Standard Model 1/4

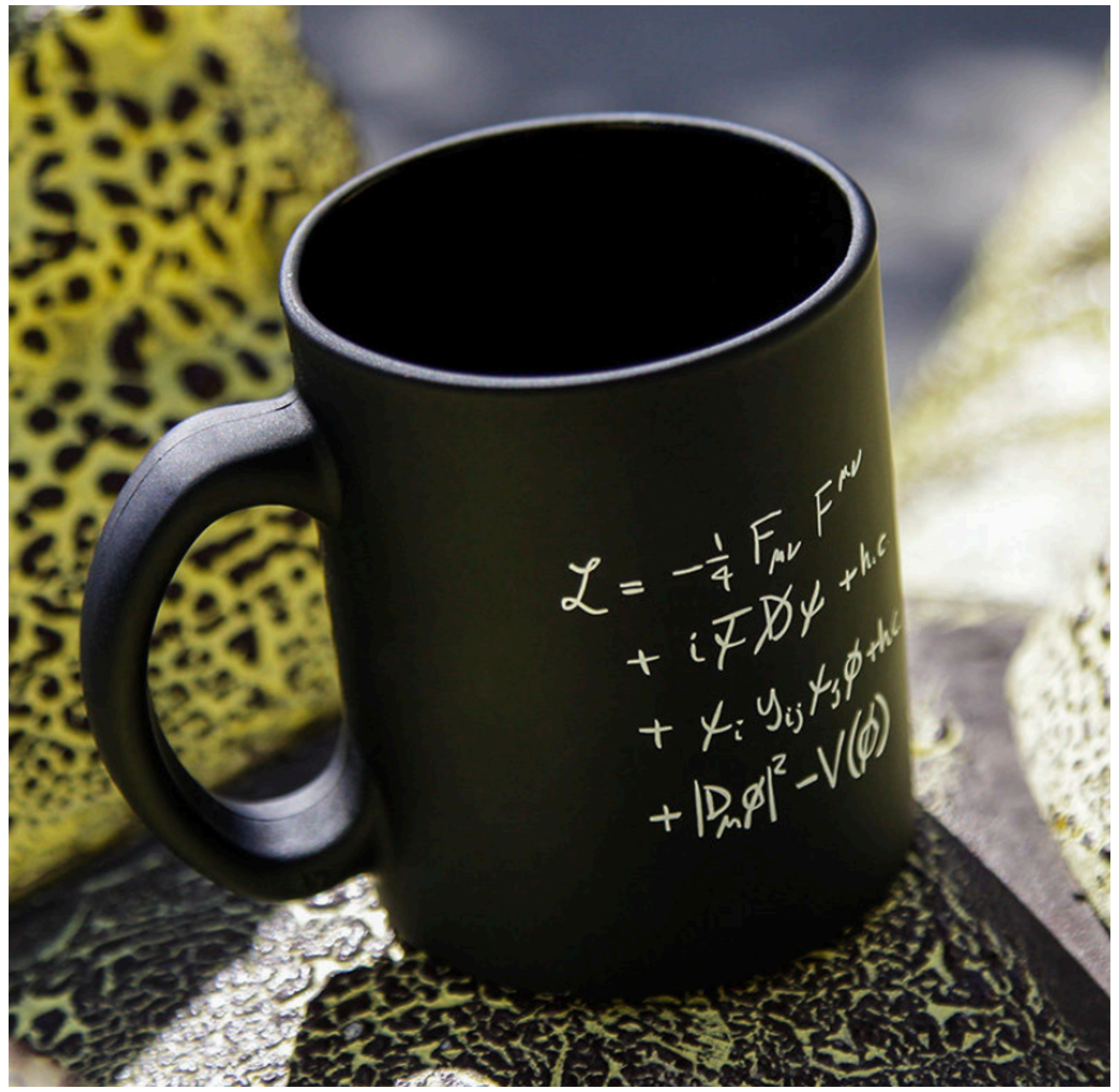
Andreas Weiler (TU Munich)

CERN, 7/2024

Please don't hesitate to contact me via email if you have any questions or need assistance with the course material.

andreas.weiler@tum.de

		Monday	Tuesday	Wednesday	Thursday	Friday
		1/7	2/7	3/7	4/7	5/7
Week 1	09h15-10h10		Introduction	Particle World	Raw Data to Physics Results	Detectors
	10h25-11h20		Particle World	Detectors	Particle World	Raw Data to Physics Results
	11h35-12h30		Detectors	Raw Data to Physics Results	Detectors	Particle World Q&A
		8/7	9/7	10/7	11/7	12/7
Week 2	09h15-10h10	Detectors	Accelerators & Beam Dynamics	Statistics	Nuclear Physics	Theoretical Particle Physics
	10h25-11h20	Accelerators & Beam Dynamics	Magnet superconductivity	Accelerators & Beam Dynamics	Theoretical Particle Physics	Statistics
	11h35-12h30	Magnet superconductivity	Statistics	Theoretical Particle Physics	Statistics	Nuclear Physics
		15/7	16/7	17/7	18/7	19/7
Week 3	09h15-10h10	Theoretical Particle Physics	Theoretical Particle Physics	Future High Energy Colliders	Astroparticle Physics	Cosmology
	10h25-11h20	Standard Model	Physics & Medical Applications	Standard Model	Cosmology	Future High Energy Colliders
	11h35-12h30	Physics & Medical Applications	Standard Model	Astroparticle Physics	Standard Model	Cosmology
		22/7	23/7	24/7	25/7	26/7
Week 4	09h15-10h10	RF superconductivity	Electronics, DAQ and Triggers	Heavy Ions	Neutrino Physics	Physics at Hadron Colliders
	10h25-11h20	Predictions at Hadron Colliders	RF superconductivity	Electronics, DAQ and Triggers	Physics at Hadron Colliders	Heavy Ions
	11h35-12h30	Electronics, DAQ and Triggers	Predictions at Hadron Colliders	Physics at Hadron Colliders	Heavy Ions	Physics at Hadron Colliders
		29/7	30/7	31/7	1/8	2/8
Week 5	09h15-10h10	Quantum Gravity	Beyond the Standard Model	Antimatter in the Lab	Beyond the Standard Model	Flavour Physics
	10h25-11h20	Physics at Lepton Colliders	Accelerator Operation & Design	Flavour Physics	Antimatter in the Lab	Beyond the Standard Model
	11h35-12h30	Accelerator Operation & Design	Physics at Lepton Colliders	Beyond the Standard Model	Flavour Physics	Close out



1

$$-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$
$$\tfrac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$

2

$$M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H -$$
$$\tfrac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\tfrac{2M^2}{g^2} +$$
$$\tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - i g c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu -$$
$$W^-_\nu \partial_\nu W^+_\mu)] - i g s_w [\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha [H^3 + H \phi^0 \phi^0 + 2 H \phi^+ \phi^-] -$$
$$\tfrac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4 H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] -$$
$$g M W^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \tfrac{1}{2} i g [W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) -$$
$$W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \tfrac{1}{2} g [W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ -$$
$$\phi^+ \partial_\mu H)] + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - i g \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) +$$
$$i g s_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - i g \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) +$$
$$i g s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \tfrac{1}{4} g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] -$$
$$\tfrac{1}{4} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- +$$
$$W^-_\mu \phi^+) - \tfrac{1}{2} i g^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- +$$
$$W^-_\mu \phi^+) + \tfrac{1}{2} i g^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- -$$
$$g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma\partial + m^\lambda_e) e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m^\lambda_u) u^\lambda_j -$$

3

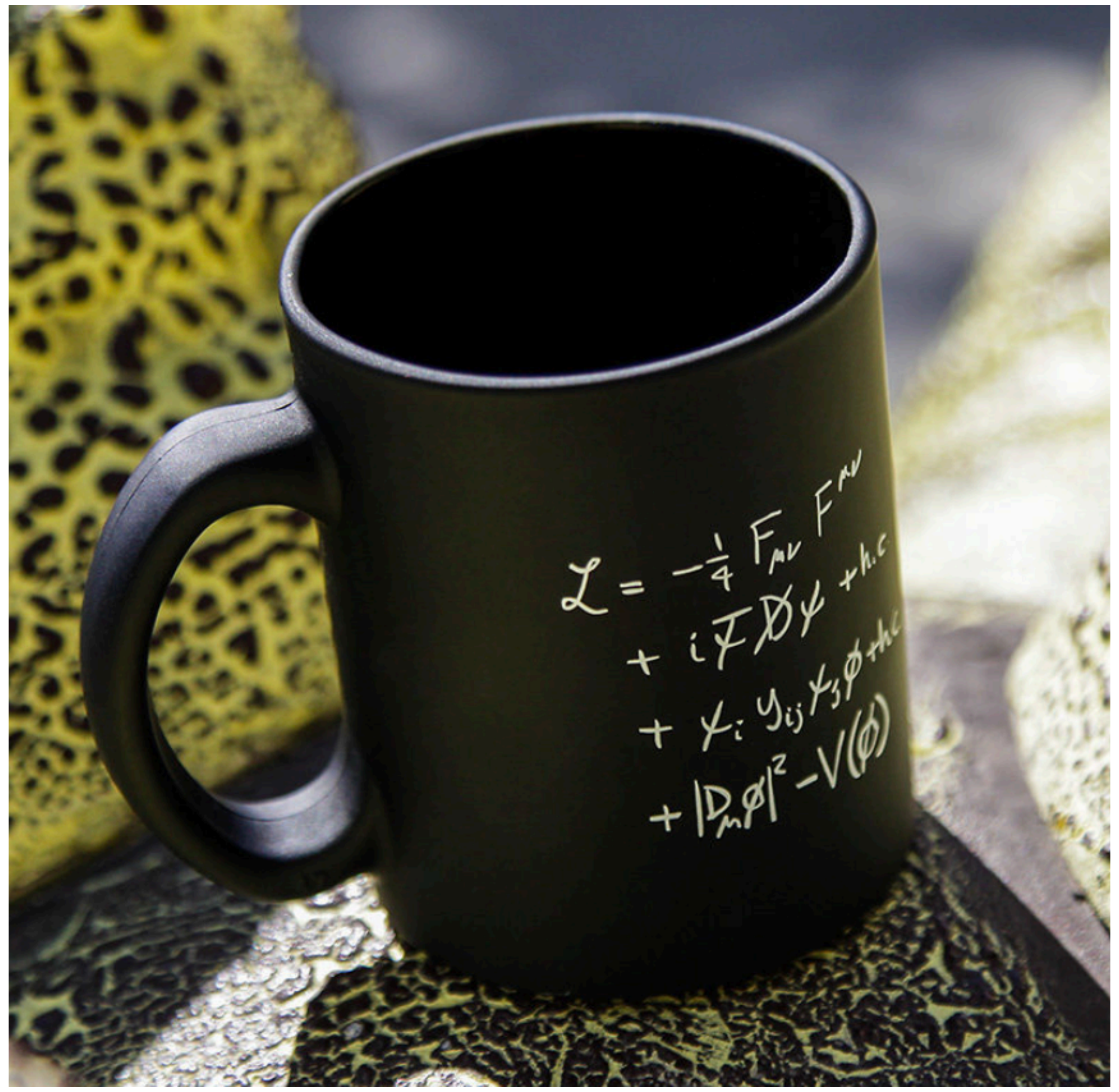
$$\bar{d}^\lambda_j (\gamma\partial + m^\lambda_d) d^\lambda_j + i g s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)] +$$
$$\tfrac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (\tfrac{4}{3} s_w^2 -$$
$$1 - \gamma^5) u^\lambda_j) + (\bar{d}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3} s_w^2 - \gamma^5) d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) +$$
$$(\bar{u}^\lambda_j \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa} \gamma^\mu (1 +$$
$$\gamma^5) u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} \tfrac{m^\lambda_e}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] -$$

4

$$\tfrac{g}{2} \tfrac{m^\lambda_e}{M} [H (\bar{e}^\lambda e^\lambda) + i \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}} \phi^+ [-m^\kappa_d (\bar{u}^\lambda_j C_{\lambda\kappa} (1 - \gamma^5) d^\kappa_j) +$$
$$m^\lambda_u (\bar{u}^\lambda_j C_{\lambda\kappa} (1 + \gamma^5) d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}} \phi^- [m^\lambda_d (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1 + \gamma^5) u^\kappa_j) - m^\kappa_u (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1 -$$
$$\gamma^5) u^\kappa_j] - \tfrac{g}{2} \tfrac{m^\lambda_u}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2} \tfrac{m^\lambda_d}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m^\lambda_u}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) -$$
$$\tfrac{ig}{2} \tfrac{m^\lambda_d}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 -$$

5

$$\tfrac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + i g c_w W^+_\mu (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + i g s_w W^+_\mu (\partial_\mu \bar{Y} X^- -$$
$$\partial_\mu \bar{X}^+ Y) + i g c_w W^-_\mu (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + i g s_w W^-_\mu (\partial_\mu \bar{X}^- Y -$$
$$\partial_\mu \bar{Y} X^+) + i g c_w Z^0_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + i g s_w A_\mu (\partial_\mu \bar{X}^+ X^+ -$$
$$\partial_\mu \bar{X}^- X^-) - \tfrac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2} \bar{X}^0 X^0 H] +$$
$$\tfrac{1-2c_w^2}{2c_w} i g M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \tfrac{1}{2c_w} i g M [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] +$$
$$i g M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \tfrac{1}{2} i g M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$



1

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu$$

2

$$- \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H -$$
$$\frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h[\frac{2M^2}{g^2} +$$
$$\frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu -$$
$$W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$
$$\frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] -$$
$$gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$
$$\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$
$$igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$
$$\frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^1 s_w^2 A_\mu A_\mu \phi^+\phi^-$$

3

$$- \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j -$$
$$\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] +$$
$$\frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 -$$
$$1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$$
$$(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+$$
$$\gamma^5)u^\lambda_j)] +$$

4

$$\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] -$$
$$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) +$$
$$m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-$$
$$\gamma^5)u^\kappa_j] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) -$$
$$\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) +$$

5

$$\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 -$$
$$\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- -$$
$$\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y -$$
$$\partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ -$$
$$\partial_\mu\bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] +$$
$$\frac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] +$$
$$igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]$$

NOT ONLY GOD KNOWS, I KNOW, AND BY THE END OF THE SEMESTER, YOU WILL KNOW.

- SIDNEY COLEMAN -

Overview

Very little formalism at first, will introduce some during the next lectures

Natural units

reminder: $\hbar = c = 1$

We are expressing every **dimensionful** quantity in terms of **energy** (units of electronVolt = eV)

Energy ~ Mass ~ 1/Length ~ 1/Time

Energy	Mass	Length	Time
1 GeV	$1.8 \cdot 10^{-27}$kg	$0.2 \cdot 10^{-15}$m	$6.6 \cdot 10^{-25}$s

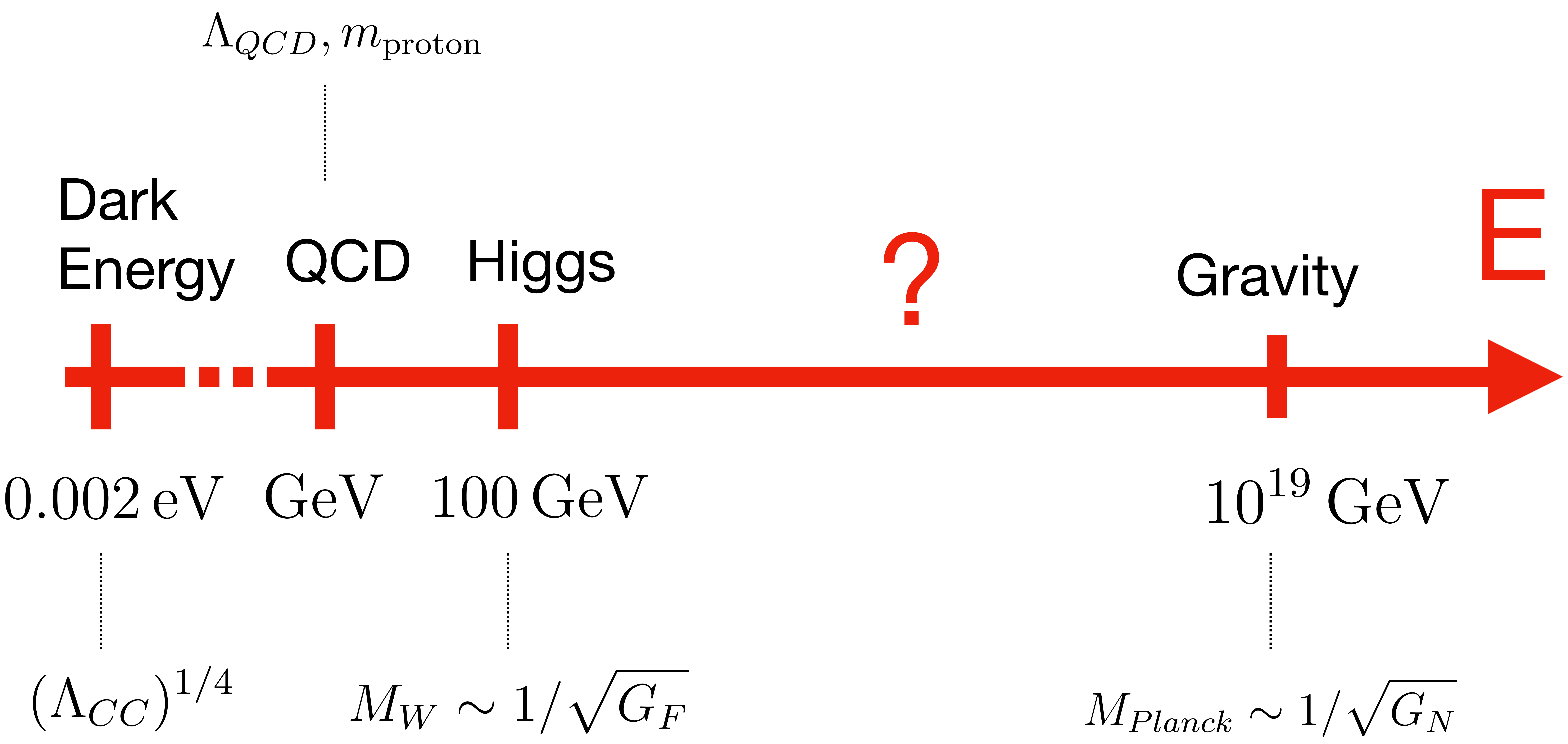
e.g. $E = mc^2$

Conversion Table for natural and MKSA Units

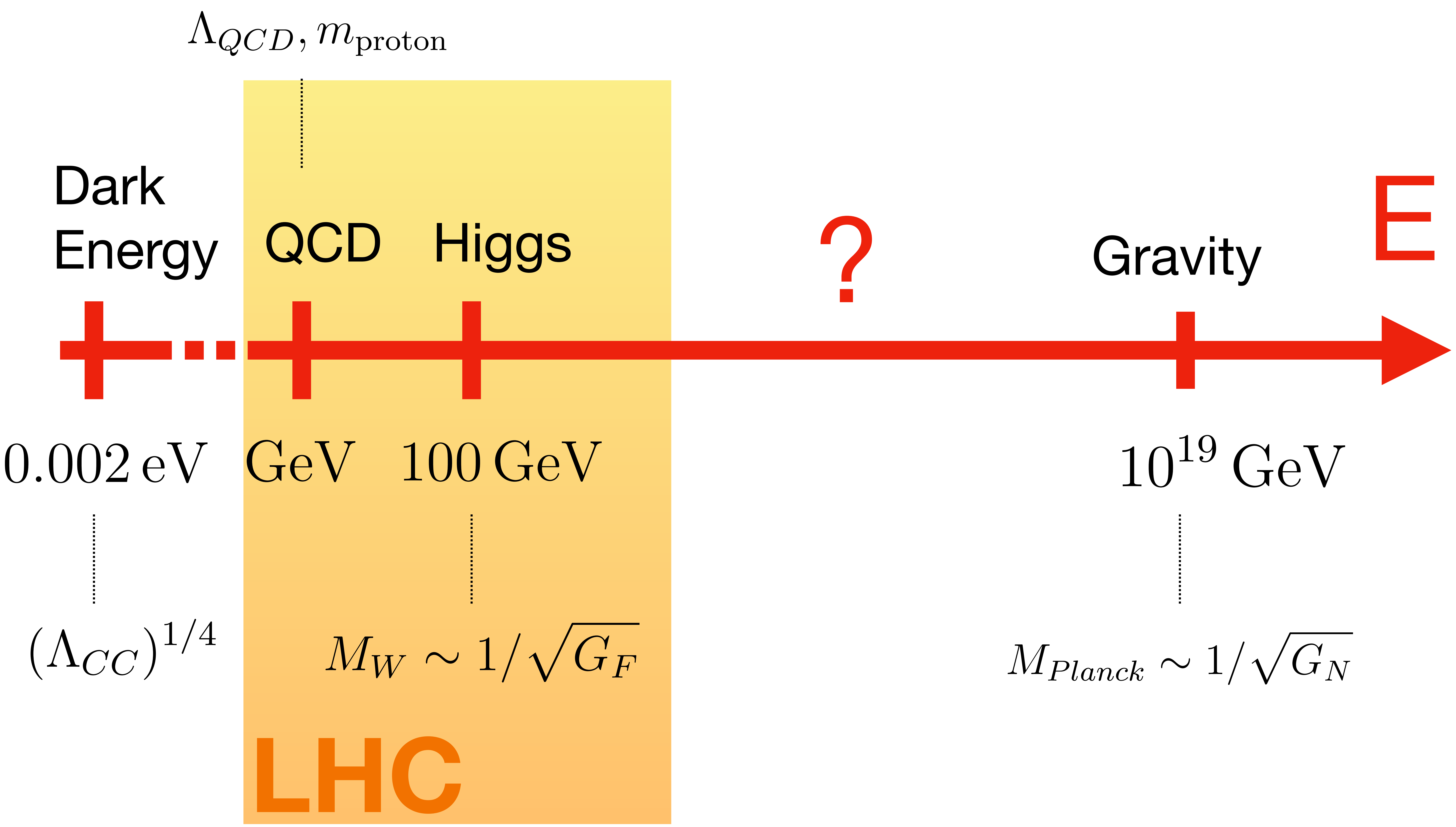
Natural units defined by: $\hbar = c = 1$ (and $4\pi\varepsilon_0 = 1$). Remaining unit is choosen to be Energy (eV).

