RF Superconductivity
part1 : fundamental

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CERN Summer Student Lecture 2024

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Outline

- Introduction: why superconducting RF for accelerators?
- Superconductors in thermal equilibrium
  - Bardeen-Cooper-Schrieffer theory
  - Superconductors and Higgs mechanism
- Response against Radio Frequency
  - Linear response theory
  - Residual resistance
- Field limitations
  - Physics of phase transition
  - Fundamental challenges
- Conclusion

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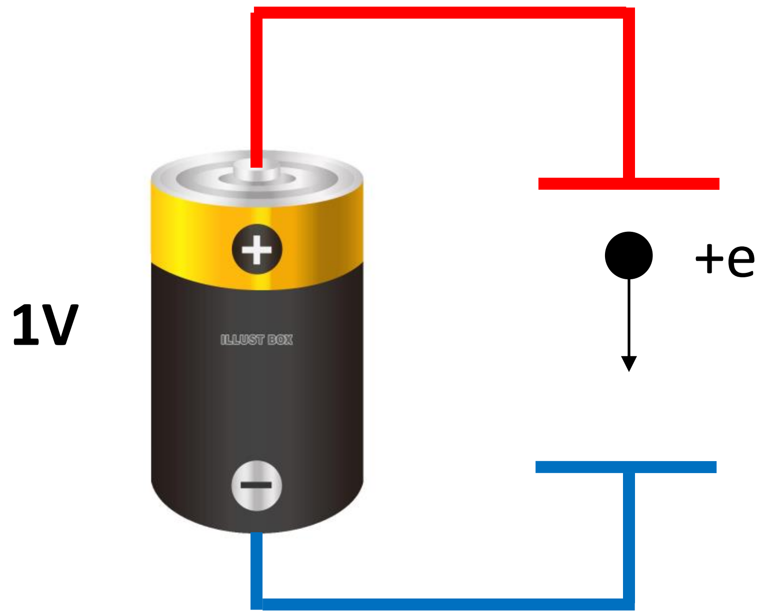
How to accelerate charged particles



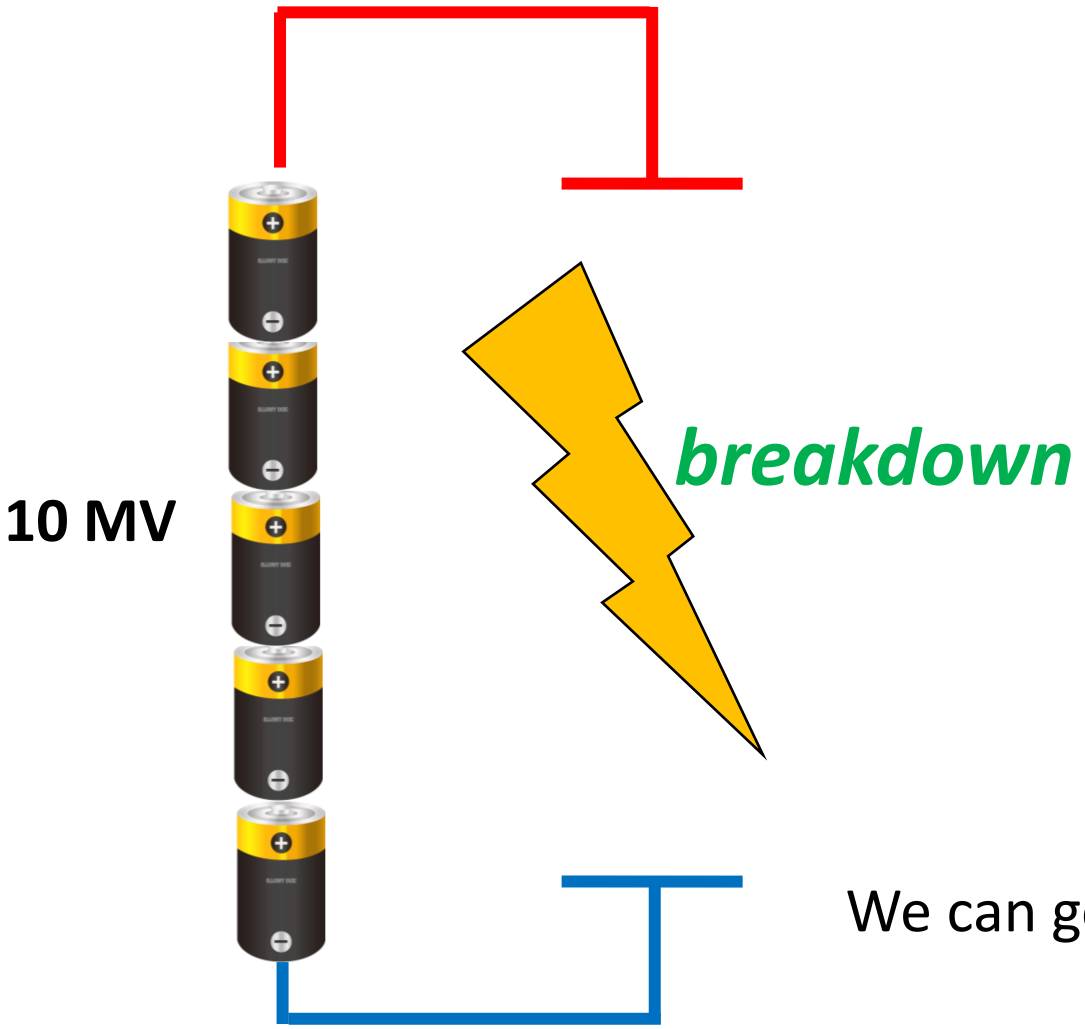
Electron's rest mass
in the natural unit
$m(c^2) = 511$ keV

Kinetic energy of a charge +e (1.6×10^{-19}C) accelerated by 1 V

$E = 1$ **eV**

Modern science ≫ MeV (Neutrons>**1GeV**, hard X-rays>**10GeV**, Higgs boson>**125+90 GeV**)

DC cannot provide high accelerating gradient ($\boldsymbol{E_{acc}}$)



We can generate high DC voltage but is limited to O(10MV)

→ For GeV science **RadioFrequency (RF)** is one option

Particle acceleration with RF resonant cavities

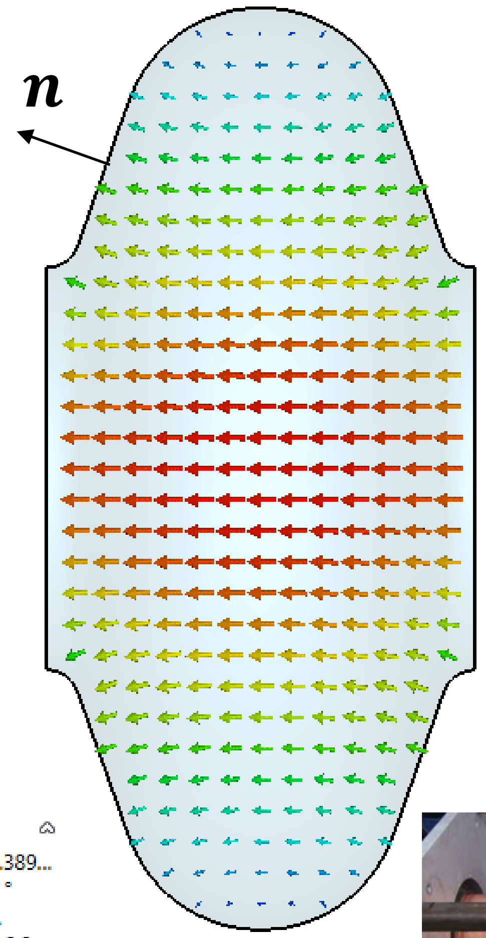
Maxwell equation

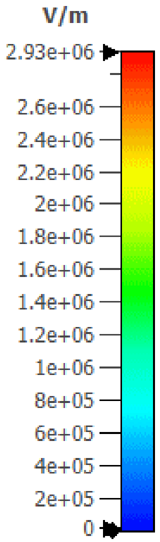
$$
\begin{cases}
\nabla \cdot \boldsymbol{E} = 0 \\
\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \cdot \boldsymbol{B} = 0 \\
\nabla \times \boldsymbol{B} = \dfrac{1}{c^2}\dfrac{\partial \boldsymbol{E}}{\partial t}
\end{cases}
$$

Boundary condition

$$
\begin{cases}
\boldsymbol{n} \times \boldsymbol{E} = 0 \\
\boldsymbol{n} \cdot \boldsymbol{B} = 0
\end{cases}
$$

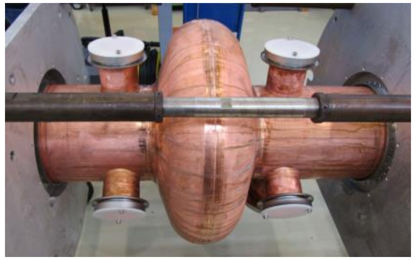
LHC

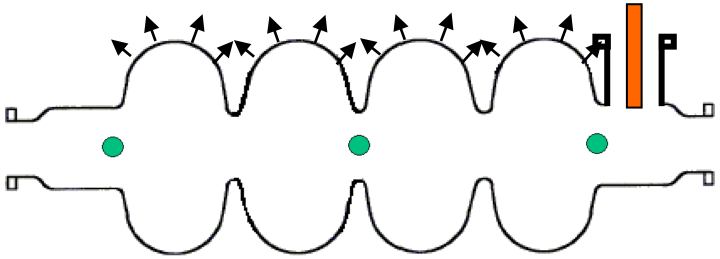




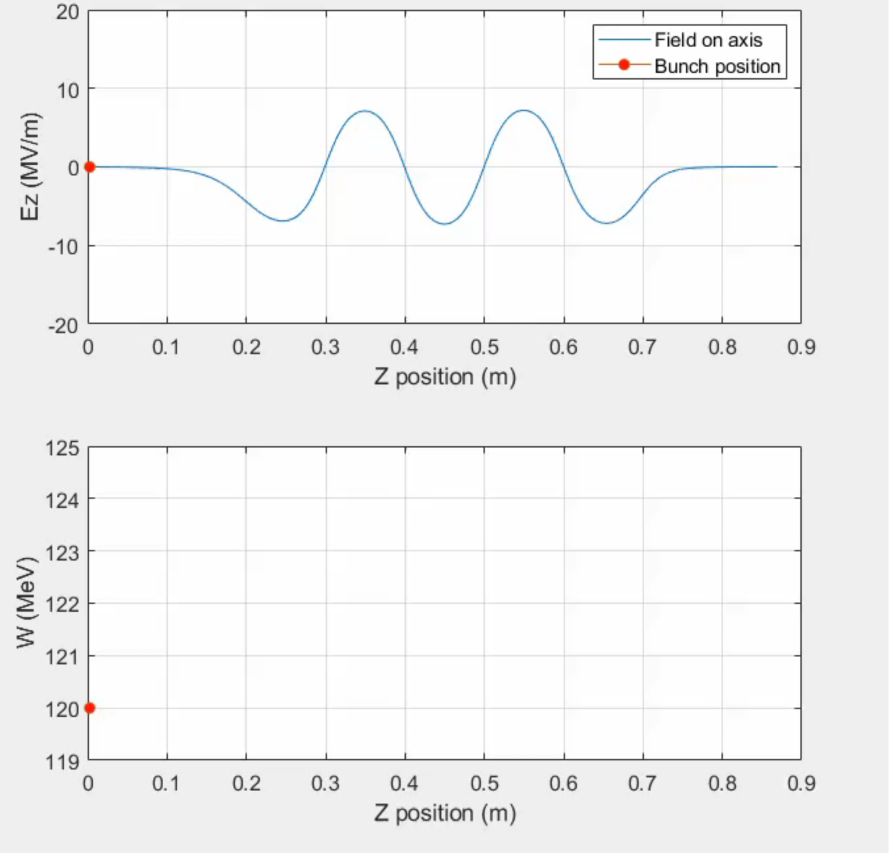
Mode 1 E-Field

Frequency	0.389...
Phase	1 °
Cross section	A
Cutplane at Y	0.0...
Maximum on Plane (Pl...	2.932...
Maximum (Plot)	2.930...





Courtesy: F. Bouly



Our interest: (unloaded) quality factor

Higher Q → higher field E_{acc} with smaller power dissipation P_c

Geometrical

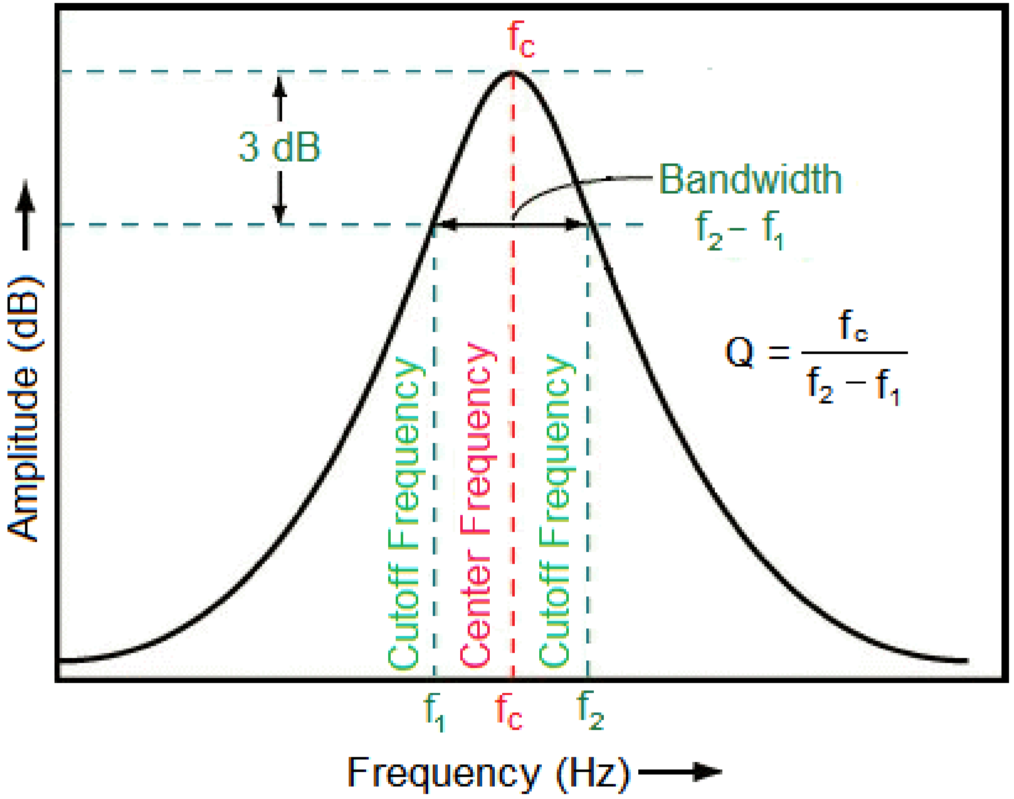
$$Q_0 = \frac{\omega U}{P_c} = \frac{\kappa E_{acc}^2}{P_c}$$

Smaller surface resistance R_s
→ high Q & low P_c

Experimental observable $$Q_0 = \frac{G}{R_s}$$ Geometrical; From material

Experimental observable $$P_c = \frac{\kappa R_s}{G} E_{acc}^2$$

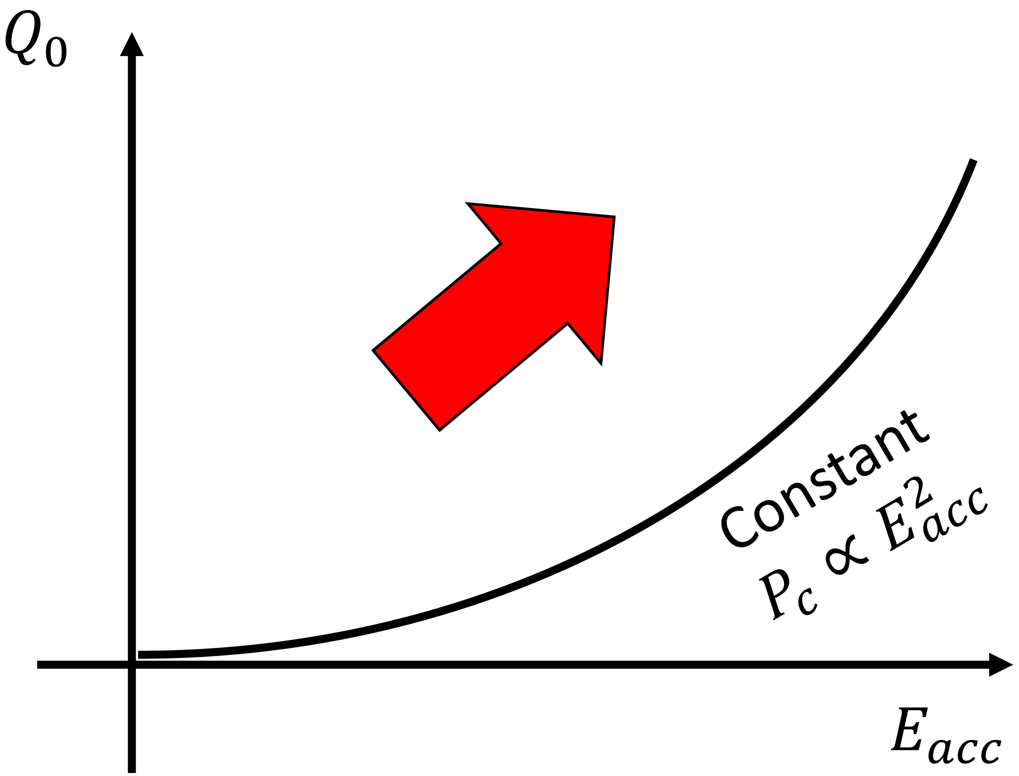
http://lossenderosstudio.com/glossary.php?index=q



G is a geometrical factor

- Elliptical cavity $G \sim 250\ \Omega$
- Spoke cavity $G \sim 133\ \Omega$
- Quarter-wave resonator $G \sim 30\ \Omega$

High-Q (Q_0) and high-gradient (E_{acc}) is the keyword



One of our goals in SRF is to go

High-gradient: E_{acc}

with lower power consumption P_c

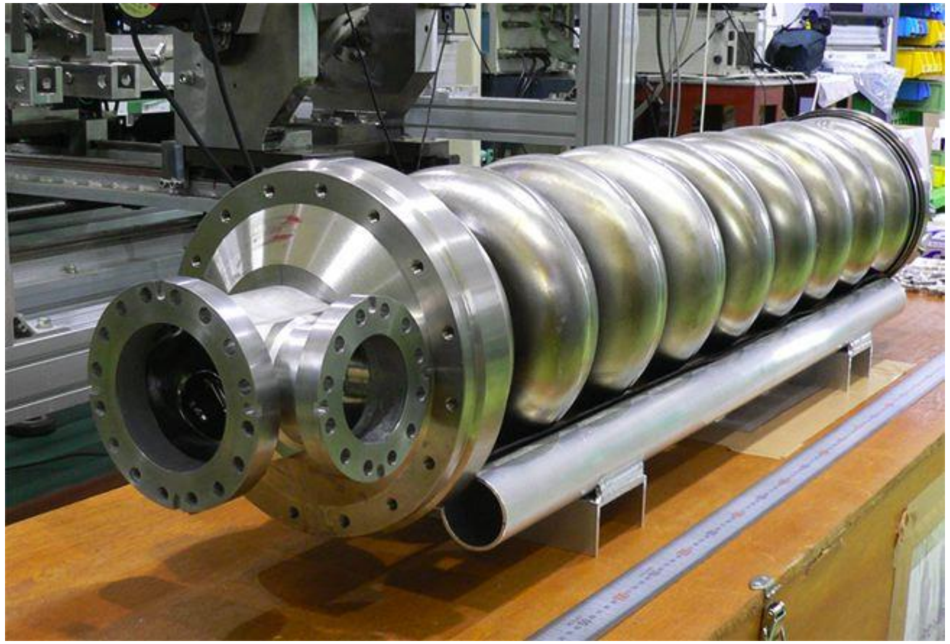
High-Q: $Q_0 = \frac{G}{R_S}$

We first consider lower $\boldsymbol{R_s}$

Superconducting

cavity

cryogenics



Cryolab
@CERN

https://www.123rf.com/stock-photo/old_radio.html?sti=mbqiuc5egnu4ynov6q|&mediapopup=41753240

Superconducting cavity for $R_s \to 0$?

