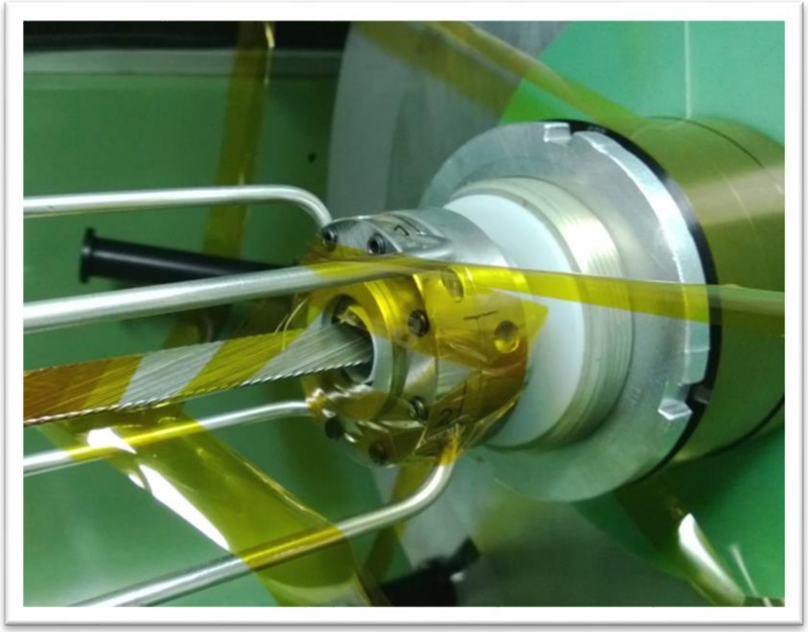


- Strands wound on spools mounted on a rotating drum
- Strands twisted around a conical mandrel into rolls
- The rolls compact the cable and provide the final shape



The cable insulation

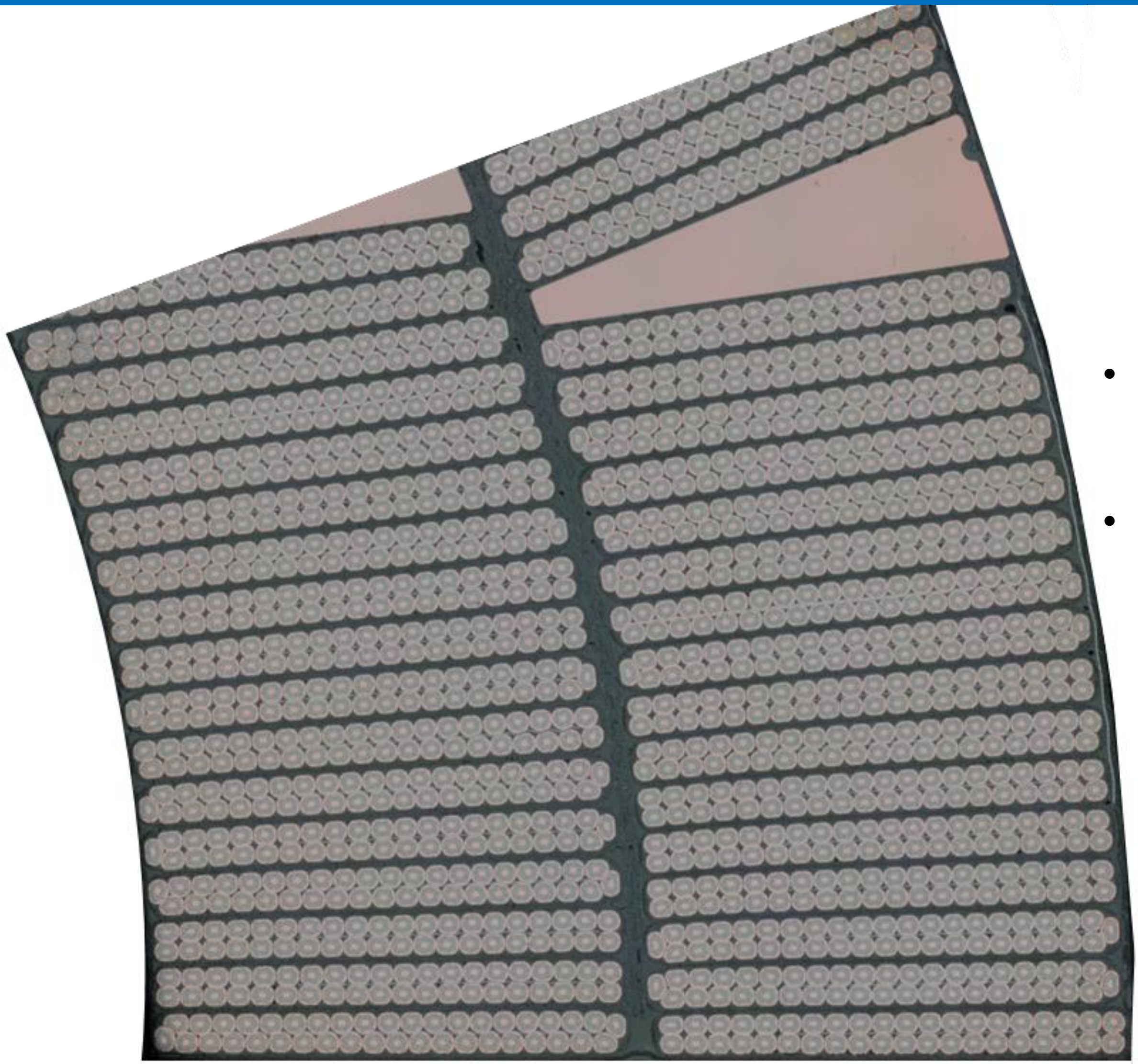
- The cable insulation must feature
  - Good **electrical properties** to withstand turn-to-turn *V* after a quench
  - Good **mechanical properties** to withstand high pressure conditions
  - **Porosity** to allow penetration of helium (or epoxy)
  - **Radiation hardness**