Quantity	Symbol	natural units	MKSA
Length	ℓ	1/eV	$1.9732705 \cdot 10^{-7}$ m $\approx 0.2\ \mu$m
Mass	m	1 eV	$1.7826627 \cdot 10^{-36}$ kg
Time	t	1/eV	$6.5821220 \cdot 10^{-16}$ s $\approx .66$ fs
Frequency	ν	1 eV	$1.5192669 \cdot 10^{15}$ Hz
Speed	v	1	$2.99792458 \cdot 10^{8}$ m/s
Momentum	p	1 eV	$5.3442883 \cdot 10^{-28}$ kg·m/s
Force	F	1 eV^2	$8.1194003 \cdot 10^{-13}$ N
Power	P	1 eV^2	0.24341350 mW
Energy	E	1 eV	$1.6021773 \cdot 10^{-19}$ J
Charge	q	1	$1.8755468 \cdot 10^{-18}$ C
Charge density	ρ	1 eV^3	244.10013 C/m^3
Current	I	1 eV	2.8494561 mA
Current density	J	1 eV^3	$7.3179379 \cdot 10^{10}$ A/m^2
Electric field	E	1 eV^2	432.90844 V/mm
Potential	Φ	1 eV	85.424546 mV
Polarization	P	1 eV^2	$4.8167560 \cdot 10^{-5}$ C/m^2
Conductivity	σ	1 eV	$1.6904124 \cdot 10^{5}$ S/m
Resistance	R	1	29.979246 Ω
Capacitance	C	1/eV	$2.1955596 \cdot 10^{-17}$ F
Magnetic flux	ϕ	1	$5.6227478 \cdot 10^{-17}$ Wb
Magnetic induction	B	1 eV^2	1.4440271 mT
Magnetization	M	1 eV^2	$1.4440271 \cdot 10^{4}$ A/m
Inductance	L	1/eV	$1.9732705 \cdot 10^{-14}$ H
some constants:			
Planck's quantum	$\hbar$	1	$1.05457266 \cdot 10^{-34}$ J·s
$h = 2\pi\hbar$	h	2π	$6.6260755 \cdot 10^{-34}$ J·s
Charge of electron	e	$8.5424546 \cdot 10^{-2}$	$1.60217733 \cdot 10^{-19}$ C
Bohr radius, $\hbar^2/me^2$	a_0	$2.6817268 \cdot 10^{-4}$/eV	$5.29177249 \cdot 10^{-11}$ m
Energy 1 electron Volt	eV	1 eV	$1.60217733 \cdot 10^{-19}$ J
Rydberg energy, $e^2/2a_0$	E_{Ryd}	13.605698 eV	$2.1798741 \cdot 10^{-18}$ J
Hartree energy, e^2/a_0	E_{h}	27.211396 eV	$4.3597482 \cdot 10^{-18}$ J
Speed of light	c	1	$2.99792458 \cdot 10^{8}$ m/s
Permeability of vacuum	μ_0	4π	$4\pi \cdot 10^{-7}$ H/m
Permittivity of vacuum	ε_0	$1/4\pi$	$8.854187817 \cdot 10^{-12}$ F/m
Bohr magneton	μ_B	$8.3585815 \cdot 10^{-8}$/eV	$9.2740154 \cdot 10^{-24}$ J/T
Mass of electron	m_e	510.99906 keV	$9.1093897 \cdot 10^{-31}$ kg
Mass of proton	m_p	938.27234 MeV	$1.6726231 \cdot 10^{-27}$ kg
Mass of neutron	m_n	939.56563 MeV	$1.6749286 \cdot 10^{-27}$ kg
Gravitation constant	G	$6.70711 \cdot 10^{-57}$/eV^2	$6.67259 \cdot 10^{-11}$ N·m^2/kg^2

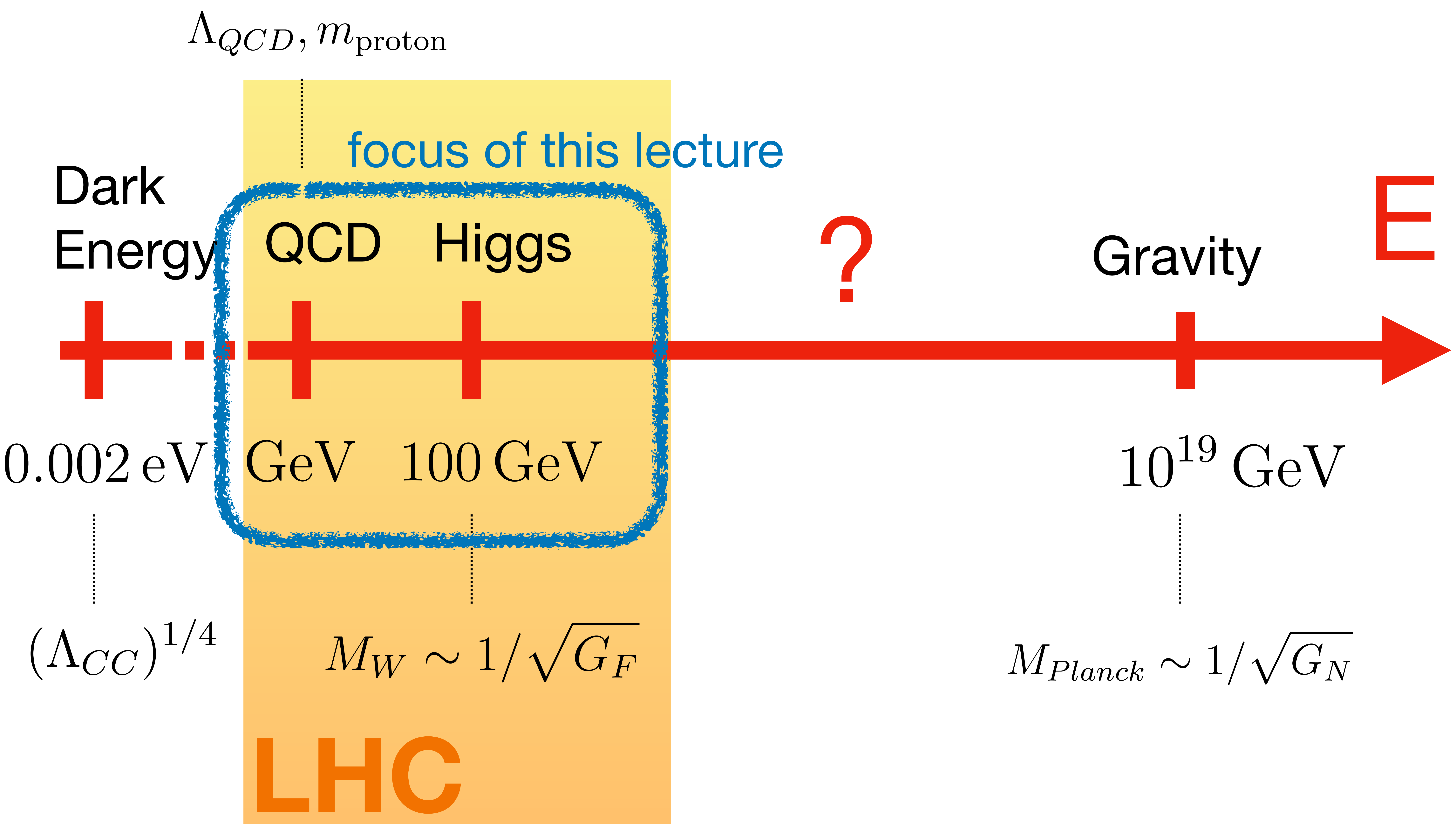
Fundamental scales

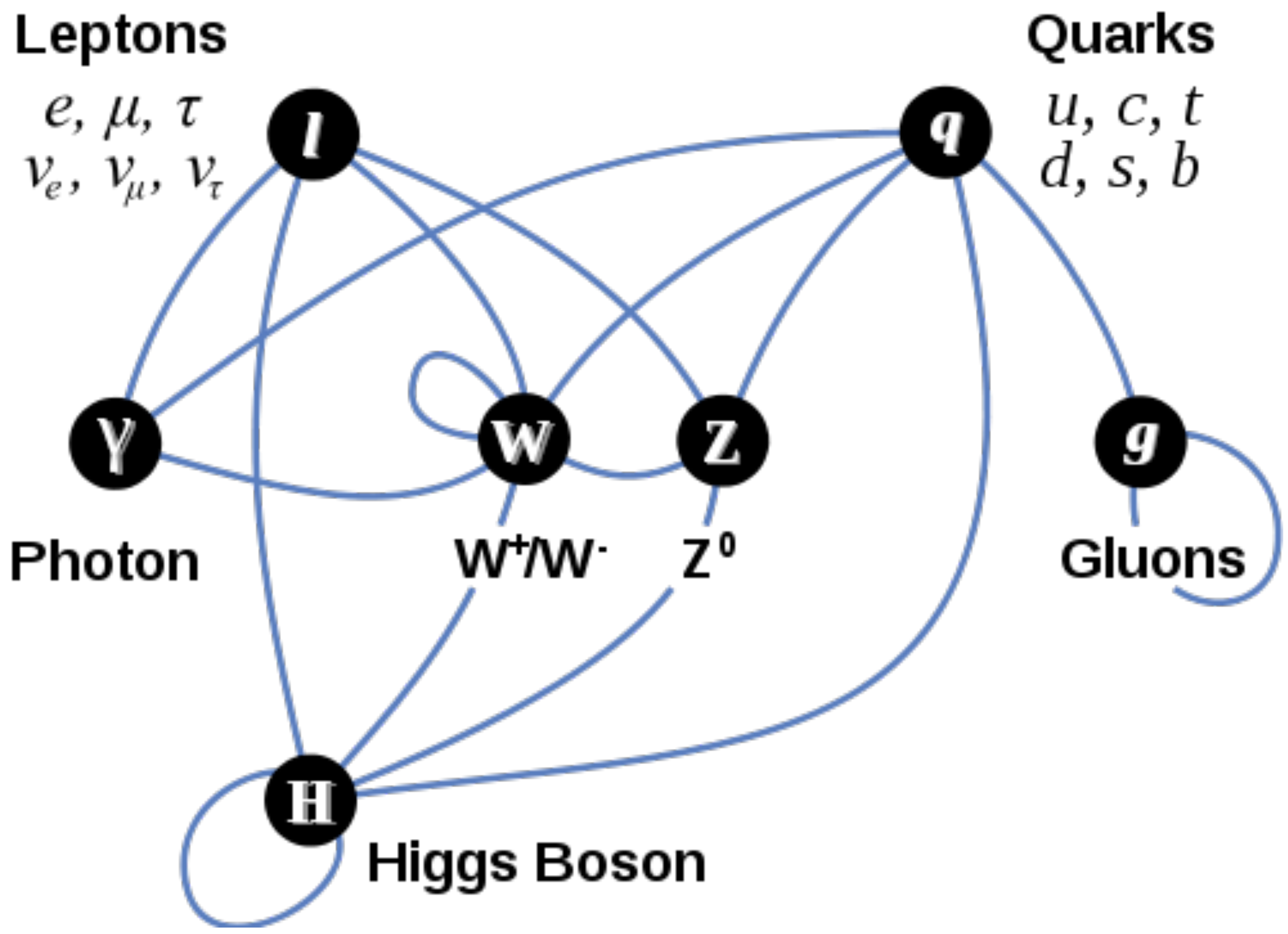


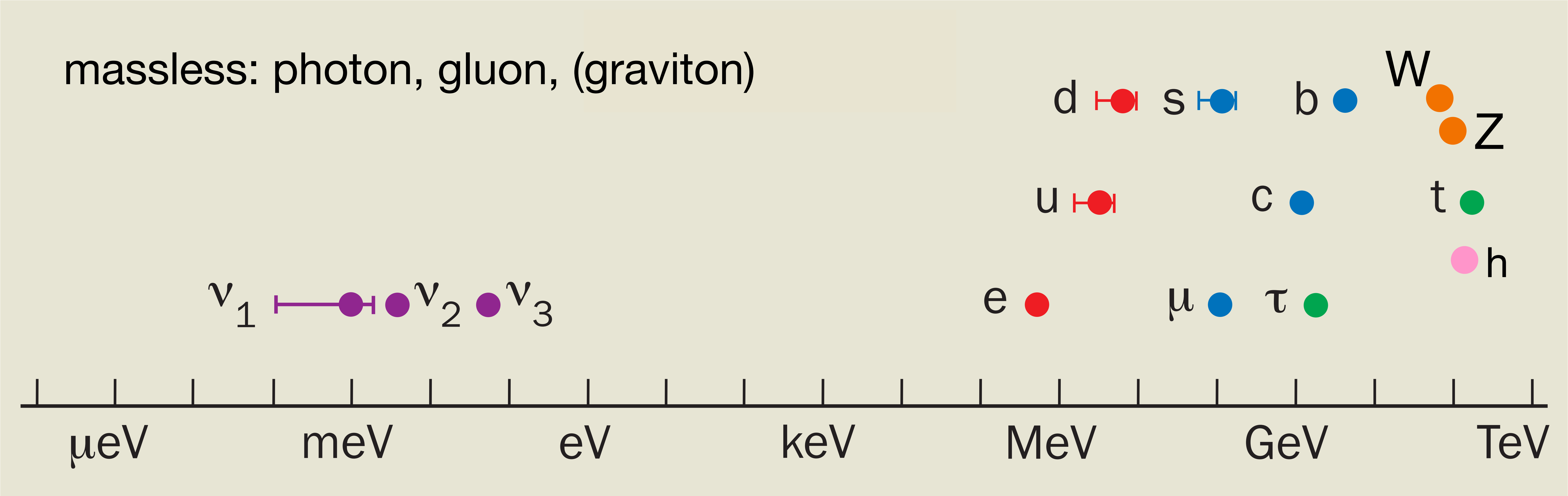
Fundamental scales



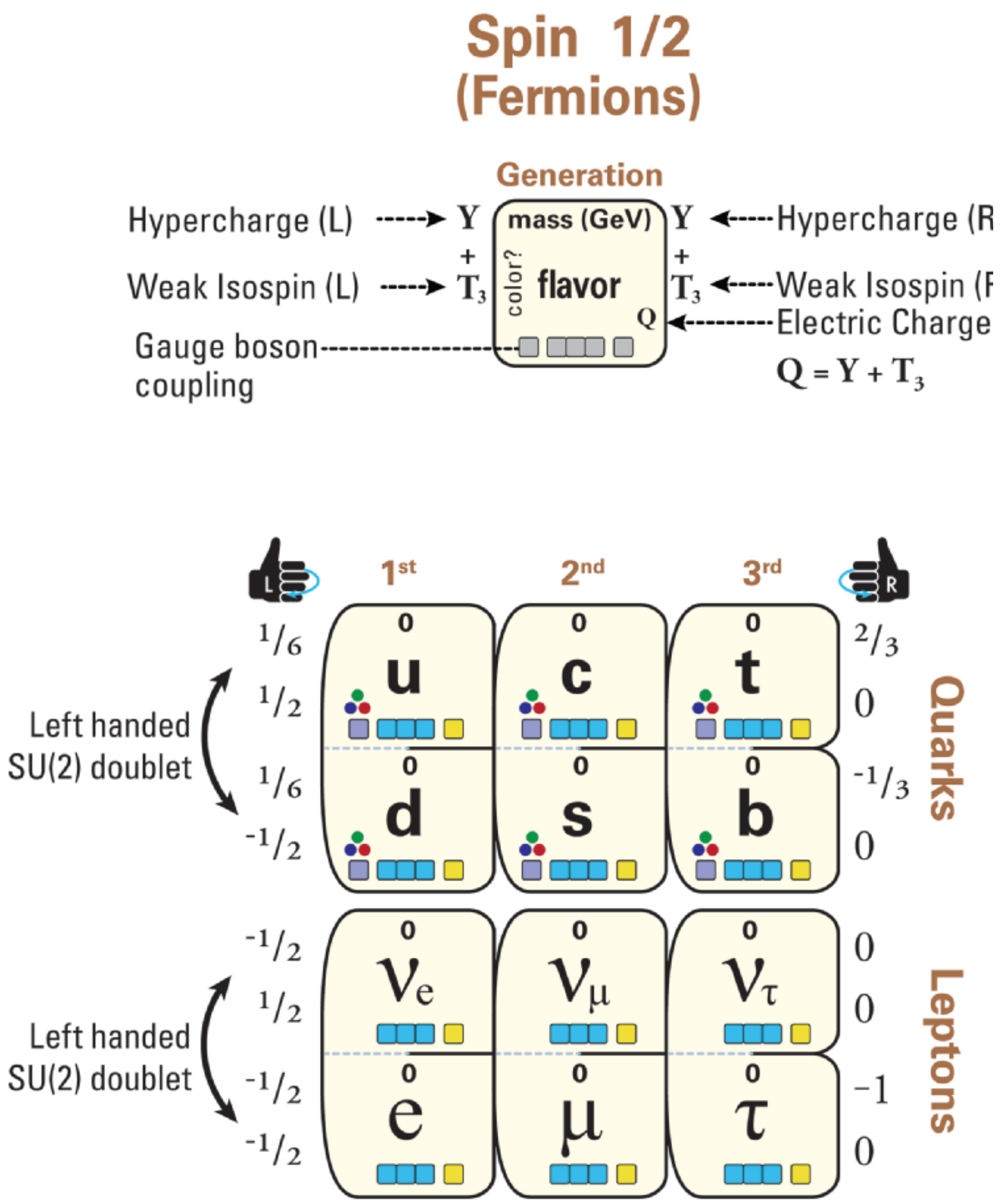
Fundamental scales




Leptons
e, μ, τ
νe, νμ, ντ
Quarks
u, c, t
d, s, b
Photon
W+/W-
Z0
Gluons
Higgs Boson


massless: photon, gluon, (graviton)
d
s
b
W
Z
u
c
t
h
ν1
ν2
ν3
e
μ
τ
μeV
meV
eV
keV
MeV
GeV
TeV

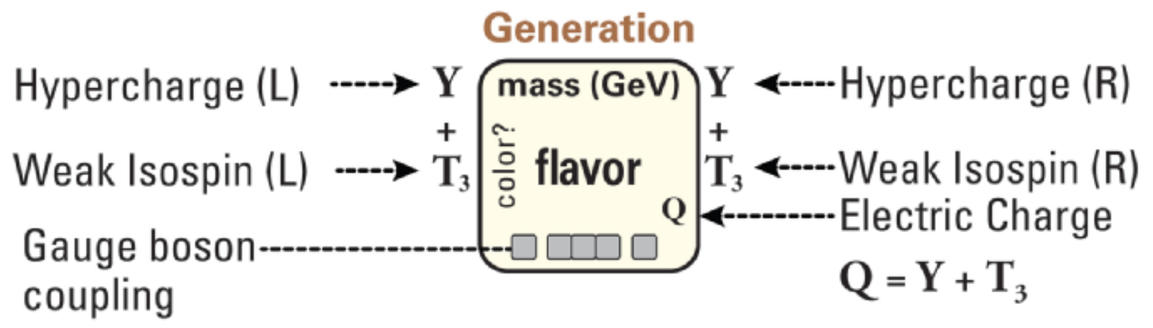
Matter

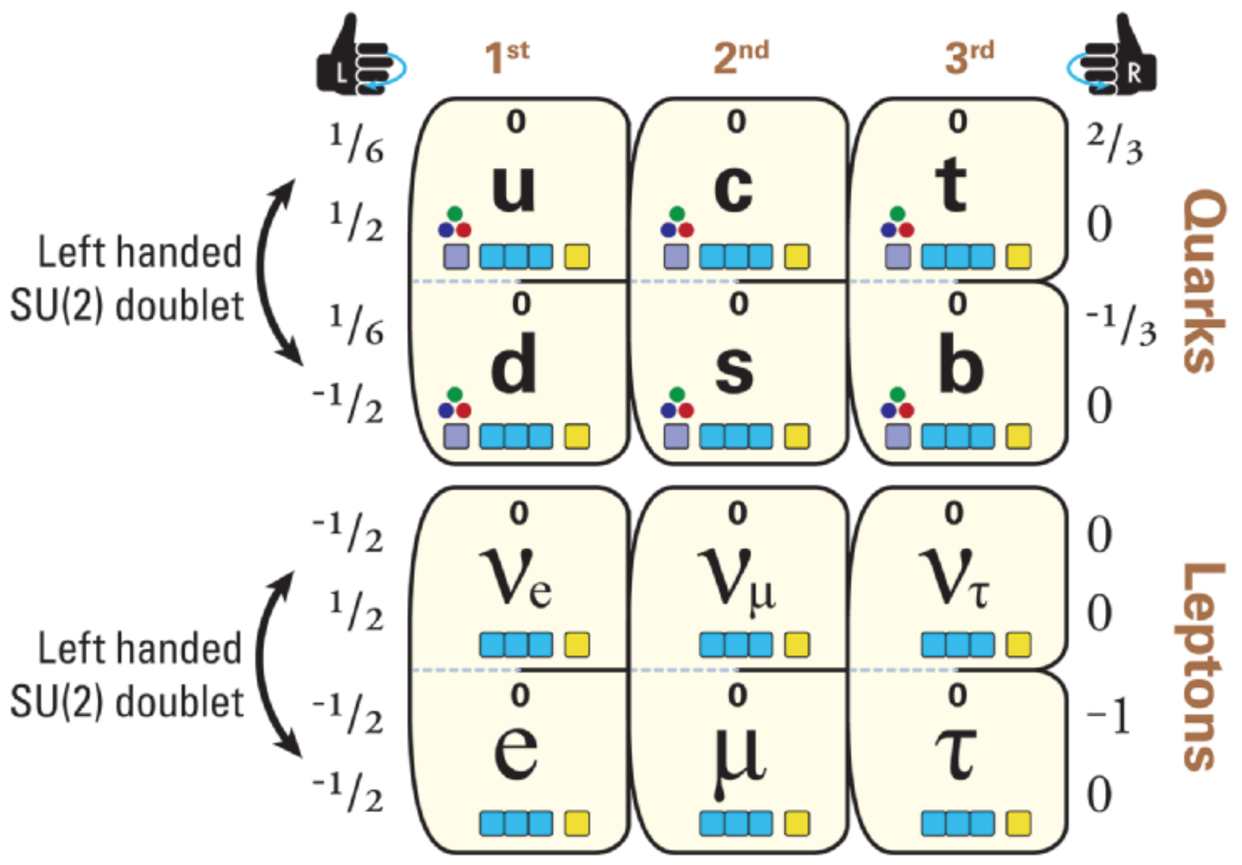


Spin 1/2
(Fermions)
Generation
Hypercharge (L)
Y
+
Weak Isospin (L)
T3
mass (GeV)
color?
flavor
Q
Y
+
T3
Hypercharge (R
Weak Isospin (F
Electric Charge
Q = Y + T3
Gauge boson coupling
1st
2nd
3rd
Left handed SU(2) doublet
1/6
1/2
1/6
-1/2
u
c
t
d
s
b
2/3
0
-1/3
0
Quarks
Left handed SU(2) doublet
-1/2
1/2
-1/2
-1/2
νe
νμ
ντ
e
μ
τ
0
0
-1
0
Leptons

Matter

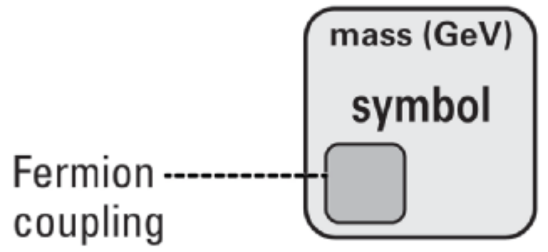
Spin 1/2
(Fermions)





