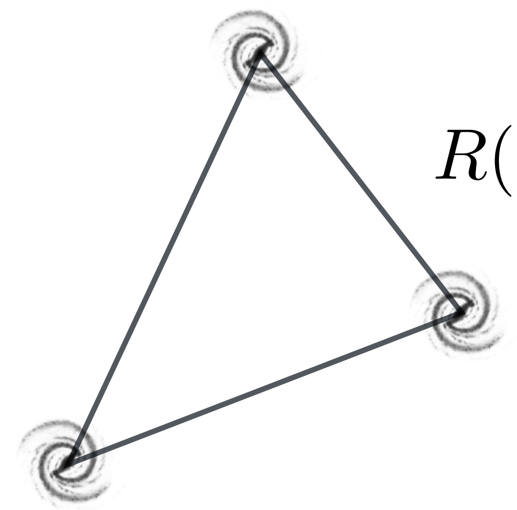
# Recap of Lecture 1

- The Universe is expanding:



$$R(t) = a(t)\,R_0$$

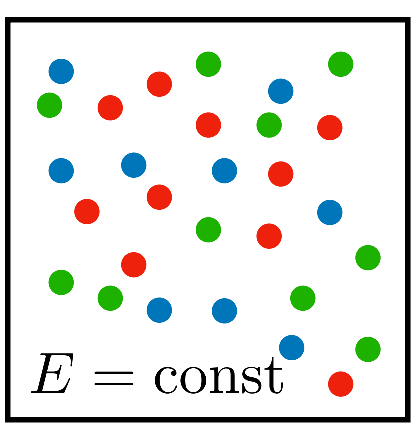
Scale factor

- The rate of expansion is determined by the Friedmann equation:

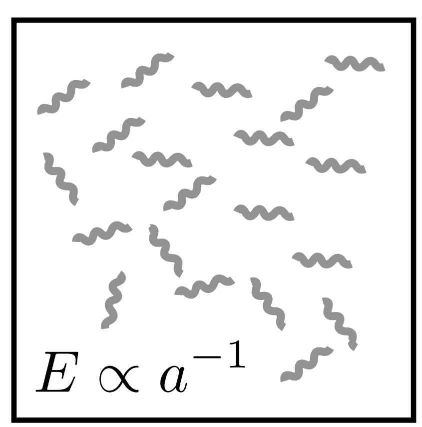
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

- The energy density of matter, radiation and dark energy dilutes as

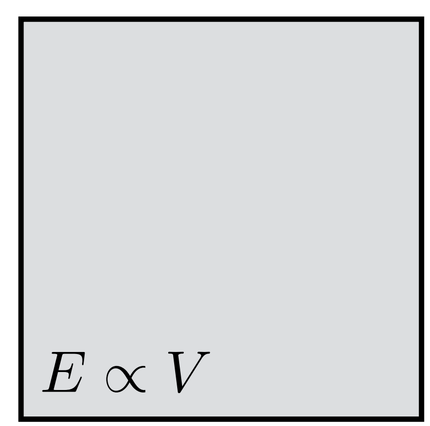
$\rho_m \propto a^{-3}$




$\rho_r \propto a^{-4}$

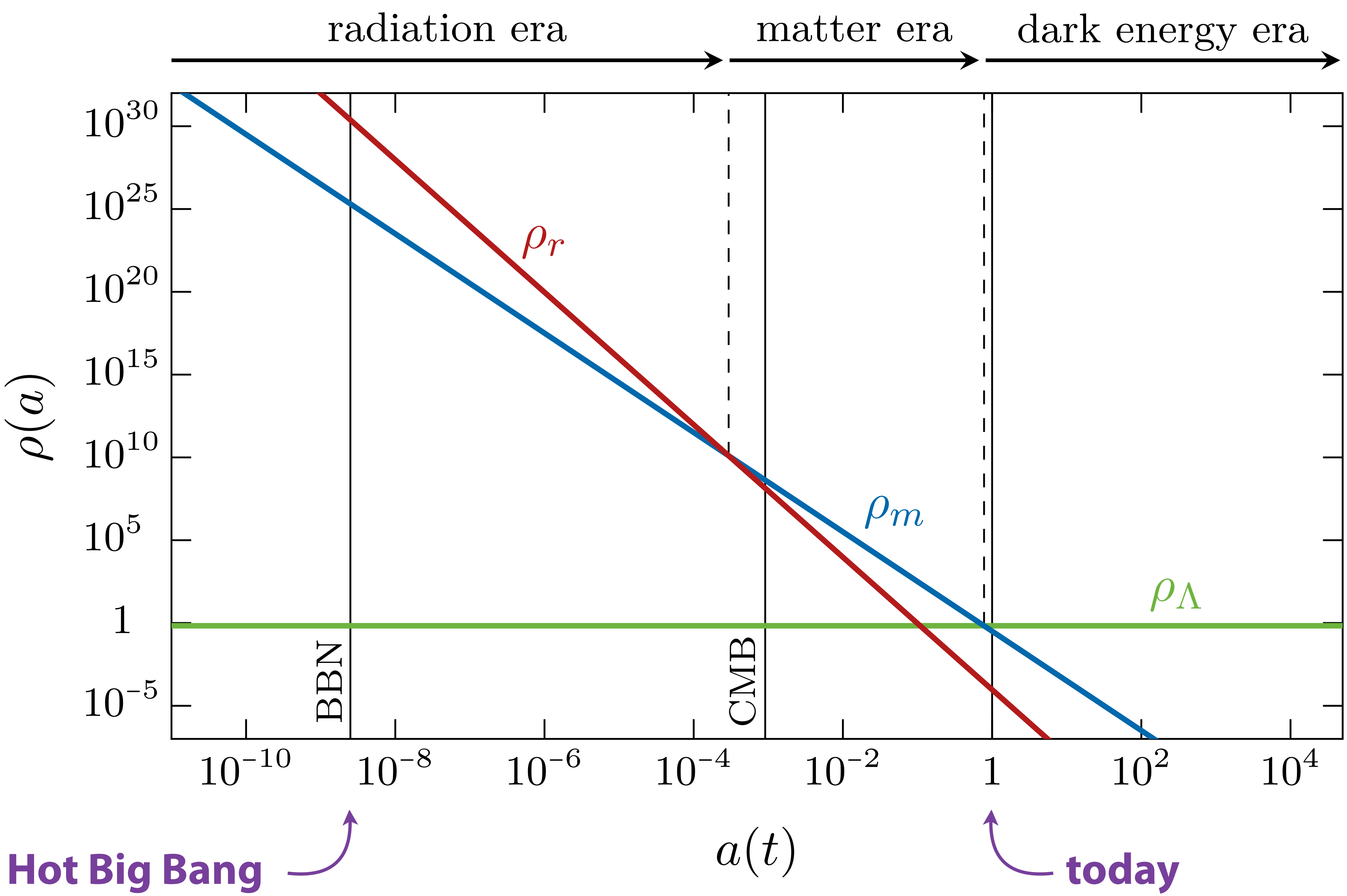



$\rho_\Lambda \propto a^0$



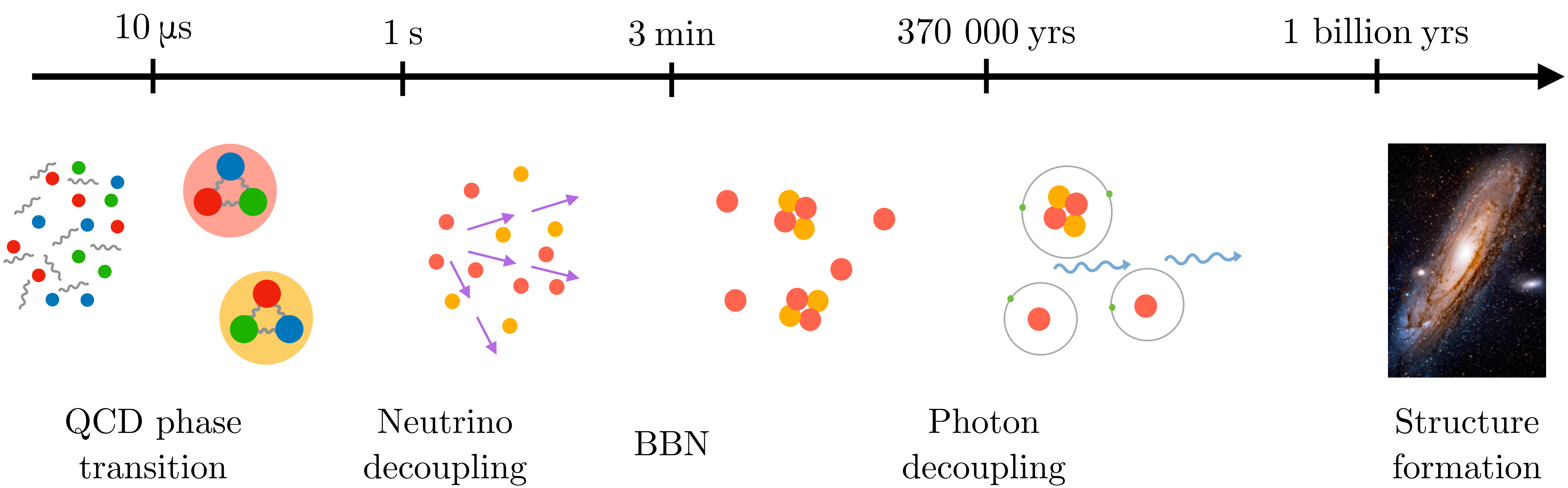
# Recap of Lecture 1

- The Universe started hot and dense, but then cooled and diluted:



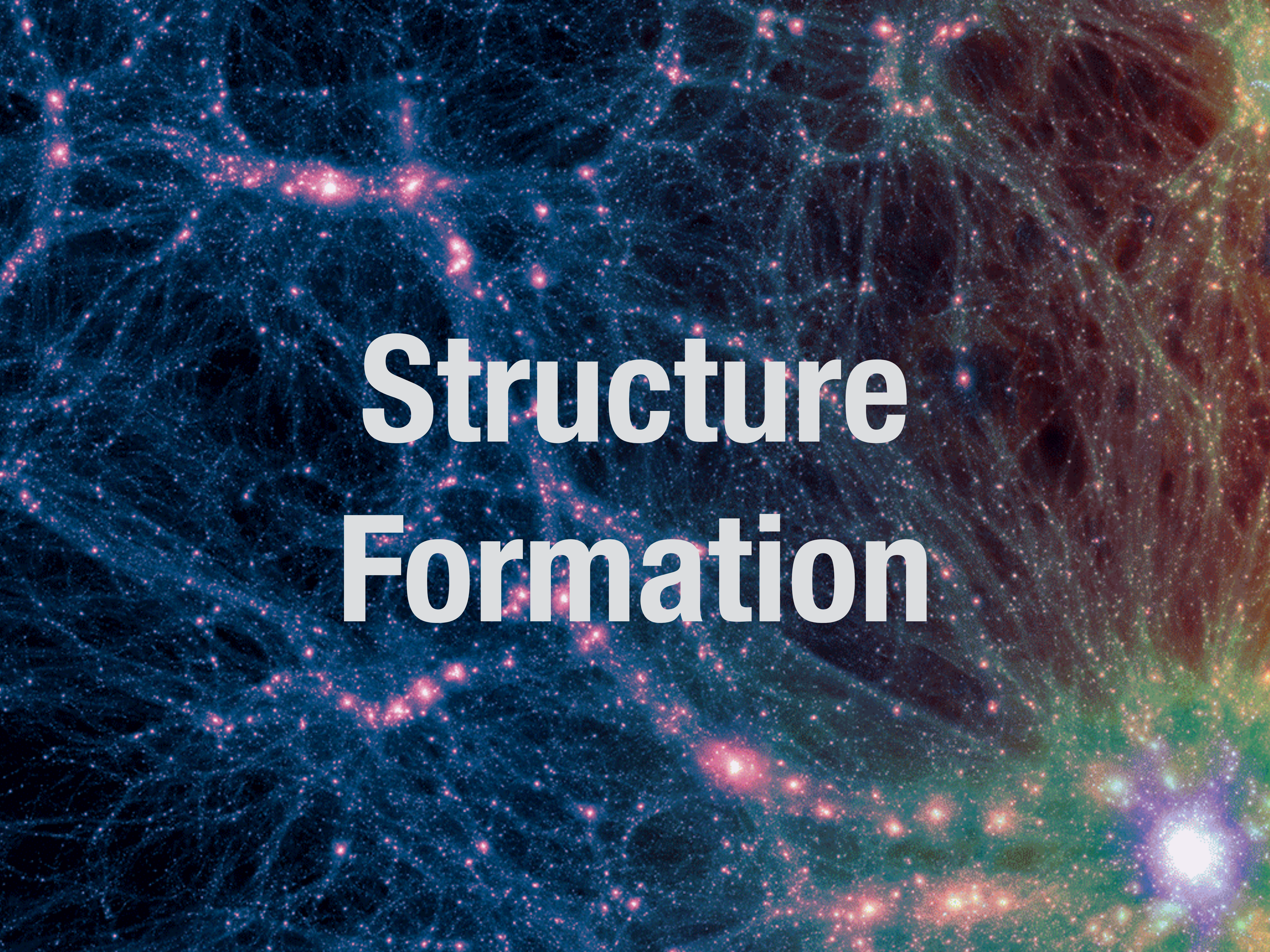
# Recap of Lecture 1

This history of the Universe is an observational fact:



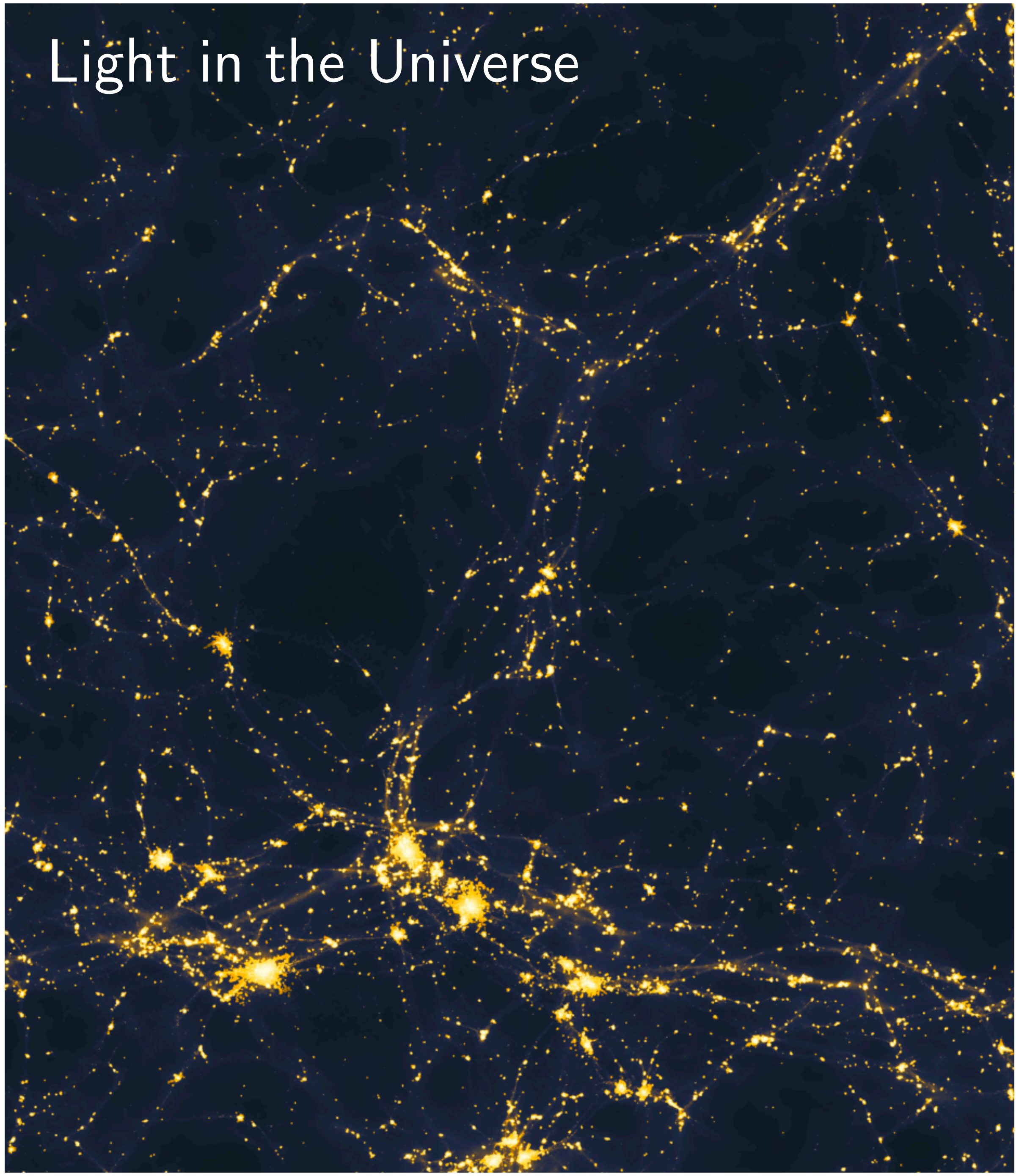

- The basic picture has been confirmed by many independent observations.
- Many precise details are probed by measurements of the CMB.

***Questions?***



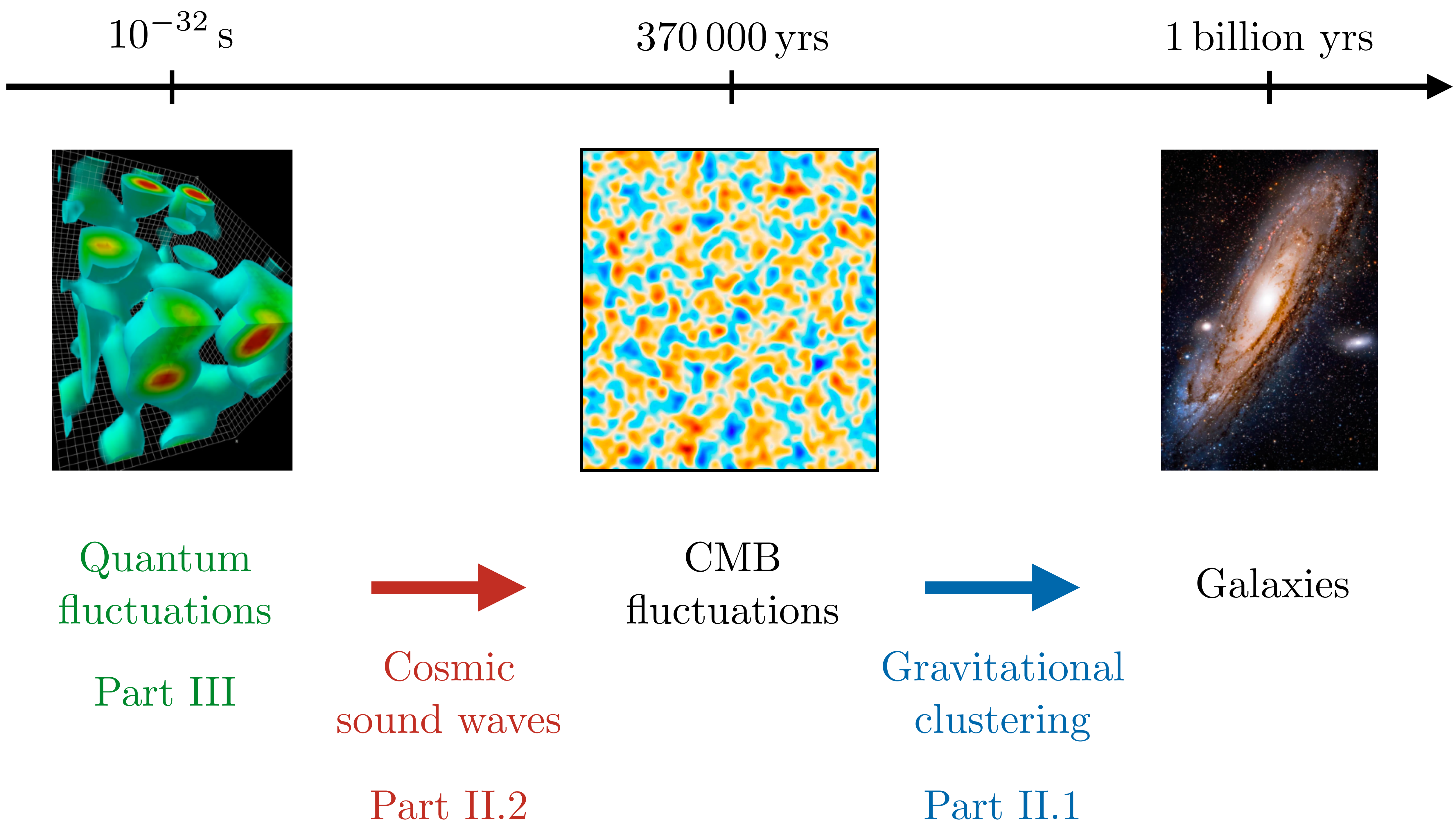
# Structure Formation

How did the structure in the Universe form?