Ohm's law

Applied DC electric field E

DC Current J

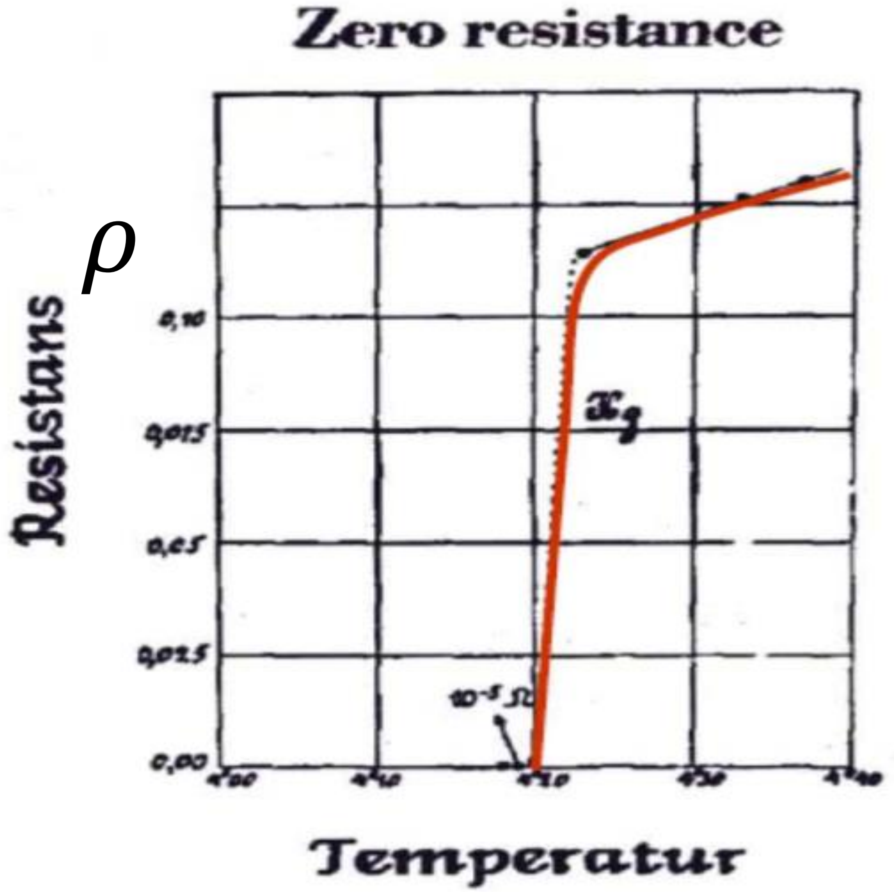
DC resistivity ρ

$$\rho \equiv \frac{E}{J}$$

DC conductivity σ

$$\sigma = \frac{1}{\rho} \equiv \frac{J}{E}$$

Cool down the resistor…



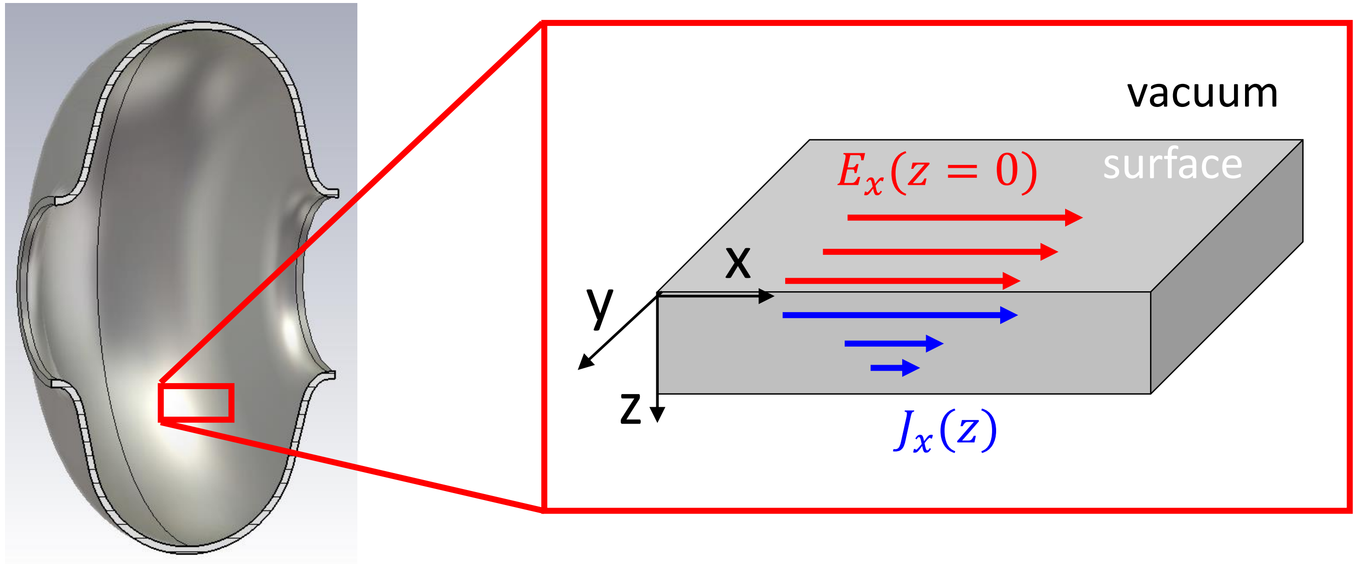
Heike Kamerlingh Onnes

Nobel prize in 1913

$\boldsymbol{\rho = 0}$ below transition temperature T_c

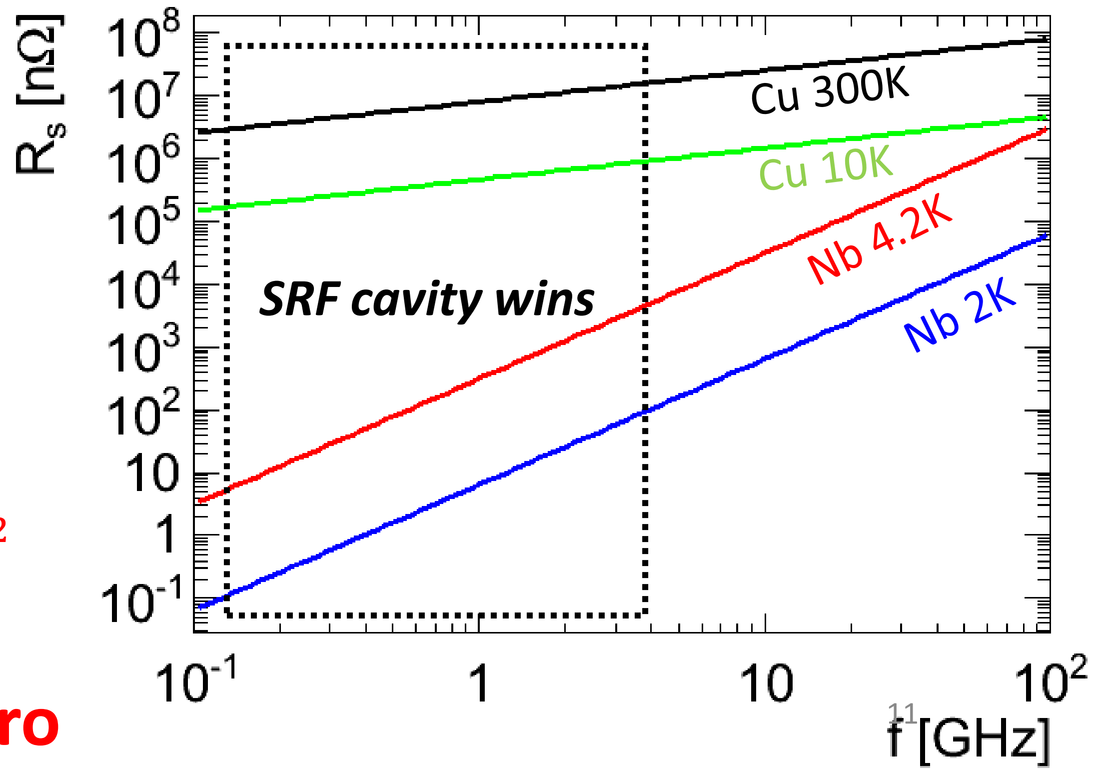
RF resistance R_s is non zero

Materials provide boundary conditions with finite power dissipation



Local surface resistance

$$R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right)$$



Normal conducting (Cu) $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} \propto f^{1/2}$

Super- conducting (Nb) $R_s = \frac{Af^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right) \propto f^2$

Superconducting R_s is small but non zero

After this lecture, you will be able to answer…

1. What is the superconductivity? Keyword: Higgs mechanism
2. What are the intrinsic origins of finite R_s in SRF cavities?
3. What is the fundamental limitation of the field E_{acc} inside SRF cavities?

I also list up questions → report assignment ☺

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Challengers for **microscopic** theory of superconductors

J. Schmalian, arxiv:1008.0447

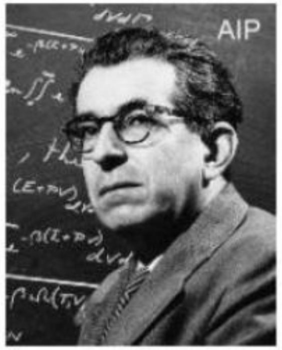
Albert Einstein
(1879-1955)

Niels Bohr
(1885-1962)

Ralph Kronig
(1905-1995)

John Bardeen
(1908-1991)

Werner Heisenberg
(1901-1976)



Fritz London
(1900-1954)

Lev D. Landau
(1908-1968)

Felix Bloch
(1905-1983)

Léon Brillouin
(1889 -1969)

Max Born
(1882-1970)

Herbert Fröhlich
(1905-1991)

Richard Feynman
(1918-1988)

A lot of models…all failed ☹

Development of quantum field theory in many body problems was necessary…

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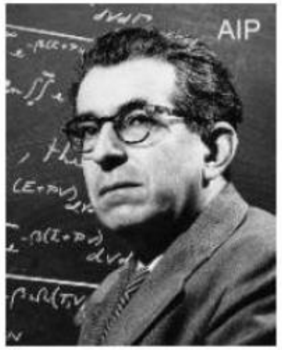

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Richard Feynman
(1918-1988)

Feynman tried to get superconductivity by **perturbation theory** including attraction forces between electrons caused by lattice vibration → **failed** ☹

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J. Schmalian, arxiv:1008.0447

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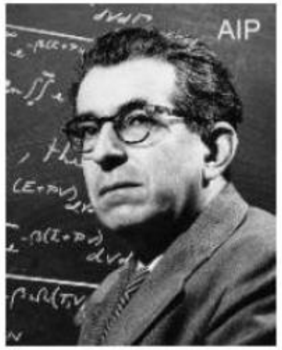

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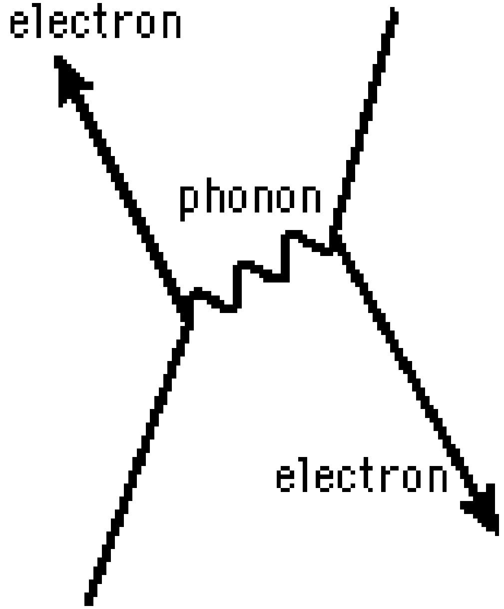
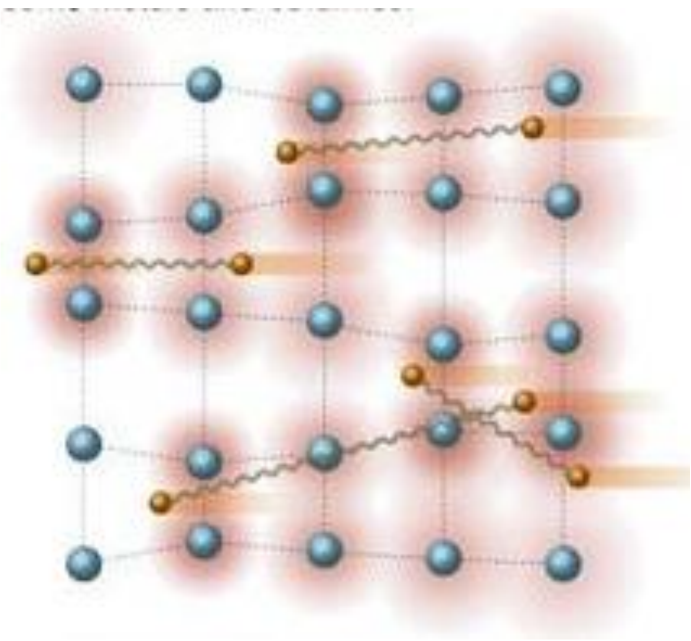
Bardeen and Fröhlich had a good idea but needed young talents

- Many body problem (Quantum field theory)
- Application of techniques developed in **particle physics**

Theory of superconductor in equilibrium





John Bardeen

Leon Cooper

John Robert Schrieffer

<u>Cooper pair: Composite boson</u>

Two electrons are bounded by something (phonon) → effective Hamiltonian $\mathcal{H}_{BCS}$

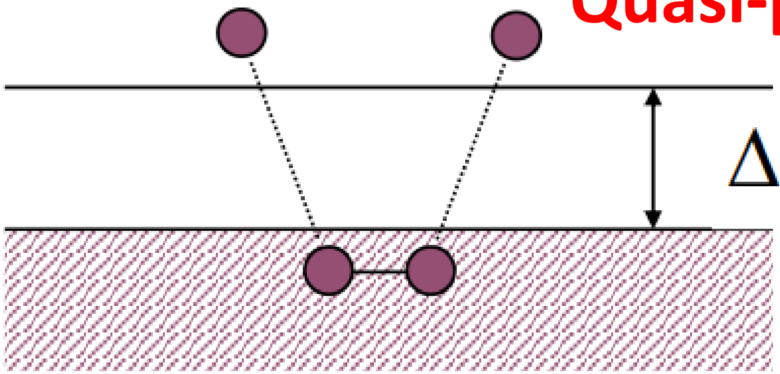
Mean field approximation + Variational method (+other approximations…)

$$\mathcal{H}_{BCS}|\Phi_0\rangle = E|\Phi_0\rangle$$

Non-perturbative!

Solution: superconducting gap

Quasi-particles

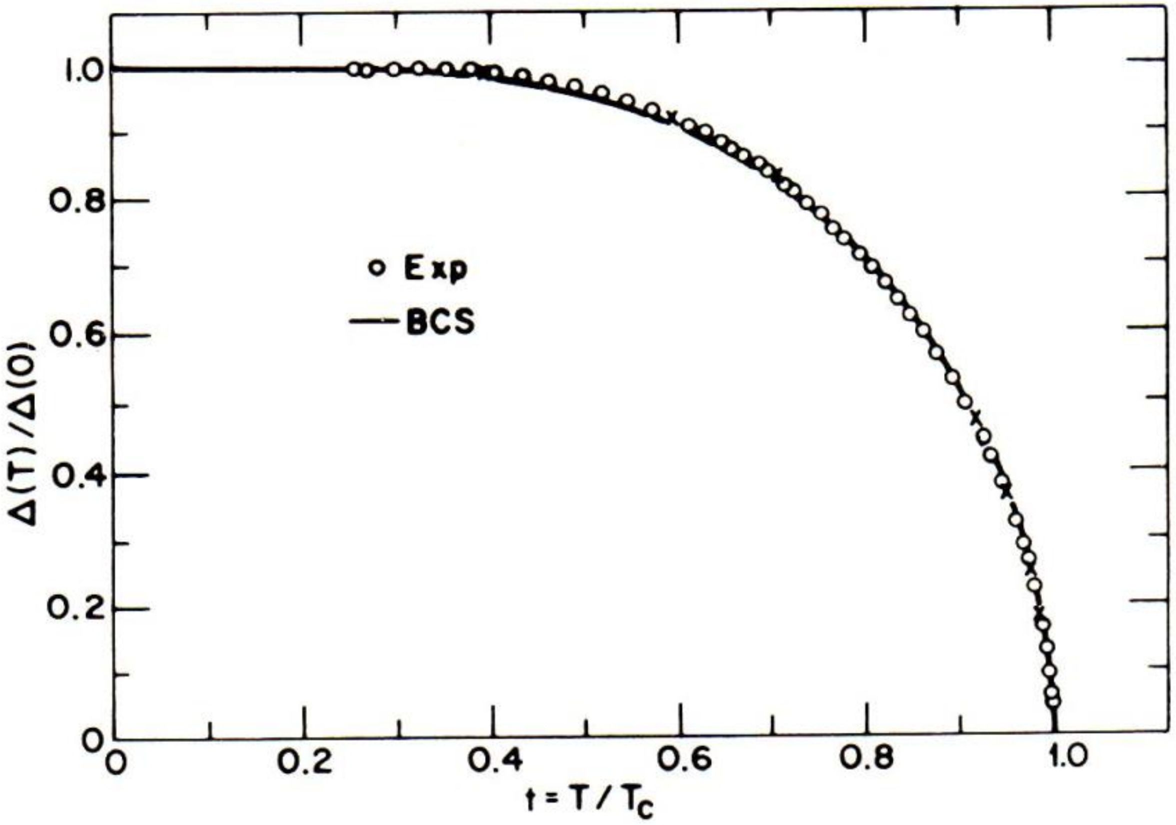


$\Delta_o \propto \Psi_o$

Scaler wavefunction

Self-consistent gap equation

$$\Delta = N(E_F)V \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_B T}\right) d\xi$$



- The Cooper pair needs certain amount of energy to be broken
- The cause of Ohmic loss, stochastic scattering of one single electron by phonon or impurity **cannot break the pair**

  →No DC loss

The Equilibrium state of conventional superconductor was understood !

→ In this lecture, we try to obtain qualitative insight of the phenomenon

Electrons in a ***perfect*** metal are free (or independent)

Q2 Electron-electron scattering?
→Pauli's exclusion principle
Cf. Fermi-liquid theory by Landau

Perfectly periodic potential by ions does ***NOT*** scatter electrons (Bloch's theorem) Q1

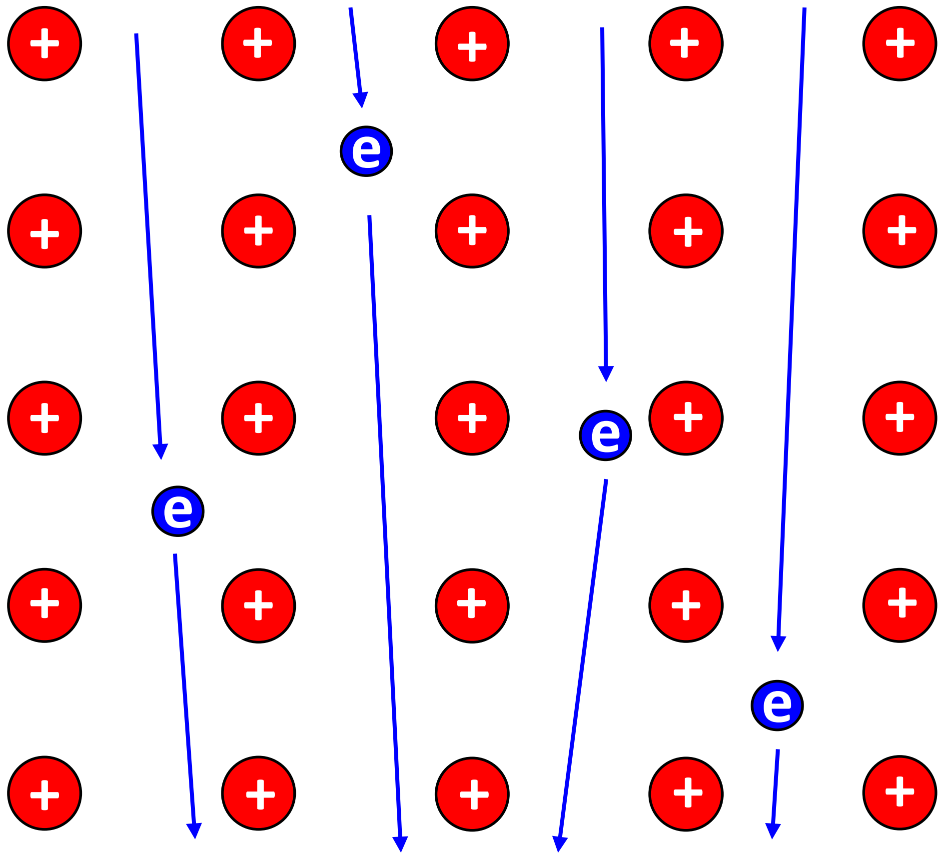
Check this

These electrons are ***NOT*** our favorite elementary particle of

$$m = 511\ \mathrm{keV}$$

These electrons are ***dressed*** by complicated electromagnetic property of metals to have an effective mass m^*given by a band structure

→ ***Quasi-particles***



In reality, imperfection causes quasi-particle scattering

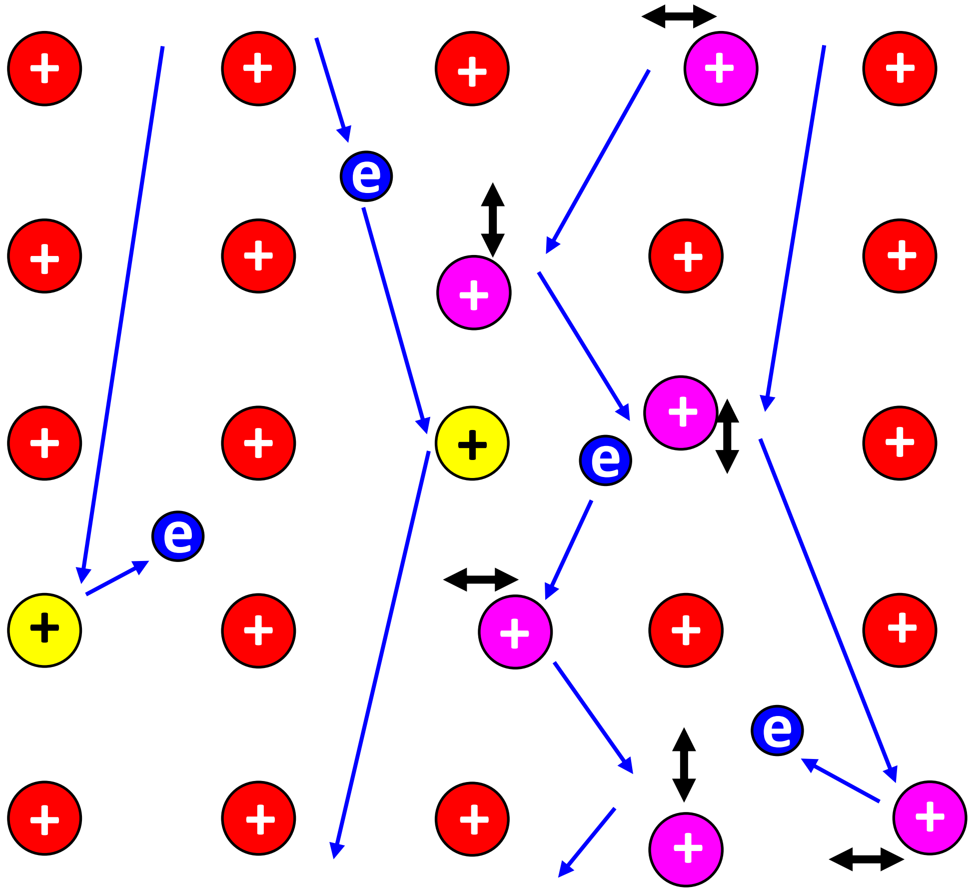
Electrons in real metals show Ohmic loss

Imperfections causes ***local*** scattering

1. Impurity, defects (scattering time τ_{def})
2. Lattice vibration, phonon (τ_{ph})

Total scattering time

$$\frac{1}{\tau} = \frac{1}{\tau_{\mathrm{def}}} + \frac{1}{\tau_{ph}}$$



Macroscopic phenomenology (Drude model)

An electron accelerated by an electric field

$$m^* \frac{dv}{dt} = -eE$$

is scattered by imperfections per τ, and its velocity relaxes to a mean velocity

$$\langle v \rangle = -\frac{e}{m^*} E\tau$$

Electric current is a collective flow of n electrons

$$j = -en\langle v \rangle = \boxed{\frac{e^2 n\tau}{m^*}} E$$

Electrical conductivity σ

Ohm's law

$$\boxed{\boldsymbol{j} = \sigma \boldsymbol{E}}$$

Paired electrons can avoid Ohmic loss

If electrons ***in a distance*** (>39 nm) are bounded, ***local*** (< 0.5 nm) scattering can be avoided

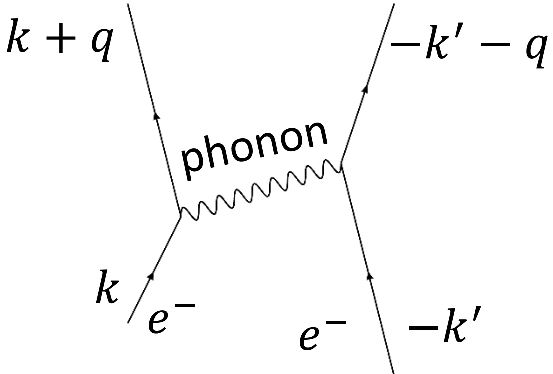
Any small attractive interaction V between electrons can lead to a ***Cooper pair*** coupled with an energy 2Δ, below critical temperature T_c

BCS gap equation (1957)

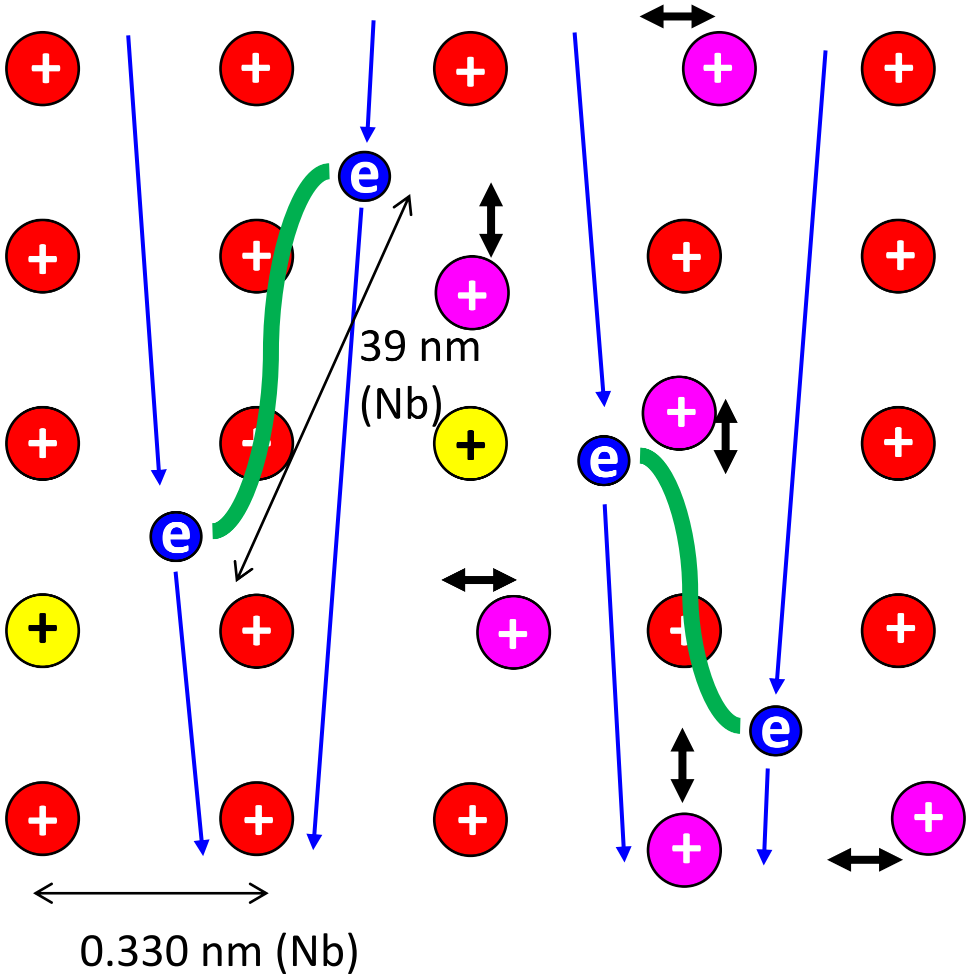
Non-perturbative!

$$\Delta = n(E_F)V \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_B T}\right) d\xi$$

Classical superconductors' attractive potential is from ***longitudinal mode of lattice vibration***



If energy transfer $|\epsilon_{k+q} - \epsilon_k|$ is smaller than phonon energy the interaction is attractive (Flöhlich)
→ Eliashberg's strong coupling superconductor (1960)



Implication of ***no*** scattering?

