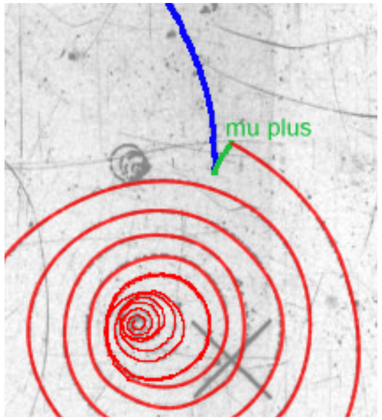
Standard Model 3/4

Andreas Weiler (TU Munich)

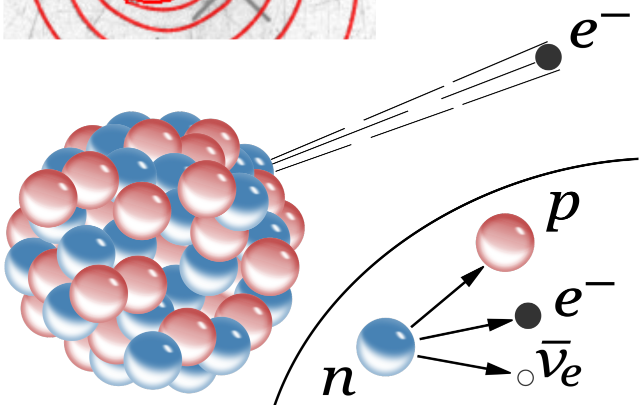
CERN, 7/2024

The weak interactions

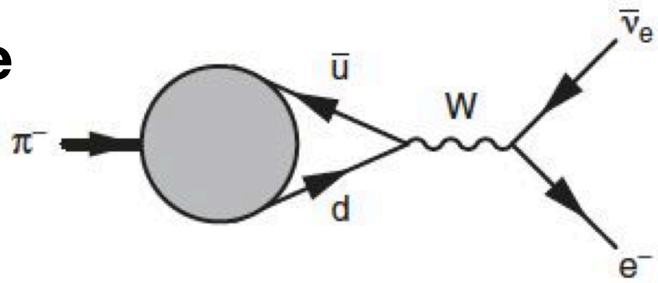
We want to explain these processes



- muon decay (lifetime: $\tau \approx 10^{-6} s$)
- neutron decay (lifetime: $\tau \approx 877\, s$)
- charged pion decay (lifetime: $\tau \approx 10^{-8}\, s$)



How can one interaction be responsible for such different life-times?



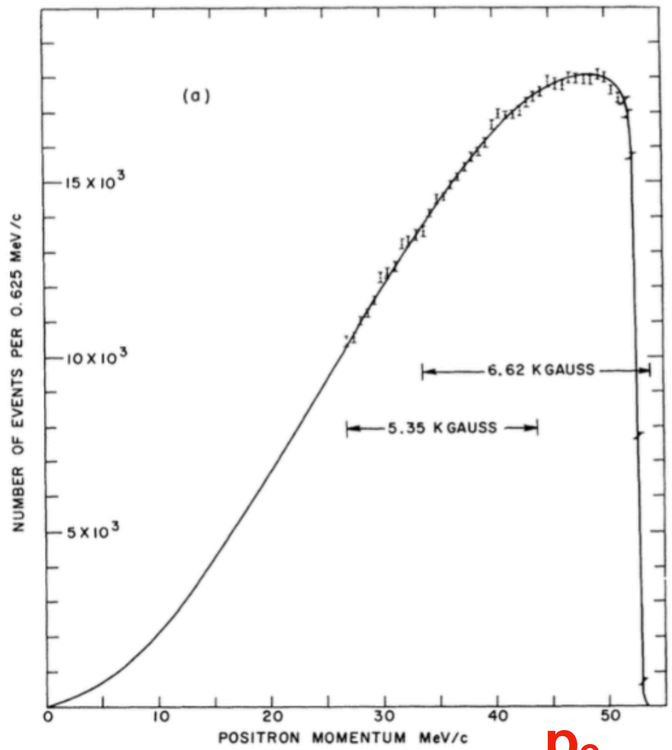
Muon decay

We observe a muon decaying into an electron: $\mu^- \rightarrow e^- + X$

X is something undetected.

On the right you see the electron spectrum
Can X be just one particle?

No, because in two-body decays
p_e would be fixed!



p_e
(electron momentum)

47.4.2. ***Two-body decays*** :

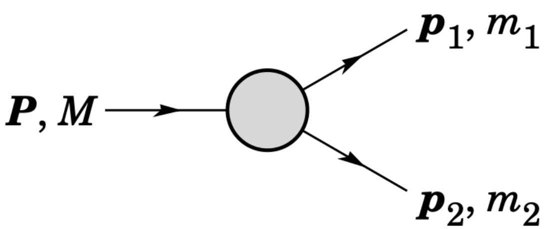


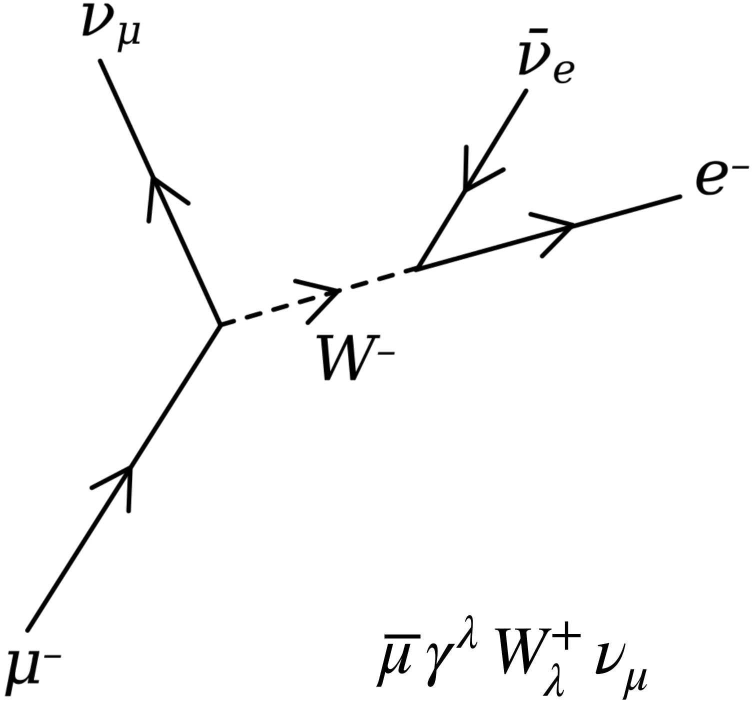
Figure 47.1: Definitions of variables for two-body decays.

In the rest frame of a particle of mass M, decaying into 2 particles labeled 1 and 2,

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \ ,$$

$$|\boldsymbol{p}_1| = |\boldsymbol{p}_2| \ = \frac{\left[\left(M^2 - (m_1 + m_2)^2\right)\left(M^2 - (m_1 - m_2)^2\right)\right]^{1/2}}{2M} \ ,$$

Muon decay (1st draft)



$$\mu^- \to e^- + \nu_\mu + \nu_e$$

$$\overline{\mu}\,\gamma^\lambda\, W_\lambda^+\, \nu_\mu$$

$$\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}^\dagger \gamma_0 (i\partial_\mu - g \begin{pmatrix} 0 \\ \nu_\mu \end{pmatrix}) \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$$

Muon decay (1st draft)*

* chiral structure will come later.