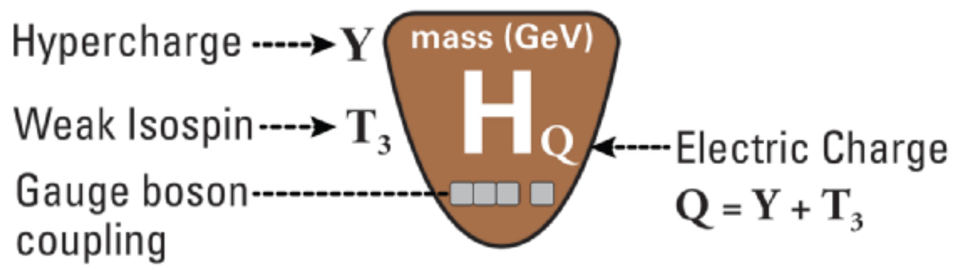
Forces

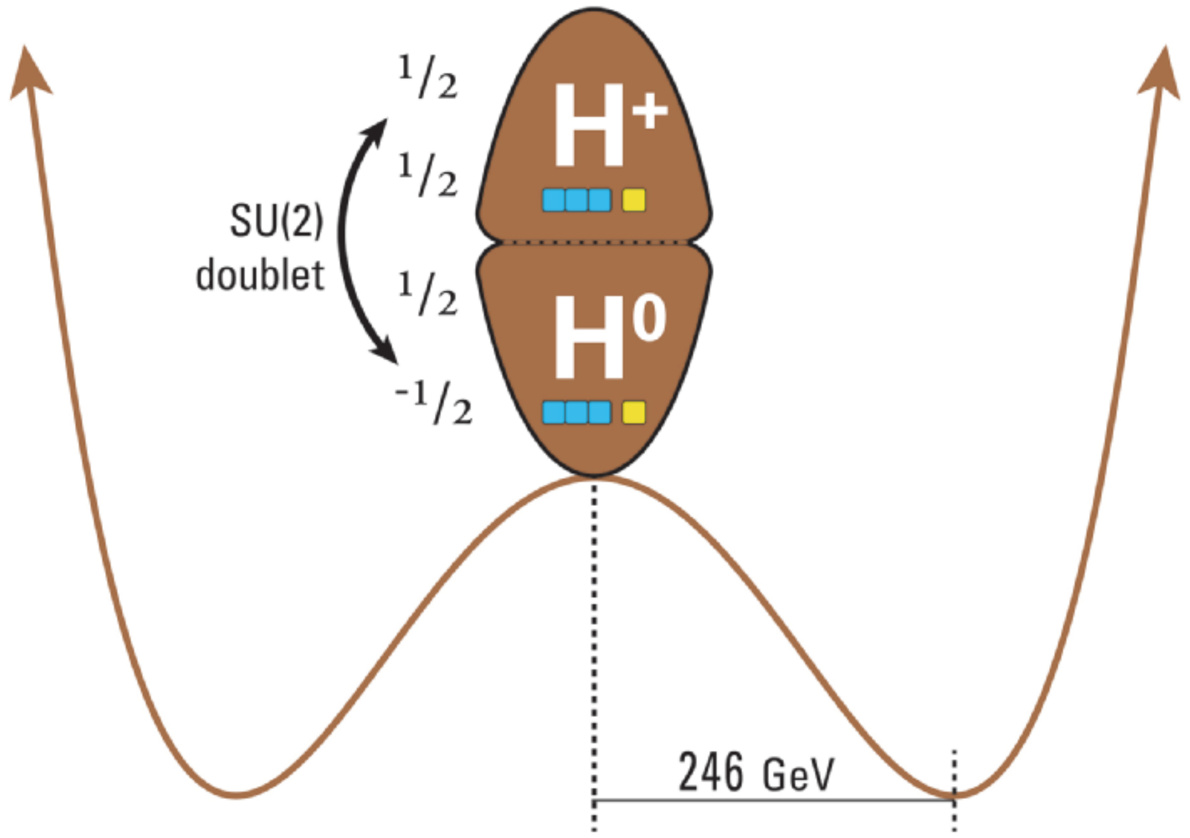
Spin 1
(Gauge Bosons)



Higgs

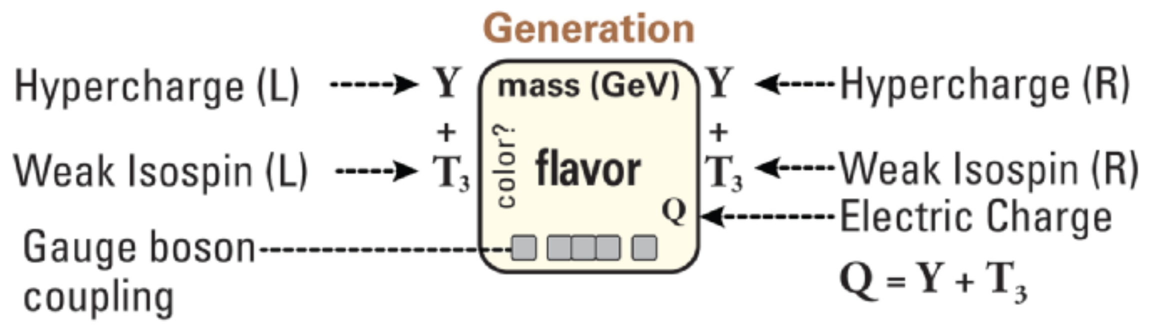
Spin 0
(Higgs Boson)

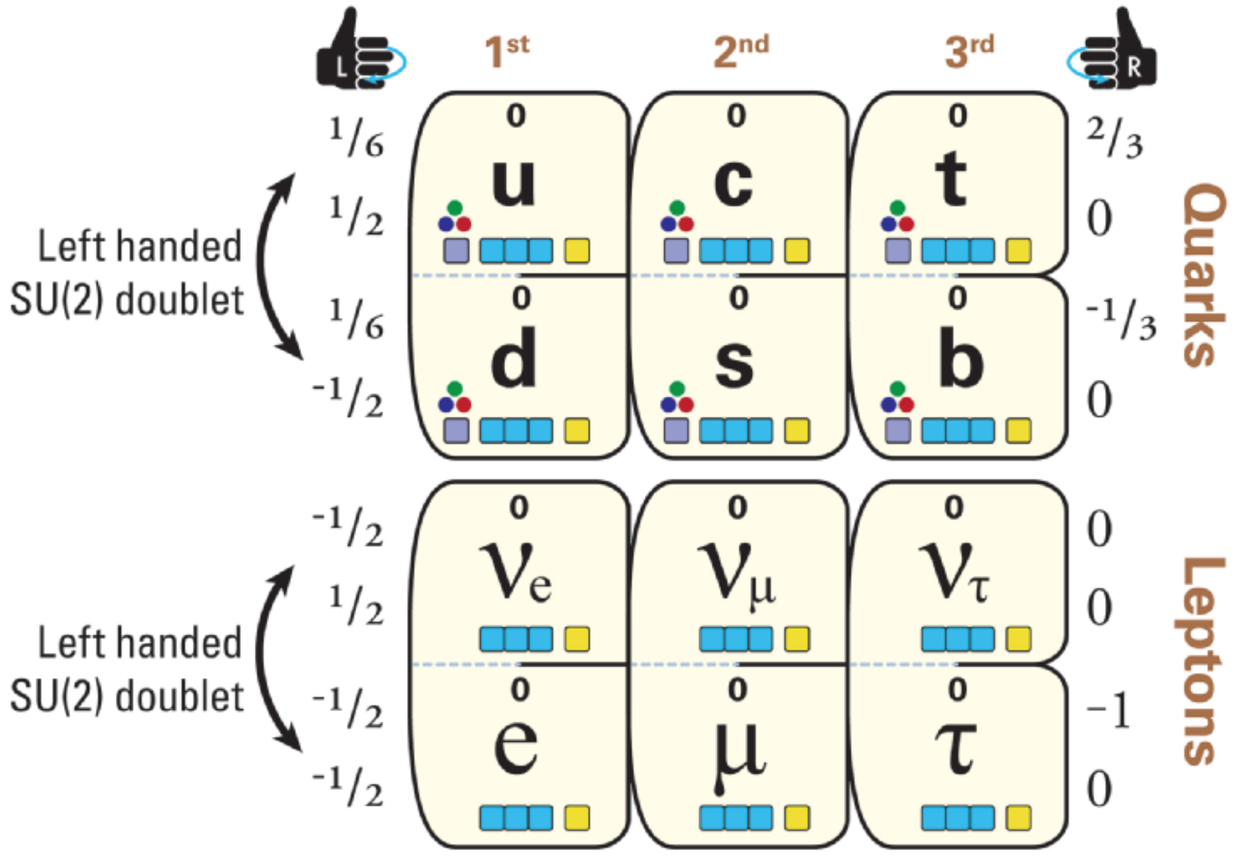




Matter

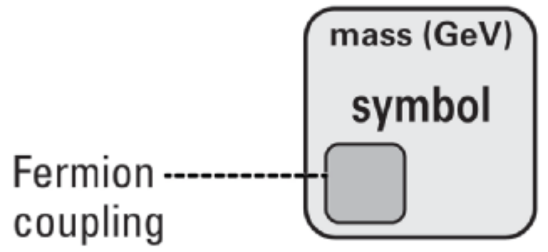
Spin 1/2
(Fermions)

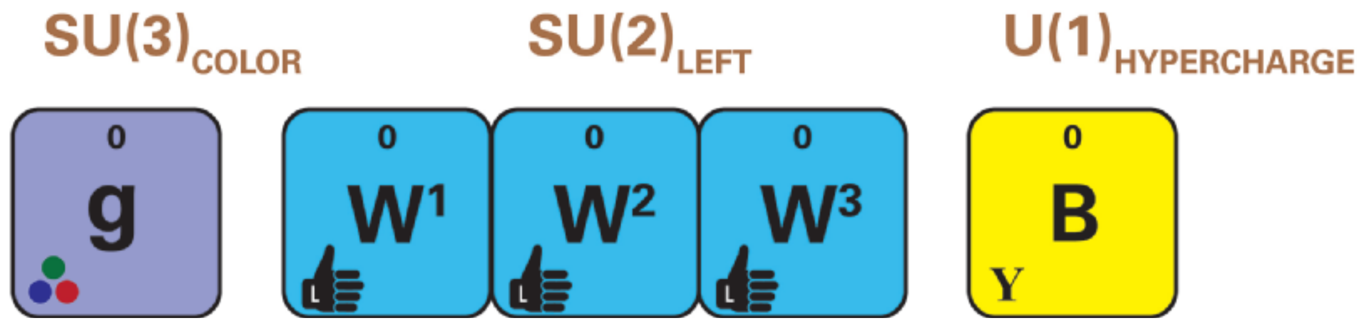




Forces

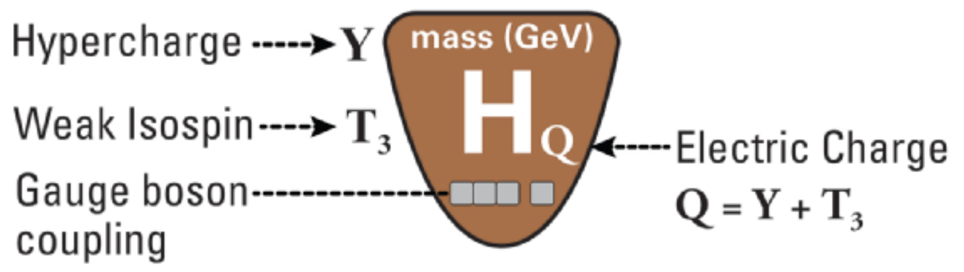
Spin 1
(Gauge Bosons)

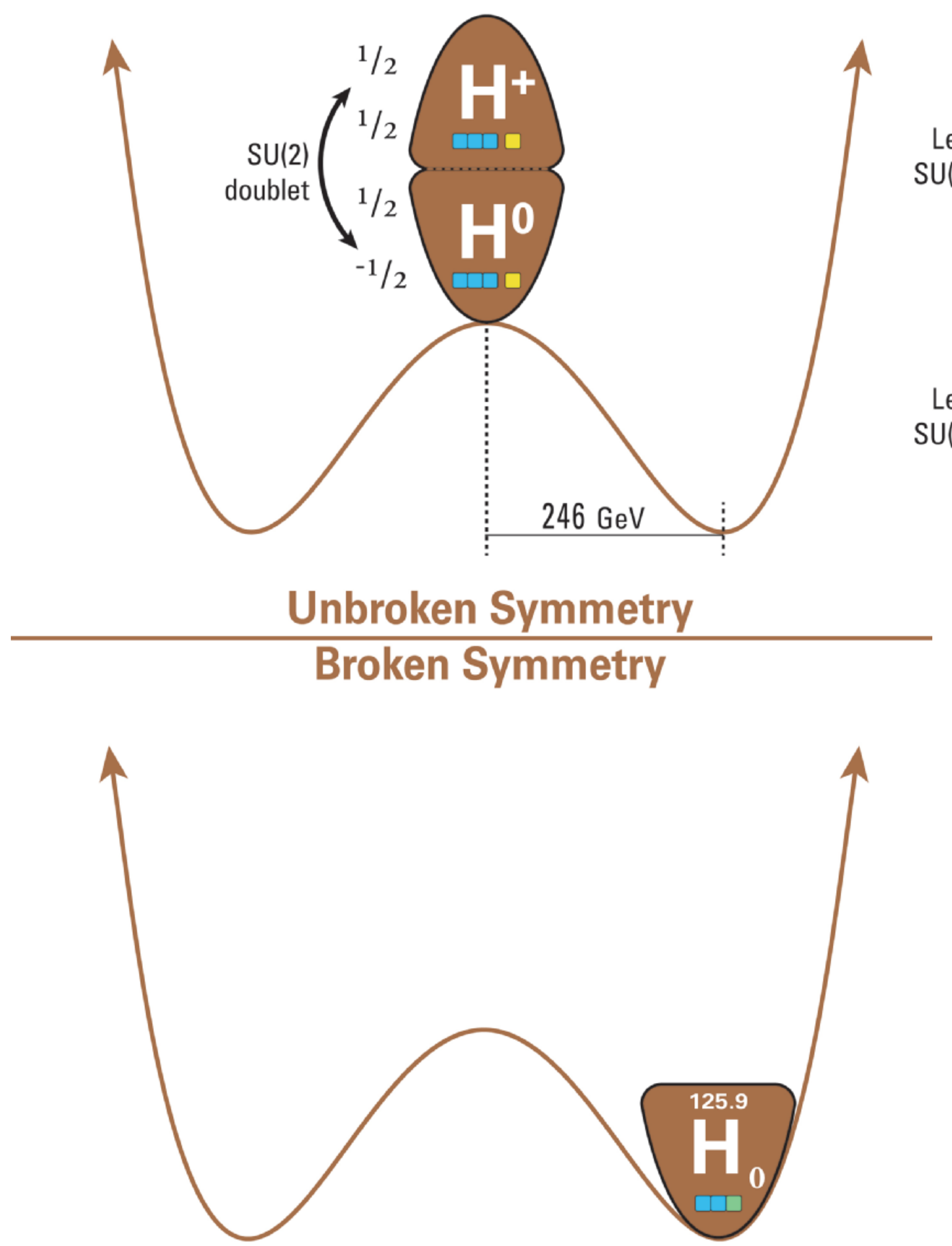




Higgs

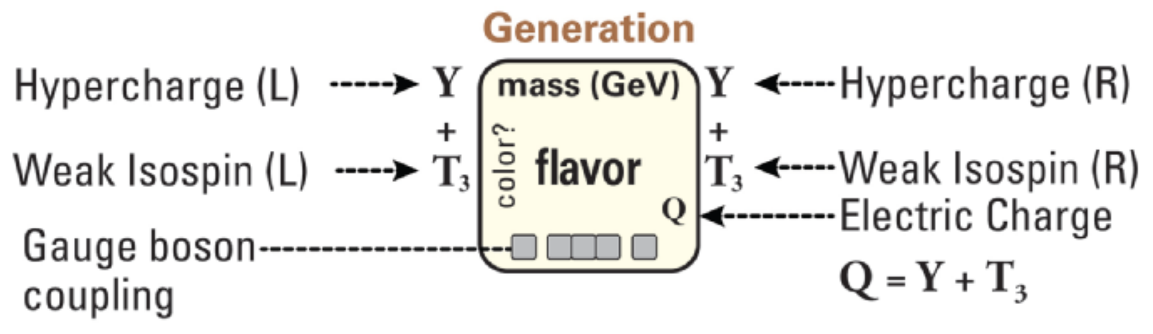
Spin 0 (Higgs Boson)

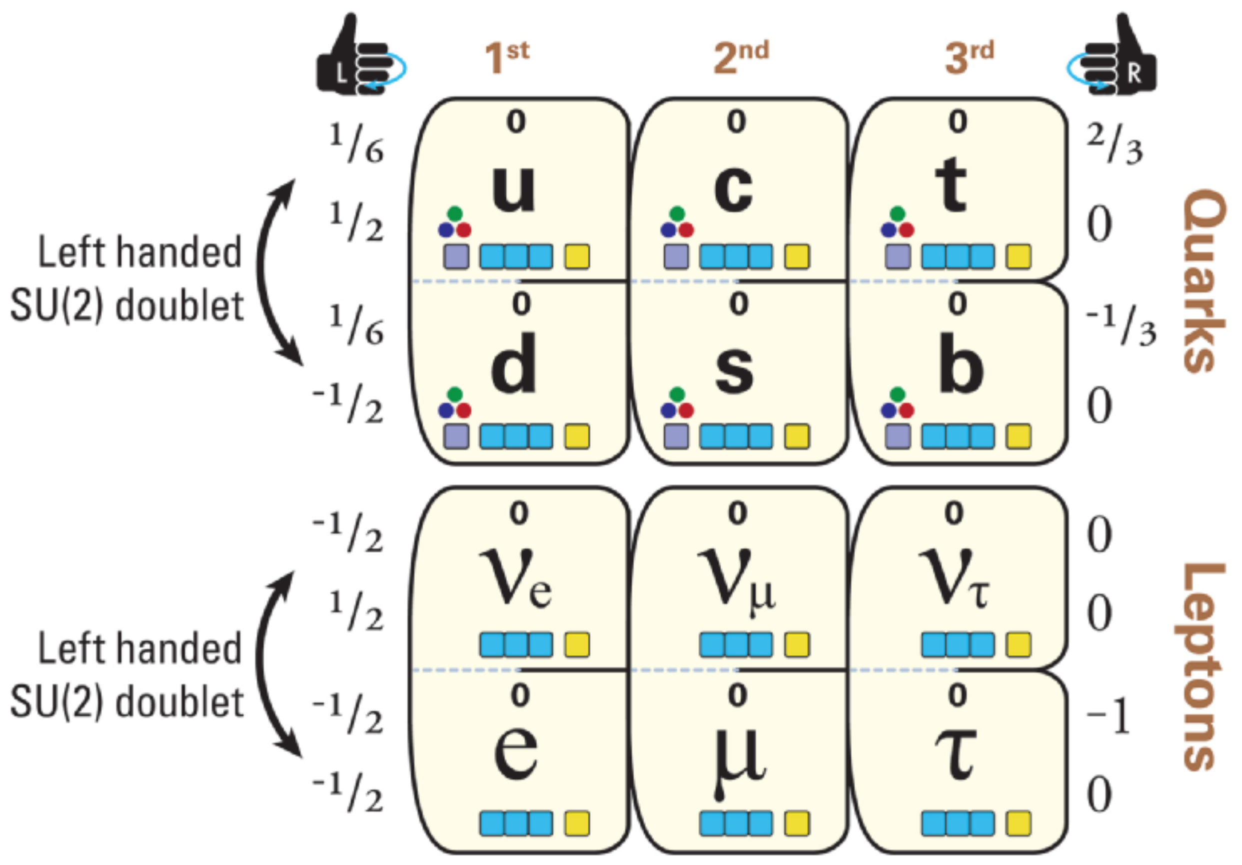




Matter

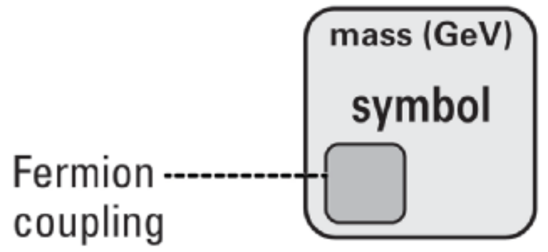
Spin 1/2 (Fermions)

