Quantum Gravity

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CERN Summer Student Lectures
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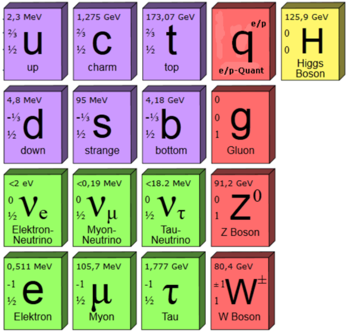
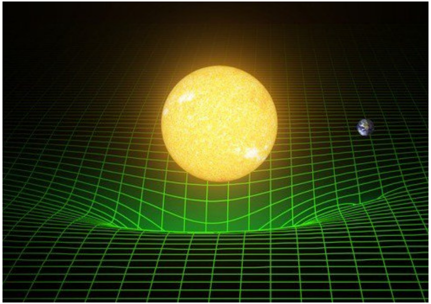
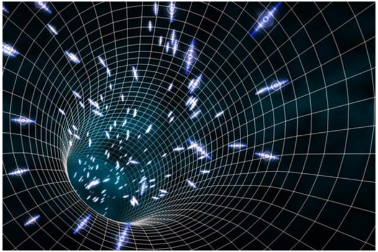
Outline

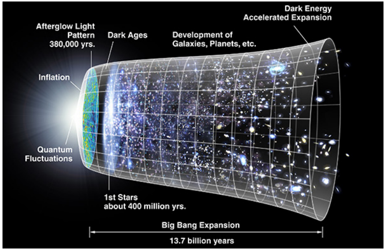
1. Introduction
2. Lessons from black holes
3. The holographic correspondence

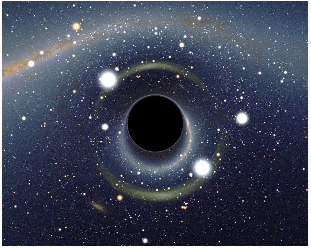
Quantum Mechanics + General Relativity= ?











Space-time at short scales, scattering at $E \gtrsim M_{planck} = 10^{19}$ GeV

Cosmology

Black holes

Recent developments: interesting connections with other fields (strongly coupled gauge theories, condensed matter, statistical mechanics, quantum information,...)

General Relativity

Dynamical variables: metric of space-time

$$g_{\mu\nu}(x)$$

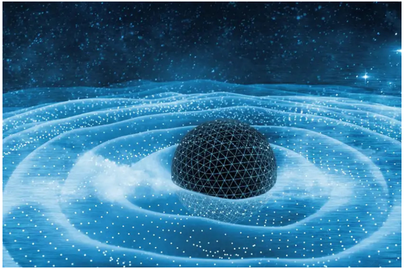
Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int \sqrt{g}(R - 2\Lambda) + [\text{matter}]$$

Quantum fluctuations of space-time



$$g_{\mu\nu}(x) \qquad \Rightarrow \qquad \hat{g}_{\mu\nu}(x)$$

Wavefunction for space-time metric

$$\Psi[g_{\mu\nu}(x)]$$

Path integral over metrics

$$\int [dg_{\mu\nu}] \; e^{\frac{i}{\hbar} S[g]}$$

Perturbative quantization

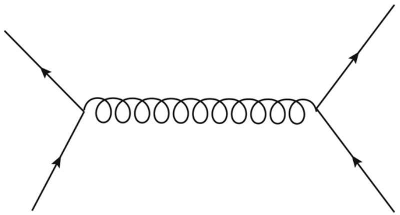
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h_{\mu\nu} \ll 1$$

Linearize Einstein equations$\Rightarrow$ wave equation for $h_{\mu\nu}$ $\Rightarrow$ gravitational waves

After quantization:

gravitational waves $\Rightarrow$ massless spin 2 particles (gravitons)