Polyimide insulation for Nb-Ti

Fiber glass insulation for Nb_3Sn

Filling ratio and current density



Coil :
≈ 1/3 superconductor
≈ 1/3 copper
≈ 1/3 insulation

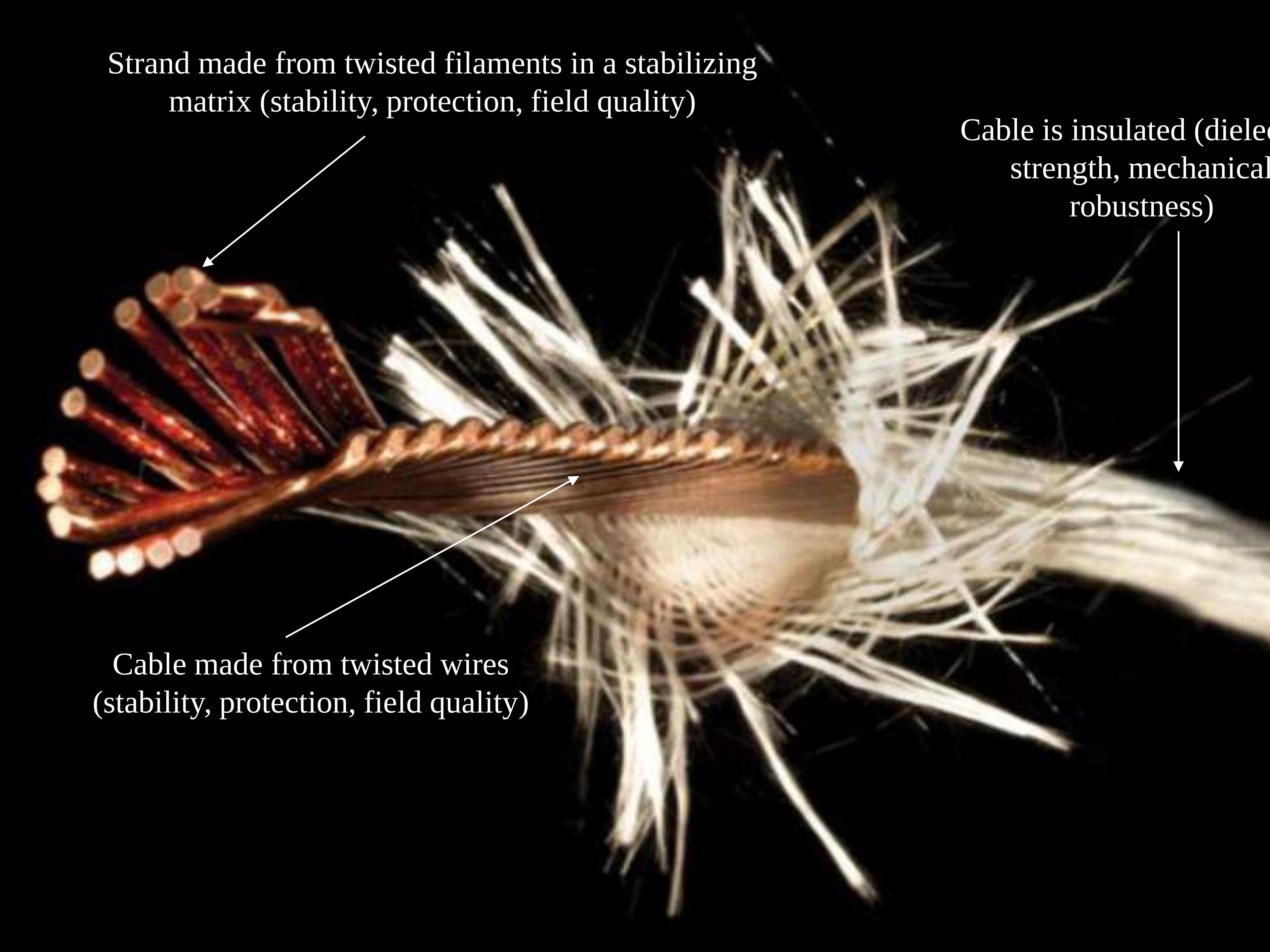
- Engineering current density is the current divided by the strand area (Cu+sc)
- Overall current density is the current divided by the total area (Cu+sc+ins)

Current density (A/mm²)

	LHC-MB		11 T DS
	IL	OL	
J_{sc}	1259	1817	1655
$J_{engineering}$	475	616	770
$J_{overall}$	349	430	522

Summary

- $\boldsymbol{p = eB\rho}$ → More energy?
  - Either brute force (longer collider)
  - Or technological development (higher magnetic field)
- Basic magnetic elements in the 'arc' of a circular accelerator:
  - **Dipoles**: magnetic field steers (bends) the particles in a ~ circular orbit
  - **Quadrupoles**: keep the particles in the orbit, providing a linear force that keep them focused acting as a spring.
- Superconductivity is destroyed by **temperature, current density, magnetic field**
  - Critical surface is $j(B,T)$ giving values below which the superconducting state exists
- For making magnets it is fundamental to have penetration of magnetic field (type II). Practical superconductors came only 50 years after the discovery of superconductivity


Strand made from twisted filaments in a stabilizing matrix (stability, protection, field quality)
Cable is insulated (diele
strength, mechanical
robustness)
Cable made from twisted wires (stability, protection, field quality)

References

- K.-H. Mess, P. Schmuser, S. Wolff, "*Superconducting accelerator magnets*", Singapore: World Scientific, 1996.
- Martin N. Wilson, "*Superconducting Magnets*", 1983.
- Fred M. Asner, "*High Field Superconducting Magnets*", 1999.
- P. Ferracin, E. Todesco, S. Prestemon, "*Superconducting accelerator magnets*", US Particle Accelerator School, www.uspas.fnal.gov.
- E. Todesco, "Masterclass -Design of superconducting magnets for particle accelerators", https://indico.cern.ch/category/12408/
- A. Devred, "*Practical low-temperature superconductors for electromagnets*", CERN-2004-006, 2006.
- Presentations from Luca Bottura and Martin Wilson

