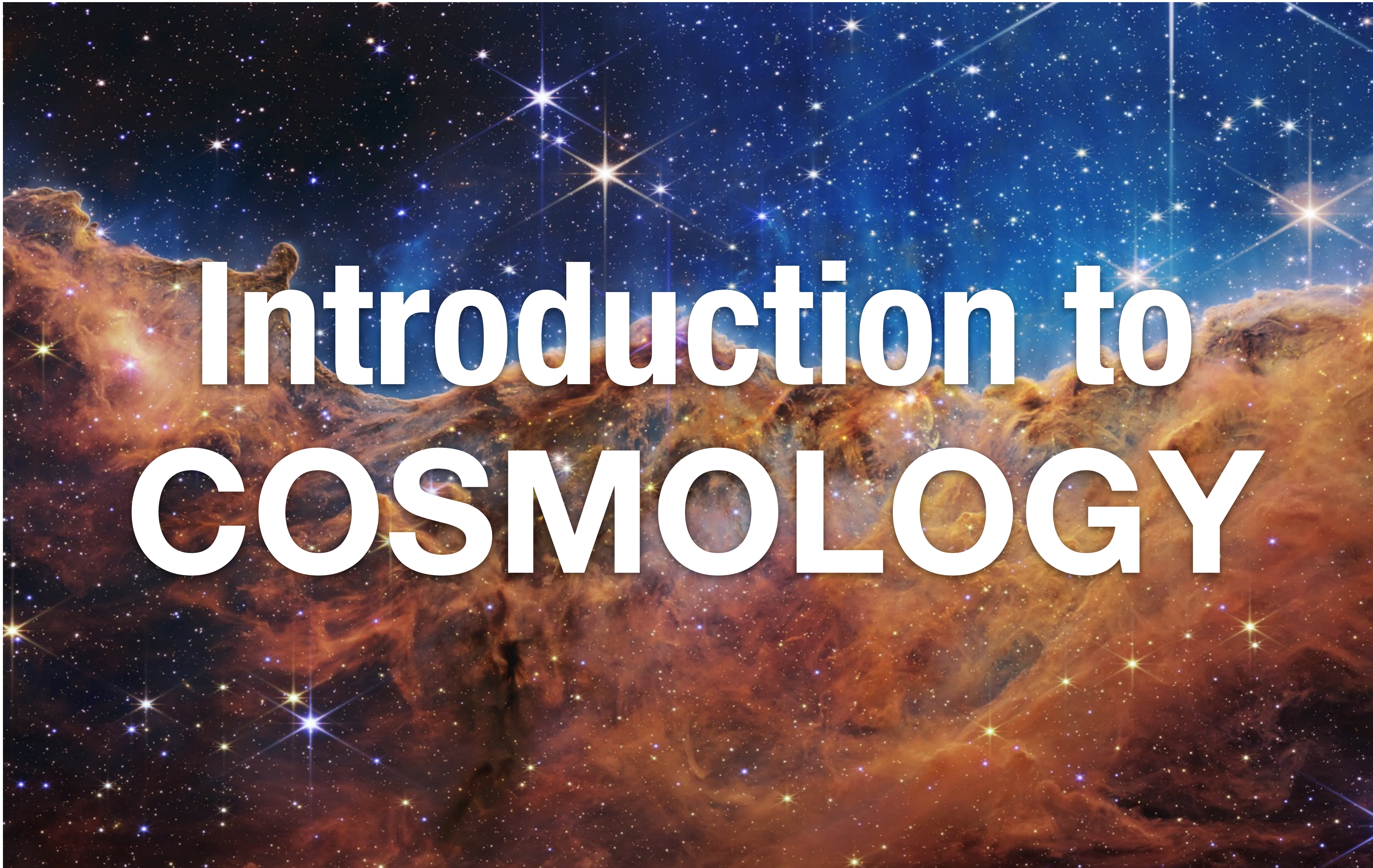

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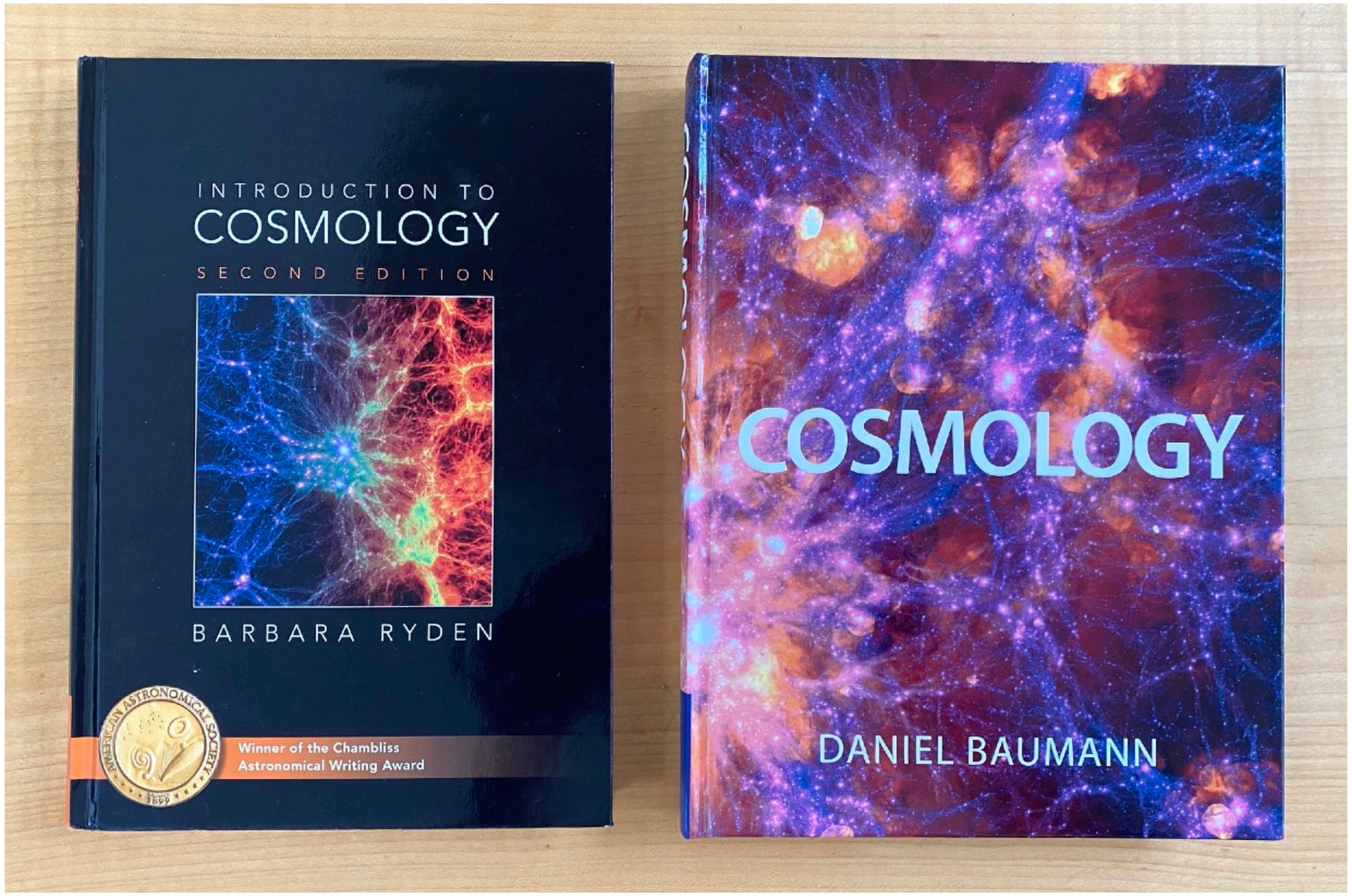
CERN Lectures
July 2024

Disclaimer:

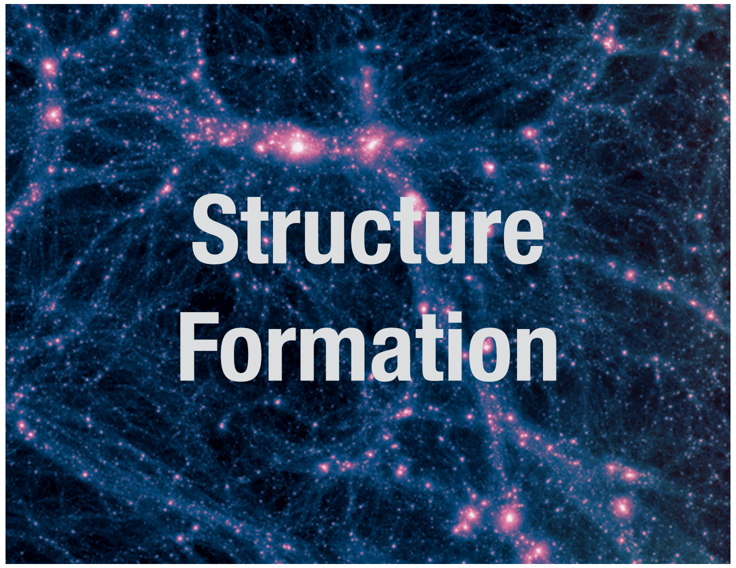
Teaching all of cosmology in 3 hours is an impossible task.
I will focus on the big picture and suppress technical details.

More can be found in:

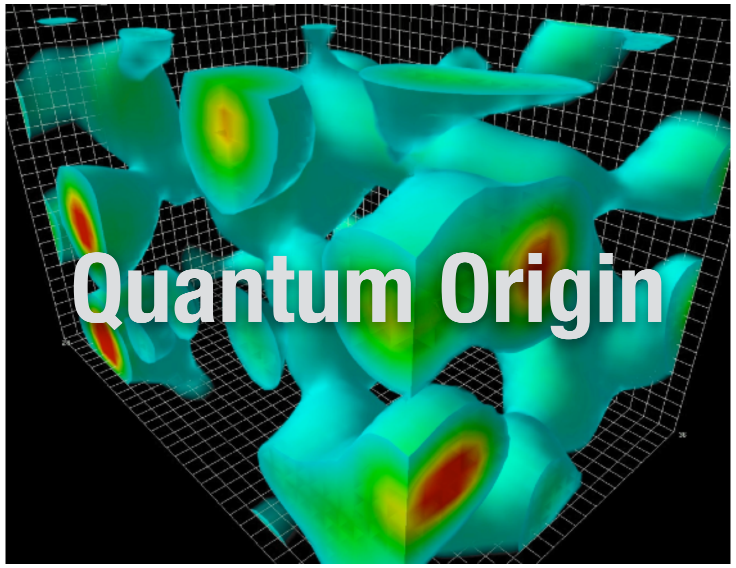


OUTLINE:

- Hubble-Lemaitre law
- Friedmann equation
- Matter and radiation
- Dark energy
- Nucleosynthesis
- Cosmic microwave background



- Density fluctuations
- Gravitational clustering
- Galaxy formation
- Power spectrum
- Cosmic sound waves
- CMB anisotropies