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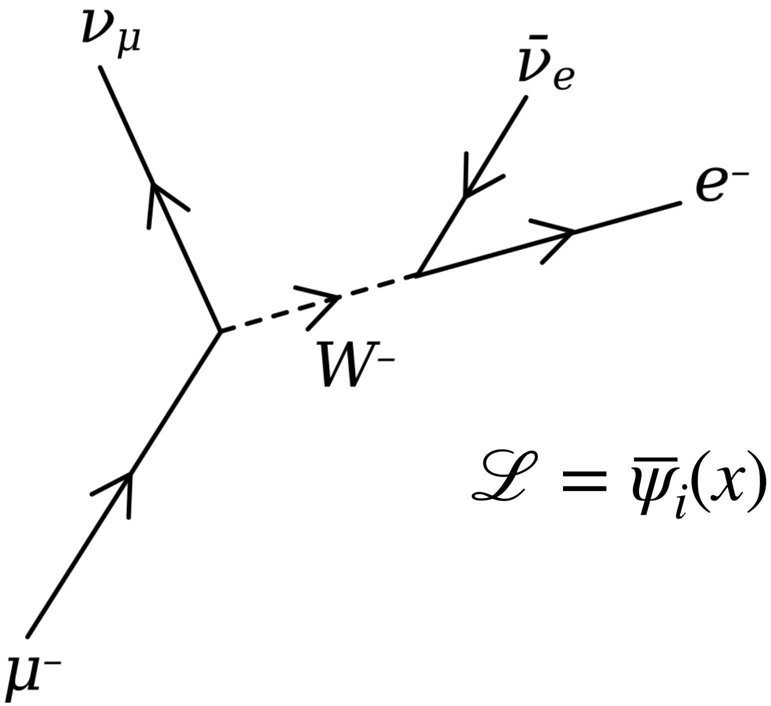

Use non-abelian gauge theory template to describe this:

Plan: replace colors (r, g, b) with (ν_μ, μ)

$$\mathscr{L} = \overline{\psi}_i(x)\,\gamma^\mu(i\partial_\mu - g_s\, A^a_\mu(x) T^a_{ij} - m)\,\psi_j(x)$$

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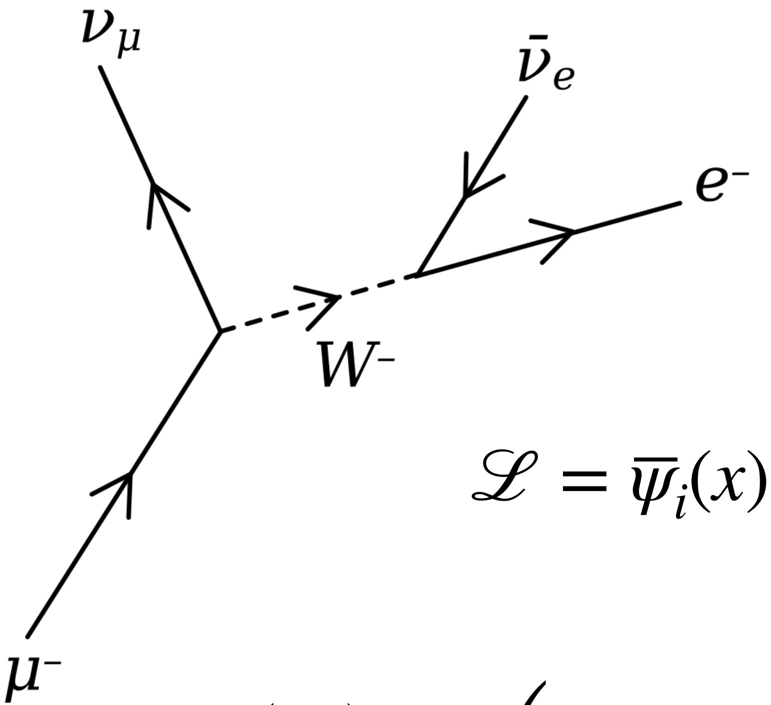

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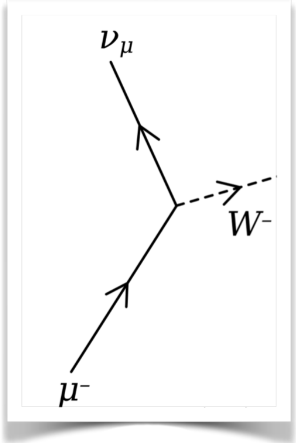
$$\mathscr{L} = \begin{pmatrix} \bar{\nu}_\mu \\ \bar{\mu} \end{pmatrix} \gamma_L^\lambda \left(i\partial_\lambda - g \begin{pmatrix} 0 & W_\lambda^+ \\ W_\lambda^- & 0 \end{pmatrix} + \ldots \right) \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} = g\bar{\nu}_\mu \gamma_L^\lambda W_\lambda^+ \mu + \ldots$$

Muon decay (1st draft)



"Charged current" (the W boson carries electric charge) interactions for the muon and electron

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Since this is an non-abelian theory, this is universal!

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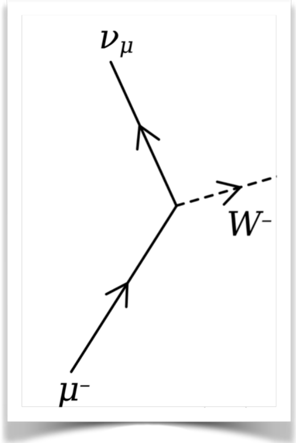

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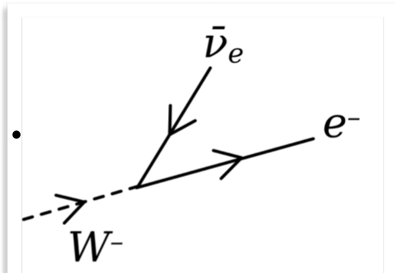
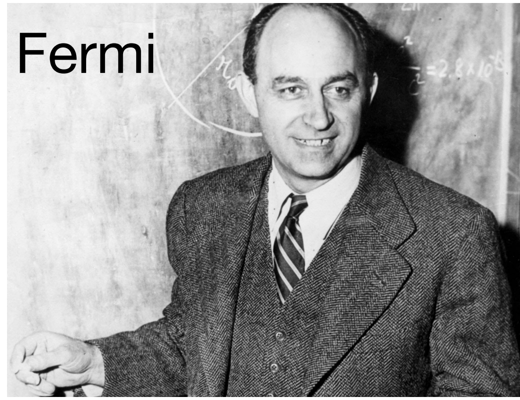
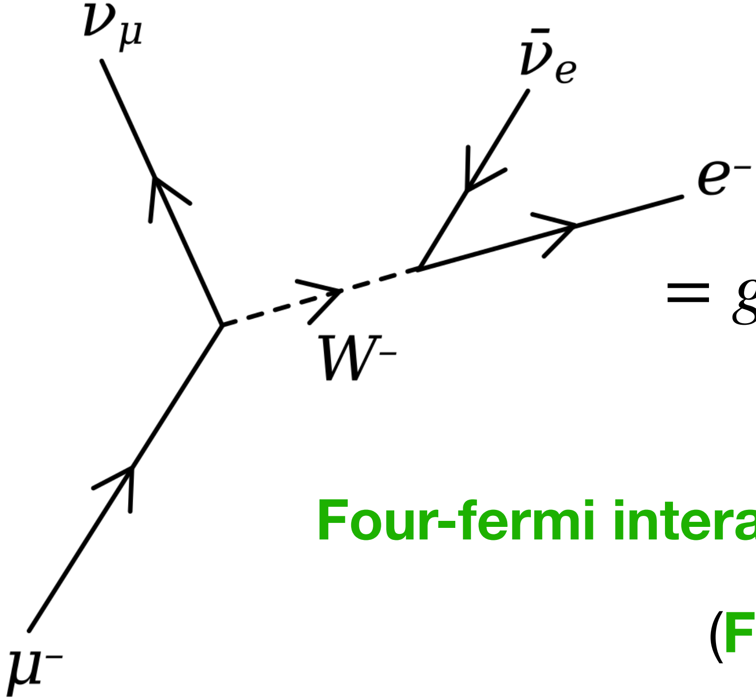




Muon decay “calculation”



$k \ll m_W^2$

$$= g^2 (\bar{\nu}_\mu \gamma_L^\lambda \mu) \frac{-i}{k^2 - m_W^2} (\bar{\nu}_e \gamma_{L\lambda} e) \approx i \frac{g^2}{m_W^2} (\bar{\nu}_\mu \gamma_L^\lambda \mu)(\bar{\nu}_e \gamma_{L\lambda} e)$$

Four-fermi interaction is an effective field theory for weak decays.

