Standard Model 2/4

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CERN, 7/2024

Recap

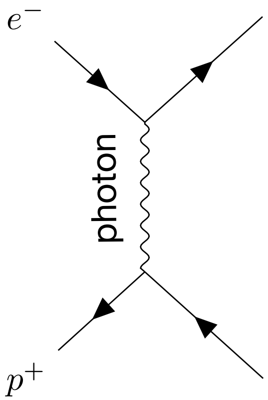
- Can you show why the photon (or the gluon) turns out to be massless in a gauge invariant theory?

- How many polarizations does a photon or a gluon have? (*Hint*: it has spin 1 and travels with the speed of light). How many entries are in the photon field $A^\mu(x), \quad \mu = 0,1,2,3$?

Example: Coulomb potential

electron-proton potential is attractive

QED



$$= (-ie)^2(\bar{u}\gamma^\mu u)\frac{-i\eta^{\mu\nu}}{k^2}(\bar{v}\gamma_\nu v).$$

$$V(r) = -\frac{e^2}{4\pi r}$$

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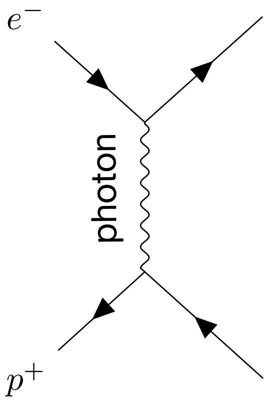

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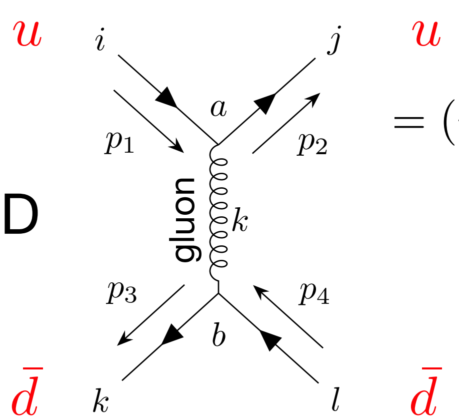

$$= (ig_s)^2 T^a_{ji} T^a_{kl} \times \bar{u}_j \gamma^\mu u_i \frac{-i\eta_{\mu\nu}}{k^2} \bar{v}_k \gamma^\nu v_l$$

quark-quark potential is only attractive for color neutral combinations *

* QCD is strongly coupled at low energies, perturbative calculations are not reliable

Example: Coulomb potential

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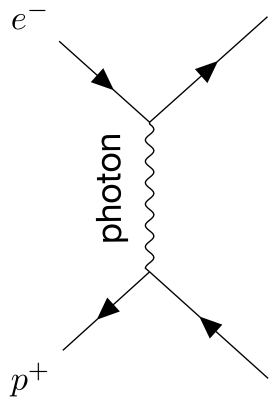

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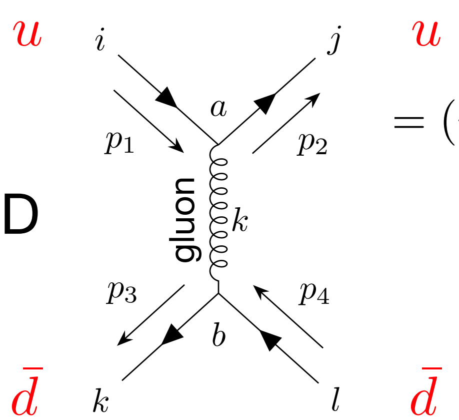

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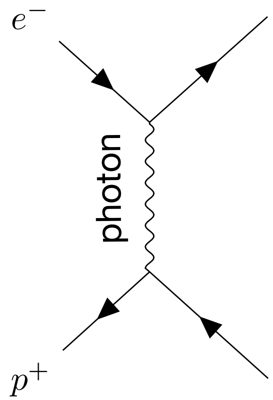

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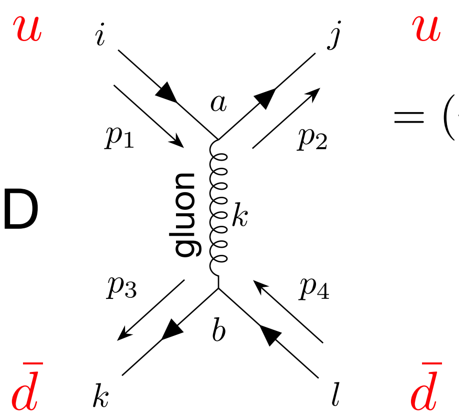

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quark-quark potential is only attractive for color neutral combinations *

$$V(r) = \frac{1}{6}\frac{g_s^2}{4\pi r} \qquad \text{(color octet)} \qquad u(r)\bar{d}(g) \to u(r)\bar{d}(g)$$

$$V(r) = -\frac{4}{3}\frac{g_s^2}{4\pi r}. \qquad \text{(color singlet)} \qquad u(r)\bar{d}(r) \to u(b)\bar{d}(b)$$

* QCD is strongly coupled at low energies, perturbative calculations are not reliable

Kinetic term for SU(N) gauge boson

We can cannot recycle the Maxwell action. The Lagrangian would not be invariant under a local SU(N) transformation

$$A_\mu(x) \to U(x) A_\mu(x) U(x)^\dagger - \frac{i}{g}(\partial_\mu U(x)) U(x)^\dagger$$

Field strength now contains a non-abelian contribution

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

It transforms homogeneously

$$F_{\mu\nu} \to U(x)\, F_{\mu\nu}\, U^{-1}(x)$$

and we can build an invariant Lagrangian

$$\mathscr{L} = -\frac{1}{4}\mathrm{Tr}(\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}) = \ldots + \mathrm{gAAA} + \mathrm{g}^2\mathrm{AAAA}$$

Note: Gluons carry colour charge and do interact with themselves.

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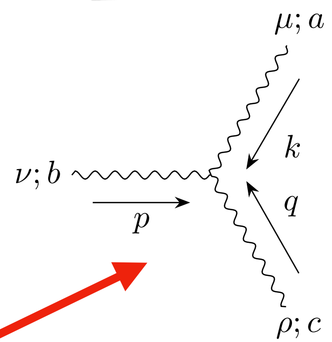

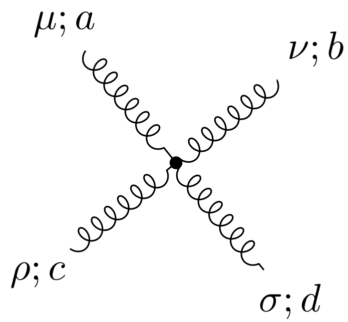

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$$\mathscr{L} = ?$$

Which theory is realized in nature? SU(N) ? U(1) ? Which particles?

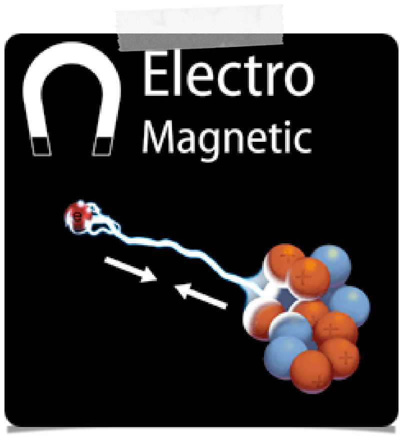
How can we discover the Lagrangian of the universe?

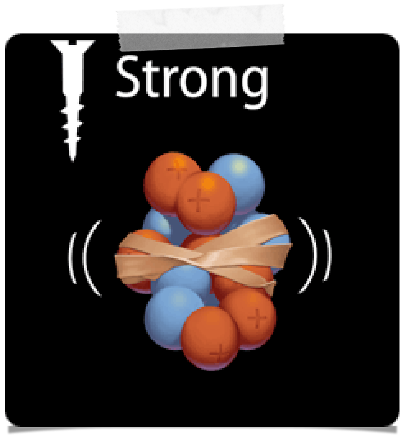
We need experiments!

The strong and the electromagnetic interactions

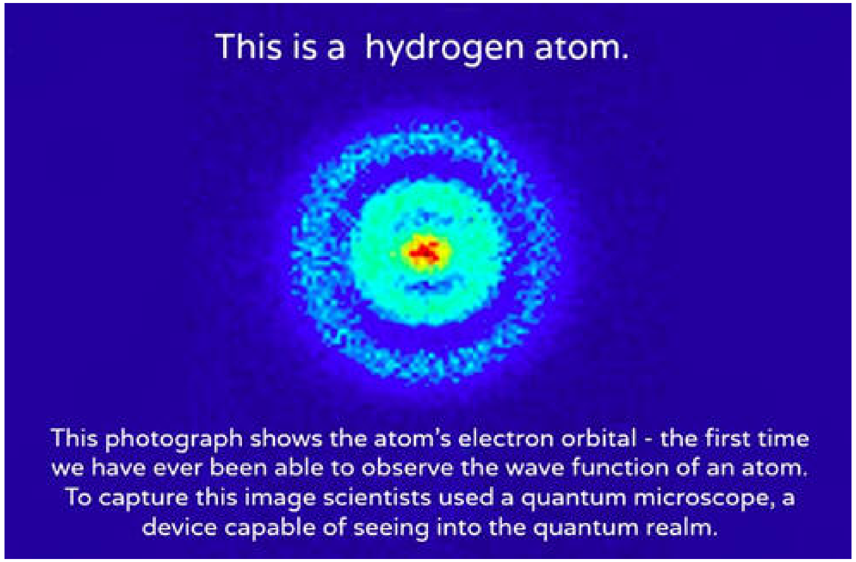
$U(1)_{em}$ **$SU(3)_C$**

- Why was evidence of electromagnetic interactions discovered before evidence of strong interactions?
- Why have we never seen a free quark, unlike electrons or protons?
- How can we test predictions about quarks if we don't observe them as free particles?





QED binds electrons and nuclei inside atoms and molecules



But quarks are fundamental objects, *not* the composite nuclei

$$p = (uud), \quad n = (udd), \ldots$$

Charges:

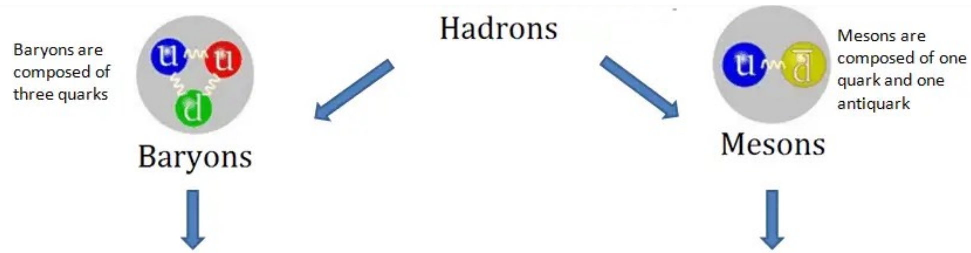
up-quark: + 2/3
down-quark: -1/3

electron: -1

$$u(x) \to e^{i\frac{2}{3}e\alpha(x)}u(x), \quad d(x) \to e^{-i\frac{1}{3}e\alpha(x)}d(x), \quad e(x) \to e^{-ie\alpha(x)}e(x)$$

QCD binds quarks into hadrons

$p = (uud), \;\; n = (udd), \;\; \pi^+ = (\bar{d}u), \ldots$ and hundreds more.