No scattering

$$m^* \frac{\partial \langle v \rangle}{\partial t} = -eE$$

generates super-current

$$j_s = -en_s \langle v \rangle$$

$$\rightarrow \frac{\partial j_s}{\partial t} - \frac{n_s e^2}{m^*} E = 0$$

Apply $\nabla \times$ from the left

$$\frac{\partial}{\partial t}(\nabla \times j_s) - \frac{n_s e^2}{m^*} \nabla \times E = 0$$

$\sim \nabla \times \boldsymbol{B}/\mu_0$; $-\frac{\partial \boldsymbol{B}}{\partial t}$

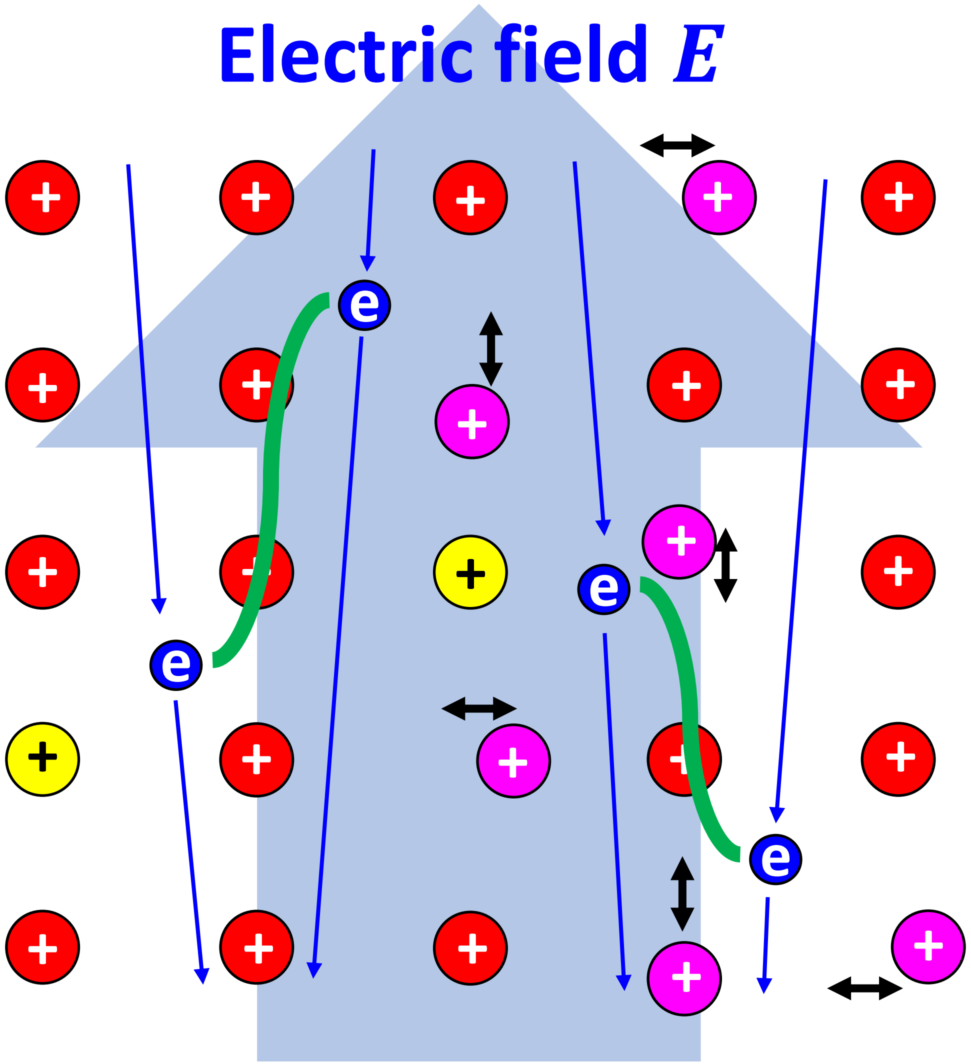
leads to

$$\frac{\partial}{\partial t}\left[\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2} \boldsymbol{B}\right] = 0$$

$$\lambda_L^2 \equiv \frac{m^*}{n_s e^2 \mu_0}$$

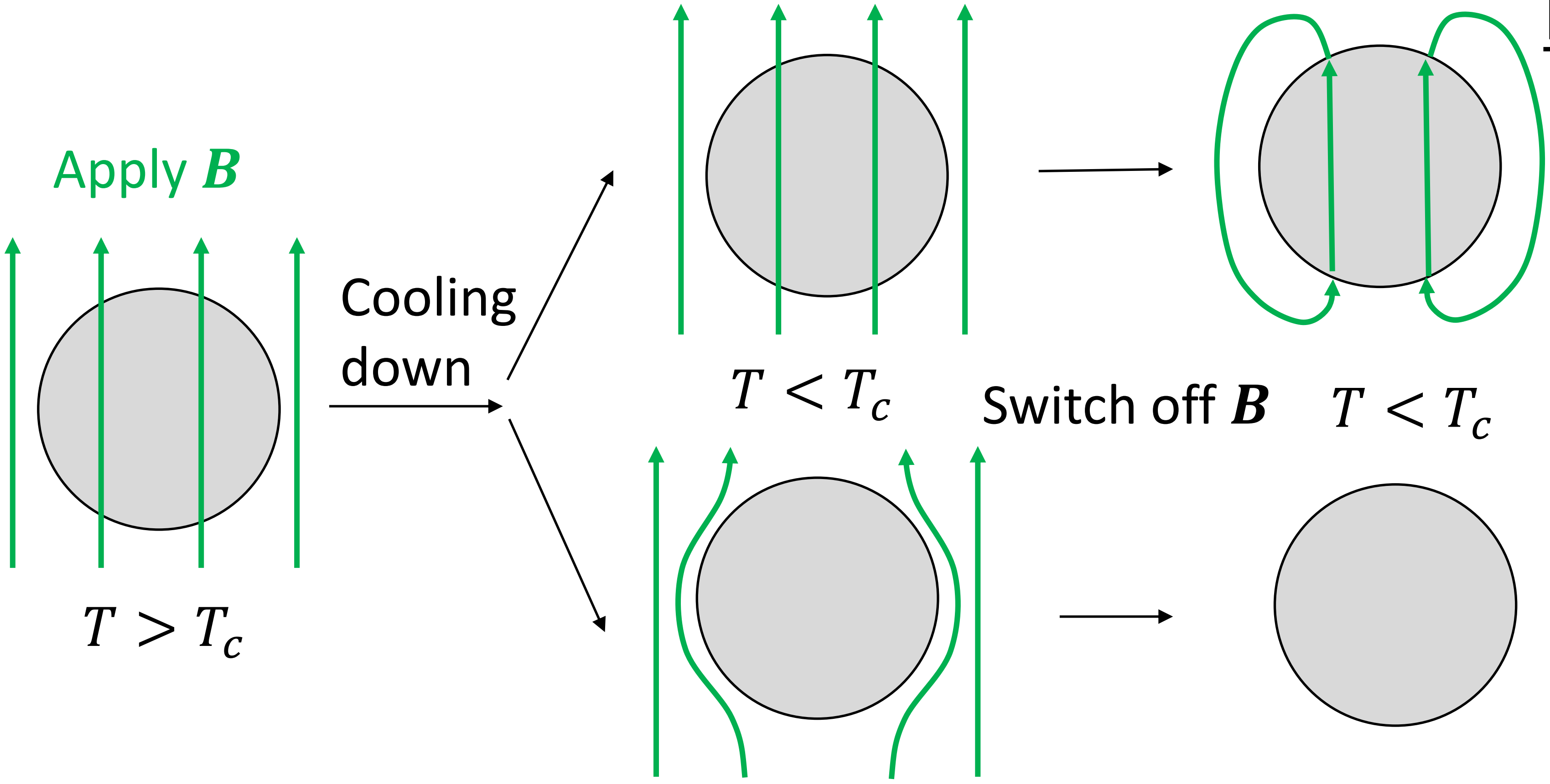
Constant of time

→ Initial condition before phase transition $T > T_c$ must be preserved



Superconductor ≠ Perfect electric conductor

Meissner effect **differentiates them**



Perfect Electric Conductor

$$\nabla^2\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) - \frac{1}{\lambda_L^2}\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) = 0$$

Preserve initial condition!

Superconductor

Zero field!

$$\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2}\boldsymbol{B} = 0$$

London equation

Additional constraint (broken Gauge symmetry)

Superconductivity is a thermodynamical state which expels magnetic fields and cannot be explained by classical electrodynamics → quantum field theory!

Cross-over of particle physics and condensed matter physics

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

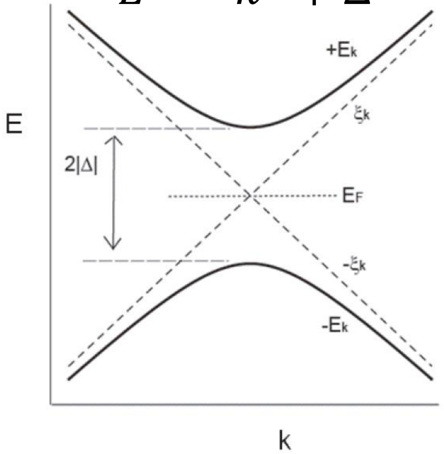
Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

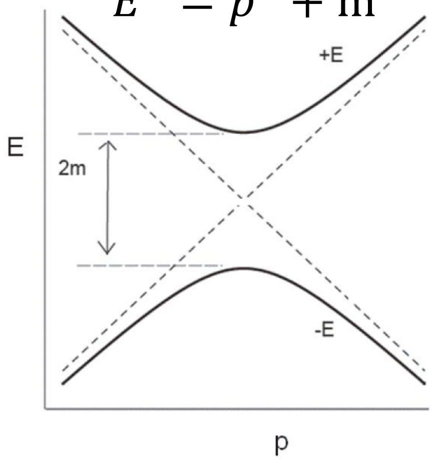
Superconductivity

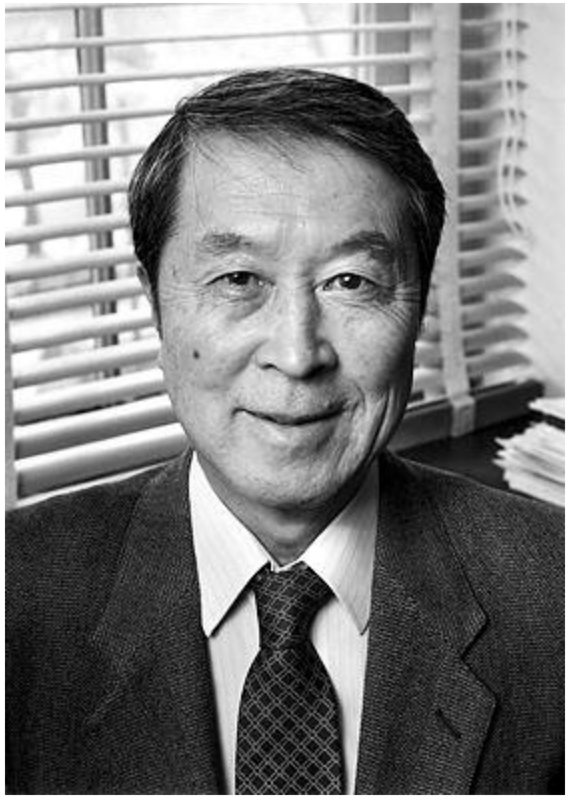
$$E^2 = k^2 + \Delta^2$$



Particle physics

$$E^2 = p^2 + \mathrm{m}^2$$





Yoichiro Nambu

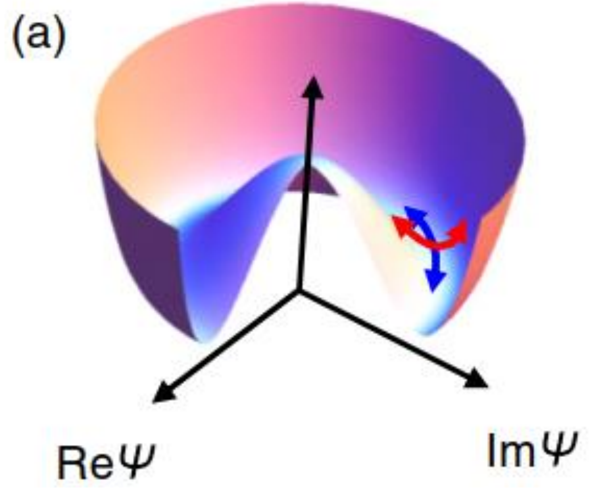
The vacuum is similar to the superconducting state

Particle mass = superconducting gap (gauge symmetry is broken in the ground state)

→ Chiral symmetry breaking, Higgs mechanism, Electroweak theory

→ Origin of mass

Spontaneous gauge symmetry breaking



R. Matsunaga et al PRL 111 057002 (2013)

<u>Ginzburg-Landau theory ($T \rightarrow T_c$ of BCS theory, $\Psi = \Delta$)</u>

$$F = (\nabla \times A)^2 + \frac{\hbar^2}{4m_e}|(\nabla + ieA)\Psi|^2 + \frac{g}{4}(|\Psi|^2 - v^2)^2$$ $\sim\phi^4$ theory

EM energy   Scaler Kinetic energy   Scaler potential

Excitation around potential minimum v at fixed gauge (Unitary gauge)

$$\Psi(\mathrm{x}) \rightarrow v + \phi(x)$$

Kinetic term

$$|(\nabla + ieA)\Psi|^2 = |\nabla\phi|^2 + e^2v^2|A|^2 + \cdots$$

Gauge field gains mass: Nambu-Goldston mode is absorbed by photon

$$e^2v^2|A|^2 \equiv m_v|A|^2$$
$$(\nabla^2 - m_v^2)A = 0$$

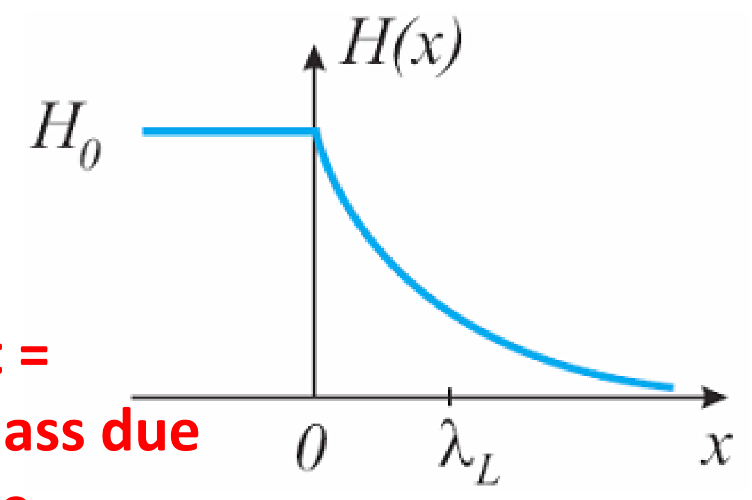
Massive vector boson eq. ↔London eq.

→ Massive photon → finite interaction length: penetration depth

$$\lambda_L = \frac{1}{m_v}$$

Higgs mode ϕ has a mass $m_S = v\sqrt{g}$: coherence length

$$\xi_0 = \frac{1}{m_s}$$



Meissner effect = photon gains mass due to broken Gauge symmetry

Spontaneous gauge symmetry breaking

Ginzburg-Landau theory ($T \to T_c$ of BCS theory, $\Psi = \Delta$)

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland

(Received 31 August 1964)

Q3 Read it!

$|(\nabla + ieA)\Psi|^2 = |\nabla\phi|^2 + e^2v^2|A|^2 + \cdots$

Gauge f… …dston mode is absorbed by photon

$\ldots A|^2 \equiv m_v|A|^2$ Massive vector boson eq.

$\ldots - m_v^2)A = 0$ $\leftrightarrow$London eq.

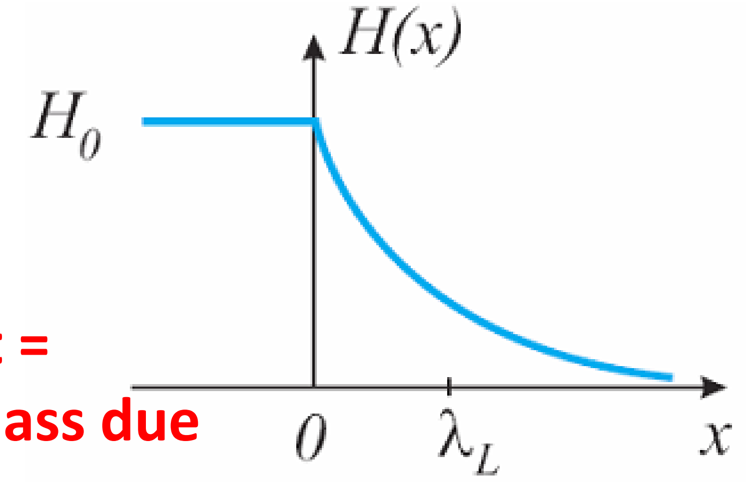
→ Mas… …tion length: penetration depth

$\lambda_L = \frac{1}{m_v}$

Higgs m… …g : coherence length

$\xi_0 = \frac{1}{m_s}$

Meissner effect = photon gains mass due to broken Gauge symmetry



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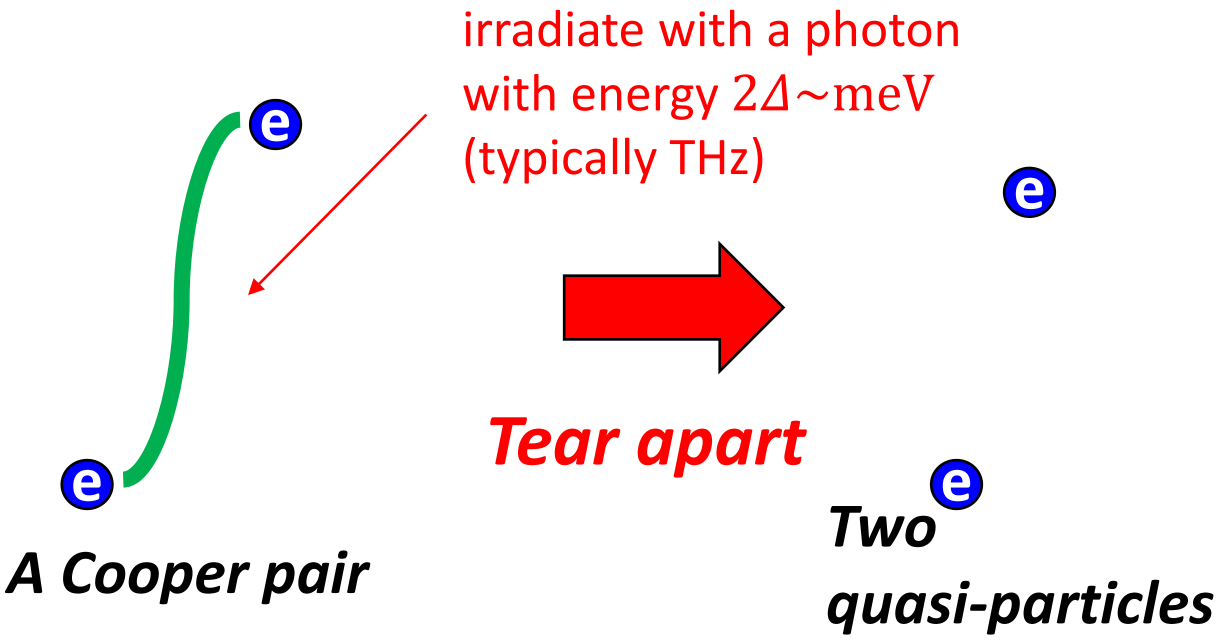
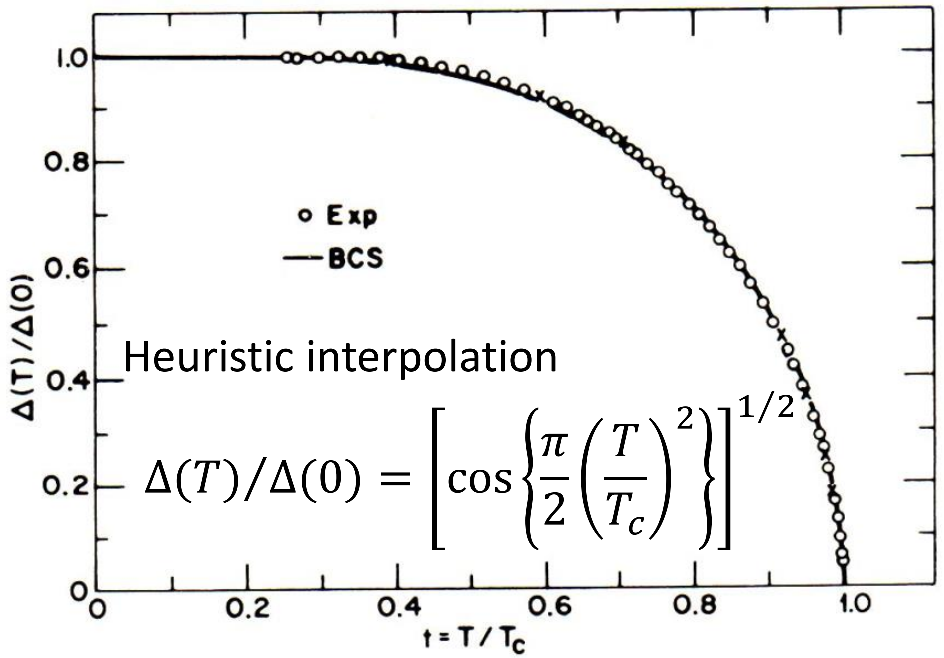
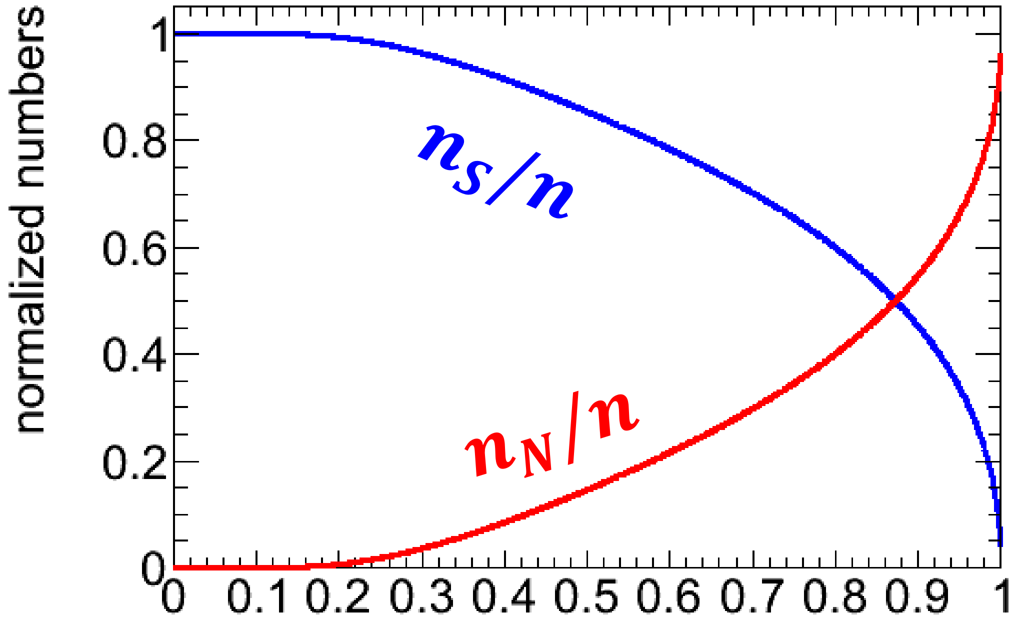
Superconducting gap