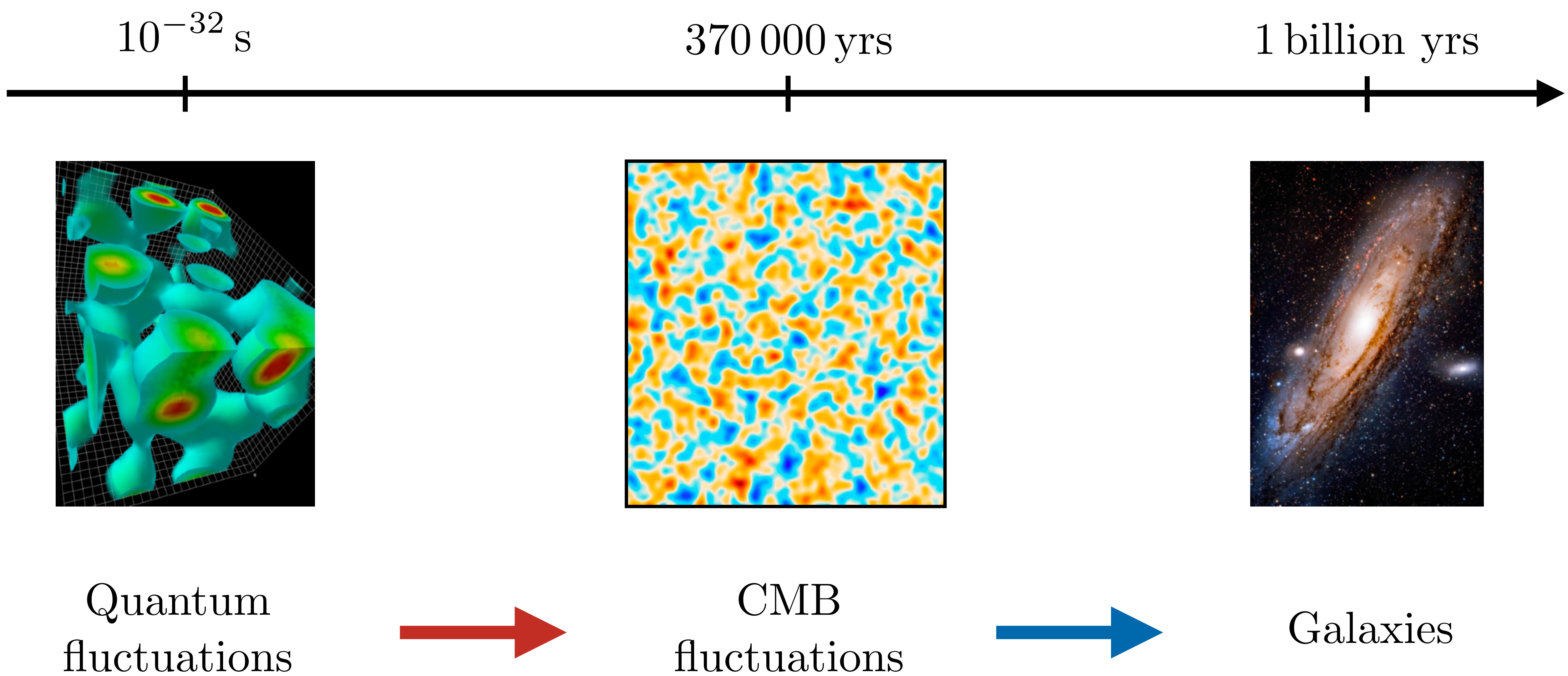




The large-scale structure of the Universe isn't randomly distributed, but has spatial correlations. What created these correlations?

Our best answer to these questions involves a fascination connection between the physics of the very small and the very large:



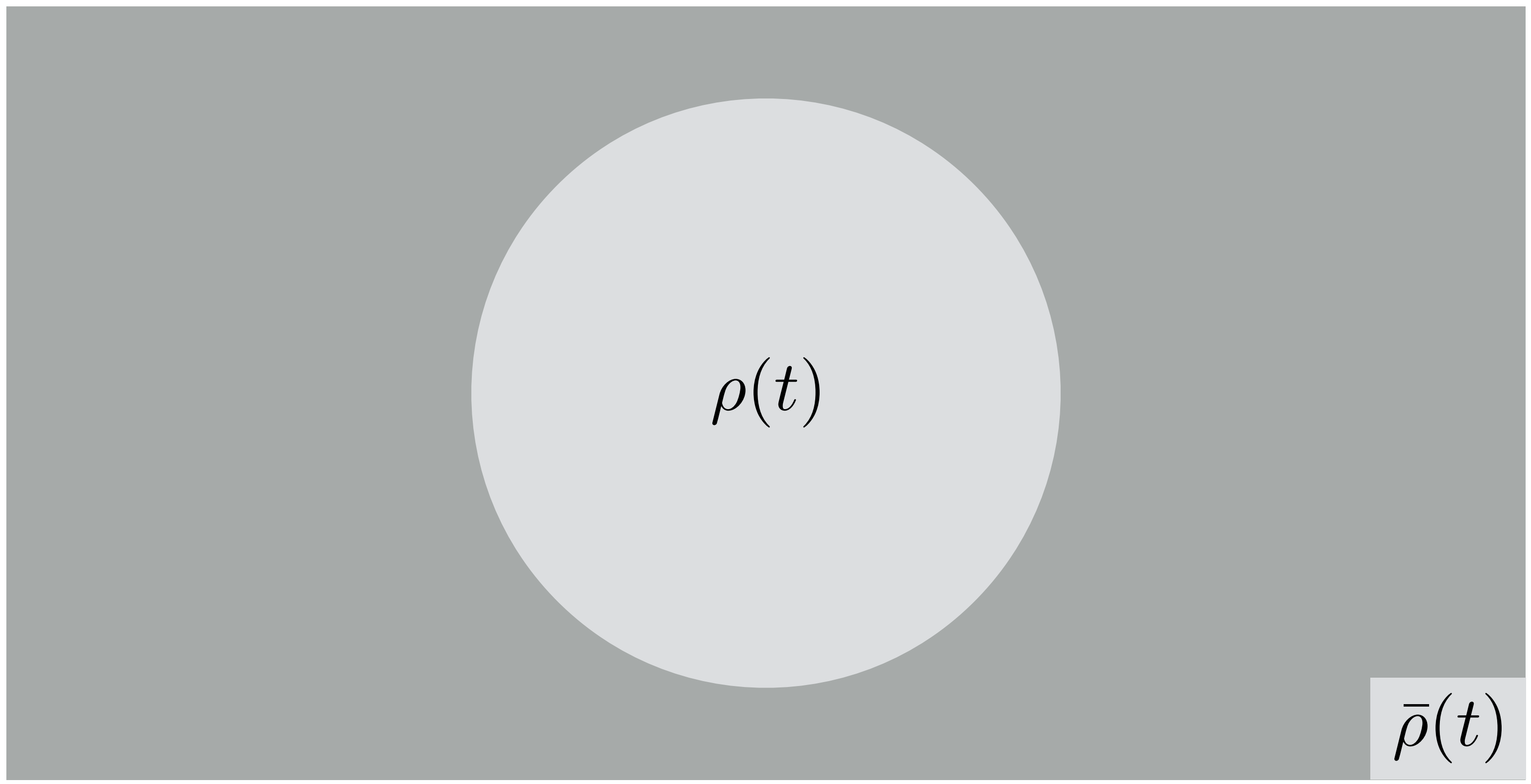
Our best answer to these questions involves a fascination connection between the physics of the very small and the very large:




This allows us to use cosmological observations to learn about short-distance/ high-energy particle physics.

# Gravitational Clustering

Consider a **spherical overdensity** in a homogeneous Universe:




We are interested in the evolution of the **density contrast**:

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

Galaxies form when the density contrast reaches a critical value.

# Gravitational Clustering

The evolution of the density contrast can be derived using Newtonian gravity (see appendix). Here, I just quote the results:

- In a **static universe**, the density contrast grows exponentially:

$$\delta(t) \propto e^{t/\tau}$$

- In an **expanding universe**, the growth is slower:

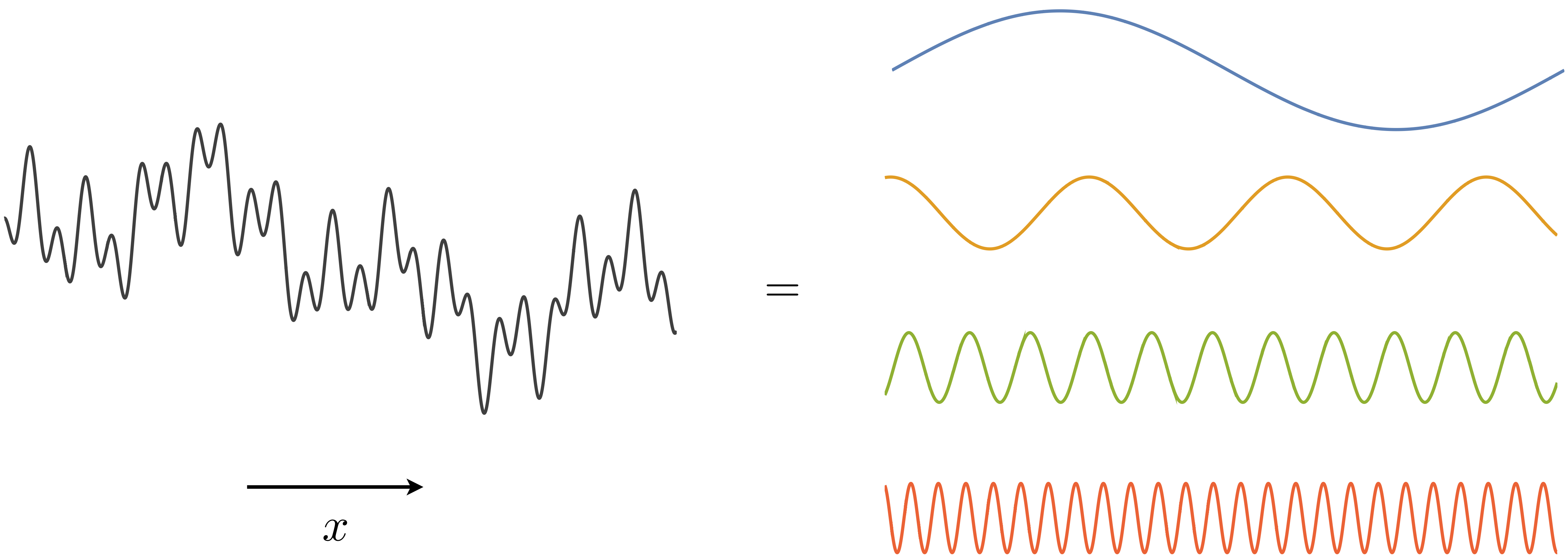
$$\delta(t) \propto t^{2/3} \qquad \text{during the matter era}$$

$$\delta(t) \propto \ln t \qquad \text{during the radiation era}$$

The clustering of matter only begins after matter-radiation equality.

# Fourier Modes

In reality, density perturbations are not spherically symmetric.
A general density fluctuation can be decomposed into its **Fourier modes**:




$$= \sum_k \delta_k \sin(kx + \phi_k) \qquad k = 2\pi/\lambda$$

Each Fourier mode satisfies the same equation of motion as a spherically symmetric overdensity.

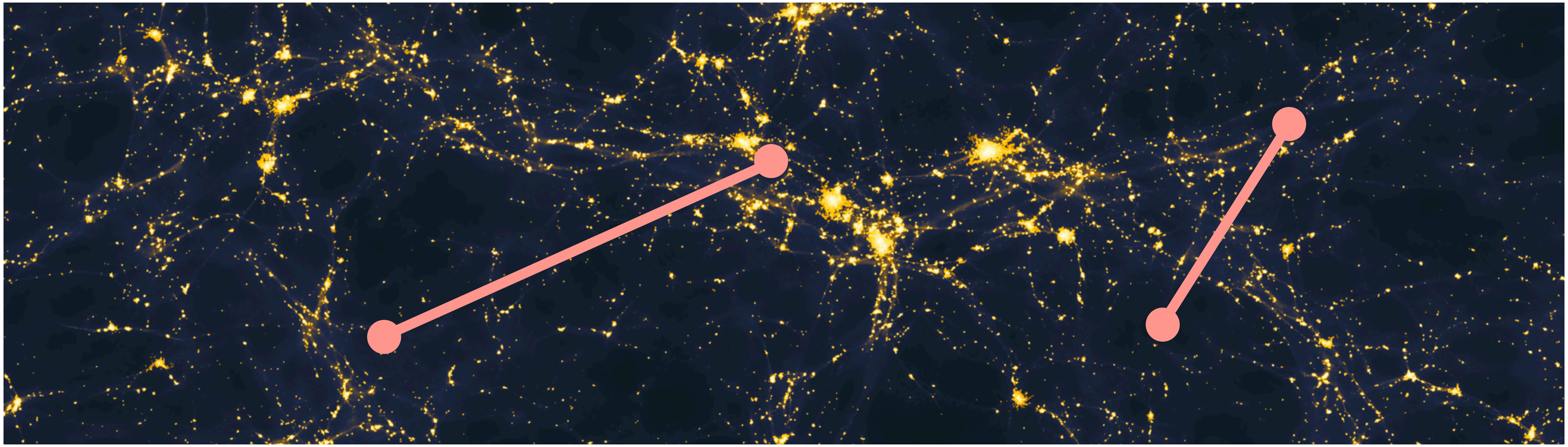
# Power Spectrum

The power spectrum is the square of the Fourier amplitude:

$$P(k) = |\delta_k|^2$$

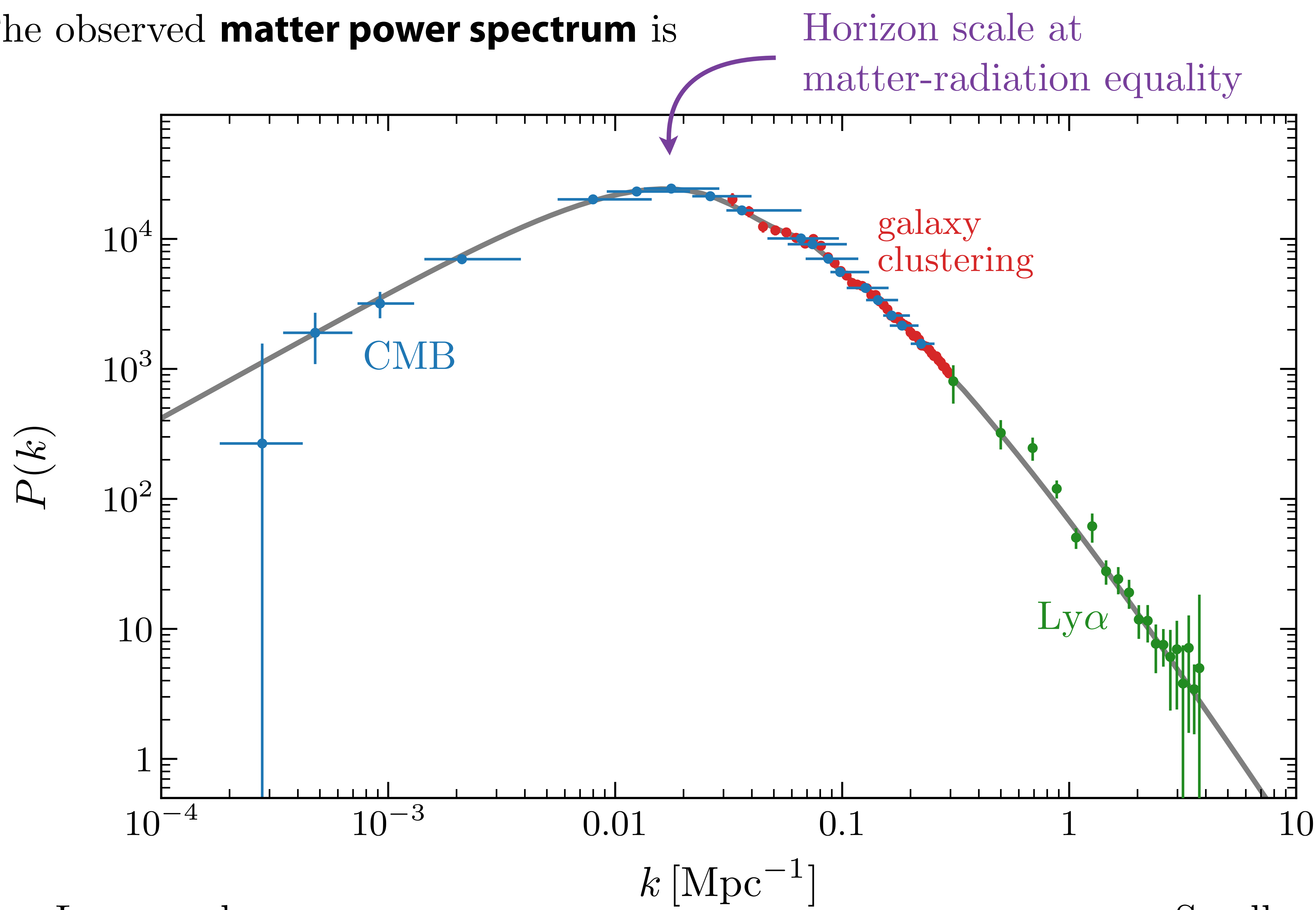
The Fourier transform of the power spectrum is the two-point correlation function. This is the main statistic of cosmological correlations.