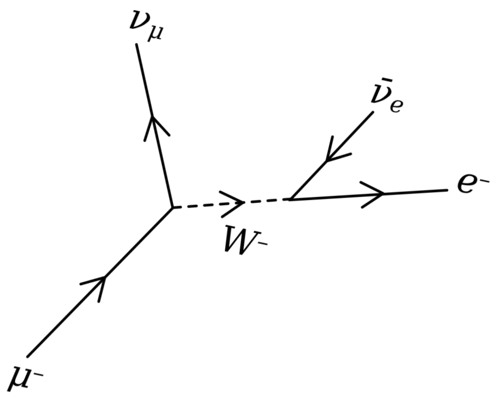
(**Fermi theory)**

We can now use *dimensional analysis* to estimate the decay width Γ.

Dimensional analysis estimate of muon life-time

$\Gamma = 1/\tau_{lifetime}$ has dimensions of Energy

$$\Gamma \propto \Big|$$



$$\Big|^2 \approx \frac{g^4}{m_W^4}$$

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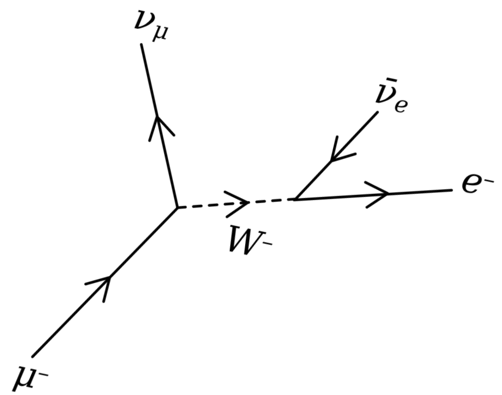
$\Gamma \propto \Bigg|$ 

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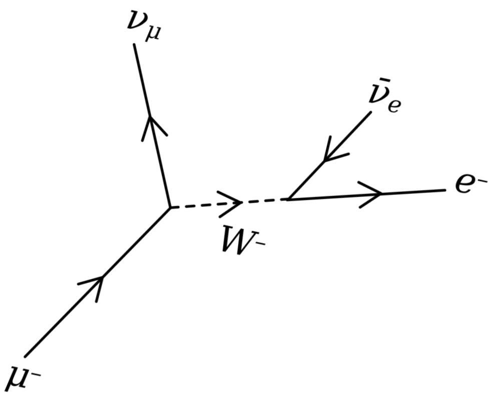

$\Big|^2 \approx \dfrac{g^4}{m_W^4} \Rightarrow \Gamma \sim \dfrac{g^4}{m_W^4} m_\mu^5$

Full calculation: $$\Gamma(\mu) = \left(\frac{g^4}{32 m_W^4}\right)\frac{m_\mu^5}{192\pi^3} = G_F^2 \frac{m_\mu^5}{192\pi^3}$$

$= G_F^2$

Muon lifetime: $\Gamma \sim 10^{-19} GeV$ or $\tau = 1/\Gamma \sim 10^{-6} s$

Muon decay vs. neutron decay

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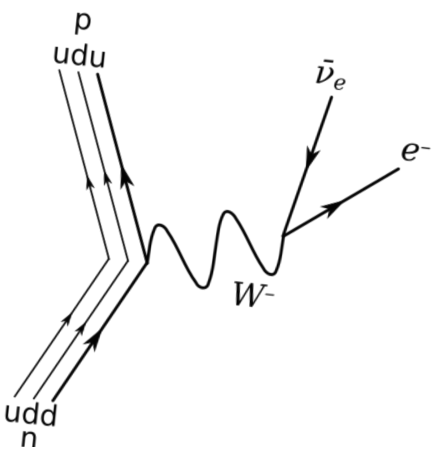
neutron lifetime: What replaces m_μ ? The energy released in neutron decay!



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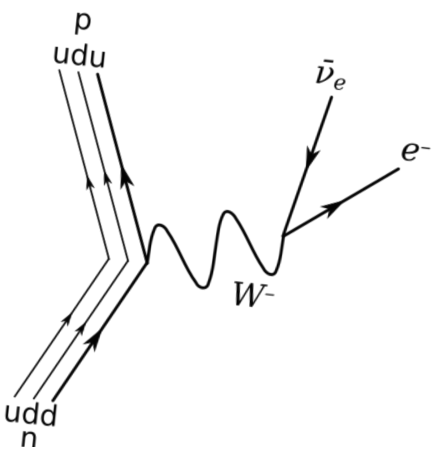

$$\mathscr{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \gamma_L^\lambda (i\partial_\lambda - g \begin{pmatrix} 0 & W_\lambda^+ \\ W_\lambda^- & 0 \end{pmatrix}) \begin{pmatrix} u \\ d \end{pmatrix} = g\bar{u}\,\gamma_L^\lambda W_\lambda^+ d + \ldots$$

$$\Gamma(n) \approx \xi\, G_F^2 \frac{(m_n - m_p)^5}{\pi^3} \approx 10^{-28}\,\text{GeV} \qquad \tau = 1/\Gamma \sim 10^3 s$$

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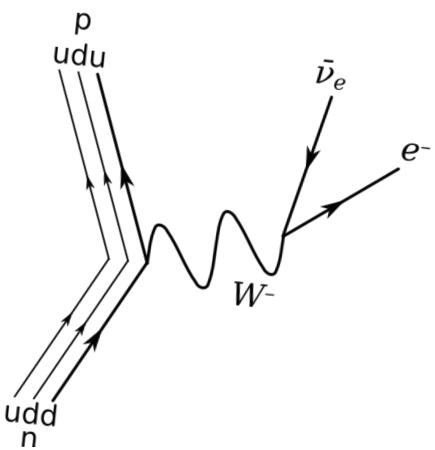

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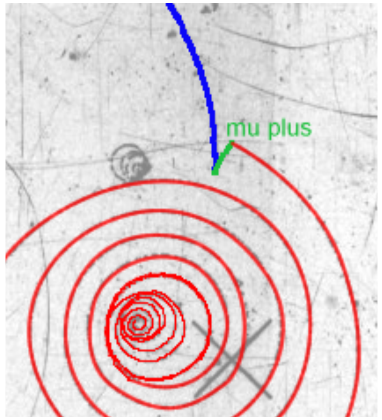

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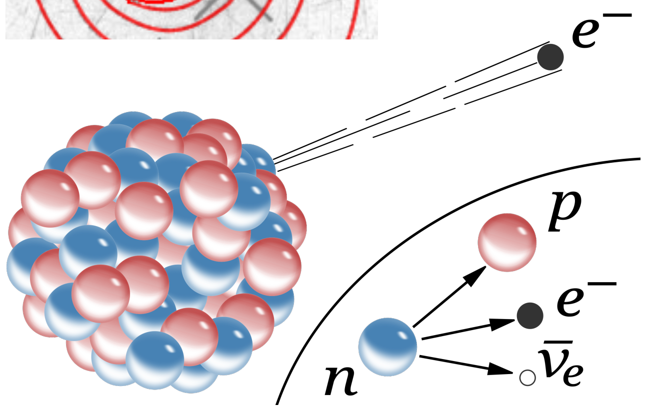

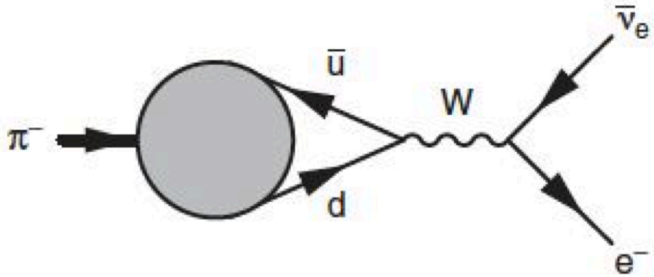

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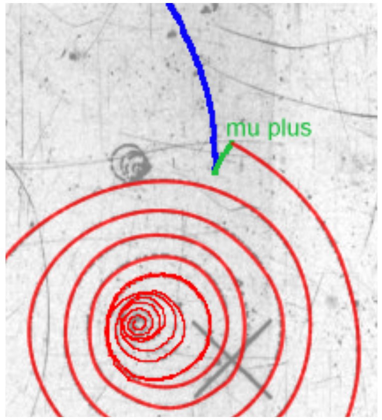