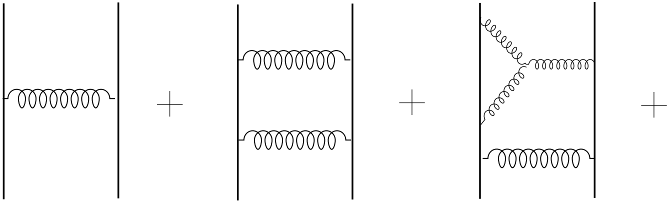


Graviton propagator

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle = \frac{K_{\mu\nu,\rho\sigma}(p)}{p^2}$$

Gravitational force mediated by graviton exchange

Loops and non-renormalizability

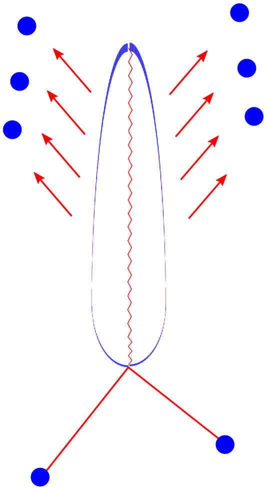


Integrals over loop momenta $\Rightarrow$ UV divergences

As we go to higher loops, we encounted new types of divergences $\Rightarrow$ need an infinite number of counterterms $\Rightarrow$ standard QFT techniques of *renormalization* do not work

Quantum Gravity is (perturbatively) non-renormalizable

High energy behavior



Higher loop diagrams become important at higher energies

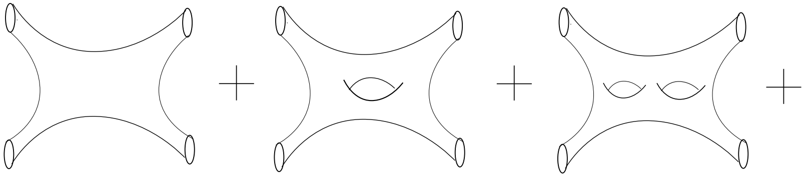
Perturbative expansion breaks down at $E \sim M_{planck}$

Scattering of particles at $E \gtrsim M_{planck} \Rightarrow$ black hole formation

Black hole will eventually evaporate into particles

Can we compute S-matrix for this process?

String Theory



Particles are 1-dimensional extended objects (strings)

At low energies looks like General Relativity + matter

Unique theory, no adjustable parameters

Extended nature of string $\Rightarrow$ effective UV cutoff$\Rightarrow$ all diagrams finite

Until now, the only UV finite theory of perturbatively interacting gravitons

However, string theory not well understood beyond perturbation theory

UV vs IR

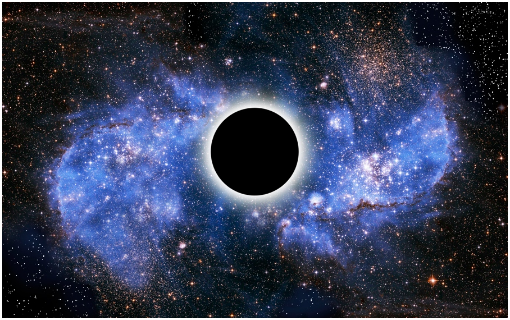
So far we discussed difficulties with UV behavior of Quantum Gravity: quantum effects become important in the UV and impossible to control by conventional techniques.

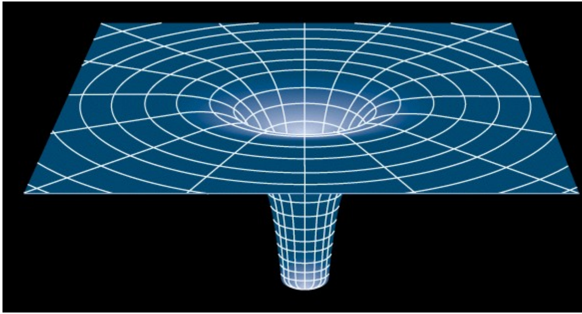
The standard approach is to expect some new physics that will kick-in at the UV and will cure these problems.