$$\xi(r) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} P(k)\, e^{ikr}$$



# Power Spectrum

The observed **matter power spectrum** is

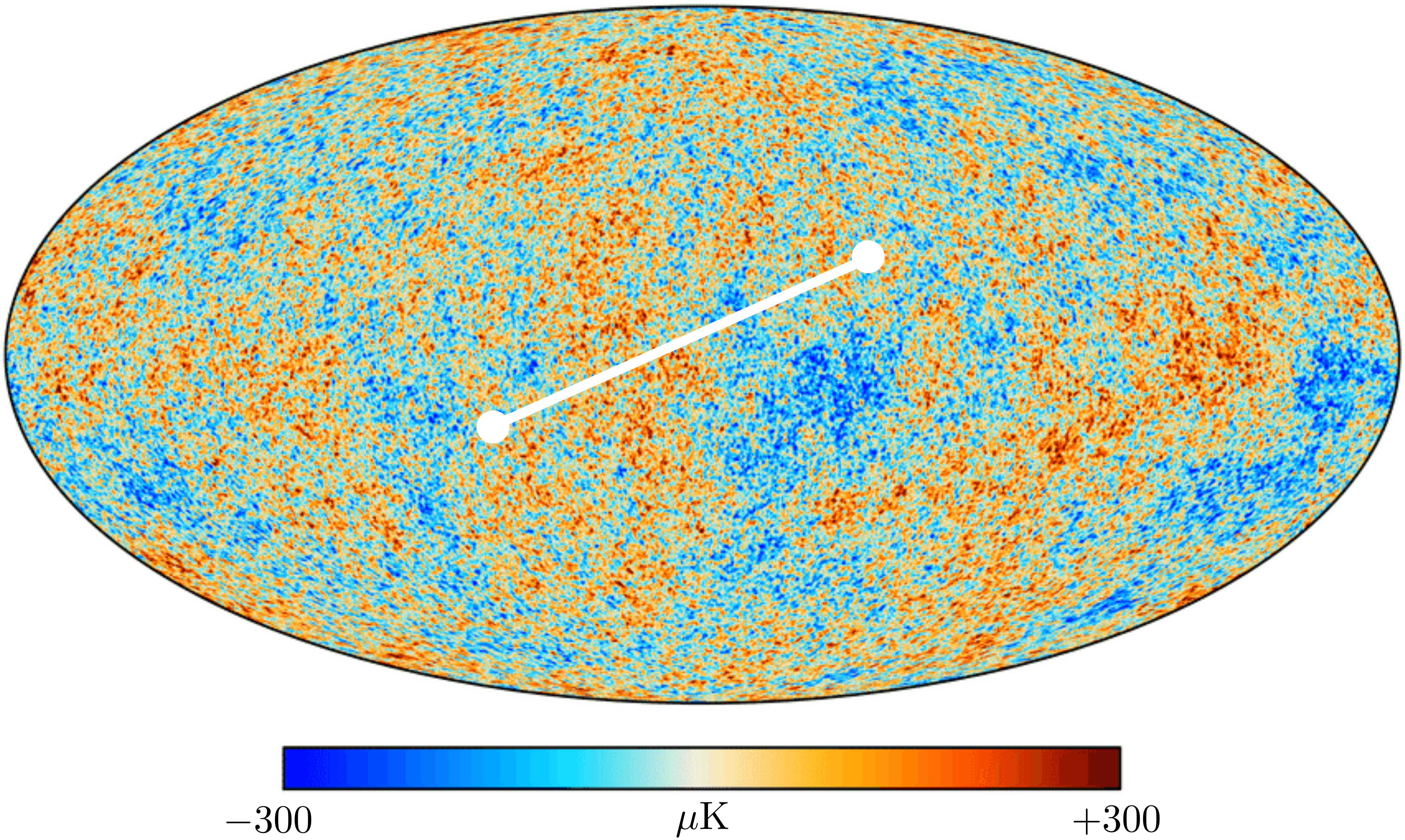



Large scales

Small scales

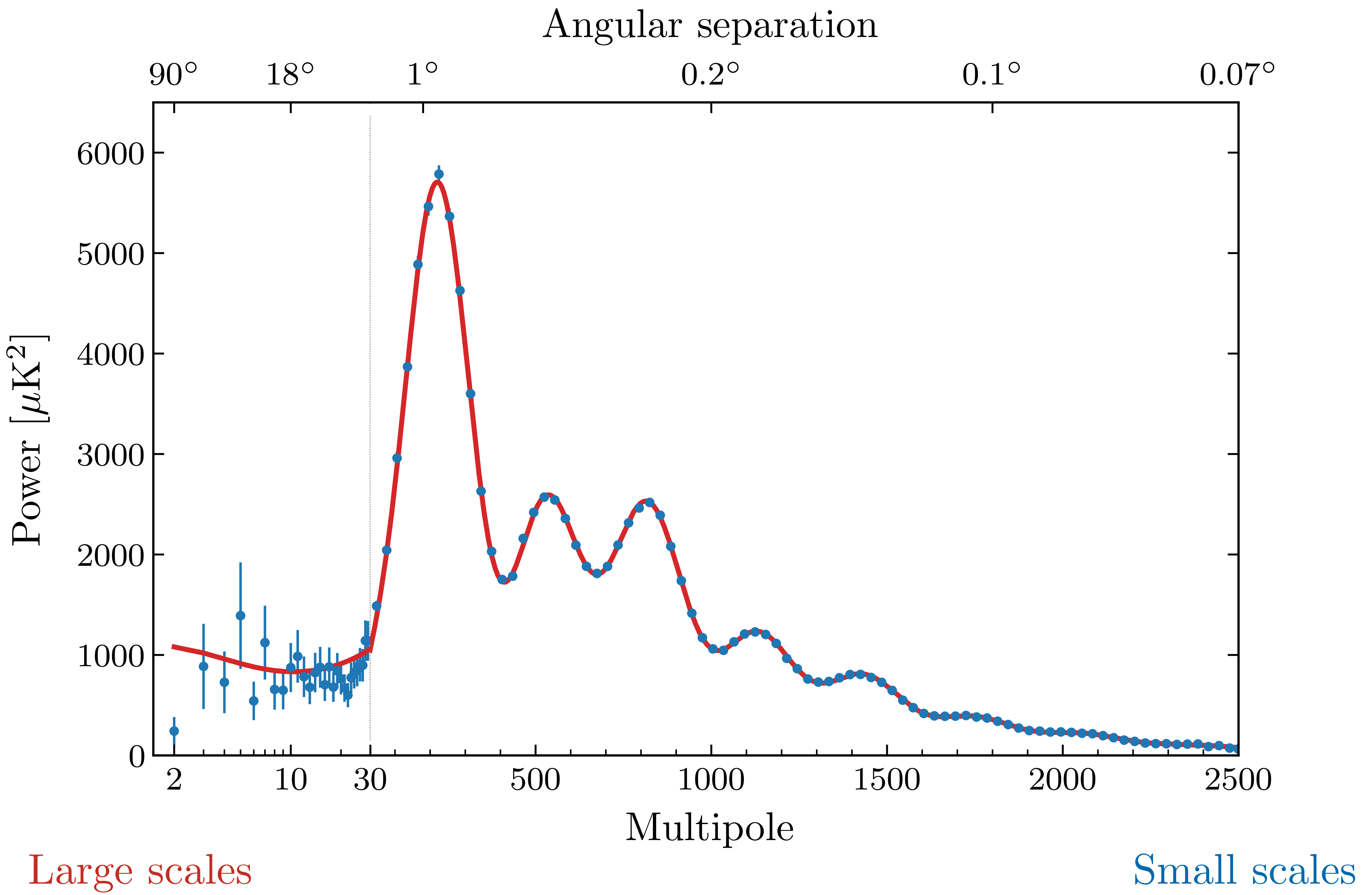
***Questions?***

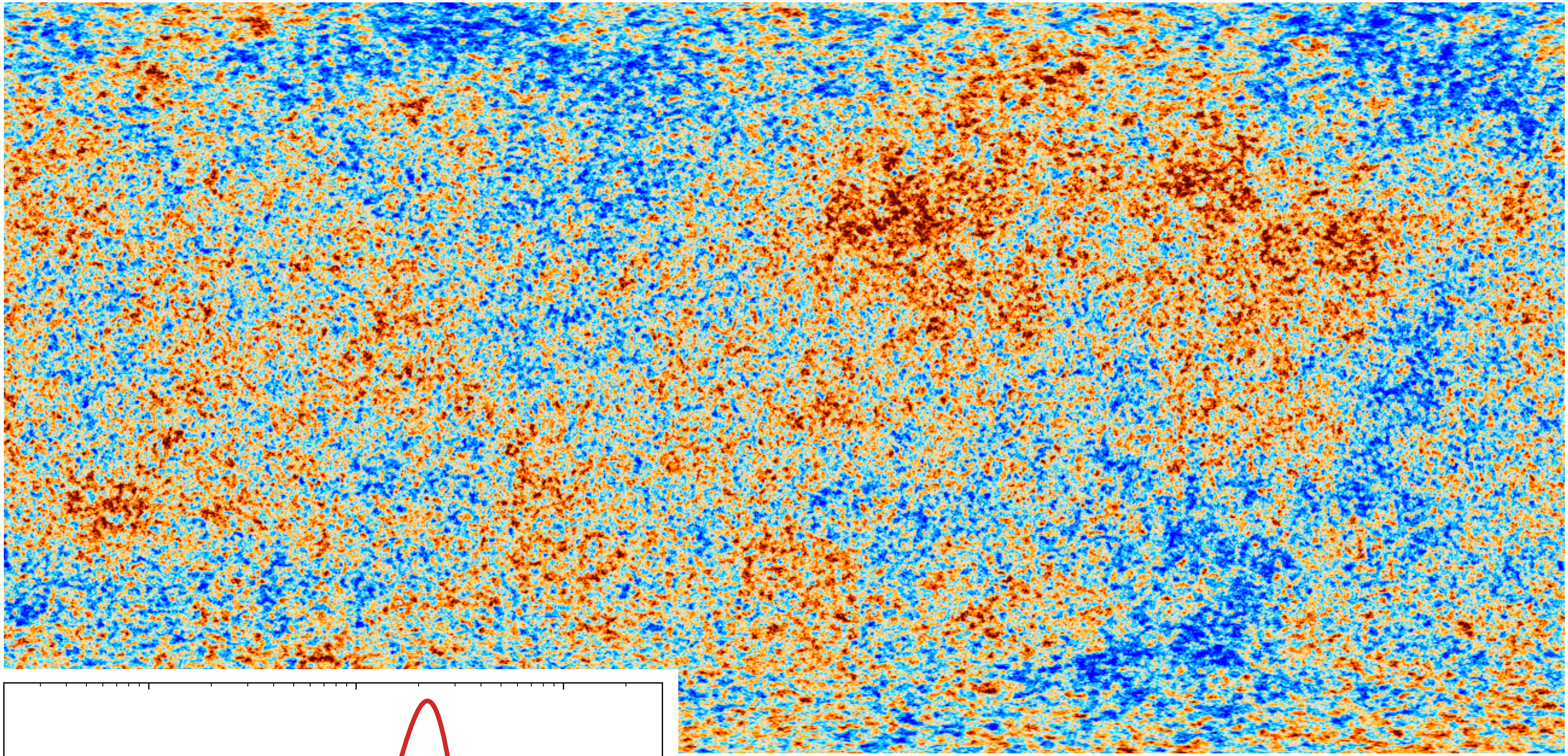
The CMB has tiny variations in its intensity, corresponding to small density fluctuations in the primordial plasma:




These fluctuations aren't random, but are highly correlated.

The observed **CMB power spectrum** is





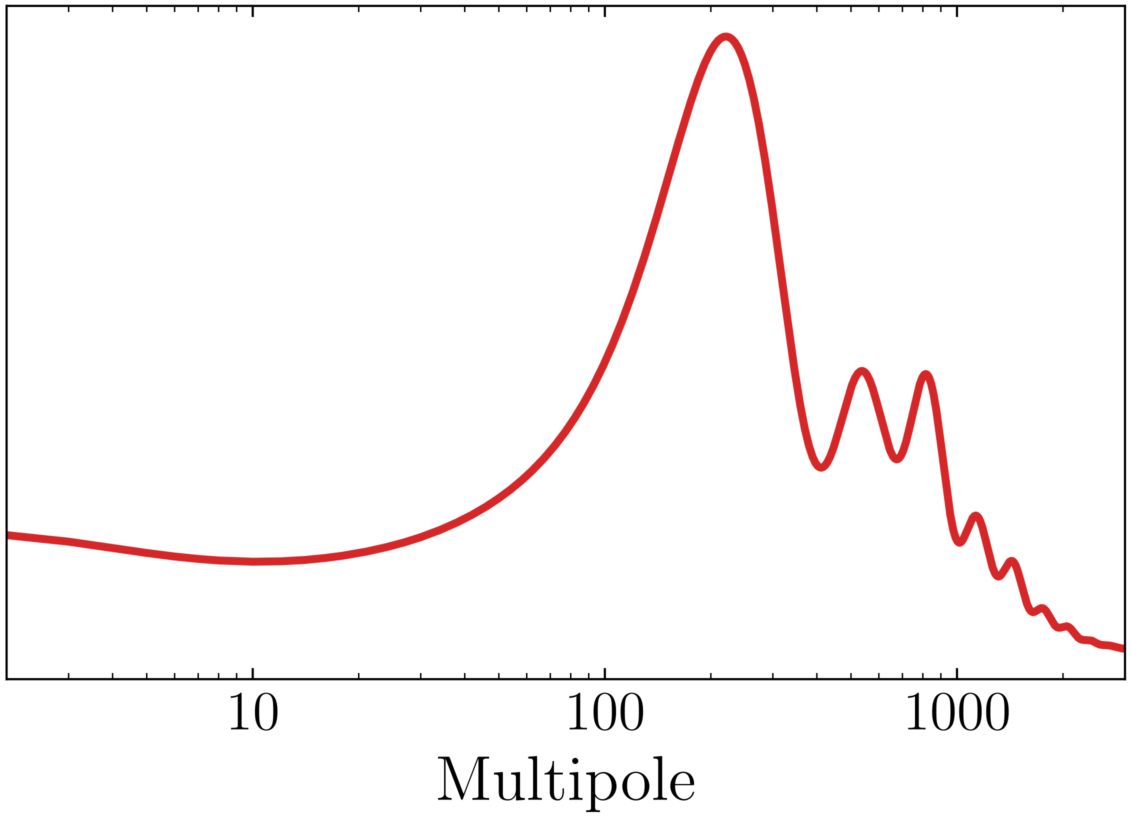



Figure courtesy of Mathew Madhavacheril

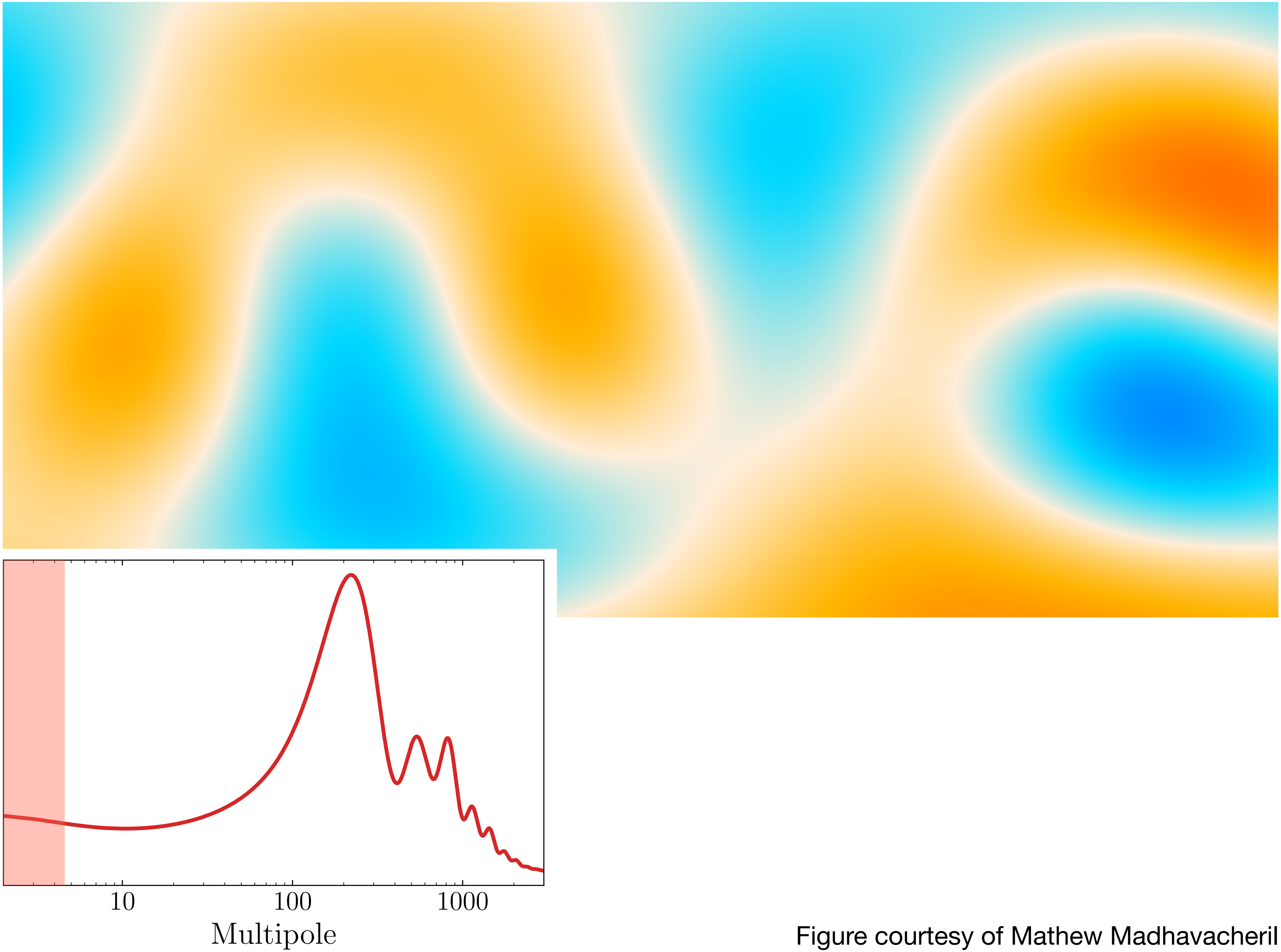
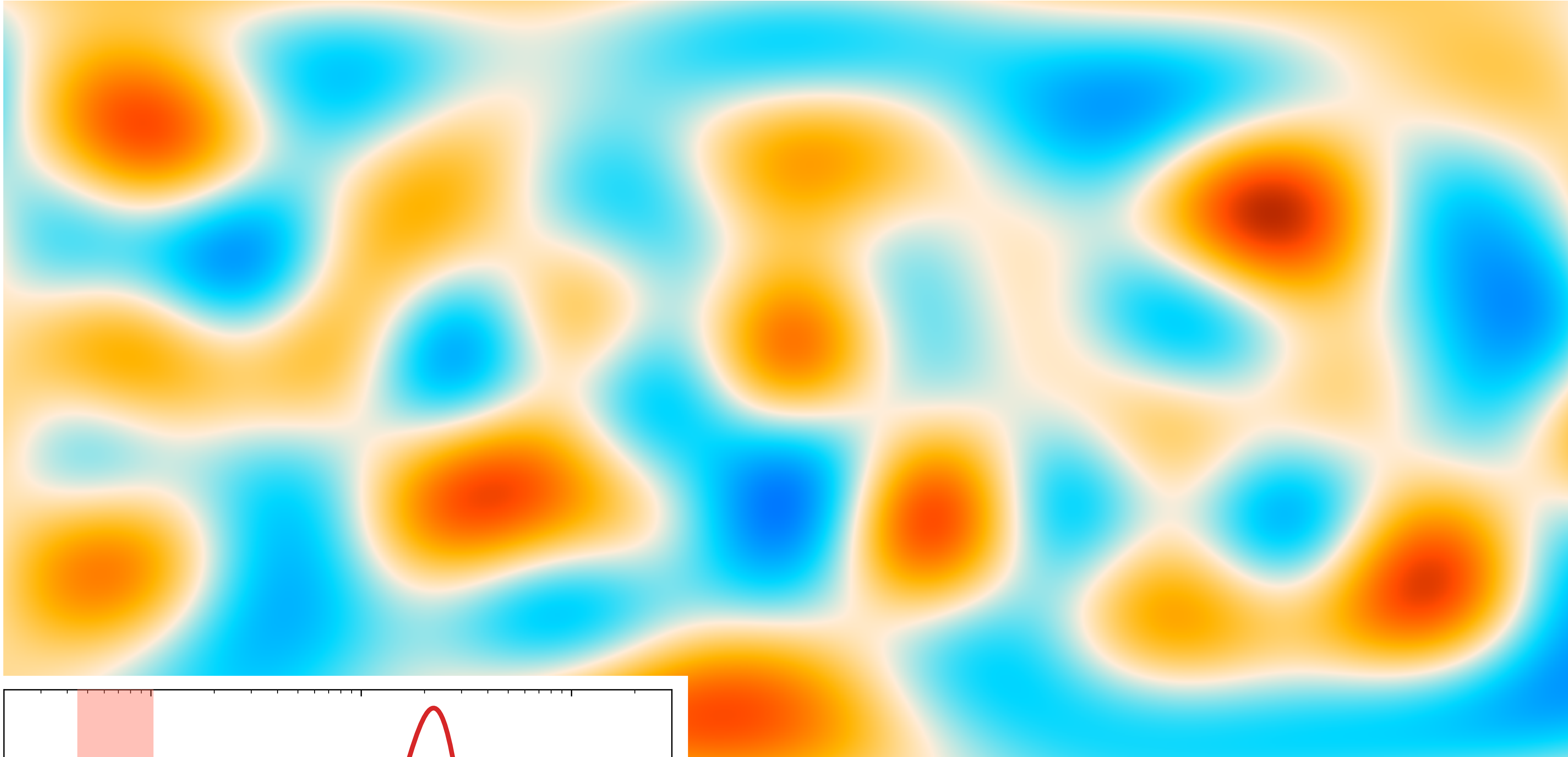