Nucleons

Particle	Mass (MeV/c^2)	τ (sec)
p	938.2	$>10^{11}$
n	939.5	10^3

Hyperons

Particle	Mass (MeV/c^2)	τ (sec)
Λ	1115	2.6×10^{-10}
Σ^+	1189	0.8×10^{-10}
Σ^0	1192	10^{-14}
Σ^-	1197	1.6×10^{-10}
Ξ^0	1314	3×10^{-10}
Ξ^-	1321	1.8×10^{-10}
Ω^-	1675	1.3×10^{-10}

Pions

Particle	Mass (MeV/c^2)	τ (sec)
π^-, π^+	139	2.5×10^{-8}
π^0	135	1.8×10^{-16}

Kaons

Particle	Mass (MeV/c^2)	τ (sec)
K^-, K^+	494	1.2×10^{-8}
K^0	498	
η	550	10^{-18}

Coupling “constants” : QED

Classical physics: forces depend on distances

Quantum field theory: charges also depend on distances

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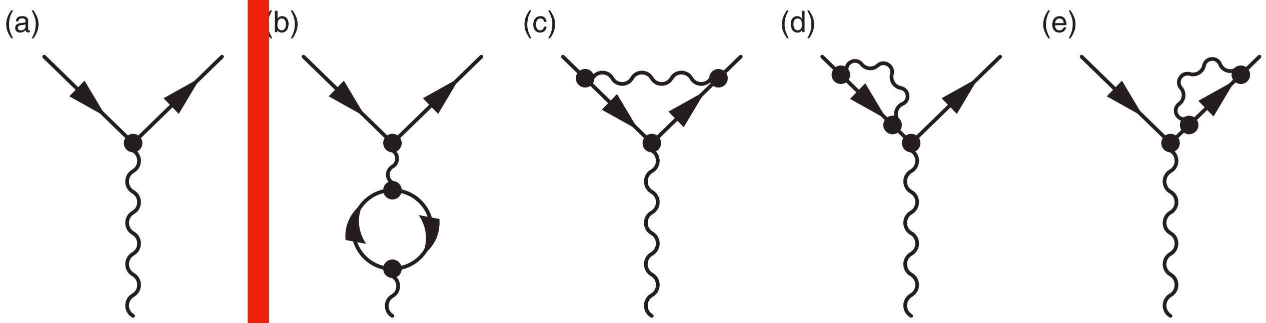

+ …

classical contribution

e

quantum fluctuations

$$e^3 \int d^4k \frac{1}{k - m_e} \frac{1}{k - m_e} \frac{1}{k^2} \rightarrow \infty$$

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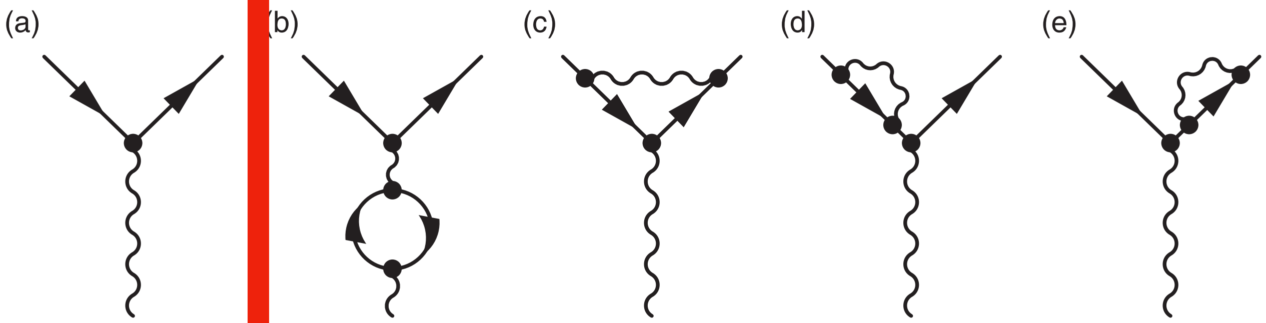
intuitive picture



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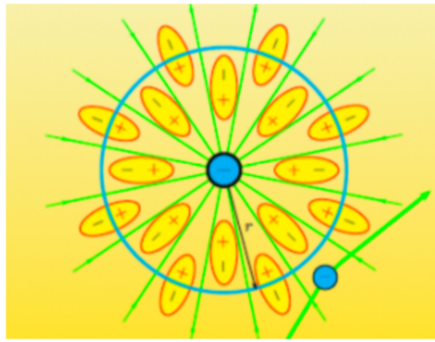

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$$e^3 \int d^4k \frac{1}{k - m_e} \frac{1}{k - m_e} \frac{1}{k^2} \to \infty$$

The vacuum screens the electric charge
-> infrared free

charge weaker at lower E
at larger r

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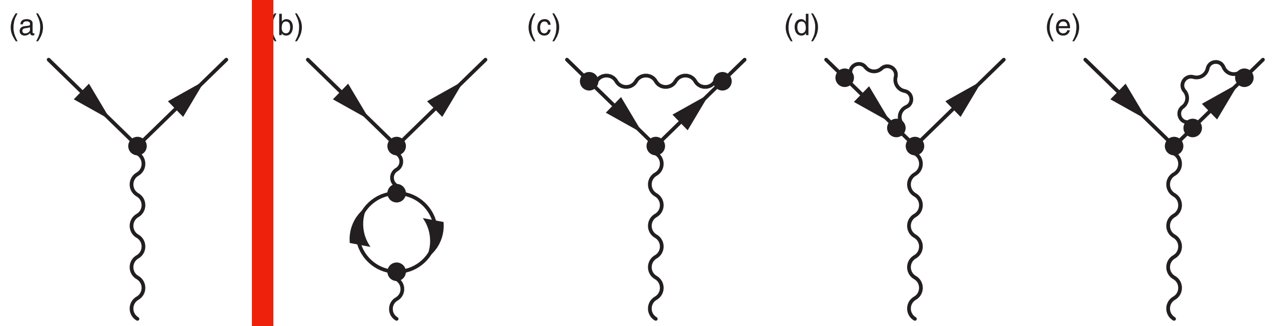

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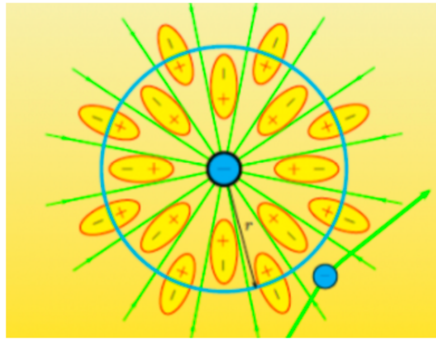

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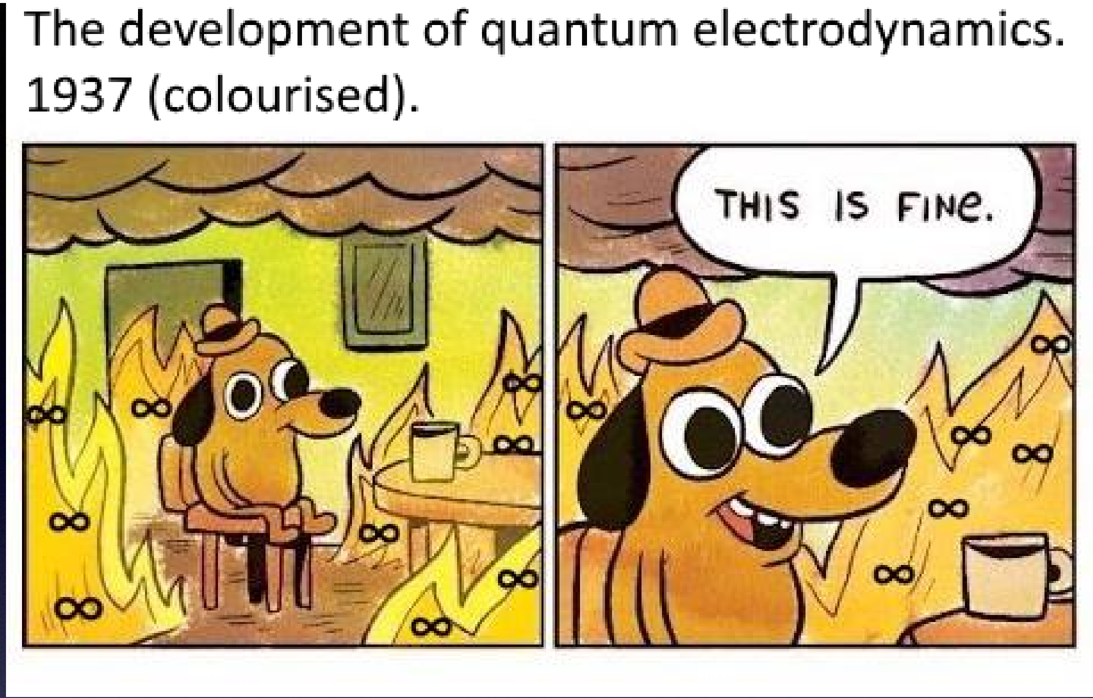
$\infty - \infty$ = finite ?

∞

The development of quantum electrodynamics.
1937 (colourised).

∞

∞



∞

Modern view: “**Ignorance is no shame**”.

We can’t trust our QFT up to infinite energy, so we should not include virtual particles up to infinite energy. We introduce a maximum energy (a cut-off) to **regularize** the theory. We compare with the measurement to determine the value of classical + regularized virtual (= **renormalize**).

This is a good thing: for example, Feynman, Schwinger, Tomonaga, and others who developed quantum electrodynamics did not have to know about the top quark.

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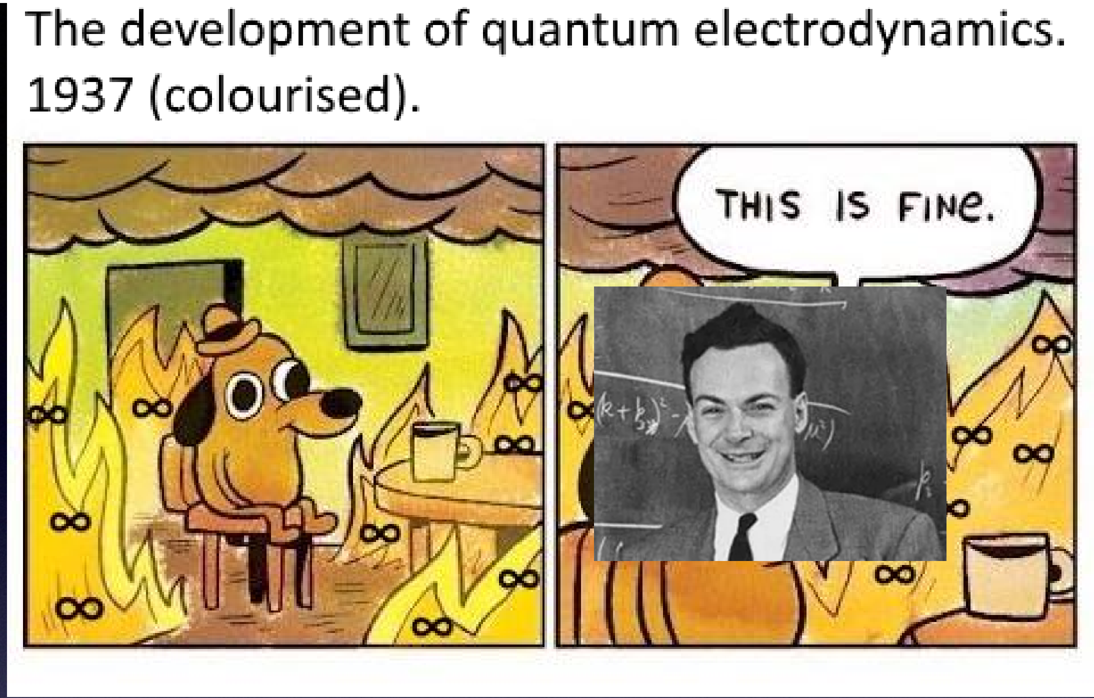