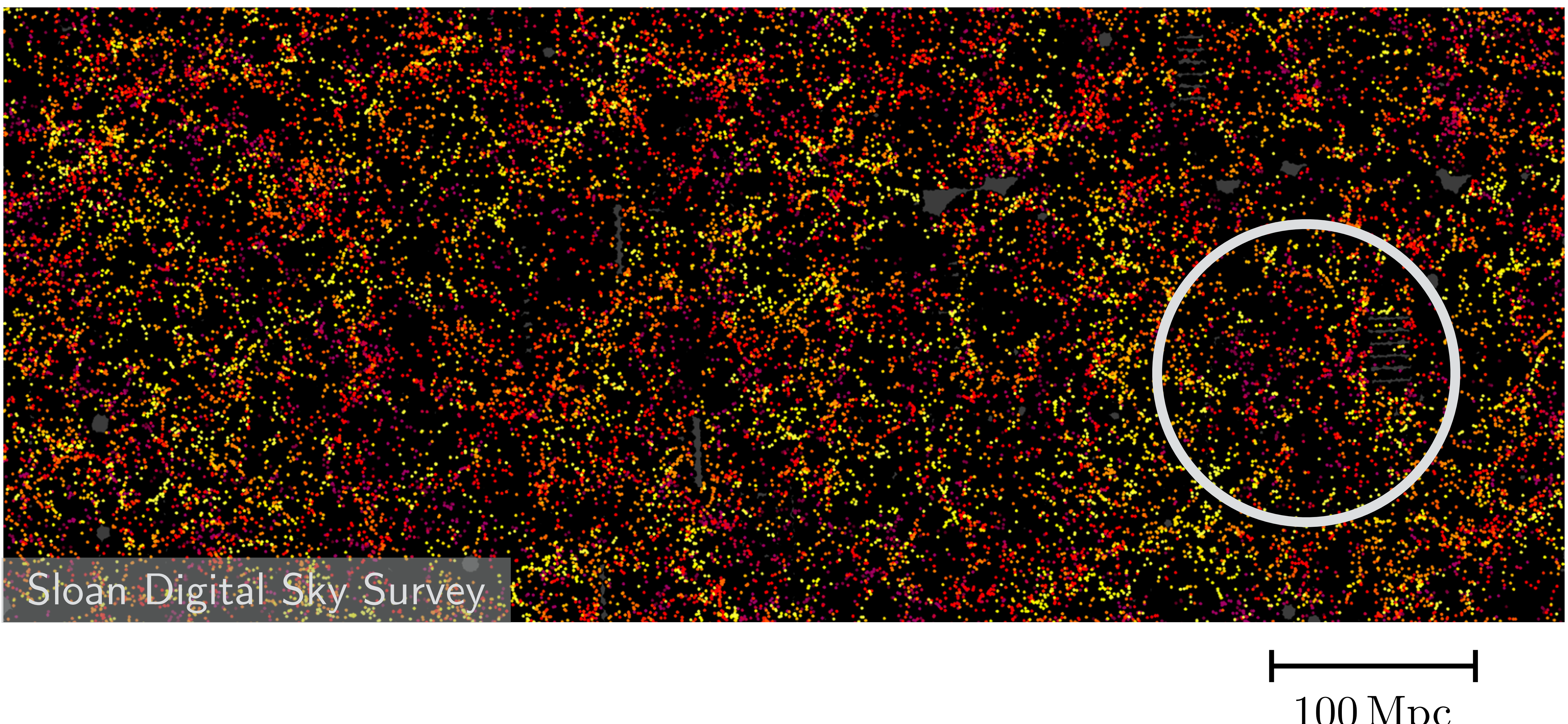
- Horizon problem
- Cosmological inflation
- Quantum fluctuations
- Primordial perturbations

The Expanding Universe

Goal of this Lecture

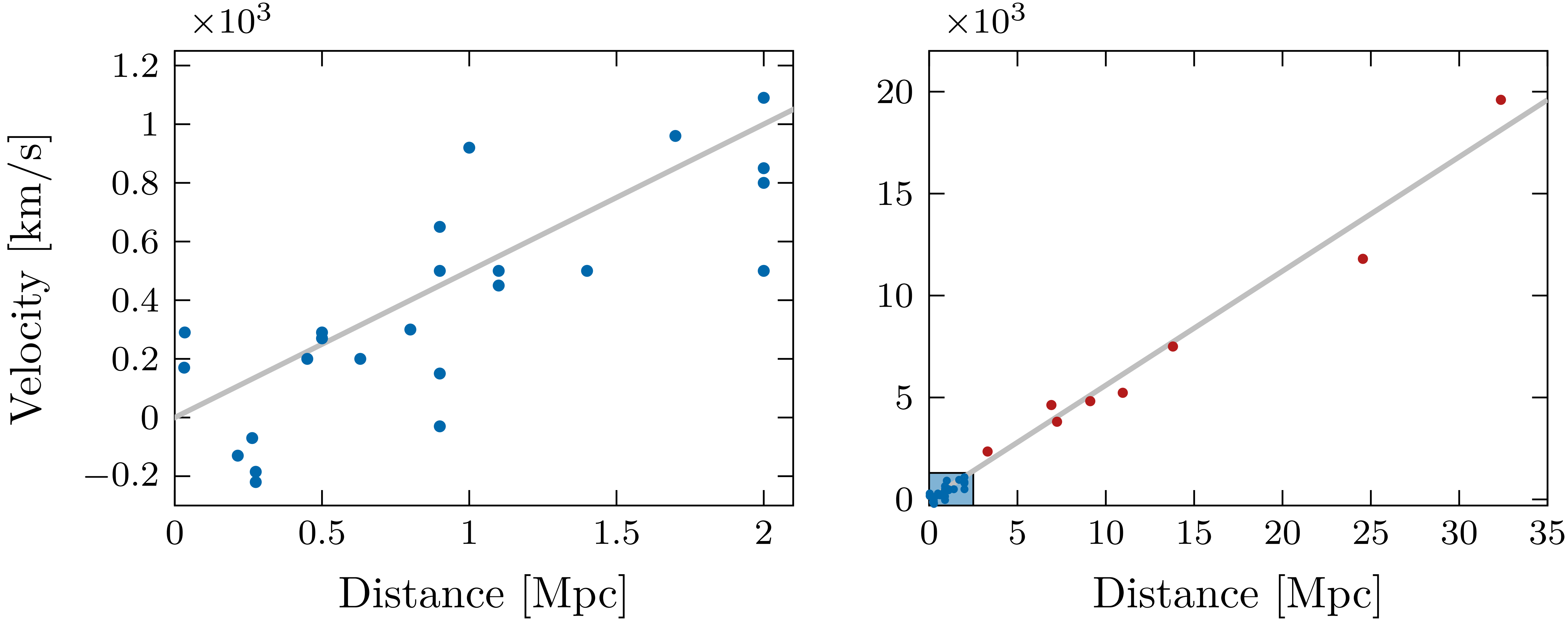
We wish to derive, and then solve, the equation governing the evolution of the entire Universe.

This is possible because, on large scales, the Universe is homogeneous and isotropic, and therefore allows for a simple mathematical description.



Hubble's Law

In 1929, Hubble discovered the **velocity-distance relation of** galaxies:



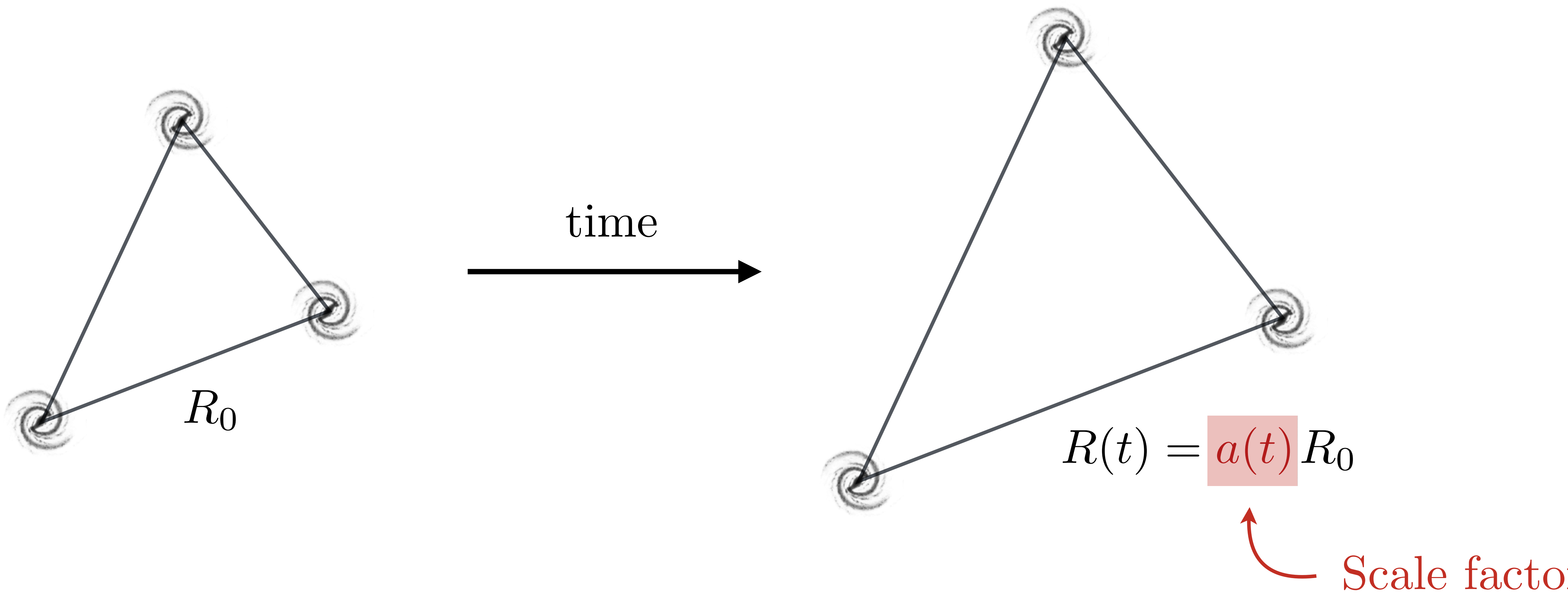
$$V = H_0 R$$

Inferred from redshifts ⟶ V

R ⟵ Inferred from luminosities

Hubble's Law

In General Relativity, this can be interpreted as the expansion of space:



$$V \equiv \dot{R} = \frac{\dot{a}}{a} R \equiv H R$$

Hubble parameter

Hubble's Law

The observed value of the **Hubble constant** is

$$H_0 = (68 \pm 2)\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

where $\mathrm{Mpc} = 3.2 \times 10^6$ light-years.

The corresponding **Hubble time** and **Hubble distance** are

$$t_H \equiv H_0^{-1} = (14.38 \pm 0.42)\,\mathrm{Gyrs}$$

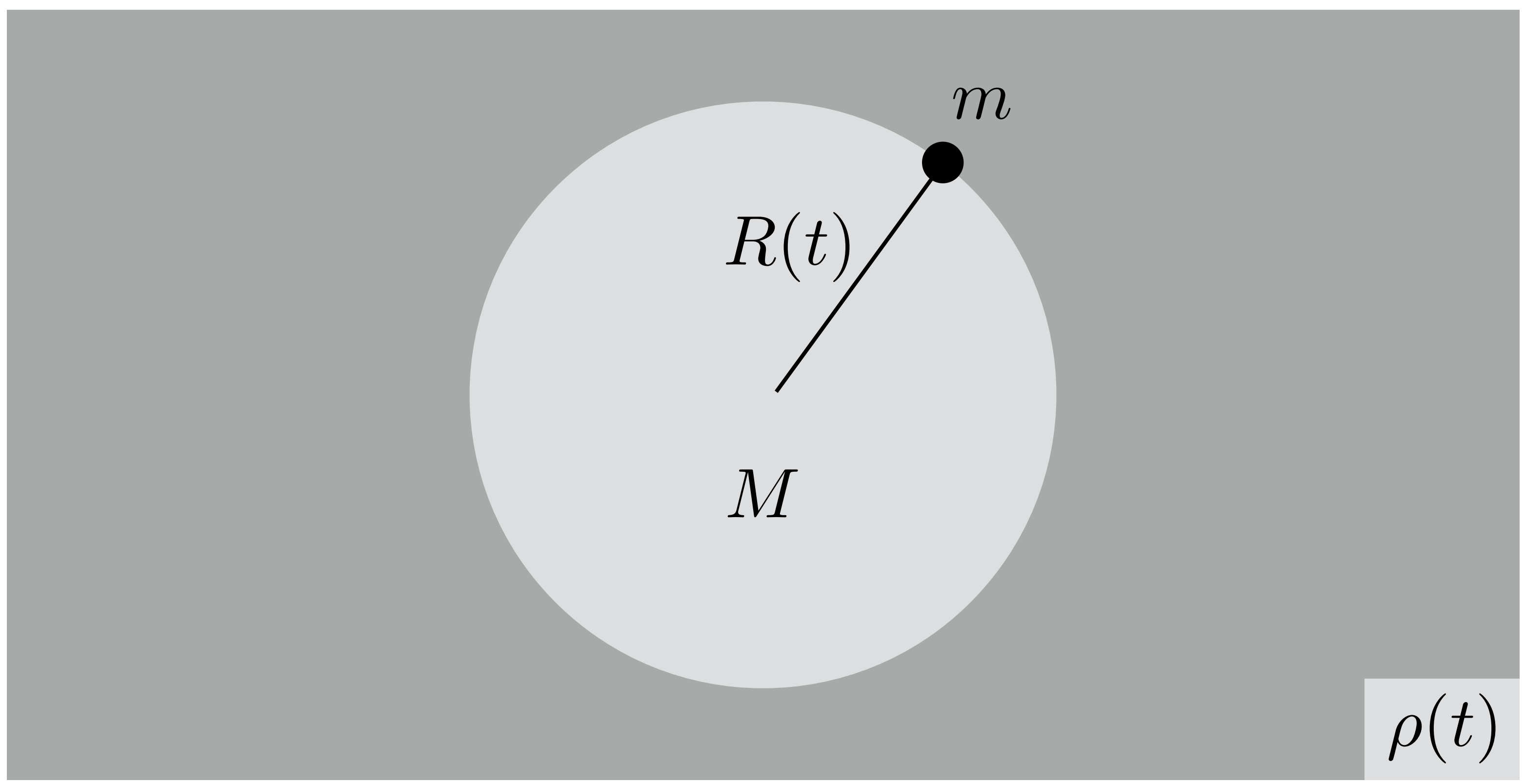
$$d_H \equiv cH_0^{-1} = (4380 \pm 130)\,\mathrm{Mpc} \sim 10^{26}\,\mathrm{m}$$

These are the characteristic **age** and **size** of the universe.