Figure courtesy of Mathew Madhavacheril



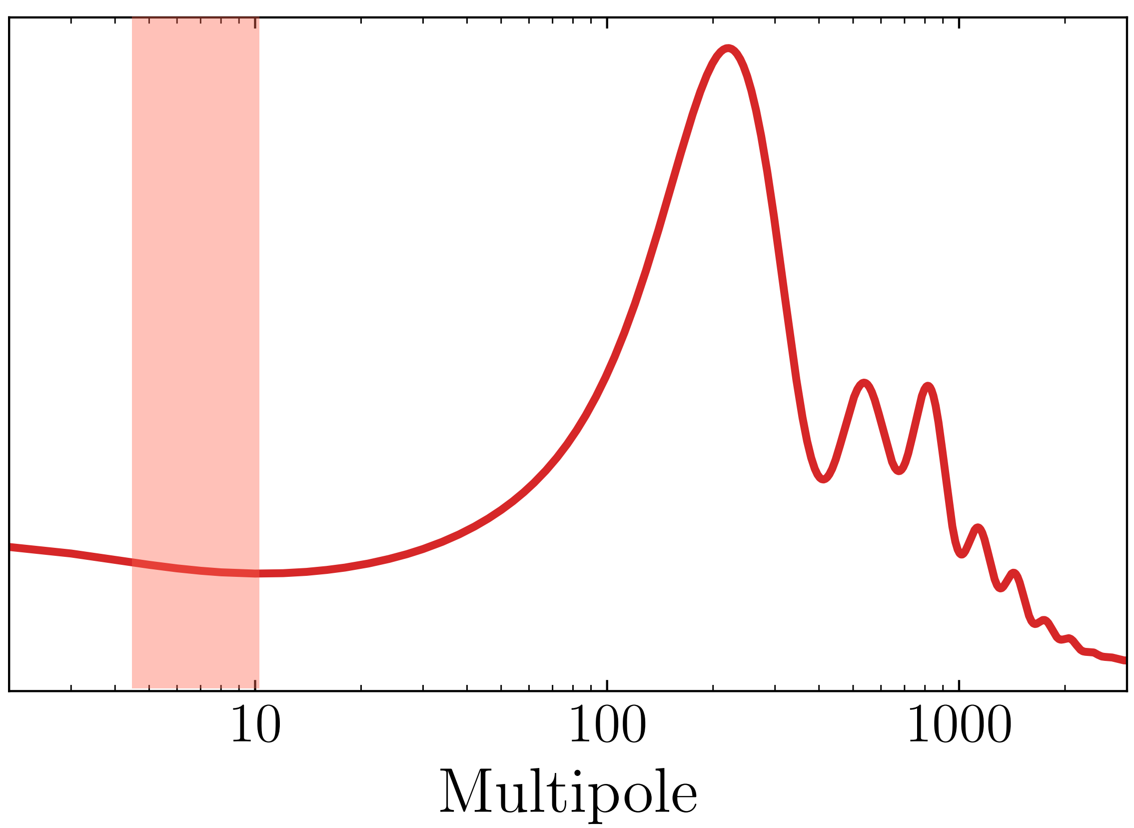



Figure courtesy of Mathew Madhavacheril

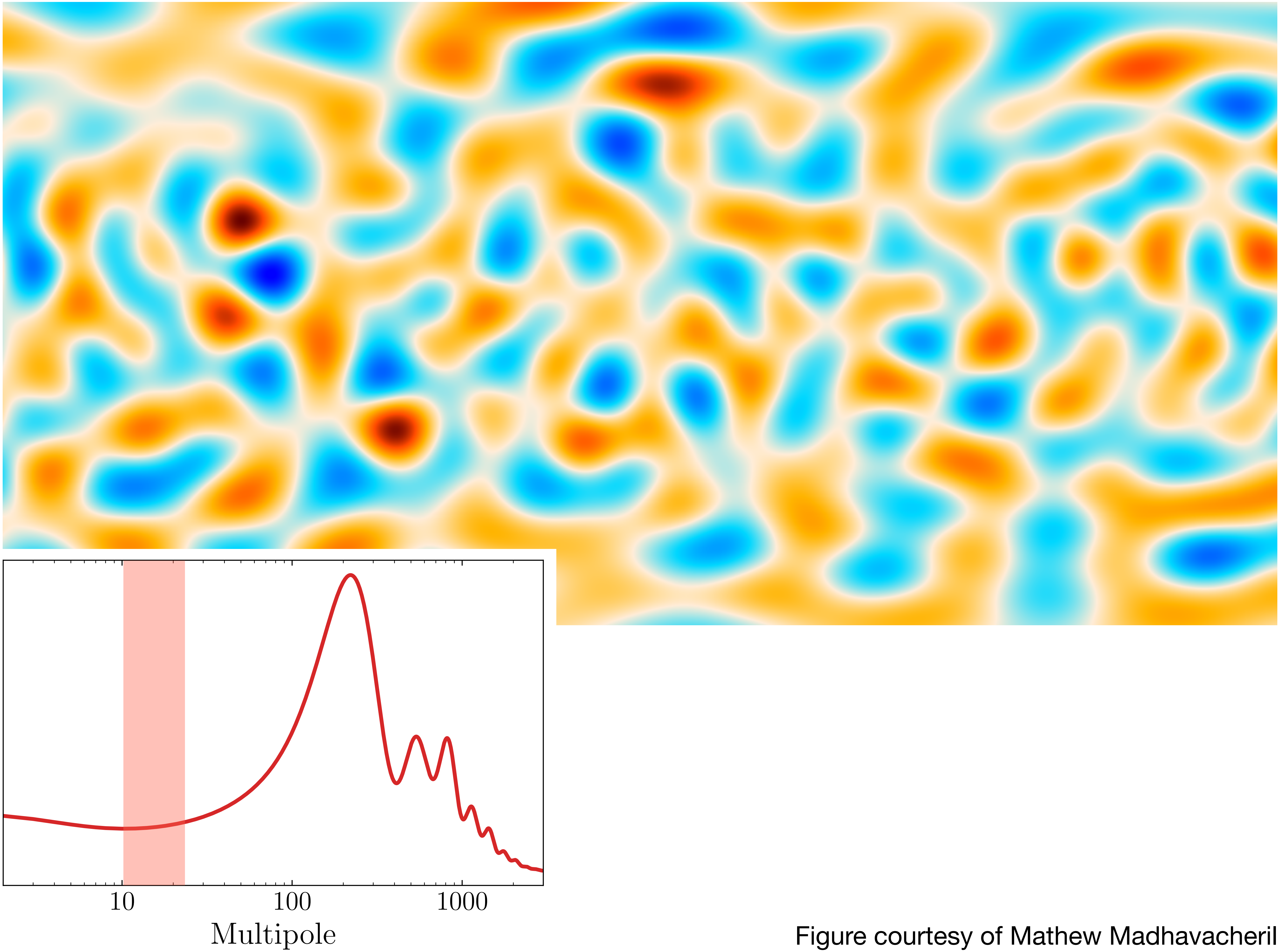



Figure courtesy of Mathew Madhavacheril

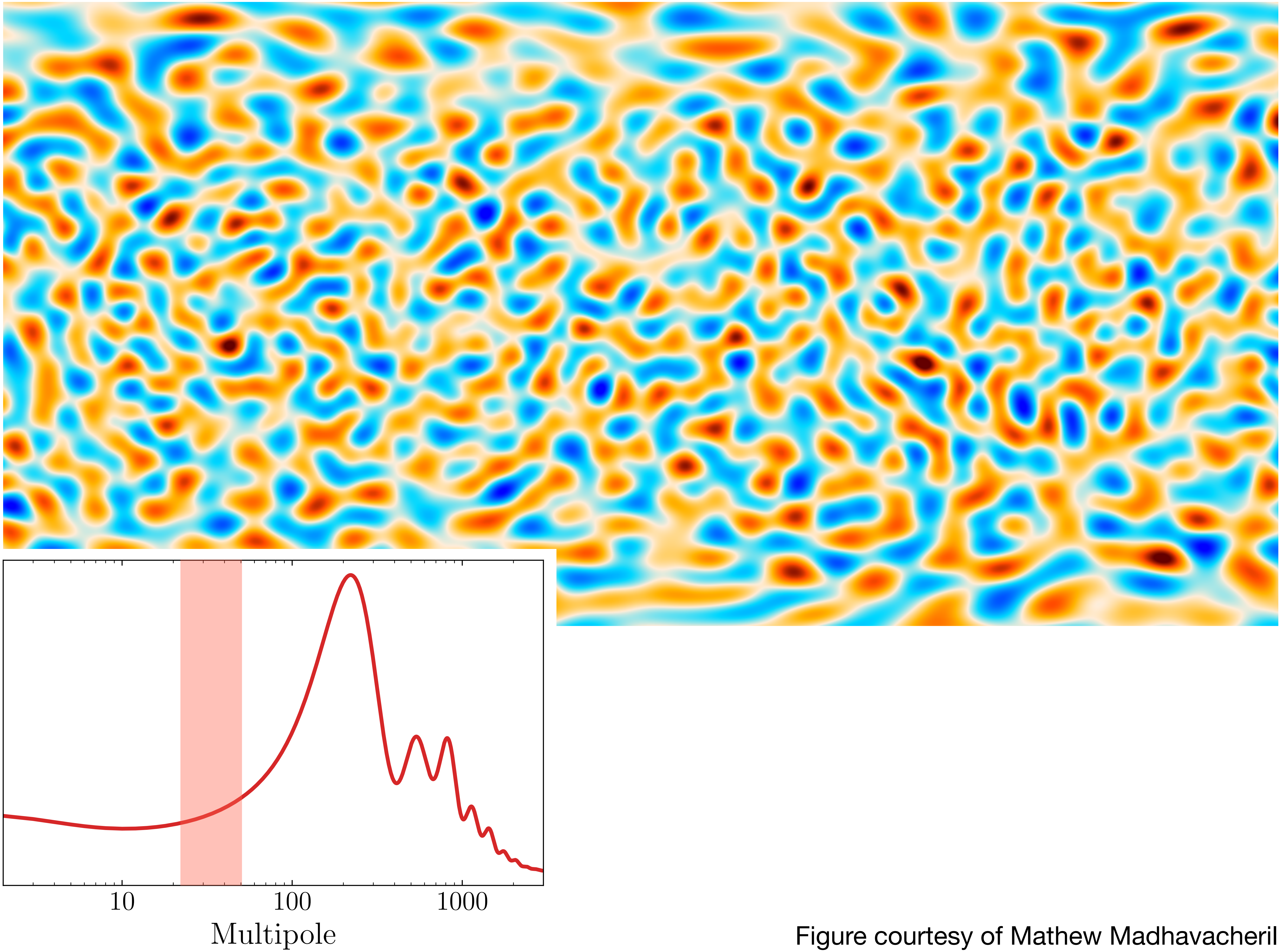



Figure courtesy of Mathew Madhavacheril

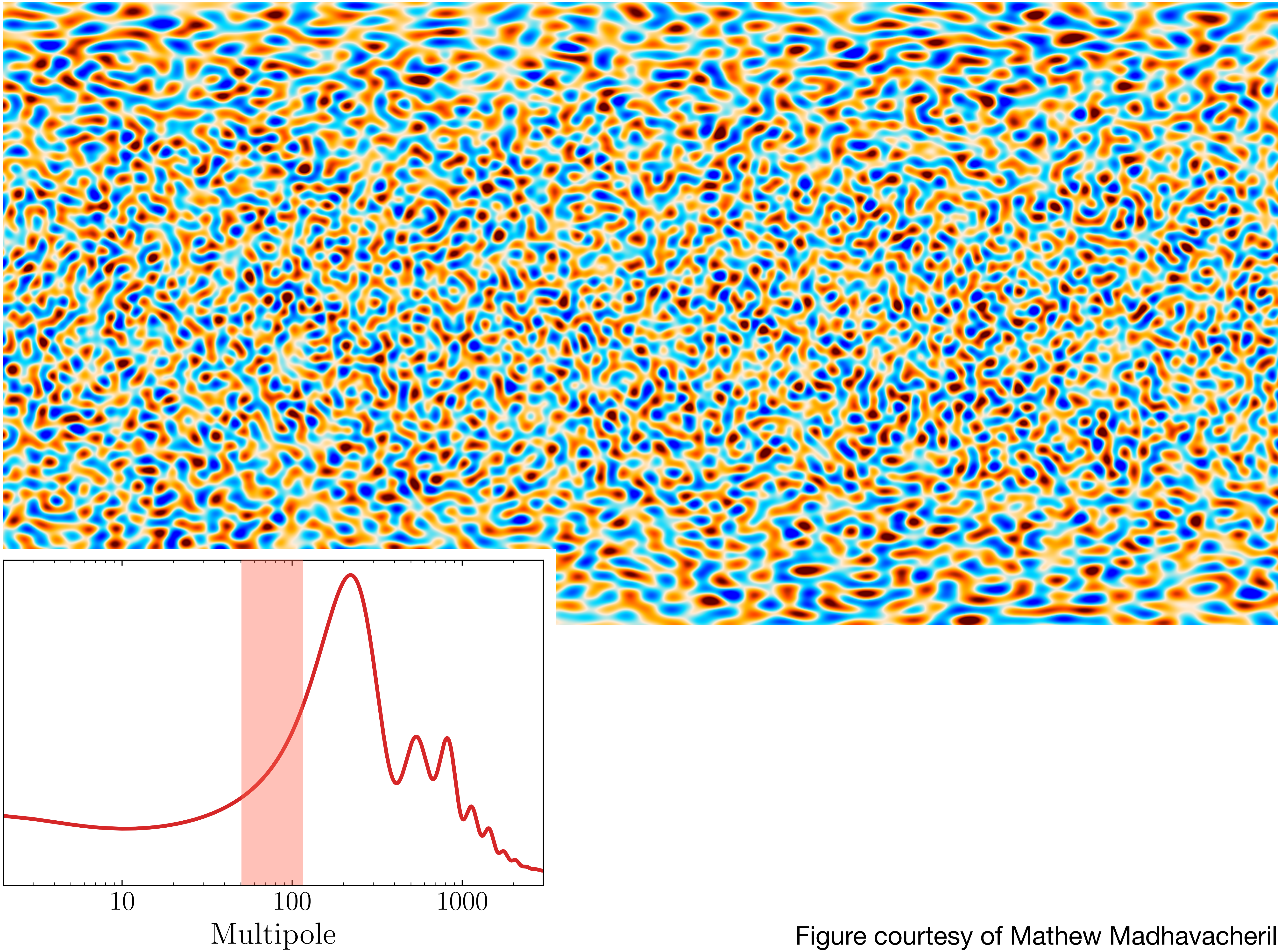



Figure courtesy of Mathew Madhavacheril

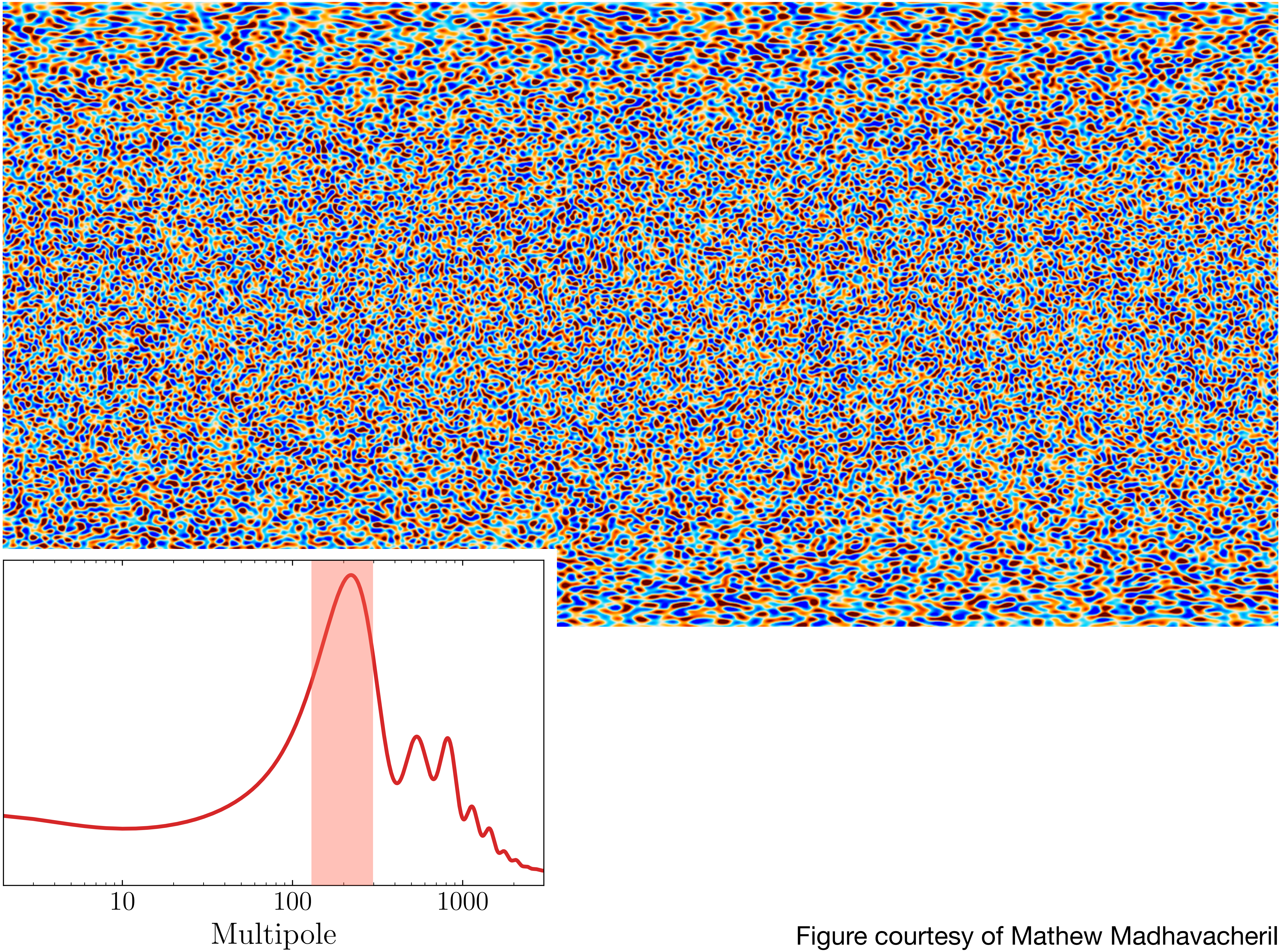



Figure courtesy of Mathew Madhavacheril

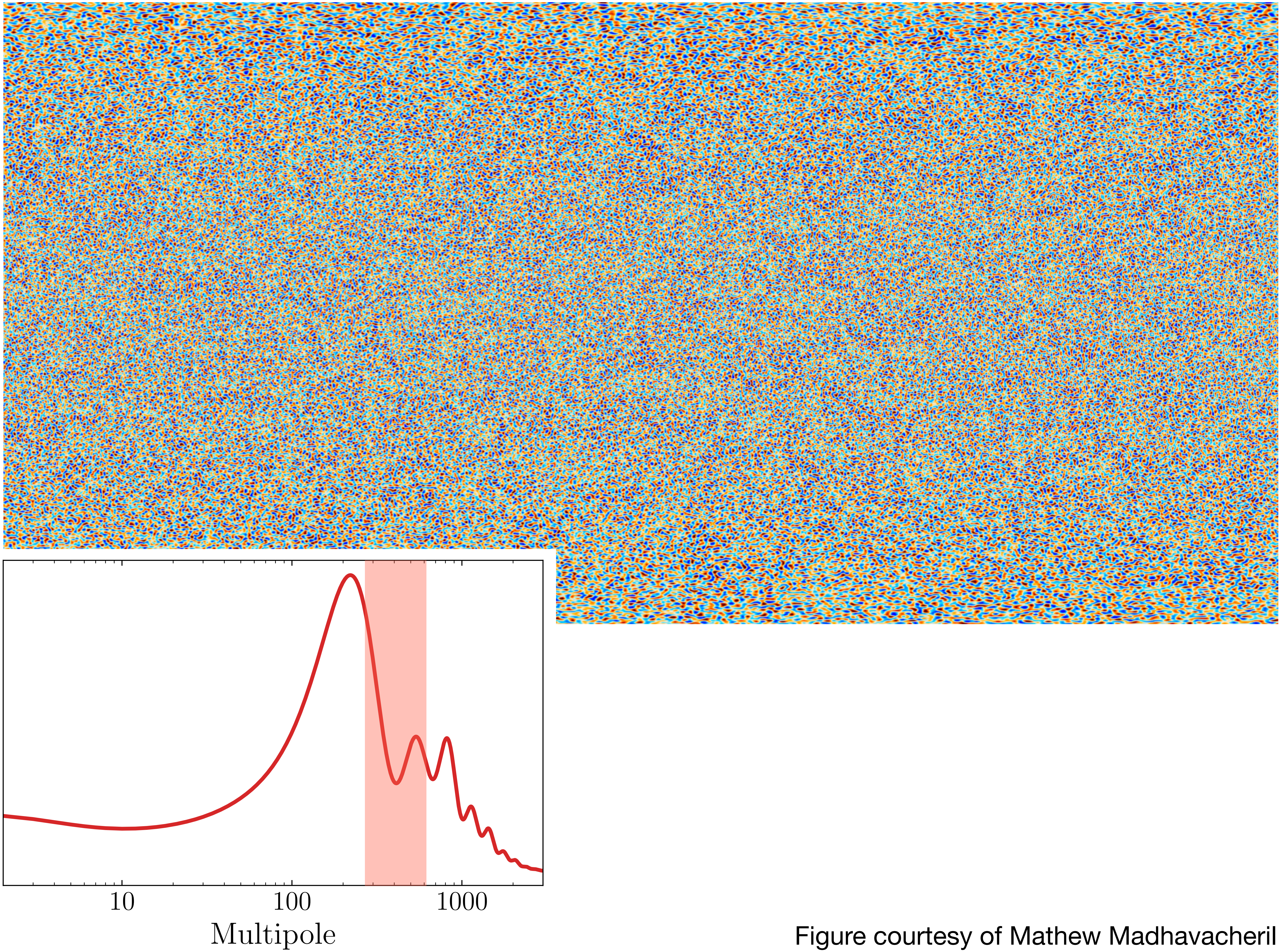



Figure courtesy of Mathew Madhavacheril

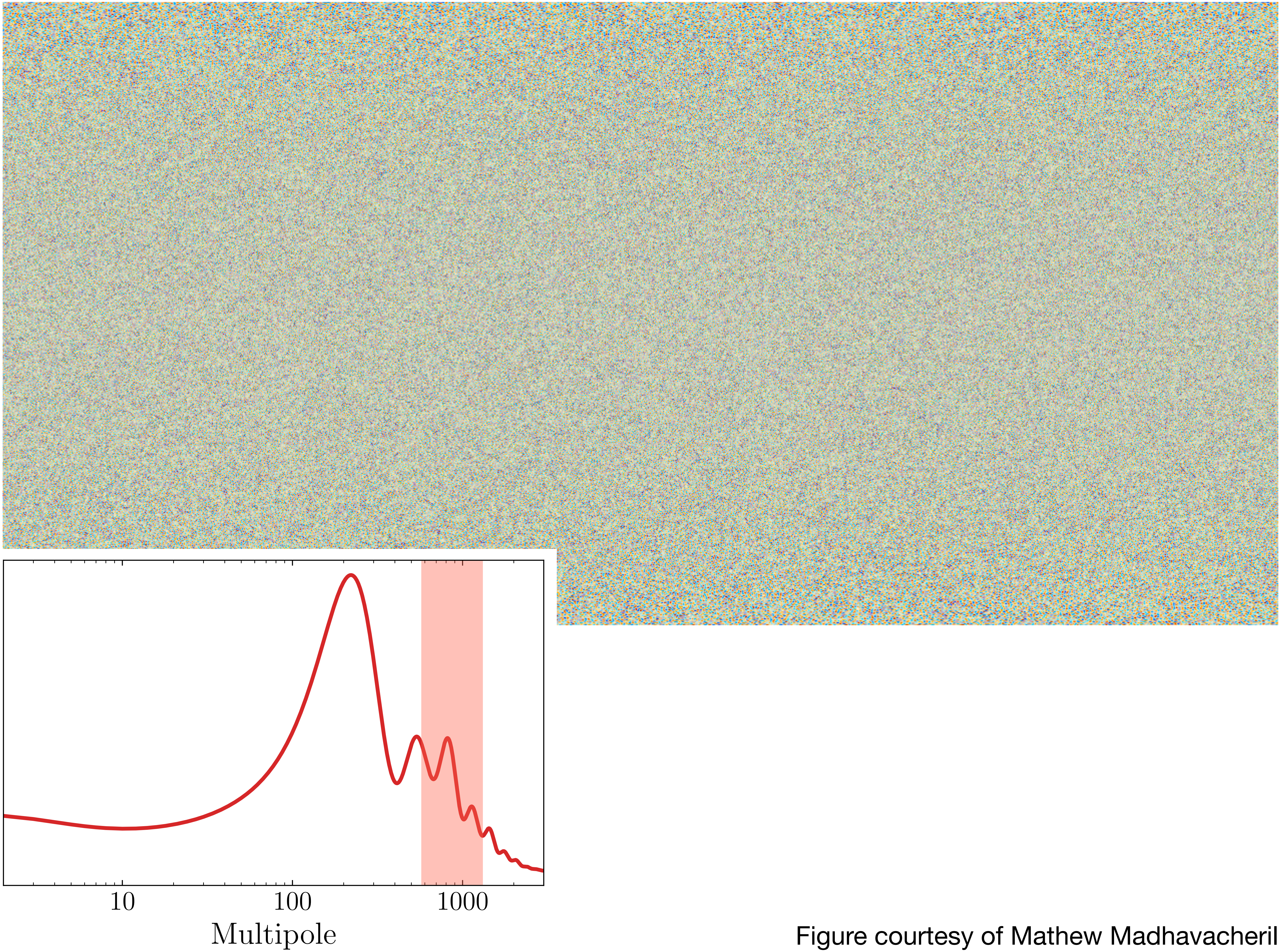



Figure courtesy of Mathew Madhavacheril