However recent developments suggest a more dramatic and interesting idea: a modification of the fundamental nature of space-time and gravity, not only in UV but even at macroscopic scales

Space-time and gravity may be *emergent* rather than fundamental notions. Thus trying to “quantize General Relativity” may not be the correct starting point.

Classical black holes

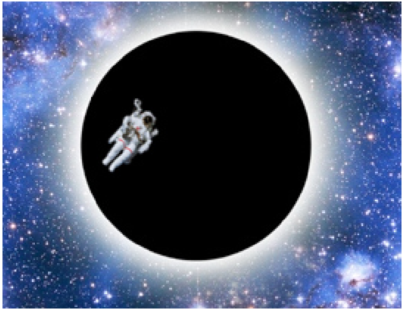




$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Horizon: $r = 2GM$
Singularity: $r = 0$

An infalling observer

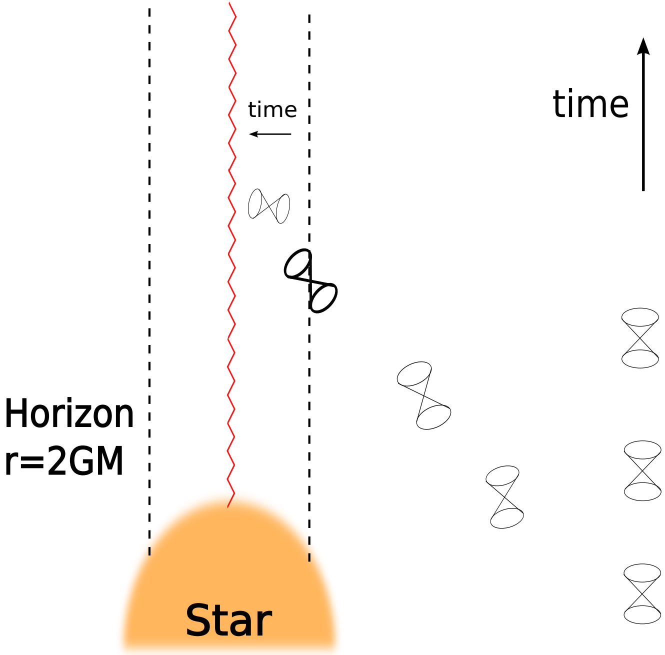


The curvature of spacetime near the black hole horizon is

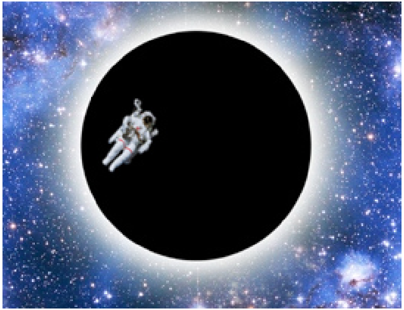
$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \frac{1}{(GM)^4}$$

For a large black hole this curvature is very low

Equivalence Principle: A freely falling observer will not notice anything when crossing the horizon of a big black hole

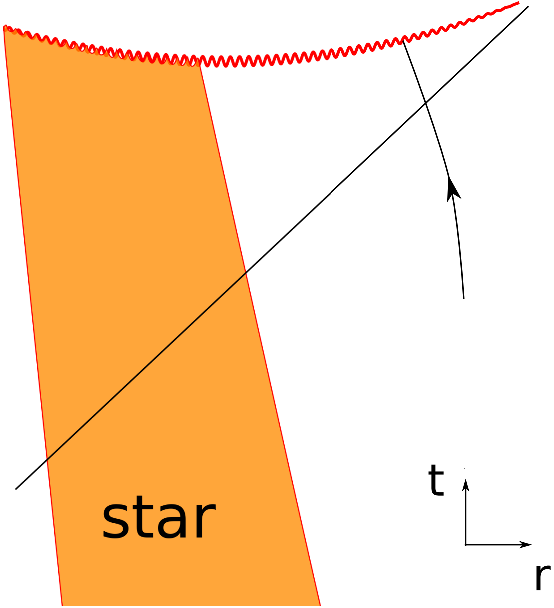

time
time
Horizon
r=2GM
Star

Time dilation



Asymptotic observer: infalling particles “slow down” near horizon, it takes ∞ amount of asymptotic time to cross horizon