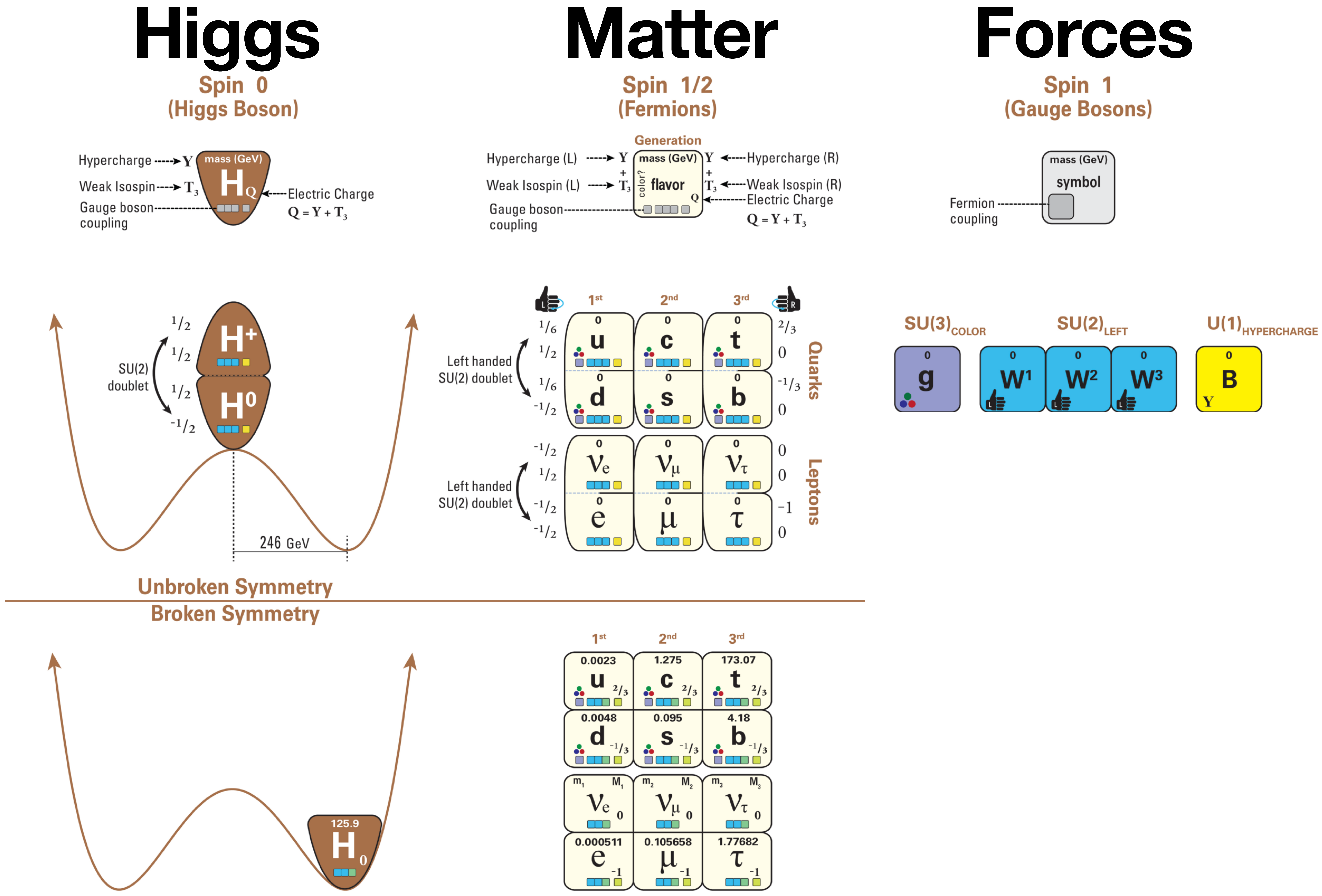




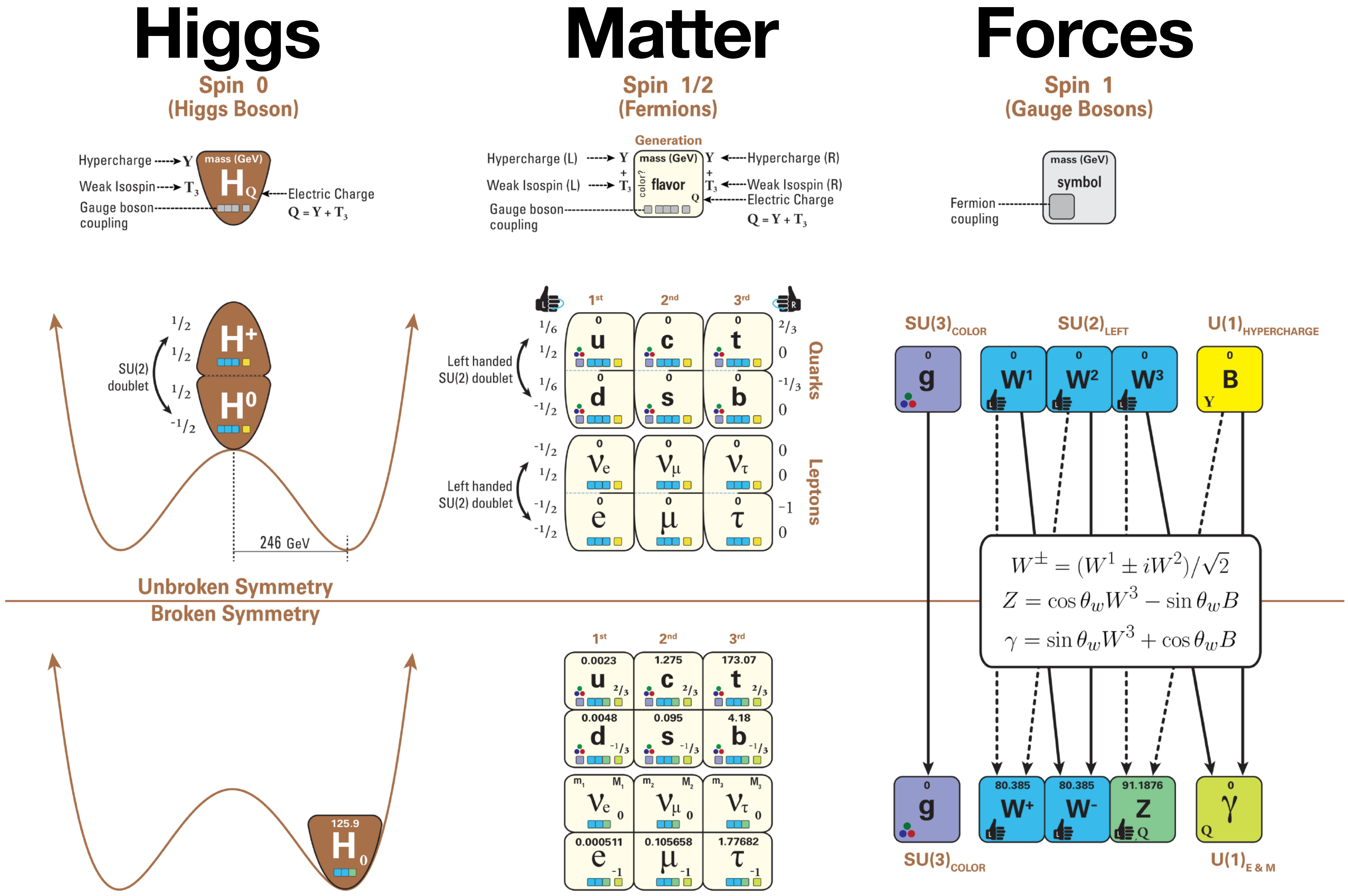
Forces

Spin 1 (Gauge Bosons)





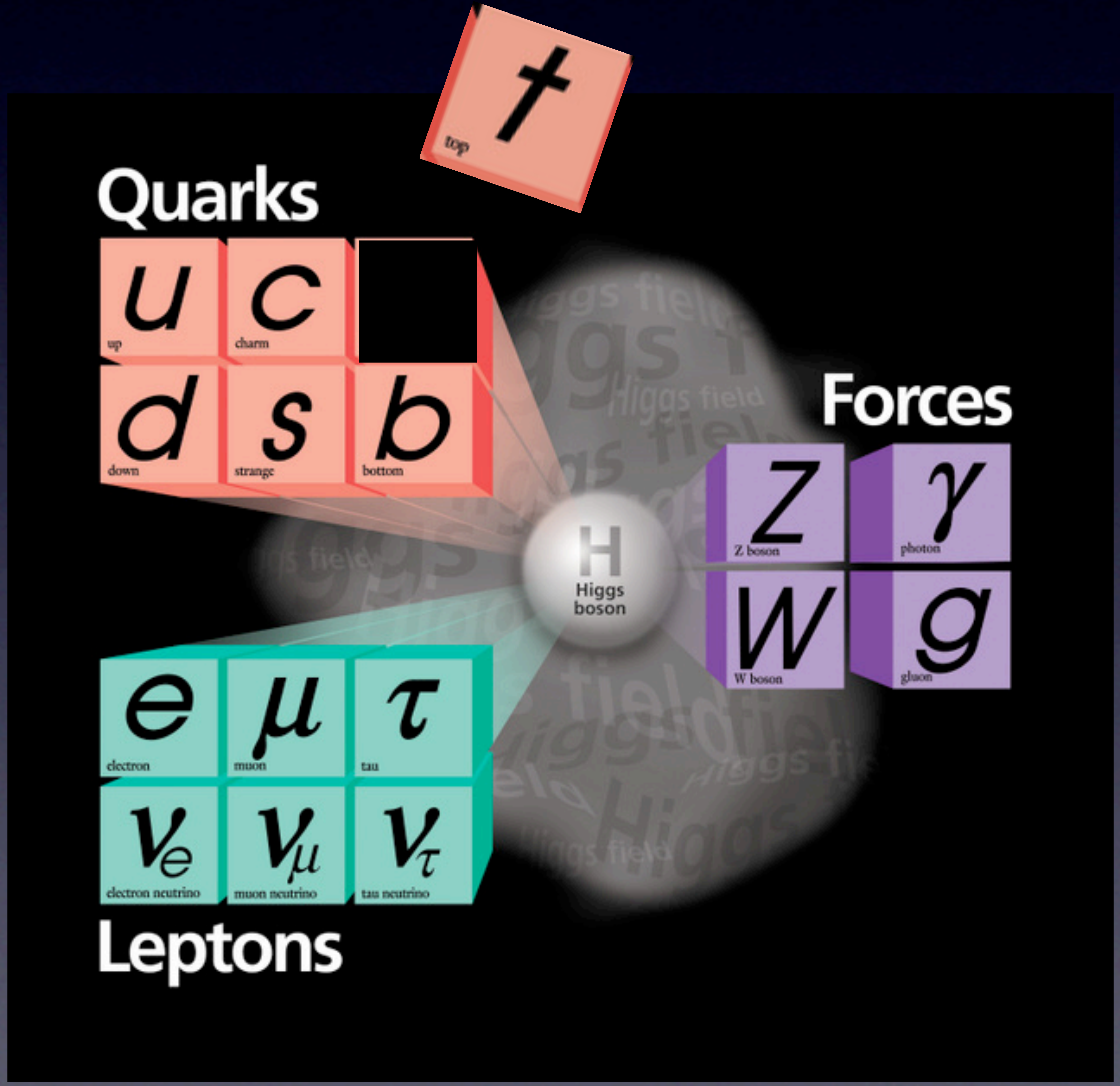
Higgs
Spin 0
(Higgs Boson)
Matter
Spin 1/2
(Fermions)
Forces
Spin 1
(Gauge Bosons)
Hypercharge
Weak Isospin
Gauge boson coupling
mass (GeV)
Electric Charge
Q = Y + T3
Generation
Hypercharge (L)
Weak Isospin (L)
Gauge boson coupling
mass (GeV)
flavor
color?
Hypercharge (R)
Weak Isospin (R)
Electric Charge
Q = Y + T3
mass (GeV)
symbol
Fermion coupling
SU(2) doublet
H+
H0
246 GeV
1st
2nd
3rd
Left handed SU(2) doublet
Left handed SU(2) doublet
u
c
t
d
s
b
νe
νμ
ντ
e
μ
τ
Quarks
Leptons
SU(3)COLOR
SU(2)LEFT
U(1)HYPERCHARGE
g
W1
W2
W3
B
Unbroken Symmetry
Broken Symmetry
0.0023
1.275
173.07
0.0048
0.095
4.18
0.000511
0.105658
1.77682
125.9
H



Higgs
Spin 0
(Higgs Boson)
Matter
Spin 1/2
(Fermions)
Forces
Spin 1
(Gauge Bosons)
Hypercharge → Y
Weak Isospin → T3
Gauge boson coupling
mass (GeV)
H Q
Electric Charge
Q = Y + T3
Generation
Hypercharge (L)
Weak Isospin (L)
Gauge boson coupling
mass (GeV)
color?
flavor
Hypercharge (R)
Weak Isospin (R)
Electric Charge
Q = Y + T3
mass (GeV)
symbol
Fermion coupling
SU(2) doublet
H+
H0
1/2
1/2
1/2
-1/2
246 GeV
1st
2nd
3rd
Left handed SU(2) doublet
u c t
d s b
1/6
1/2
1/6
-1/2
2/3
0
-1/3
0
Quarks
νe νμ ντ
e μ τ
-1/2
1/2
-1/2
-1/2
0
0
-1
0
Leptons
SU(3)COLOR
SU(2)LEFT
U(1)HYPERCHARGE
g
W1
W2
W3
B
Y
W± = (W1 ± iW2)/√2
Z = cos θw W3 − sin θw B
γ = sin θw W3 + cos θw B
Unbroken Symmetry
Broken Symmetry
125.9
H 0
0.0023 u 2/3
1.275 c 2/3
173.07 t 2/3
0.0048 d -1/3
0.095 s -1/3
4.18 b -1/3
m1 M1 νe 0
m2 M2 νμ 0
m3 M3 ντ 0
0.000511 e -1
0.105658 μ -1
1.77682 τ -1
0 g
80.385 W+
80.385 W-
91.1876 Z Q
0 γ Q
SU(3)COLOR
U(1)E & M

Standard Model

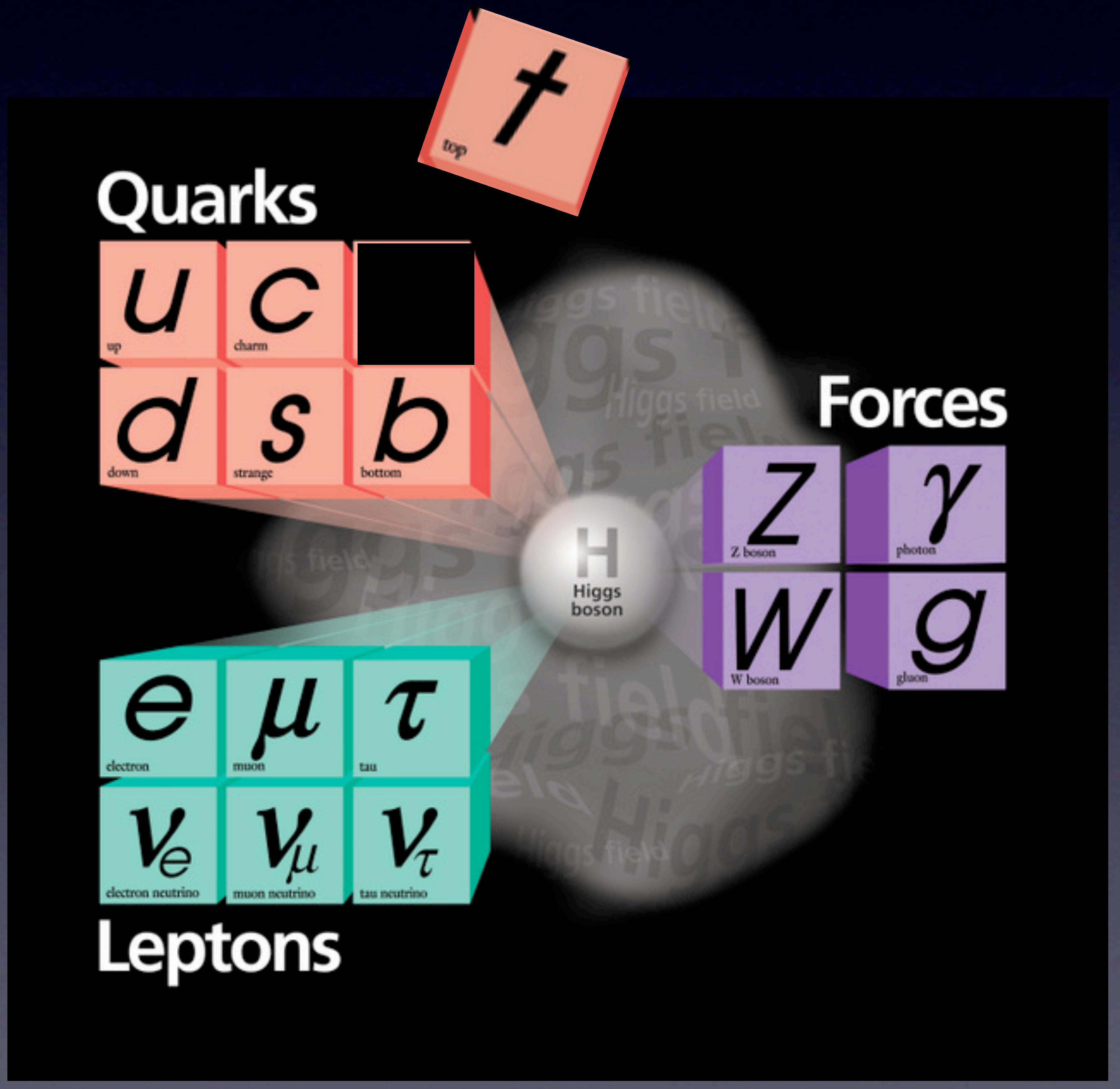
top quark, Tevatron Fermilab 1995

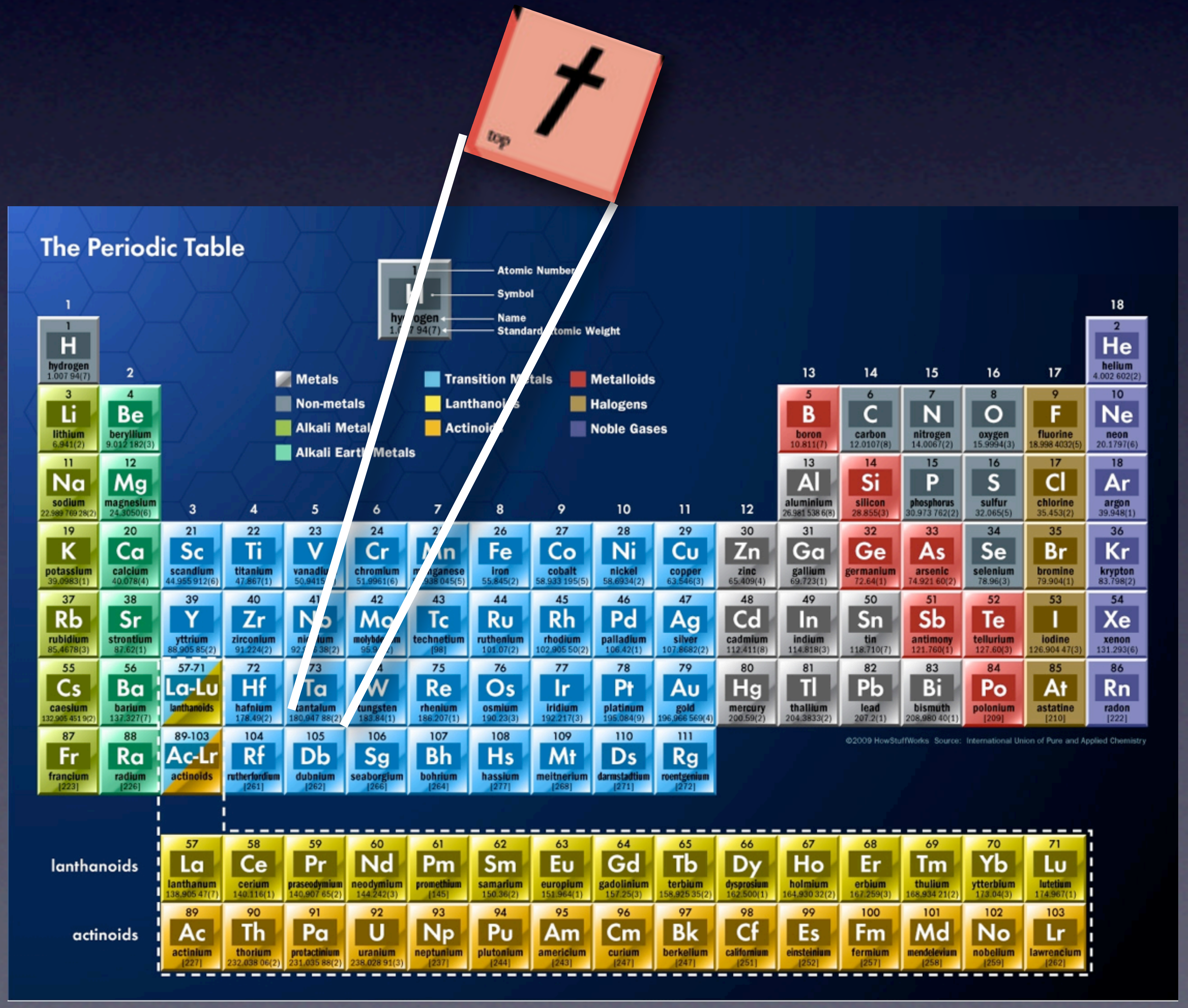


Standard Model

= subatomic Taxonomy?

top quark, Tevatron Fermilab 1995





Standard Model is not about particles!

Standard Model is not about particles!

Principles !

What are the rules at short and long distances?

Quantum Mechanics, Lorentz-Invariance, Locality, Unitarity, Global Symmetries, Gauge Redundancies, Conservation Laws, Spontaneous Symmetry Breaking…

SM = QM + relativity + symmetry + low energy expansion

QFT

$$SU(3)_c \times SU(2)_W \times U(1)_Y$$

+ Higgs mechanism

accidental symmetries $U(1)_B \times U(1)_L$

Weltformel

quantum mechanics — spacetime — gravity

$$W = \int_{k<\Lambda} [Dg][DA][D\psi][D\Phi] \exp\left\{ i \int d^4x \sqrt{-g} \left[\frac{m_p^2}{2} R \right.\right.$$