At finite temperature $0 < T < T_c$, these two states are ***in thermal equilibrium***

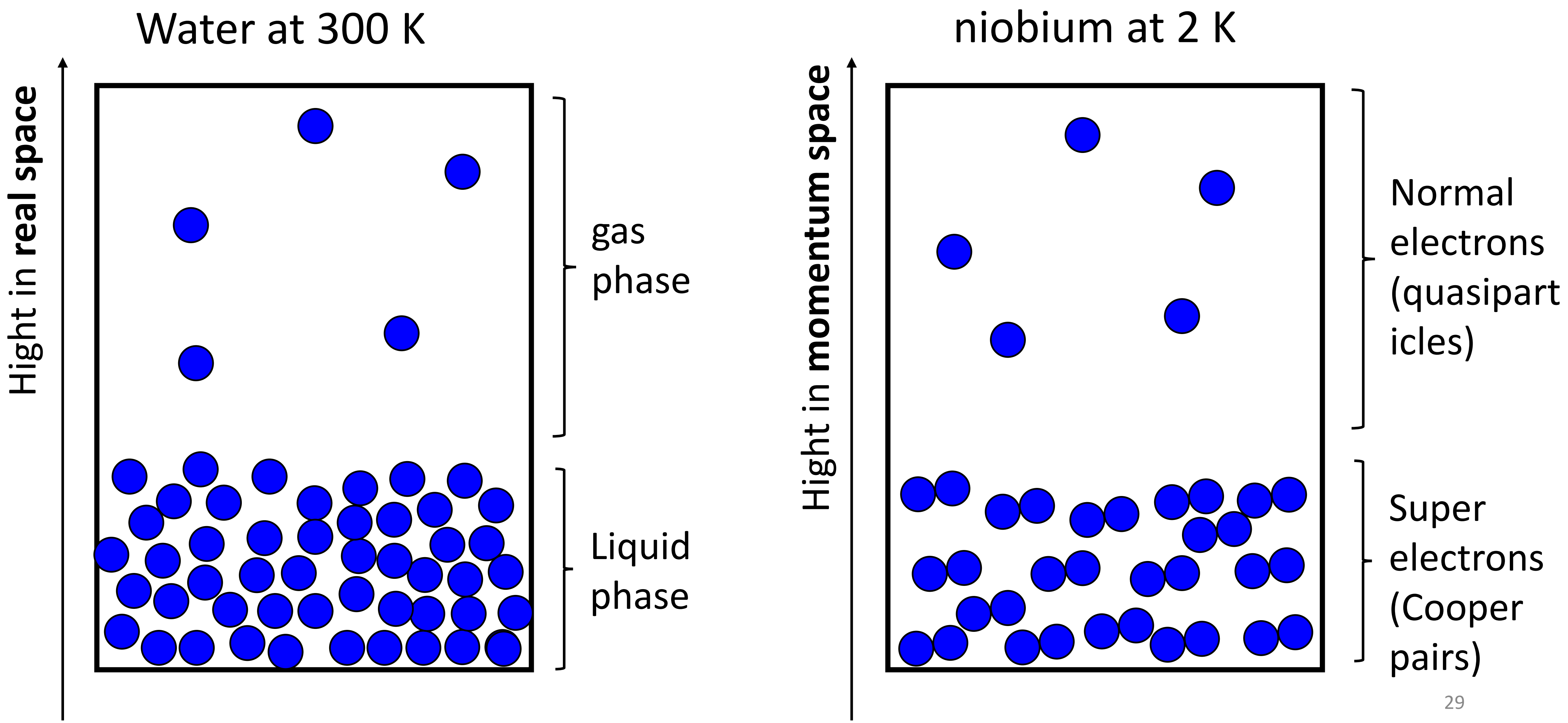
\# of quasiparticles: $n_N \sim \exp\left(-\frac{\Delta}{k_B T}\right)$

\# of electrons in Cooper pairs: $n_S \sim n - n_N$

T/T_c

Quasi-particles (~normal conducting electrons) still exist if $T > 0$

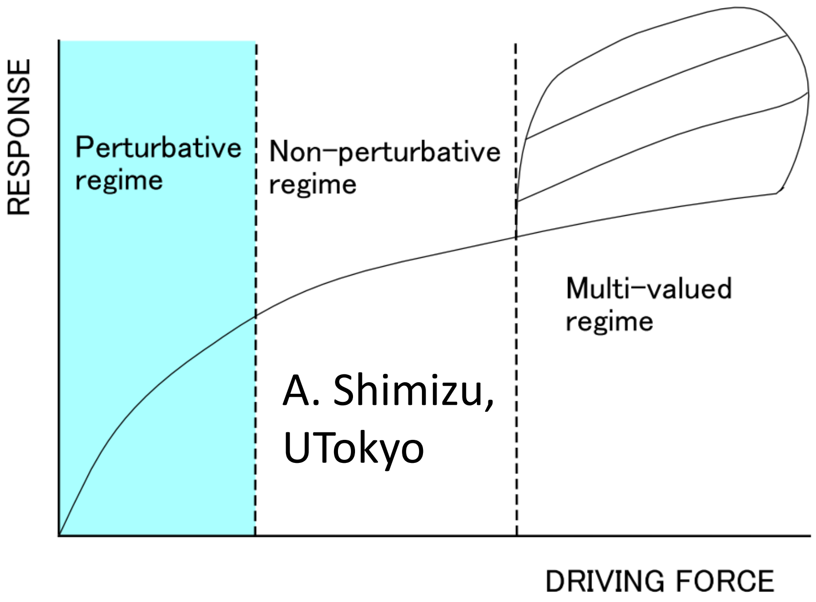
Why normal and super electrons at a time?

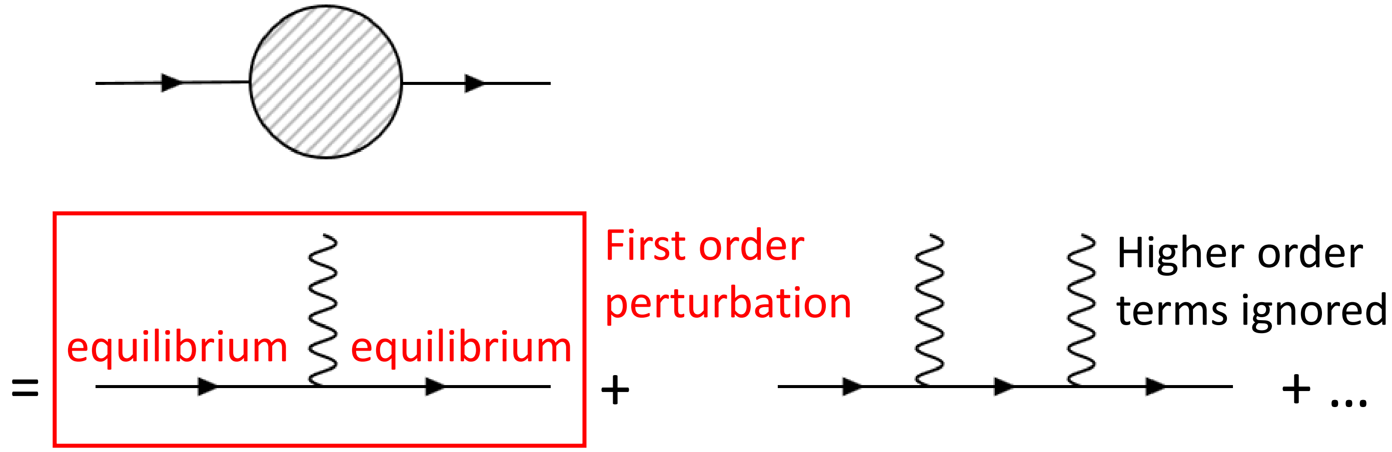


Linear response to RF→ BCS resistance R_{BCS}

Quamtum mechanical ***derivation*** of R_s requires quantum many body theory

$$\mathcal{H} = \mathcal{H}_{BCS} + \boxed{\mathcal{H}_{RF}(t)}$$ If the RF field is "small"





The responding current is given by the equilibrium state

$$J(q) = -\frac{1}{c}K(q)A(q)$$

Quantum ***derivation*** *of* Ohm's law is equally complicated…

$$\sigma = -\frac{1}{i\omega}[\Phi^R(\omega) - \Phi^R(0)]$$

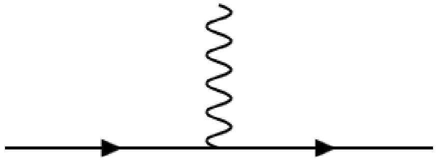
$$\Phi^R = \frac{i}{\hbar V}\theta(t)\langle \hat{J}(t)\hat{J}(0) - \hat{J}(0)\hat{J}(t)\rangle$$

$$\rightarrow \sigma = \frac{ne^2\tau_{\boldsymbol{k}}}{m}\frac{\widetilde{\rho_0}}{\rho_0}$$

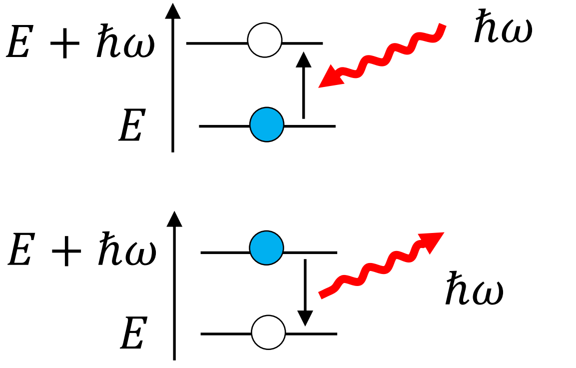
Introduction to ***quantum*** mechanical derivation: ***Integrate*** contribution of all the quasi-particles

Fermi's Golden rule [Z. Physik 266 p.209 (1974)]

$$R_s \propto P \cong \Sigma_{p,p',\hbar\vec{k}} \left| \quad \right|^2 \propto \text{(photon energy) x (net \# of absorbed photons)}$$



$$= \hbar\omega(n_+ - n_-) = \hbar\omega \int_{\Delta}^{\infty} dE\,[f(E) - f(E+\hbar\omega)] \times N(E)N(E+\hbar\omega)$$



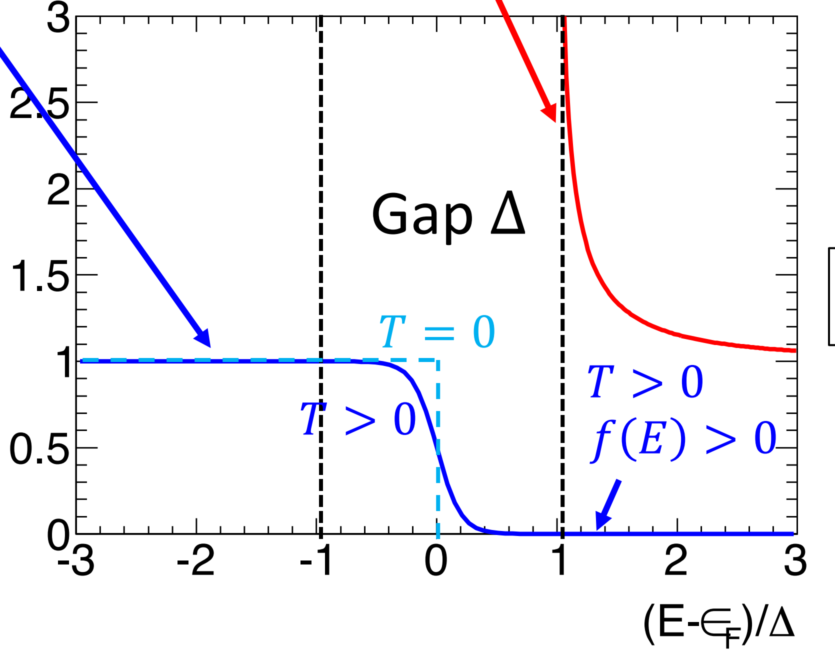
All quasiparticles $\Delta < E < \infty$

$N(E)$: density of states (how many quantum states at energy E: a kind of degeneracy)

$f(E)$: distribution function (how many electrons are in one state at energy E)

Fermi-Dirac function in equilibrium state

$$f(E) = \frac{1}{\exp(-\mathrm{E}/k_B T) + 1}$$



Q4 Follow the math

$$R_s \propto \frac{\omega^{1.5}}{T} \exp\left(-\frac{\Delta}{k_B T}\right)$$

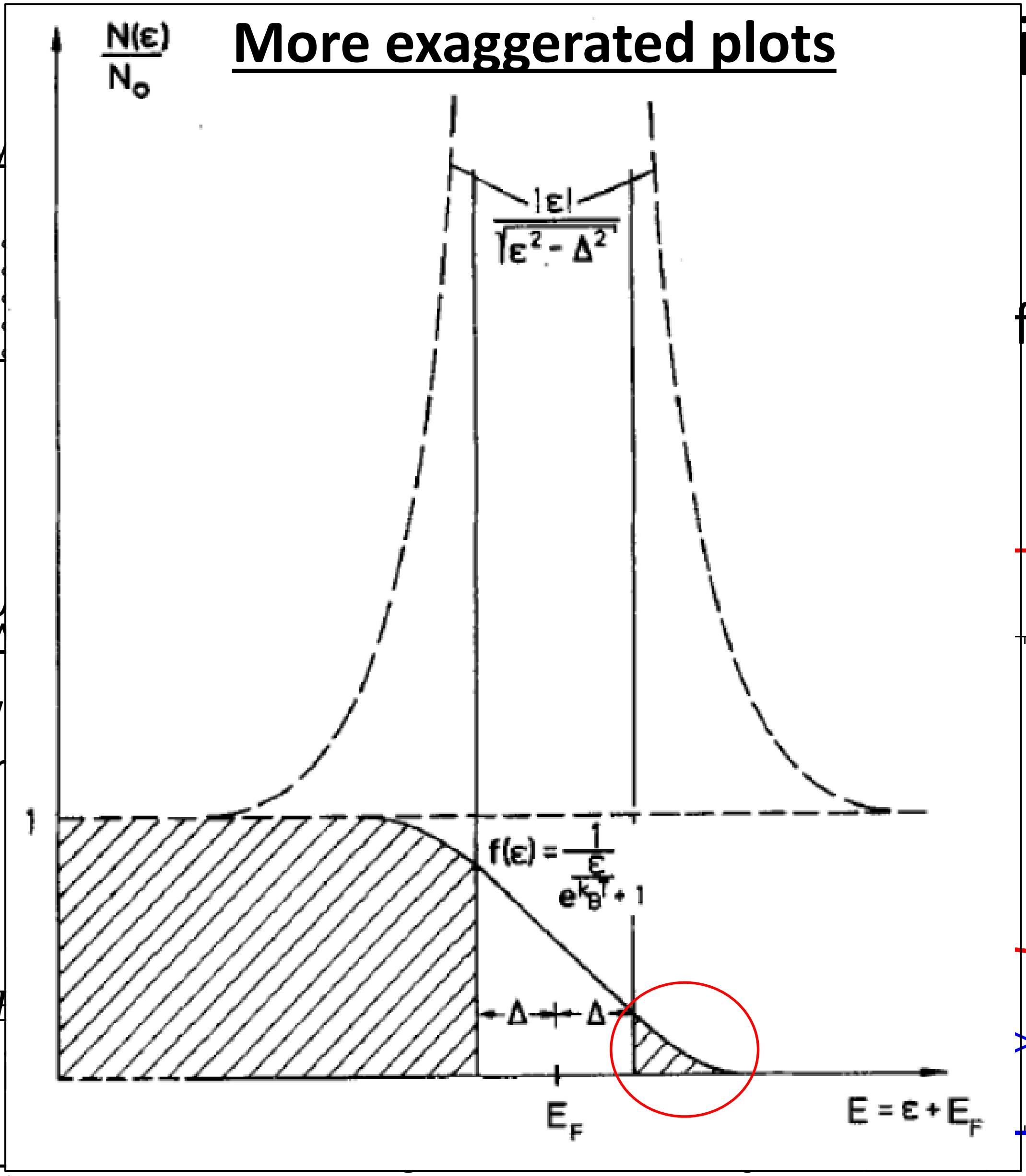
$$R_s(T=0) = 0$$

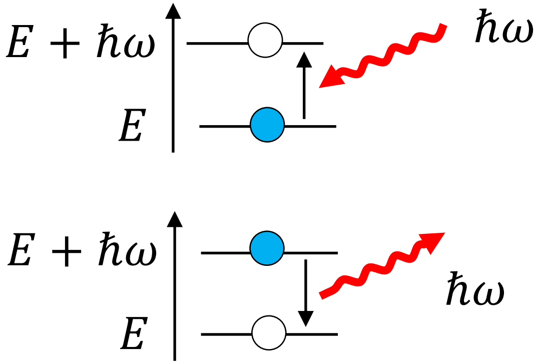
Introduction to ***quantum*** mechanical derivation:
Integrate … i-particles

Fermi's Golden rule [Z. Phy…

$R_s \propto P \cong \Sigma_{p,p',h\vec{k}}$ … f absorbed photons)

$= \hbar\omega(n_+ - n_-) = \hbar\omega$ … $+\,\hbar\omega)$





All quasiparticles
$\Delta < E < \infty$

$N(E)$: density of states (how … states at energy E: a kind of degener…

$f(E)$: distribution function … electrons are in one state at energy E…

Fermi-Dirac function in equ…

$$f(E) = \frac{1}{\exp(-E/k_BT) + 1}$$

> 0

3

(E-∈F)/Δ

Q4	Follow the math

$$R_s \propto \frac{\omega^{1.5}}{T}\exp\left(-\frac{\Delta}{k_BT}\right)$$

$R_s(T = 0) = 0$

Reality in the literature…complete picture until 1970s

Mattis and Bardeen Phys Rev 111 2 1958

$$\mathbf{j}(\mathbf{r},t)=\sum_{\omega}\frac{e^2N(0)v_0}{2\pi^2\hbar c}\times\int\frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}_\omega(r')]I(\omega,R,T)e^{-R/l}dr'}{R^4}$$

$$I(\omega,R,T)=-\pi i\int_{\epsilon_0-\hbar\omega}^{\epsilon_0}[1-2f(E+\hbar\omega)]\times[g(E)\cos(\alpha\epsilon_2)-i\sin(\alpha\epsilon_2)]e^{i\alpha\epsilon_1}dE$$
$$-\pi i\int_{\epsilon_0}^{\infty}\{[1-2f(E+\hbar\omega)]\times[g(E)\cos(\alpha\epsilon_2)-i\sin(\alpha\epsilon_2)]e^{i\alpha\epsilon_1}-[1-2f(E)]\times[g(E)\cos(\alpha\epsilon_1)+i\sin(\alpha\epsilon_1)]e^{-i\alpha\epsilon_2}\}dE,$$

$$\frac{\sigma_1}{\sigma_N}=\frac{2}{\hbar\omega}\int_{\epsilon_0}^{\infty}[f(E)-f(E+\hbar\omega)]g(E)dE+\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega}^{-\epsilon_0}[1-2f(E+\hbar\omega)]g(E)dE,$$

$$\frac{\sigma_2}{\sigma_N}=\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega,\,-\epsilon_0}^{\epsilon_0}\frac{[1-2f(E+\hbar\omega)](E^2+\epsilon_0^2+\hbar\omega E)}{(\epsilon_0^2-E^2)^{\frac{1}{2}}[(E+\hbar\omega)^2-\epsilon_0^2]^{\frac{1}{2}}}$$

Abrikosov et at JTEP 35 182 1959

$$\mathbf{j}(\mathbf{k},\omega)=\frac{3e^2N\mathbf{A}(\mathbf{k},\omega)}{32mc}\int_{-1}^{1}d\cos\theta\sin^2\theta\int_{-\xi_0}^{\xi_0}d\xi\left[\left(1-\frac{\xi_1\xi_2+\Delta^2}{\varepsilon_1\varepsilon_2}\right)\left(\tanh\frac{\varepsilon_1}{2T}+\tanh\frac{\varepsilon_2}{2T}\right)\left(\frac{1}{\varepsilon_1+\varepsilon_2+\omega+i\delta}+\frac{1}{\varepsilon_1+\varepsilon_2-\omega-i\delta}\right)\right.$$
$$\left.+\left(1+\frac{\xi_1\xi_2+\Delta^2}{\varepsilon_1\varepsilon_2}\right)\left(\tanh\frac{\varepsilon_1}{2T}-\tanh\frac{\varepsilon_2}{2T}\right)\left(\frac{1}{\varepsilon_1-\varepsilon_2+\omega+i\delta}+\frac{1}{\varepsilon_1-\varepsilon_2-\omega-i\delta}\right)\right]-\frac{e^2}{mc}N\mathbf{A}(\mathbf{k},\omega).$$

$$\frac{Z(\omega)}{R_n}=2\left(\frac{\omega}{\pi\Delta}\right)^{1/3}\left[\frac{4}{3\pi}\sinh\frac{\omega}{2T}K_0\left(\frac{\omega}{2T}\right)e^{-\Delta/T}-i\right].$$

Strong coupling theory

(Eliashberg JTEP 11 696 1960; Nam Phys Rev 156 470 1967; Marsiglio et al PRB 50 7203 1994)

$$\sigma_1(\nu)=\frac{ne^2}{m}\frac{1}{2\nu}\left(\int_0^D d\omega\left[\tanh\frac{\beta(\omega+\nu)}{2}-\tanh\frac{\beta\omega}{2}\right]g(\omega,\nu)+\int_{-\nu}^{0}d\omega\tanh\frac{\beta(\omega+\nu)}{2}g(\omega,\nu)\right),\qquad(1)$$

$$g(\omega,\nu)=\mathrm{Im}\left(\frac{1-N(\omega)N(\omega+\nu)-P(\omega)P(\omega+\nu)}{\epsilon(\omega)+\epsilon(\omega+\nu)+i/\tau}+\frac{1+N^*(\omega)N(\omega+\nu)+P^*(\omega)P(\omega+\nu)}{\epsilon^*(\omega)-\epsilon(\omega+\nu)-i/\tau}\right)$$

$$\epsilon(\omega)\equiv\sqrt{\tilde{\omega}^2(\omega+i\delta)-\phi^2(\omega+i\delta)}$$