Infalling observer: Reaches horizon in finite amount of *proper time*, and then falls into singularity after $\tau \sim GM$.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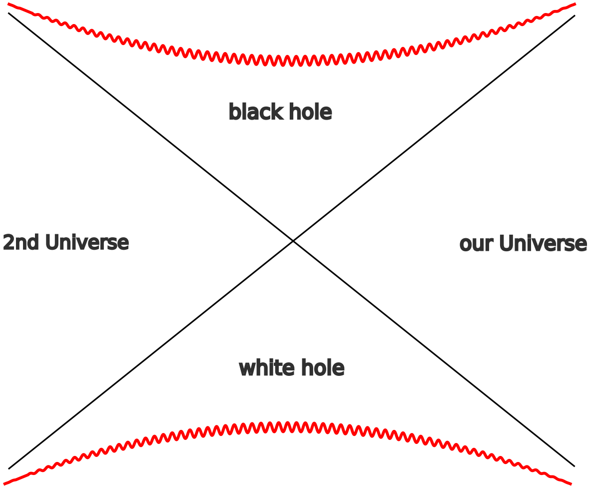
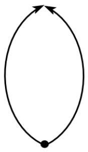

black hole
2nd Universe
our Universe
white hole

Quantum vacuum fluctuations

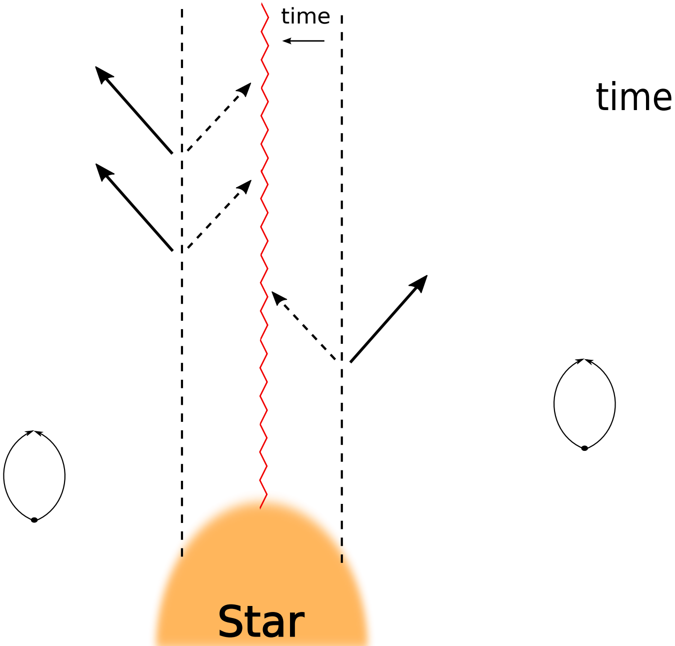


Virtual Particles

Uncertainty principle for the quantum fields

$$[\phi(0,x),\pi(0,x')] = i\hbar\delta(x-x')$$

Quantum vacuum fluctuations of the field around $\phi = 0$. We can think of them in terms of pairs of virtual particles which spontaneously materialize out of the vacuum and quickly annihilate.


time
time
Star

Hawking radiation

Hawking (1974):Black holes emit (almost) black-body radiation at temperature

$$T = \frac{1}{8\pi GM}$$

For solar mass BH, $T \approx 60$ nanoKelvin

Negative specific heat

Black hole entropy

From

$$T = \frac{1}{8\pi GM} \qquad , \qquad \left(\frac{\partial S}{\partial E}\right) = \frac{1}{T}$$

we find

$$S = 4\pi GM^2$$

or using

$$R = 2GM$$

we get

$$S = \frac{A}{4G}$$

Bekenstein-Hawking entropy. Universality of area-entropy.