Friedmann Equation

Let us now determine the **evolution of the scale factor**.
We will do this using Newtonian gravity (but comment on GR corrections).

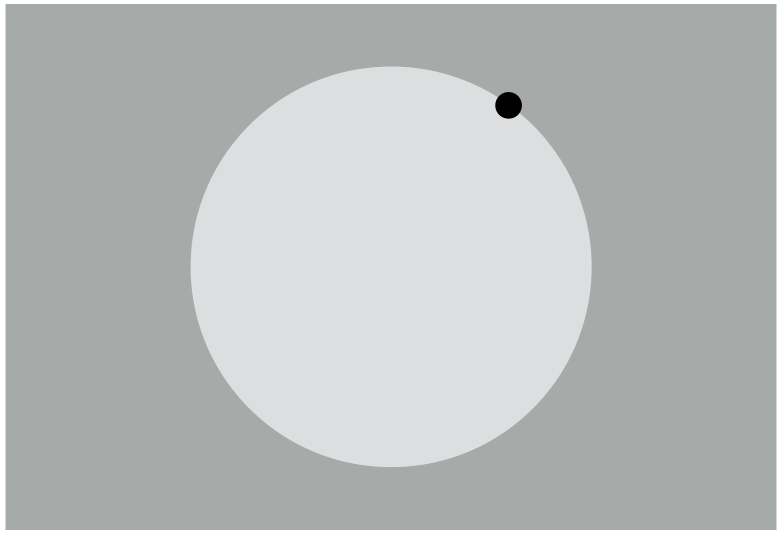
Consider a spherical region in a homogeneous universe:



The force on a test particle on the surface of the sphere is

$$F = -\frac{GMm}{R^2}$$

Friedmann Equation



The acceleration of the test particle is

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2}$$

Integrating this gives the conservation of energy:

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = U = \text{const.}$$

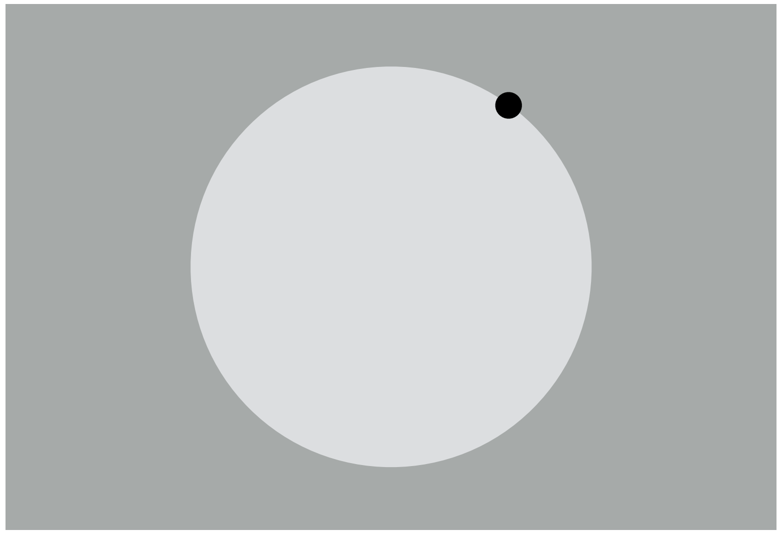
Substituting $M = \frac{4\pi}{3}R^3\rho$ and $R(t) = a(t)R_0$, we find

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2U}{R_0^2}\frac{1}{a^2}}$$

Friedmann Equation

Hubble parameter

Friedmann Equation

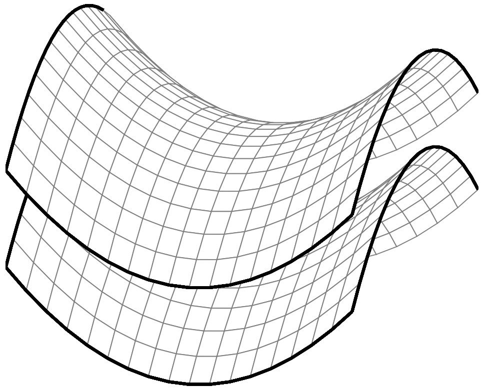


$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2U}{R_0^2}\frac{1}{a^2}$$

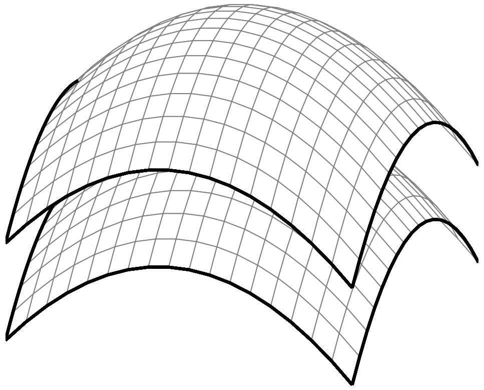
There are three different cases:

$U > 0$	$U < 0$	$U = 0$
expands forever	stops and recollapses	limiting case

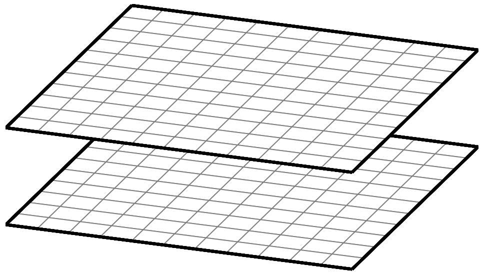
In GR, these correspond to:



negatively curved



positively curved



flat

Observations favor a **flat universe**.

Friedmann Equation

For a flat universe, the Friedmann equation reduces to

$$H^2 = \frac{8\pi G}{3}\rho$$

From the measured value of the Hubble constant, we infer the average **density** of the Universe today:

$$\rho_0 = \frac{3H_0^2}{8\pi G} \quad = \quad 0.8 \times 10^{-29}\,\mathrm{grams\,cm^{-3}}$$

$$= \quad 1.3 \times 10^{11}\, M_\odot\,\mathrm{Mpc^{-3}}$$

$$= \quad 5.1\,\mathrm{protons\,m^{-3}}$$

Friedmann Equation

In GR, the Friedmann equation is derived from the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Curvature tensor   Energy-momentum tensor

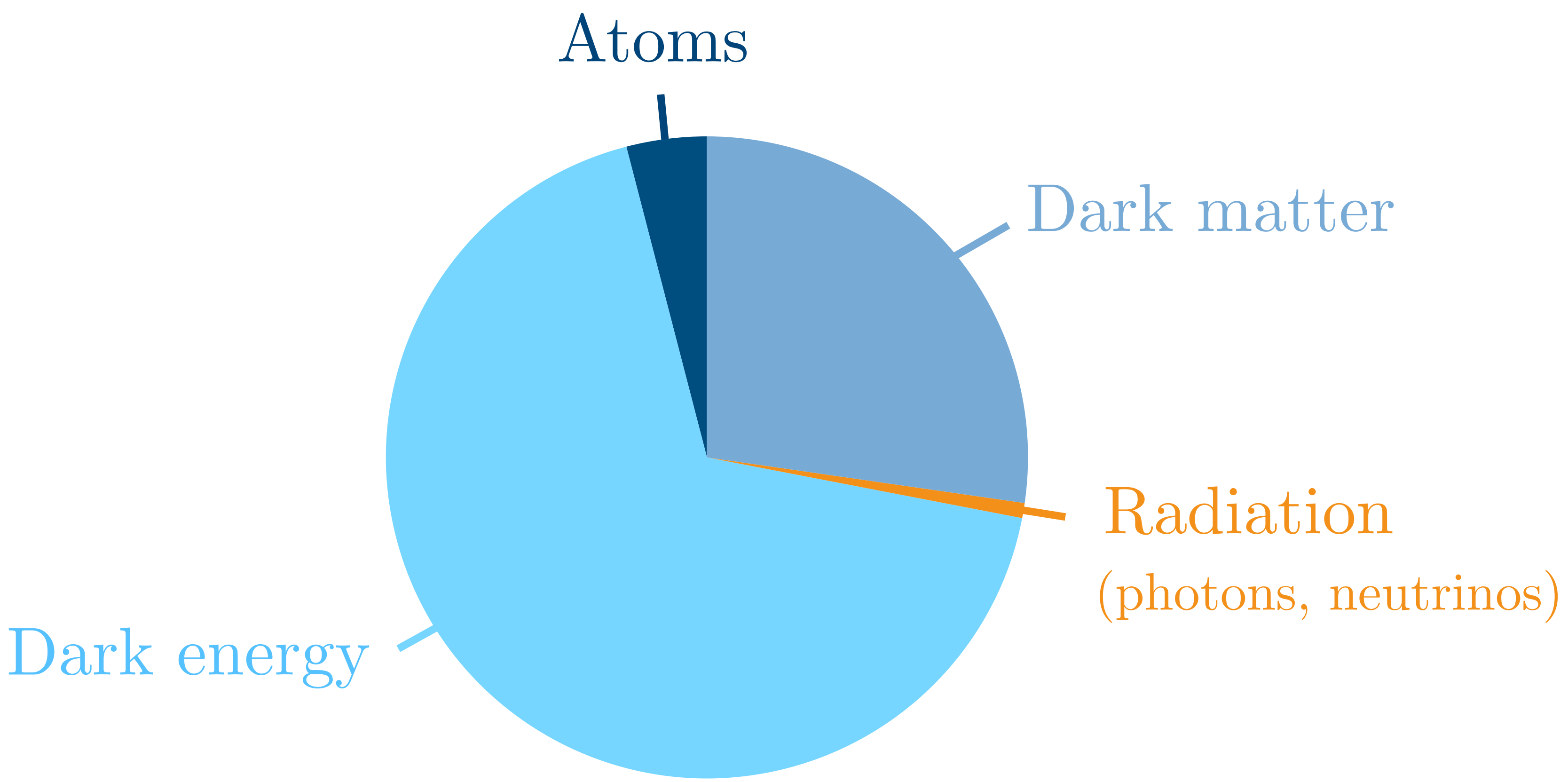
$$\boxed{H^2 = \frac{8\pi G}{3} \rho}$$

where the mass density is replaced by the **energy density** $\varepsilon \equiv \rho c^2$.
(In natural units ($c \equiv 1$), we use ρ and ε interchangeably.)