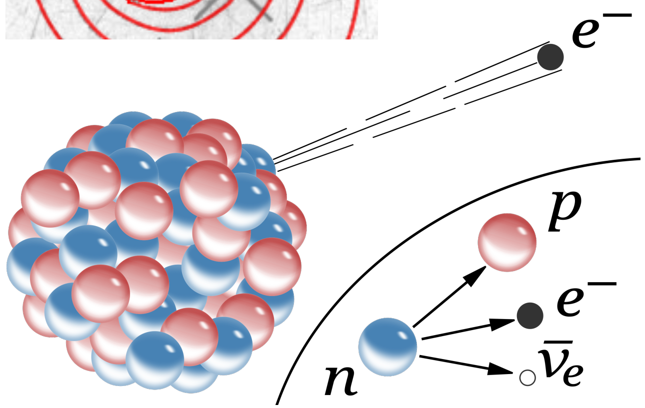

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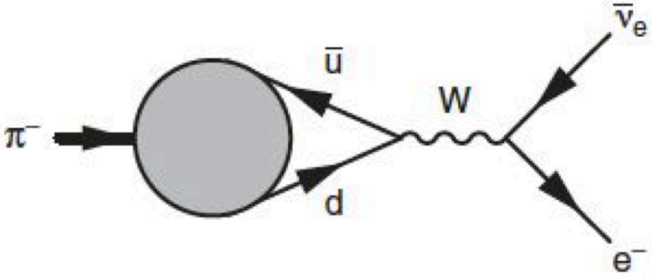
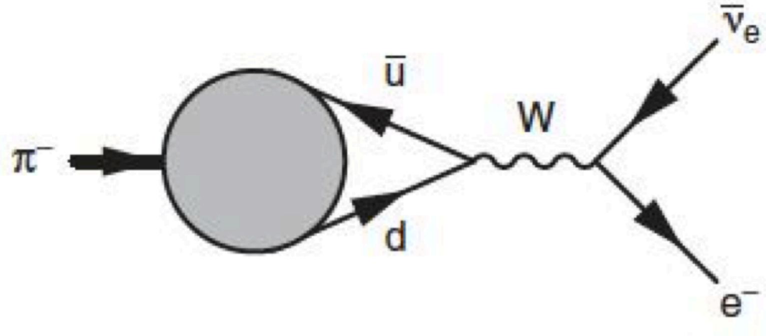


Charged pion decay



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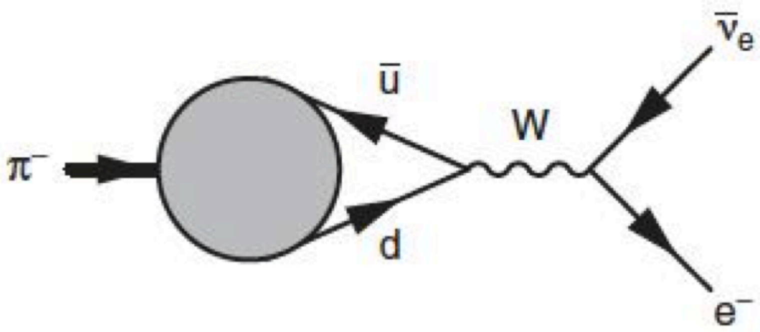
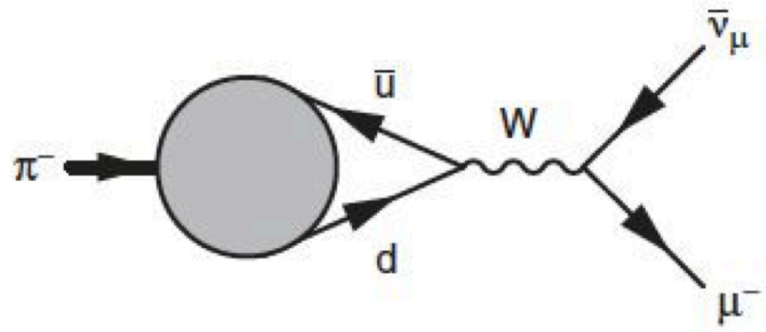
The paradox: charged pion decay

We measure a lifetime: $\tau \approx 10^{-8}\, s$

Two contributions:





How important do we expect each one to be?

Using dimensional analysis, we predict

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_e)} = \frac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5} \sim 10^3 \quad \text{vs. experiment} \quad \frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_e)}\Big|_{EXP} \approx 10^{-4}$$

Chirality of weak interactions

Why is pion decay different than to muon & neutron decays?

Pion is a scalar!   Muon and neutron are spin 1/2 fermions.

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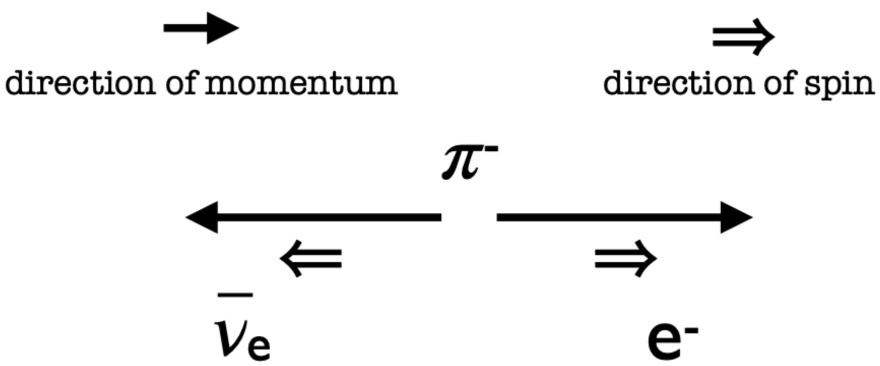

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If the original **interaction was left-handed** then we need a mass term insertion!

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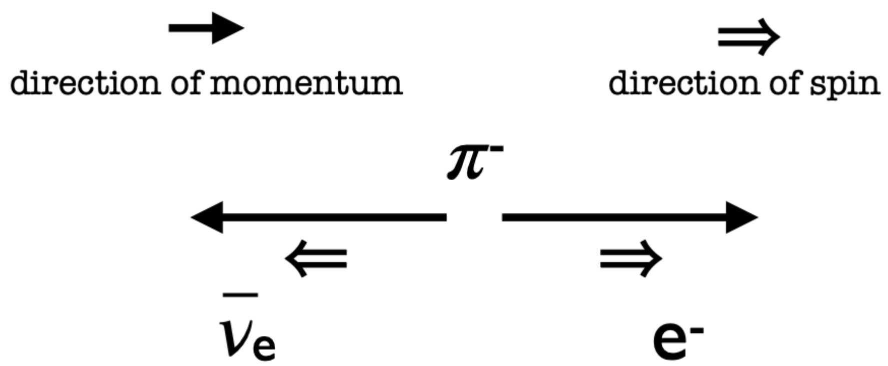

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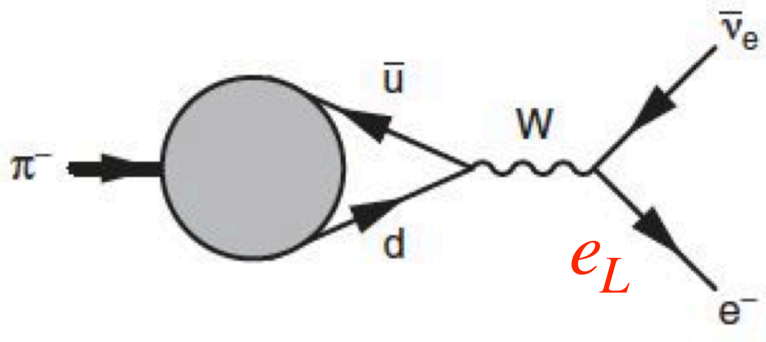
$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$