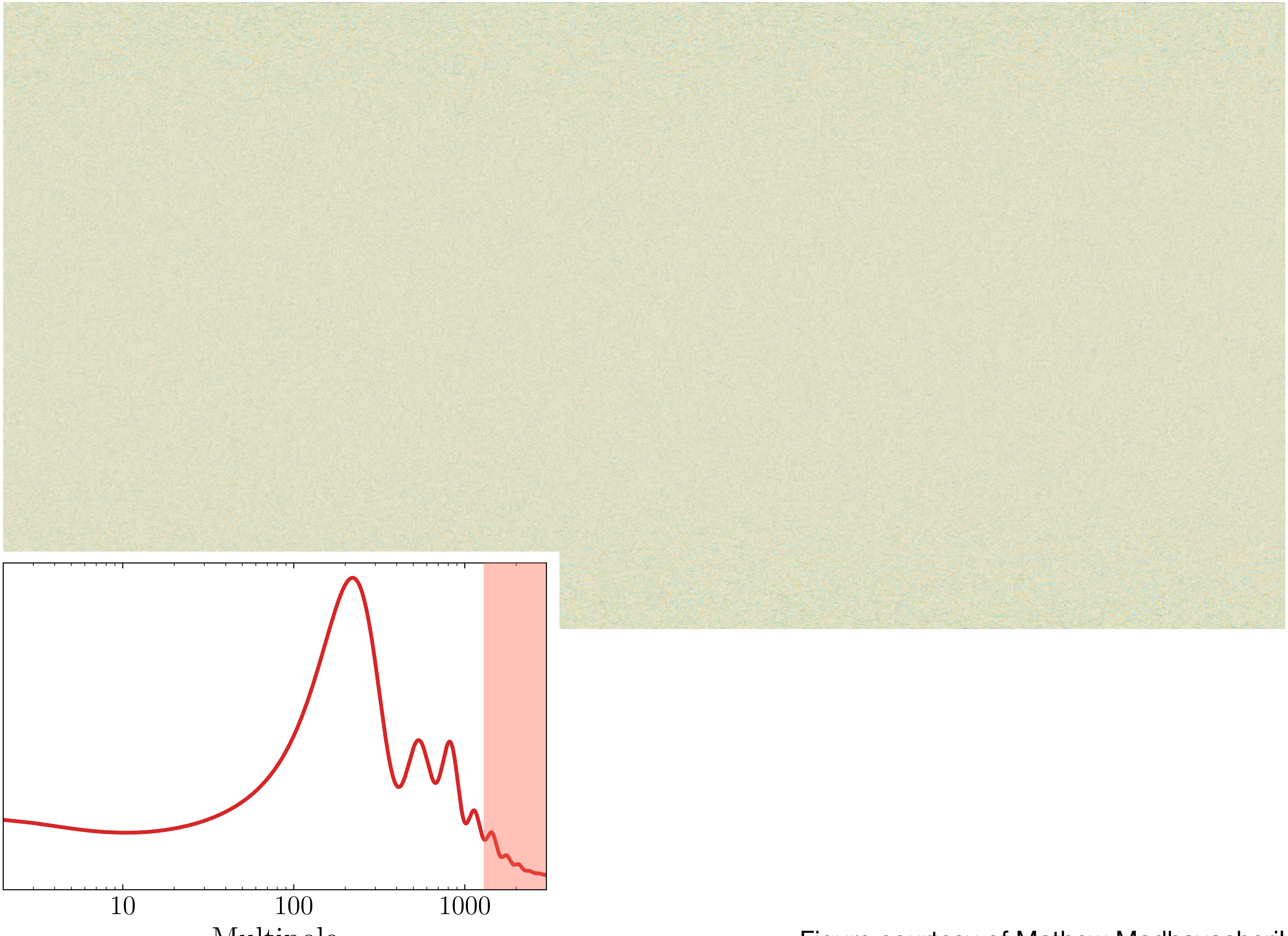
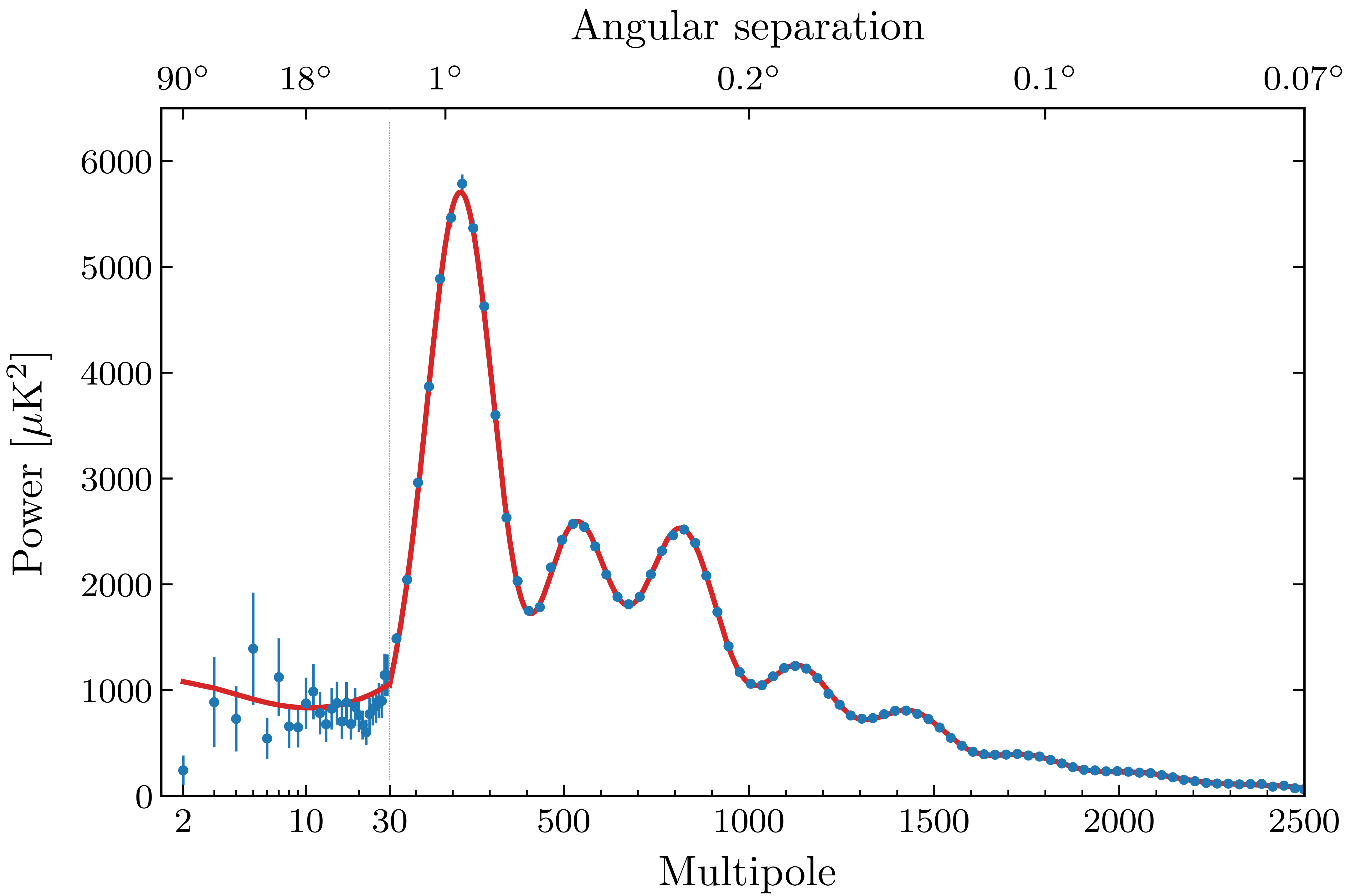



Figure courtesy of Mathew Madhavacheril

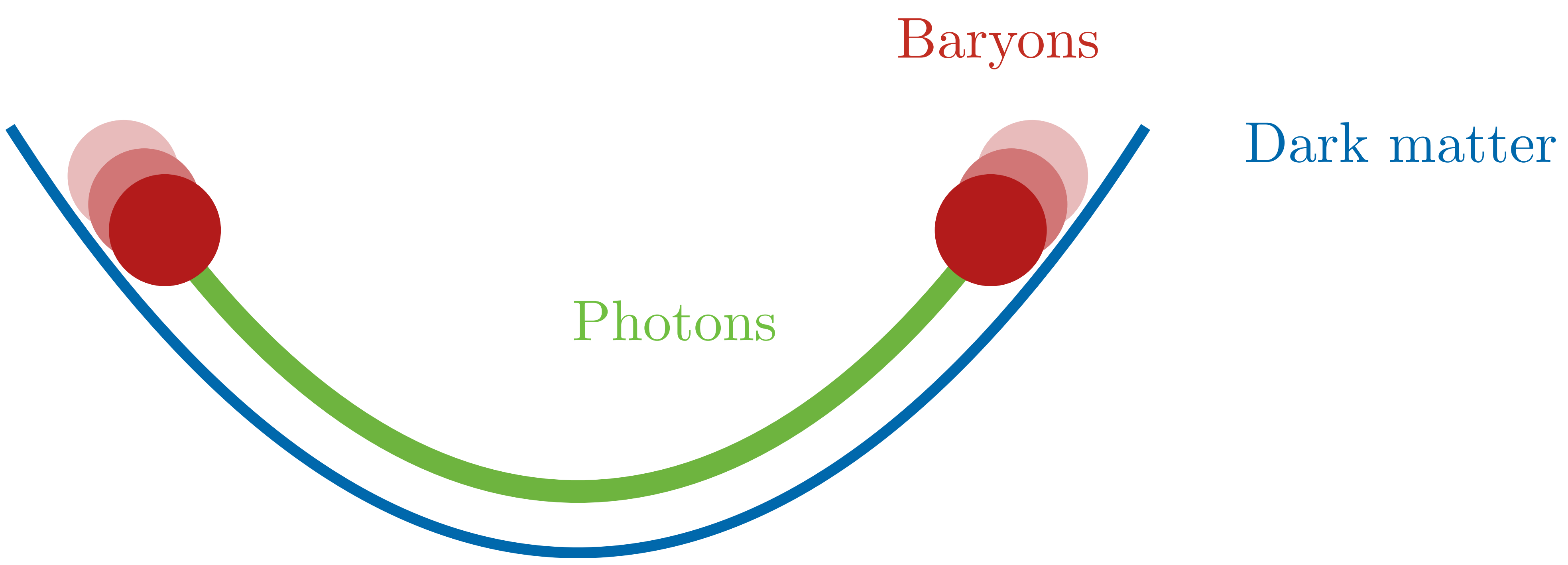
# CMB Power Spectrum




What created the features in the power spectrum?

# Photon-Baryon Fluid

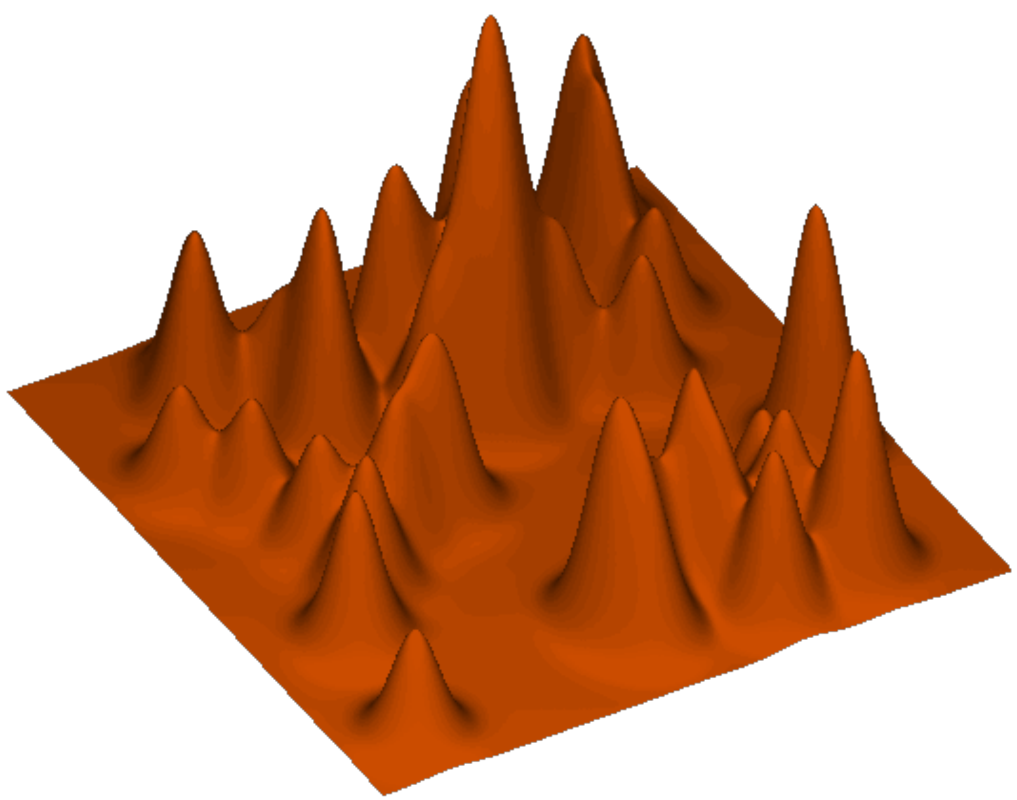
At early times, photons and baryons (mostly protons and electrons) are strongly coupled and act as a single fluid:



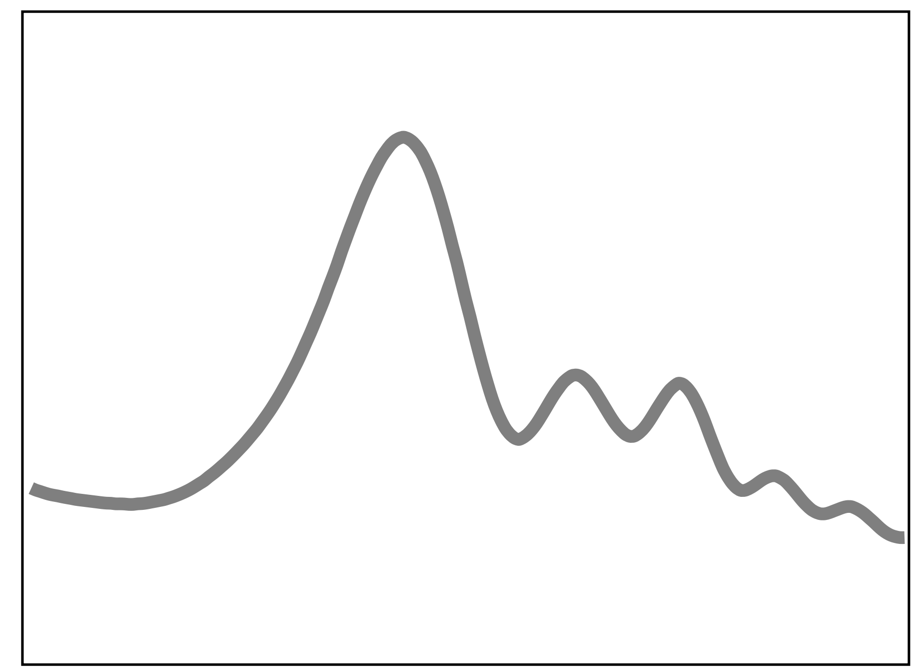

The **photon pressure** prevents the collapse of density fluctuations.
This allows for **sound waves** (like for density fluctuations in air).