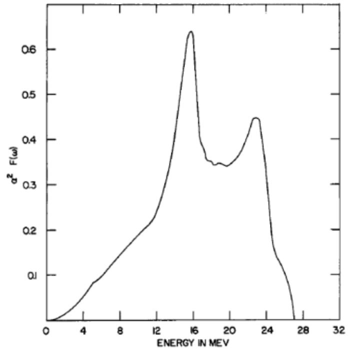
$$N(\omega)\equiv\tilde{\omega}(\omega+i\delta)/\epsilon(\omega),$$

$$P(\omega)\equiv\phi(\omega+i\delta)/\epsilon(\omega).$$

$$\tilde{\omega}(\omega)=\omega+i\pi T\sum_{m=0}^{\infty}\frac{\tilde{\omega}(i\omega_m)}{[\tilde{\omega}^2(i\omega_m)-\phi^2(i\omega_m)]^{1/2}}[\lambda(\omega-i\omega_m)-\lambda(\omega+i\omega_m)]+i\pi\int_{-\infty}^{\infty}dz\frac{\tilde{\omega}(\omega-z)}{[\tilde{\omega}^2(\omega-z)-\phi^2(\omega-z)]^{1/2}}\alpha^2F(z)[N(z)+f(z-\omega)],$$

$$\phi(\omega)=i\pi T\sum_{m=0}^{\infty}\frac{\phi(i\omega_m)}{[\tilde{\omega}^2(i\omega_m)-\phi^2(i\omega_m)]^{1/2}}[\lambda(\omega-i\omega_m)+\lambda(\omega+i\omega_m)-2\mu^*]+i\pi\int_{-\infty}^{\infty}dz\frac{\phi(\omega-z)}{[\tilde{\omega}^2(\omega-z)-\phi^2(\omega-z)]^{1/2}}\alpha^2F(z)[N(z)+f(z-\omega)].$$

Electron-phonon spectral function $\alpha^2F(\omega)$



Wolf, J Low Temp Phys 40 19 1980

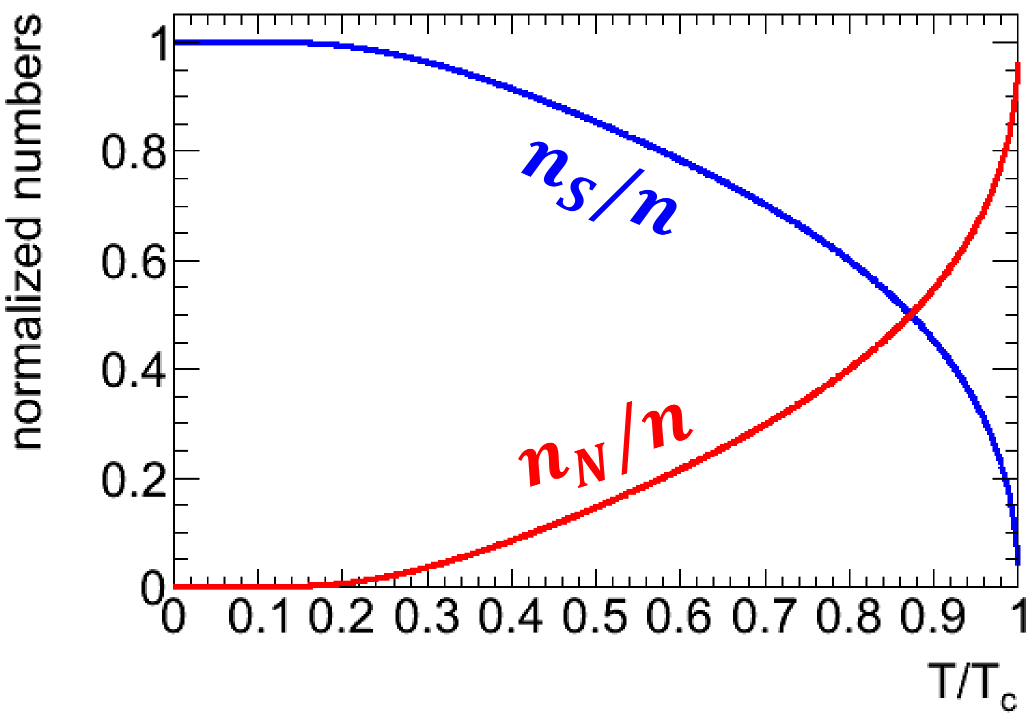
Good news: classical model works very well

Supercurrent

$$\frac{\partial j_s}{\partial t} - \frac{n_S e^2}{m^*} E = 0$$

$$j_s = j_0 \exp(i\omega t)$$

$$j_s = -i \frac{n_S e^2}{m^* \omega} E \qquad \equiv \sigma_S$$



Normal current

Ohm's law →

$$j_N = \frac{n_N e^2 \tau}{m^*} E \qquad \equiv \sigma_N$$

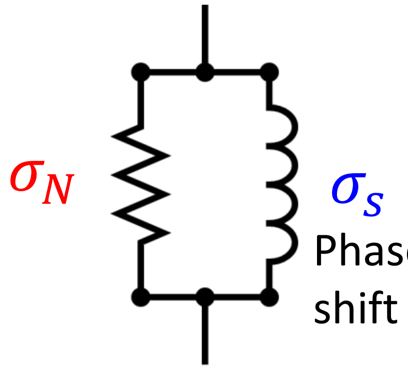
Total current induced by RF

$$j = j_S + j_N \quad \rightarrow \quad j = (\sigma_N - i\sigma_S)E$$

Dissipation by quasi-particles →resistive

Inertia of Cooper pairs → inductive

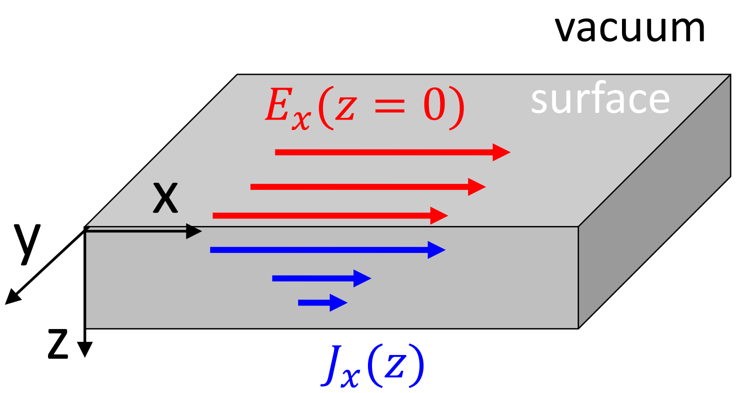
Equivalent circuit



Phase shift $\pi/2$

Two fluid model

Surface resistance of superconductor



$$\begin{cases} j_x = (\sigma_N - i\sigma_S)E_x \\ E_x(z) = E_0 \exp(-z/\lambda_L) \end{cases}$$

Q5 Follow the math

$$\rightarrow R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right) \sim \frac{1}{2}\frac{\sigma_N}{\sigma_S}\sqrt{\frac{\omega\mu_0}{\sigma_S}} = \frac{\mu_0^2}{2}\lambda_L^3\sigma_N\omega^2 > 0$$

$$\sigma_N = \frac{e^2 n_N \tau}{m^*} \propto n_N \propto \exp\left(-\frac{\Delta}{k_B \boldsymbol{T}}\right)$$

Lessons

- One origin of the finite R_s of superconductors is quasi-particles
- Quasi-particles are thermally activated from Cooper pairs at $0 < T < T_c$
- R_s exponentially decreases by lower T because quasi-particles are frozen out
- Higher RF frequency increases $R_s \sim \omega^2$

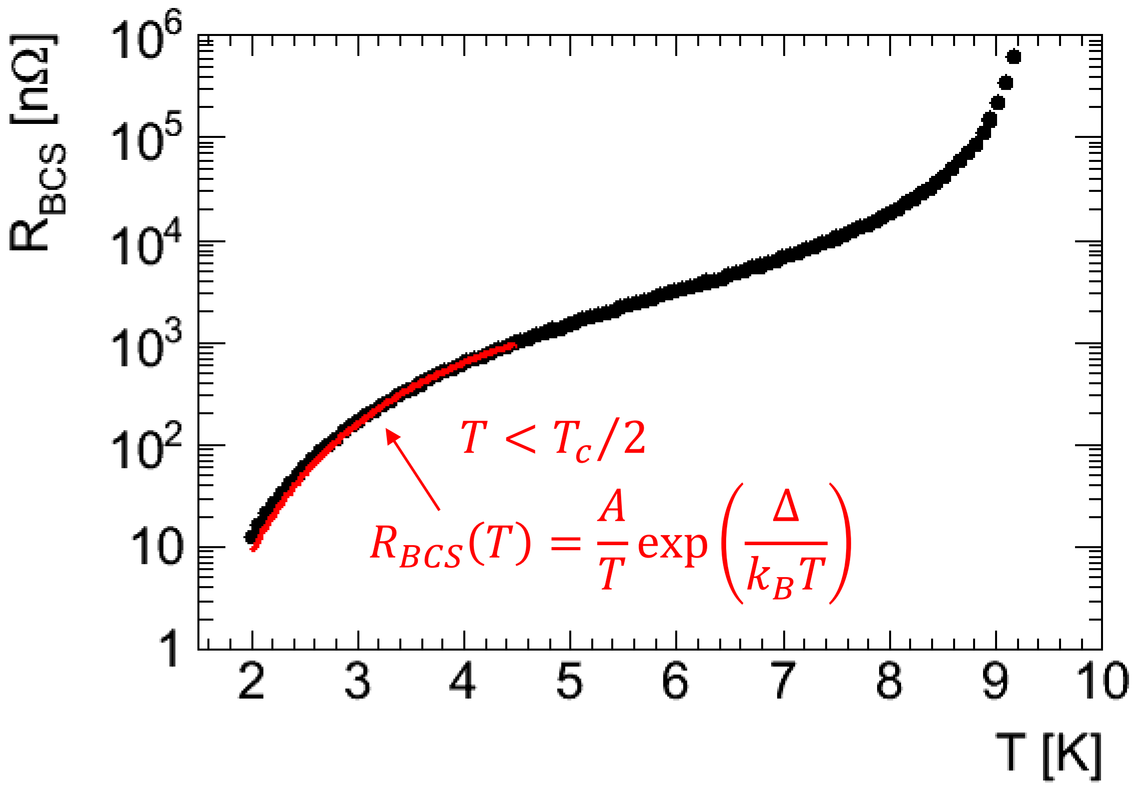
Classical understanding is sufficient in most of the SRF activities

Superconducting cavities: $R_{BCS}(T, f)$

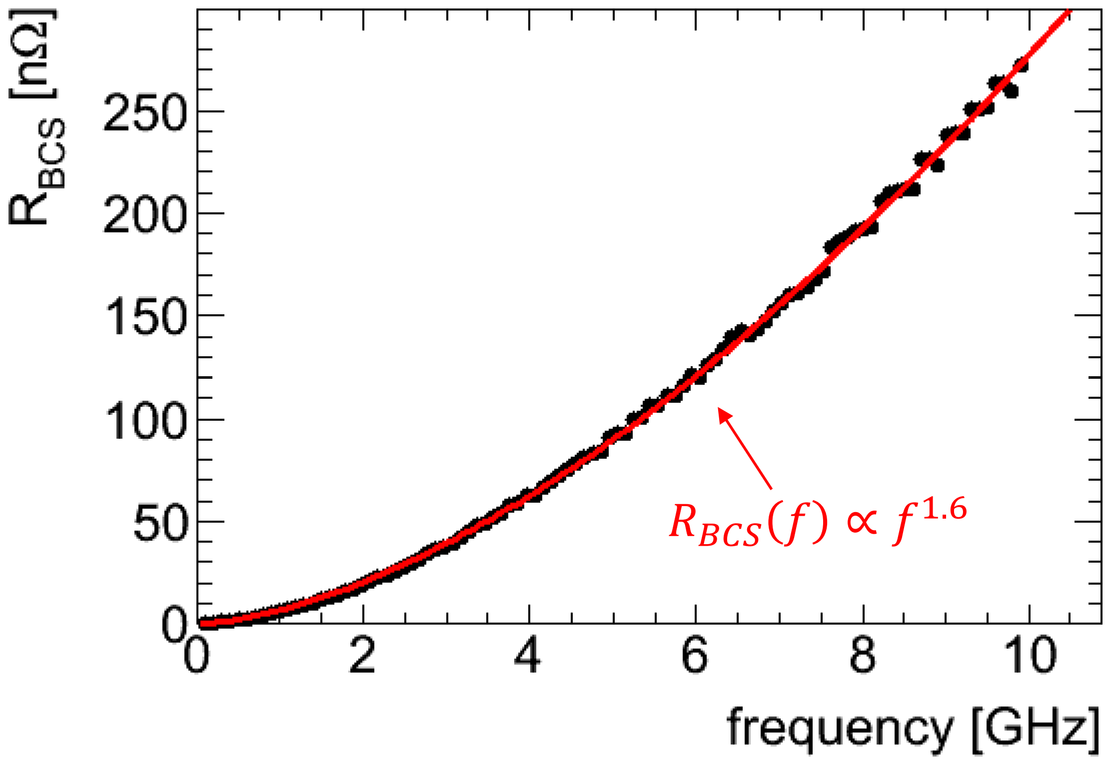
- Halbritter, KFK-Ext.03/70-06 (1970), https://publikationen.bibliothek.kit.edu/270004230: Fortran66 code for all (ξ, λ, l)

  Detail phonon-electron interaction is not included → BCS (weak coupling limit) + phenomenological parameter $\alpha = \Delta/k_B T_c$

<u>Temperature dependence is exponential</u>



<u>Frequency dependence between $f^{1.5}$ and f^2</u>

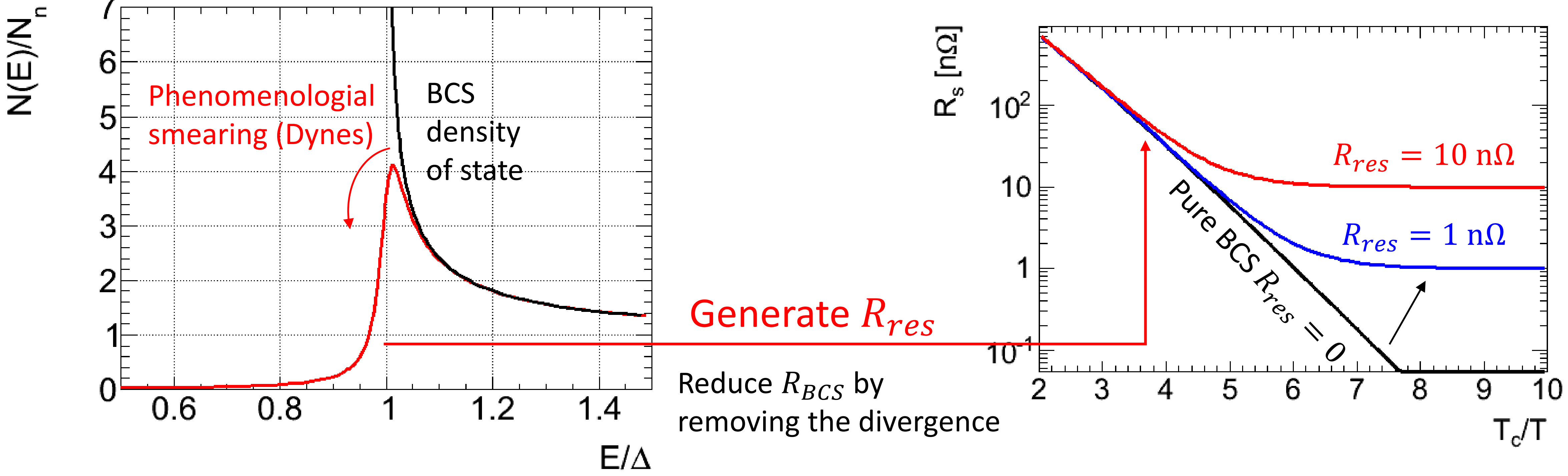


Classically derived two-fluid model works fine to explain quantum calculation of BCS

→ **Practically, we can use the two fluid model to interpret data in your lab**

Smearing of Density of States and residual resistance

In reality $R_s \sim R_{\mathrm{BCS}}(T) + R_{\mathrm{res}}$



<u>Remark: DoS smearing is not the only cause of residual resistance</u>

- Lossy oxides?
- Hydride?
- Grain boundaries??
- Influence of magnetic vortex

etc…

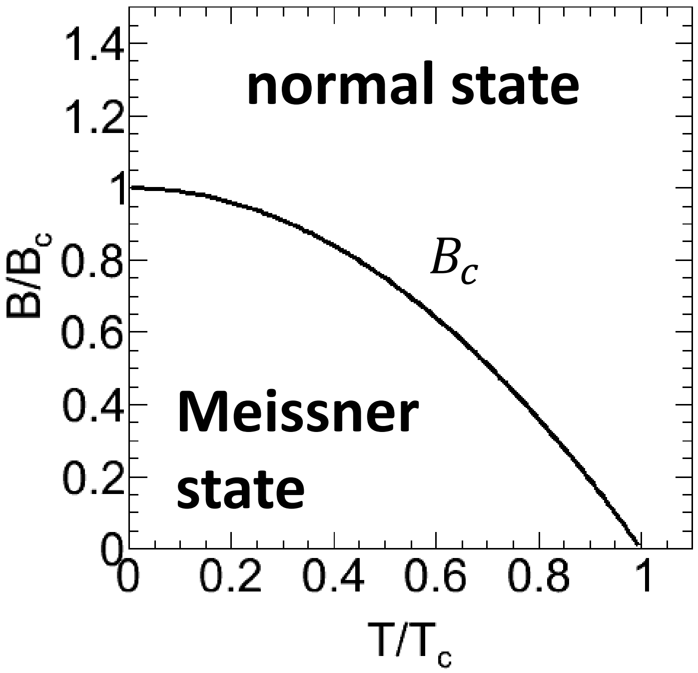
Forget about practicalities ☺

Let's focus on fundamental aspect of ***topological defect***

Under strong but ***static*** magnetic field: Type-I vs Type-II

<u>Type-I</u>

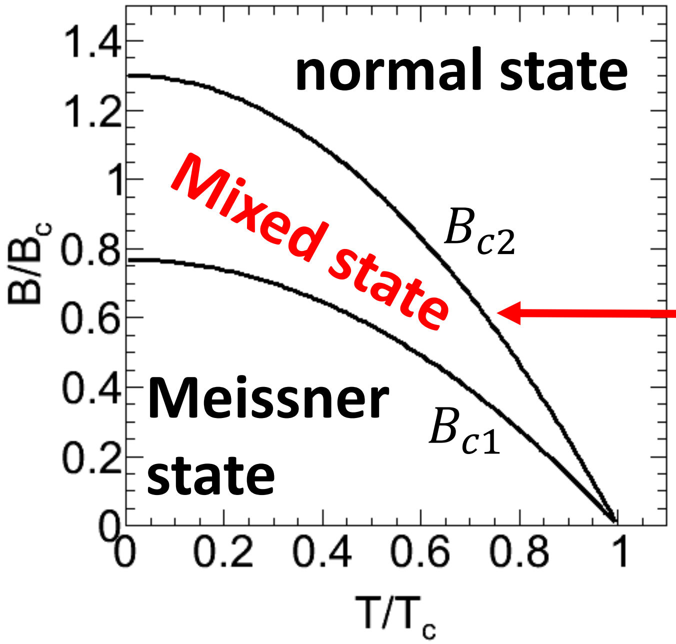
$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Pb} \sim \frac{28\ \text{nm}}{71\ \text{nm}} \sim 0.40$$

<u>Type-2</u>

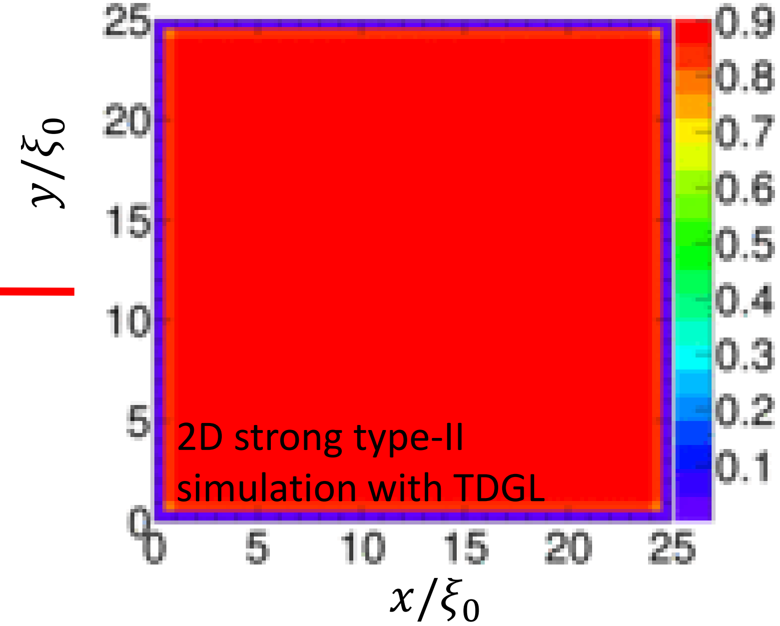
$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Nb} \sim \frac{36\ \text{nm}}{39\ \text{nm}} \sim 0.92$$

Stabilized by NC/SC boundary energy

$$\frac{1}{2\mu_0}(\xi_0 B_c^2 - \lambda_L B^2) < 0 \text{ for } B > B_{c1}$$



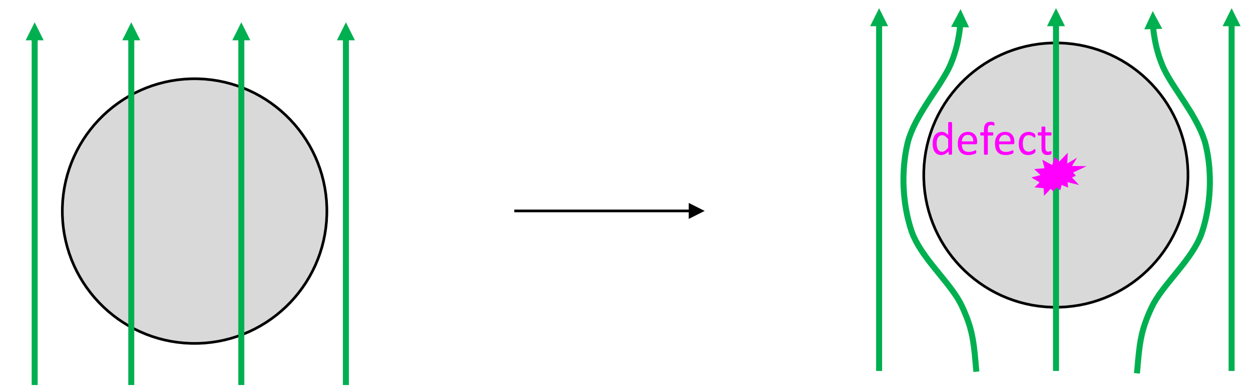
Without pinning centers, type-II ***traps*** magnetic flux if $B > B_{c1}$