Black Hole entropy

$$S = \frac{A}{4G}$$

$$S_{\rm solarBH} \approx 10^{77} \qquad \text{while} \qquad S_{\rm sun} \approx 10^{57}$$

In Statistical Mechanics

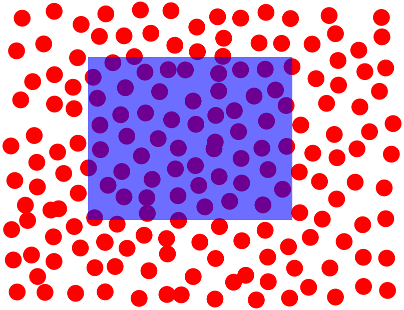
$$S = \log N$$

Predicted number of states for solar-mass BH: $N = e^{10^{77}}$,

In general relativity we have only 1 state (no hair theorem)

What are the "microstates" of a black hole?

Volume vs Area



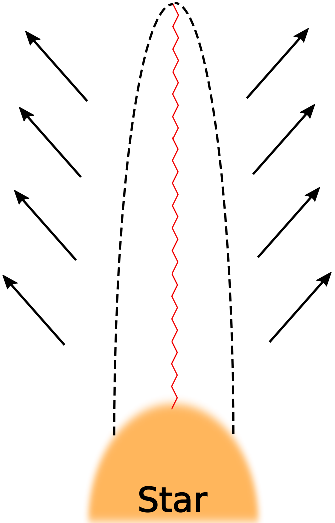
In most physical systems entropy is *extensive*,i.e. proportional to the volume of the system/region. Degrees of freedom are spread out in space.

Black hole area law suggests that degrees of freedom of quantum gravity are spread out on the horizon, hence in one lower dimension

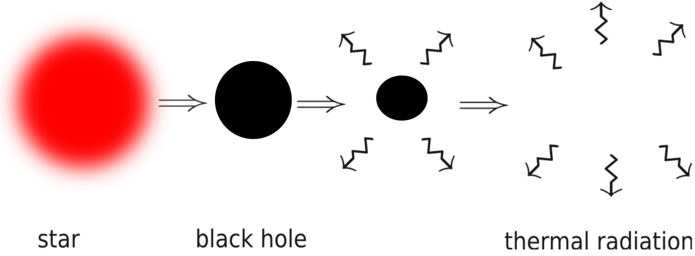
Hawking radiation carries away energy

Where is this energy coming from?

Black hole evaporation



Black Hole Information paradox



$$|\Psi\rangle \quad \rightarrow \quad \rho = \frac{e^{-\frac{1}{T}H}}{Z}$$

Black Hole Information paradox

If final radiation is thermal, where is the information of the star which formed the black hole?

In Quantum Mechanics time evolution is Unitary

$$i\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle$$

or

$$|\Psi\rangle_{\rm final} = e^{-iHt}|\Psi\rangle_{\rm initial}$$

A given $|\Psi\rangle_{\rm final}$ corresponds to a unique $|\Psi\rangle_{\rm initial}$

A pure state $|\Psi\rangle_{\rm initial}$ cannot evolve into a mixed state $\rho = \frac{e^{-\beta H}}{Z}$

We have identified a fundamental conflict between General Relativity and Quantum Mechanics.

Should we give up Unitarity in Quantum Mechanics?

Do quantum effects destroy smoothness of horizon?

Corrections to Hawking's computation?

Imagine burning a piece of coal. Radiation *appears* to be thermal

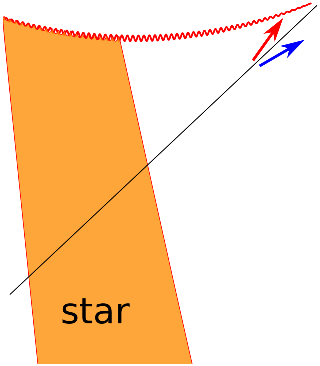
Small correlations between photons (of size e^{-S})

Accurate measurement of correlations $\Rightarrow$ full information of initial state

No information loss

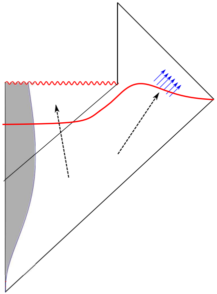
Could this happen during black hole evaporation?