Outline

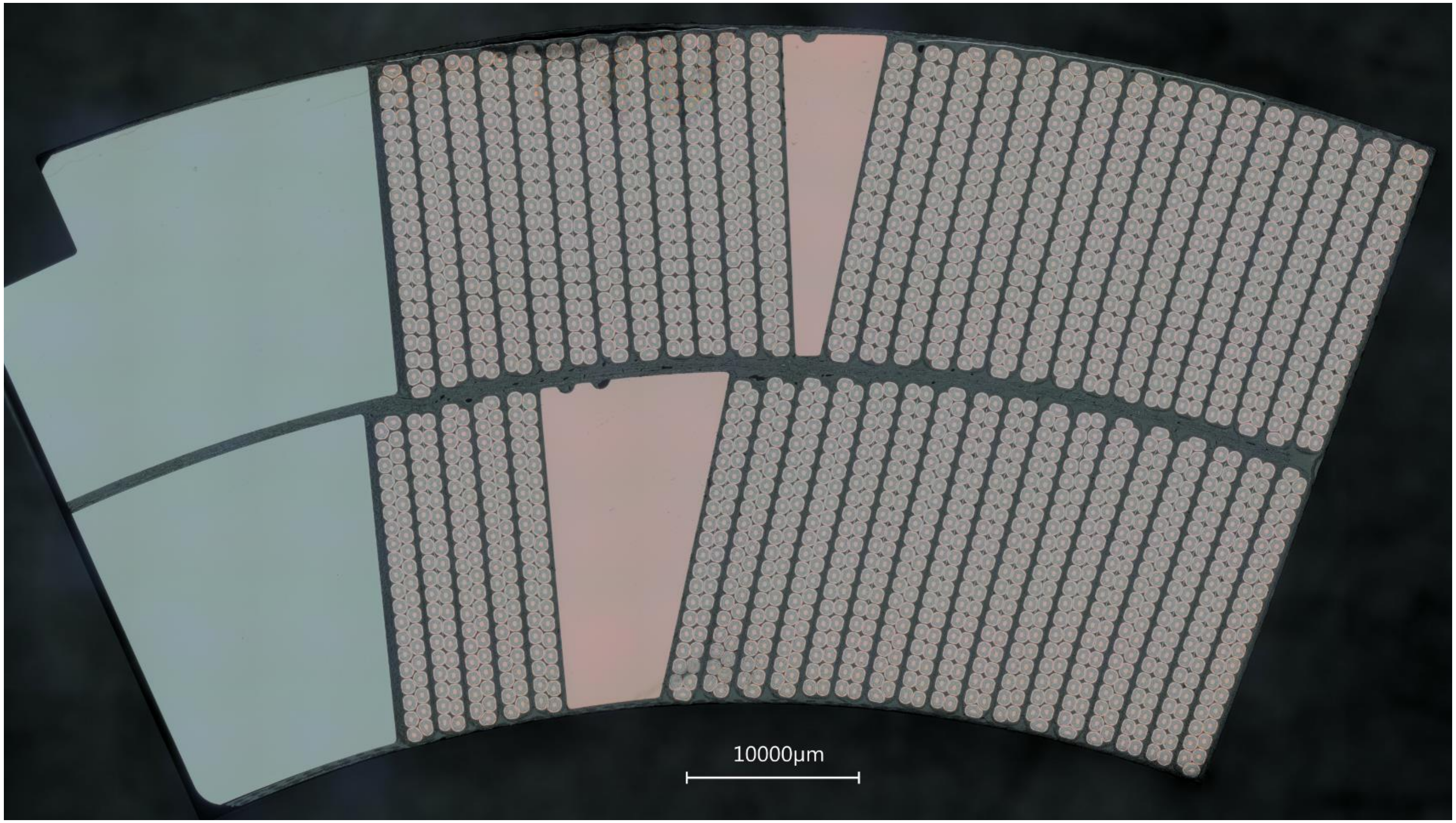
- Part I
  - Particle accelerators, magnets and the need of superconductors
  - Magnetic design and coil fabrication
- Part II
  - Mechanical design and assembly
  - Quench, training and protection
  - Outlook, what brings the future

Introduction

- The magnetic design is one of the first steps in the design of a superconducting magnet development
- It starts from the **requirements**(from accelerator physicists, researchers, medical doctors…others)
- A field **shape**: Dipole, quadrupole, etc
- A field **magnitude** usually with low temperature superconductors from 5 to 20 T
- A field **homogeneity**, uniformity inside a solenoid, harmonics in a accelerator magnet
- A given **aperture** (and **volume**), some cm diameter for accelerator magnets, much more for detectors and fusion magnets

Magnetic design and coil

- How do we create a **perfect field**?
- How do we **express** field and its “**imperfections**”?
- How do we design a coil to **minimize field errors**?



How to create a dipole field?

Perfect dipole: intercepting circle/ellipses

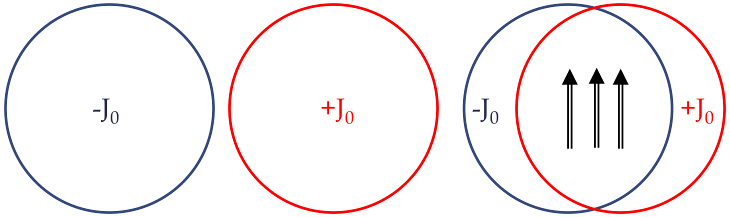
- Within a cylinder carrying j_0, the field is perpendicular to the radial direction and proportional to the distance to the centre r:

$$B = -\frac{\mu_0 j_0 r}{2}$$

- Combining the effect of two intersecting cylinders

$$B_x = \frac{\mu_0 j_0 r}{2}\{-r_1 \sin\theta_1 + r_2 \sin\theta_2\} = 0$$

$$B_y = \frac{\mu_0 j_0 r}{2}\{-r_1 \cos\theta_1 + r_2 \cos\theta_2\} = -\frac{\mu_0 j_0}{2} w$$

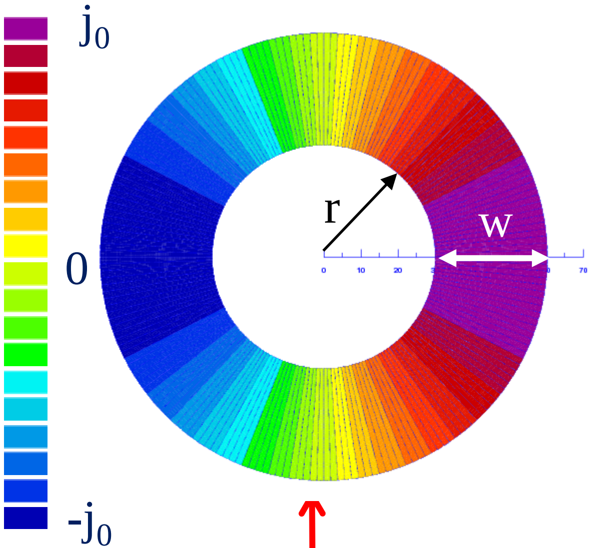


But…

- The aperture is not circular
- Not easy to simulate with a flat cable
- Similar proof for intercepting ellipses

How to create a dipole field?

Perfect dipole: thick shell with cosθ current distribution



- If we assume a current distribution proportional to the angle

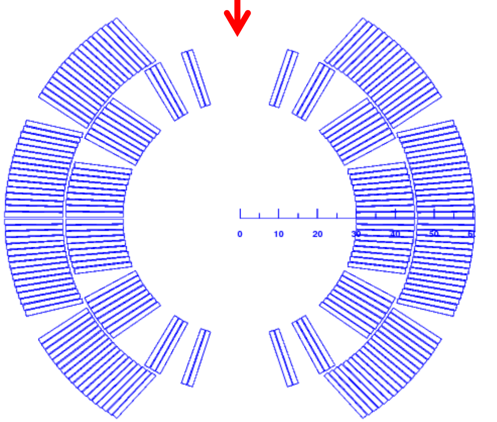
$$j(\theta) = j_0 cos(\theta)$$

- The generated dipole field is

$$B_y = -4\frac{\mu_0 j_0}{2\pi}\int_0^{\pi/2}\int_r^{r+w}\frac{cos^2(\theta)}{\rho}\rho d\rho d\theta = -\frac{\mu_0 j_0}{2}w$$

In a dipole:
B ∝ current density (obvious)
B ∝ coil width w (less obvious)
B independent of the aperture r (surprising)