To solve the Friedmann equation, we need to know how the energy density evolves as the Universe expands.

Cosmic Inventory

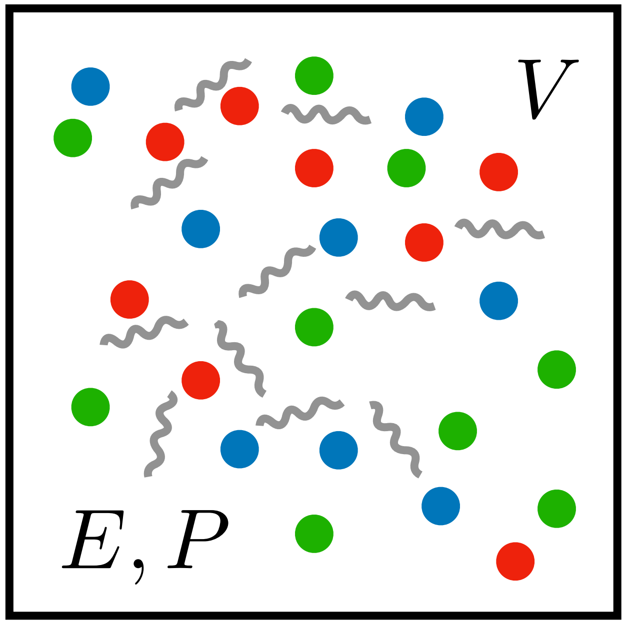
The Universe is filled with four different types of energy:



We need to determine how each of these components evolves and sources the expansion of the Universe.

Fluid Equation

Consider a fluid in a box:



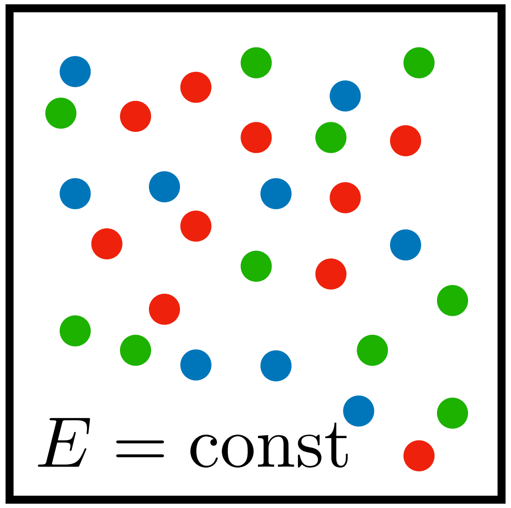
The energy density is $\rho = E/V$

How does this evolve with increasing scale factor?

This can be derived from the First Law of thermodynamics (see appendix), or we can guess it.

Matter

For a pressureless fluid (= **matter**), we get



$$\rho \equiv \frac{E}{V} \propto a^{-3}$$

Most of the matter in the Universe is non-luminous **dark matter**.
Ordinary matter is less than 5%.

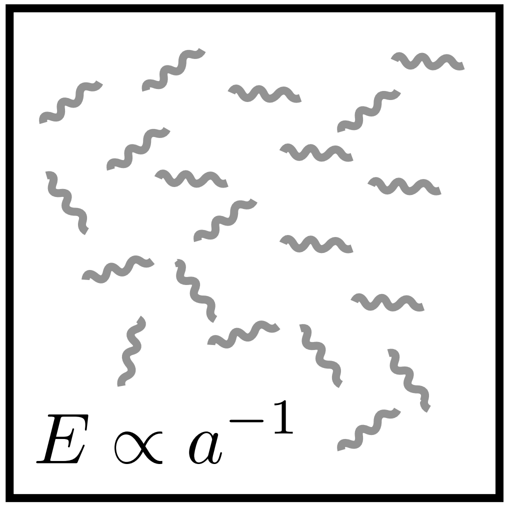
Feeding $\rho \propto a^{-3}$ into the Friedmann equation, we find

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho \propto a^{-3} \longrightarrow a \propto t^{2/3}$$

Radiation

For a relativistic fluid (= **radiation**), we get

$$\rho \equiv \frac{E}{V} \propto a^{-4}$$



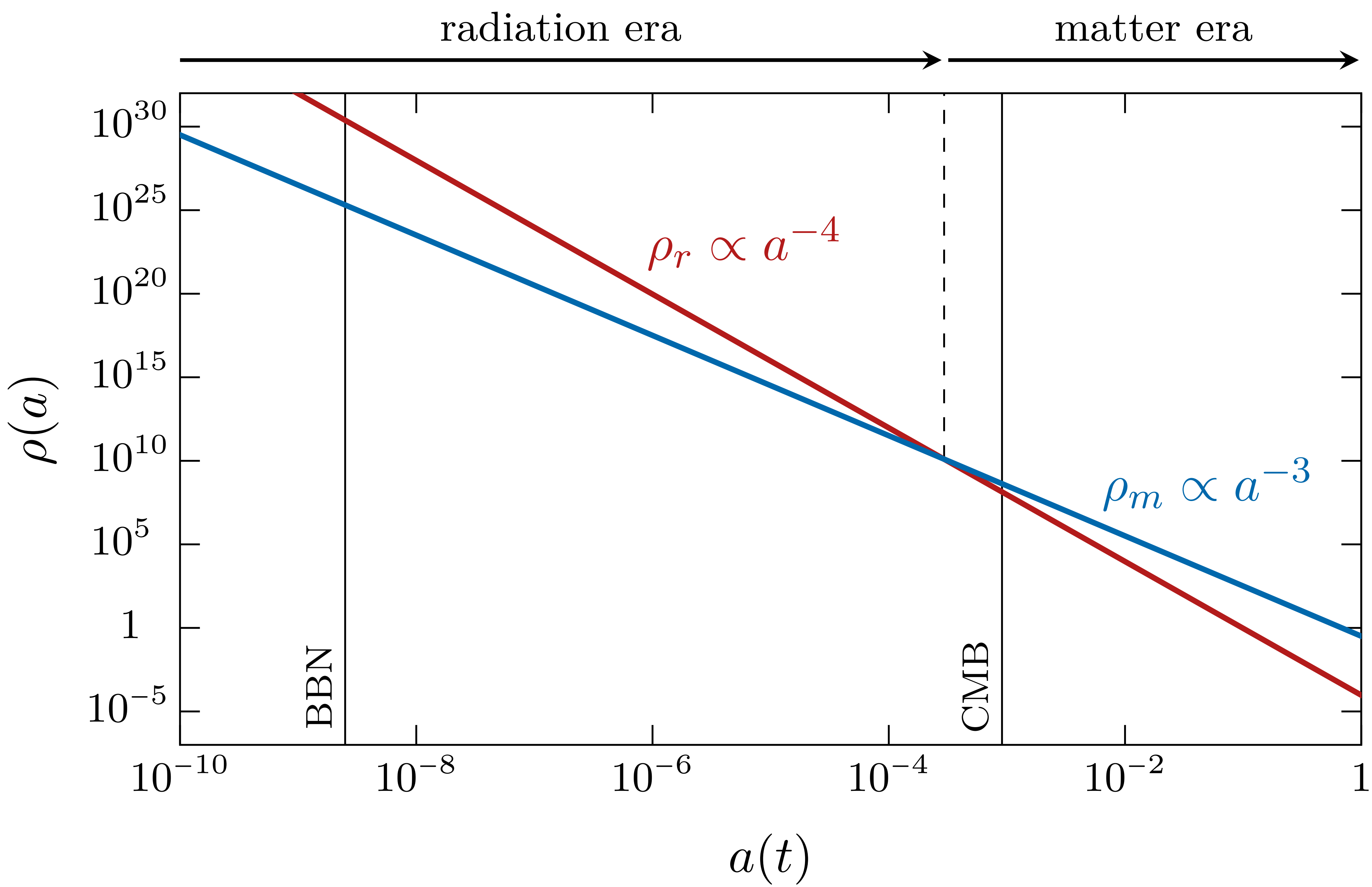
Most of the radiation in the Universe are **photons** from the early universe (= **cosmic microwave background**). Starlight is less than 0.1%.

Feeding $\rho \propto a^{-4}$ into the Friedmann equation, we find

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho \propto a^{-4} \quad \longrightarrow \quad a \propto t^{1/2}$$

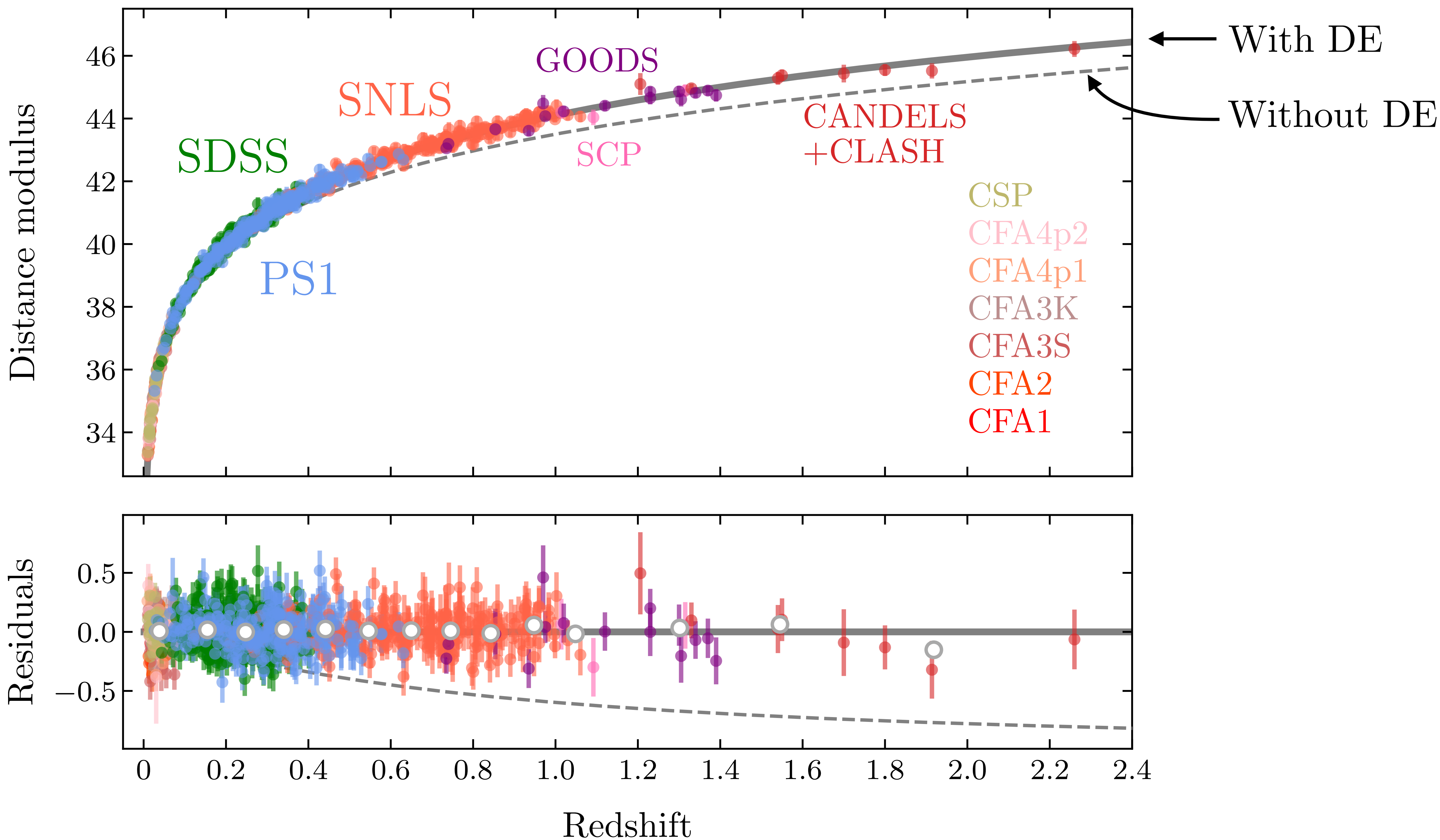
Matter and Radiation

The Universe was first dominated by radiation, then by matter:



Dark Energy

In 1998, it was discovered that the Universe is **accelerating**.
The source of the acceleration is unknown. We call it **dark energy**.