Burning vs black hole evaporation



The Hawking particles are produced near the horizon (empty space). How do they encode and carry away the information of the state?

Quantum Cloning



$$|\Psi\rangle \nrightarrow |\Psi\rangle \otimes |\Psi\rangle$$

(no-cloning theorem)

Additional refinements $\Rightarrow$ Firewall paradox

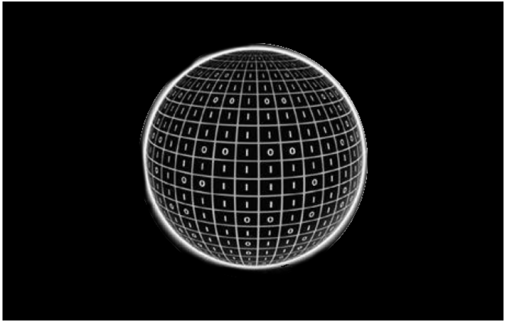
Black Hole Information paradox

Fundamental conflict between General Relativity and Quantum Mechanics.

Seems to be a sharp paradox within low-energy effective field theory, not sensitive to details of UV completion

Attempts to resolve it have led to insights towards identifying fundamental principles of Quantum Gravity.

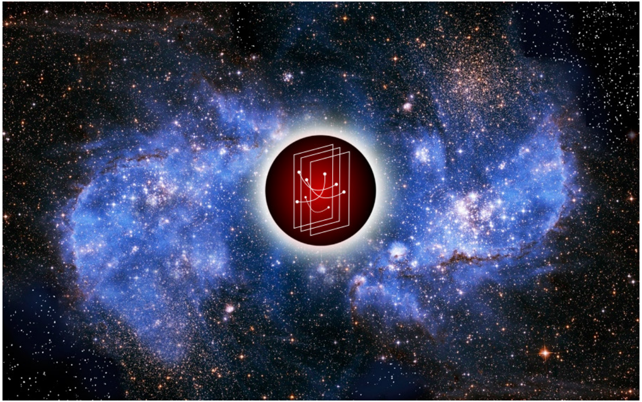
The Holographic Principle



$$S = \frac{A}{4G}$$

The fundamental degrees of freedom of quantum gravity leave on the "boundary" of space. The space-time we experience is a holographic representation.

Black holes in string theory



In the 90's it was discovered that string theory contains objects called "D-branes". They provide a microscopic description of (supersymmetric) black holes.

Representation of black hole microstates, entropy counting.

The AdS/CFT correspondence

Mathematically precise realization of the idea of holography for Quantum Gravity in a universe with negative cosmological constant (anti-de Sitter space)

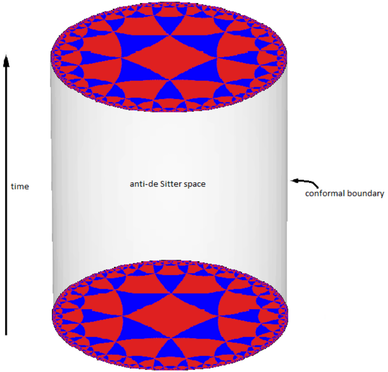
The fundamental degrees of freedom of gravity live on the boundary of space

In many examples, we know what these degrees of freedom are, and what are their dynamical laws

To this day it is the only mathematically consistent and complete framework describing Quantum Gravity. For example, in principle it can be used to compute the Black Hole S-matrix discussed earlier.

The AdS/CFT correspondence

A $d+1$-dimensional theory of gravity in a Universe with negative cosmological constant, is equivalent to a d-dimensional large N $SU(N)$ gauge theory without gravity



Questions about quantum gravity can be translated in the QFT

QFT is strongly coupled

Emergence of extra AdS-dimension from QFT remains mysterious (related to RG-scale)

The AdS/CFT correspondence

Space-time and gravity are emergent from the collective behavior of the degrees of freedom of the lower dimensional gauge theory.