From ideal to practical configuration

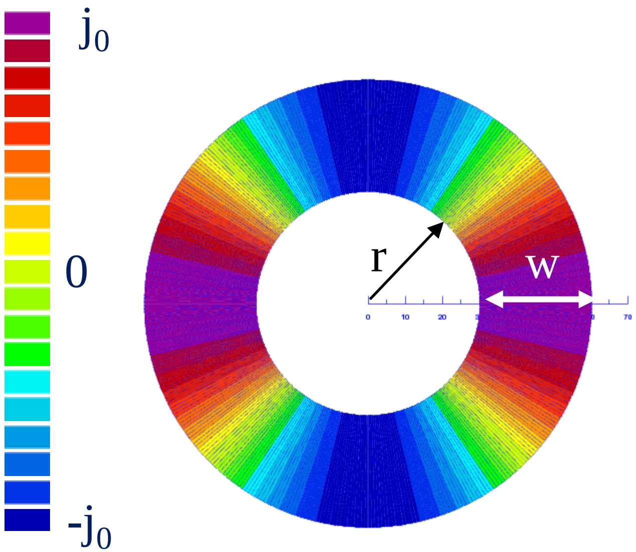


- A bit easier to reproduce with a flat cable (Rectangular cross-section and constant *J)*
  - More **layers** and **wedges** to reduce *J* towards the 90 degrees plane
  - It will not be a perfect field…but it can be pretty close!

Perfect 2n-pole field

- **Four intercepting** circles/ellipses and a **cos2θ** current distribution generate a perfect quadrupole field

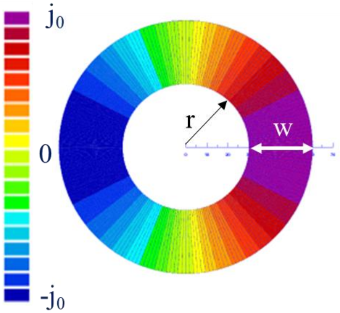
$$G = \frac{B_y}{r} = -\frac{\mu_0 j_0}{2} \ln\left(1 + \frac{w}{r}\right)$$

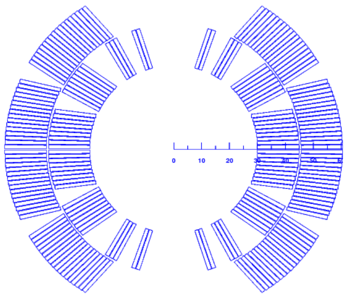


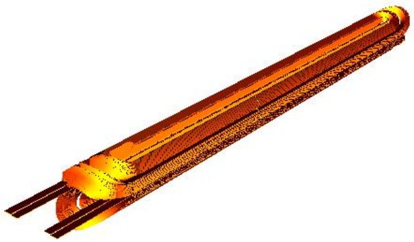
- And so on…
  - Perfect sextupole: **cos3θ** or **3** intersecting ellipses
  - Perfect 2n-poles: **cos(nθ)** or **n** intersecting ellipses

From ideal to real configurations

- 'The solution' to go from the ideal cosθ current distribution to a windable configuration → Approximation of the cos-theta layout by sectors with uniform current density







- Now we can use the multipolar expansion to **optimize** our "practical" **cross-section**
  - The first allowed harmonic in a dipole configuration is B_3

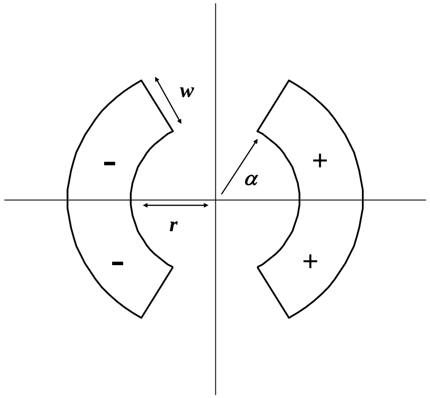
$$B_3 = \frac{\mu_0 j R_{ref}^2}{\pi} \frac{\sin(3\alpha)}{3} \left(\frac{1}{r} - \frac{1}{r+w} \right)$$

for $\alpha=\pi/3$ (i.e. a 60° sector coil) one has $B_3=0$

  - The second allowed harmonic in a dipole configuration is B_5

$$B_5 = \frac{\mu_0 j R_{ref}^4}{\pi} \frac{\sin(5\alpha)}{5} \left(\frac{1}{r^3} - \frac{1}{(r+w)^3} \right)$$

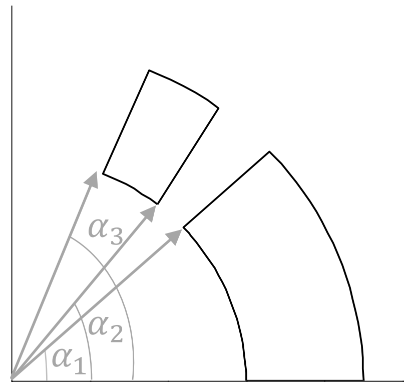
for $\alpha=\pi/5$ (i.e. a 36° sector coil) or for $\alpha=2\pi/5$ (i.e. a 72° sector coil) one has $B_5=0$



Dipole sector coils

- With one sector, we can only set to zero one multipole
- With two sectors, equations to set to zero $B_{3,}$ B_5 *and* B_7

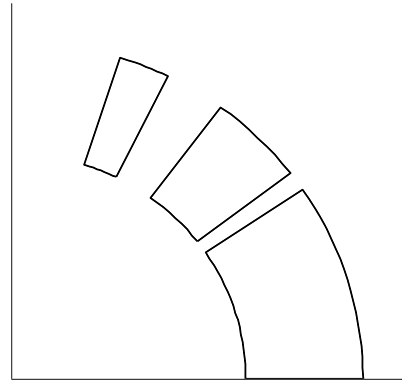
$$\begin{cases} \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0 \\ \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0 \end{cases}$$