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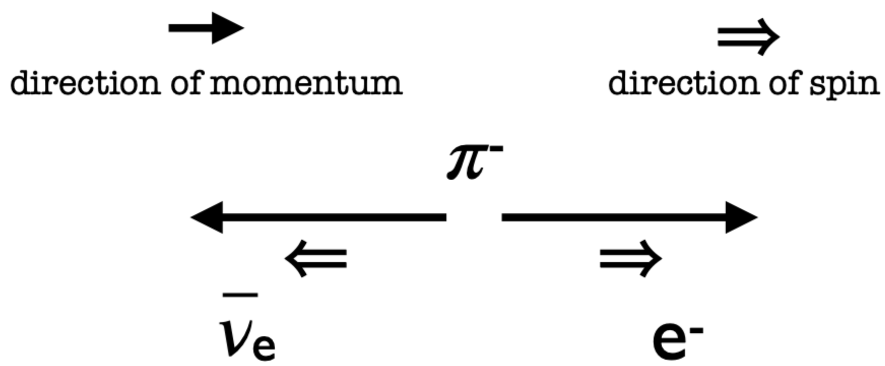

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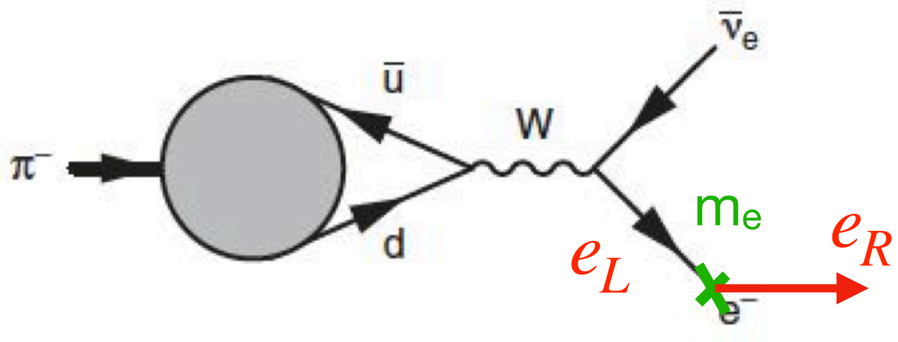

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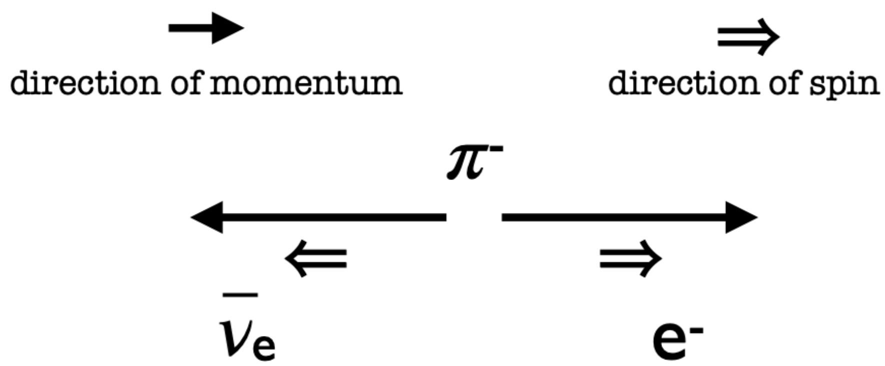

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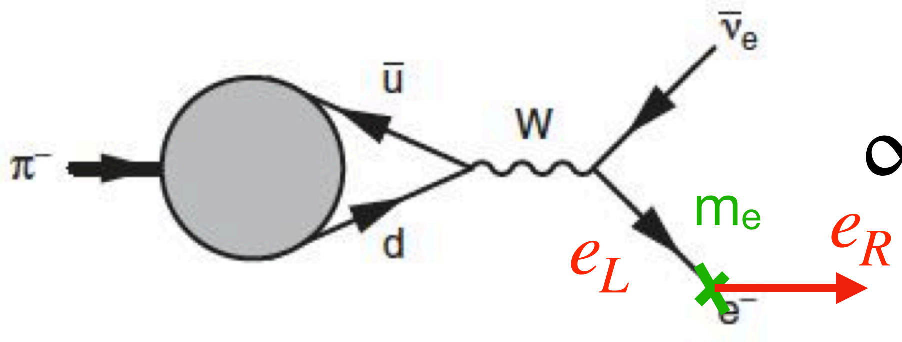

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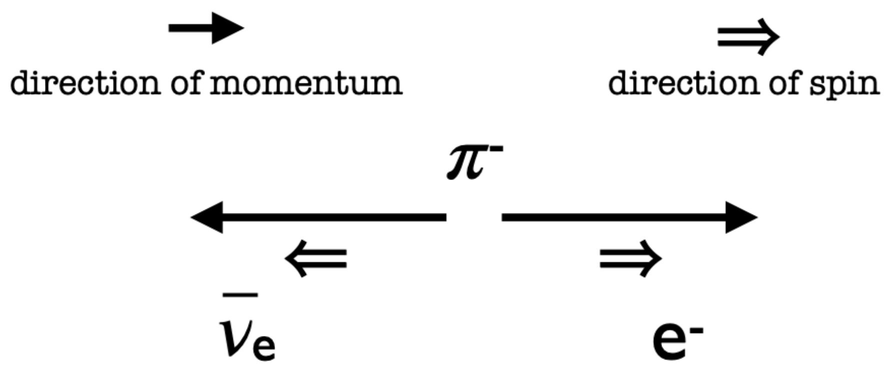

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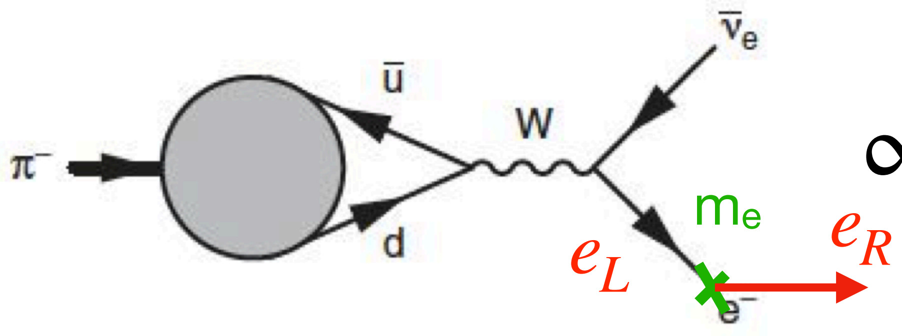

$\propto m_e$ ⇒

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4}_{\text{obs}}$$

phase space factor included

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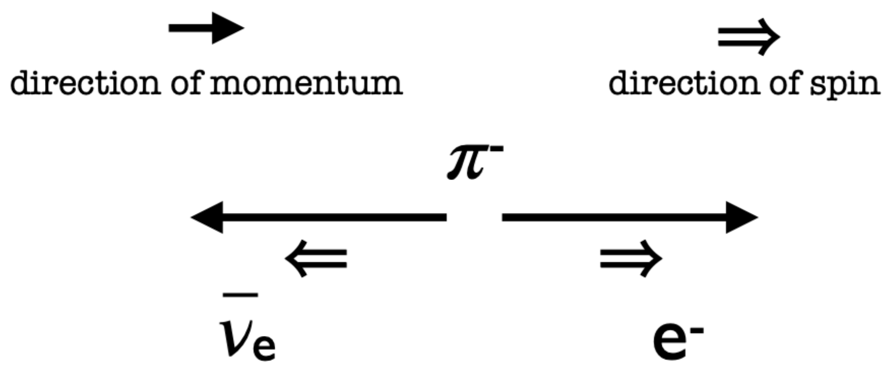