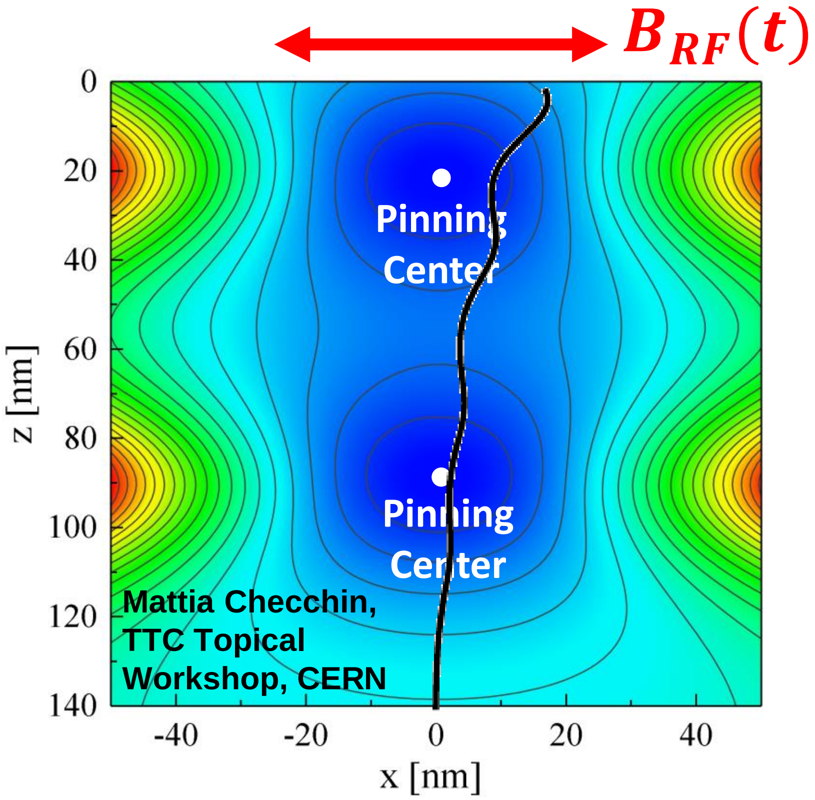
Type-II superconductors become energetically favorable to create normal conducting boundaries inside if $B > B_{c1}$

→ How to maximize interface area? → **Quantized flux** $\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}$ Wb

R_{res} contribution from magnetic flux oscillation

Flux expulsion may not be perfect





Phenomenological equation of motion (Bardeen Stephen)

$$M\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \eta\frac{\partial \boldsymbol{u}}{\partial t} - \epsilon\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + \nabla U(z,u) = \boldsymbol{J}_{RF}(z,u) \times \boldsymbol{B}_{\text{ext}}$$

Effective inertia | Effective viscosity | Effective tension | **Pinning potential** | **Lorentz force drives flux oscillation**

This flux oscillation can cause substantial power dissipation

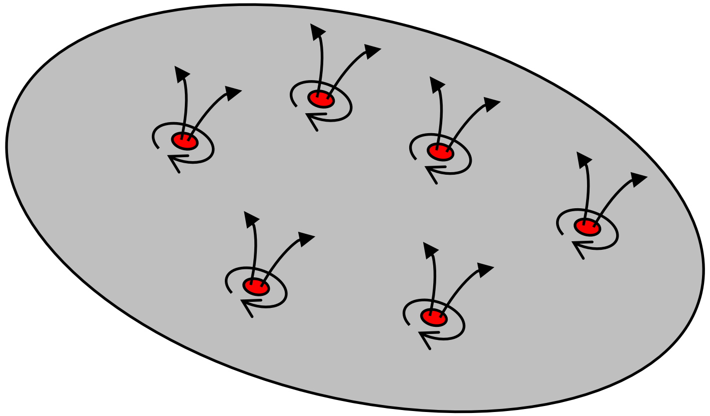
Simple approximation

$$M\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \eta\frac{\partial \boldsymbol{u}}{\partial t} - \epsilon\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + \nabla U(z,u) = \boldsymbol{J}_{RF}(z,u) \times \boldsymbol{B}_{\mathrm{ext}}$$

Static model

$$\rightarrow \quad R_{\mathrm{mag}} \sim N \times \pi\xi_0^2 \times R_{\mathrm{n}} \sim \frac{B_{\mathrm{ext}}}{2B_{c2}} R_{\mathrm{n}}$$

- N: Flux number density
- $\pi\xi_0^2$: Normal conducting area
- R_{n}: Normal conducting surface resistance



D. Longuevergne, AM
arXiv:2009.07007
S, Calatroni and R. Vaglio,
PRAB 22, 022001 (2019)

Earth field B_{ext}=50uT
B_{c2}~400 mT (Nb)
R_n~1.3 mΩ at 1.3 GHz (Nb)

$$R_{\mathrm{mag}} \sim 80\mathrm{n\Omega} > R_{BCS}(2K) \sim 10\mathrm{n\Omega}$$

A cavity can be spoiled!

<u>Solutions</u>

1. A good **magnetic shield** (earth field 50uT → < 1uT)
2. Expel more fluxes at phase transition
3. (Reduce sensitivity of the flux oscillation against RF)

Engineering challenges!

Outline

- Introduction: why superconducting RF for accelerators?
- Superconductors in thermal equilibrium
  - Bardeen-Cooper-Schrieffer theory
  - Superconductors and Higgs mechanism
- Response against Radio Frequency
  - Linear response theory
  - Residual resistance
- Field limitations
  - Physics of phase transition
  - Fundamental challenges
- Conclusion

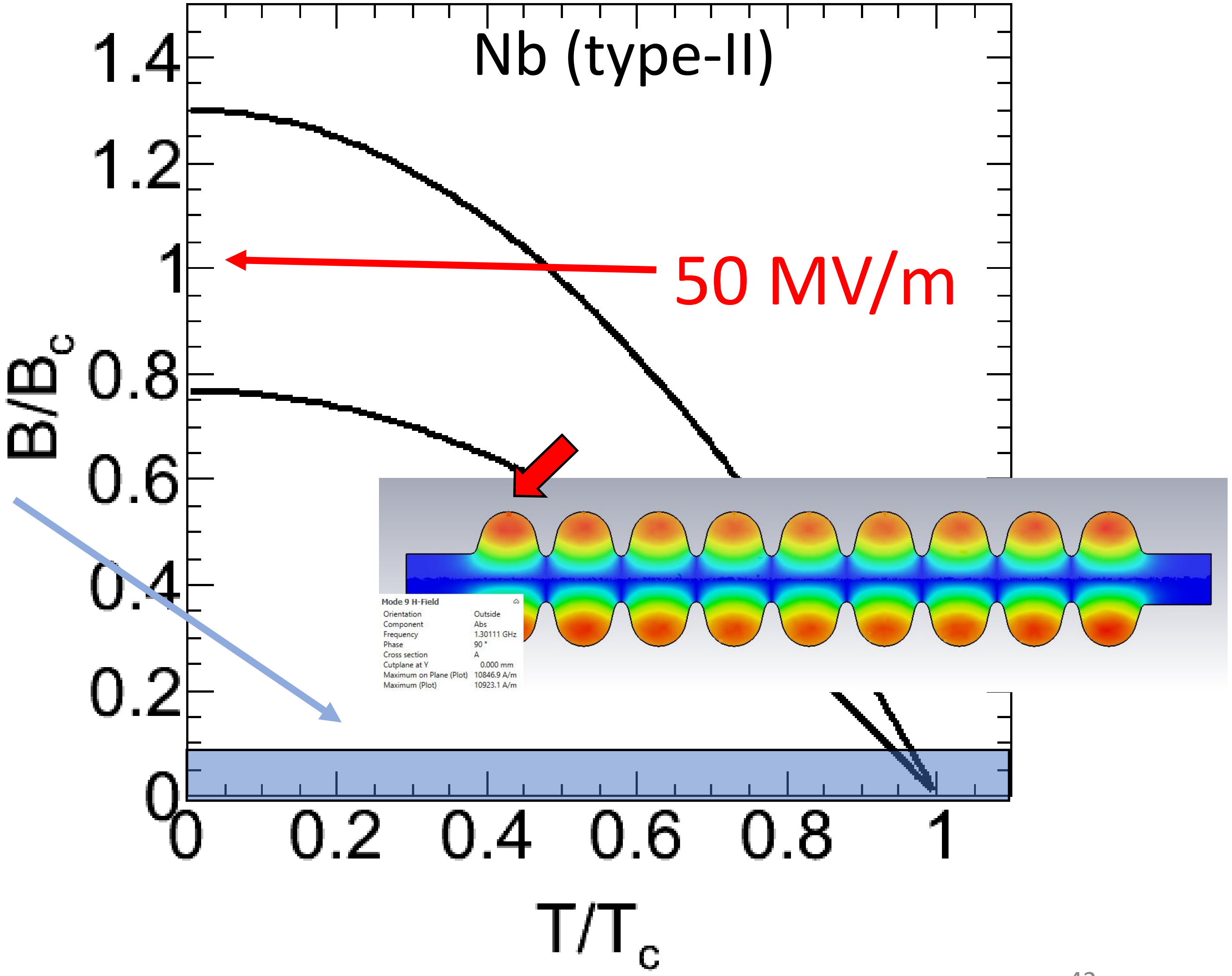
Remark: validity of linear response theory

Our formula (f<< 2Δ, T<T_c/2)

$$R_{BCS} \propto \frac{\omega^{1.5-2.0}}{T} \exp\left(-\frac{\Delta}{k_B T}\right)$$

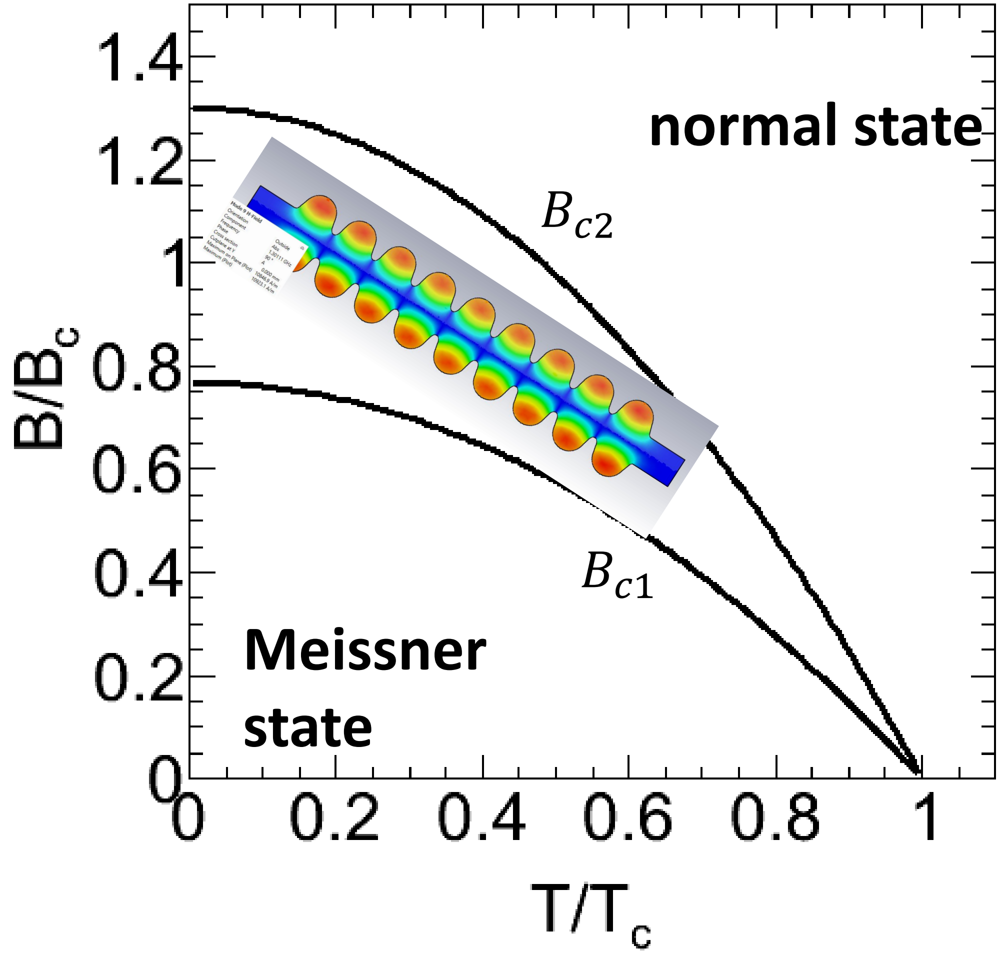
is valid for low RF field ($B_{RF} \ll B_c$) because it is 1st order perturbation (linear response)

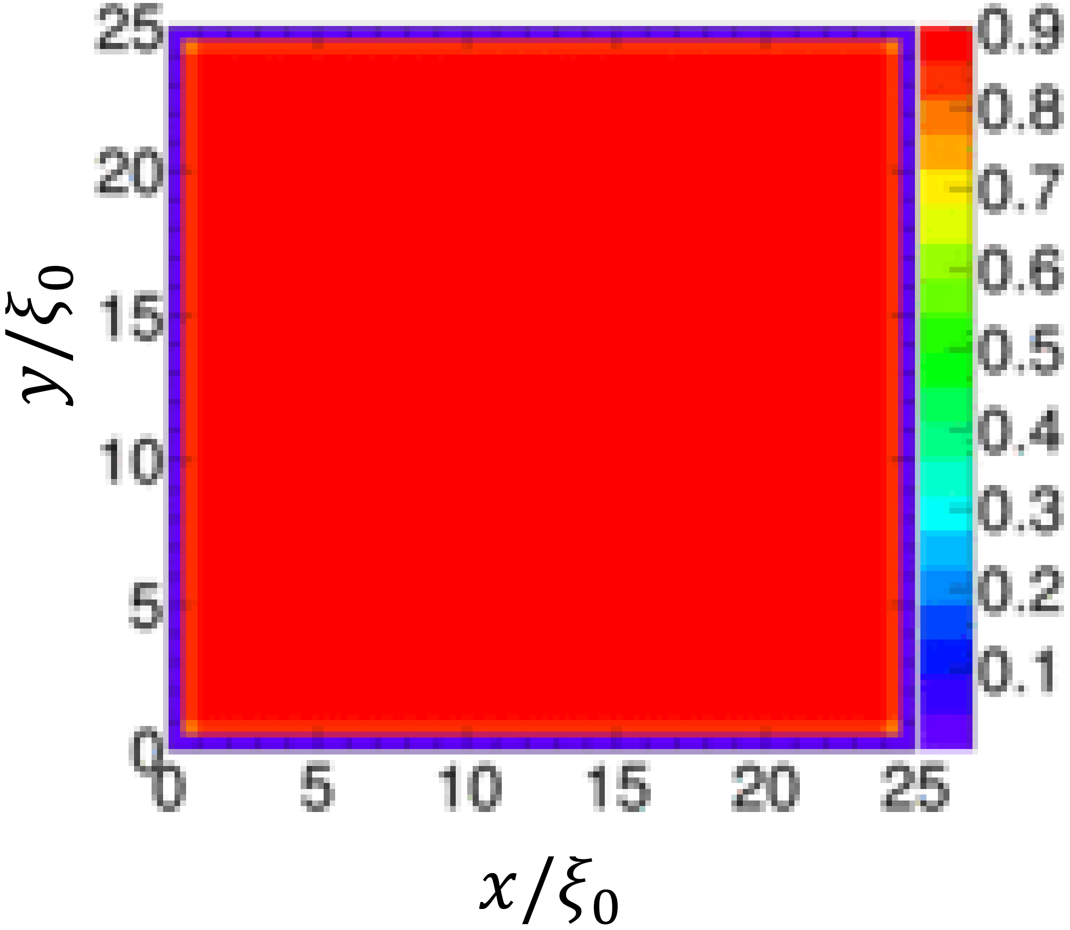
However, state-of-the-art cavities reach 50 MV/m i.e. $B_{RF} \sim B_c$



→ Fundamental challenge in condensed matter physics

The RF magnetic field exceeds B_{c1}





Does type-II superconductor dissipate too much power from flux entry & oscillation? Are type-II superconductors ***useless*** for SRF?

1^{st} order phase transition can be ***metastable***

Super-cooling of water: $T < 0$ C but still liquid

https://tenor.com/view/diy-science-hack-ice-water-gif-3448836

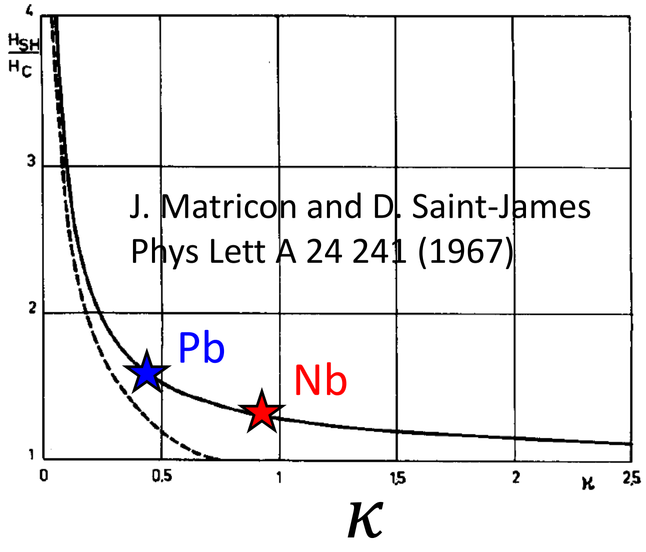
SC phase transition with a ***magnetic field*** is a 1^{st} order phase transition

→ **$B > B_{c1}$ can be a metastable super-heating state**

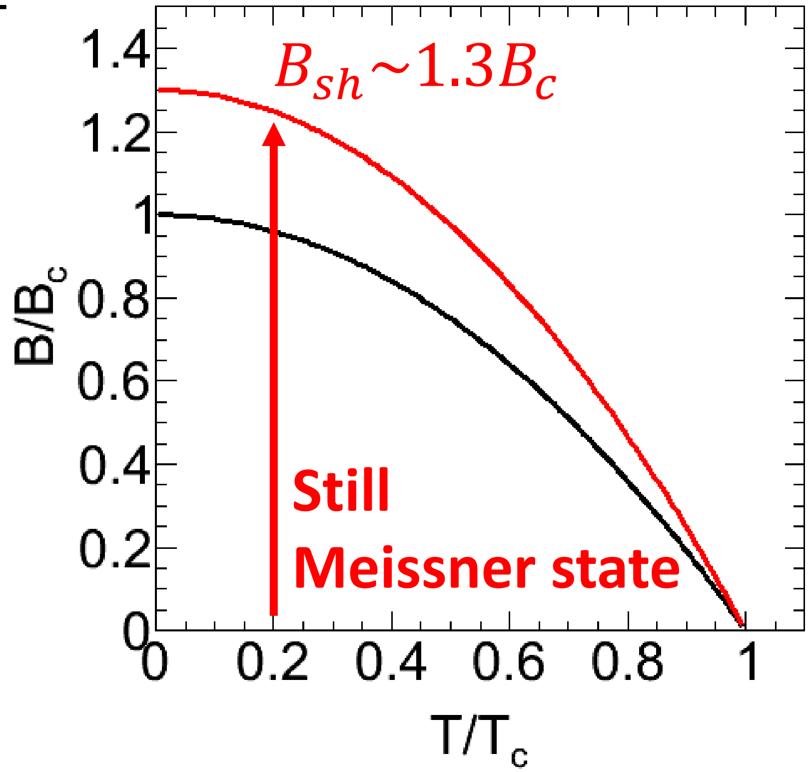
Relevant critical field for SFR: superheating field

<u>Ginzburg-Landau equation</u>

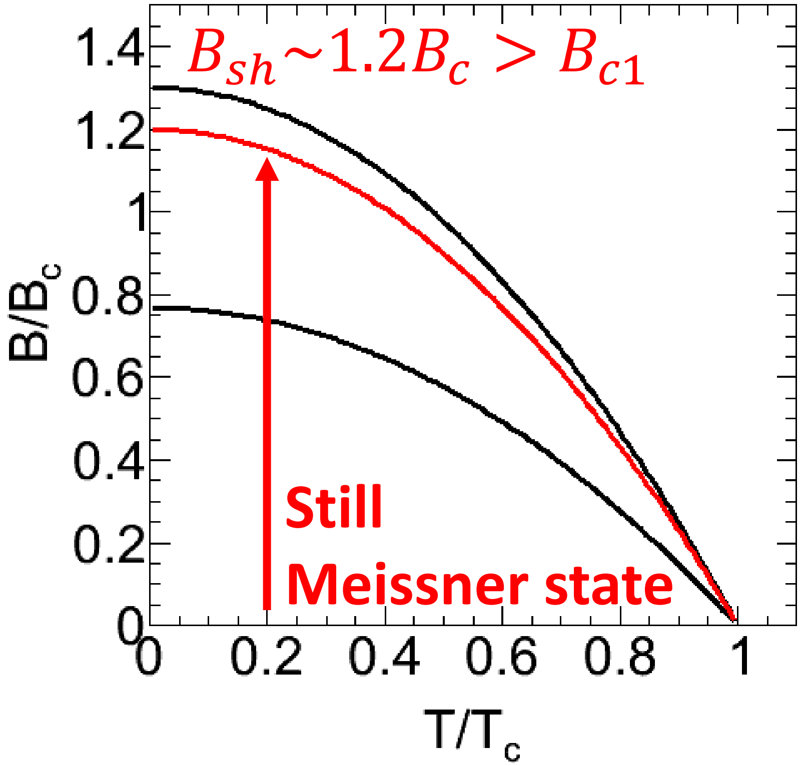
$B_{sh} > B_c$ in general



Pb (type-I)



Nb (type-II)



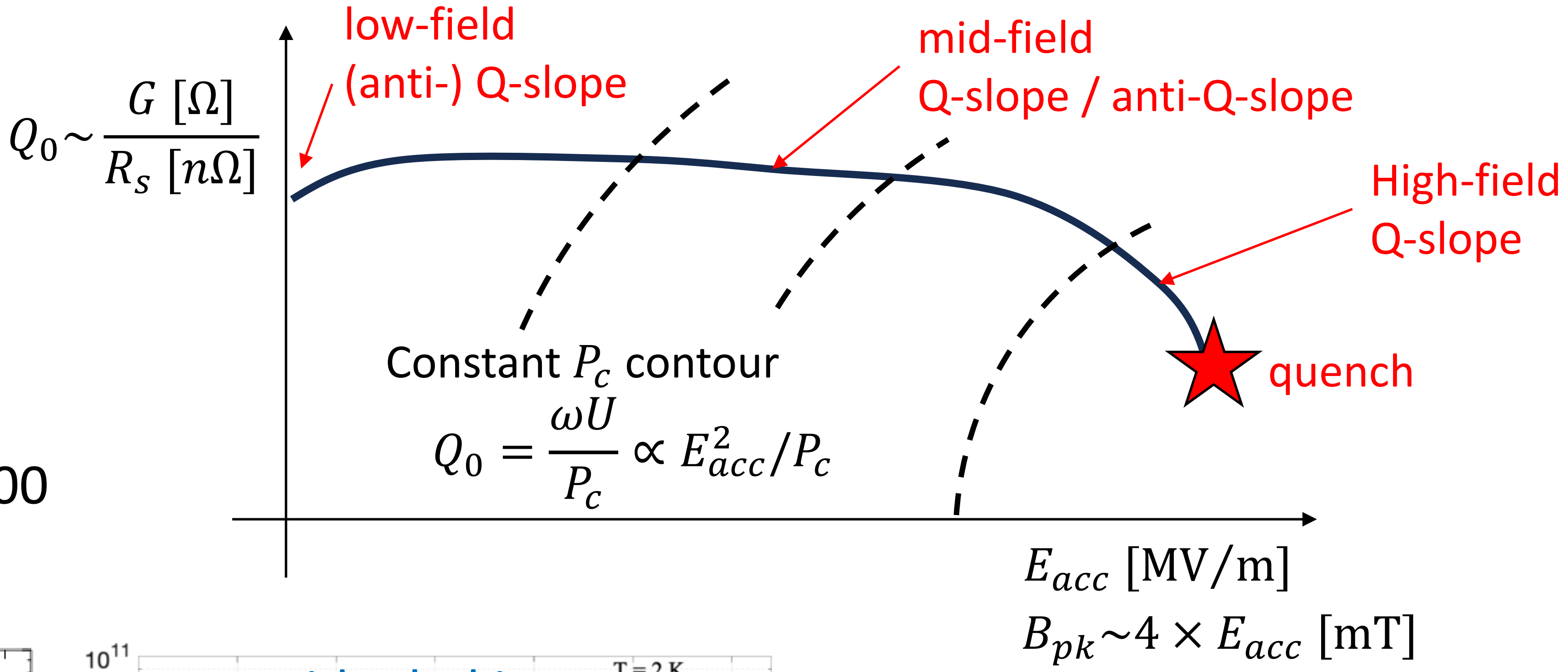
Go'rkov showed that BCS theory reproduces Ginzburg Landau equation around $\boldsymbol{T \to T_c}$