for instance (48°,60°,72°) or (36°,44°,64°) are solutions

- With three sectors, one can set to zero 5 multipoles

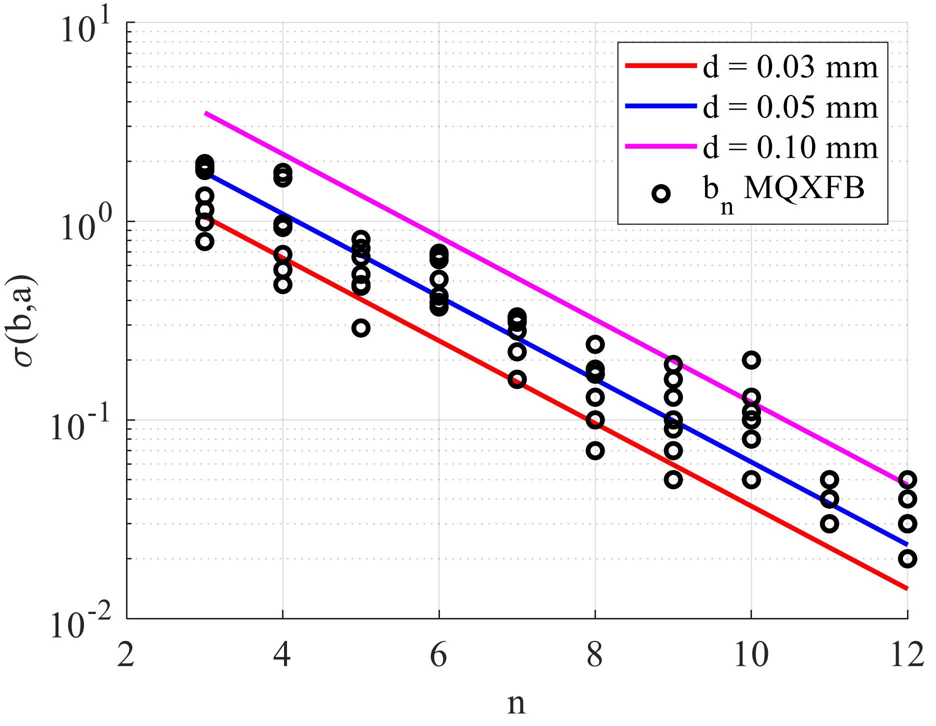
$$\sin(3\alpha_5) - \sin(3\alpha_4) + \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0$$
$$\sin(5\alpha_5) - \sin(5\alpha_4) + \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0$$
$$\sin(7\alpha_5) - \sin(7\alpha_4) + \sin(7\alpha_3) - \sin(7\alpha_2) + \sin(7\alpha_1) = 0$$
$$\sin(9\alpha_5) - \sin(9\alpha_4) + \sin(9\alpha_3) - \sin(9\alpha_2) + \sin(9\alpha_1) = 0$$
$$\sin(11\alpha_5) - \sin(11\alpha_4) + \sin(11\alpha_3) - \sin(11\alpha_2) + \sin(11\alpha_1) = 0$$



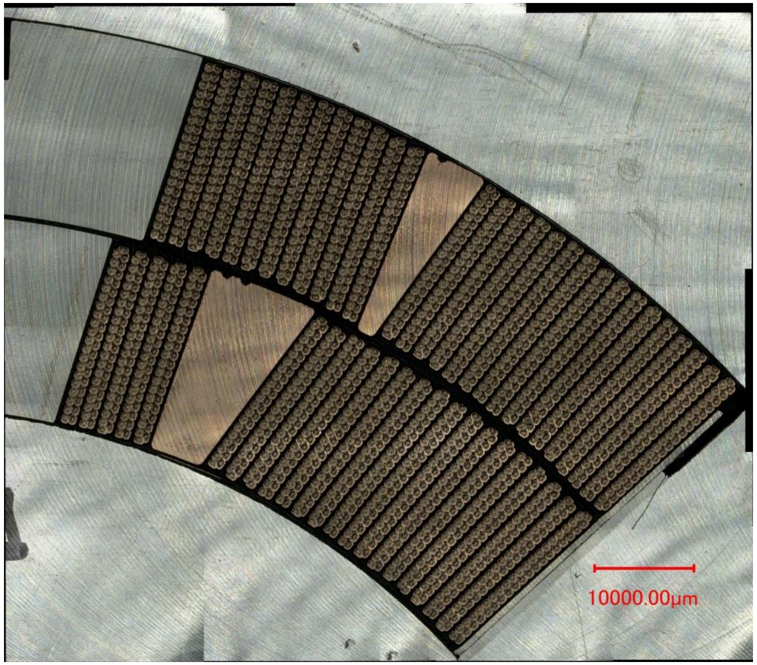
~[0°-33.3°, 37.1°- 53.1°, 63.4°- 71.8°]

Coil fabrication

- The coil: most **critical component** of a superconducting magnet
- **Cross-sectional accuracy** of few tens of micrometers over ~15 m
- Manufacturing tolerances (~30 µm on blocks position) are accounted as random components for field quality.



Cross section of a Nb_3Sn practice coil



38

Coil fabrication (Nb_3Sn)

Winding & Curing

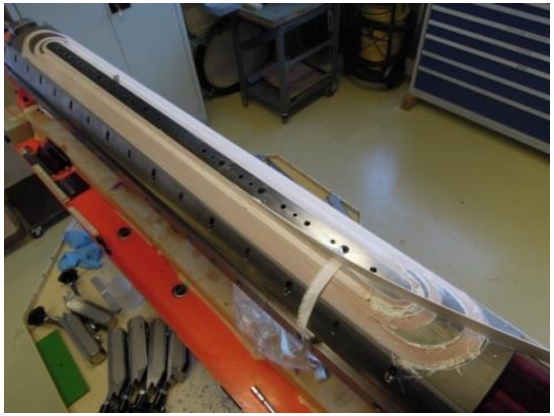
The cable is wound around a pole on a mandrel.
A ceramic binder is applied and cured (T~ 150 C) to have a rigid body easy to manipulate.

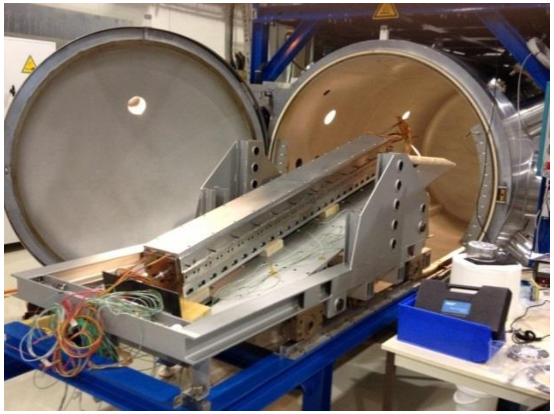
Reaction

Sn and Nb are heated to 650-700 C in vacuum or inert gas (argon) → Nb_3Sn
The cable becomes brittle

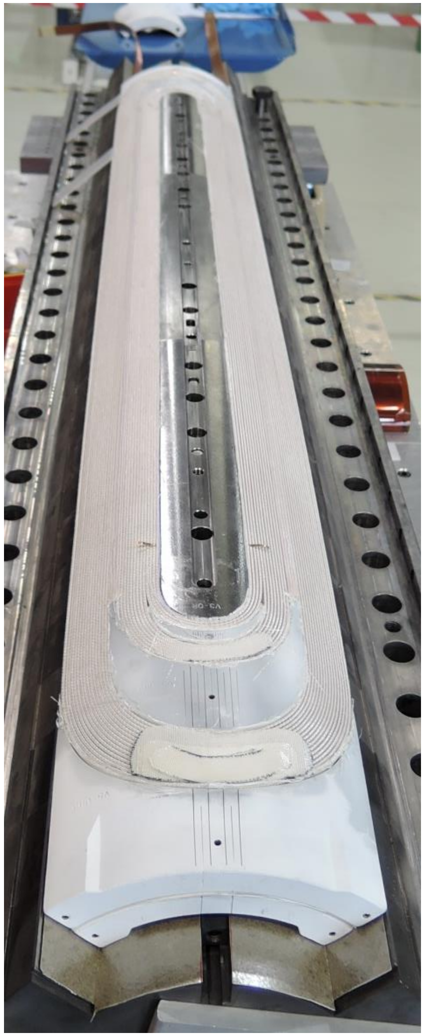
Impregnation

In order to have a **solid block**, the coil placed in a impregnation fixture
The fixture is inserted in a vacuum tank, evacuated → **epoxy injected**