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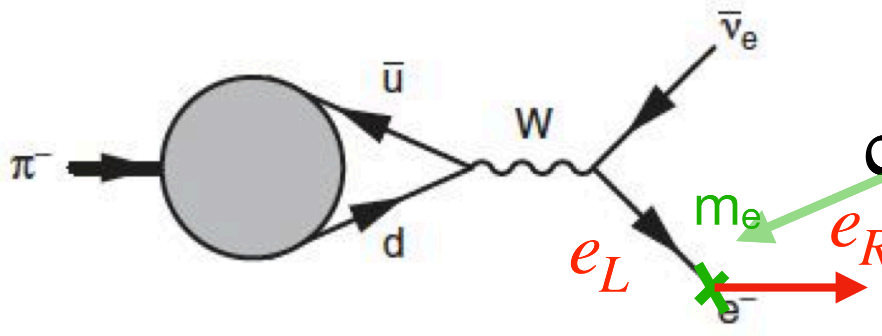

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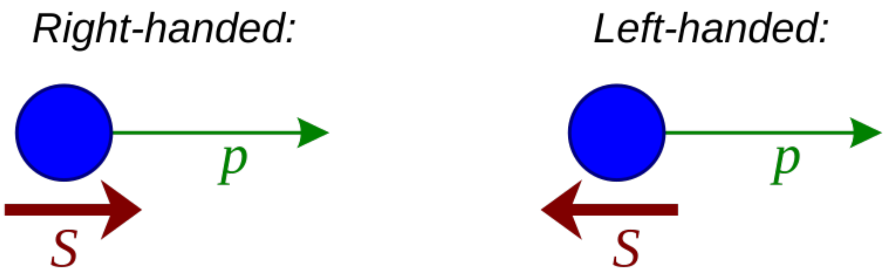
Chirality and Dirac equation

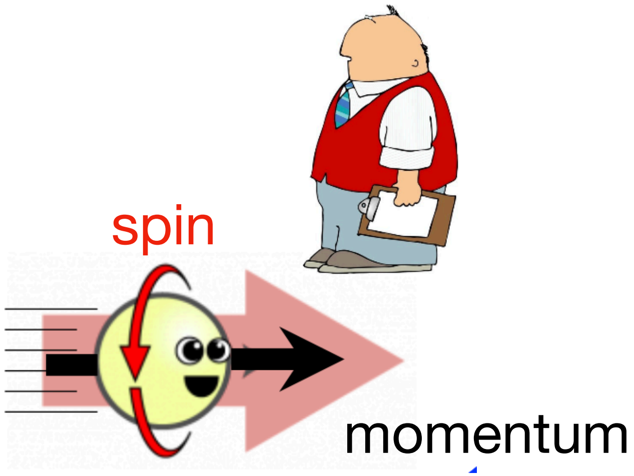


$$\mathscr{L} = \overline{\psi}(x)\,\gamma^{\mu}(i\partial_{\mu} - m)\,\psi(x) \qquad \Psi(x) = \Psi_L(x) + \Psi_R(x)$$

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^{\mu} \partial_{\mu} \psi_L + \bar{\psi}_R \gamma^{\mu} \partial_{\mu} \psi_R + m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

L and R chirality do **not** talk to each other if fermion is **massless (m=0)**!

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$



massive right-handed
spinor

(velocity < c)

picture curtesy Gian Giudice

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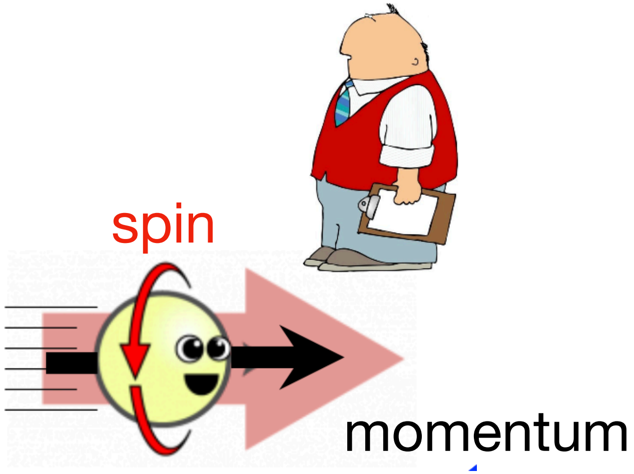
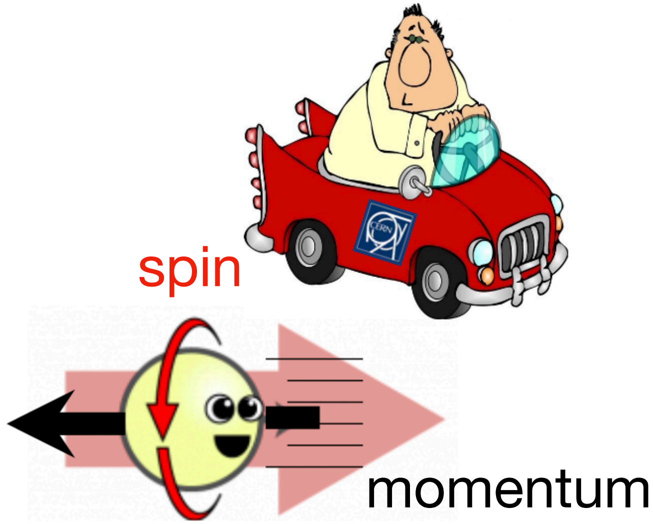


massive right-handed
spinor

(velocity < c)

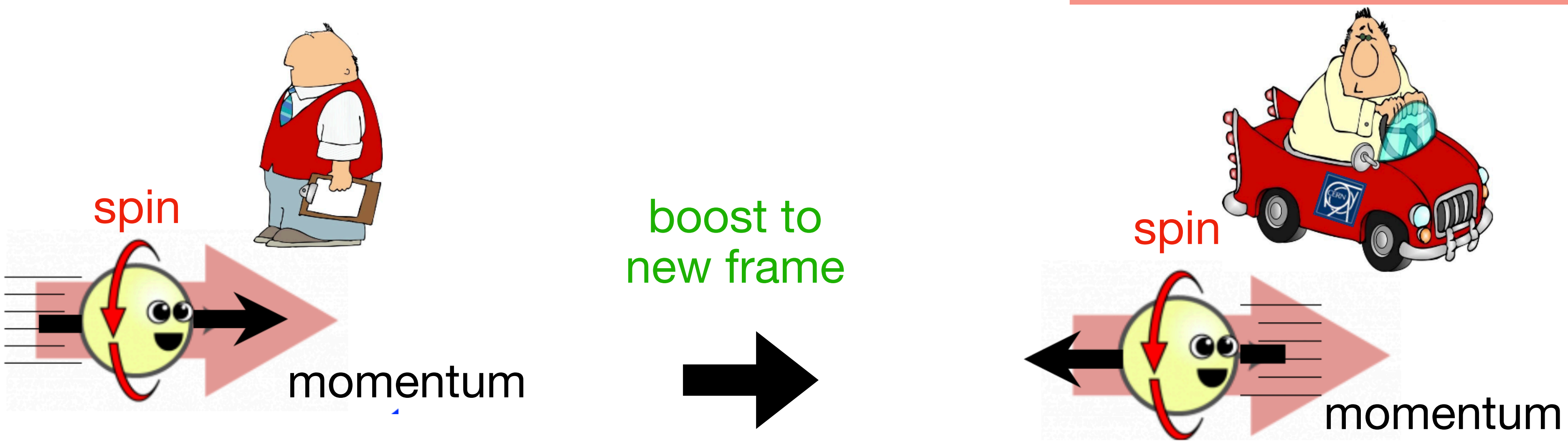
boost to
new frame



right-handed massive
spinor

picture curtesy Gian Giudice

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$



massive right-handed spinor

right-handed massive spinor

(velocity < c)

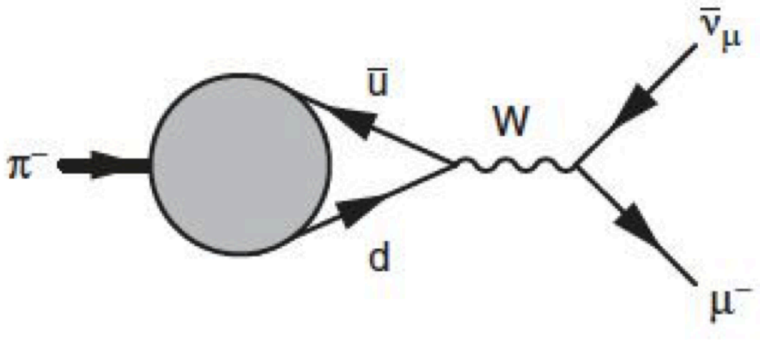
Conclusion: If our theory is able to distinguish left-handed and right-handed fermions, then we must have $m_e = 0$ (or give up on theory?).