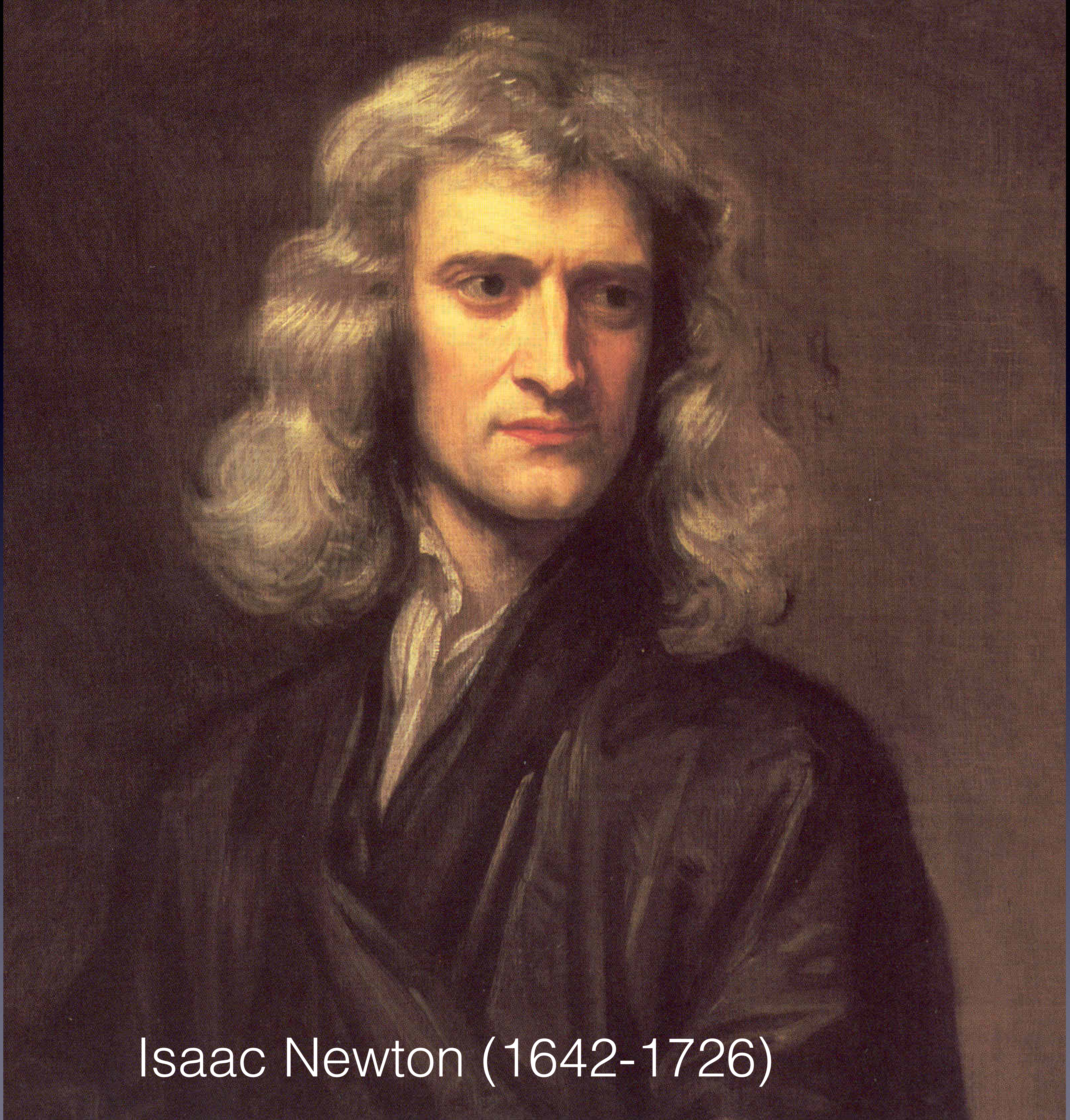
$$\left.\left. -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi}^i \gamma^\mu D_\mu \psi^i + \left(\bar{\psi}^i_L V_{ij} \Phi \psi^j_R + \text{h.c.} \right) - |D_\mu \Phi|^2 - V(\Phi) \right] \right\}$$

other forces — matter — Higgs

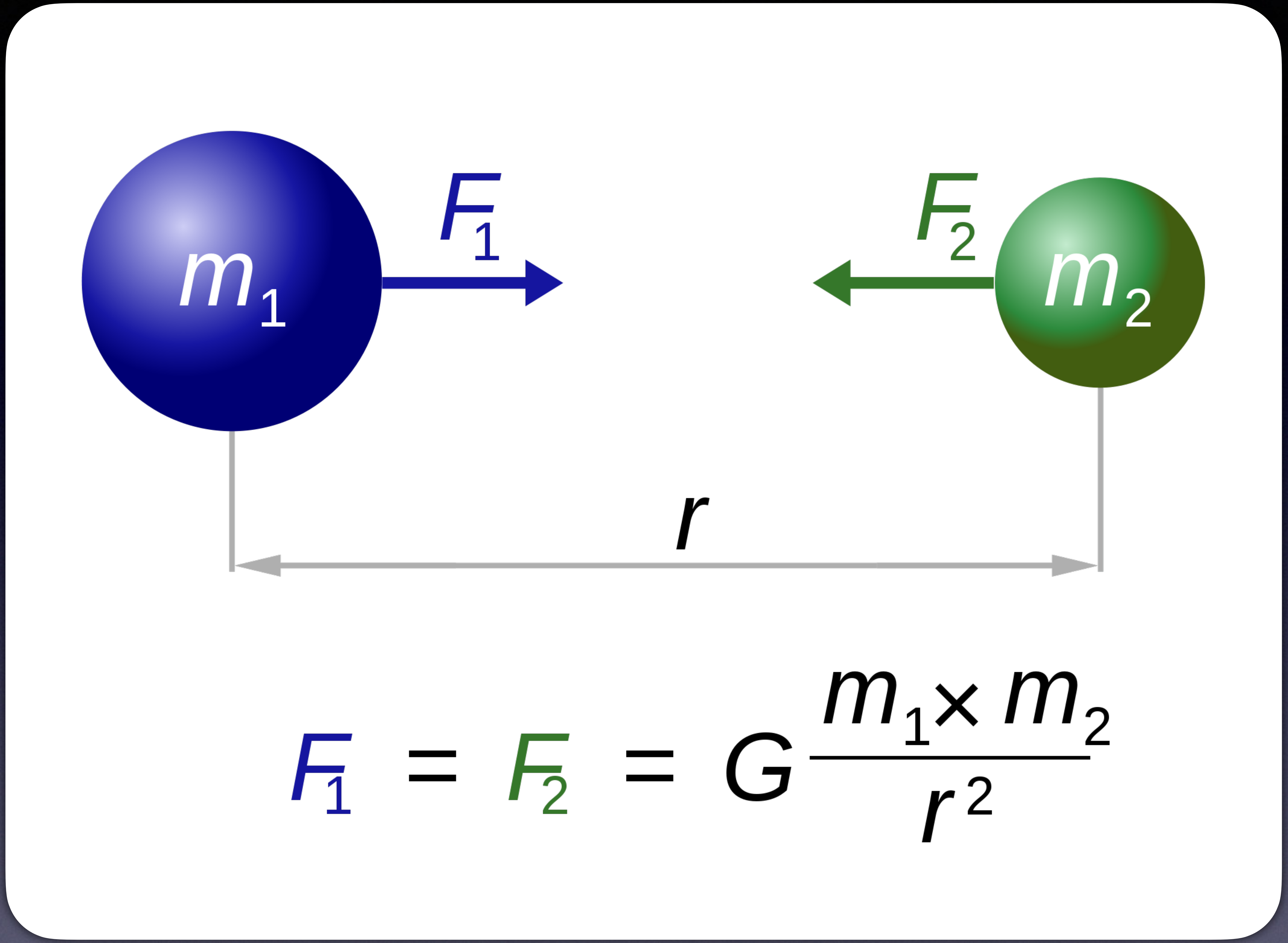
Quantum Field Theory

In **quantum field theory** the field is the fundamental object from which all properties of **matter** and **forces** emerge.

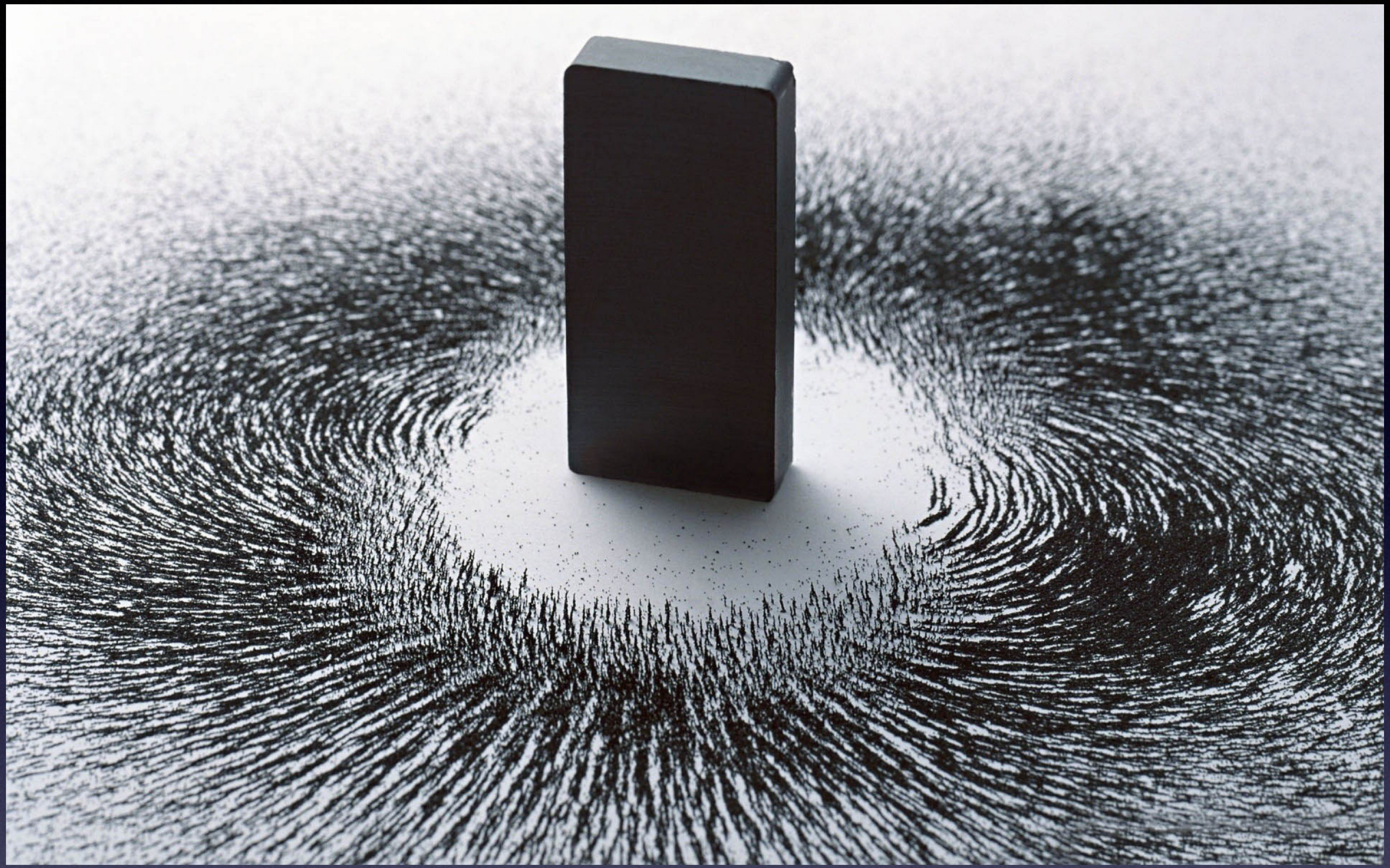
Fields



Isaac Newton (1642-1726)



Action at a distance?

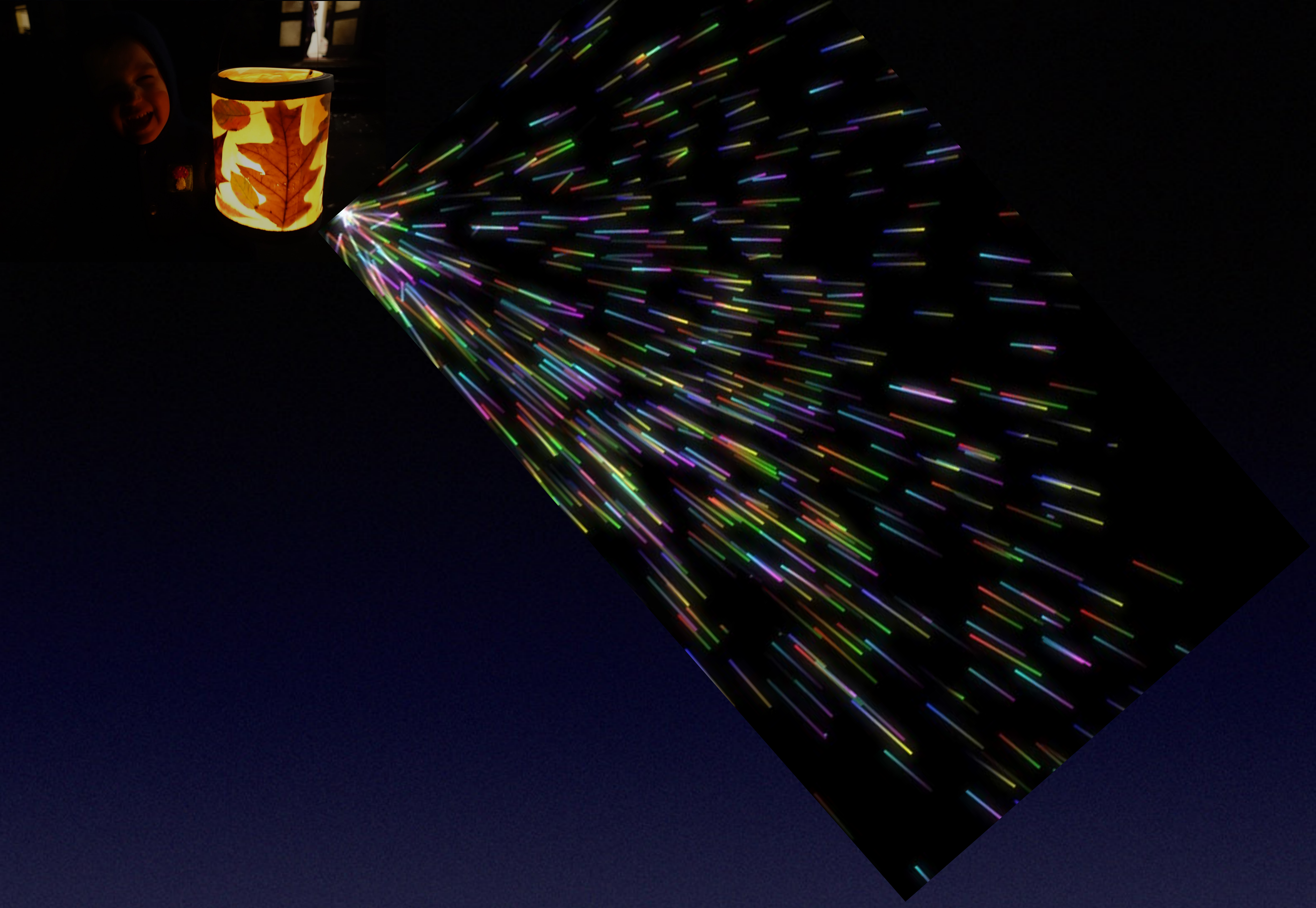


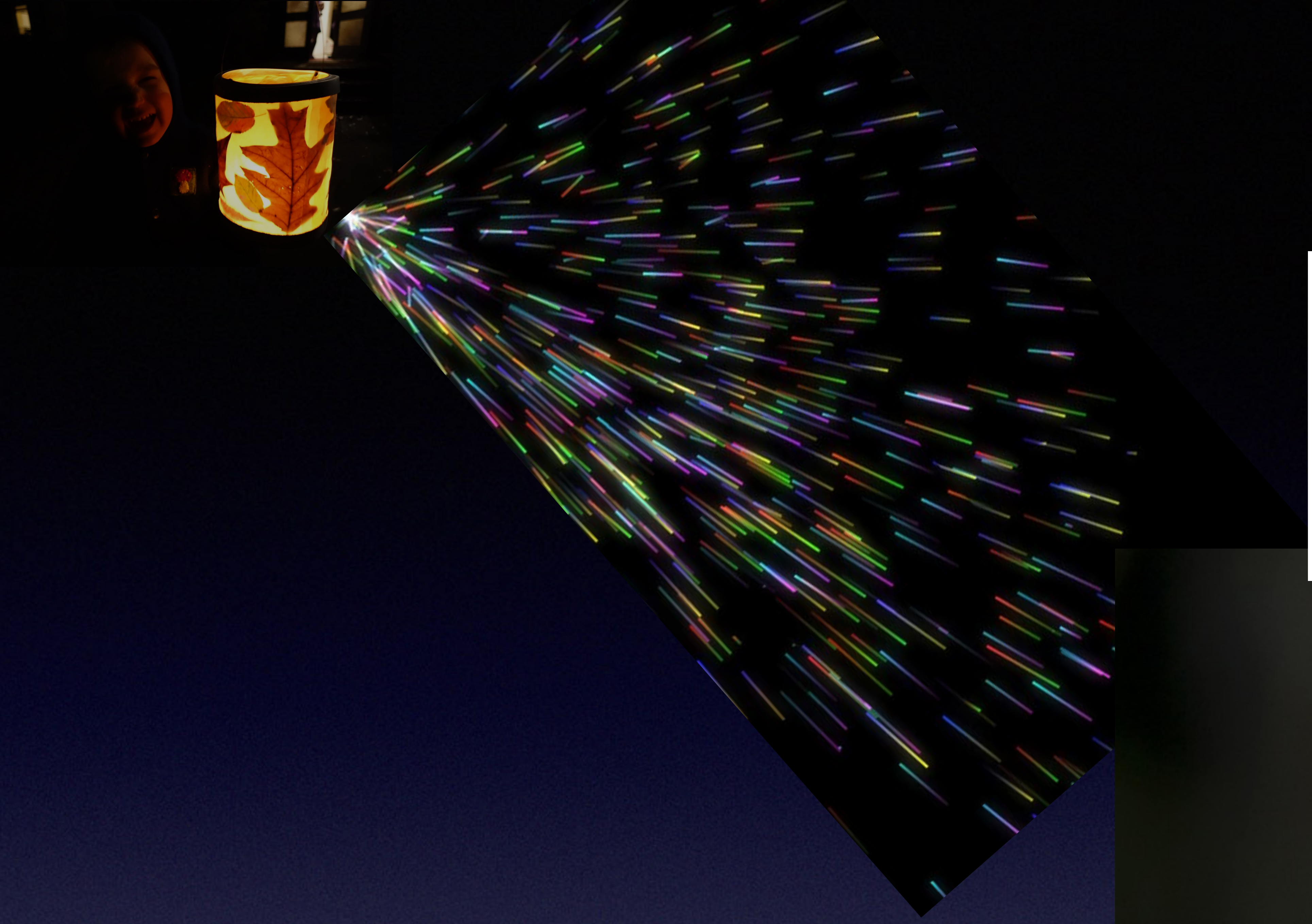
19th century: **matter** consists of particles, **forces** are mediated by space-filling fields

20th century: Everything is a field.

More precisely: everything is a **quantum field**

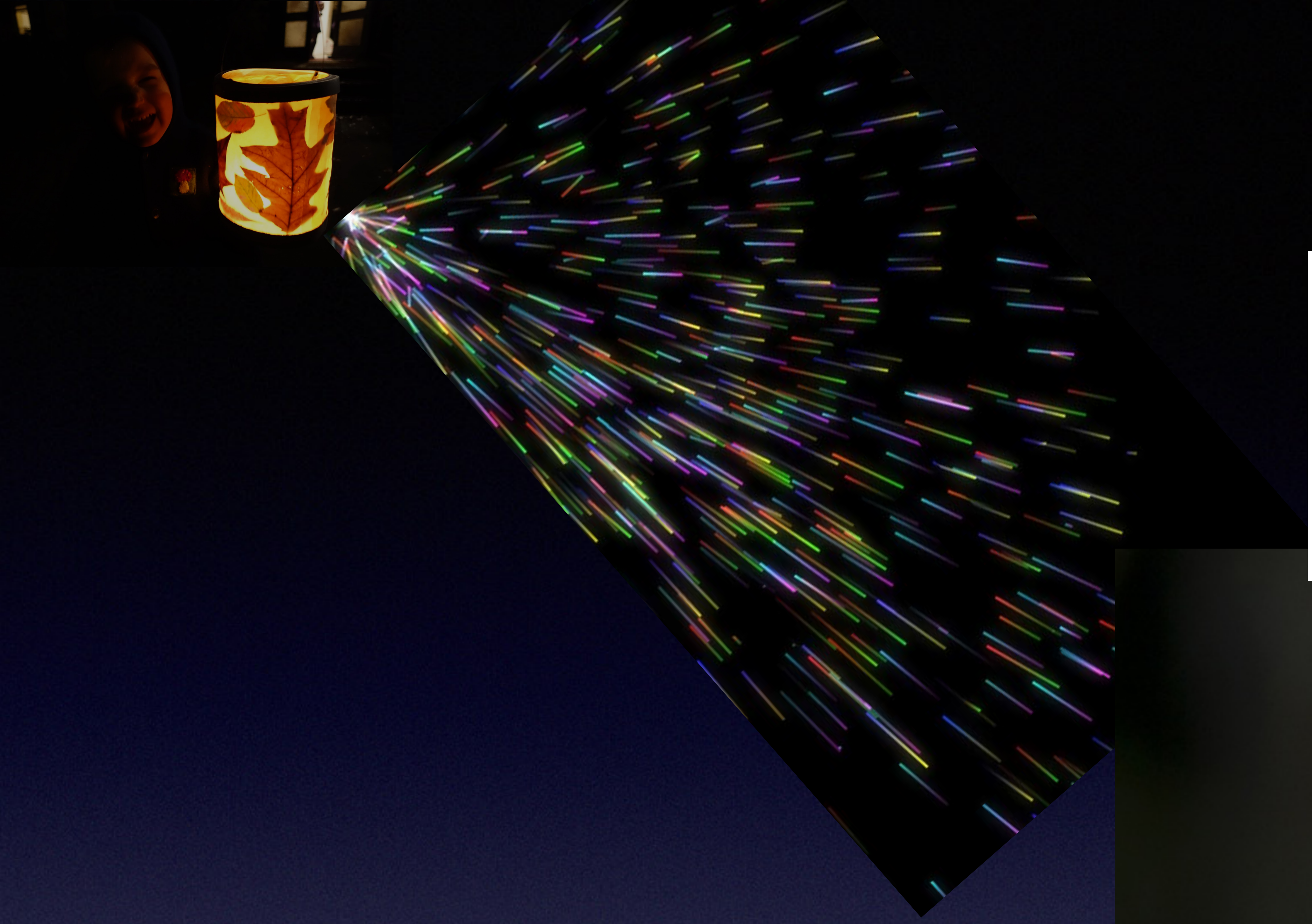
Tim's QFT book





Journal Reference:

1. Nam Mai Phan, Mei Fun Cheng, Dmitri A. Bessarab, Leonid A. Krivitsky. **Interaction of Fixed Number of Photons with Retinal Rod Cells**. *Physical Review Letters*, 2014; 112 (21) DOI: 10.1103/PhysRevLett.112.213601