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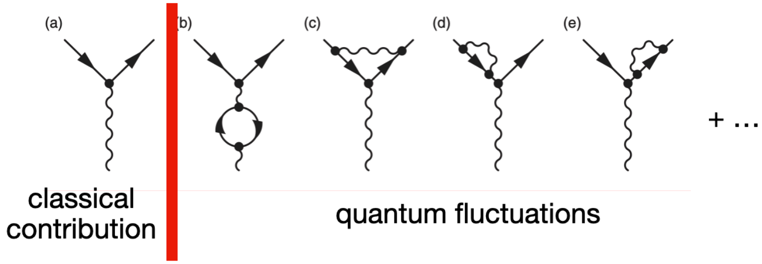

Fine structure constant: $\alpha_{QED} = \dfrac{e^2}{4\pi}$

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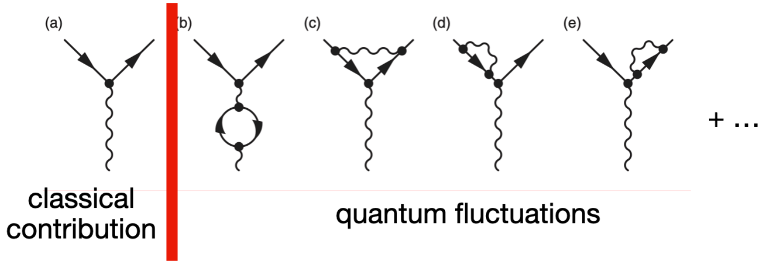

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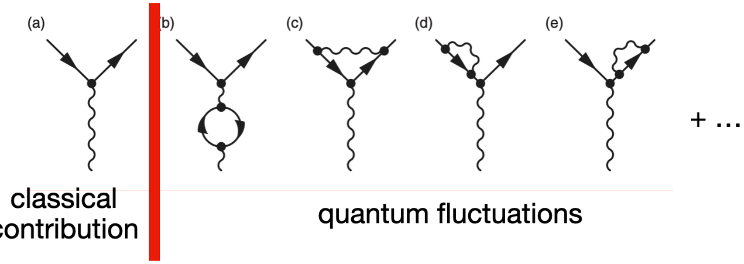

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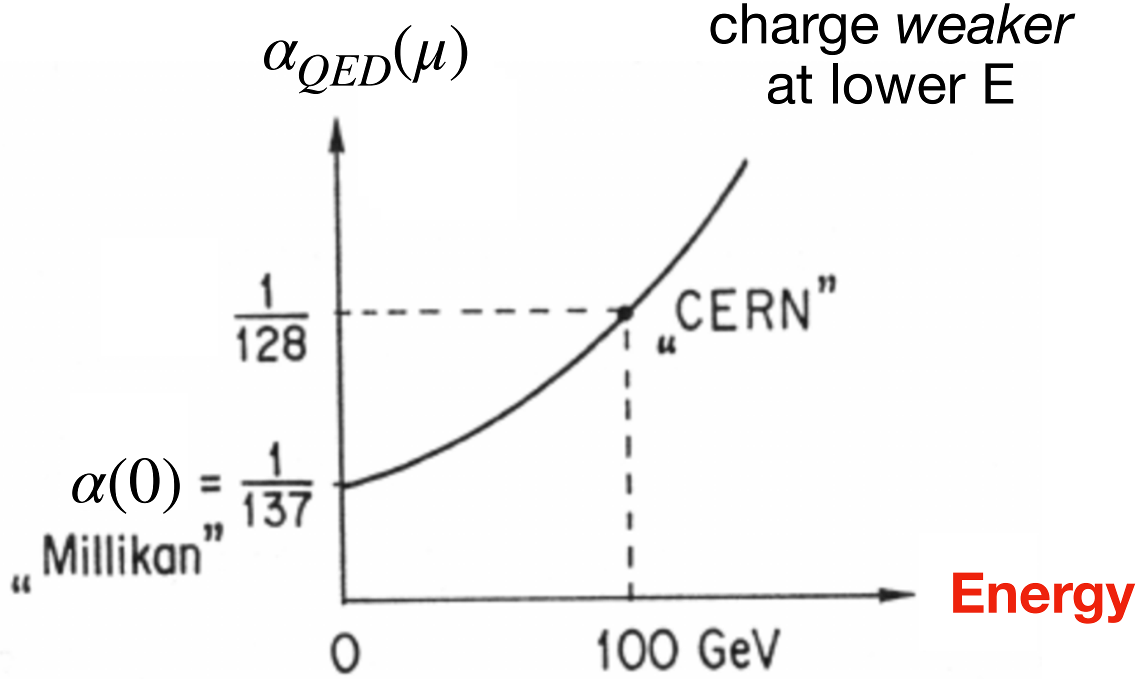

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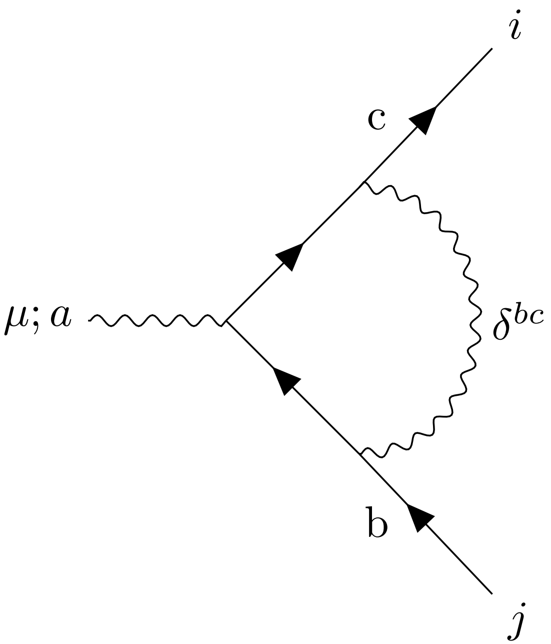

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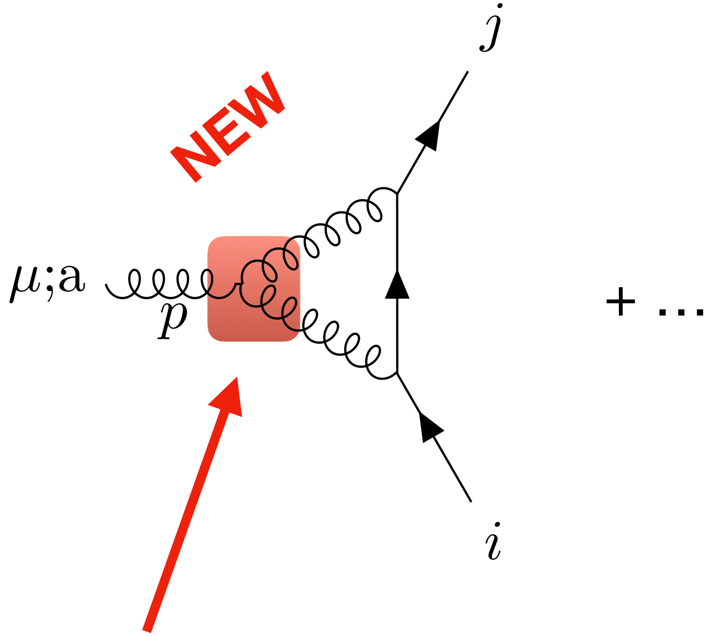

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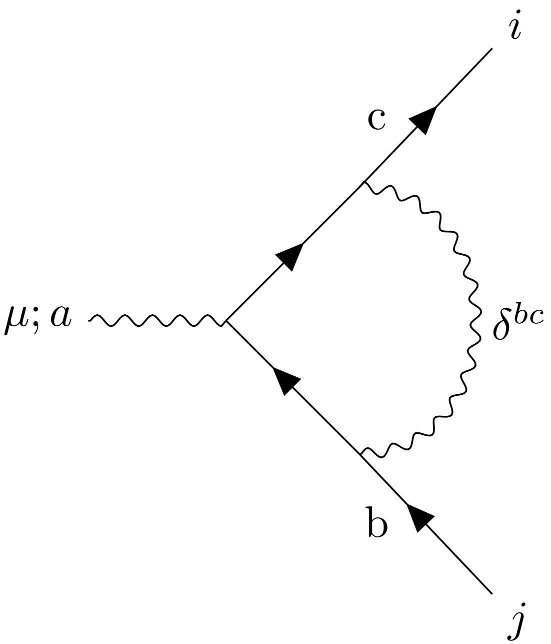

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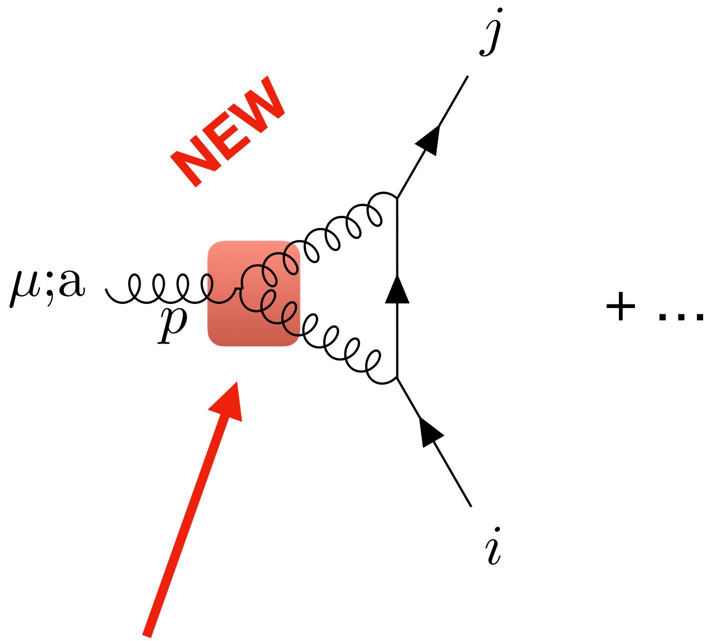

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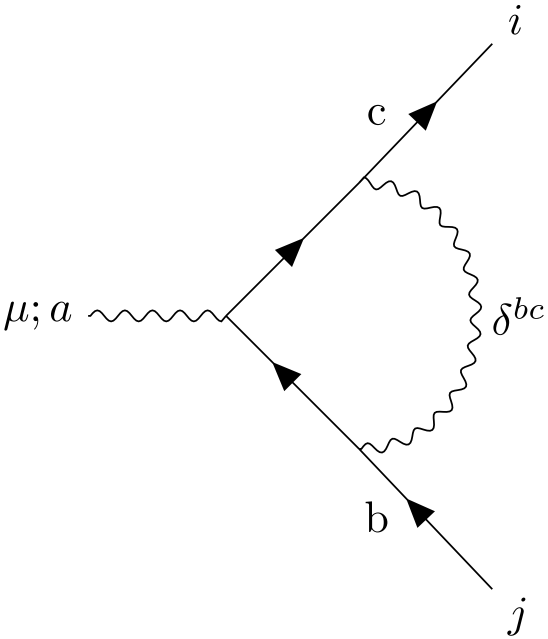