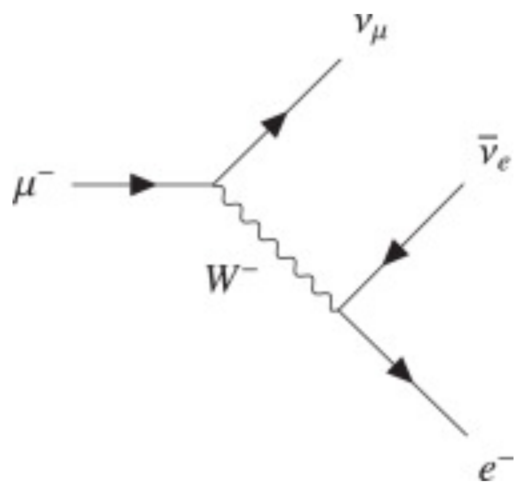
picture curtesy Gian Giudice

Muons are polarized

Clever idea by Lederman/Garwin (57)

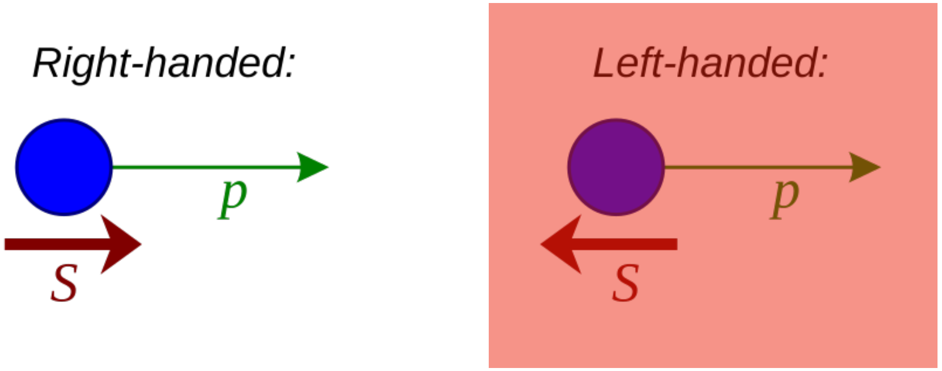
Decay pion to muon and investigate the muon decay products





The angular distribution of the electrons reveals the chirality of the interaction.

Chirality and handedness



The weak interactions only talk to left-handed particles!

Historically found this in decay of Cobalt (C.S. Wu '56)

$^{60}_{27}\mathrm{Co} \rightarrow ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$ **only left-handed (LH) e- produced**

Weak interactions: neutral boson

W_3 is charge neutral

Let's extend this to SU(2)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^{\pm} = \sigma^1 \pm i\sigma^2$$

$$\mathscr{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \gamma_L^{\lambda}(i\partial_{\lambda} - g \begin{pmatrix} W_{\lambda}^3 & W_{\lambda}^+ \\ W_{\lambda}^- & -W_{\lambda}^3 \end{pmatrix}) \begin{pmatrix} u \\ d \end{pmatrix}$$

Could the ***W_3*** be the photon?

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Could the ***W_3*** be the photon?

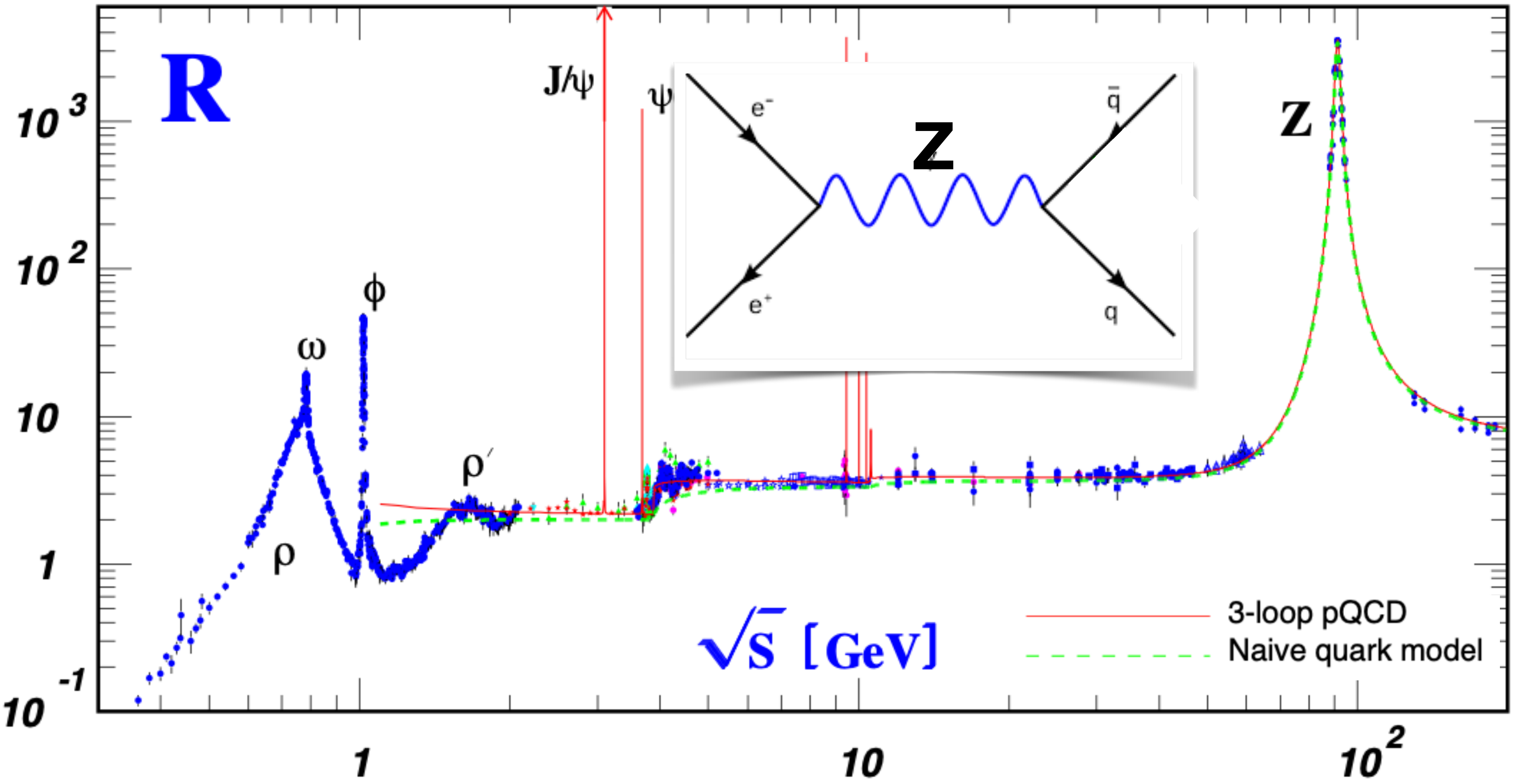
No. Would e.g. predict +1 charge for up, -1 charge for down.

New massive neutral gauge boson: this will turn into the Z-boson!

This was also called "a new neutral current process", discovered at CERN.

You have already seen the Z boson!