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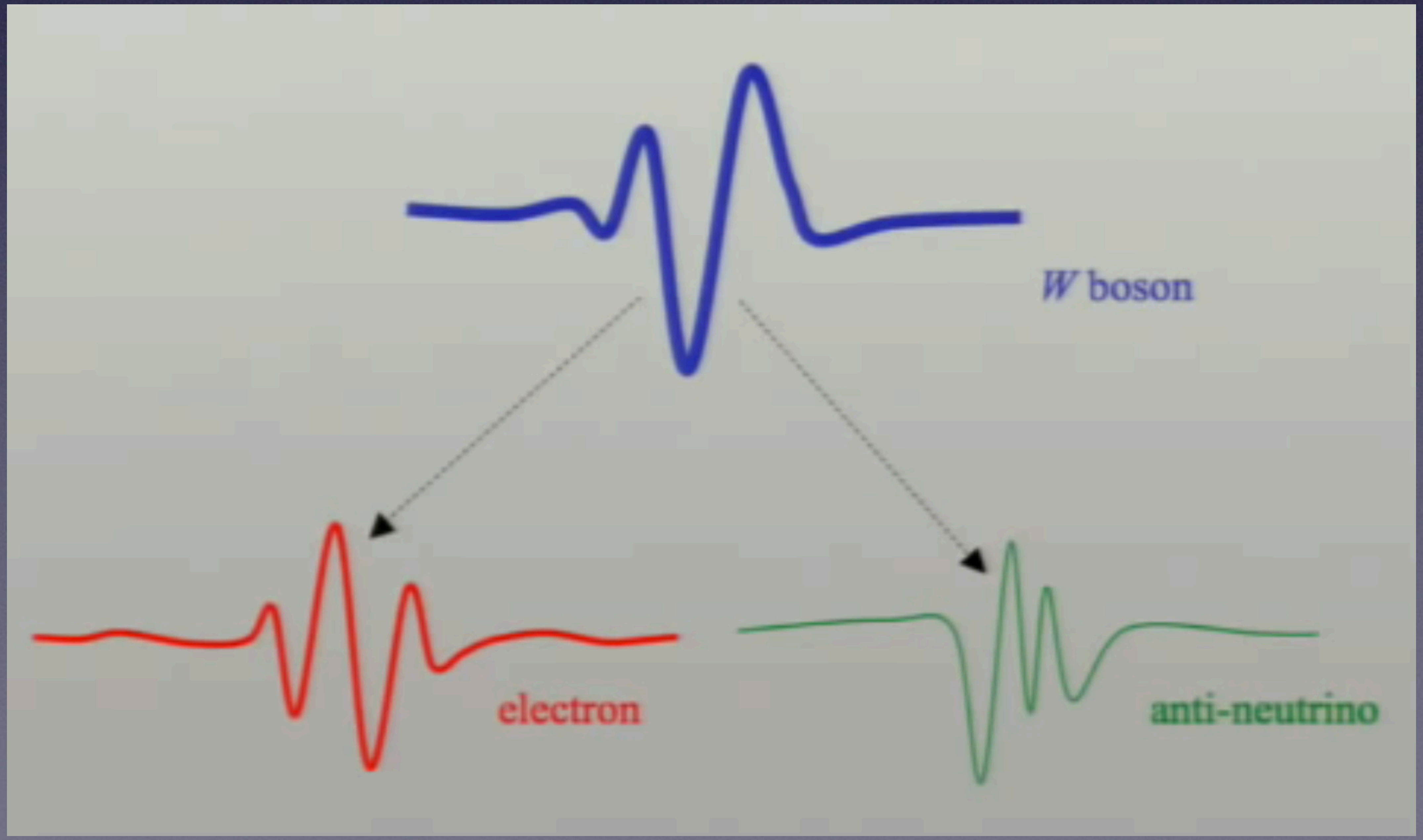
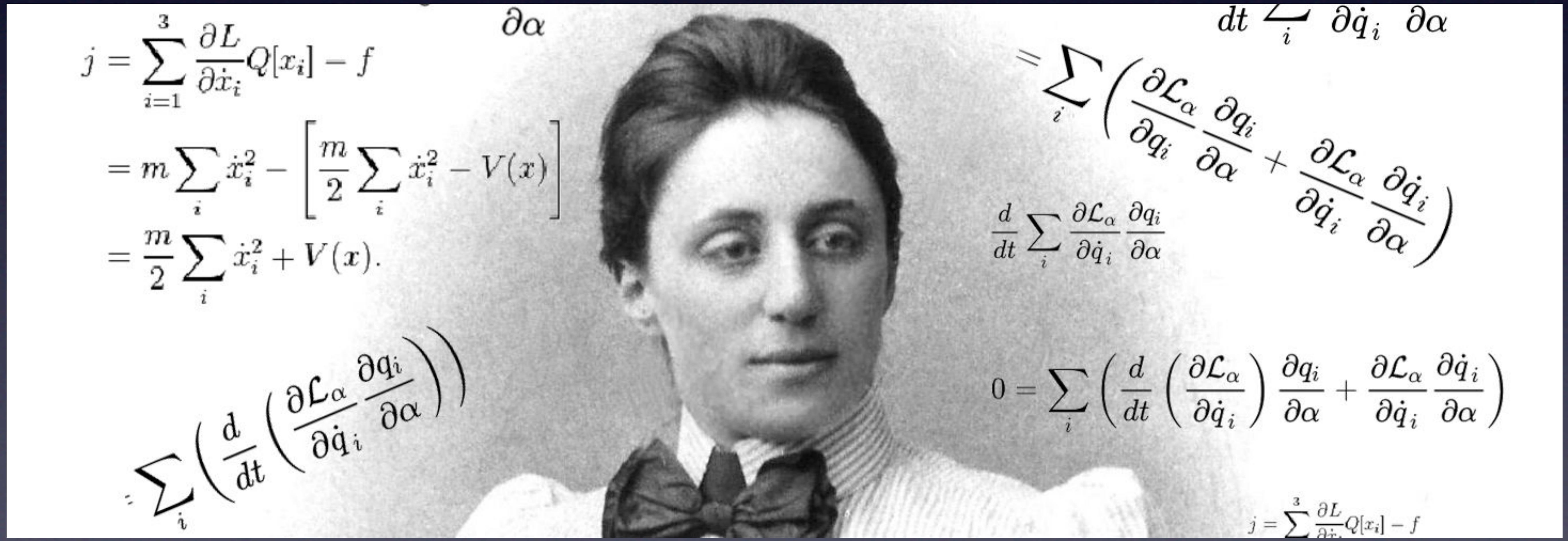
- Particles is what we see. Fields are what reality is made of.

Minimal energy to get field vibrating
= mass of particle

Couplings between different fields
= particle interactions



(global) symmetries = conservation laws

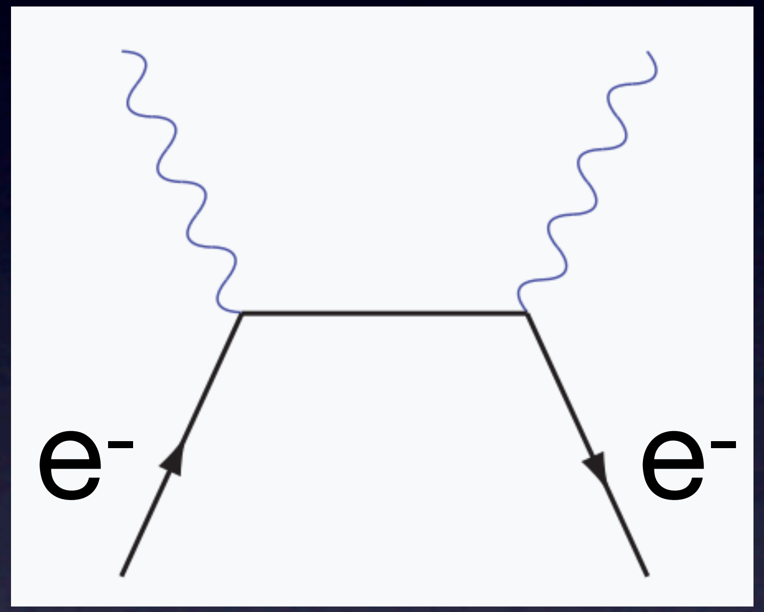


Emmy Noether “Invariante Variationsprobleme” (1918)

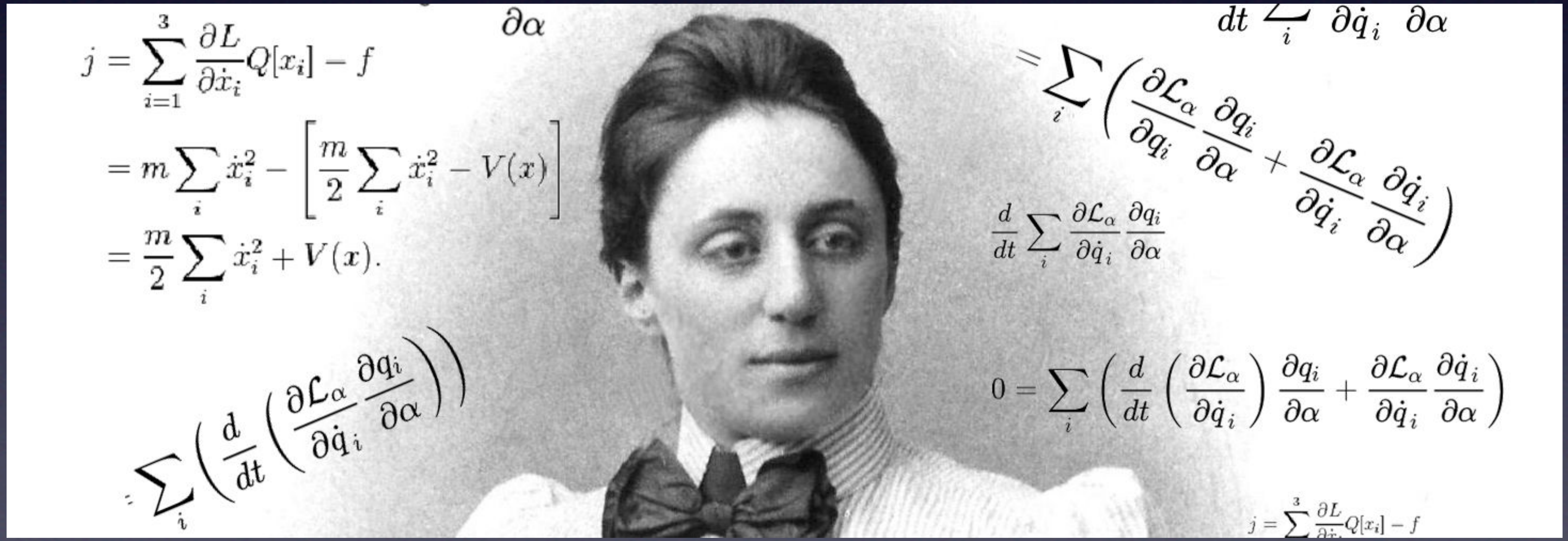
$$\psi(x) \to e^{i\alpha}\psi(x)$$

(global) symmetries = conservation laws

$$e^-\gamma \to e^-\gamma$$



(charge conservation)



Emmy Noether “Invariante Variationsprobleme” (1918)

but not

~~$e^-\gamma \to e^+\gamma$~~

local symmetries = predict form of interactions

The SM is a

$$SU(3)_c \times SU(2)_W \times U(1)_Y$$

gauge theory

required for consistency of massless spin 1:
QM + relativity impose this structure

$U(1)$ **example**

$$\mathcal{L} = \overline{\psi}(x)\,\gamma^\mu(i\partial_\mu - m)\,\psi(x)$$

$$\psi(x) \to e^{i\alpha(x)}\,\psi(x)$$

$$\mathcal{L} = \overline{\psi}(x)\,\gamma^\mu(i\partial_\mu - eA_\mu(x) - m)\,\psi(x)$$

local symmetries = predict form of interactions

The SM is a

$$SU(3)_c \times SU(2)_W \times U(1)_Y$$

gauge theory

required for consistency of massless spin 1:
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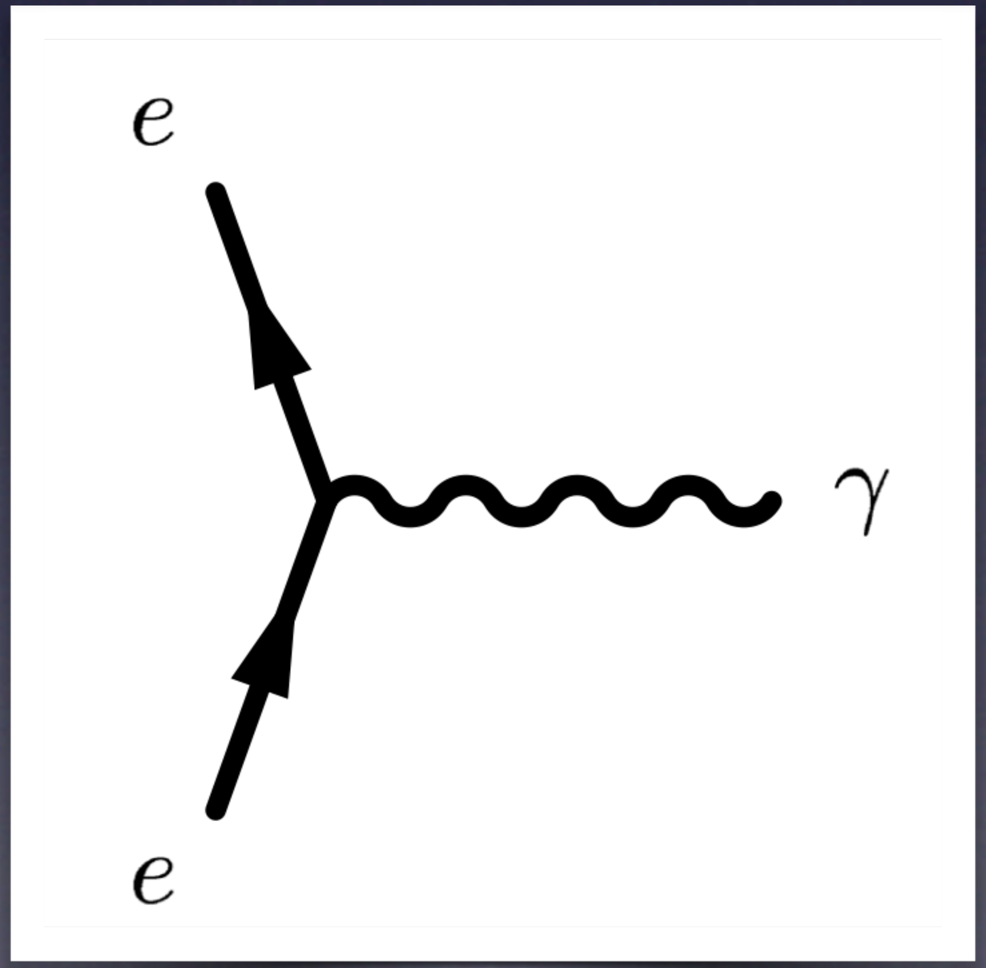
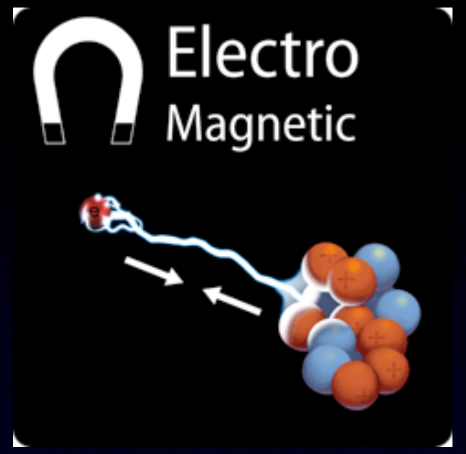
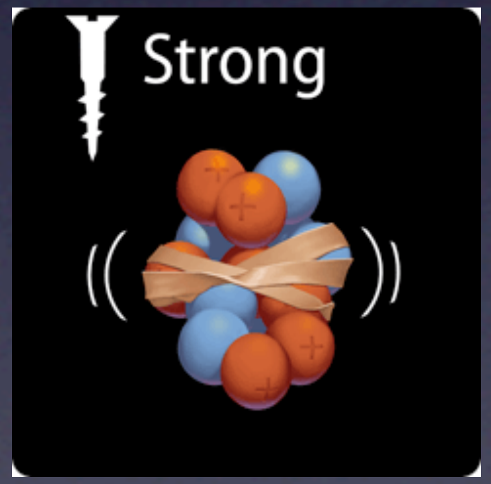


$= ie\gamma^\mu$

Four fundamental forces

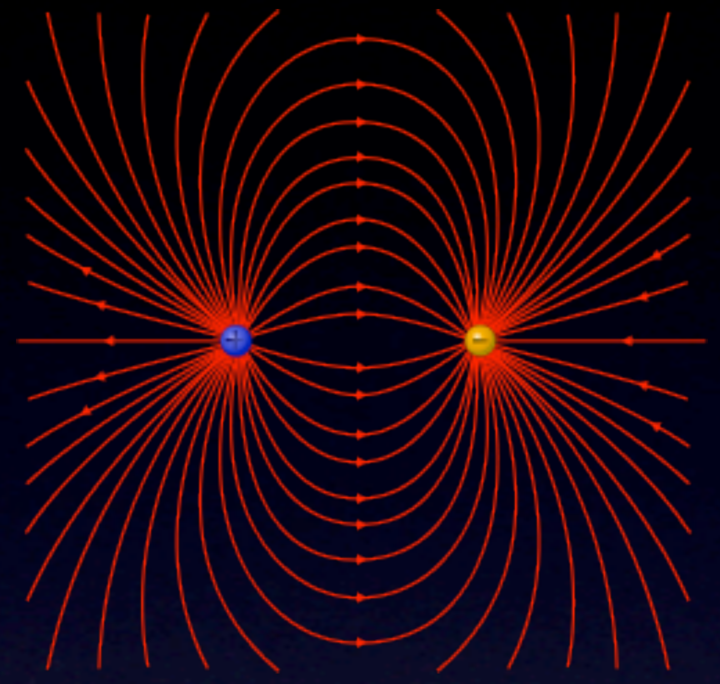
		local Symmetry	Particle	
Electromagnetic		*SU(2)xU(1)*	photon	
Weak			W,Z	spin=1
Strong		*SU(3)* color	gluon	
Gravitational		space-time diffeomorphism local SO(1,3)	graviton	spin=2

Phases of the Fundamental Interactions

Mass of force carrier

$M = 0$ {

Coulomb phase → Gravity, Electro-magnetism



$$F \propto \frac{1}{r^2}$$

Confined → **Strong**

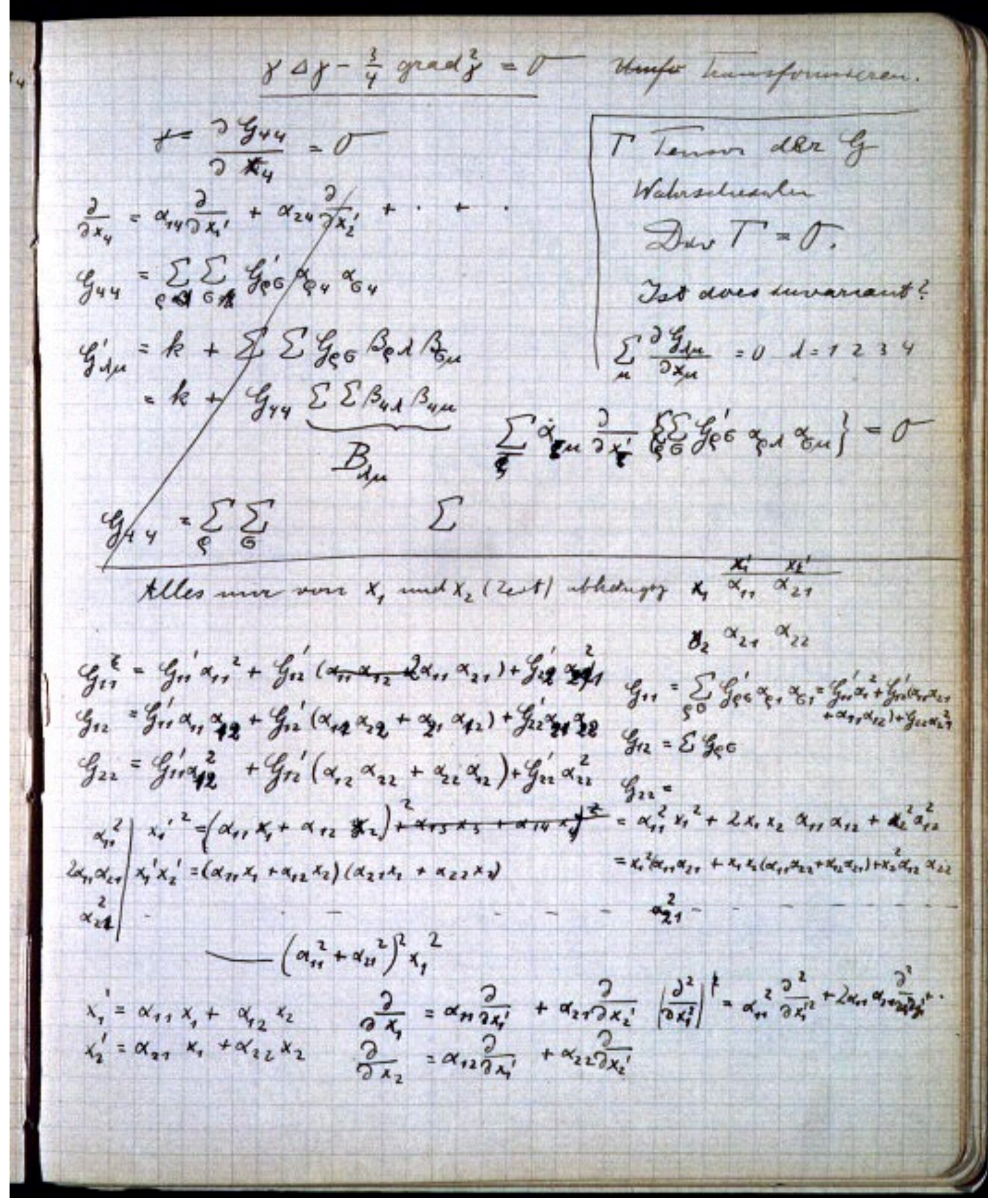
$M \neq 0$

Screened

Weak

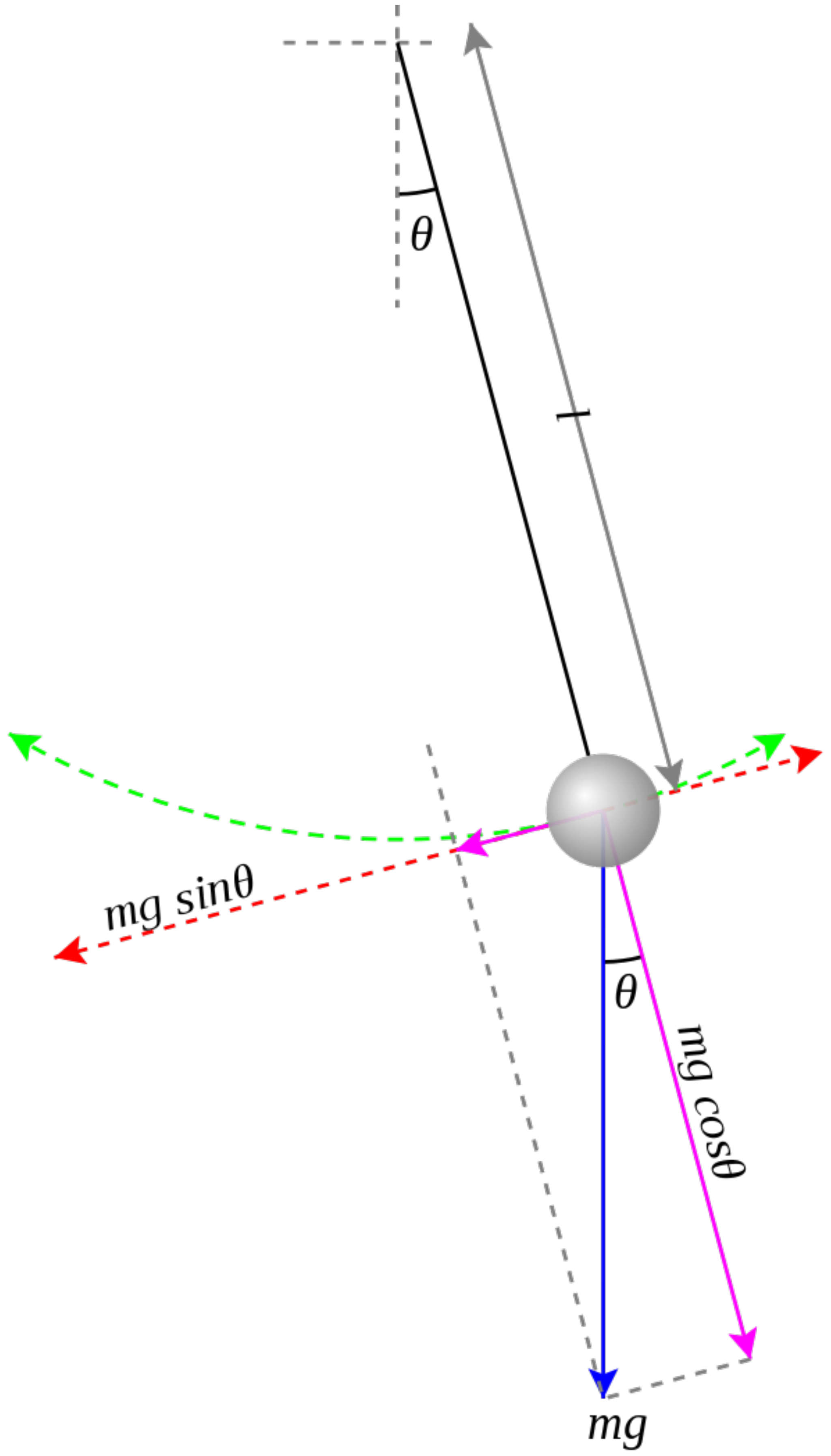
$$F \propto e^{-Mr}\left(\frac{1}{r^2} + \frac{M}{r}\right),$$

Lagrangians



$$L = \frac{m}{2}\dot{x}^2 - \frac{1}{2}m\omega^2 x^2$$

$$L = \frac{m}{2}\dot{\theta}^2 - mg(1 - \cos\theta) \approx \frac{m}{2}\dot{\theta}^2 - \frac{1}{2}mg(\theta^2 - \frac{1}{12}\theta^4 + \ldots)$$



- Lagrangians reveal symmetries by remaining **invariant** under transformations
- At low energies (small oscillations), **accidental symmetries** can appear

https://www.physicswithelliot.com/pendulum-graph

Matter (1st draft): relativistic field equation

Massive spin 1/2 particle: $\psi(x)$ **(4 component Dirac spinor)**

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \psi \qquad\qquad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

Matter (1st draft): relativistic field equation

Massive spin 1/2 particle: $\psi(x)$ **(4 component Dirac spinor)**

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \psi \qquad\qquad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{pmatrix}$$

Matter (1st draft): relativistic field equation

Massive **spin 1/2** **particle:** $\psi(x)$ **(4 component Dirac spinor)**

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \psi \qquad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{pmatrix}$$

Dirac equation: from Lagrangian

$$0 = \delta \mathscr{L} = \delta\psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \psi$$

$$\left(i\gamma^\mu \partial_\mu - m \right) \psi = 0$$

$$E = \begin{cases} +\sqrt{p^2c^2 + m^2c^4} & \text{matter} \\ -\sqrt{p^2c^2 + m^2c^4} & \text{antimatter} \end{cases}$$

Predicts anti-particle exist: positron (discovered by Anderson 1932)

How does an electron couple to photons ?