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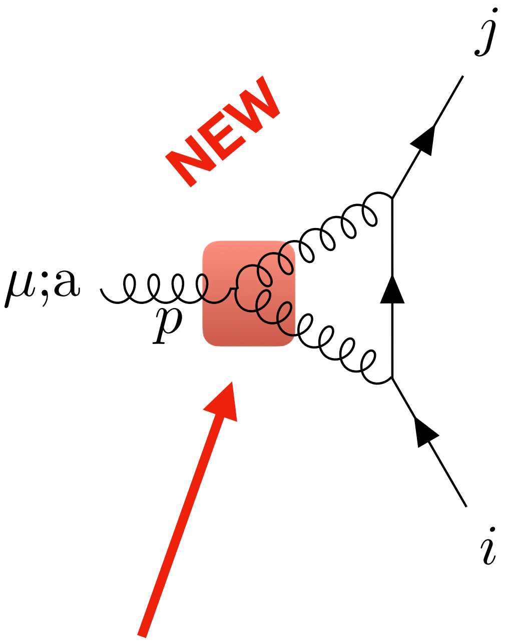

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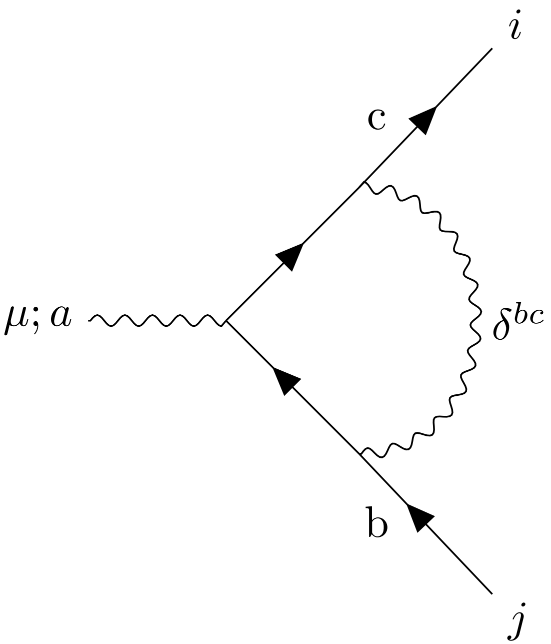

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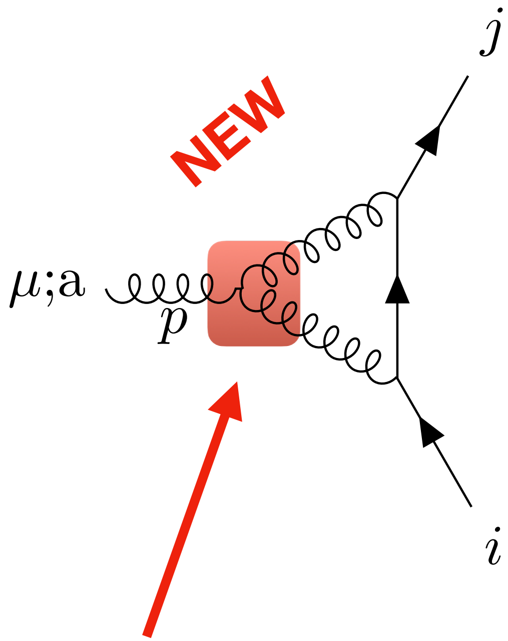

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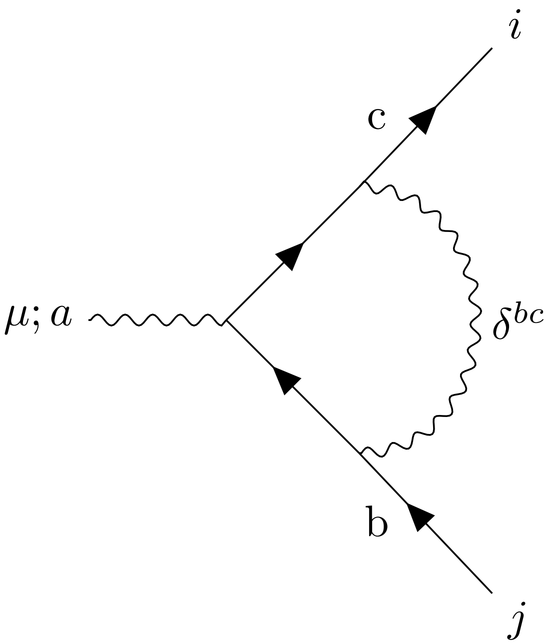

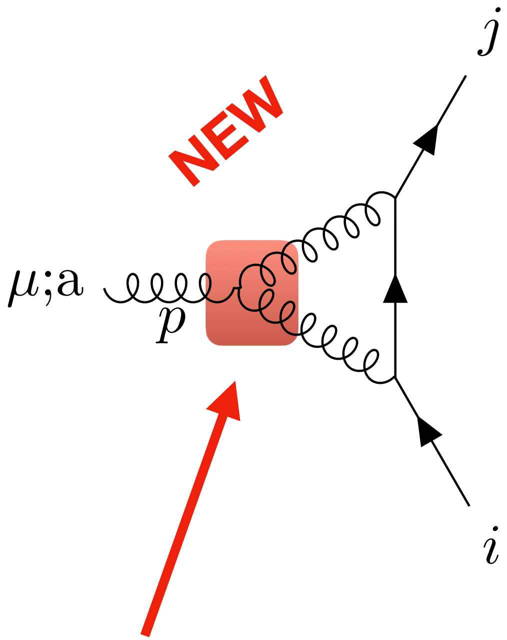

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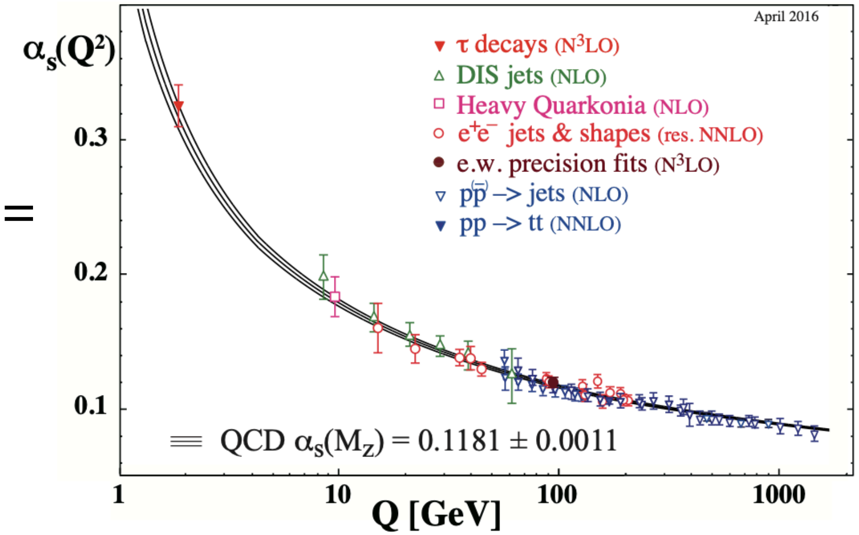


QED-like

gluons interact with themselves

Evolution of coupling constants

QED: virtual particles screen charge, *weaker* longer distances
= “infrared freedom”

infinite range 

$$V = \frac{q_1 q_2}{r}$$

Evolution of coupling constants

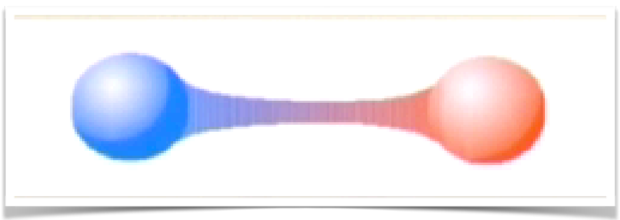
QED: virtual particles screen charge, *weaker* longer distances
= "infrared freedom"

infinite range

$$V = \frac{q_1 q_2}{r}$$

QCD: virtual particles anti-screen charge, *stronger* longer distances
= "asymptotic* freedom"

(* asymptotic means at high energies)



Cannot separate color charges!

The range of the strong interactions determined by the exchange of the lightest colorless hadron (= pion)

$$V = \frac{g_1 g_2}{r} e^{-m_\pi r} \qquad m_\pi = 125\ MeV, \quad \frac{1}{m_\pi} \approx 1\ Fermi = 10^{-13}\ cm$$

Measuring the quark charge

Can we measure the electric charge of the quarks?

Can we test that there are $N_C = 3$ colors?

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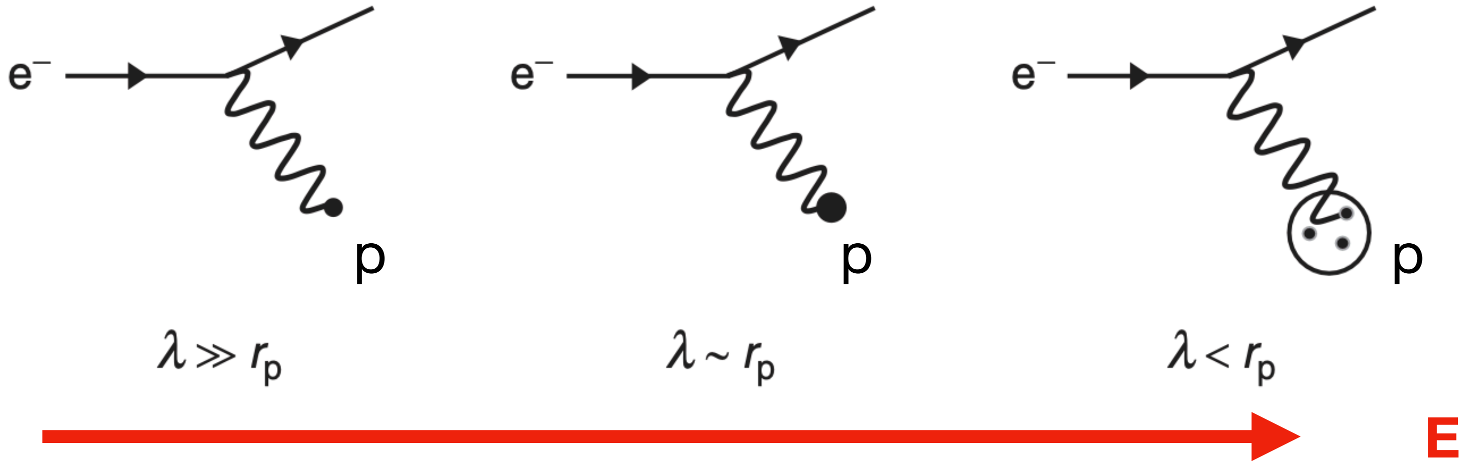
p appears pointlike *p has geometric features* *we see quarks !*



r_p : proton radius

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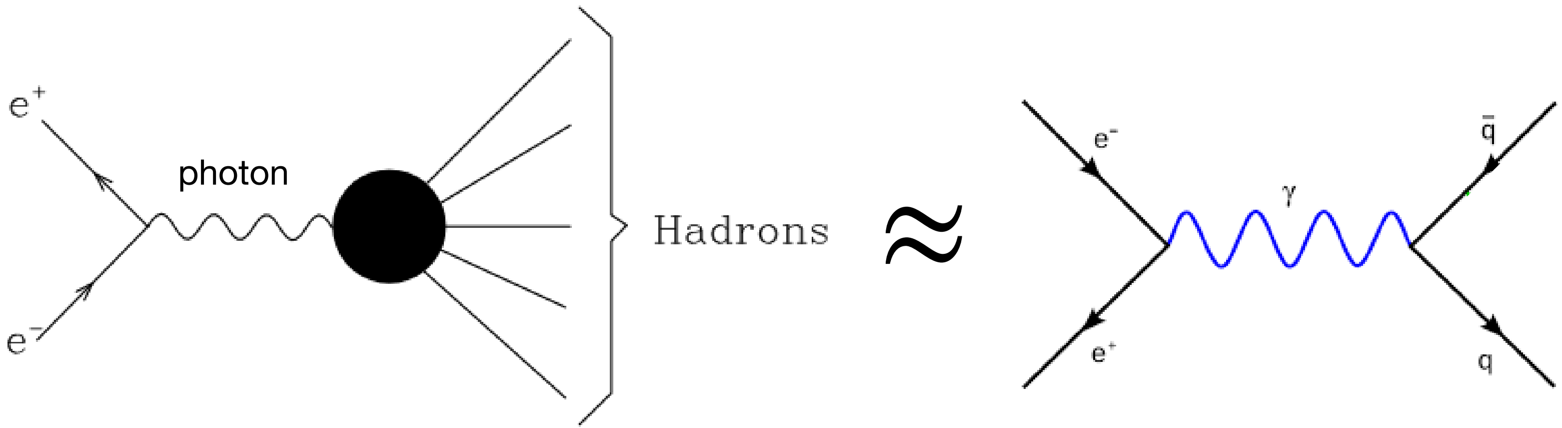