# Cosmic Sound Waves

The pattern of the CMB fluctuations is a consequence of these **sound waves**:



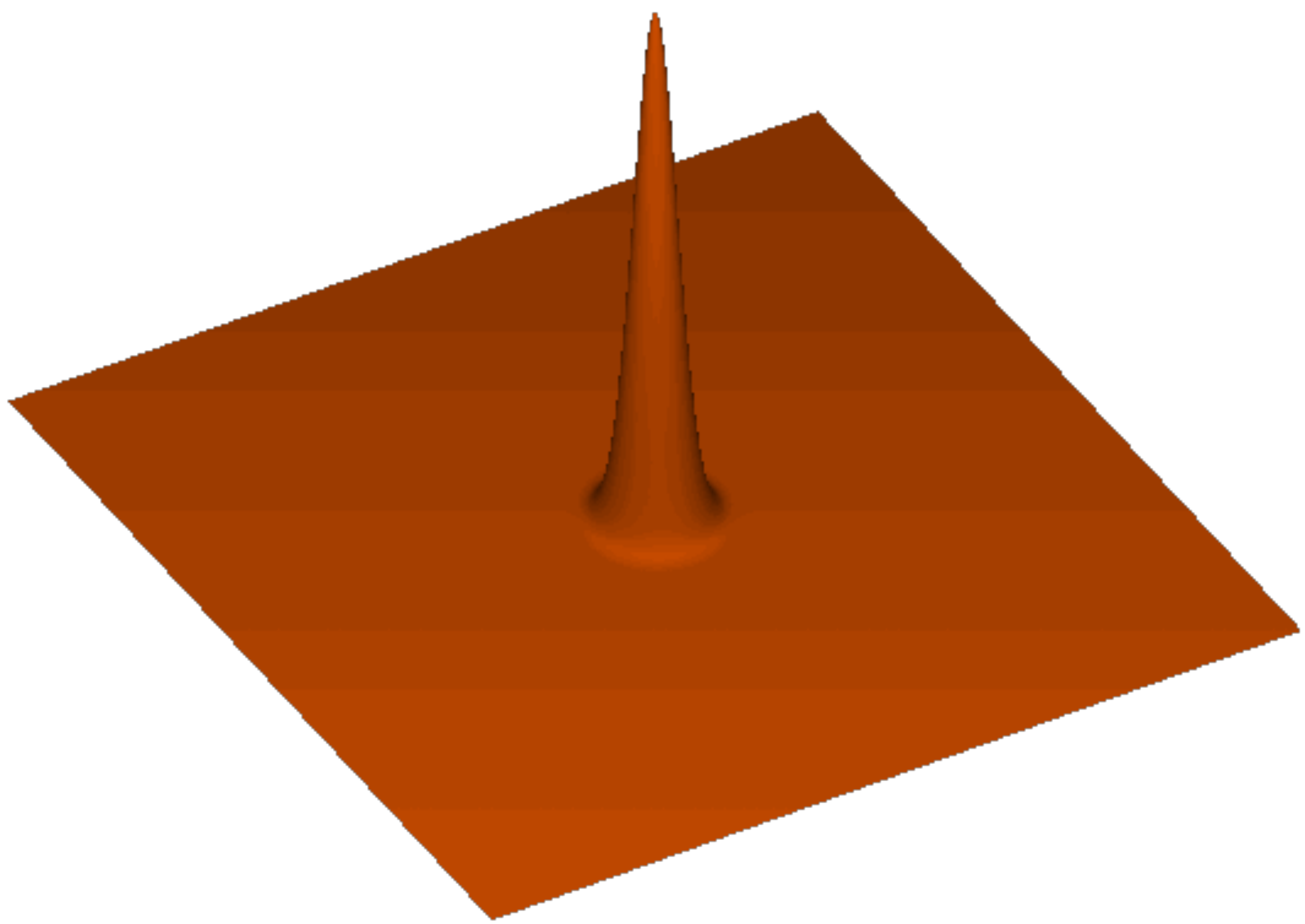
Superposition of many waves



CMB correlations

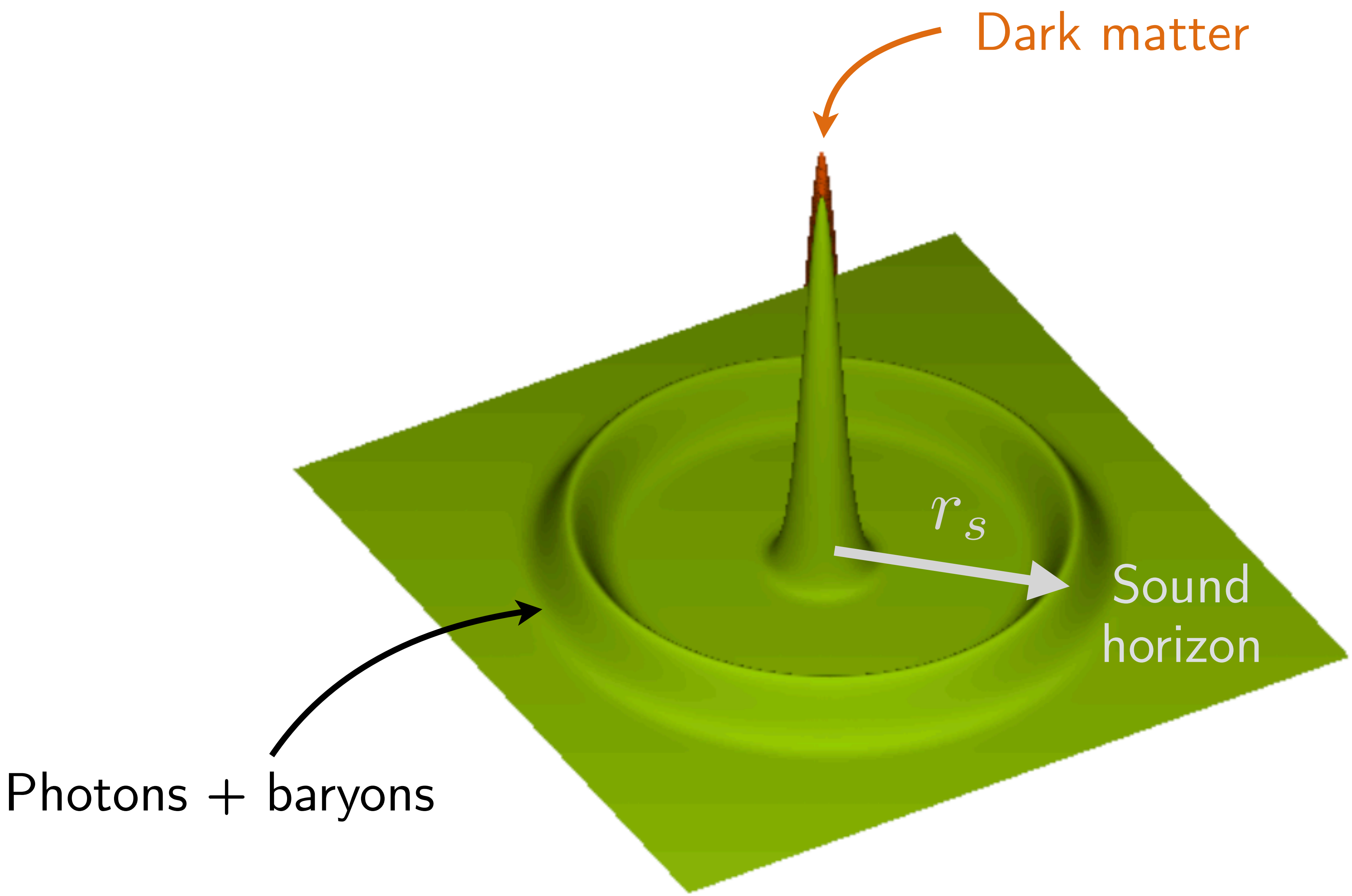
# Cosmic Sound Waves

Consider the evolution of a single localized density fluctuation:



# Cosmic Sound Waves

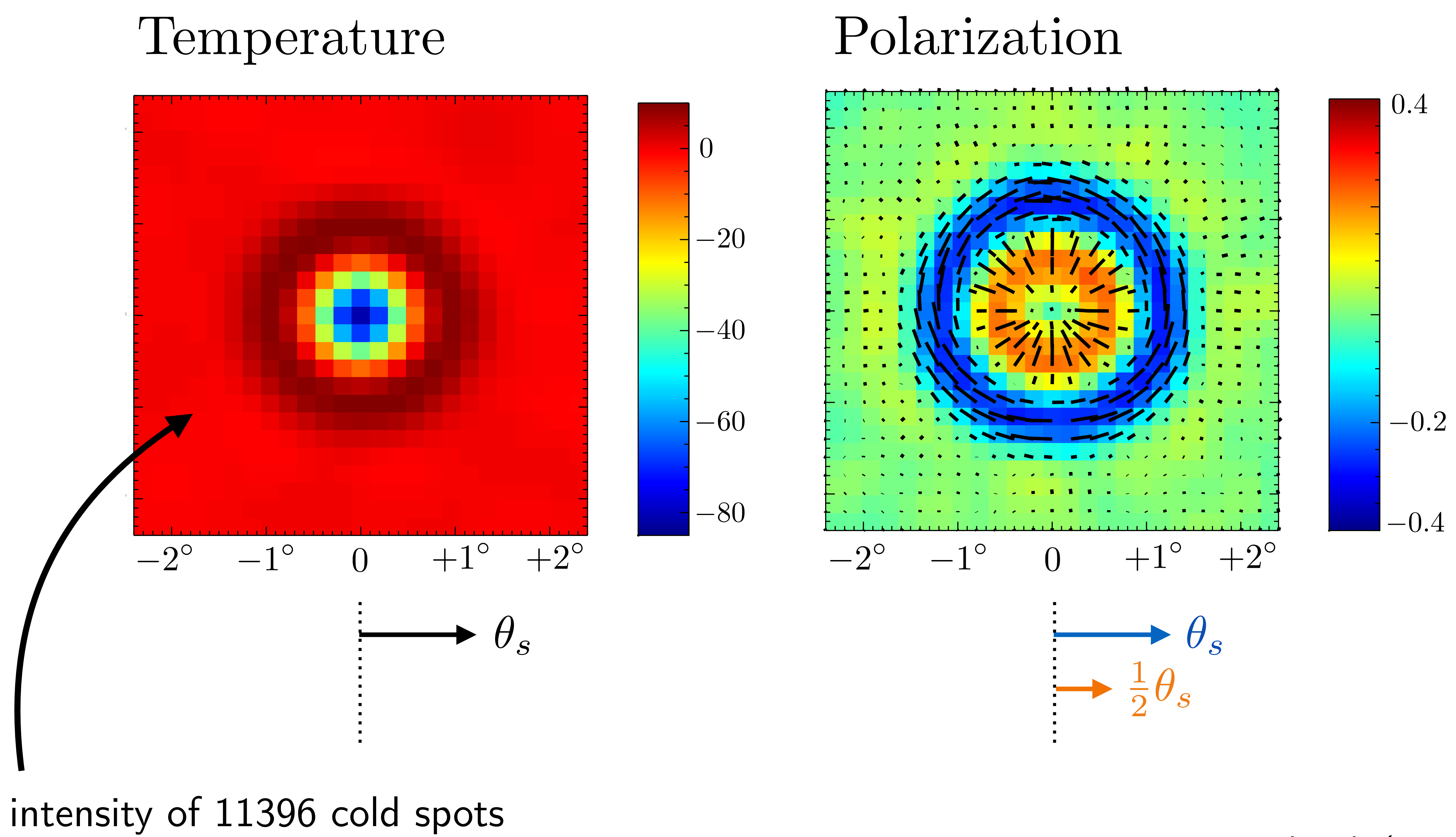
This creates a radial sound wave in the photon-baryon fluid:




The wave travels a distance of 50 000 light-years (called the **sound horizon**) before the Universe becomes transparent to light.

# CMB Anisotropies

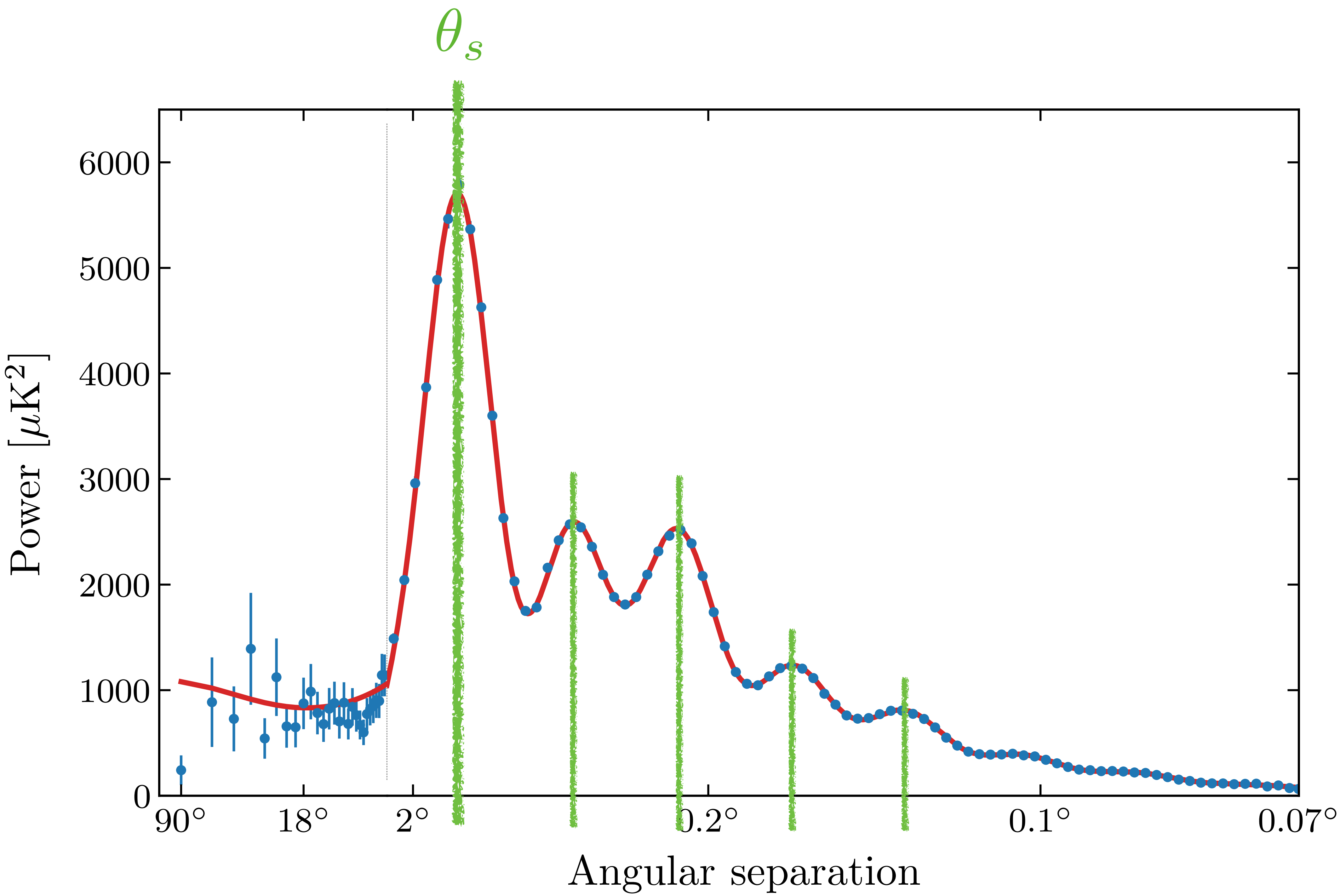
This sound horizon is imprinted in the pattern of CMB fluctuations:




Planck (2015)

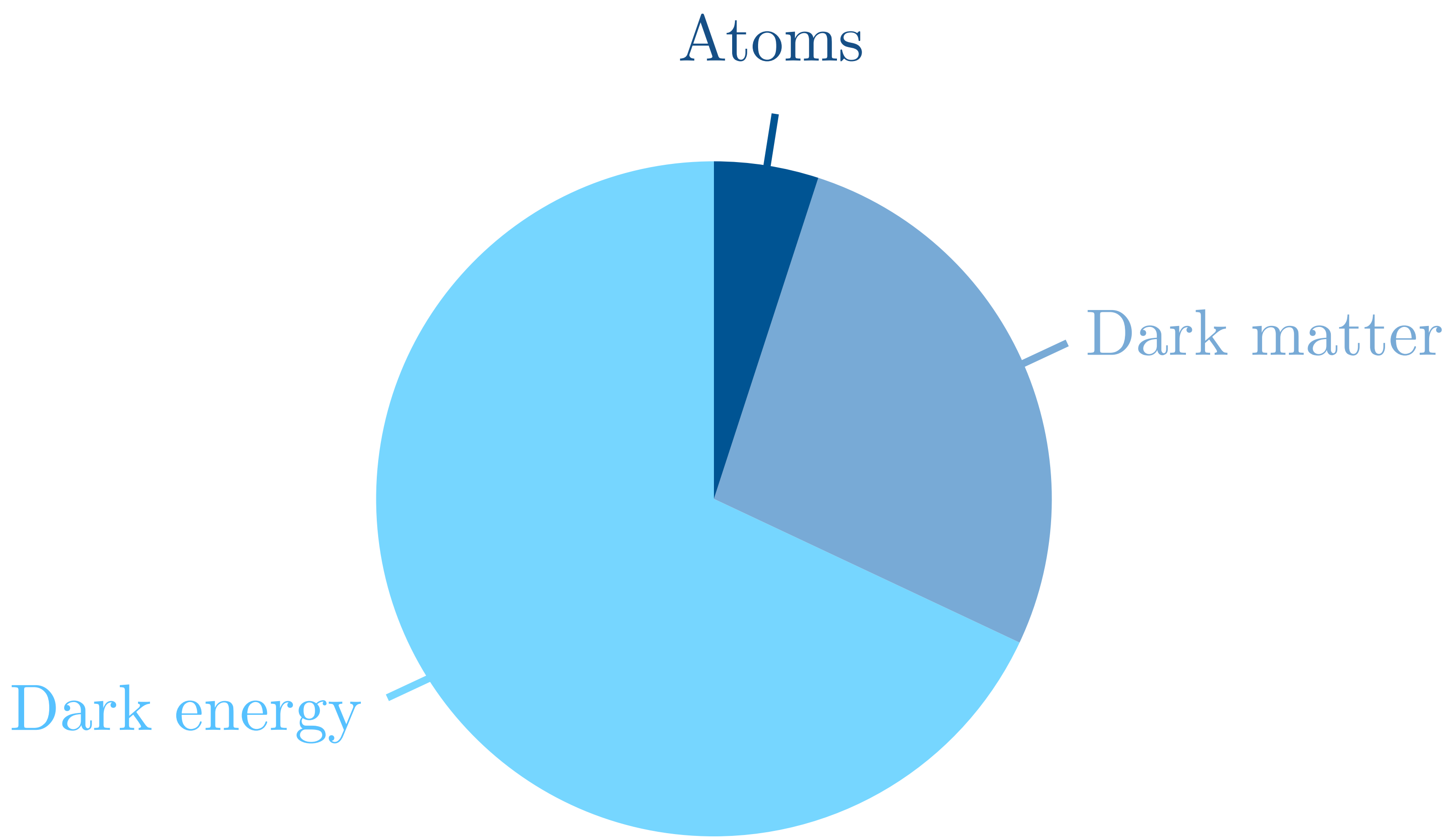
# CMB Anisotropies

This sound horizon is imprinted in the pattern of CMB fluctuations:



# CMB Anisotropies

The precise pattern of the CMB fluctuations depends on the composition of the Universe (and its initial conditions):

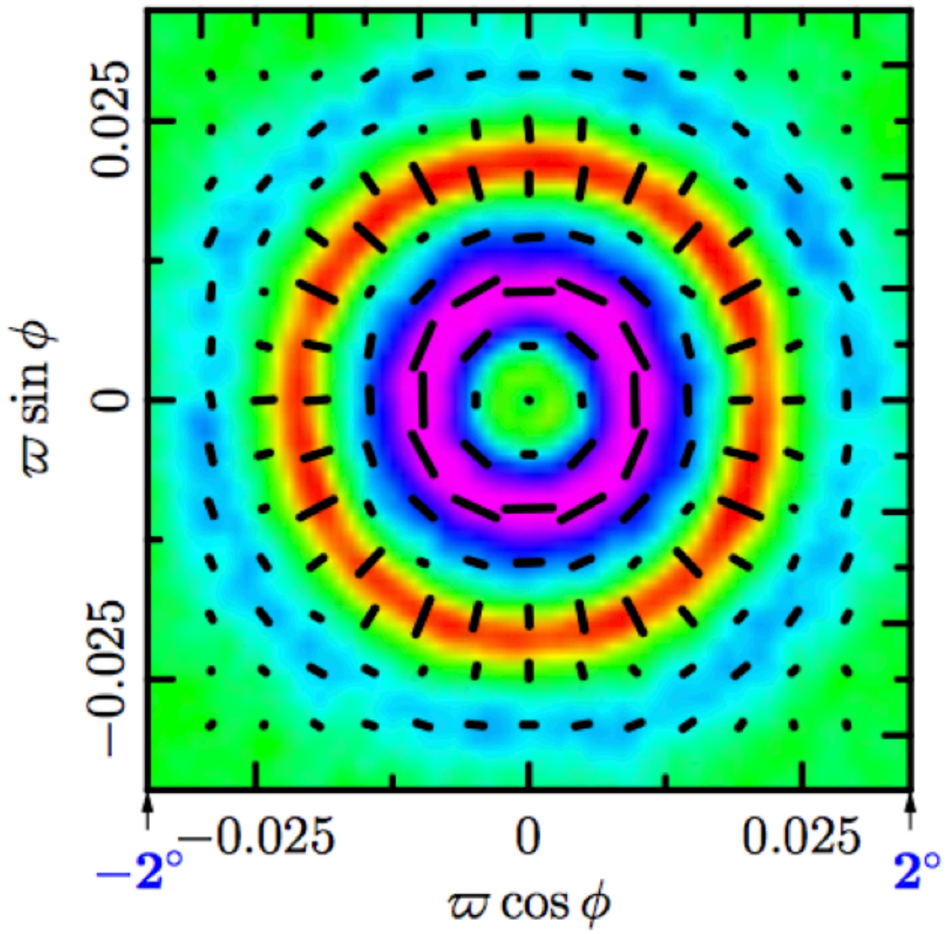



Observations of the CMB have therefore allowed us to determine the parameters of the cosmological standard model.