Coil at different manufacturing steps



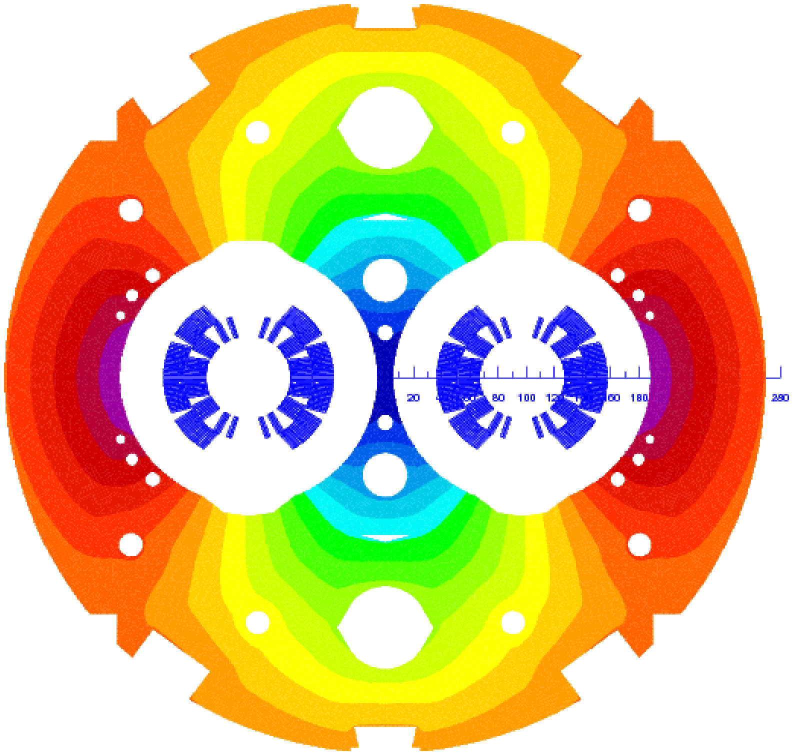
After curing

After reaction

After impregnation

The iron yoke

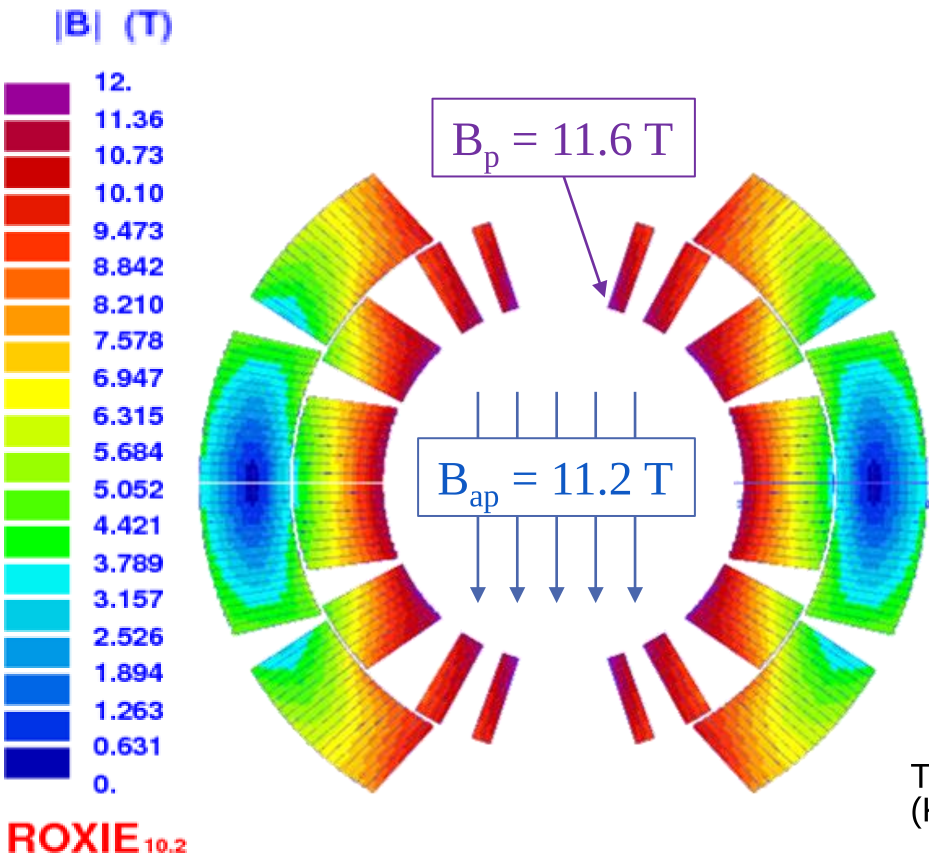
- Keep the **return magnetic flux** close to the coils, thus avoiding fringe fields
- In some cases the iron is partially or totally contributing to the **mechanical structure**
- Considerably **enhance the field** for a given current density
- The increase is relevant (10-30%), getting higher for thin coils
- This allows using lower currents, easing the protection