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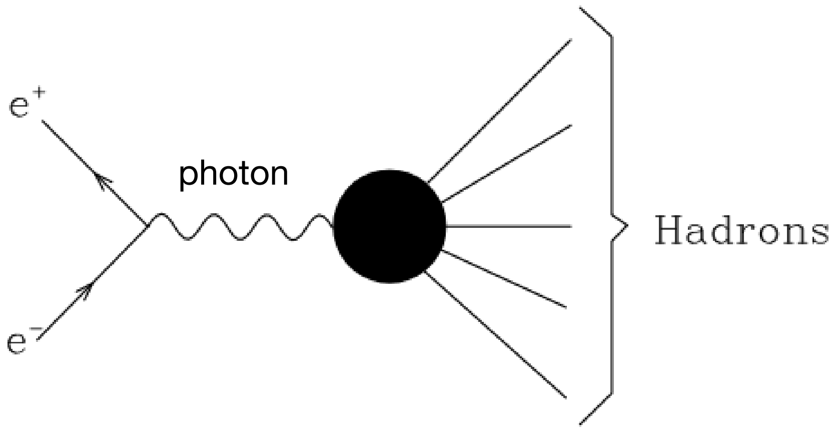
up, charm, top: +2/3
down, strange, bottom: -1/3



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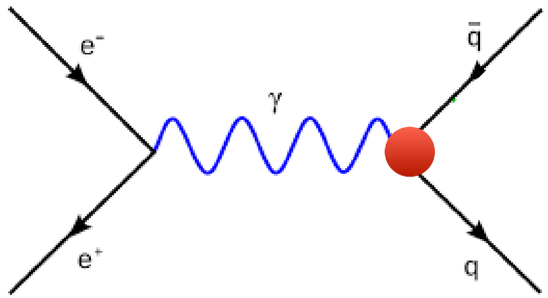
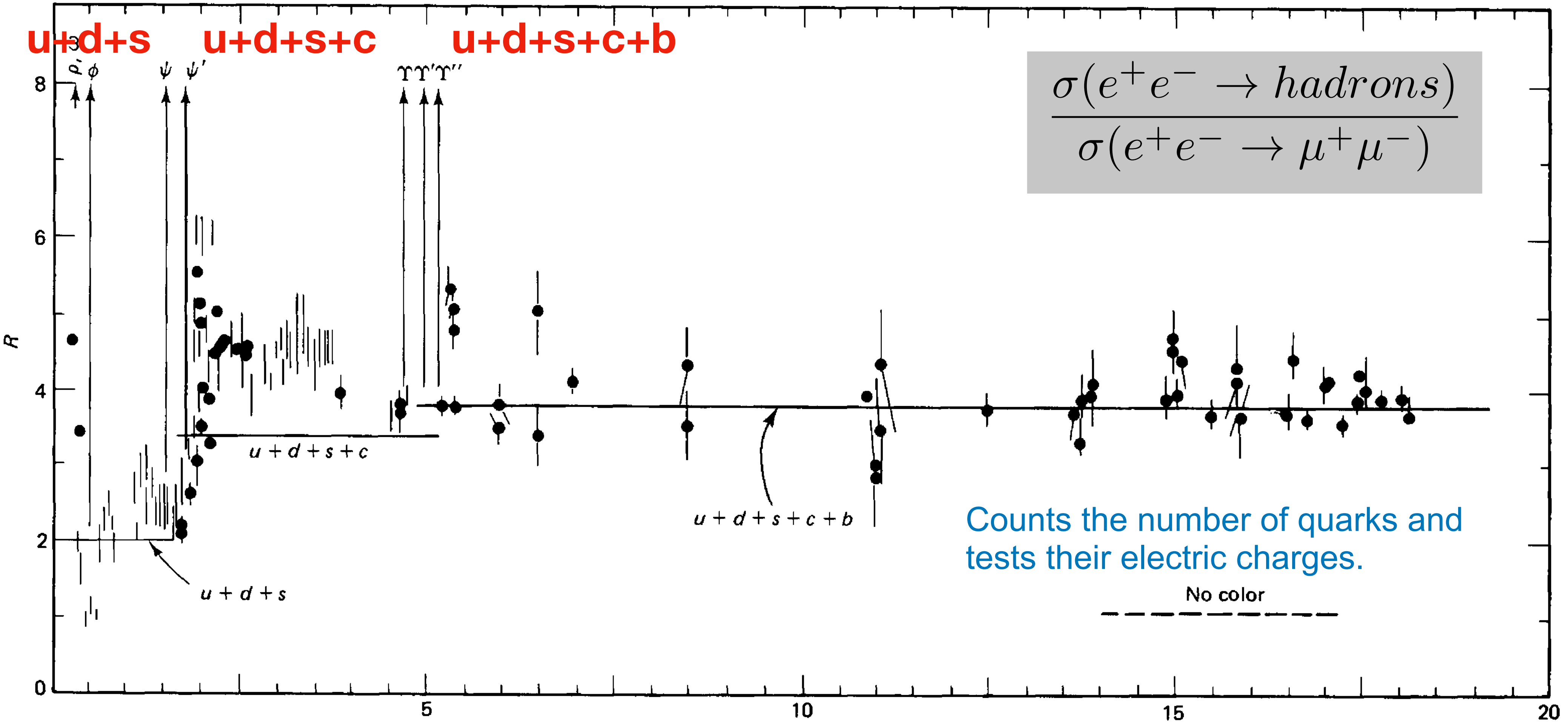