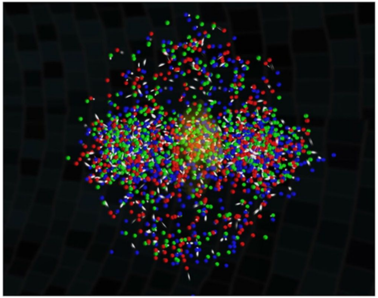
Locality is emergent and thus approximate

The metric tensor/graviton and other particles in anti-de Sitter space are “composite” objects (glueballs) in the dual gauge theory.

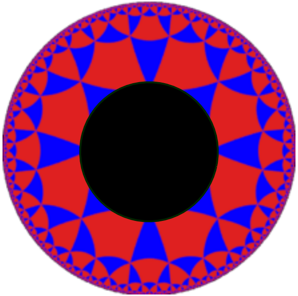
This suggests that the idea of starting with a path integral over metrics and quantizing may not be correct.

Black Holes in AdS/CFT

Quark gluon plasma



Black Hole in AdS



Understanding of black hole entropy

Dynamical properties of QGP (viscosity, etc.)

AdS/CFT and the information paradox

AdS/CFT convincingly settles that information is not destroyed during black hole evaporation

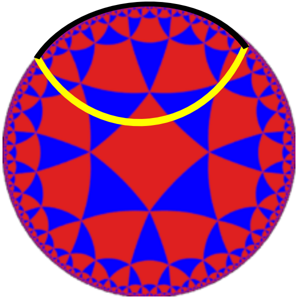
Calculations in the dual gauge theory provide hints about how information *appears* to be lost into the black hole for a long time but eventually emerges again encoded in the Hawking radiation

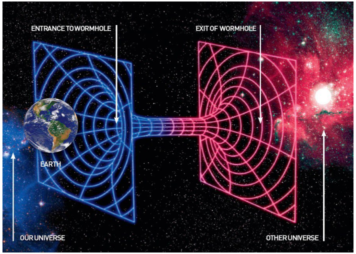
However, translating these hints directly into gravitational language and resolving the original formulation of the paradox remains work in progress

Understanding the region behind the horizon of a black hole using AdS/CFT remains an important open challenge

Can we use the dual gauge theory to study the black hole singularity?

Entanglement, quantum mechanics and spacetime





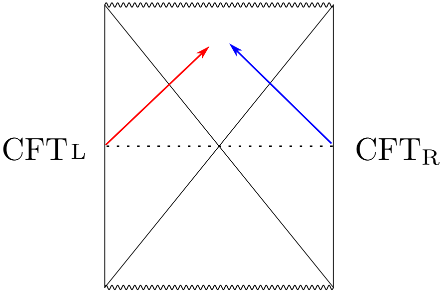
$$S_{EE} = -\mathrm{Tr}[\rho \log \rho]$$

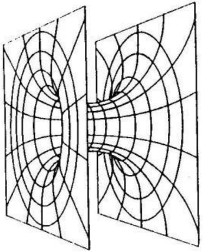
Ryu-Takayanagi proposal $\Rightarrow$ Entanglement determines geometry of spacetime

Einstein equations from dynamics of entanglement

Quantum Error Correction

Entanglement and wormholes





$$H = H_L + H_R$$

$$|\mathrm{TFD}\rangle = \sum_E \frac{e^{-\beta E/2}}{\sqrt{Z}} |E\rangle_L \otimes |E\rangle_R$$

Some open questions

Gravity and space-time may be emergent from underlying quantum degrees of freedom living in lower dimensions.

What are the general principles of this phenomenon, what are the conditions for this to happen? Does Quantum Information Theory provide us with a useful language to describe it?

What about other space-times (flat space, de Sitter, cosmology)?

What is the fate of an observer falling into a black hole? Is the horizon smooth at the quantum level?

Can time be emergent?