Model building the SM

- Lagrangians are invariant (equation of motions are covariant)

- We impose global space-time symmetries, like Lorentz and space-time translation invariance

- We build the most general Lagrangian that is allowed by **local** symmetries (gauge symmetries)

- We read of the interactions and translate them to Feynman diagrams, with this we can calculate observables like cross-sections, decay widths

$$\mathscr{L} = \overline{\psi}(x)\,\gamma^{\mu}(i\partial_{\mu} - m)\,\psi(x)$$

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$$A_{\mu}(x) \to A_{\mu}(x) - \frac{1}{e}\partial_{\mu}\alpha(x)$$

U(1) gauge redundancy

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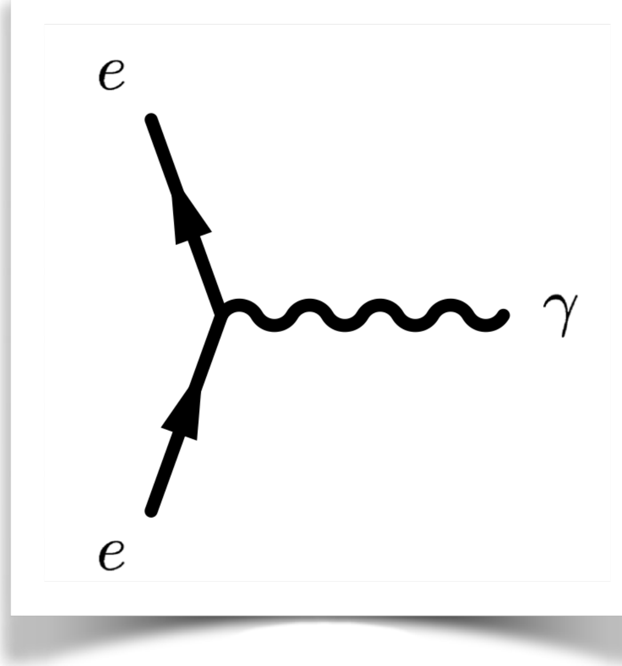

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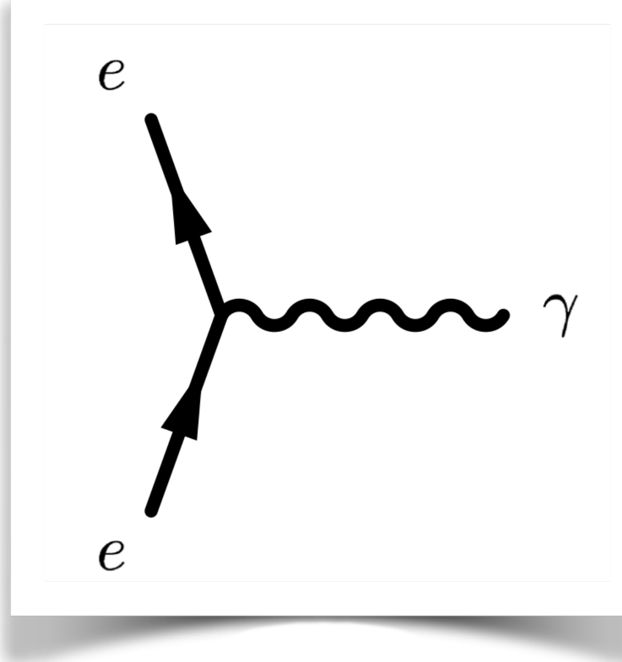
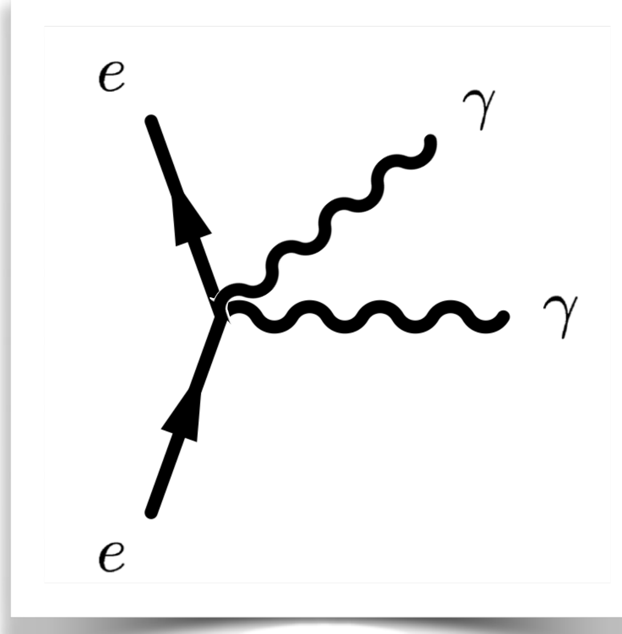


but **not** e.g.

$$\mathscr{L}_{wrong} = \overline{\psi}(x)\,\psi(x)\,e^2A_{\mu}(x)A^{\mu}(x)$$



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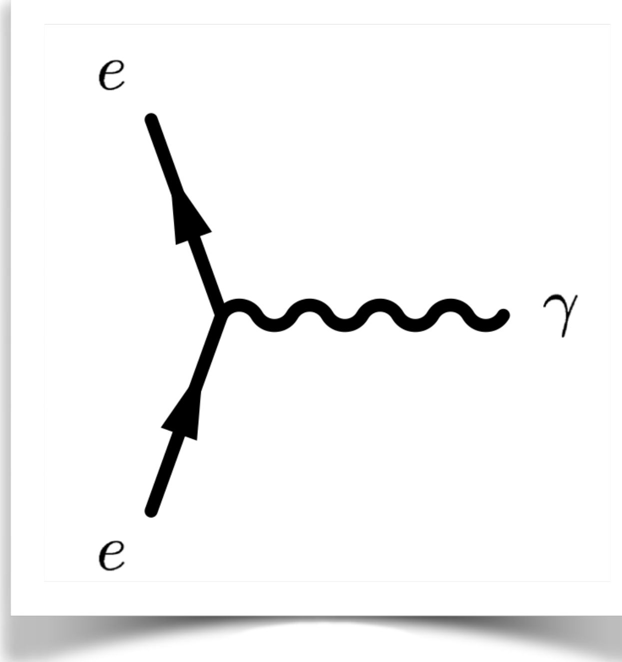

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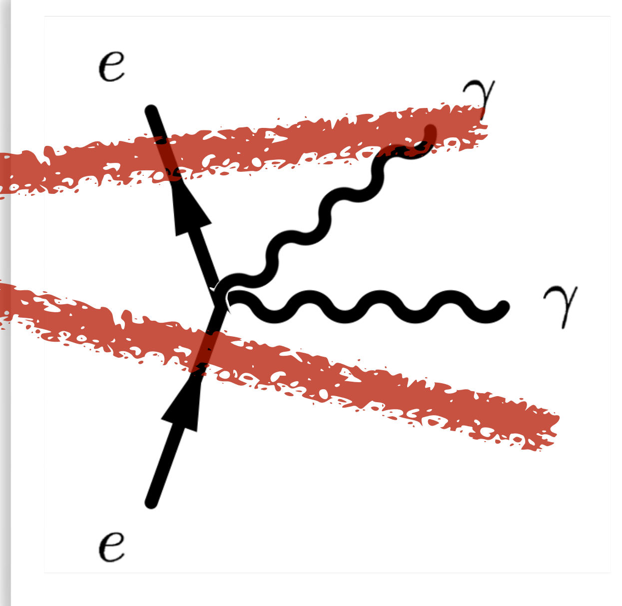


Kinetic term for the photon

We can recycle the Maxwell action, i.e. classical electromagnetism. It can be quantized to give us quantum electrodynamics. The relativistic form of the Lagrangian is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$ Field strength tensor

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$ Lagrangian invariant: $A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$

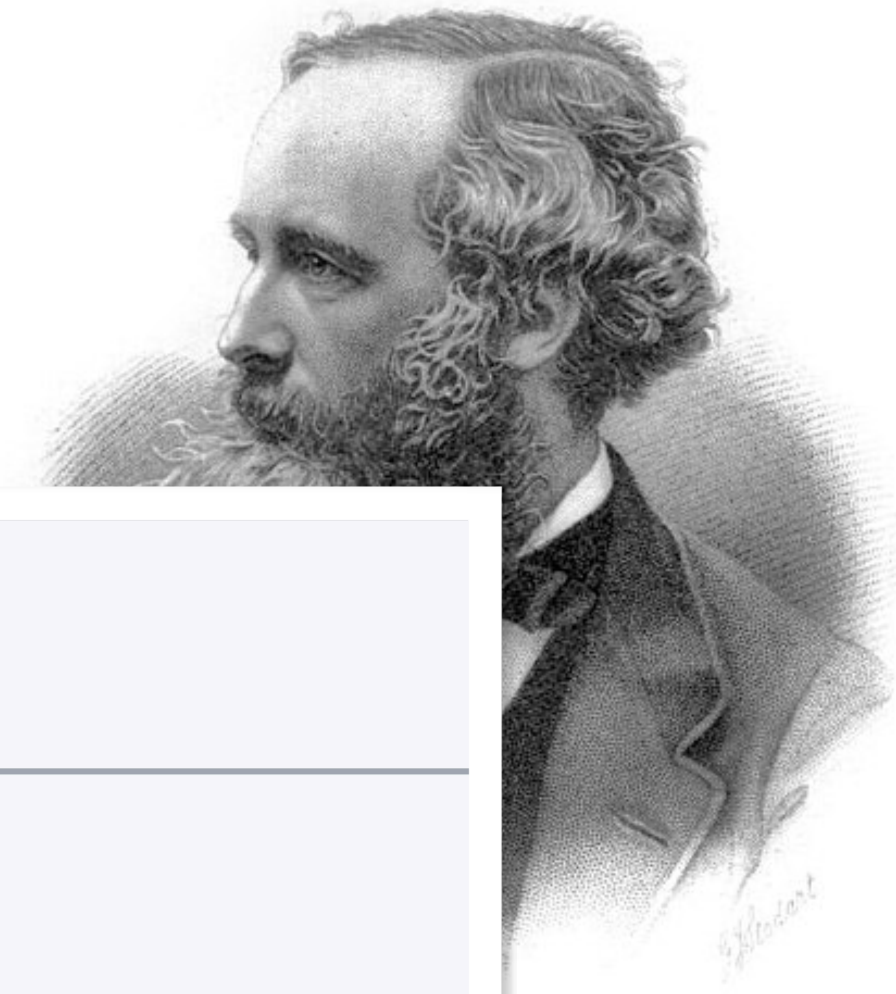
$$F^{\mu\nu} = \begin{pmatrix} 0 & -\vec{E}_x & -\vec{E}_y & -\vec{E}_z \\ \vec{E}_x & 0 & -\vec{B}_z & \vec{B}_y \\ \vec{E}_y & \vec{B}_z & 0 & -\vec{B}_x \\ \vec{E}_z & -\vec{B}_y & \vec{B}_x & 0 \end{pmatrix}$$

equation of motion:

$$\partial_\mu F^{\mu\nu} = J^\nu$$

Note: Photons do not carry charge and do not interact with themselves.

Kinetic term for the photon



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$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$$

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Non-abelian gauge symmetry

Generalize the Maxwell theory from U(1) -> SU(N)

$$e^{i\alpha} \implies e^{i\sum_a \alpha^a T^a_{ij}}$$

Now consider the presence of multiple massless spin 1 force carriers, such as the **8** gluons responsible for strong interactions or the **3** bosons involved in weak interactions.

Goal: generalize Maxwell's equations to accommodate these multiple carriers!

Instead of one Dirac field, consider N-dimensional vector of Dirac fields: $\overrightarrow{\psi}$

$$\overrightarrow{\psi} \longrightarrow U\,\overrightarrow{\psi}$$

U : matrix of **SU(N)** "**N** dimensional special unitary group" $U^\dagger U = \mathbf{1}_{N\times N}, \qquad \det(U) = 1$

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Invariants: $\vec{\psi}^\dagger \vec{\psi} \longrightarrow \vec{\psi}^\dagger U^\dagger U\,\vec{\psi} = \vec{\psi}^\dagger \vec{\psi}$

Using invariants as building blocks for our Lagrangian. $\mathscr{L} = \vec{\psi}^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m\right) \vec{\psi}$

Example: SU(2) with $U = \exp(i\alpha^a \sigma^a)$, $\quad [\sigma^a, \sigma^b] = i2\epsilon^{abc}\sigma^c$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli matrices

e.g.

$$g_1 = e^{i\alpha\sigma^1} = \begin{pmatrix} \cos(\alpha) & i\sin(\alpha) \\ i\sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

$$g_3 = e^{i\beta\sigma^3} = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{-i\beta} \end{pmatrix}$$

Note: SU(N) is **non-abelian** because two elements do not generally commute.

Exercise: Show that $g_1 g_3 - g_3 g_2 = [g_1, g_3] = \begin{pmatrix} 0 & -2\sin\alpha\sin\beta \\ 2\sin\alpha\sin\beta & 0 \end{pmatrix} \neq 0$

Matter Lagrangian

Generalize quantum electrodynamics

U(1)

$$\mathscr{L} = \overline{\psi}(x)\,\gamma^{\mu}(i\partial_{\mu} - eA_{\mu}(x) - m)\,\psi(x)$$

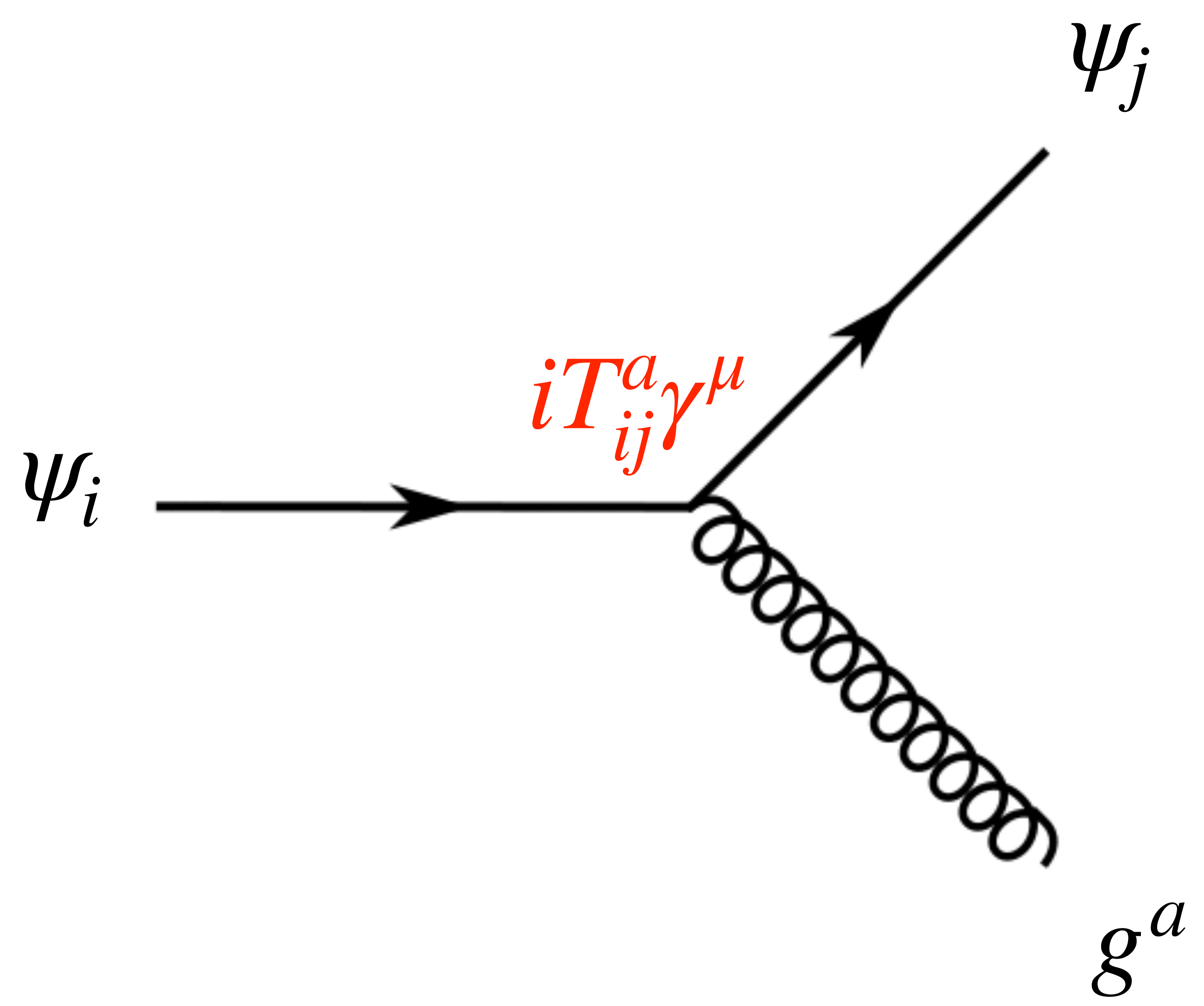
SU(N)

$$\mathscr{L} = \overline{\psi}_i(x)\,\gamma^{\mu}(i\partial_{\mu} - gA^{a}_{\mu}(x)T^{a}_{ij} - m)\,\psi_j(x)$$

Local SU(N) invariance

$$\psi(x) \rightarrow U(x)\,\psi(x) = e^{i\alpha^{a}(x)T^{a}}\,\psi(x)$$

$$A_{\mu}(x) \rightarrow U(x)A_{\mu}(x)U(x)^{\dagger} - \frac{i}{g}(\partial_{\mu}U(x))U(x)^{\dagger}$$



Matter Lagrangian

Generalize quantum electrodynamics

U(1) → **SU(N)**

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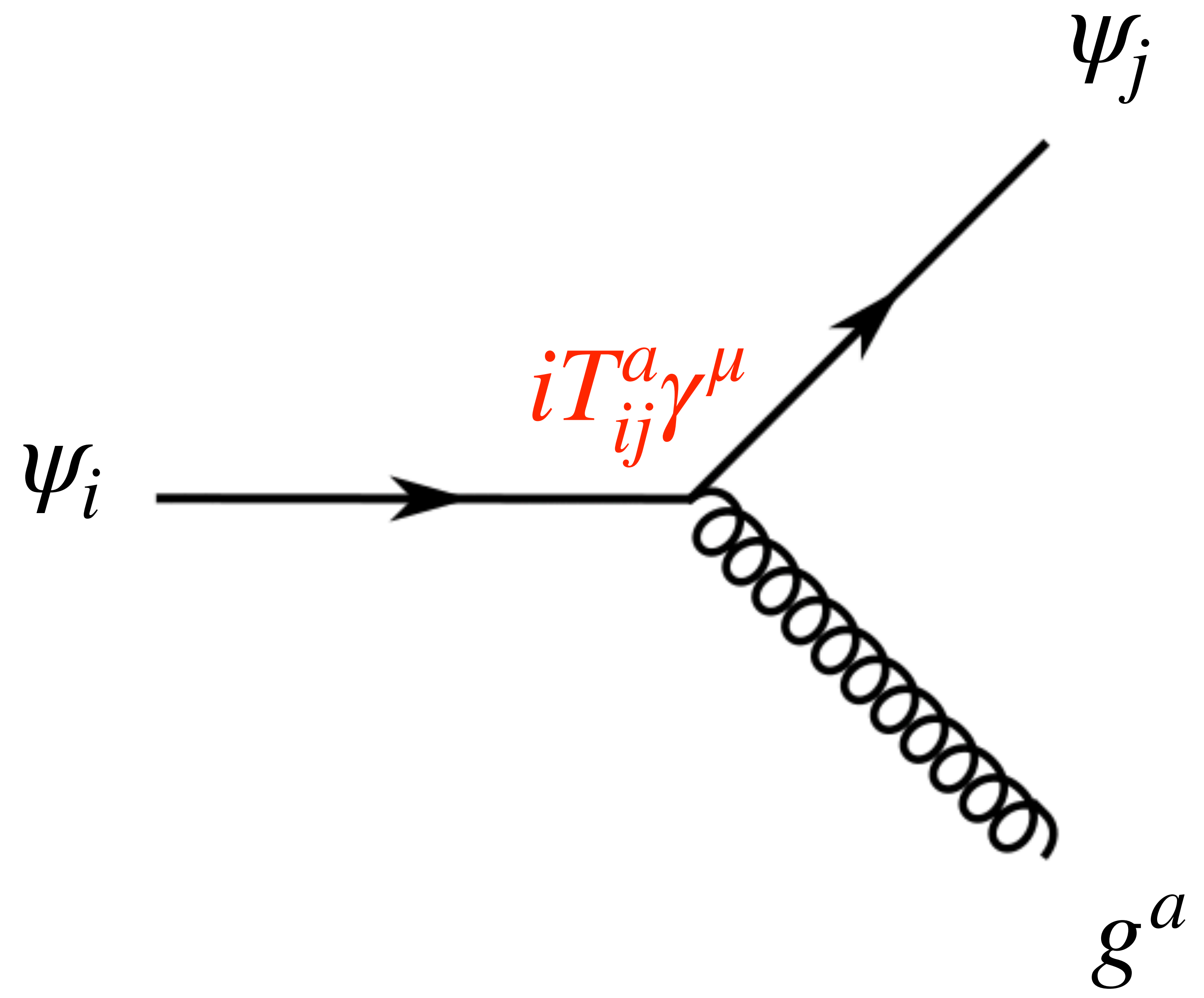
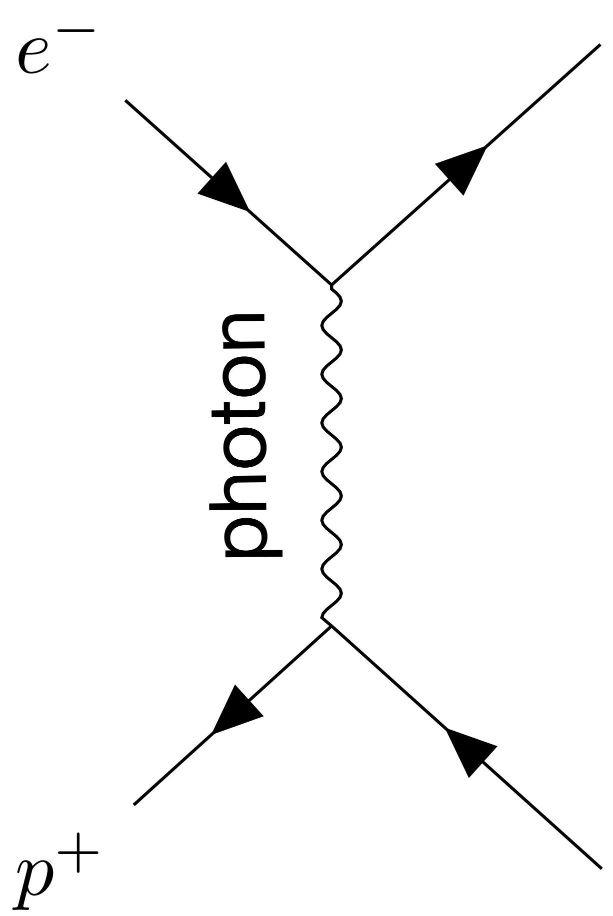