Dark Energy

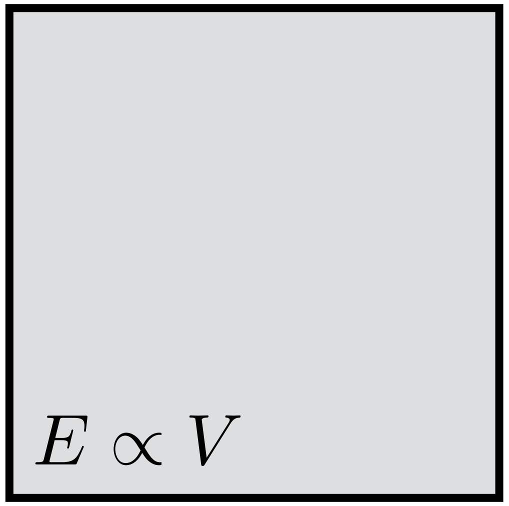
In 1998, it was discovered that the Universe is **accelerating**.
The source of the acceleration is unknown. We call it **dark energy**.

The simplest form of dark energy is a **cosmological constant**:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Moving this term to the RHS, it becomes a **vacuum energy**:

$$\boxed{\rho \equiv \frac{E}{V} = \text{const}}$$

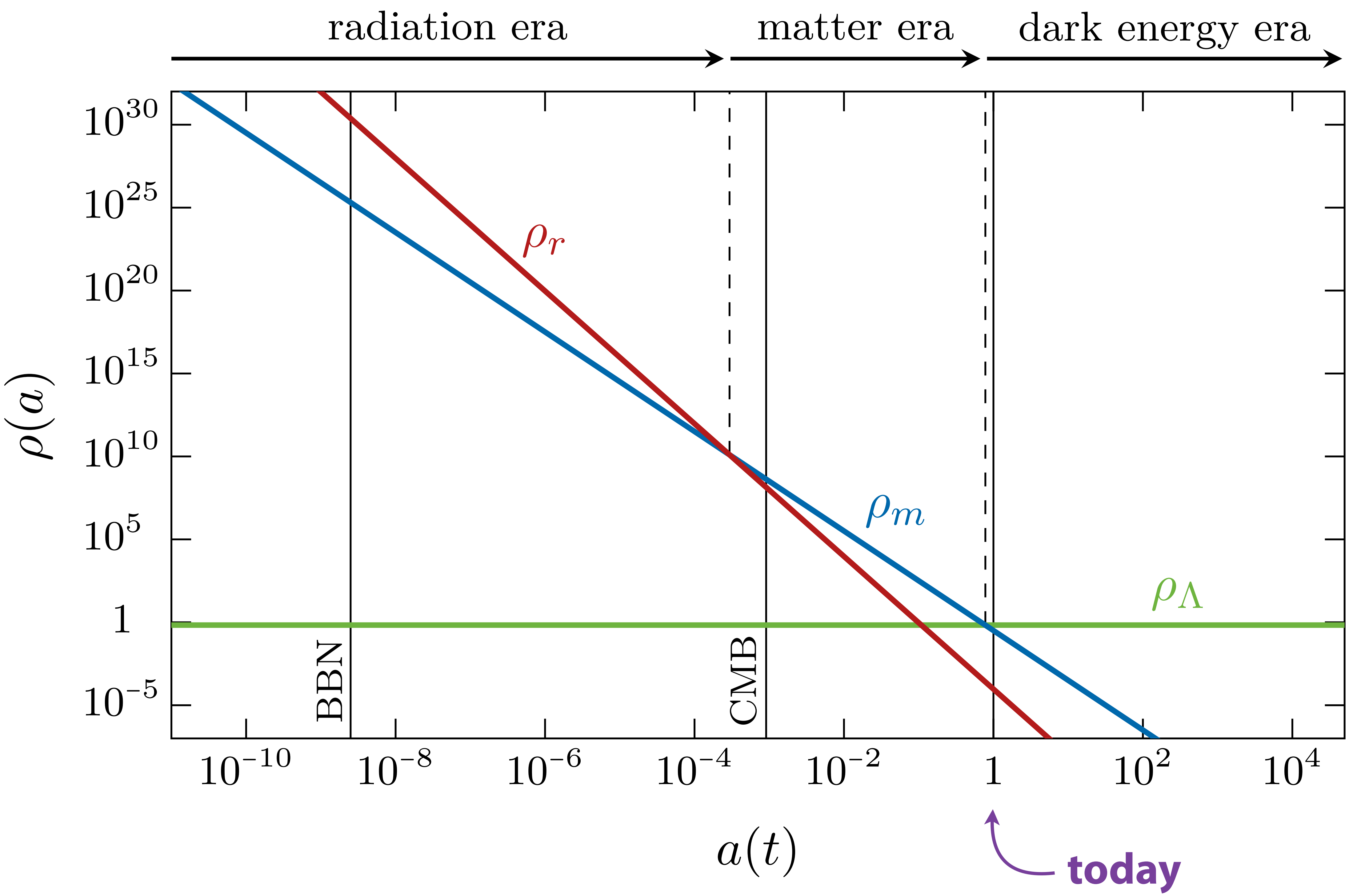


Feeding this into the Friedmann equation gives

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho = \text{const} \longrightarrow \boxed{a \propto e^{H_0 t}}$$

Dark Energy

Today, the Universe is dominated by dark energy (Why now?):



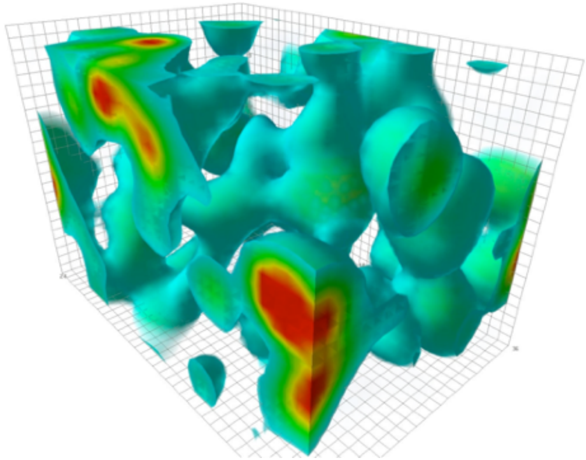
Cosmological Constant Problem

The observed value of the dark energy density is much smaller than our (naive) theoretical expectation:

$$\rho_{\mathrm{QFT}} \sim \int_0^{\Lambda_{\mathrm{UV}}} \frac{\mathrm{d}k^3}{(2\pi)^3} \frac{1}{2}\hbar\omega \sim \Lambda_{\mathrm{UV}}^4 \ggg \rho_{\mathrm{obs}} = (2 \times 10^{-3}\mathrm{eV})^4$$

Quantum zero-point fluctuations

Sum over wavelengths

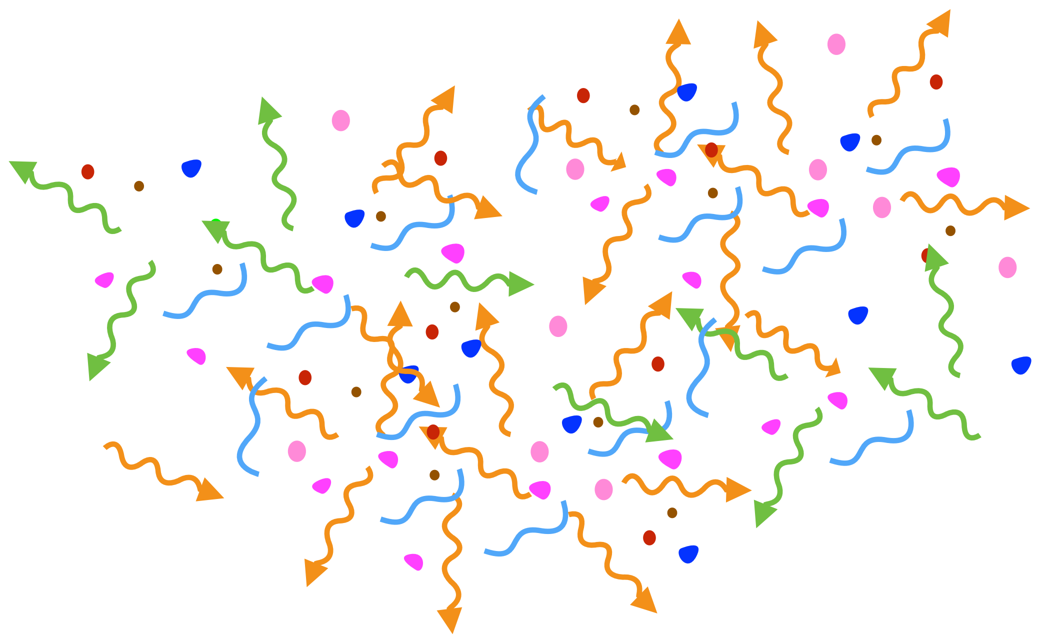


Explaining the observed dark energy density remains one of the most important open questions in fundamental physics.

Questions?

The Hot Big Bang

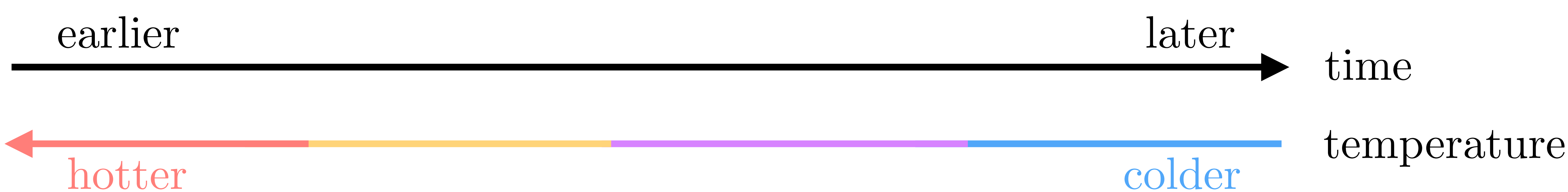
The Universe started in a hot and dense state:



$$T(t) \propto a(t)^{-1}$$

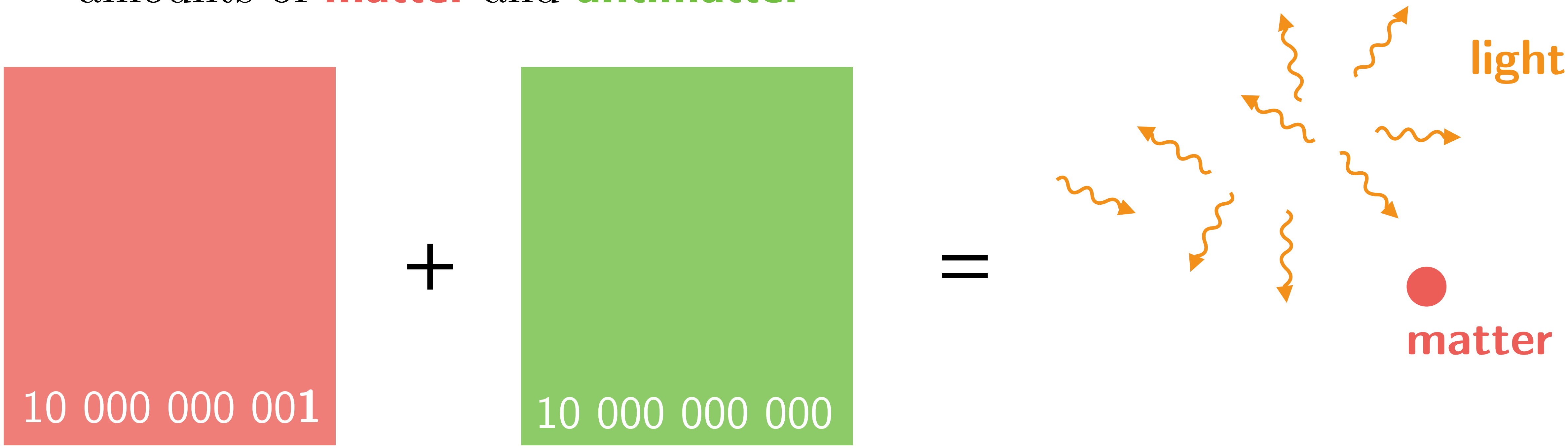
As the Universe expands, it cools.