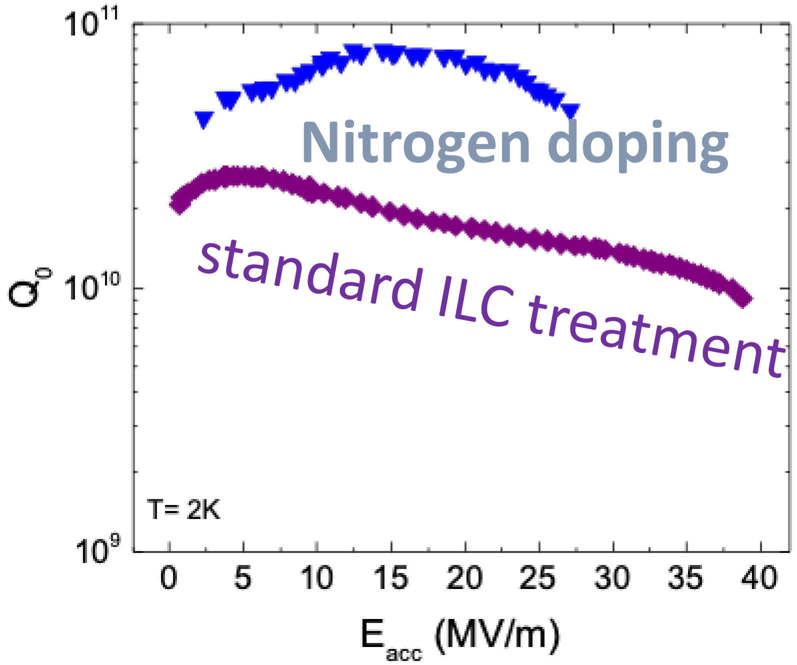
→ The validity of this B_{sh} at $T < T_c$ deserves discussion

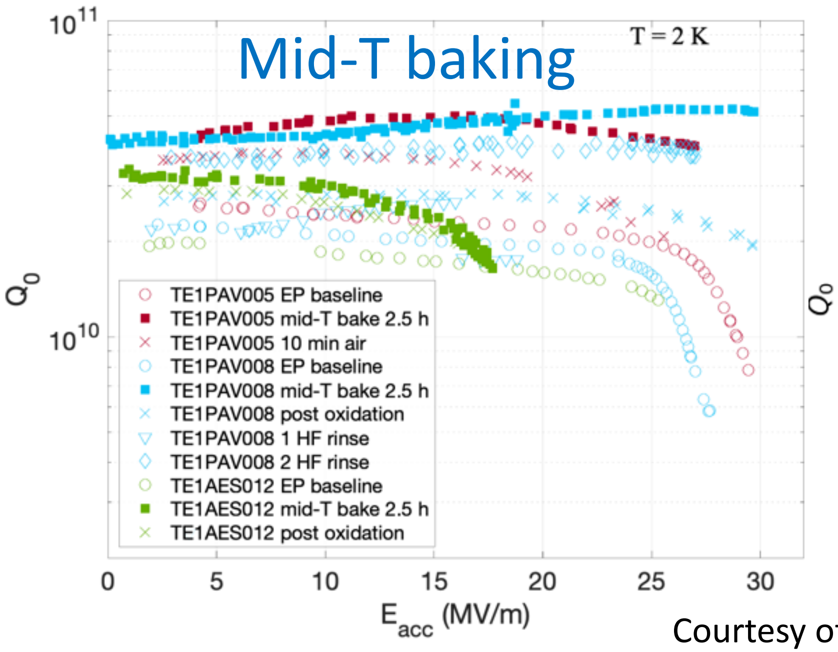
Quasi-classical formalism, influence of impurity, multilayer coating to further enhance B_{sh}, nonlinear $R_s(B_{RF})$…

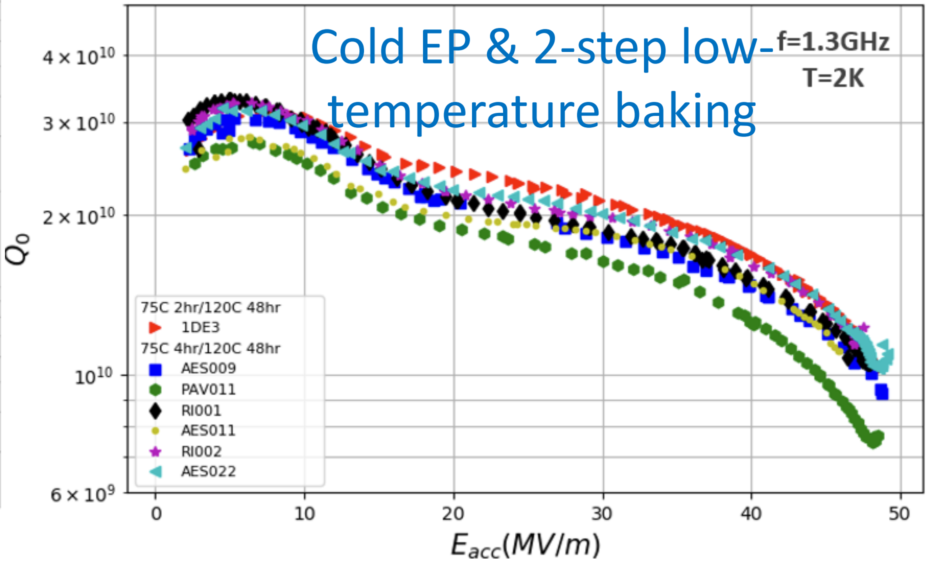
Q vs E

- Upper right is better
- Unknown causes of nonlinear behavior
- Quench limits
- Dramatic change by 100 nm surface treatment









Courtesy of Sergey Belomestnykh

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Answer to the first three questions

1. What is the superconductivity?
    1. A finite attractive interaction between independent electrons form a Cooper pair that obeys nonrelativestic U(1) Higgs mechanism
    2. Photons gain mass in superconductors due to spontaneous symmetry breaking, which leads to the Meissner effect
2. What are the fundamental origins of finite RF loss in SRF cavities?
    1. Thermally activated quasi-particles at finite temperature act like normal conducting electrons and cause a loss in RF
    2. Even at absolute zero temperature, residual resistance exists due to several different mechanisms, such as flux oscillation and subgap state’s effect, whose ultimate origins are not wholly understood
3. What are the fundamental limitations of the field inside SRF cavities?
    1. Superheating field, which exceeds thermodynamic critical fields in equilibrium state, would give a fundamental limitation
    2. The dynamic calculation of the superheating field is still an open field of fundamental research

References 1/2: textbook and reviews

- Standard textbooks on SRF
  - H. Padamsee et al “RF superconductivity for accelerators”, 2nd edition, WILEY-VCH (2008)
  - H. Padamsee “RF superconductivity”, WILEY-VCH (2009)
- Reviews on SRF
  - J. P. Turneaure et al “The surface impedance of superconductors and normal conductors: the Mattis-Bardeen theory”, J. Supercond. 4, 341-355 (1991)
  - A. Gurevich “Theory of RF superconductivity for resonant cavities”, Supercond. Sci. Technol. 30 034004 (2017)
- Introduction to solid state physics (before second quantization)
  - N. W. Ashcroft and N. D. Mermin, “Solid State Physics” Thomson Learning (1976)
- Introduction to superconductivity + minimal knowledge on condensed matter physics (but lack of SRF…)
  - S. Fujita and S. Godoy “Quantum statistical theory of superconductivity”, Springer, (1996)
- Dictionary of superconductivity
  - M. Tinkham “Introduction to superconductivity”, 2nd edition, Dover (2004)
- More advanced textbook on superconductivity
  - N. Kopnin “Theory of Nonequilibrium Superconductivity”, Oxford Science Publications (2001)

References 2/2: selected papers related to this lecture

- BCS resistance
  - J. Bardeen, L. N. Cooper, and J. R. Schrieffer,Phys. Rev.108, 1175 (1957). [Matrix elements for static magnetic field were calculated here]
  - D. C. Mattis and J. Bardeen, Phys. Rev.111,412 (1958). [1^{st} order perturbation of RF response and nonlocality, substituting matrix elements modified for RF]
  - J. Halbritter, Z. Physik266, 209 (1974) [Fermi‘s golden rule applied for constant martix element and two fluid *approximation*]
  - J. Halbritter, KFK-Ext.03/70-06 (1970) [FORTRAN66 code for BCS resistance of f<Δ/2 and arbitrary ξ_0, λ_L, l]
- Residual resistance due to flux oscillation
  - J. Bardeen and M. J. Stephen, Phys. Rev.140, A1197(1965). [Phenomenological model to describe trapped flux as a string]
  - J. I. Gittleman and B. Rosenblum, Phys. Rev. Lett.16,734 (1966). [driven-damped ordinary differential equation for flux oscillation driven by Lorentz force]
  - M. Checchin, M. Martinello, A. Grassellino, A. Roma-nenko, and J. F. Zasadzinski, Supercond. Sci. Technol.30, 3 (2017). [application of Gittleman & Rosenblum for SRF cavities]
  - A. Gurevich and G. Ciovati, Phys. Rev. B87, 054502 (2013). [keeping tension term and solved partial differential equation instead]
- Quench field
  - J. Matricon and D. Saint-James Phys Lett A 24 241 (1967). [solving Ginzburg-Landau equation to estimate superheating field]
  - F. P.-J. Lin and A. Gurevich, Phys. Rev. B 85, 054513 (2012). [solving Eilenberger equations to estimate superheating field in arbitrary impurity]
  - Vudtiwat Ngampruetikorn and J. A. Sauls, Phys. Rev. Research 1, 012015(R) (2019). [including inhomogeneity at the surface]

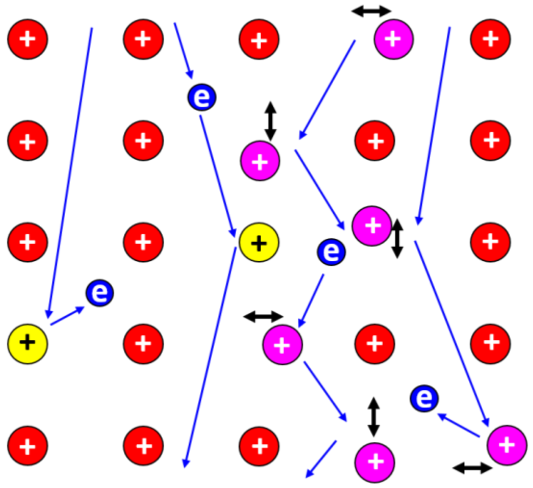
backup

Three characteristic lengths

Cf. Lattice constant of Nb is **0.330 nm**

Mean free path

$$l = \langle v \rangle \tau$$



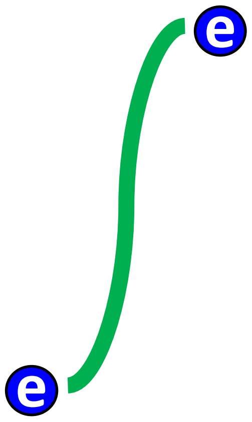
How often quasi-particles are scattered

l depends on RRR ($l \sim 2.7 \times RRR$)

RRR=300 → $l =$ **810 nm**

Coherent length

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta}$$

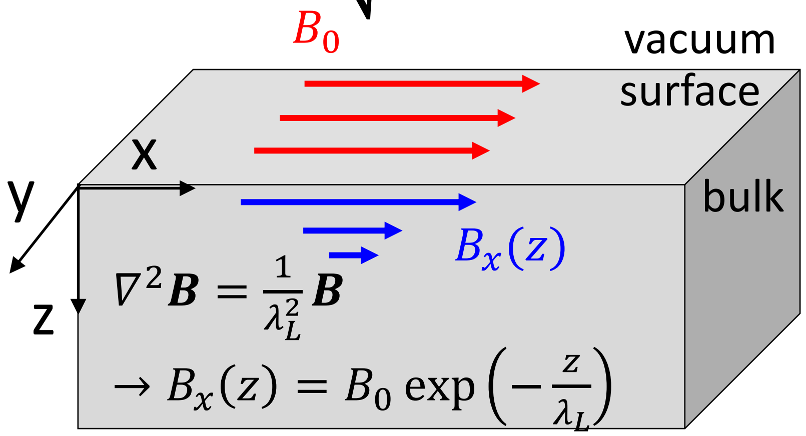


Characteristic size of Cooper pairs

$\xi_0 \sim$ **39 nm** for Nb

(London) Penetration depth

$$\lambda_L = \sqrt{\frac{m^*}{n_s e^2 \mu_0}}$$



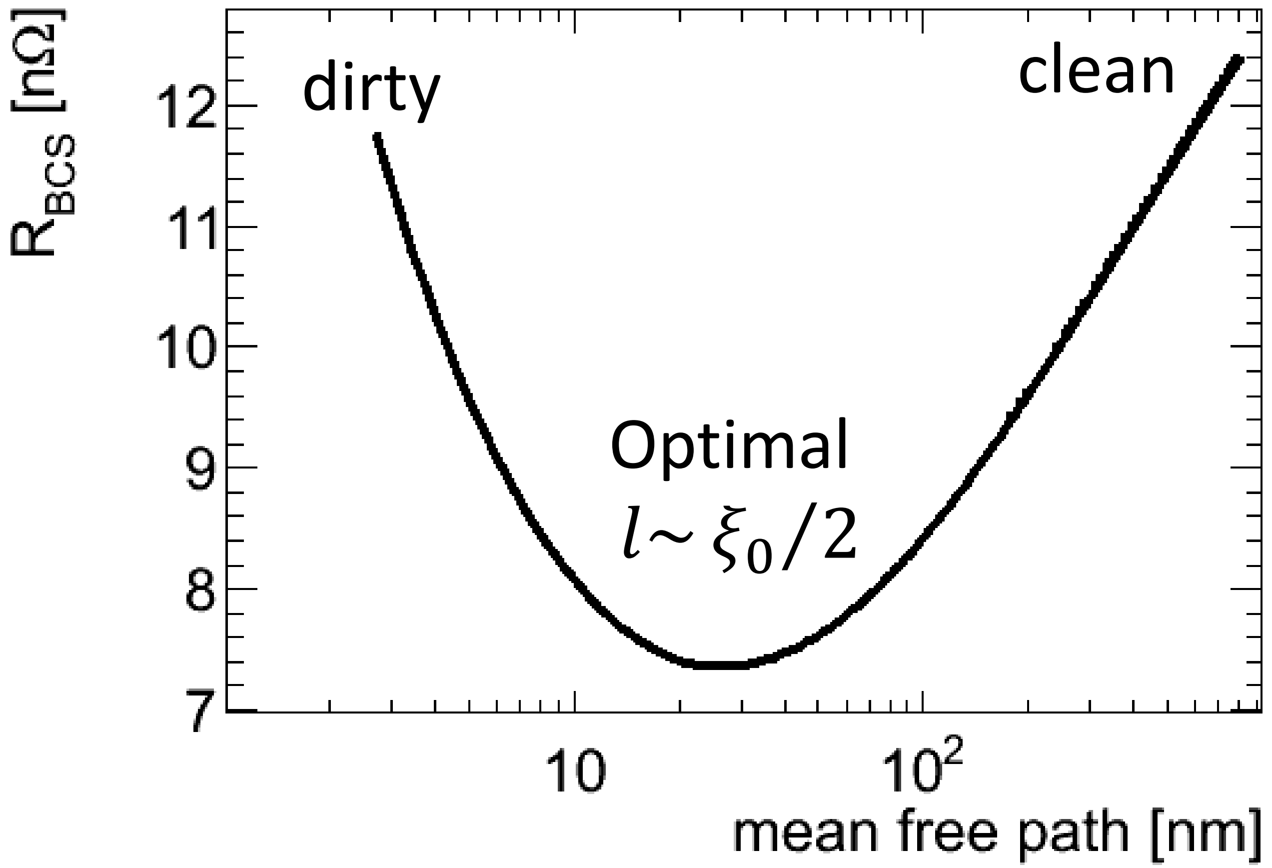
How much magnetic fields can penetrate into a superconductor

$\lambda_L \sim$ **36 nm** for Nb

R_{BCS} vs mean free path l: anomalous skin effect

$$
\begin{cases}
R_s \sim \frac{\mu_0^2}{2} \lambda_0^3 \sigma_N \omega^2 \\
\lambda_0 = \lambda_L \sqrt{1 + \frac{\pi \xi_0}{2l}} \\
\sigma_N = \frac{e^2 n \tau}{m^*} \propto l
\end{cases}
$$

$$
\rightarrow R_s \sim l \times \left(1 + \frac{\pi \xi_0}{2l}\right)^{3/2}
$$



Counter intuitively, super clean material is not ideal for SRF cavities!

→ Heat treatment, doping, etc to make ***surface*** dirty

Penetration depth vs skin depth: similar but totally different origin

Superconductor

Quantum mechanics

$$\lambda_L = \sqrt{\frac{m^*}{n_s e^2 \mu_0}}$$

$\ll$

Normal conductor

$$\delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma}}$$

From classical electrodynamics

From London equation (broken gauge symmetry)

$$\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2}\boldsymbol{B} = 0$$

Both **static** magnetic field and **RF** electromagnetic field and currents

From a RF screening effect of quasi-particles

$$\boldsymbol{j}_n = \sigma \boldsymbol{E}$$

$$\nabla \times \nabla \times \boldsymbol{E} = -\frac{\partial(\nabla \times B)}{\partial t} \sim \mu_0 \frac{\partial \boldsymbol{j}_n}{\partial t}$$

$$(= -\nabla^2 \boldsymbol{E})$$

$$E = E_0 exp(i2\pi f t)$$

$$\nabla^2 \boldsymbol{E} - \frac{1}{\delta^2}\boldsymbol{E} = 0$$

Math looks similar…

RF electromagnetic fields and currents

For niobium (<9.25K)