$\propto q_{Quark}$

Testing the quark charges and N_C

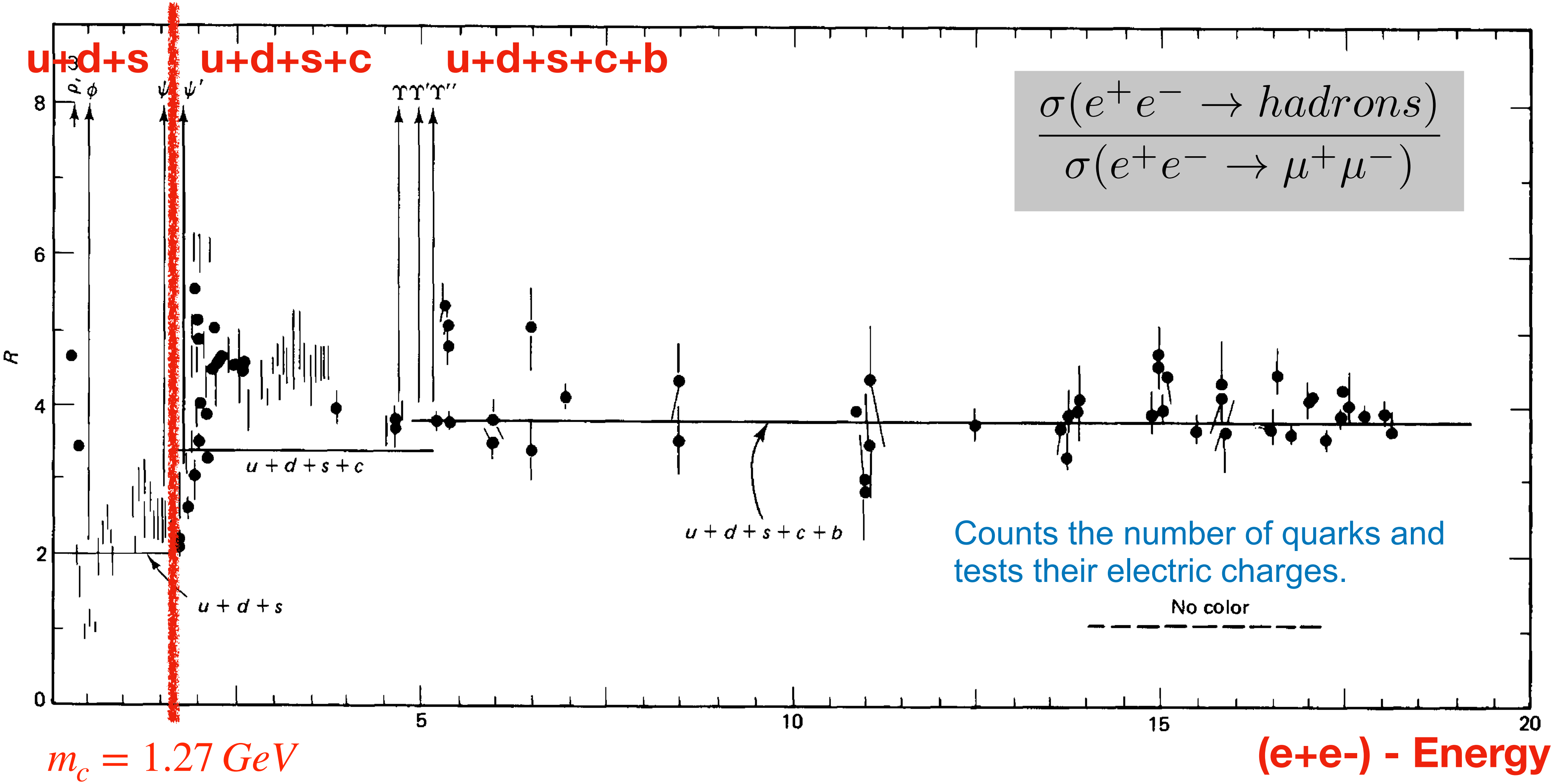


$$\frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

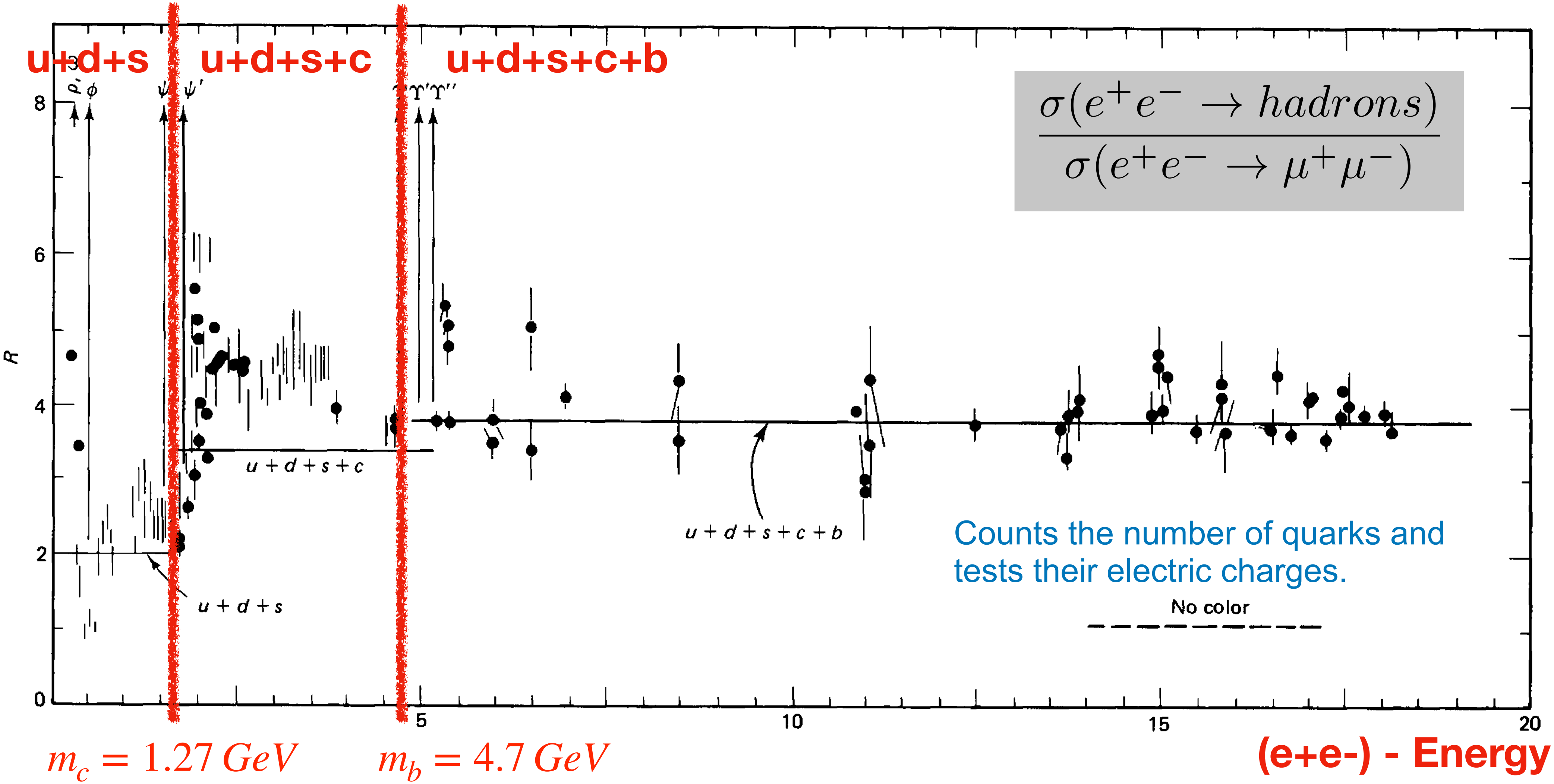
Counts the number of quarks and tests their electric charges.

(e+e-) - Energy

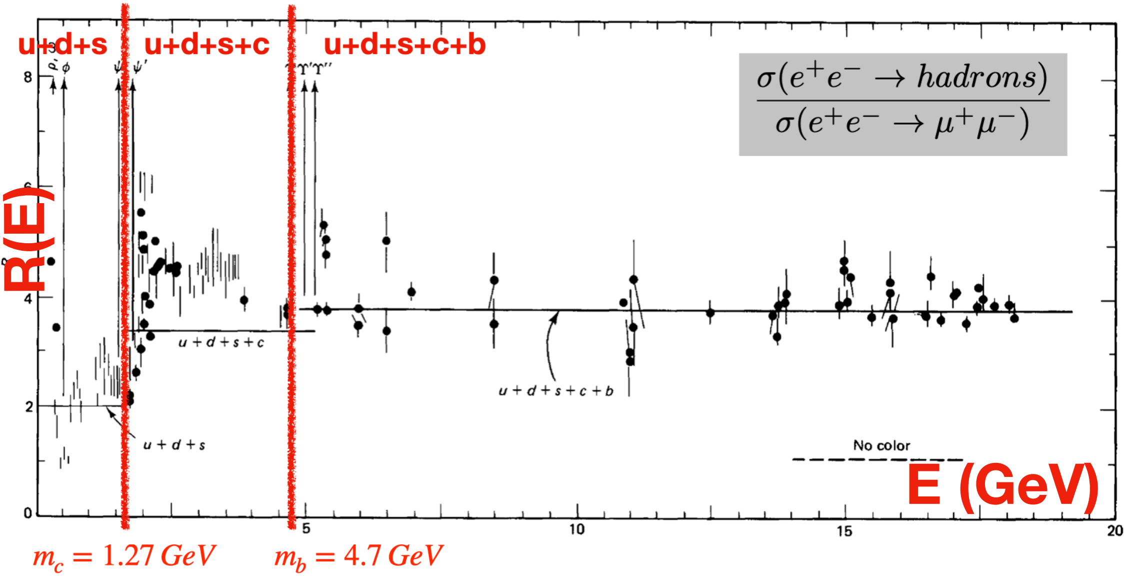
Testing the quark charges and N_C



Testing the quark charges and Nc



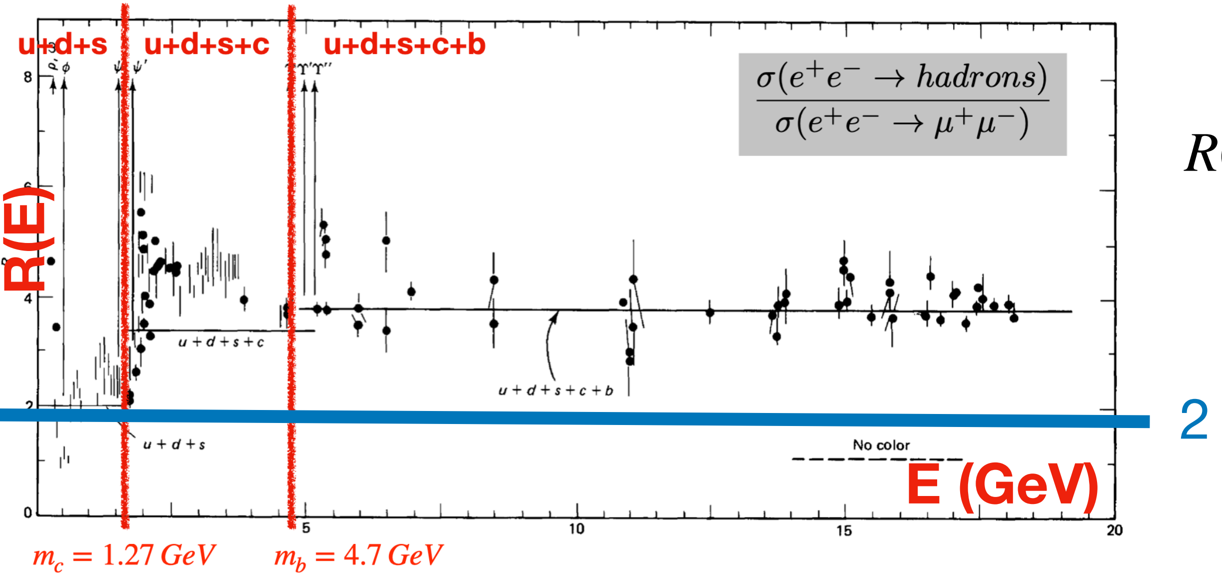
Testing the quark charges and N_C



$$R(E) = \sum_{m_i < E} Q_i^2$$

= "sum over (quark charges)^2 accessible at energy **E**"

Testing the quark charges and N_C



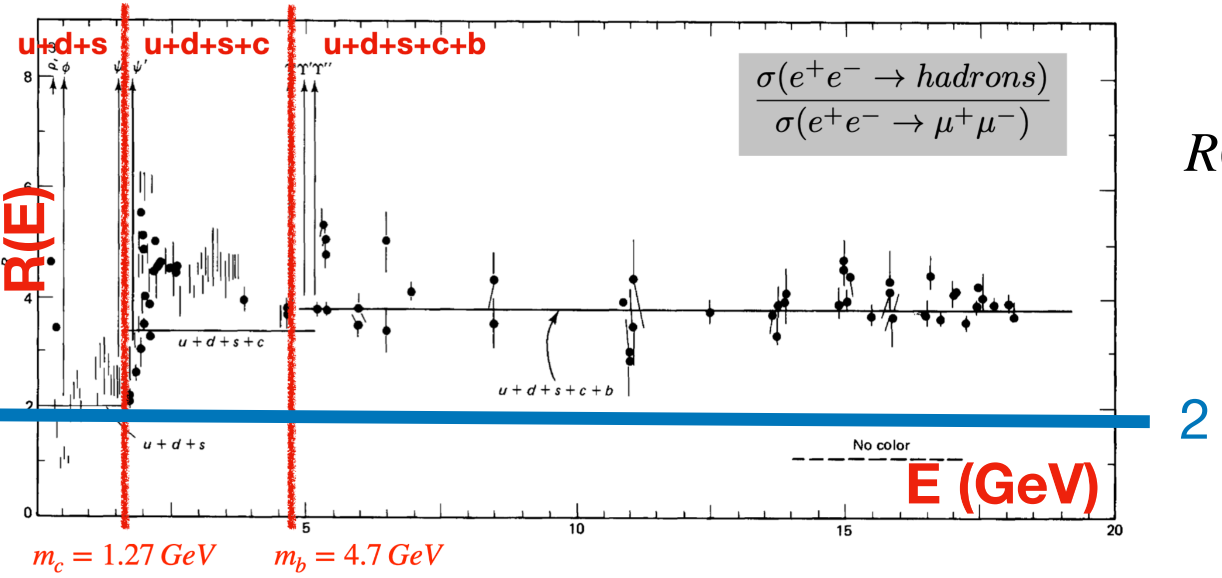
$$R(E) = \sum_{m_i<E} Q_i^2$$

= "sum over (quark charges)^2 accessible at energy **E**"

q_u **q_d** **q_s**

$$R(E = 1GeV) = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right) = 2$$

Testing the quark charges and N_C



$$R(E) = \sum_{m_i<E} Q_i^2 \quad =$$

"sum over (quark charges)^2 accessible at energy **E**"