Many interesting things happened.



$t = 10^{-19}\,\mathrm{s}$

The Universe is filled with almost equal amounts of **matter** and **antimatter**



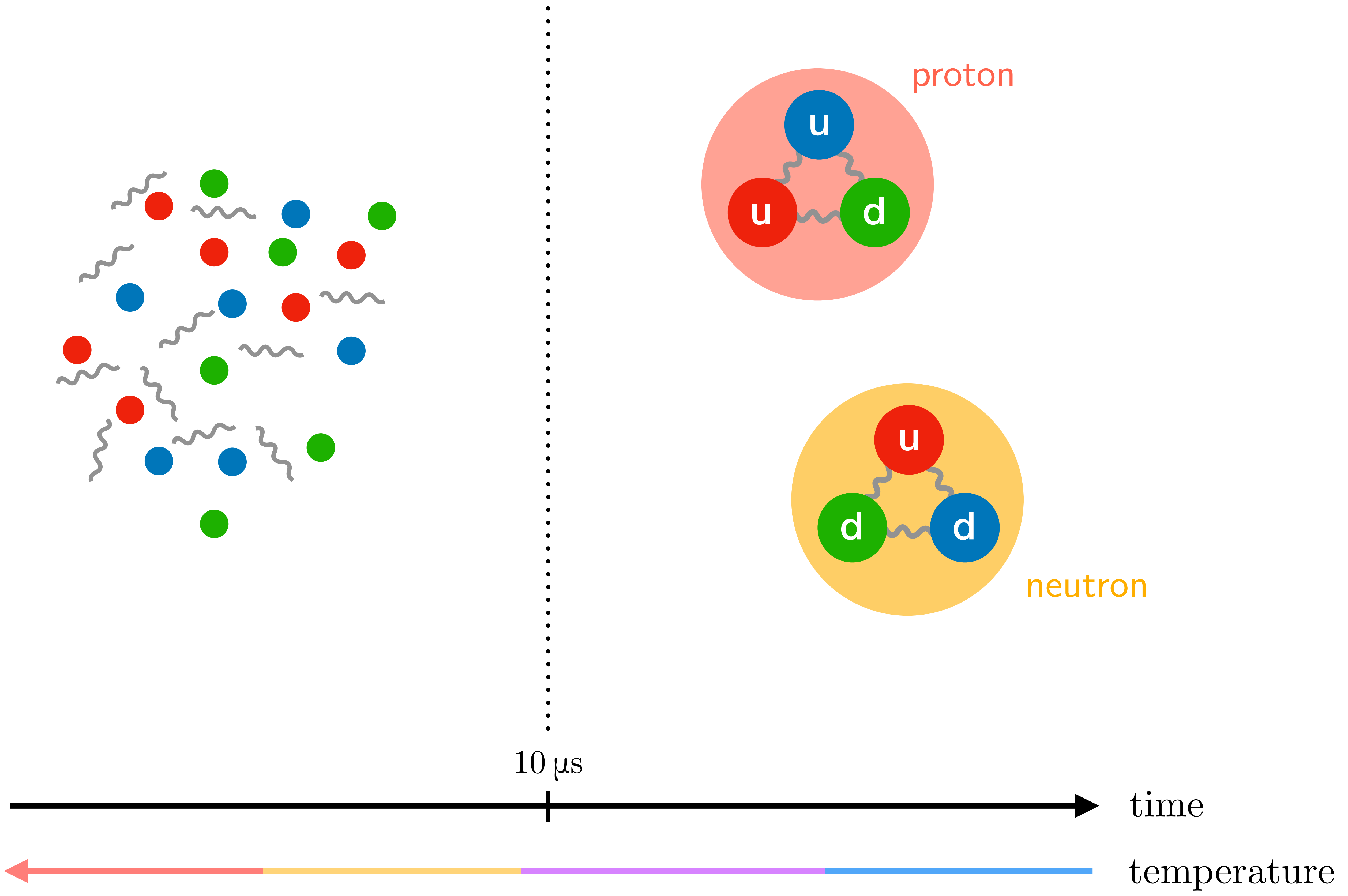
As the universe cools, matter and antimatter annihilate.

For some mysterious reason, there was initially a fraction more matter than antimatter. This matter survived the annihilation.

Without this **asymmetry** we wouldn't exist.

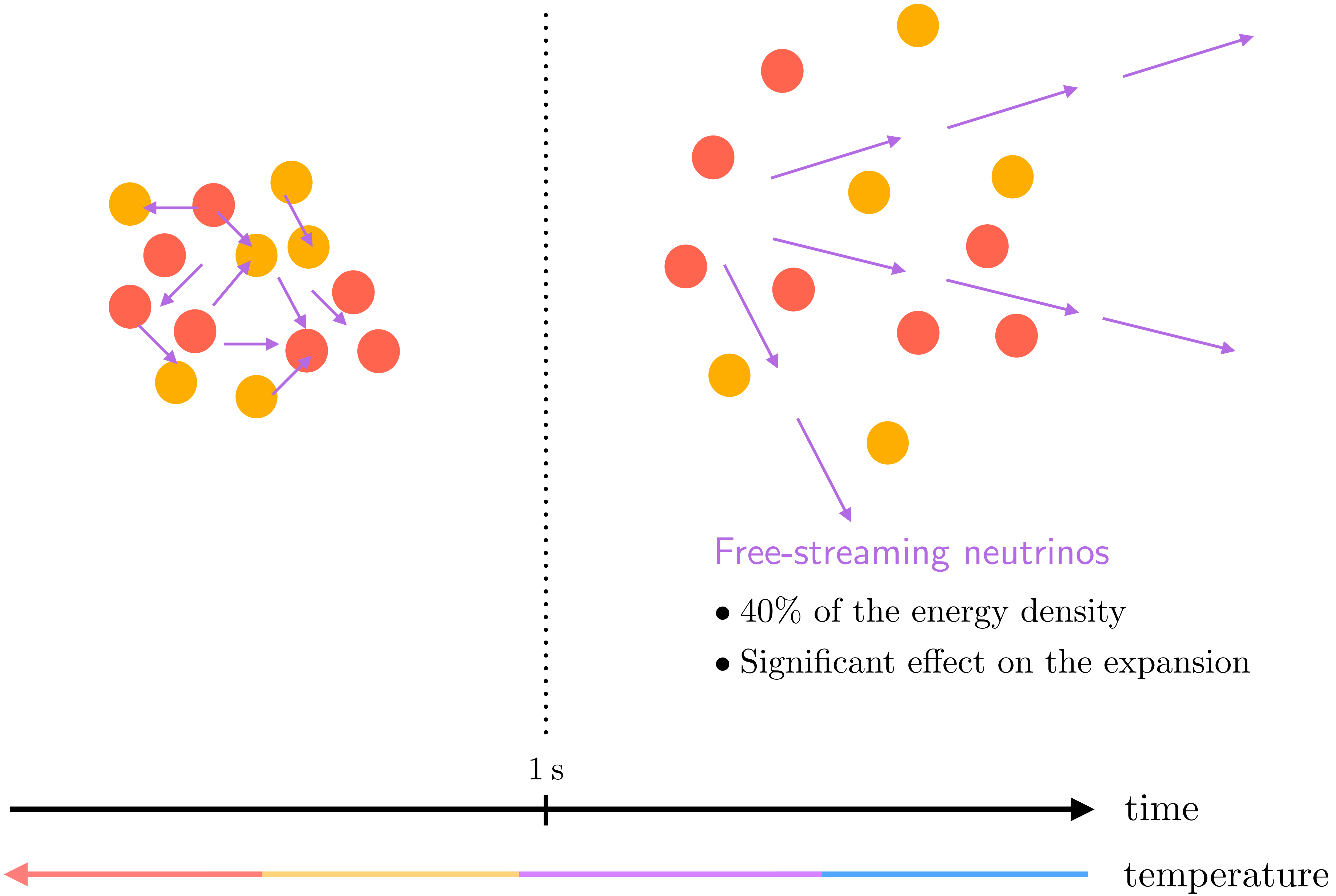
$t = 10^{-5}$ s

Quarks and gluons condense into nuclei:

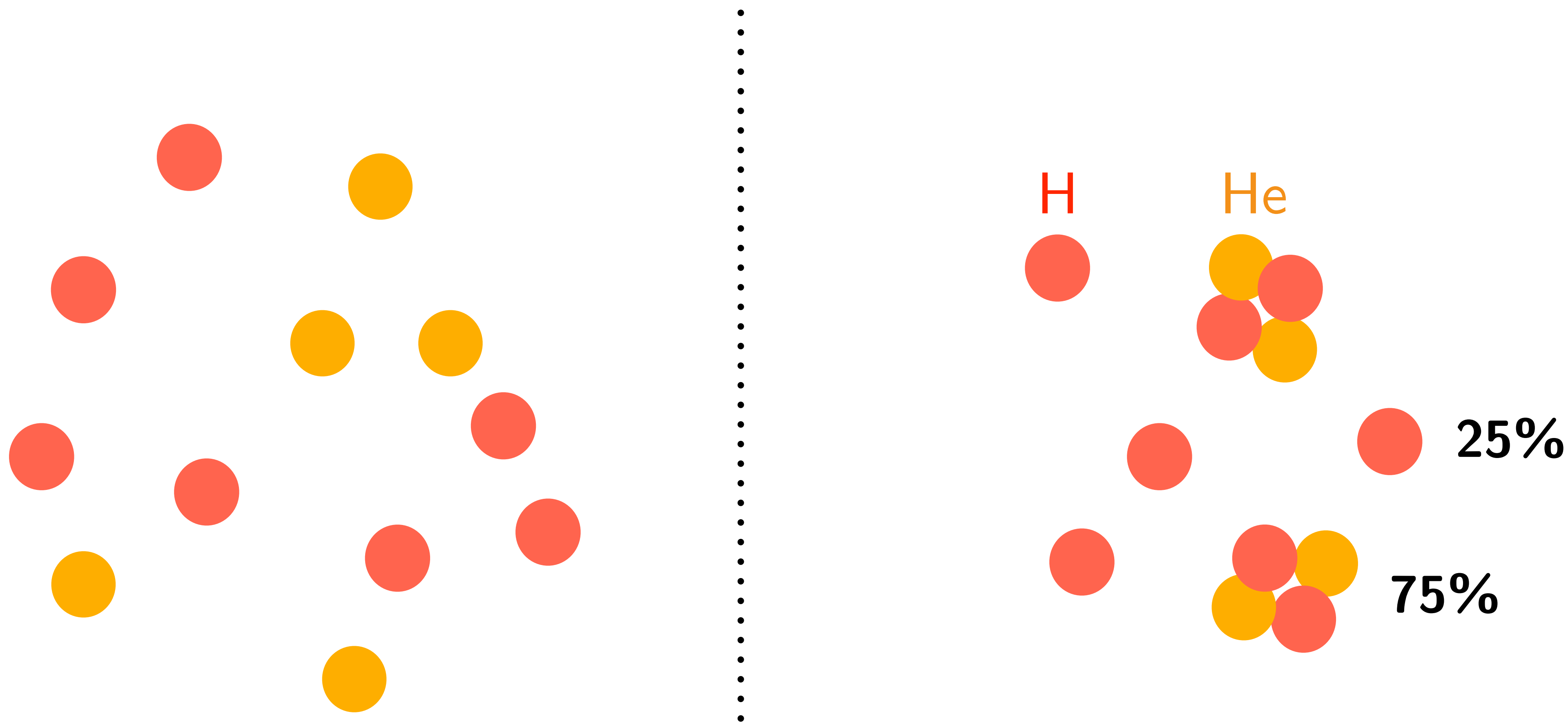


$t = 1\,\mathrm{s}$

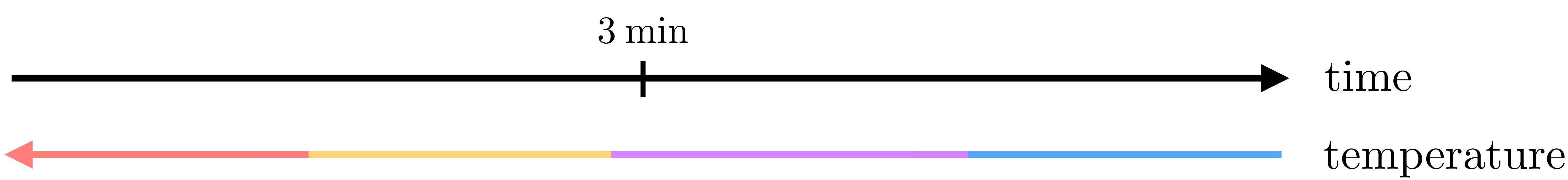
Neutrinos decouple and neutrons freeze out:



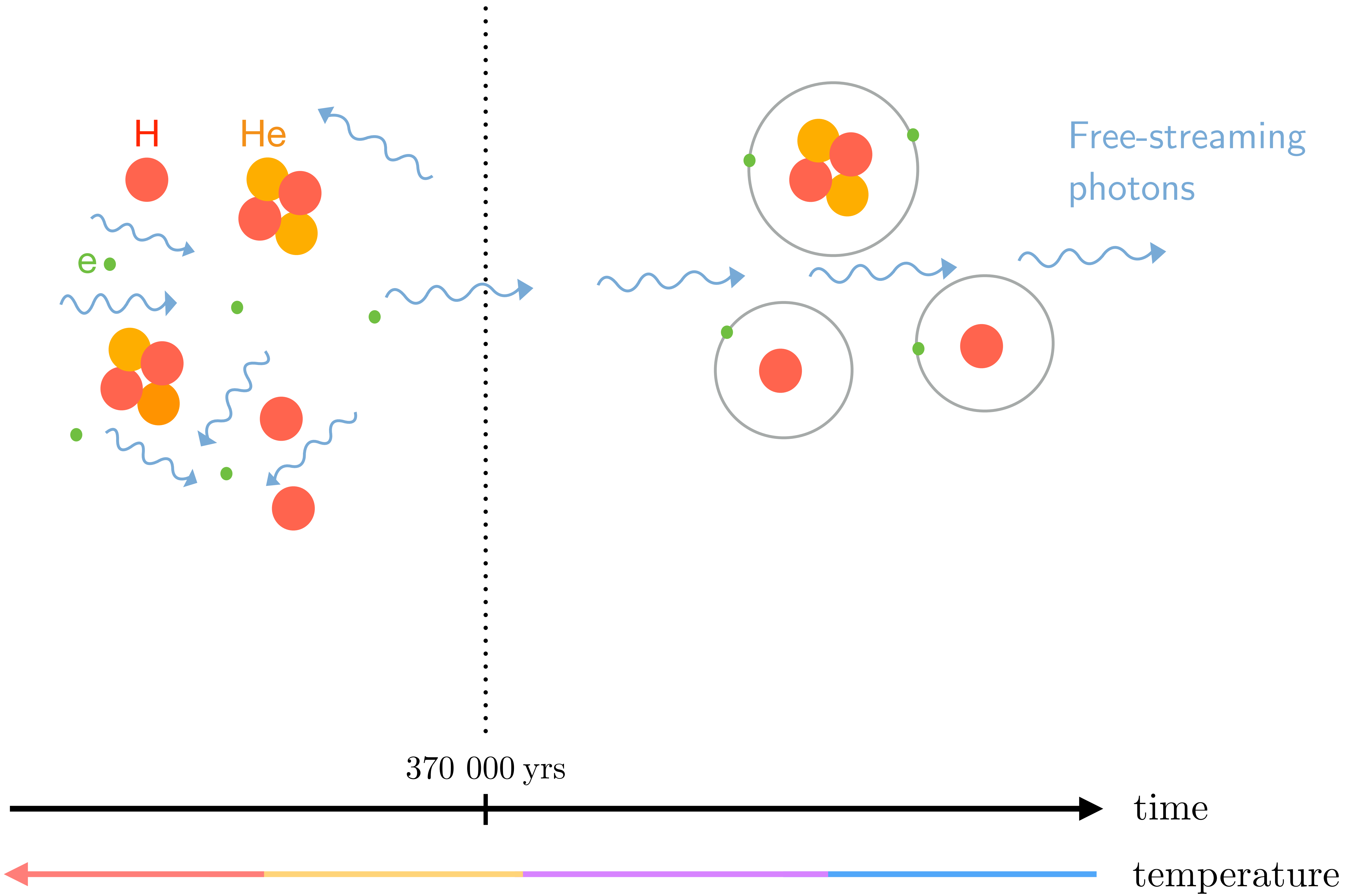
$t = 3\,\text{min}$ Light elements (H, He, and Li) form:



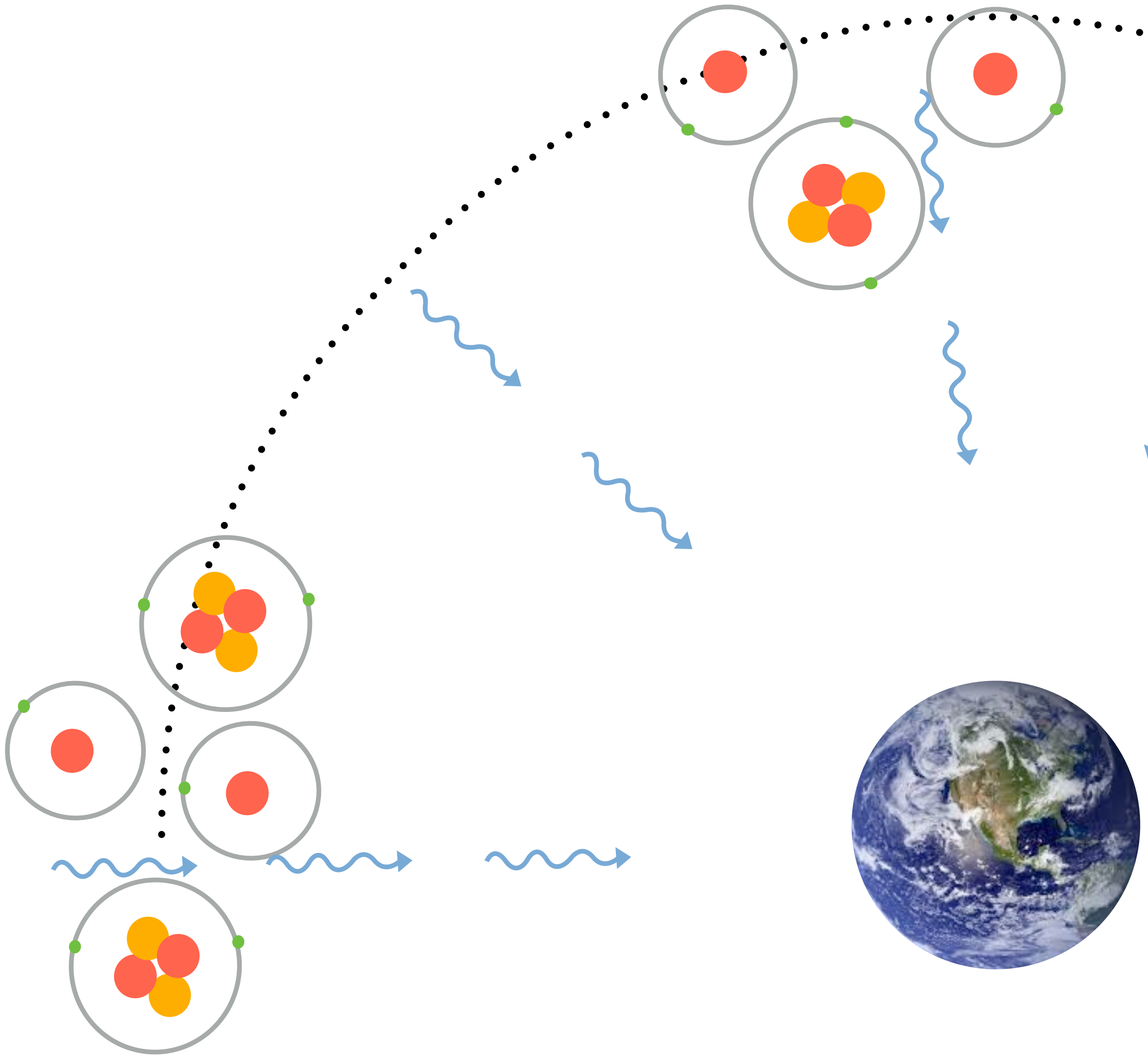
- Big Bang nucleosynthesis (BBN) predicts the correct abundances of the light elements.
- Heavier nuclei were fused inside stars.



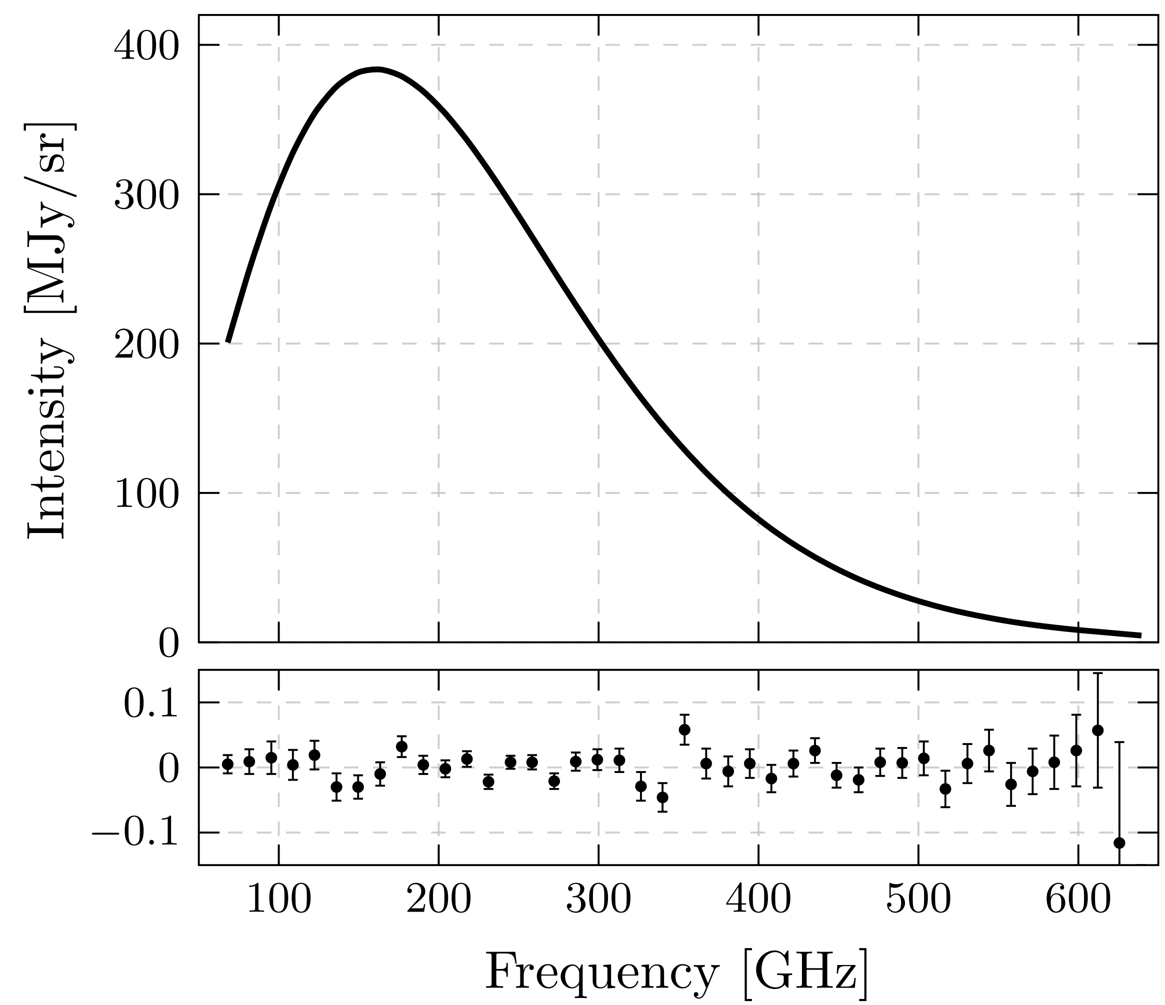
$t = 370\,000\,\text{yrs}$ Atoms form and the first light is released:



This afterglow of the Big Bang is still seen today:



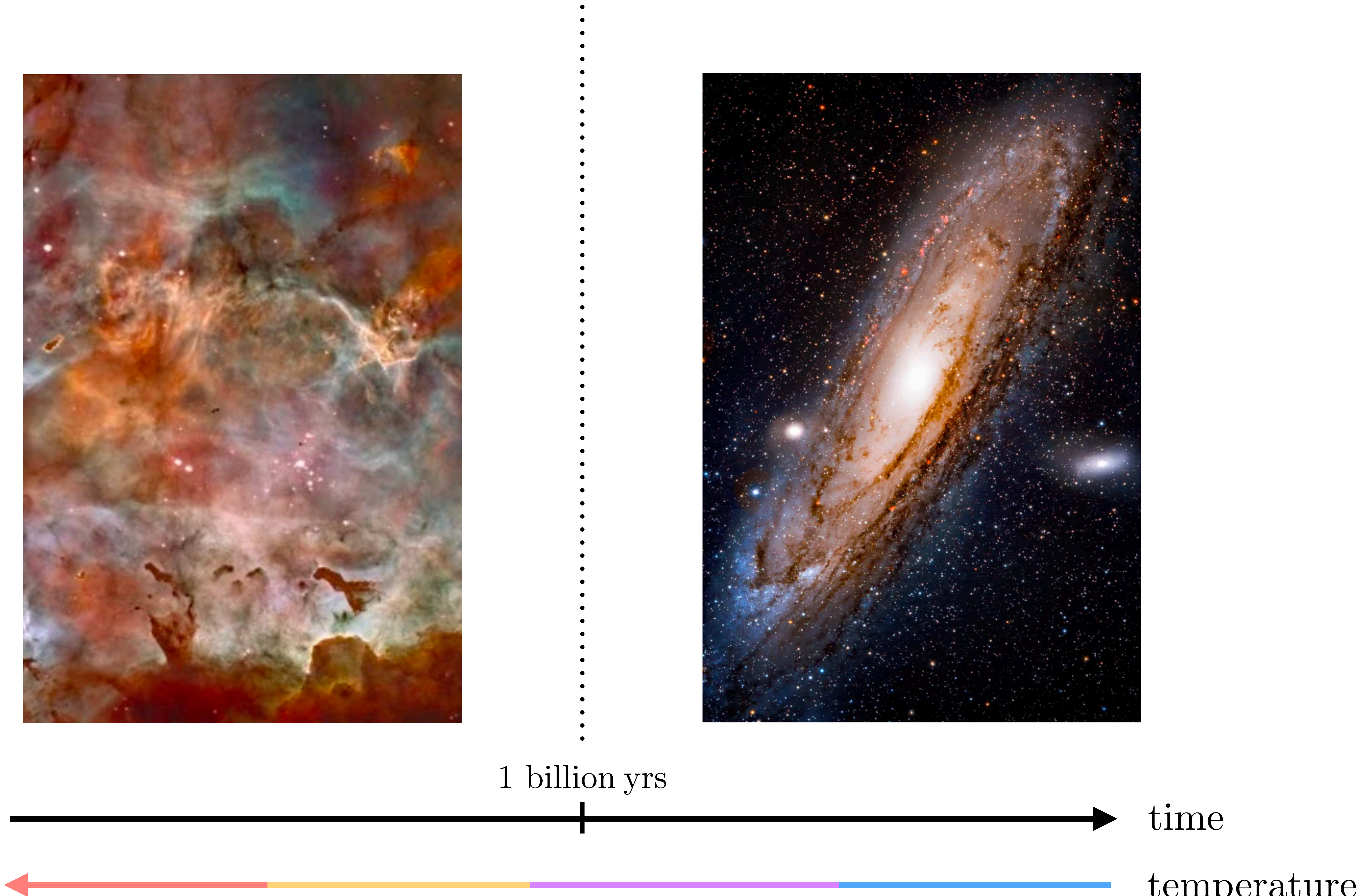
Cosmic **M**icrowave **B**ackground



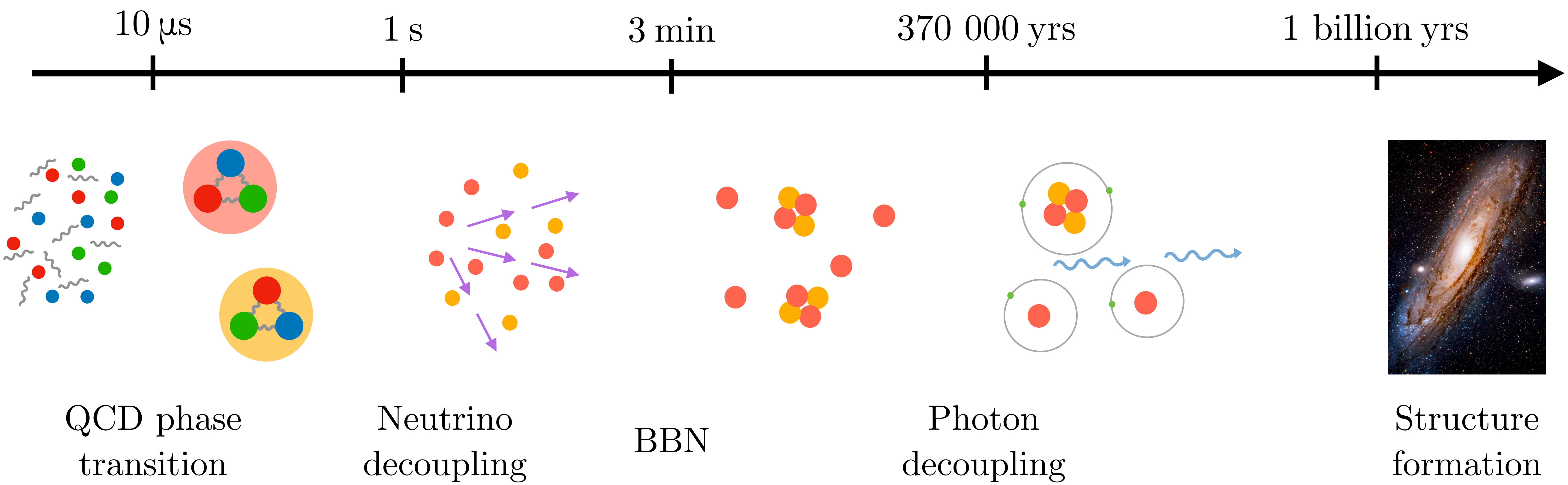
- 410 photons per cubic centimeter
- cooled by the expansion: 2.7 K
- faint microwave radiation: CMB

$t > 1$ billion yrs

Matter collapses into stars and galaxies:



This history of the Universe is an observational fact:



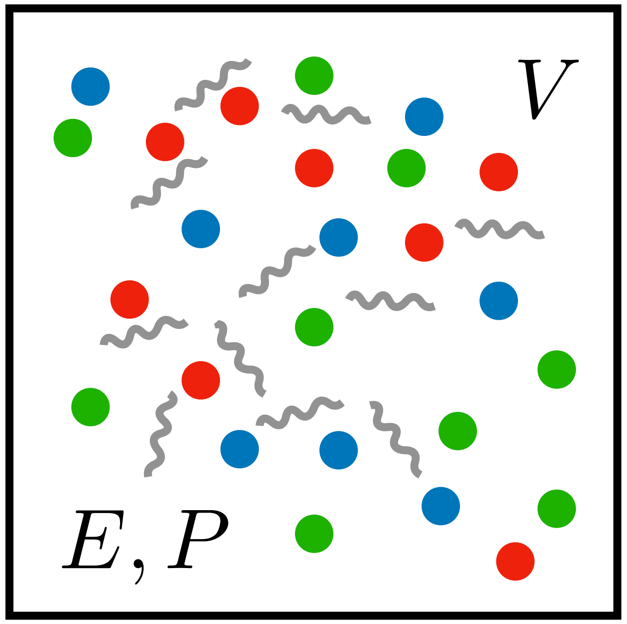
- The basic picture has been confirmed by many independent observations.
- Many precise details are probed by measurements of the CMB.

Where did it all come from?

Appendix

Fluid Equation

Consider a fluid in a box:



The First Law of thermodynamics implies

$$\mathrm{d}E = \mathrm{d}Q - P\mathrm{d}V$$

Homogeneity doesn't allow any heat flow ($\mathrm{d}Q = 0$), so that

$$\boxed{\frac{dE}{dt} = -P\frac{dV}{dt}}$$

Fluid Equation

Using $E = \rho V$ and $V \propto a^3$, we find

$$\frac{dE}{dt} = -P\frac{dV}{dt} \longrightarrow \boxed{\frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(\rho + P)}$$

Fluid Equation

Cosmological fluids are described by a constant **equation of state**:

$$w \equiv P/\rho$$

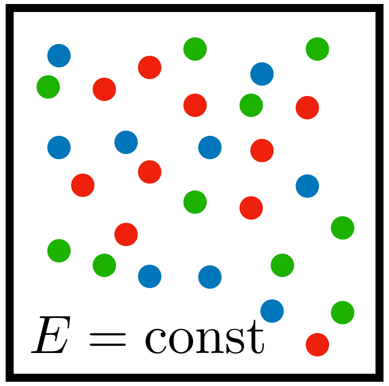
The fluid equation then becomes

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \longrightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

Equation of State

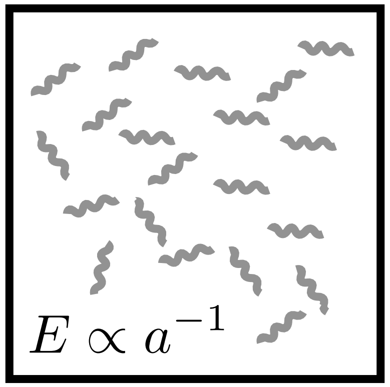
- For a pressureless fluid (= **matter**), we get

$$w = 0 \longrightarrow \boxed{\rho \propto a^{-3}}$$



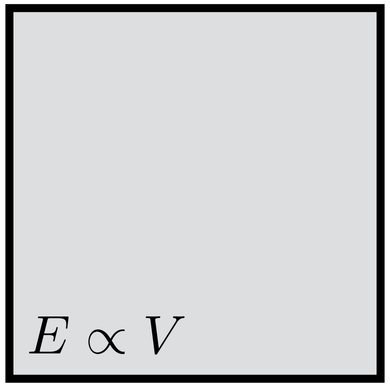
- For a relativistic fluid (= **radiation**), we get

$$w = 1/3 \longrightarrow \boxed{\rho \propto a^{-4}}$$



- For the **vacuum energy**, we have

$$w = -1 \longrightarrow \boxed{\rho \propto a^{0}}$$



Acceleration Equation

Combining the Friedmann equation with the fluid equation

$$\frac{d}{dt}\left[\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\right] \quad + \quad \frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(\rho + P)$$

we get

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)}$$

Acceleration Equation

There are two different cases:

$$P > -\frac{1}{3}\rho \qquad\qquad P < -\frac{1}{3}\rho$$

Expansion slows down

$\ddot{a} < 0$

Expansion speeds up

$\ddot{a} > 0$