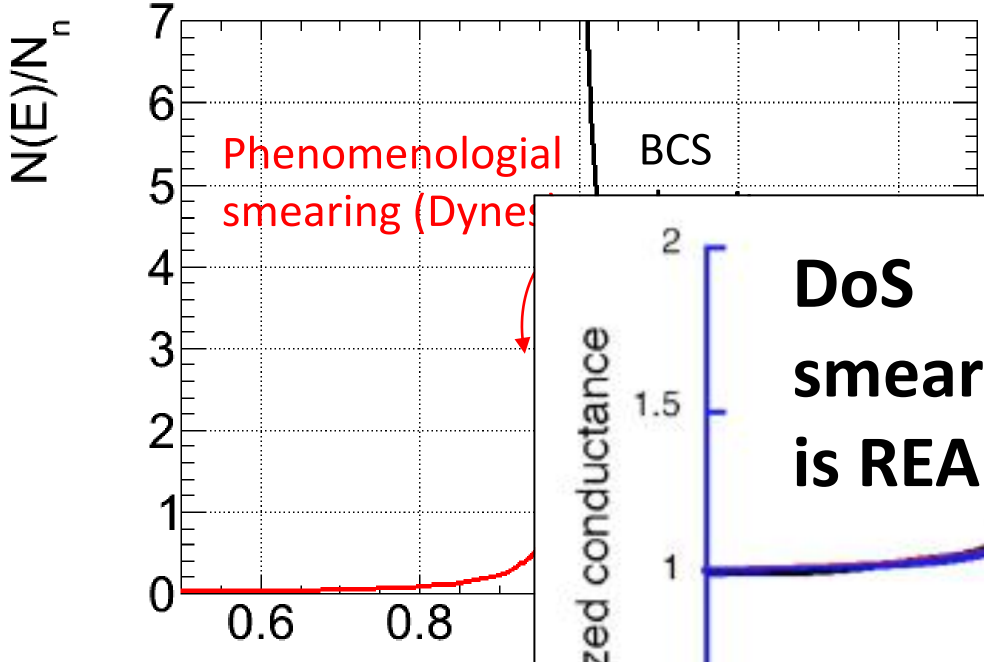
$\lambda_L \sim 36$ nm

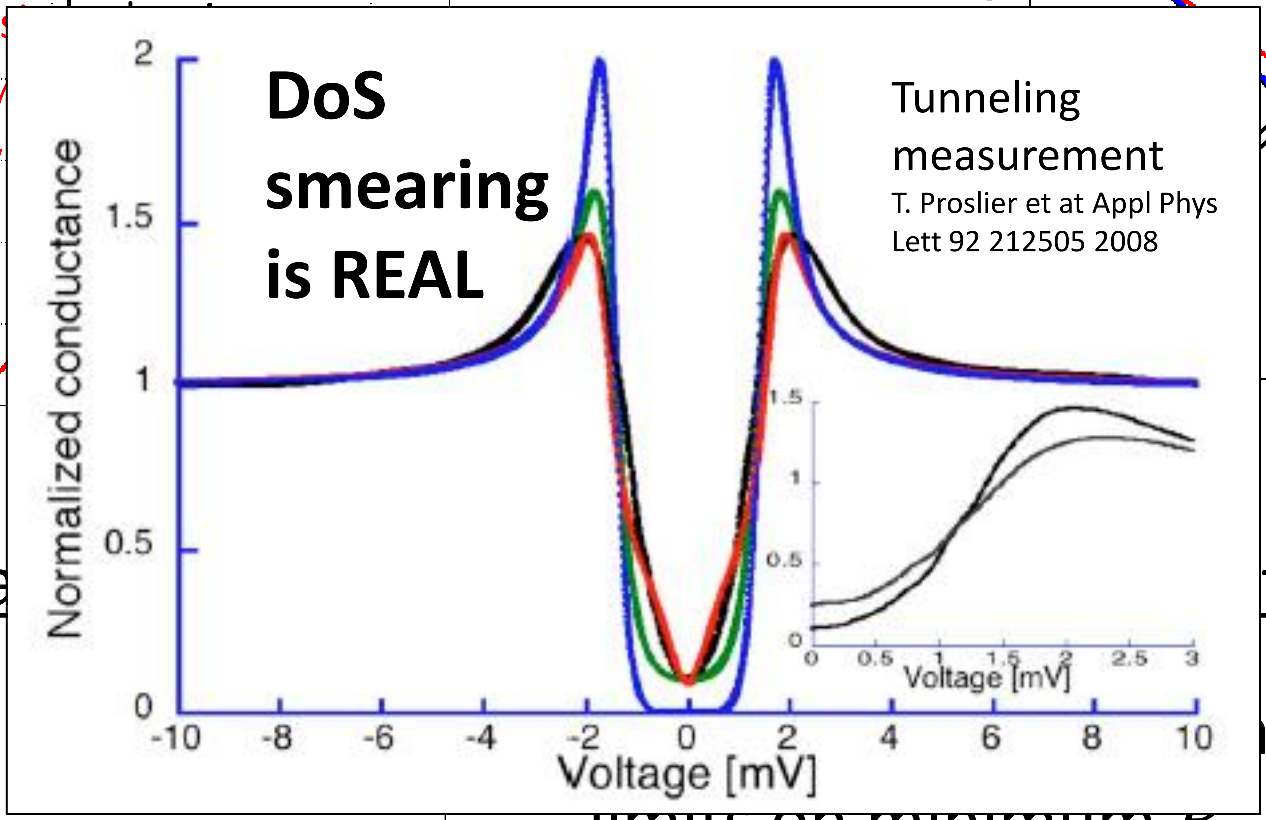
For 300K copper and $f = 0.1 - 1$ GHz

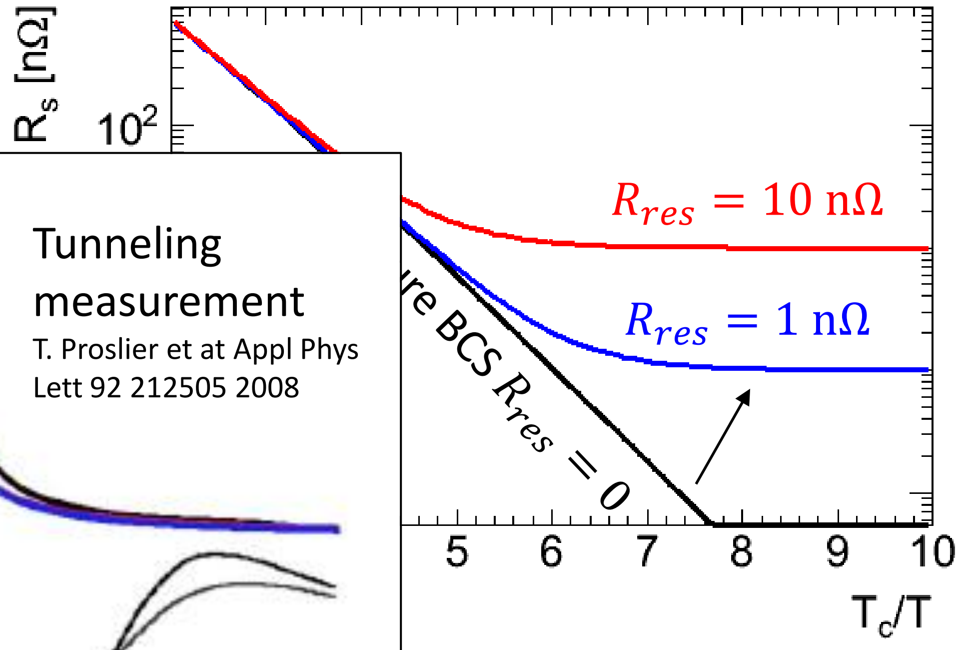
$\delta > 2$ µm

Smearing of Density of States and residual resistance

In reality $R_s \sim R_{\mathrm{BCS}}(T) + R_{\mathrm{res}}$







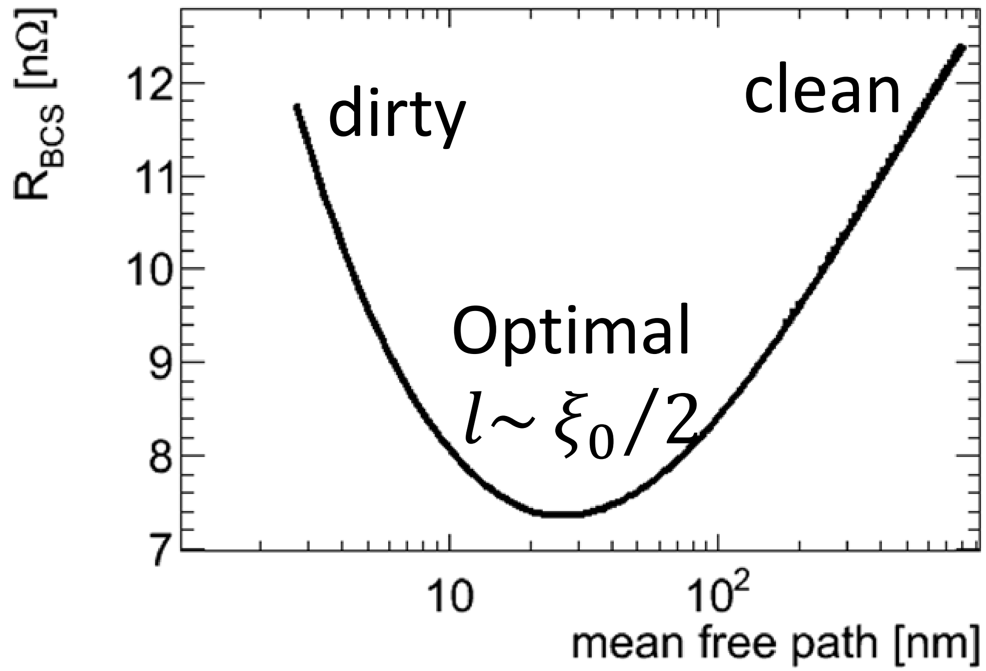
Remark: DoS smea ... tance

- Lossy oxides?
- Hydride?
- Grain boundaries??
- Influence of magnetic vortex

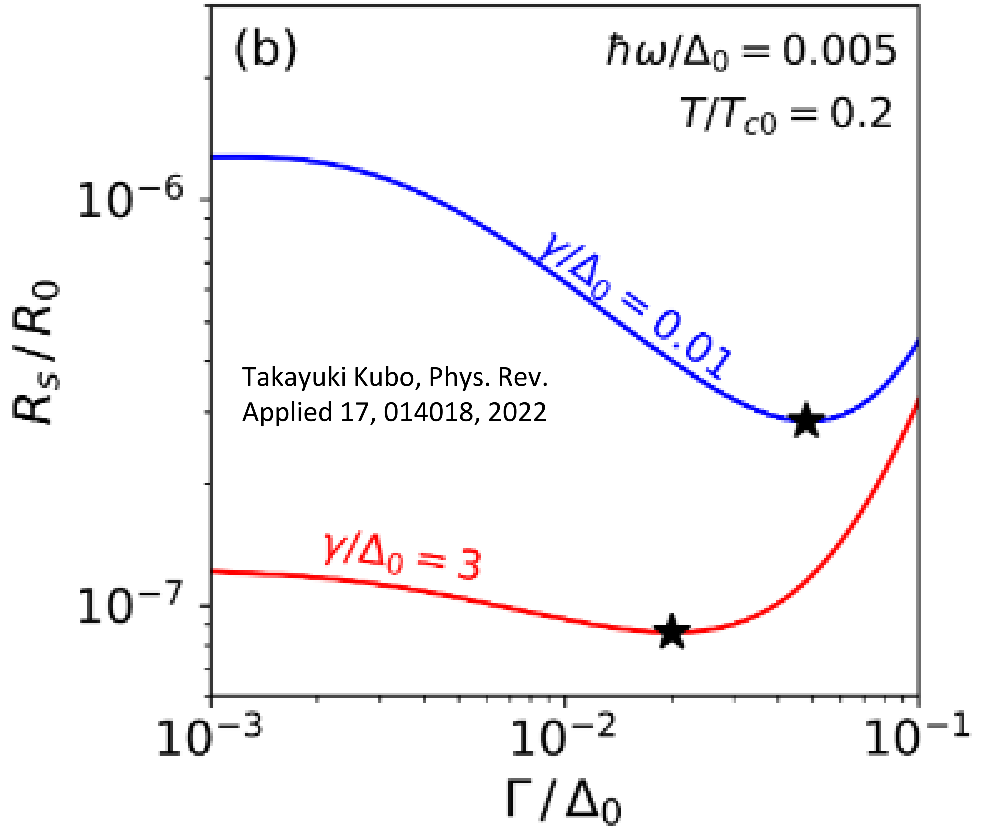
... is ultimate
limits on minimum R_s (highest Q)
after removing extrinsic effects

Minimum surface resistance from the theory

$R_{\mathrm{BCS}}(T)$ has a minimum as a function of impurity scattering (anomalous skin effect)



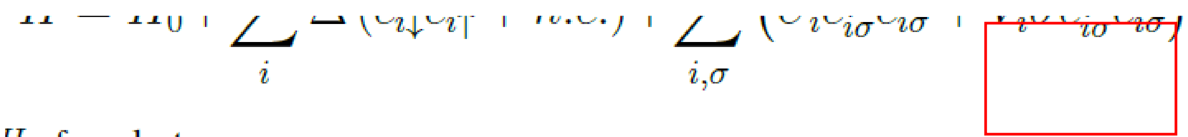
$R_{\mathrm{BCS}}(T) + R_{res}$ has a minimum as a function of Dynes parameter Γ with a given impurity scattering



<u>Fundamental question:</u>

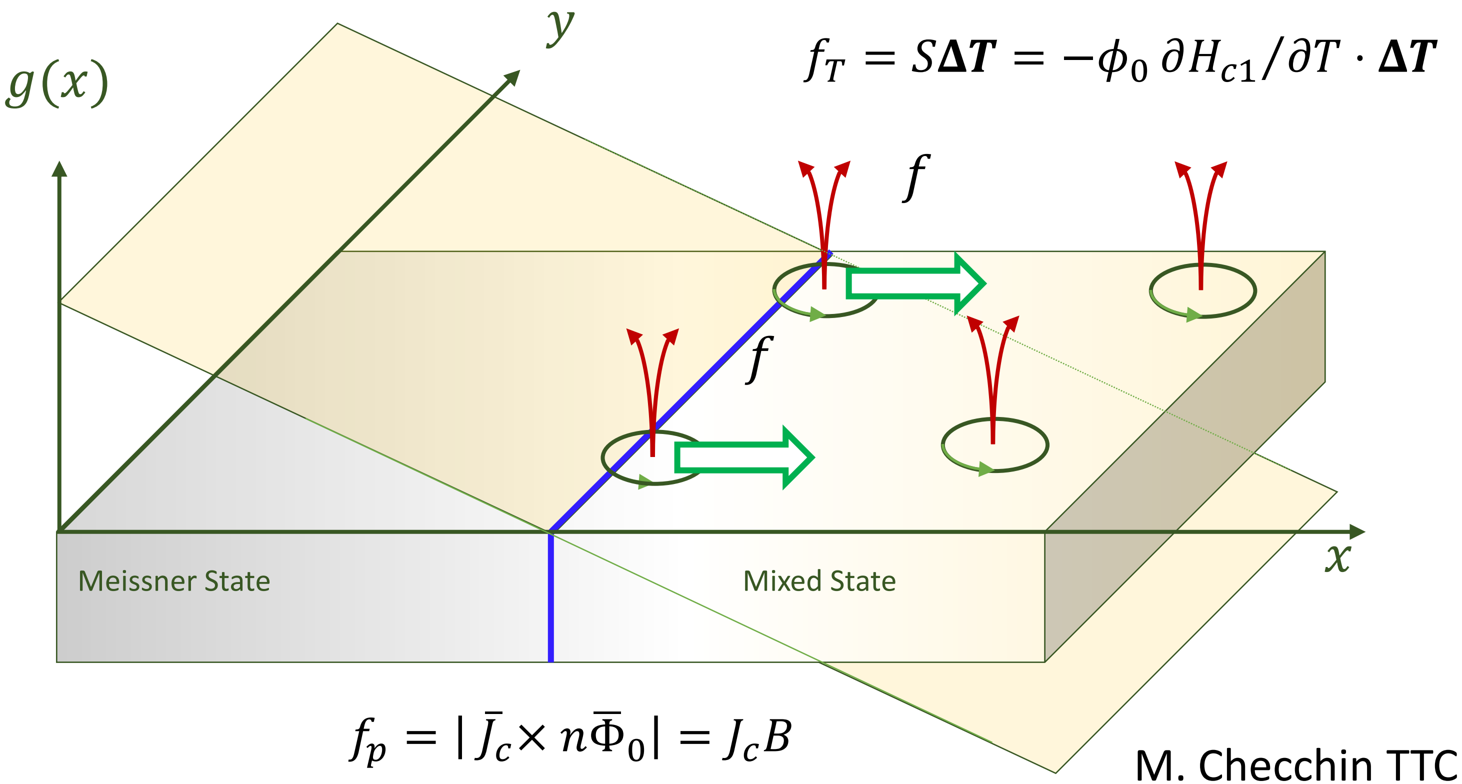
What causes the Dyne's DoS smearing?

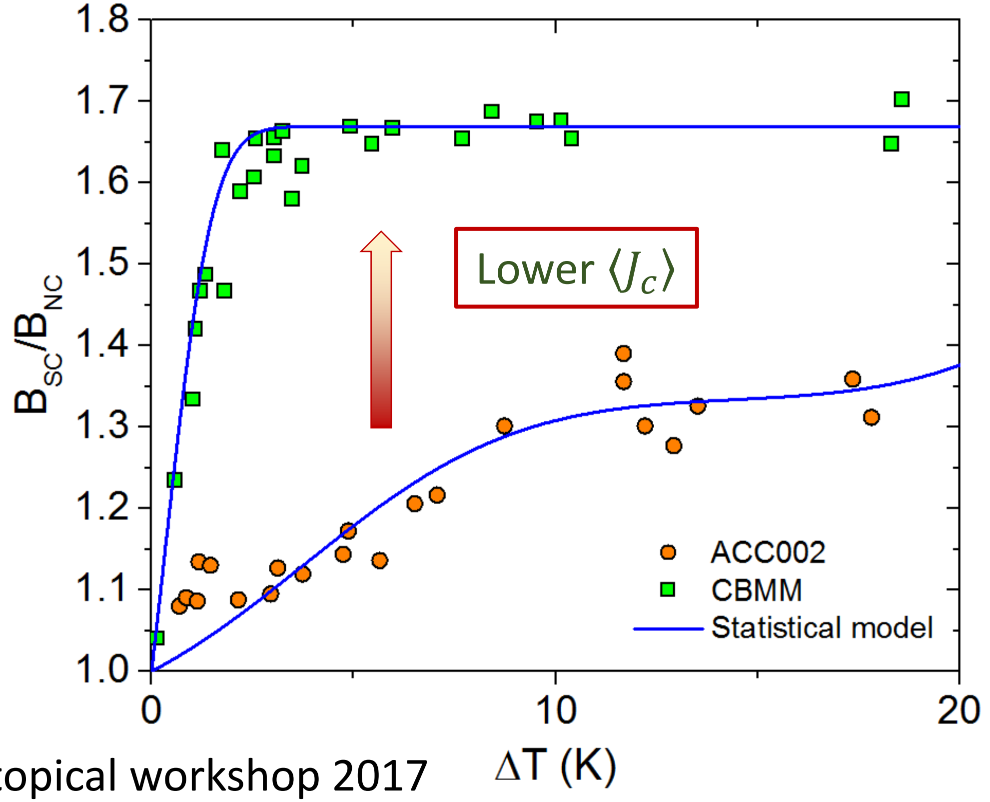
F. Herman: pair-breaking term (?) [PRB 96 014509]



H_0: free electrons.

Flux expulsion at the phase transition from NC to SC





M. Checchin TTC topical workshop 2017

- Balance between thermodynamic force f_T and pinning force f_p in the mixed state $[B_{c1}(T_c) < B_{ext} < B_{c2}(T_c)]$
- Higher thermal gradient → higher expulsion efficiency
- Statistical assumption in trapping efficiency → Material difference (J_c) reproduced
  → Cooling down with higher thermal gradient is a standard receipt in LCLS-II at SLAC

(ξ_{GL}, λ_{GL}) in Ginzburg Landau theory

BCS-Gor'kov → GL around T_c

$$\xi_{GL}(T) = 0.739[\xi_0^{-2} + 0.882(\xi_0 l)^{-1}]^{-1/2} R^{-1/2} \left(1 - \frac{T}{T_c}\right)^{-1/2}$$

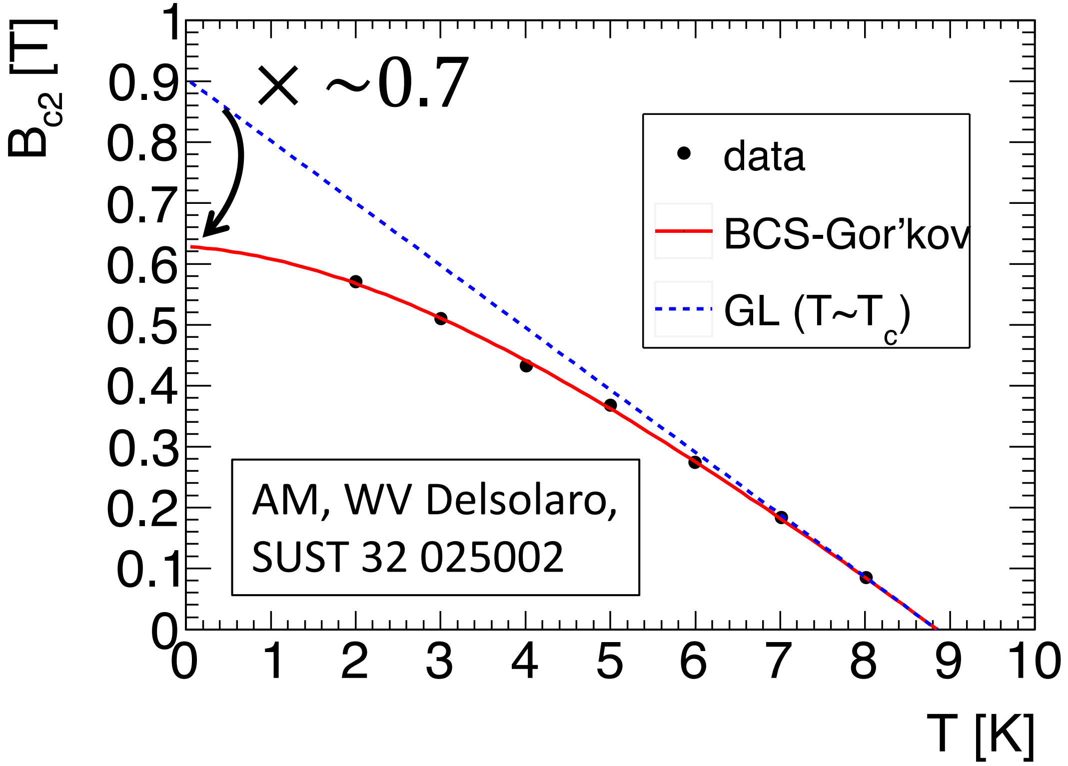
$$\lambda_{GL}(T) = 2^{-1/2}\lambda_L \left[1 + \frac{0.882\xi_0}{l}\right]^{1/2} R^{1/2} \left(1 - \frac{T}{T_c}\right)^{-1/2}$$

$$\kappa_{GL} \equiv \frac{\lambda_{GL}(T)}{\xi_{GL}(T)} = 0.957\frac{\lambda_L}{\xi_0}\left(1 + \frac{0.882\xi_0}{l}\right) R^{-1} \sim \frac{\lambda_L}{\xi_0}$$

$$1 = R(0) < R(l) < R(\infty) = 1.17$$

$$B_{c2}(T) = \frac{\Phi_0}{2\pi\xi_{GL}(T)^2}$$

→ $B_{c2}(T \to T_c) \propto 1 - (T/T_c)$



T. P. Orlando, E. J. McNiff, Jr., S. Foner, and M. R. Beasley, Phys. Rev. B 19, 454 (1979)

Superconductor is ***protected*** against ***parallel*** magnetic fields

<u>Solving London equation with the image force term</u>

(To fulfill boundary condition)

$$\nabla^2 H(x,z) - \frac{1}{\lambda^2} H(x,z) = -\frac{\phi_0}{\mu_0 \lambda^2}\left[\delta(x)\delta(z-z_0) - \delta(x)\delta(z+z_0)\right]$$

Results in two terms

1. External field term which attracts the parallel flux

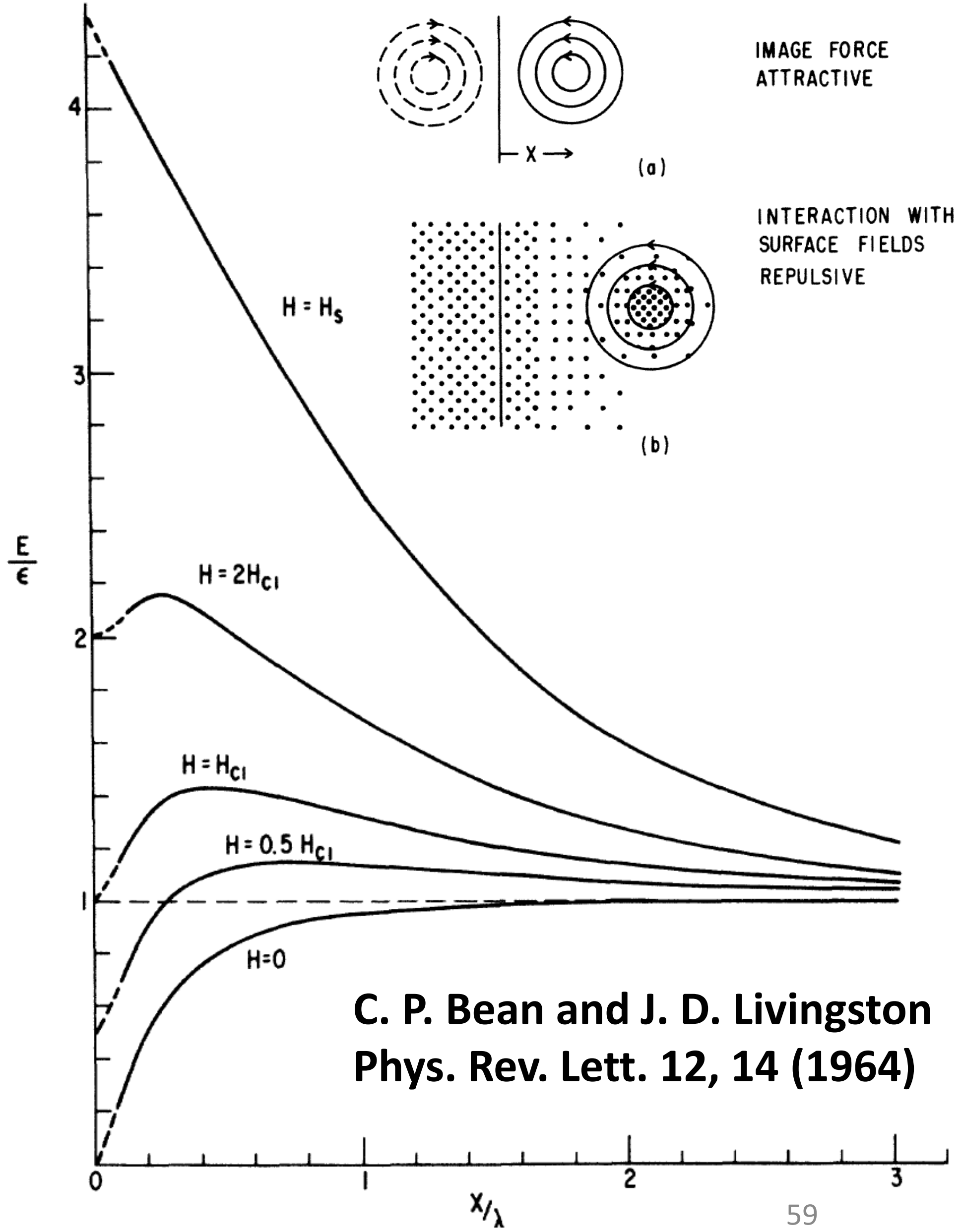
$$f_1 = \frac{\phi_0 H_0}{\lambda} \exp\left(-\frac{z_0}{\lambda}\right)$$

2. Image force term which expels the parallel flux

$$f_2(x) = \frac{\phi_0}{2\pi\mu_0\lambda^3} K_1\left(\frac{2z_0}{\lambda}\right)$$

(one particular solution using 2D Green function)

The 2nd term dominates even at $H > H_{c1}$ but to be defeated by the 1st term Above $H > H_s \sim \frac{\phi_0}{4\pi\xi\lambda} \sim \frac{H_c}{\sqrt{2}}$ the surface barrier disappears but this is still lower than superheating field H_{sh} estimated from GL theory



C. P. Bean and J. D. Livingston Phys. Rev. Lett. 12, 14 (1964)