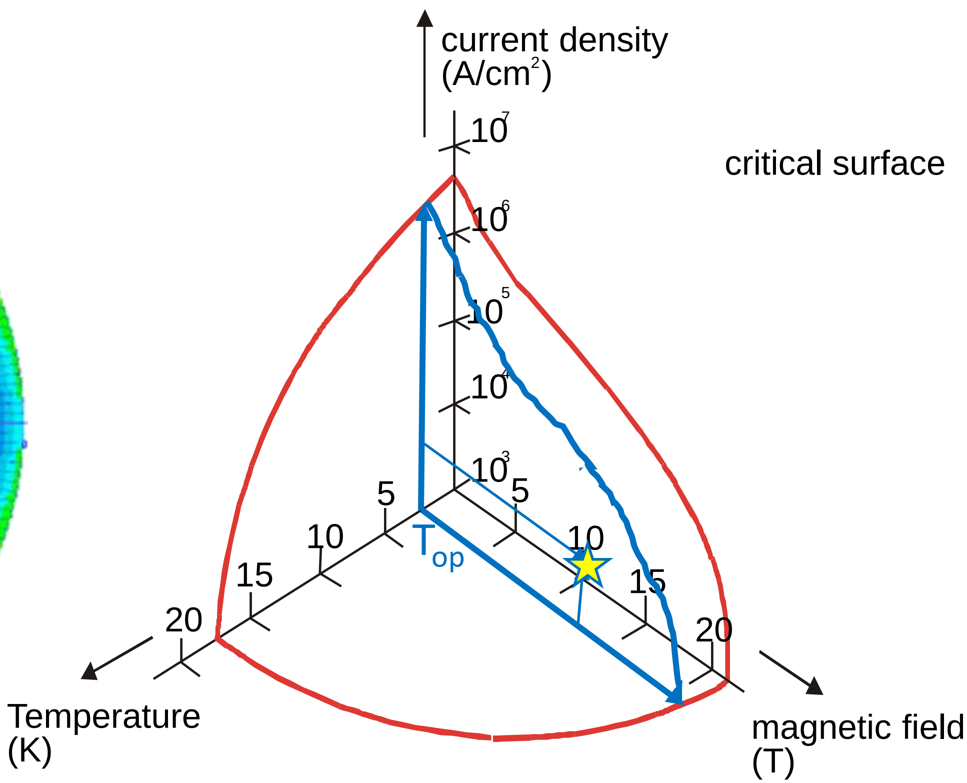


11 T Double Aperture Magnet

Peak Field, Aperture Field and Margin

- The margin of a magnet is defined with respect to its weakest point, i.e. the peak field
- What matters for the margin is the peak field in the coil, not the field in the aperture

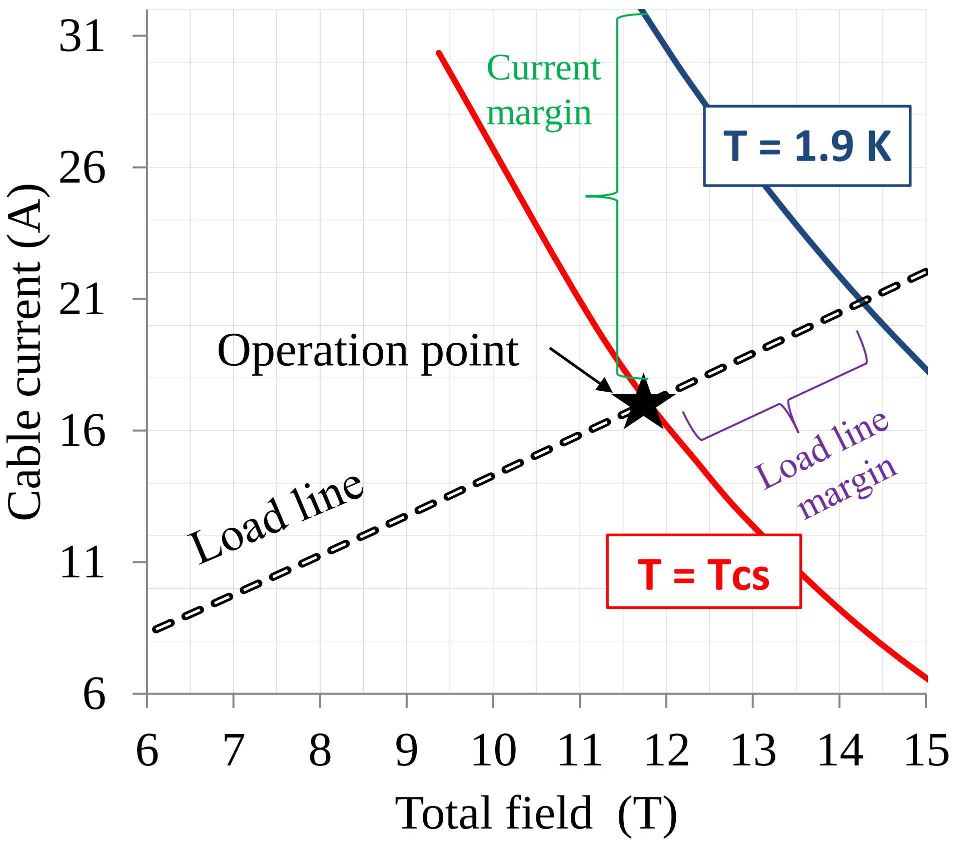




Margin

- The margin of a magnet is defined with respect to its weakest point, i.e. the peak field
- Short sample(SS) corresponds to the intersection of the load line for the peak field and the critical current density curve: ideally is the maximum performance of the magnet

- Among magnet engineers, a commonly used concept is the loadline margin
- The concept is always criticized (not physical) but never replaced: the success of a magnet judged on its ability of reaching the max performance
    - $LL_{margin} = 1 - I_{op}/I_{SS}$
- High field accelerator magnets typically are design to operate at ≈ 80% of the short sample level (20 % margin)



Summary

- **$B \propto J \cdot w$** → Two ways to increase the field:
  - Larger current density (up to a certain level, then hard limits in terms of stress and protection)
  - Larger coils (cost)

- The coil is most **critical component** of a superconducting magnet.
  - Typically, 1/3 of the coil material is superconductor, 1/3 copper and 1/3 insulation/resin

- The **iron** keeps the return magnetic flux and considerably enhances the field. In some cases it also has a mechanical function.

- The **margin** of a magnet is defined with respect to its weakest point, i.e. the peak field

References

- K.-H. Mess, P. Schmuser, S. Wolff, “*Superconducting accelerator magnets*”, Singapore: World Scientific, 1996.
- Martin N. Wilson, "*Superconducting Magnets*", 1983.
- Fred M. Asner, "*High Field Superconducting Magnets*", 1999.
- S. Russenschuck, “*Field computation for accelerator magnets*”, J. Wiley & Sons (2010).
- P. Ferracin, E. Todesco, S. Prestemon, “*Superconducting accelerator magnets*”, US Particle Accelerator School, www.uspas.fnal.gov.
- E. Todesco, “Masterclass -Design of superconducting magnets for particle accelerators”, https://indico.cern.ch/category/12408/
- A. Jain, “*Basic theory of magnets*”, CERN 98-05 (1998) 1-26
- L. Rossi, E. Todesco, “*Electromagnetic design of superconducting quadrupoles*”, Phys. Rev. ST Accel. Beams 10 (2007) 112401.
- L. Rossi and Ezio Todesco, “*Electromagnetic design of superconducting dipoles based on sector coils*”, Phys. Rev. ST Accel. Beams 9 (2006) 102401.
- CAS 2023 on Normal and Superconducting Dipoles https://indico.cern.ch/event/1227234/

Thank you

For questions, don’t hesitate!