91.2 GeV



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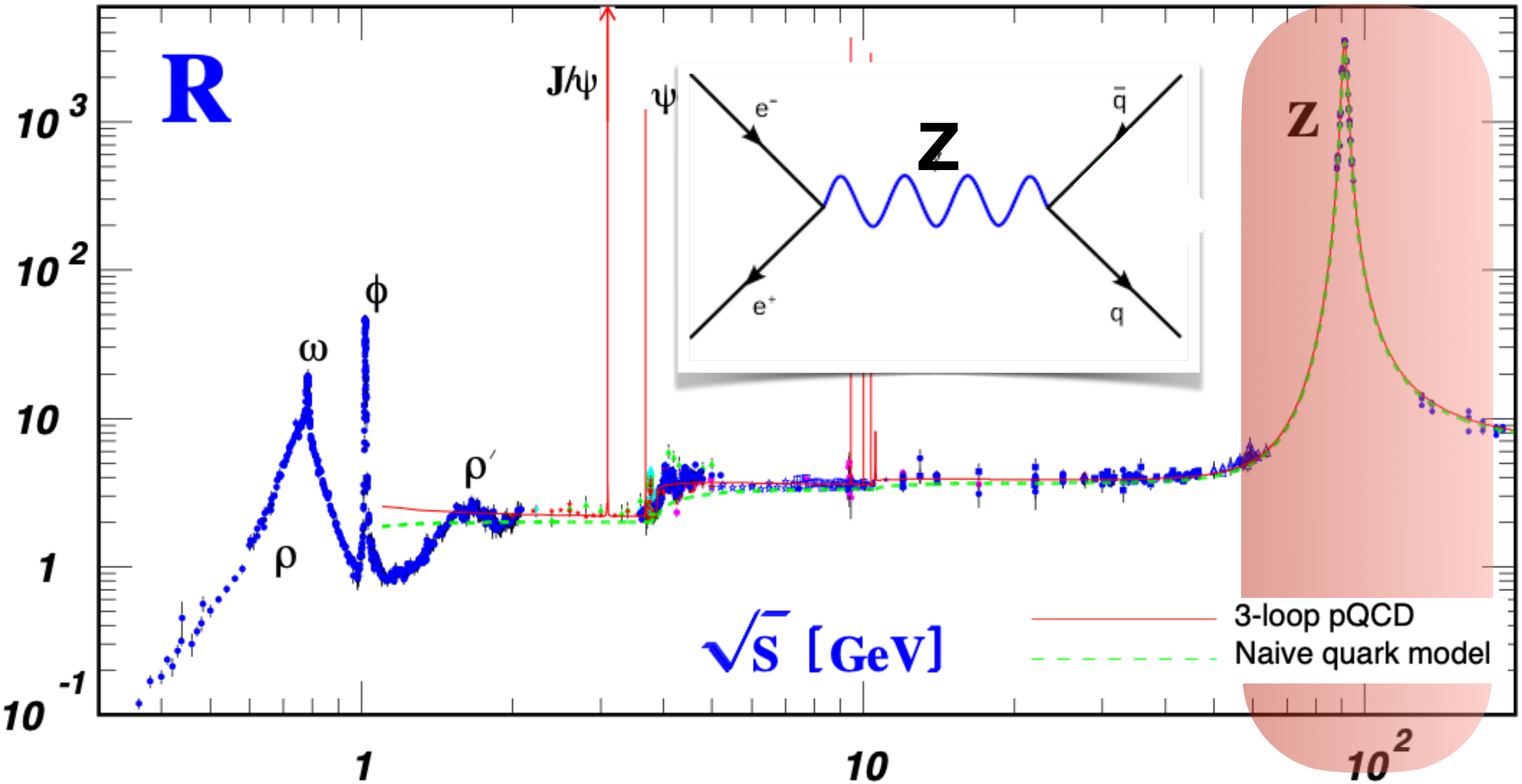
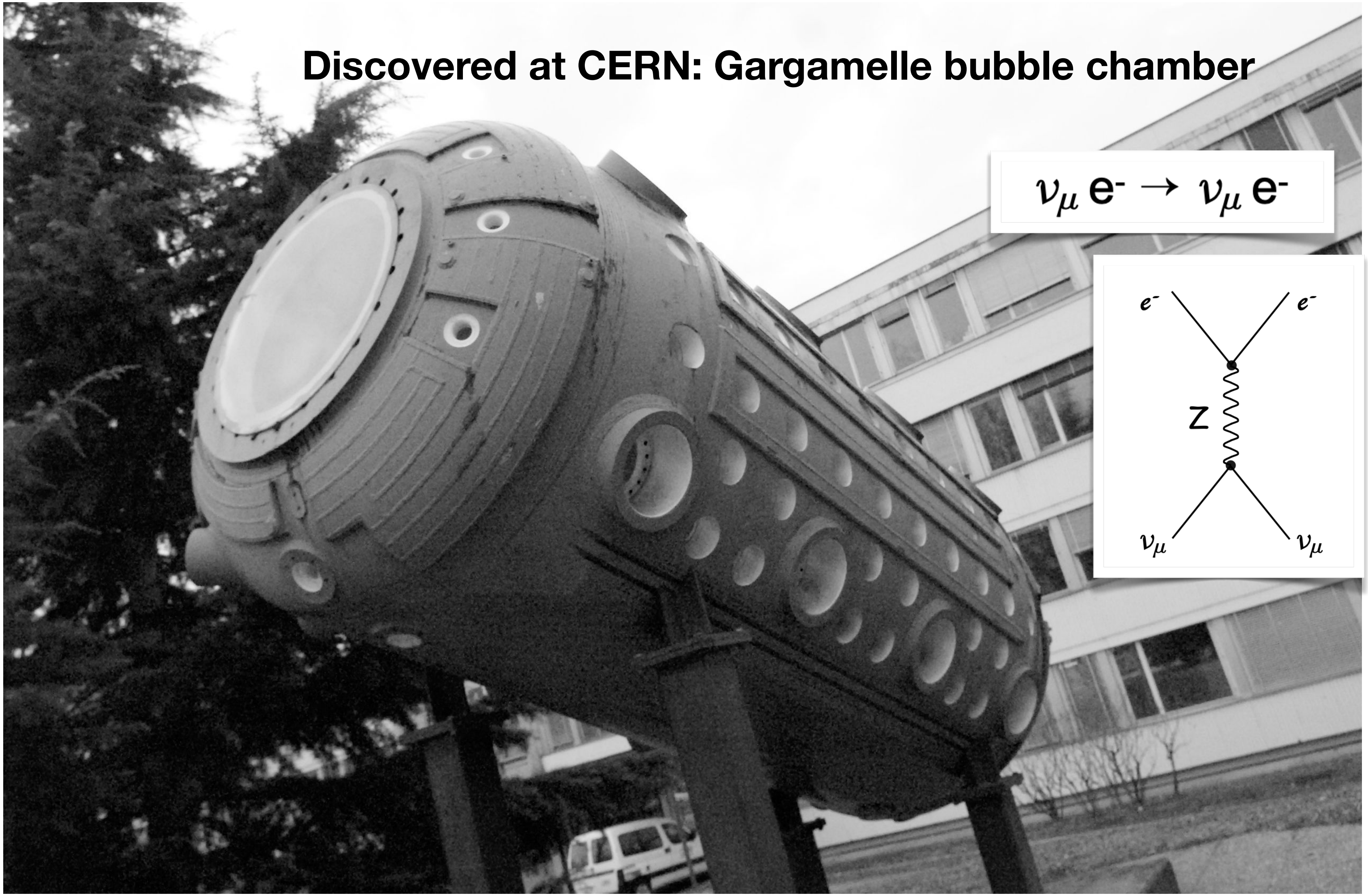


Discovered at CERN: Gargamelle bubble chamber



In front of the CERN theory group.

Weak interactions paradoxes

We need non-abelian gauge theory, but gauge bosons need to be massive!
Not SU(2) gauge invariant?

$$M_W^2 W_\mu^+ W^{\mu -} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

Since the weak interactions are chiral (only the **left-handed** particles are charged), the **left-handed** and **right-handed** particles are fundamentally different.
How can they have a mass term?

SU(2) $\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R \qquad d_R$

vs. $m(\bar{u}_L u_R + \bar{u}_R u_L)$ **not** SU(2) invariant !

Next lecture