Exercise: show invariance of (1)

Example: Coulomb potential

electron-proton potential is attractive

QED



$$= (-ie)^2(\bar{u}\gamma^\mu u)\frac{-i\eta^{\mu\nu}}{k^2}(\bar{v}\gamma_\nu v).$$

$$\Rightarrow \quad V(r) = -\frac{e^2}{4\pi r}$$

Example: Coulomb potential

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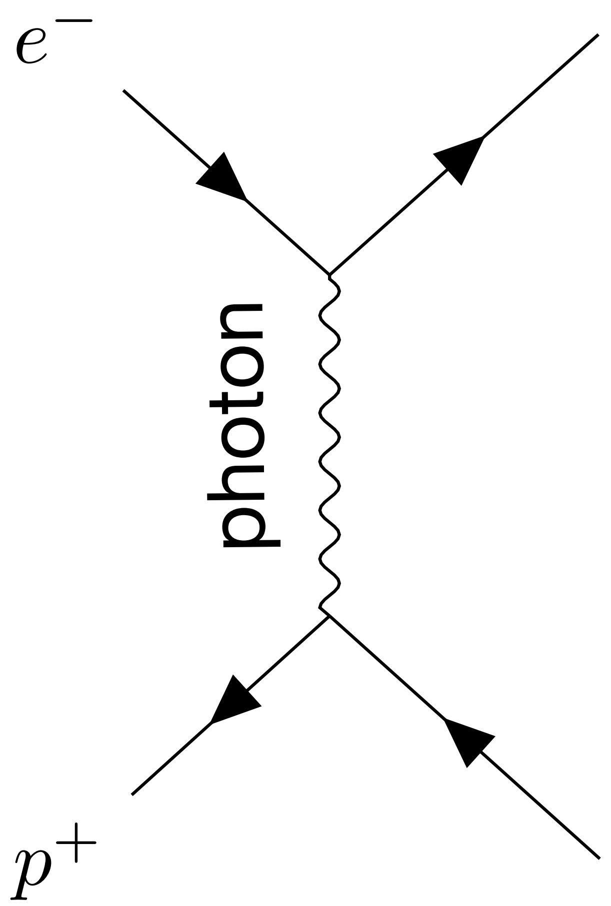

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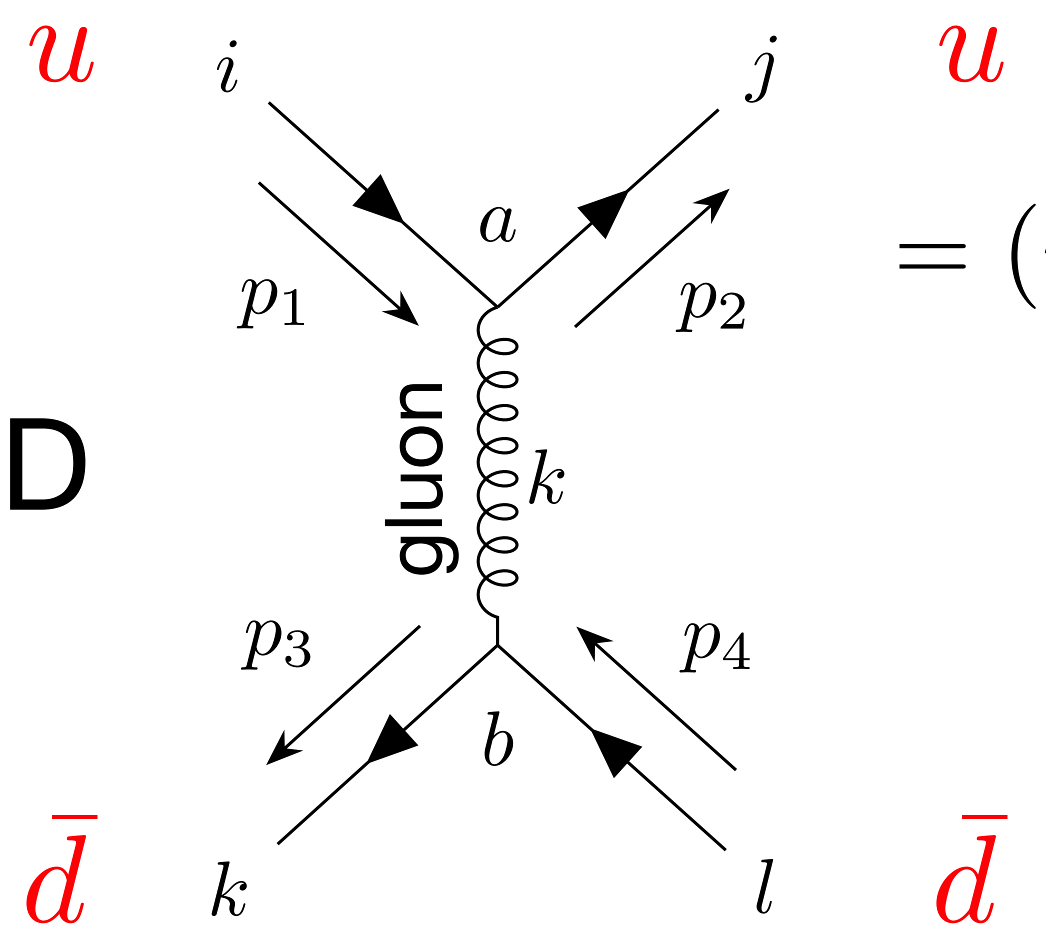
electron-proton potential is attractive

$$V(r) = -\frac{e^2}{4\pi r}$$

quark-quark potential is only attractive for color neutral combinations *

QCD



$$= (ig_s)^2 T^a_{ji} T^a_{kl} \times \bar{u}_j \gamma^\mu u_i \frac{-i\eta_{\mu\nu}}{k^2} \bar{v}_k \gamma^\nu v_l$$

* QCD is strongly coupled at low energies, perturbative calculations are not reliable

Example: Coulomb potential

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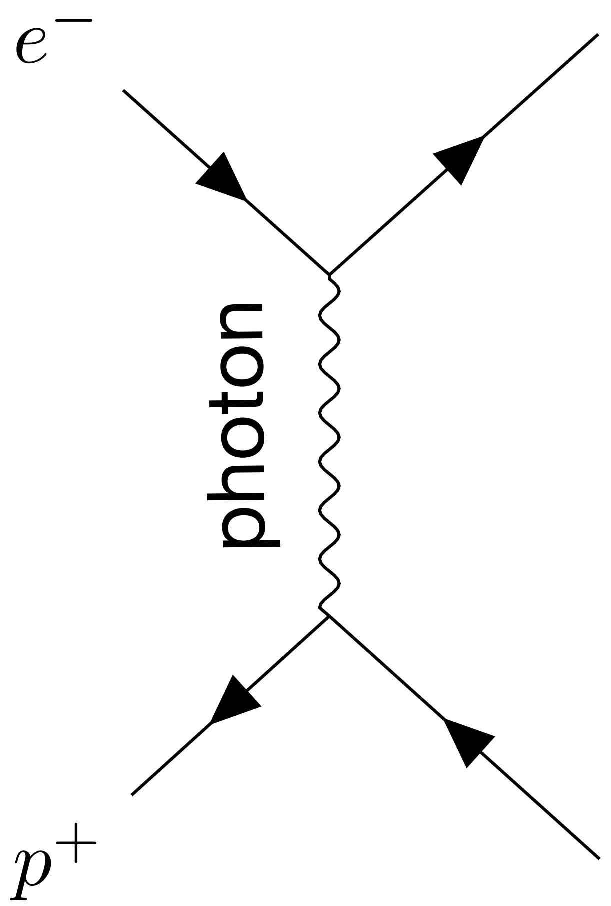

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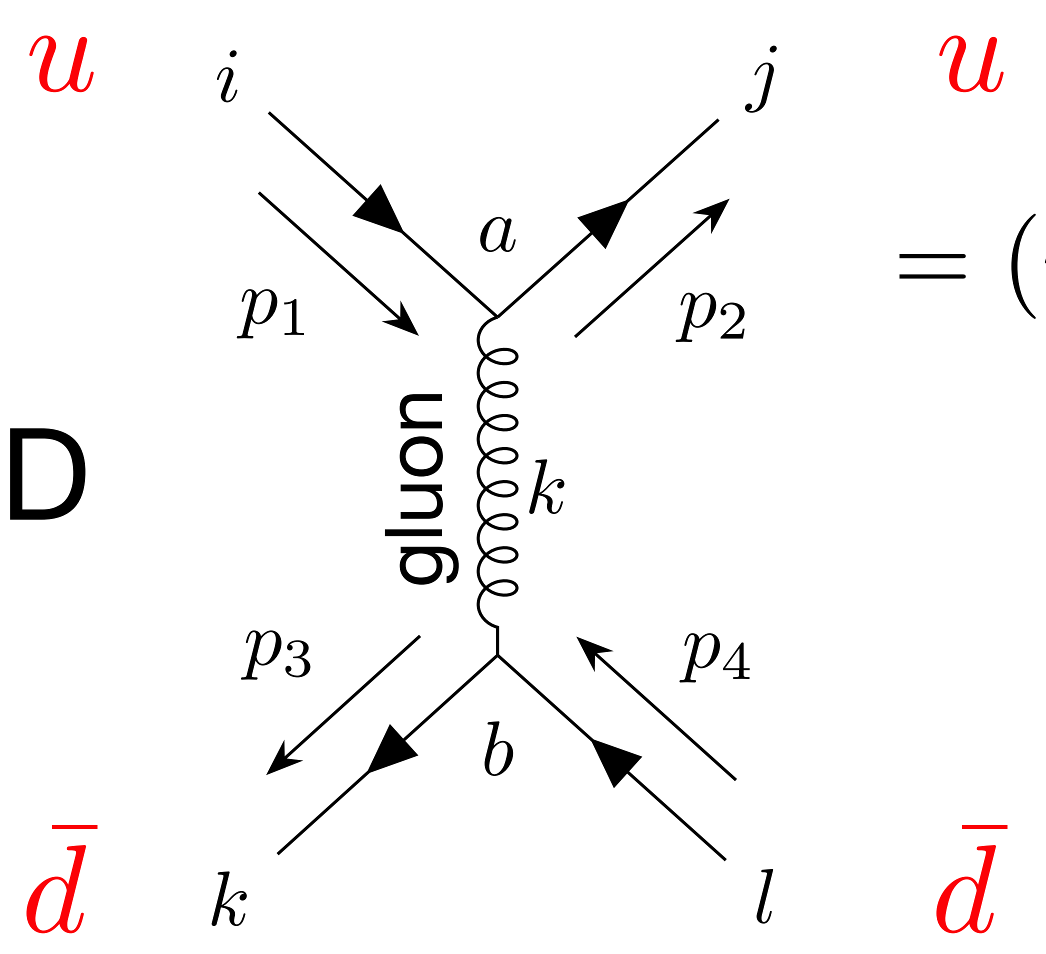

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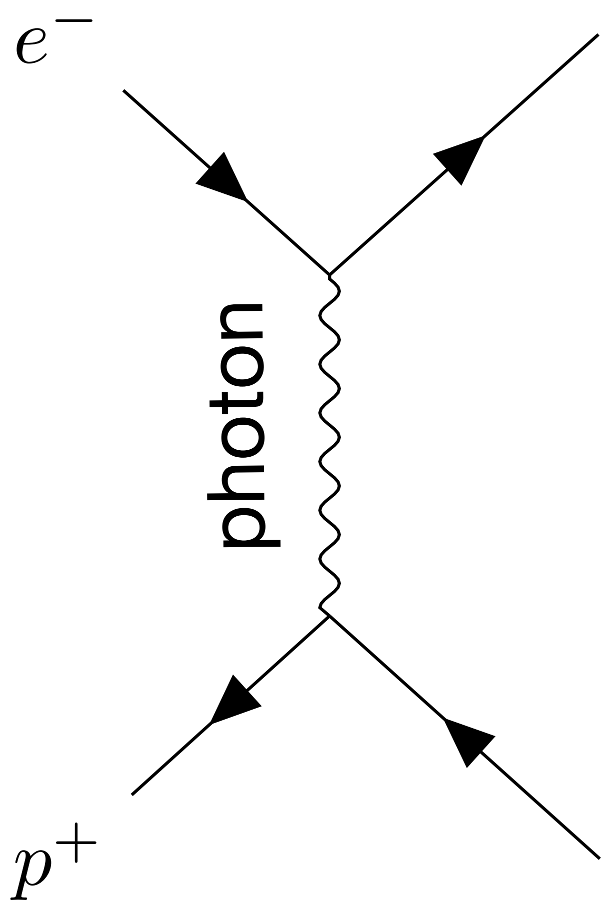

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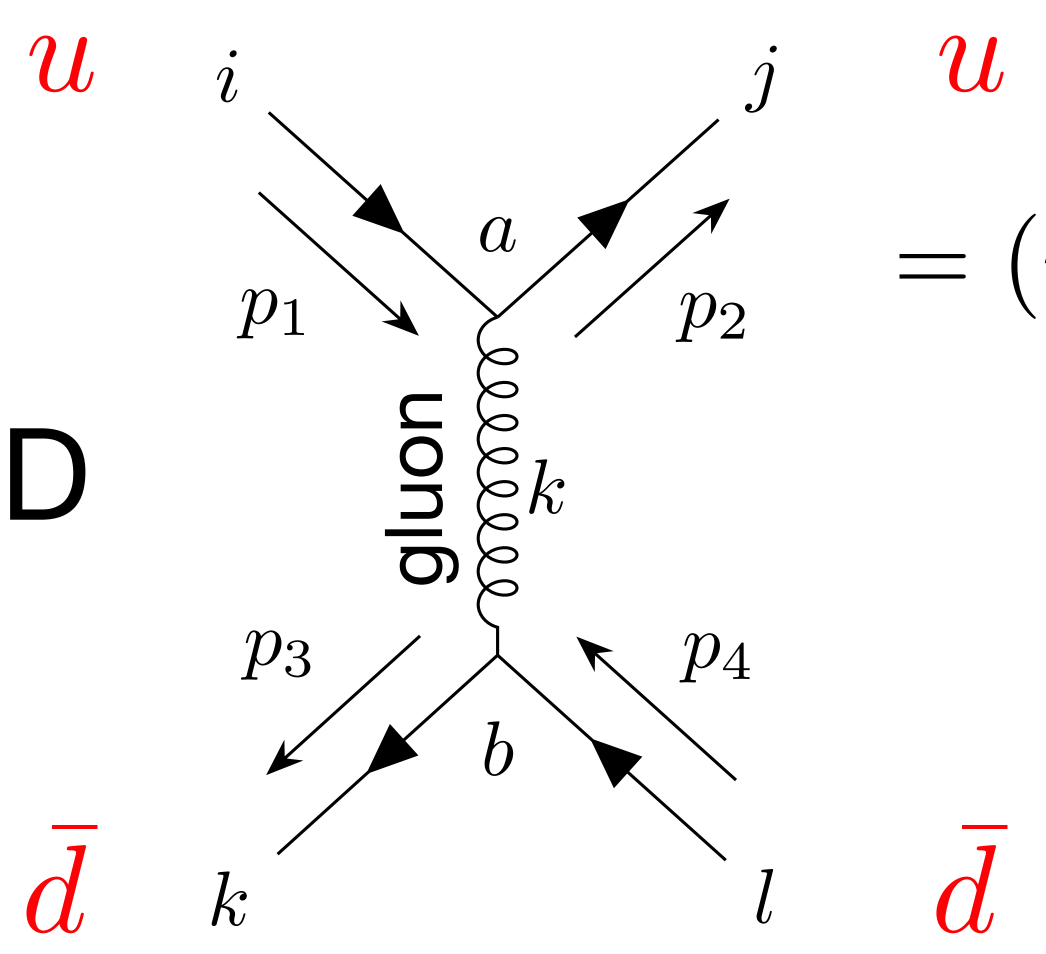

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$$V(r) = \frac{1}{6}\frac{g_s^2}{4\pi r} \qquad \text{(color octet)} \qquad u(r)\bar{d}(g) \to u(r)\bar{d}(g)$$

$$V(r) = -\frac{4}{3}\frac{g_s^2}{4\pi r}. \qquad \text{(color singlet)} \qquad u(r)\bar{d}(r) \to u(b)\bar{d}(b)$$

Kinetic term for SU(N) gauge boson

We can cannot recycle the Maxwell action. The Lagrangian would not be invariant under a local SU(N) transformation

$$A_\mu(x) \to U(x)A_\mu(x)U(x)^\dagger - \frac{i}{g}(\partial_\mu U(x))U(x)^\dagger$$

Field strength now contains a non-abelian contribution

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

It transforms homogeneously

$$F_{\mu\nu} \to U(x)\, F_{\mu\nu}\, U^{-1}(x)$$

and we can build an invariant Lagrangian

$$\mathscr{L} = -\frac{1}{4}\mathrm{Tr}(\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}) = \ldots + \mathrm{gAAA} + \mathrm{g}^2\mathrm{AAAA}$$

Note: Gluons carry colour charge and do interact with themselves.

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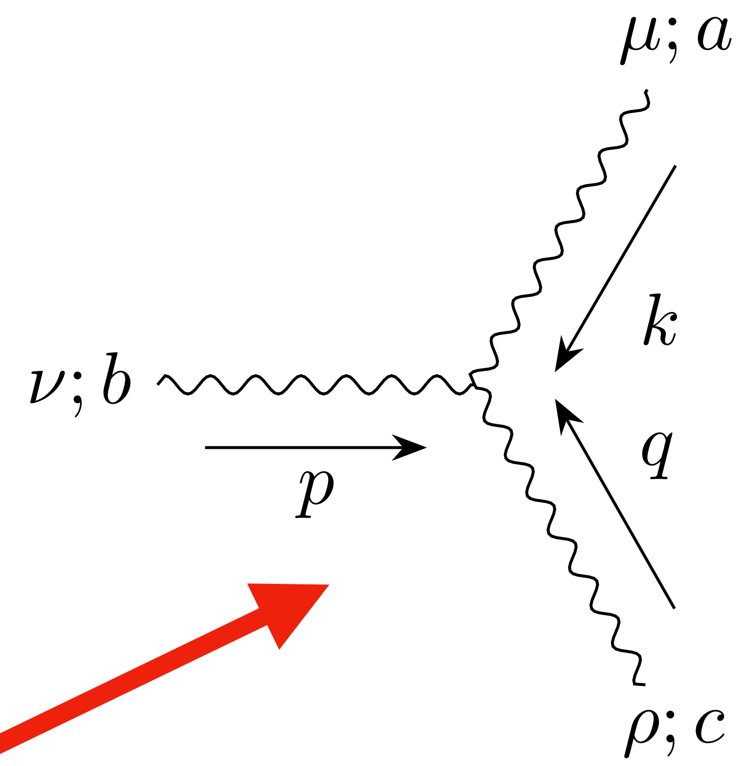

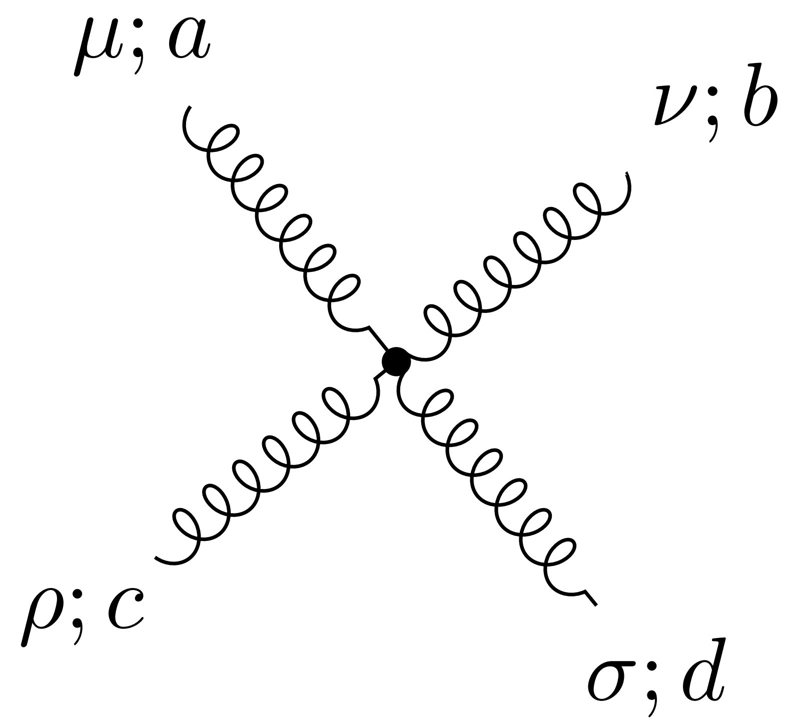

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How can we discover the Lagrangian of the universe?

We need experiments! -> next lecture