susana.izquierdo.bermudez@cern.ch

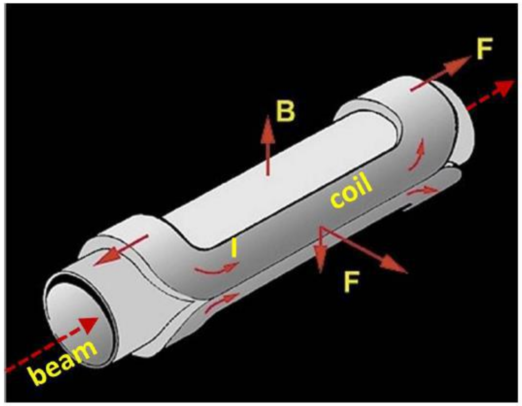
Maxwell equations

- Maxwell equations for magnetic field

$$\nabla \cdot B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \qquad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

- In absence of charge and magnetized material

  (inside a magnet)



$$\nabla \times B = \left(\frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}, \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} \right) = 0$$

- If $\frac{\partial B_z}{\partial z} = 0$ (constant longitudinal field), then

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

  - x and y perpendicular to the beam (transverse coordinates), z along the beam

Analytic functions

- If $\frac{\partial B_z}{\partial z} = 0$ Maxwell gives

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

$$\begin{cases} \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} = 0 \\ \frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} = 0 \end{cases}$$

Cauchy-Riemann conditions

and therefore, the function $B_y + iB_x$ is analytic

$$B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{\infty} C_n (x+iy)^{n-1} \qquad (x,y) \in D$$

where C_n are complex coefficients

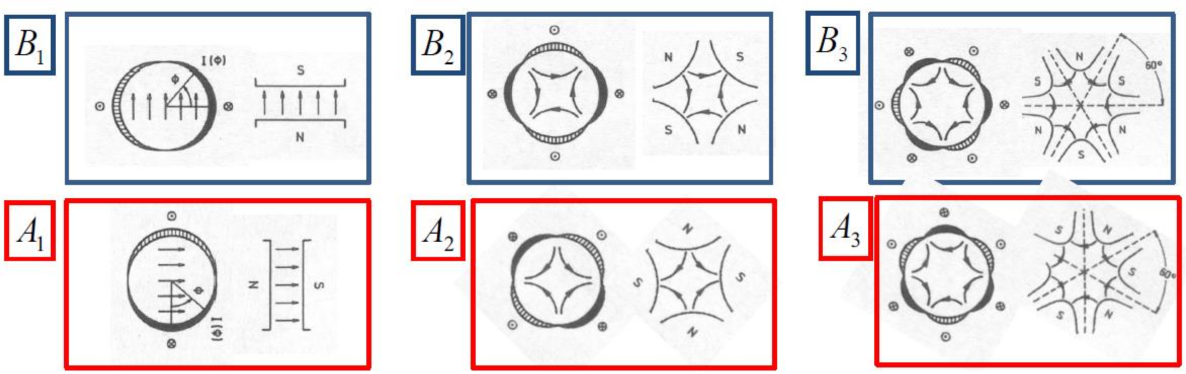
- Advantage: we reduce the description of the field to a (simple) series of complex coefficients

Field harmonics

- The field can be described as a (simple) series of complex coefficients, each coefficient corresponds to a “pure” multipolar field

$$B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{\infty} C_n (x+iy)^{n-1} = \sum_{n=1}^{\infty} (B_n + iA_n)(x+iy)^{n-1}$$

- Magnets usually aim at generating a single multipole
  - Dipole, quadrupole, sextupole, octupole, decapole, dodecapole …



By K.-H. Mess et al.

Field harmonics

- The field harmonics are rewritten as

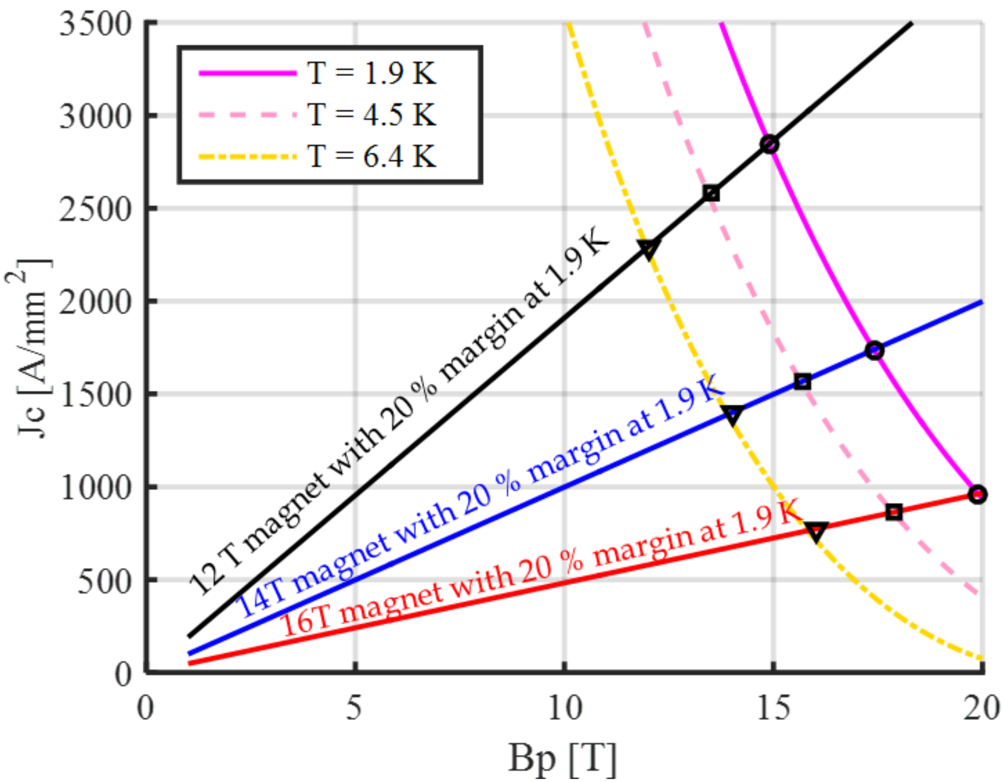
$$B_y + iB_x = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n)\left(\frac{x+iy}{R_{ref}}\right)^{n-1}$$

  - We factorize the main component (B_1 for dipoles, B_2 for quadrupoles)
  - We introduce a reference radius R_{ref} to have dimensionless coefficients (usually chosen as 2/3 of the aperture radius)
  - We factorize 10^{-4} since the deviations from ideal field in superconducting magnets for particle accelerators have to be ~0.01%

- The coefficients b_n, a_n are called normalized multipoles
  - b_n are the normal, a_n are the skew (adimensional)

Considerations on margin

- For Nb_3Sn and Nb-Ti the temperature margin depends only on the loadline margin and very weakly on the field.
- For a given a material and an operational temperature, load line margin and temperature margin are equivalent
- For a given LL margin, Nb_3Sn T margin is about 2.5 times greater than NbTi T margin



Temperature margins at 20% on loadline		
Operational temperature	1.9 K	4.2 K
Nb-Ti	2.1 K	1.2 K
Nb_3Sn	4.5 K	3.0 K