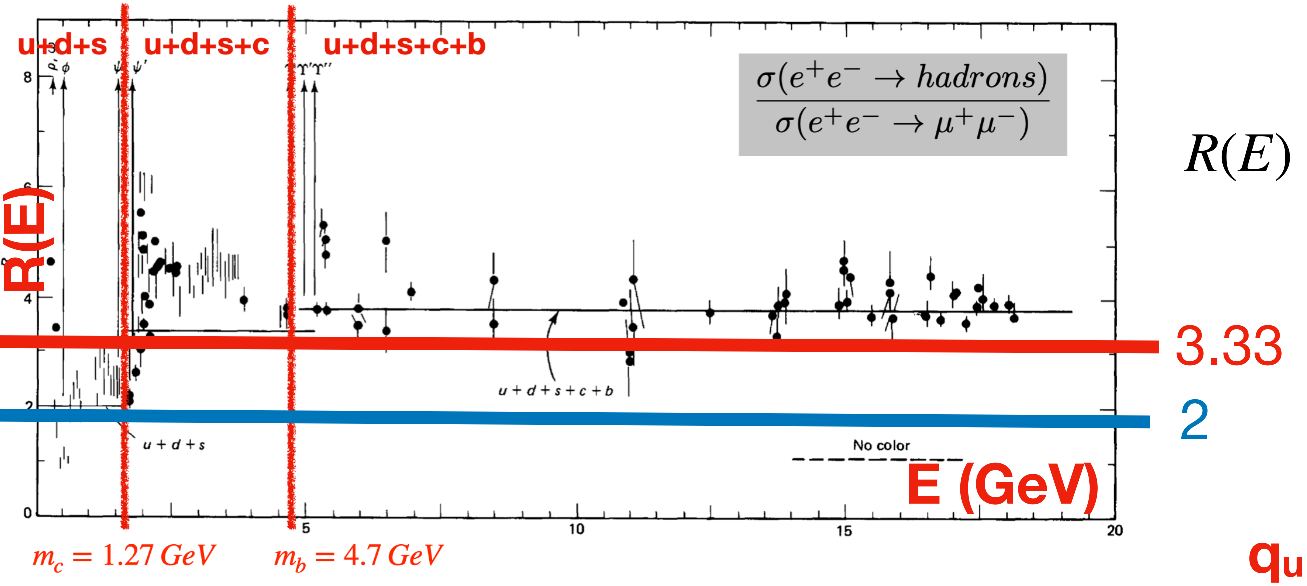
because there are 3 colors

$SU(3)_c$

q_u q_d q_s

$$R(E = 1GeV) = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right) = 2$$

Testing the quark charges and N_C



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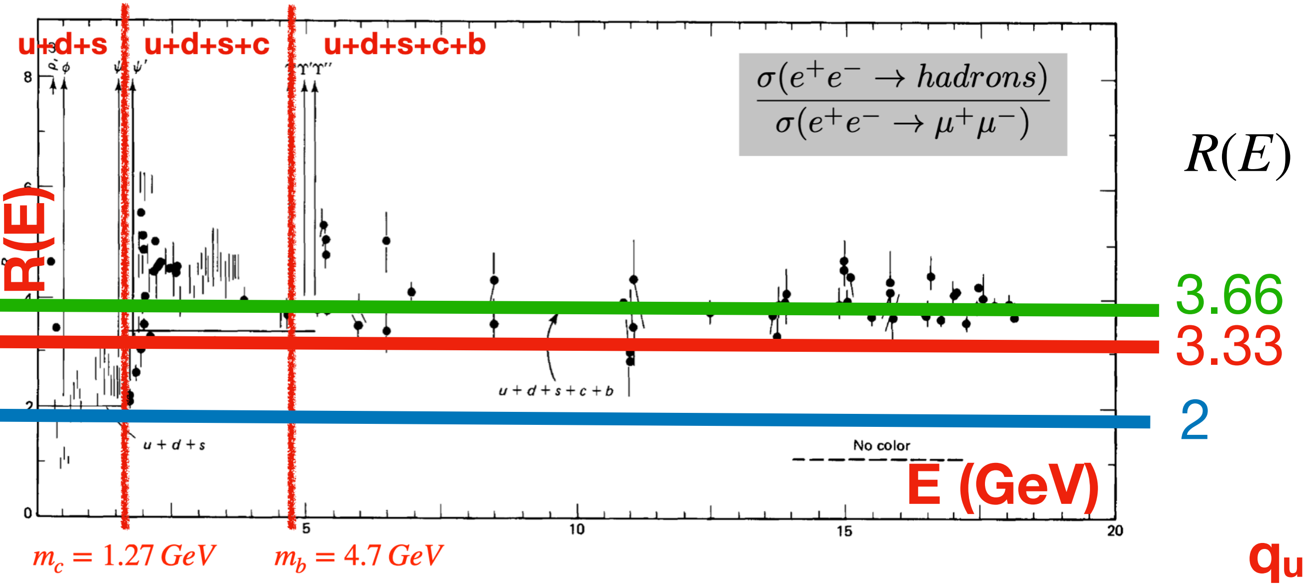
q_c

$$R(1.2 < E < 4.6) = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \right) = 10/3 \approx 3.33$$

Testing the quark charges and N_c



$$R(E) = \sum_{m_i<E} Q_i^2$$ = "sum over (quark charges)^2 accessible at energy **E**"

because there are 3 colors

SU(3)c

q_u **q_d** **q_s**

$$R(E = 1GeV) = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right) = 2$$

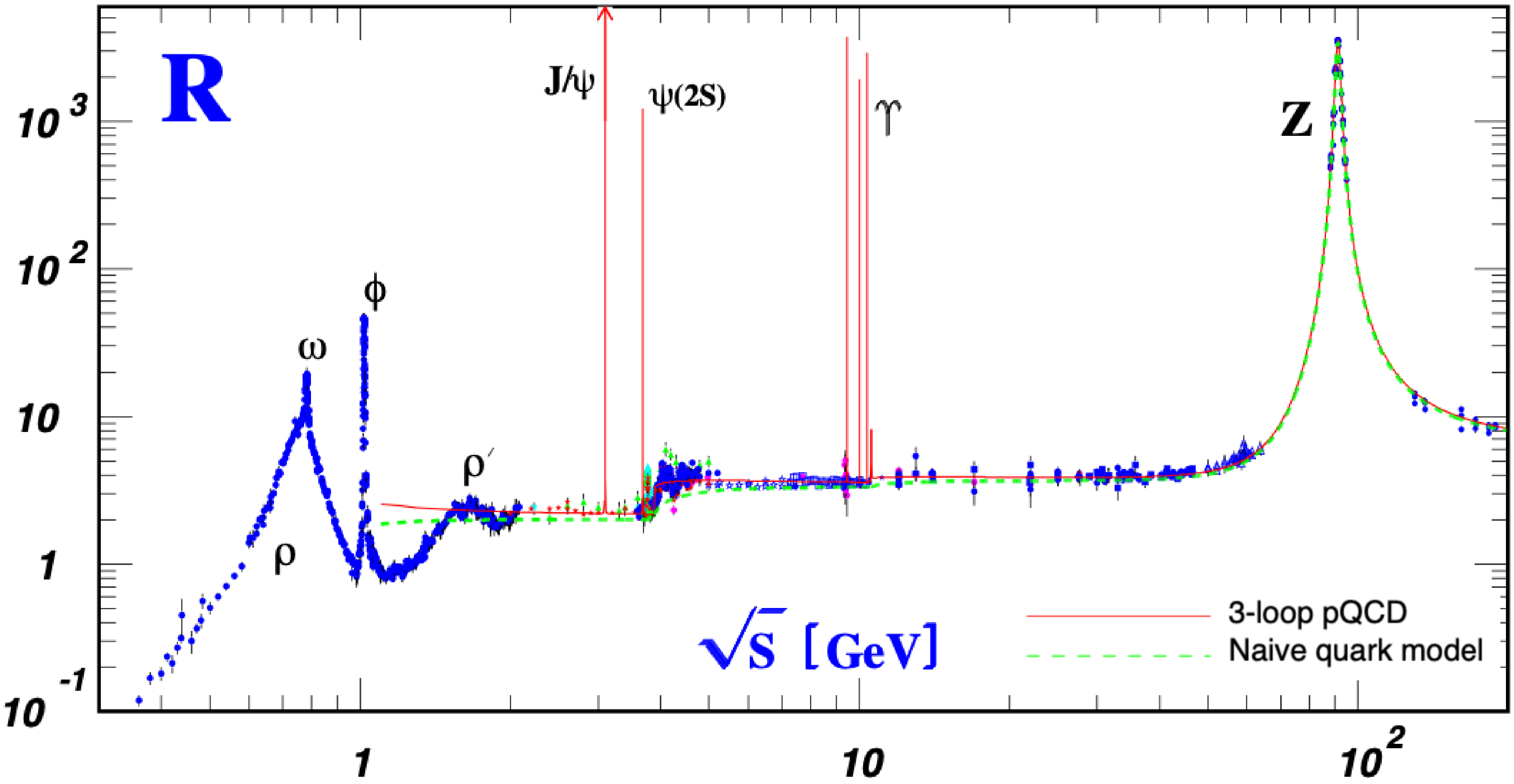
q_c

$$R(1.2 < E < 4.6) = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \right) = 10/3 \approx 3.33$$

q_b

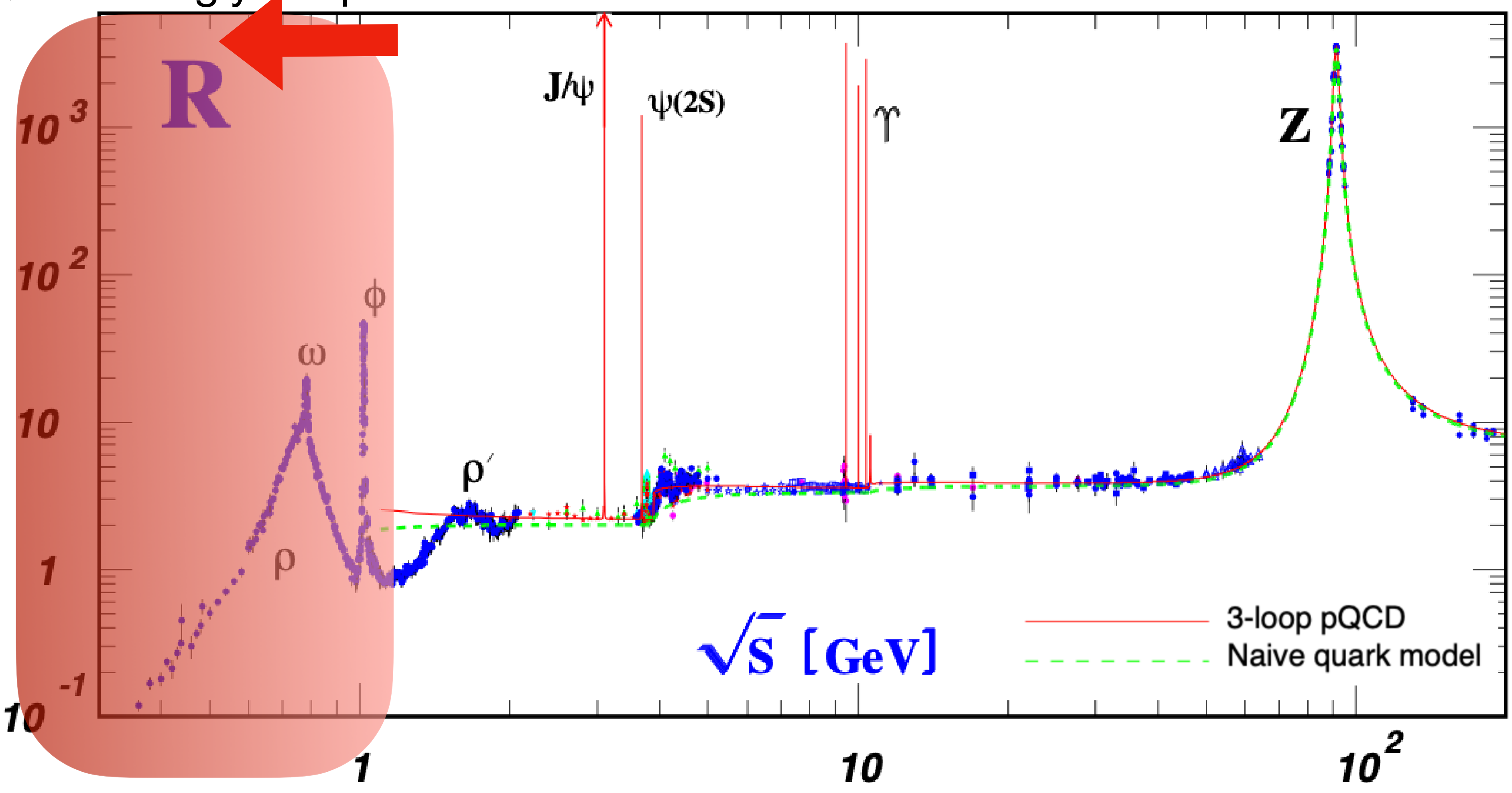
$$R(E > 4.7GeV) = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right) = 11/3$$