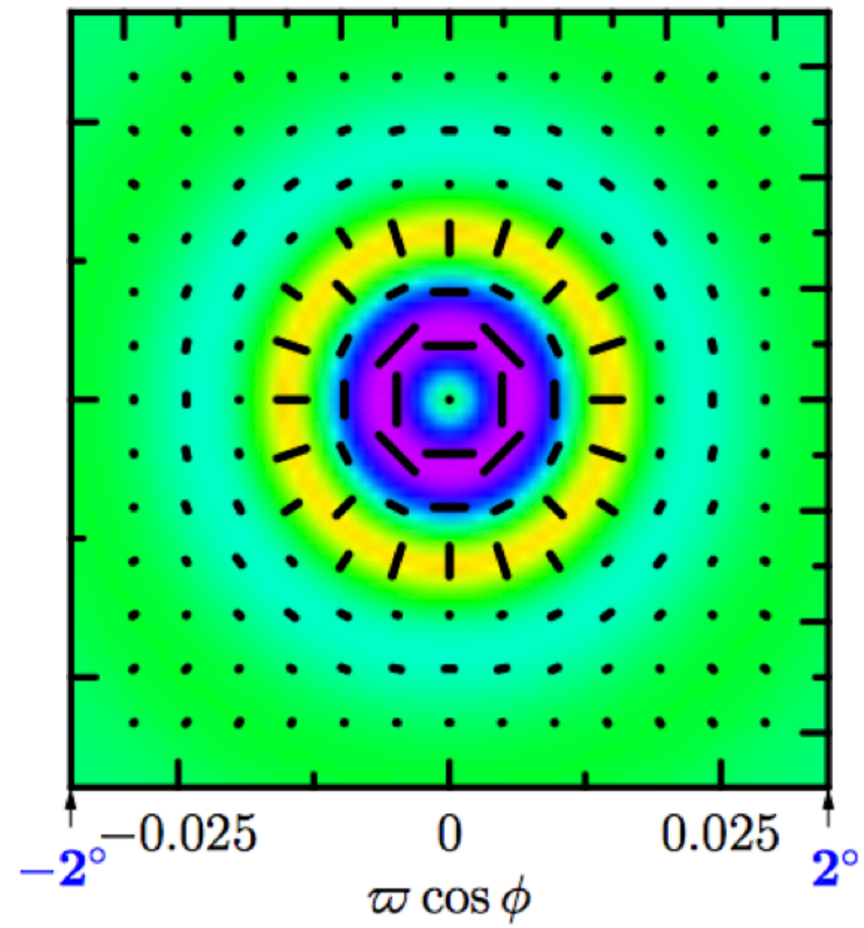
# Dark Matter

Without dark matter, the data would look very different:

Data

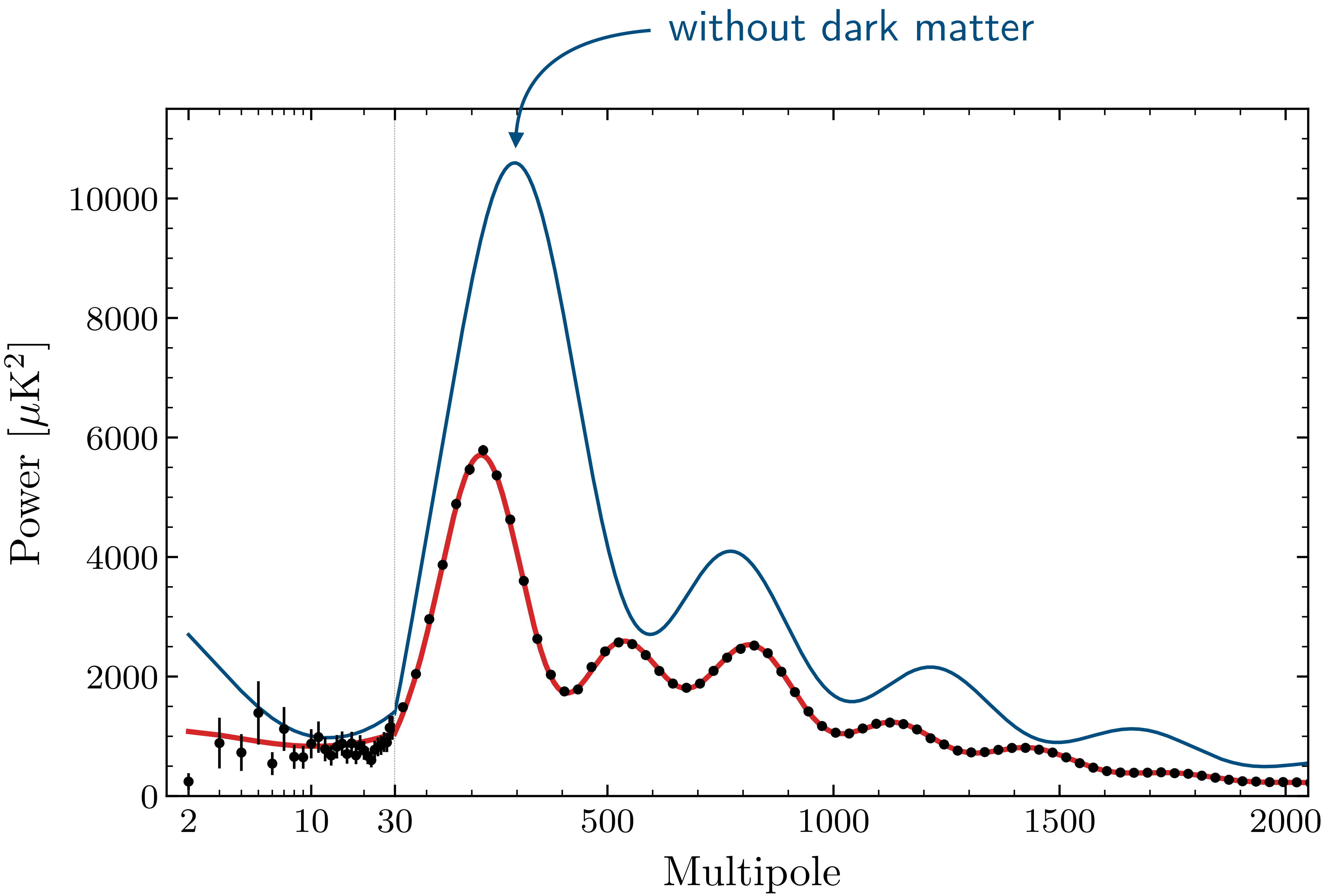



No Dark Matter




Figures courtesy of Zhiqi Huang and Dick Bond [ACT collaboration]

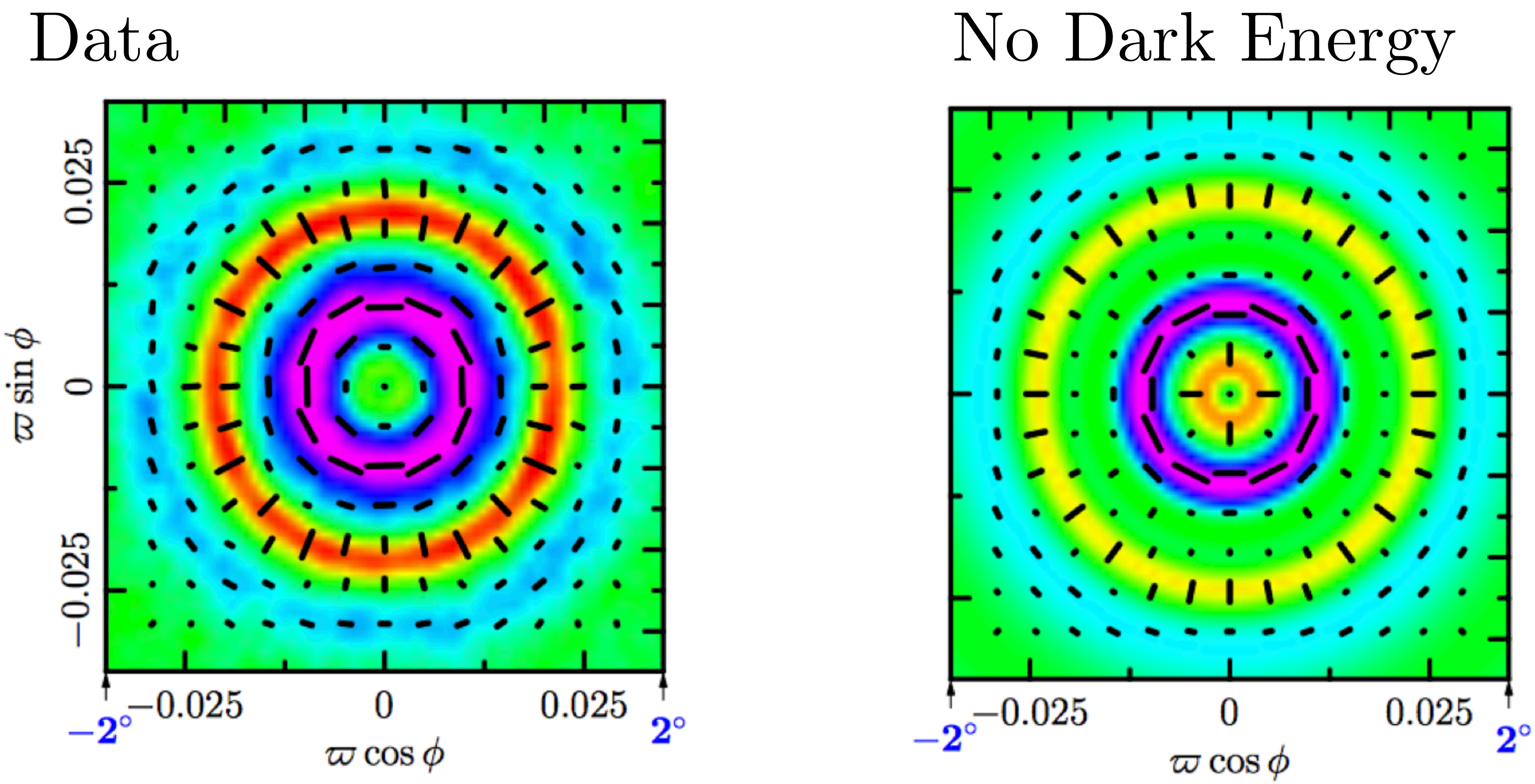
This can also be seen in the power spectrum: $\Omega_{\mathrm{m}} = 0.32$




Planck (2018)

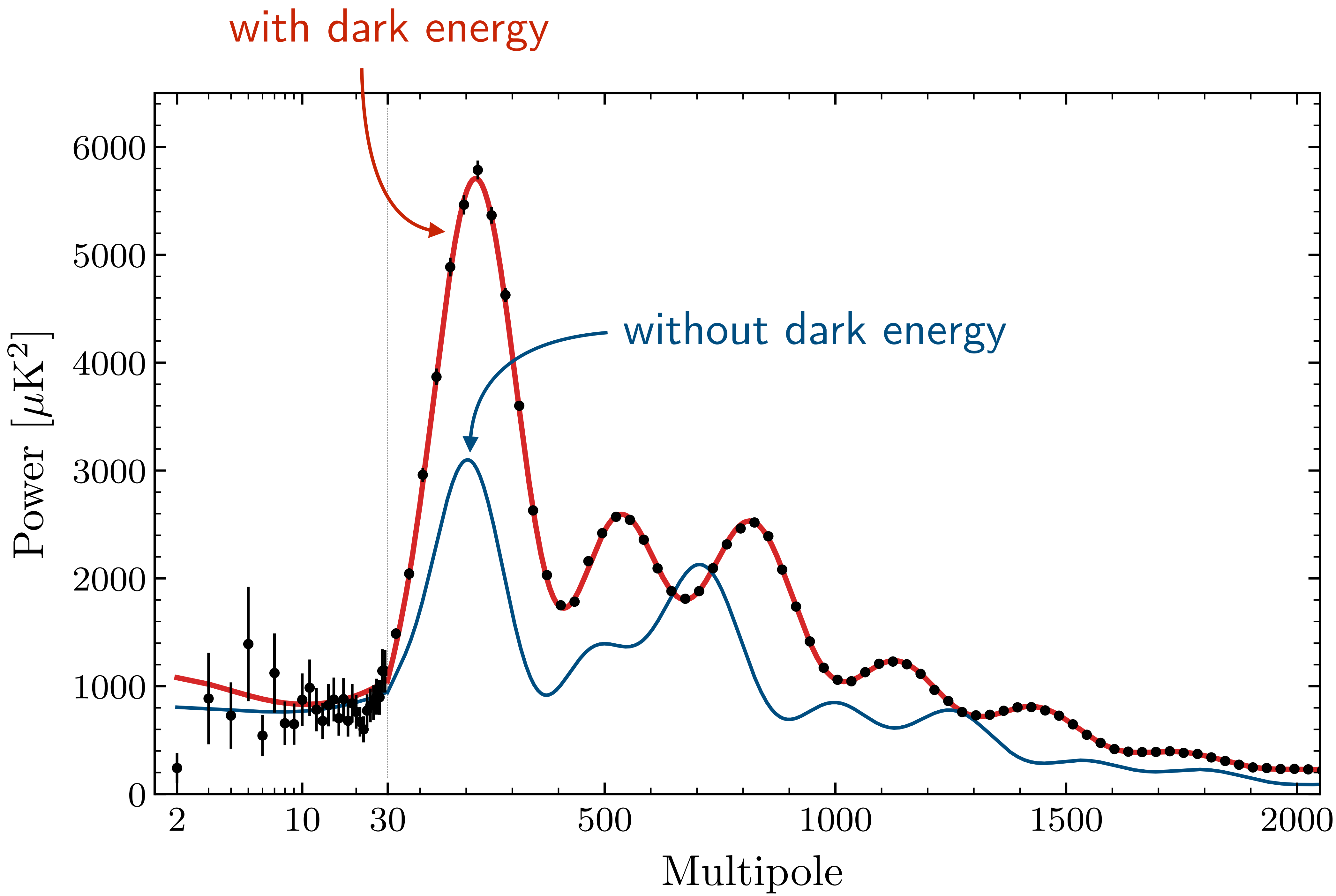
# Dark Energy

Without dark energy, the data would look very different:



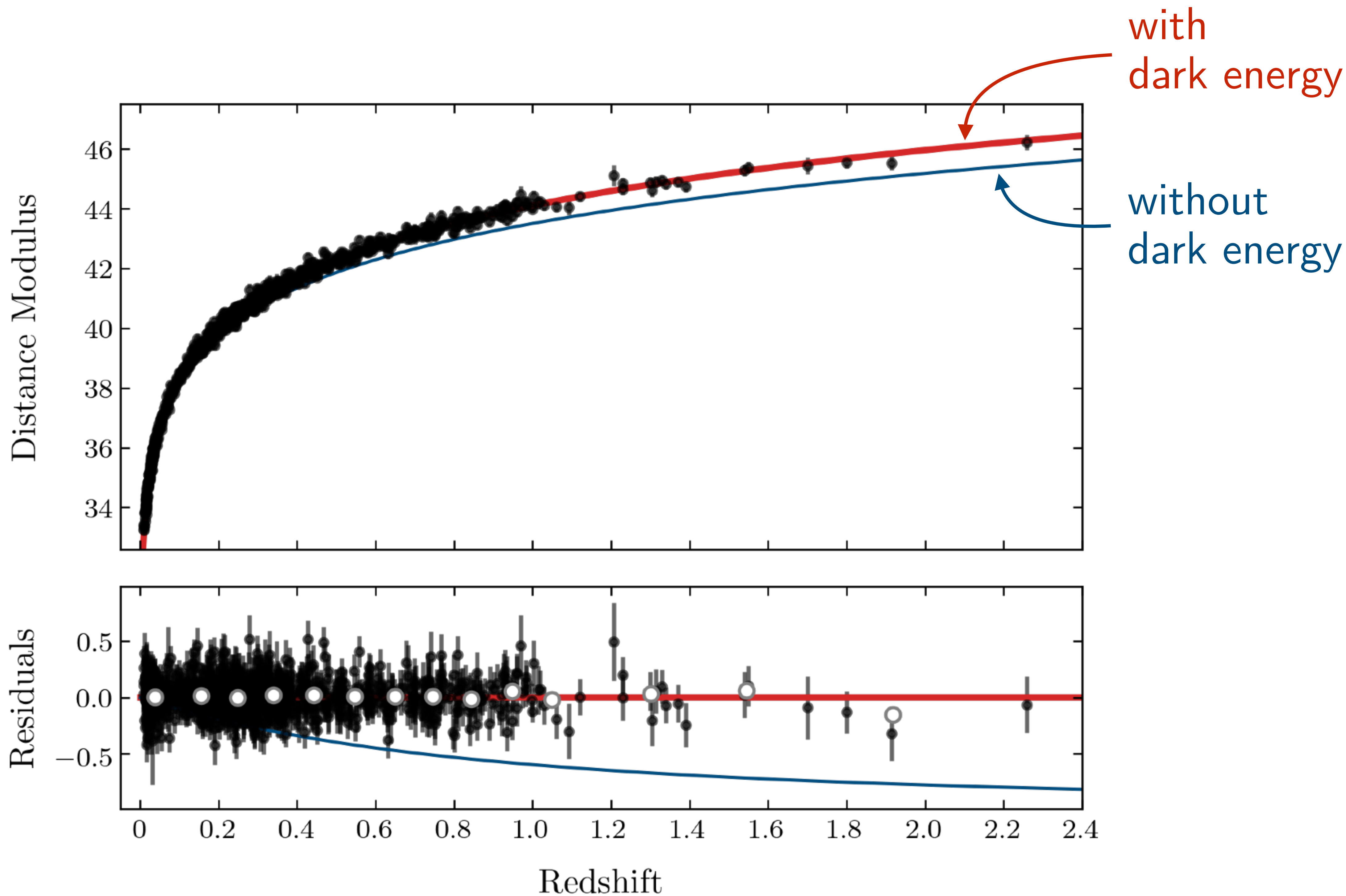

Figures courtesy of Zhiqi Huang and Dick Bond [ACT collaboration]

This can also be seen in the power spectrum: $\Omega_\Lambda = 0.68$




Planck (2018)