Modern version of the measurement



Modern version of the measurement

QCD strongly coupled



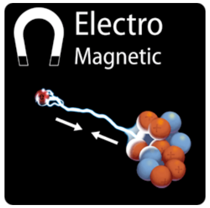
More on QCD at colliders:

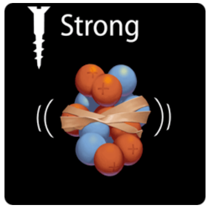
10:25 → 11:20 **Making Predictions at Hadron Colliders 1/2**

Speaker: Alexander Yohei Huss (CERN)

SM without weak interactions

Summary



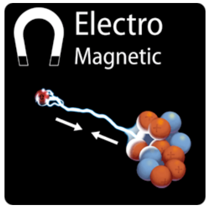


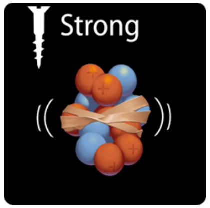
- Symmetry: $SU(3) \times U(1)_{em}$
- Particles: $u = \mathbf{3}_{2/3}, \quad d = \mathbf{3}_{1/3}, \quad e = \mathbf{1}_{-1}$ (per generation)

$$\mathcal{L}_{QCD+QED}(g_S, e, m_{u_i}, m_{d_i}, m_{e_i})$$

SM without weak interactions

Summary

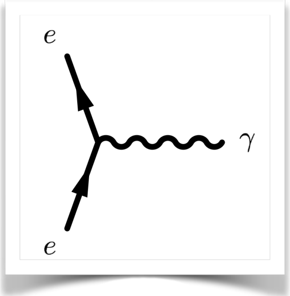


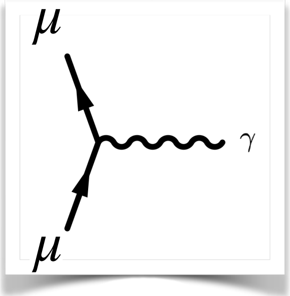


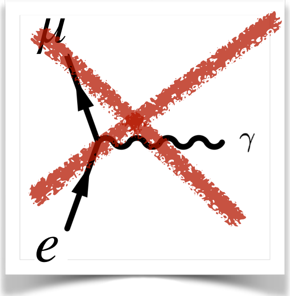
- Symmetry: $SU({\color{red}3}) \times U(1)_{em}$
- Particles: $u = {\color{red}\mathbf{3}}_{2/3}, \quad d = {\color{red}\mathbf{3}}_{1/3}, \quad e = \mathbf{1}_{-1}$ (per generation)

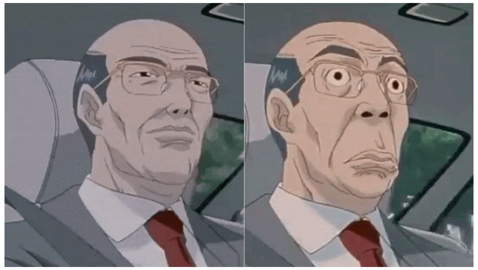
$$\mathcal{L}_{QCD+QED}(g_S, e, m_{u_i}, m_{d_i}, m_{e_i})$$

- One tiny problem: no way to violate individual quark or lepton number!

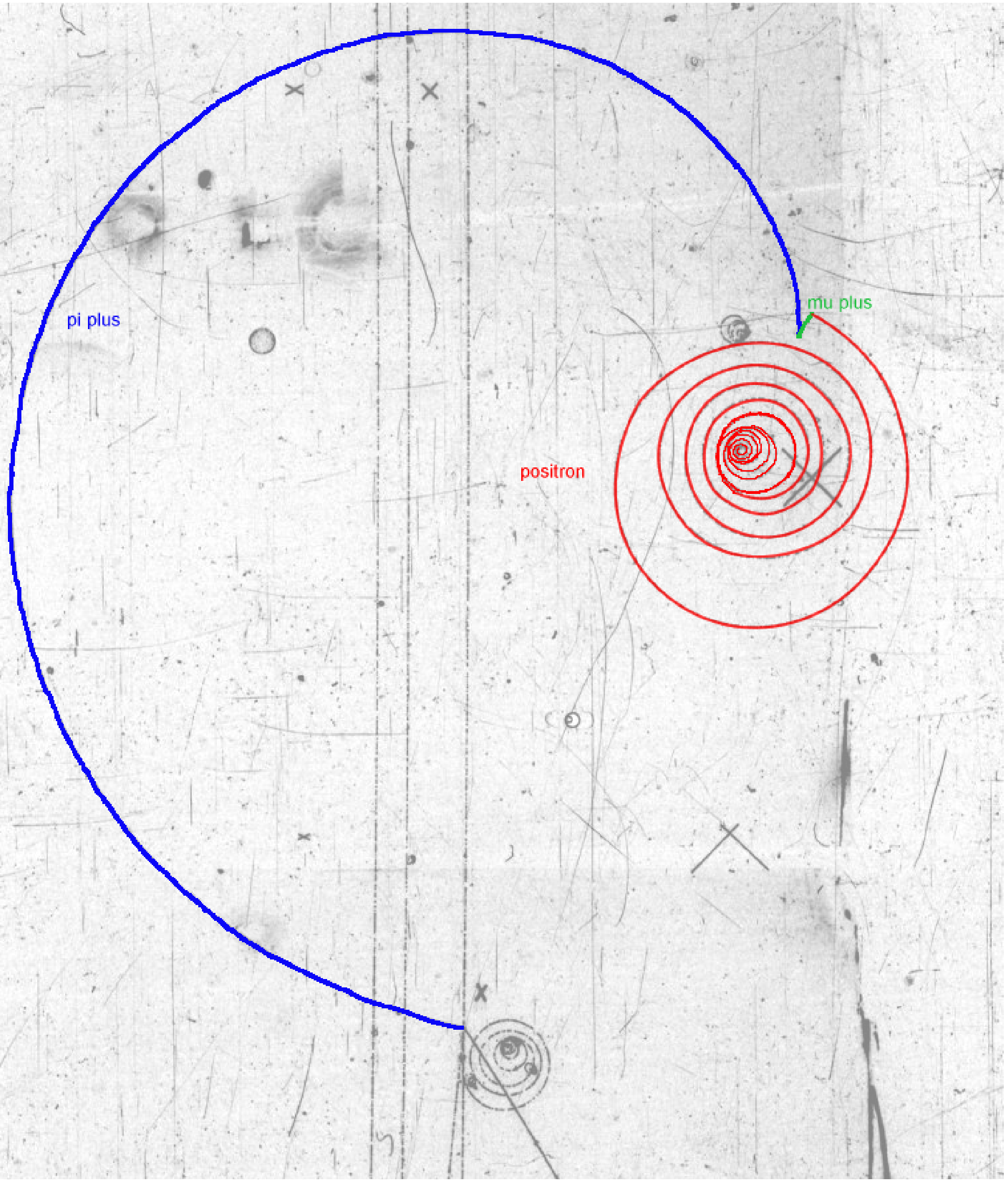








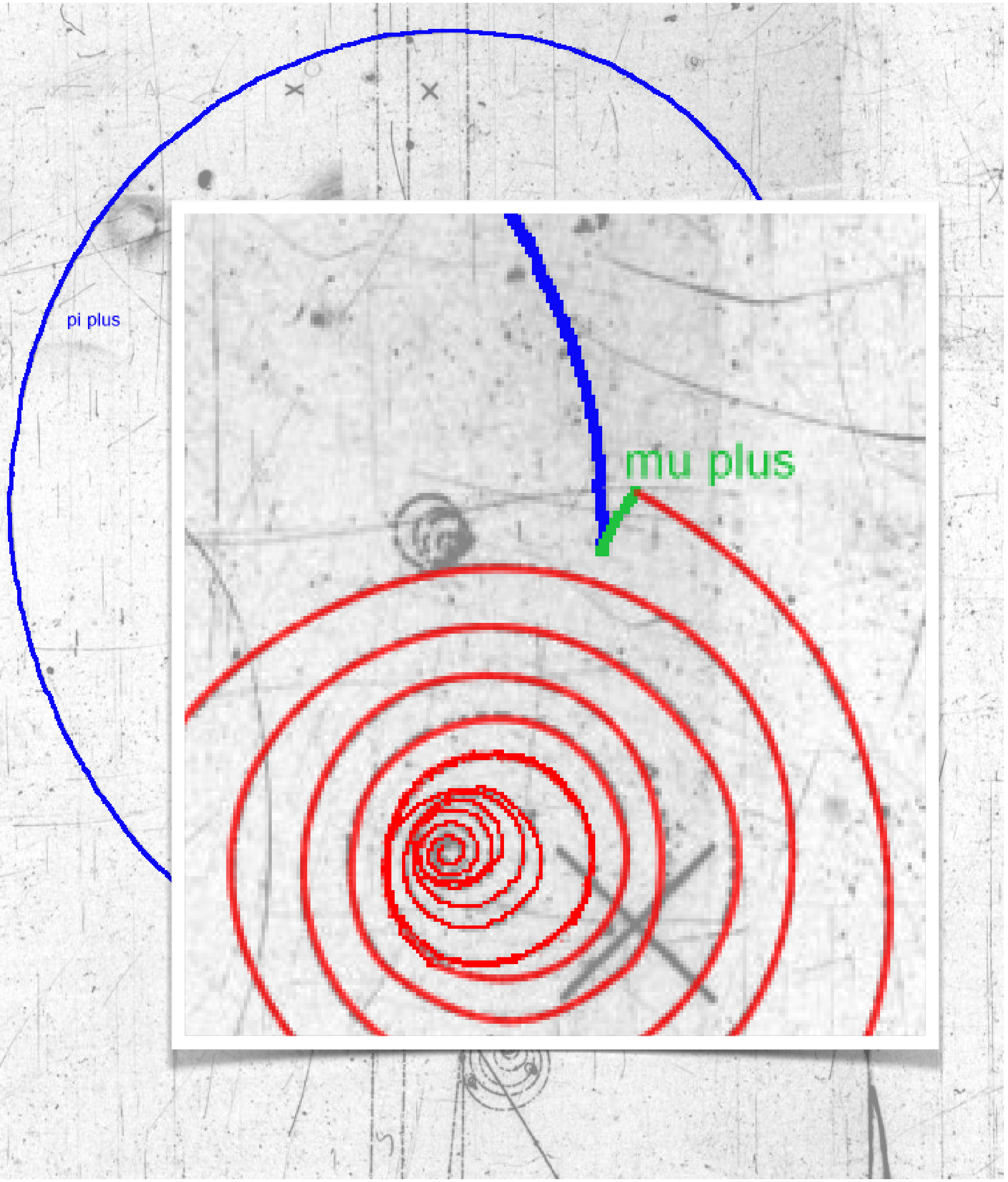
But: the muon decays!



CERN 2 metre hydrogen bubble chamber, exposed to a 10 GeV/c K+ beam (from top of the picture).

Example of a pion stopping and then decaying into a muon. This a `two-body decay' - the muon is accompanied by a neutrino moving in the opposite direction with equal and opposite momentum. Energy and momentum conservation force this momentum to be about 30 MeV/c. The **range of a muon** with this momentum in hydrogen is about a centimetre.

At the end of its short range, the muon itself decays into a positron which spirals characteristically.



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We are missing an interaction: the weak force!