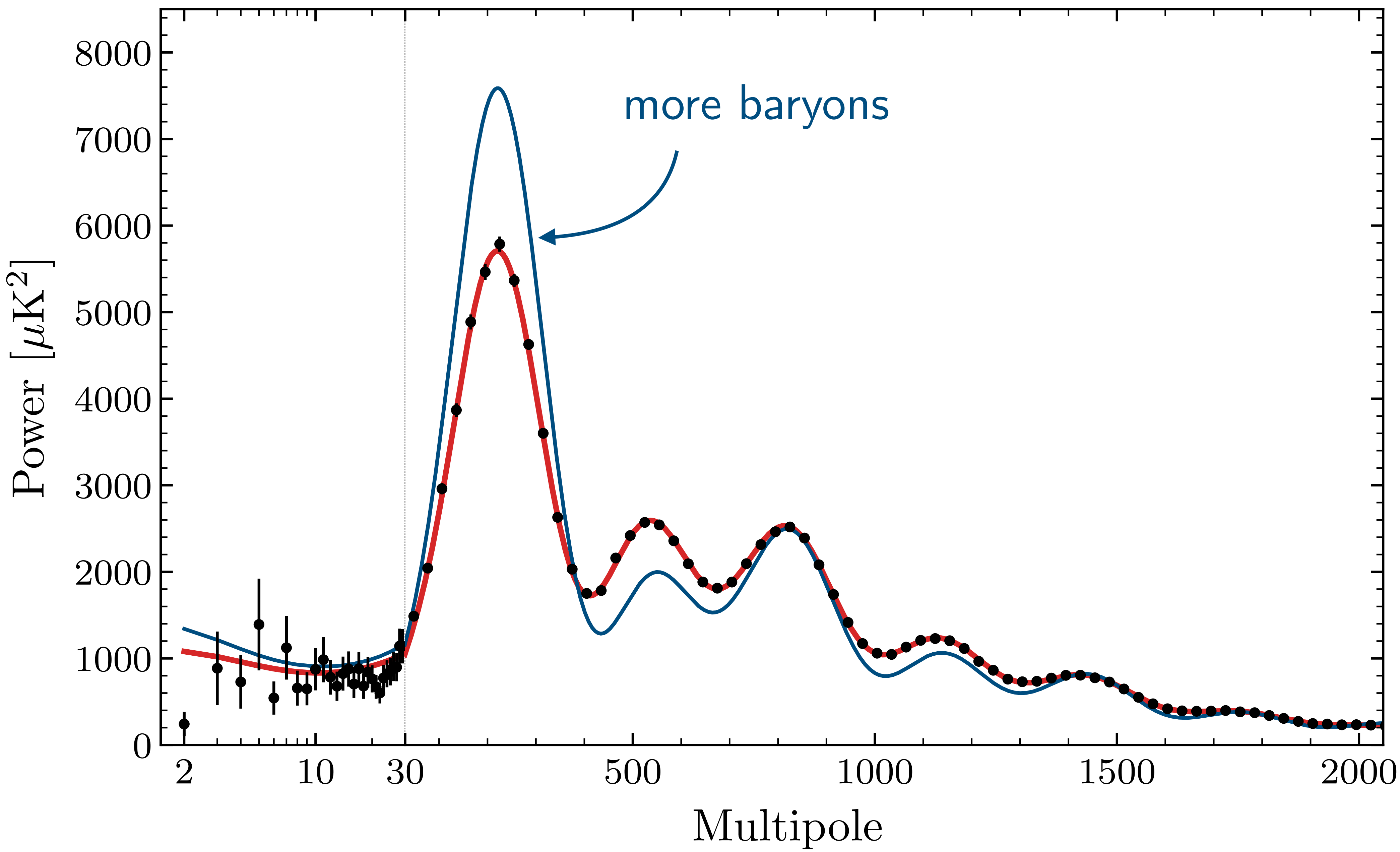
This is consistent with the direct observation of **dark energy** from supernova observations:




Riess et al (1998)
Perlmutter et al (1998)

# Baryons

The peak heights depend on the baryon density: $\Omega_b = 0.04$

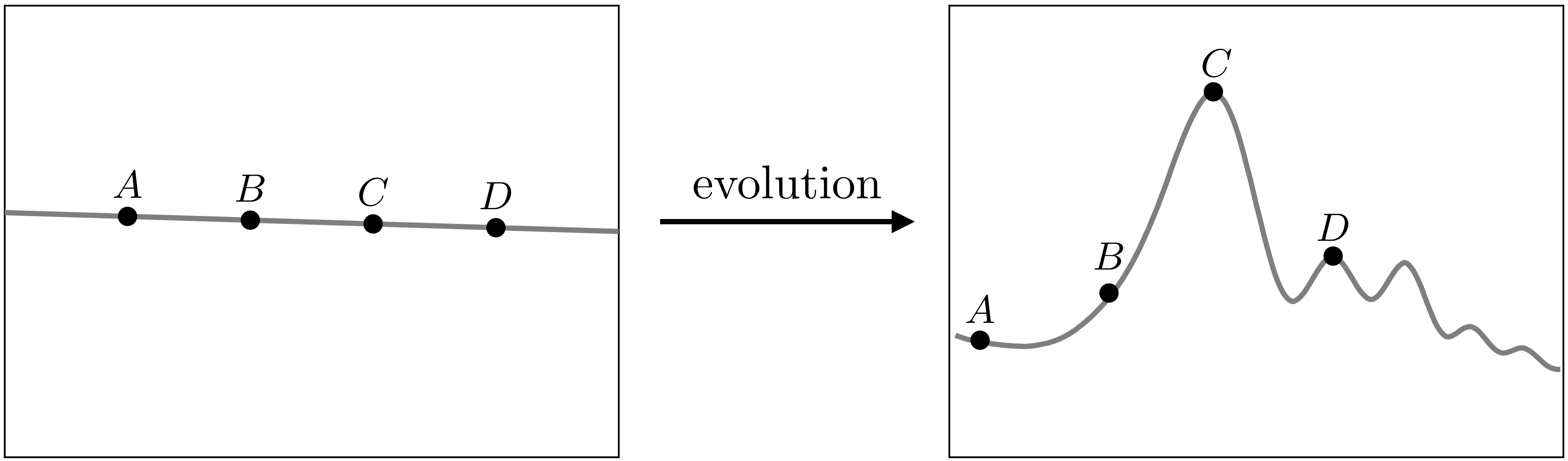



The measured baryon density is consistent with BBN.

Planck (2018)

# Initial Conditions

The CMB power spectrum also probes the initial conditions:



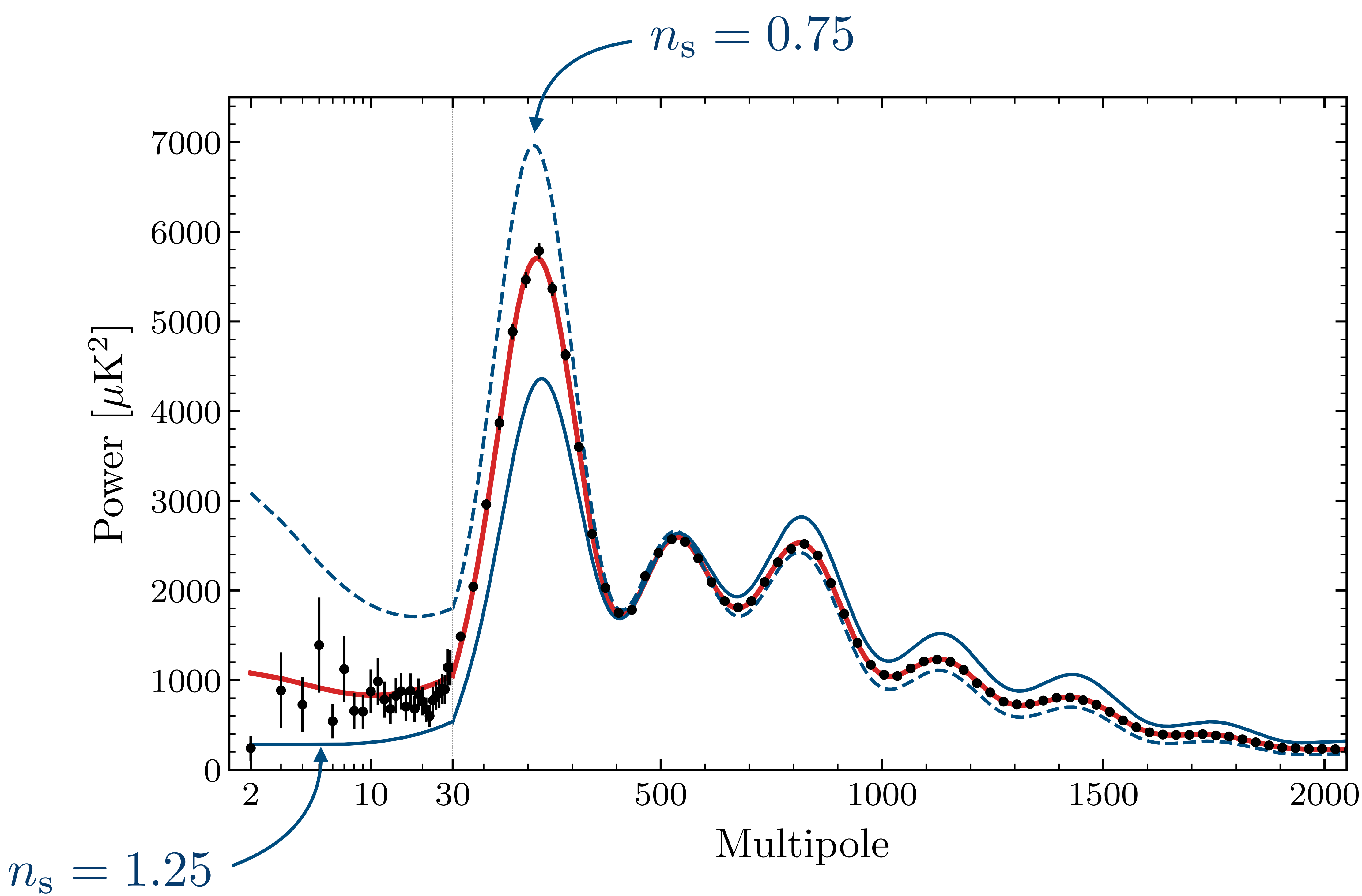

$$A_\mathrm{s} \left(\frac{k}{k_0}\right)^{n_\mathrm{s}-1}$$

Amplitude

scale-dependence

$$A_\mathrm{s} = 2.20 \times 10^{-9}$$

The primordial power spectrum is close to scale invariant: $n_s = 0.96$




WMAP (2009)
Planck (2018)

The observed deviation from scale invariance is significant.

# The Standard Model

A simple 5-parameter model fits all observations:

| | |
|---|---|
| $\Omega_{\rm b} = 0.04$ | Amount of ordinary matter |
| $\Omega_{\rm m} = 0.32$ | Amount of dark matter |
| $\Omega_{\Lambda} = 0.68$ | Amount of dark energy |
| $10^9 A_{\rm s} = 2.20$ | Amplitude of density fluctuations |
| $n_{\rm s} = 0.96$ | Scale dependence of the fluctuations |

# The Standard Model

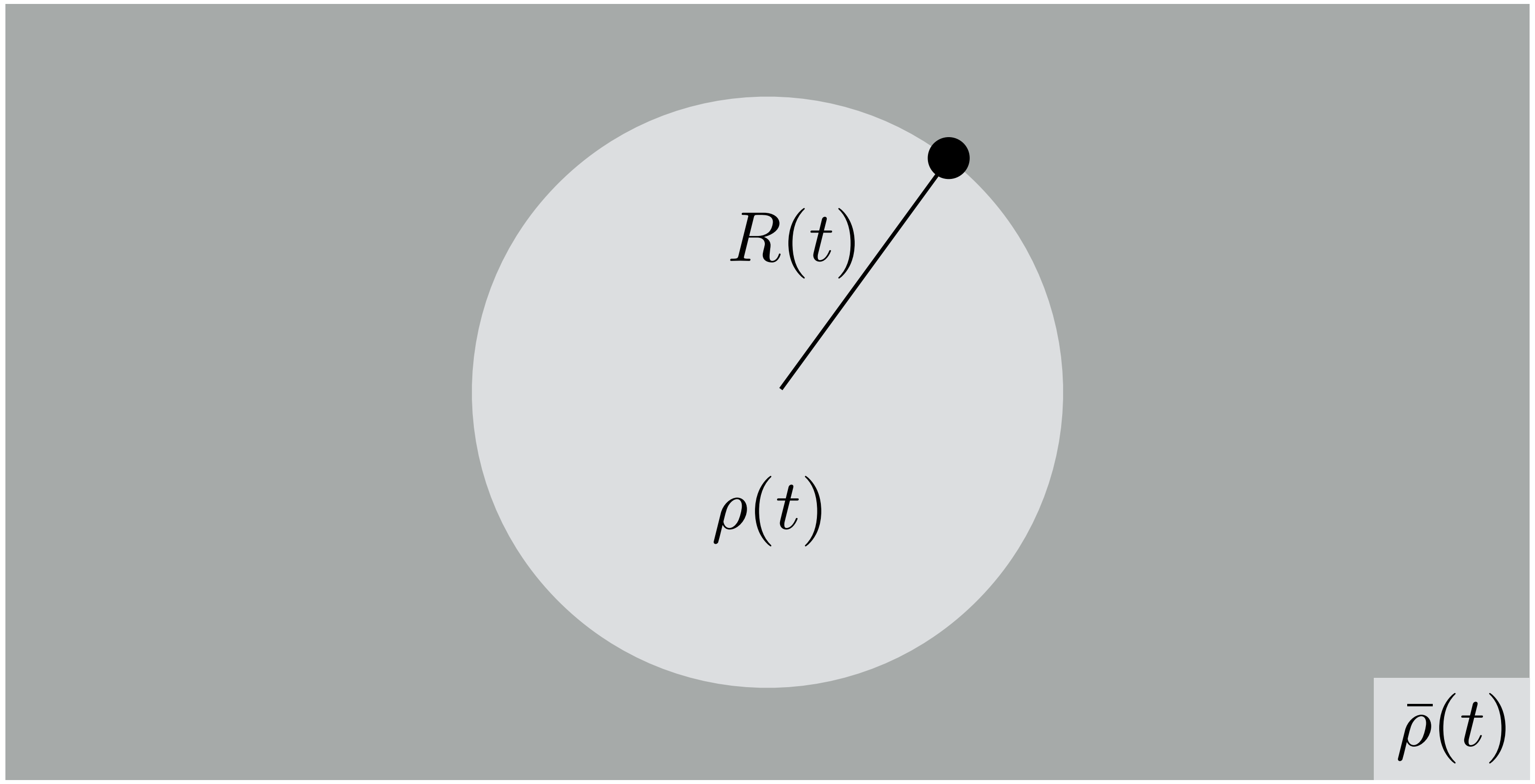
A key challenge of modern cosmology is to explain these numbers:

| | | |
|---|---|---|
| $\Omega_{\rm b} = 0.04$ | ← | Why is there more matter than antimatter? |
| $\Omega_{\rm m} = 0.32$ | ← | What is the dark matter? |
| $\Omega_{\Lambda} = 0.68$ | ← | What is the dark energy? |
| $10^9 A_{\rm s} = 2.20$ | ↖ | What was the origin of the fluctuations? |
| $n_{\rm s} = 0.96$ | ↙ | |

# *Appendix*

# Gravitational Clustering

Consider a **spherical overdensity** in a homogeneous universe:




We are interested in the evolution of the **density contrast**:

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

# Gravitational Clustering

Let us first study this overdensity in a **static universe**.

The acceleration at the sphere's surface is

$$\ddot{R} = -G\frac{\Delta M}{R^2} = -\frac{G}{R^2}\left(\frac{4\pi}{3}R^3\,\bar{\rho}\delta\right) \longrightarrow \boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G\bar{\rho}}{3}\delta(t)}$$

Conservation of mass implies $M = \frac{4\pi}{3}R^3(t)\,\bar{\rho}\,[1+\delta(t)] = \text{const}$ , so that

$$R(t) = \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3}\left[1+\delta(t)\right]^{-1/3} \approx R_0\left[1-\frac{1}{3}\delta(t)\right] \quad \text{for} \quad |\delta| \ll 1\,.$$

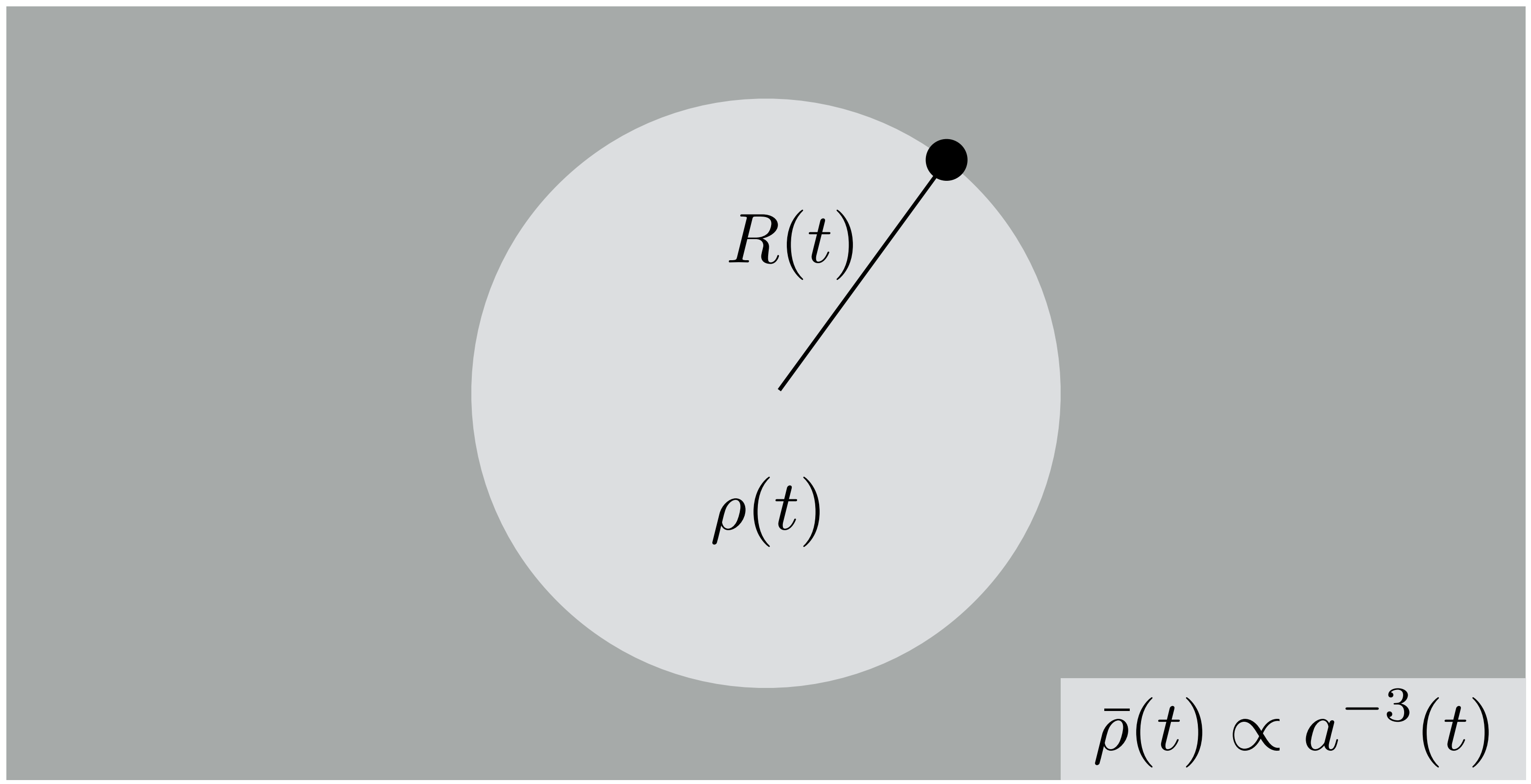
Substituting this into the equation of motion, we get

$$\boxed{\ddot{\delta} = (4\pi G\,\bar{\rho})\,\delta} \longrightarrow \delta(t) = Ae^{t/\tau} + Be^{-t/\tau}$$

Exponential growth

# Adding Expansion

Now, consider the same overdensity in an **expanding universe** with only pressureless matter:




The acceleration at the sphere's surface is

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{G}{R^2}\left(\frac{4\pi}{3}R^3\rho\right)$$
$$= -\frac{4\pi G}{3}\bar{\rho}R - \frac{4\pi G}{3}(\bar{\rho}\delta)R$$

# Adding Expansion

Mass conservation implies $M = \frac{4\pi}{3} R^3(t)\,\bar{\rho}(t)\,[1+\delta(t)] = \text{const}$, so that

$$R(t) \propto \bar{\rho}^{-1/3}(t)\Big[1+\delta(t)\Big]^{-1/3} \propto a(t)\left[1 - \frac{1}{3}\delta(t)\right]$$

Substituting this into the equation of motion, we get

$$\frac{\ddot{R}}{R} = \frac{\ddot{a}}{a} - \frac{1}{3}\ddot{\delta} - \frac{2}{3}\frac{\dot{a}}{a}\dot{\delta} \;=\; -\frac{4\pi G}{3}\bar{\rho} - \frac{4\pi G}{3}\bar{\rho}\delta$$

The density contrast therefore satisfies

Gravitational force

$$\boxed{\ddot{\delta} + 2H\dot{\delta} = (4\pi G\bar{\rho})\delta}$$

Hubble friction

# Clustering of Dark Matter

In a **matter-dominated universe**, with $a \propto t^{2/3}$ and $H = \dfrac{2}{3t}$ , we get

$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}H^2\delta \quad \longrightarrow \quad \ddot{\delta} + \frac{4}{3t}\dot{\delta} = \frac{2}{3t^2}\delta$$

The solution for the density contrast is

$$\delta(t) = At^{2/3} + Bt^{-1}$$

**Power-law growth**

In a **radiation-dominated universe**, the growth is only **logarithmic**:

$$\delta(t) = A\ln t + B$$

The clustering of matter only begins after matter-radiation equality.