Fixed Points in 5d Gauge Theories

Lorenzo Di Pietro
(Università di Trieste)

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Asymptotic Safety Meets Particle Physics & Friends

work with F. De Cesare, M. Serone
2107.00342, 2212.11848

An **open question**: is there any interacting CFT in 5d (or above) without supersymmetry?

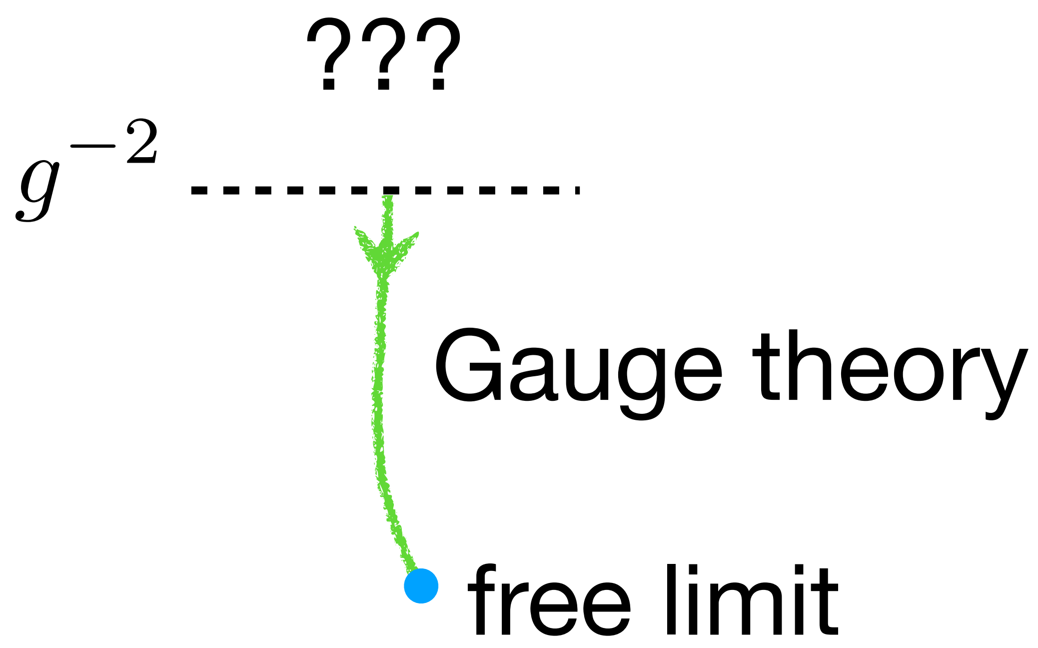
Deformations of free theory are all irrelevant.

SUSY example based on string theory construction.
UV completion of supersymmetric gauge theories.

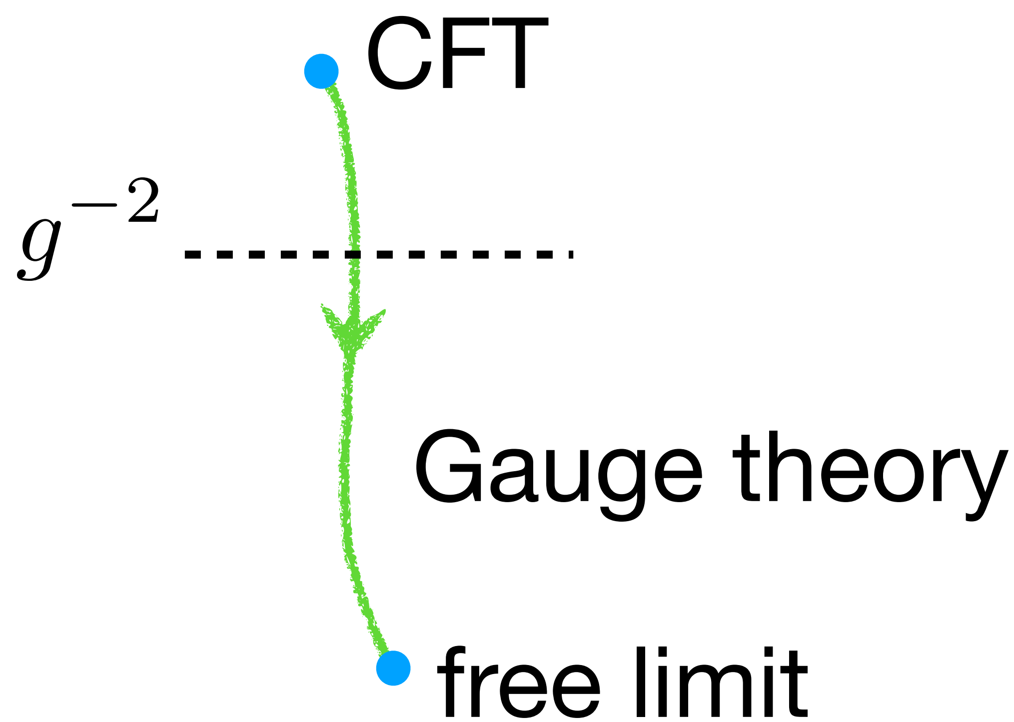
A promising setup: **non-abelian gauge theories**

$$\int d^5x \, \mathrm{tr} \frac{1}{4g^2} \left[F_{\mu\nu} F^{\mu\nu} \right] \text{ (+ matter)}$$

$$[g^2] = E^{-1}$$



Interacting UV fixed point?



CFT with a relevant deformation that corresponds to g^{-2} and flows to this EFT.

More restrictive: g^{-2} only relevant deformation.

Phase diagram:

Methods to study this problem

- Epsilon-expansion
- Lattice

  [Florio, Lopes, Matos, Penedones]
- Deformations of the SCFT

  [Benetti-Genolini, Honda, Kim, Tong, Vafa]
- Bootstrap

  [Li, Poland]
- Functional renormalization group

  [Gies]

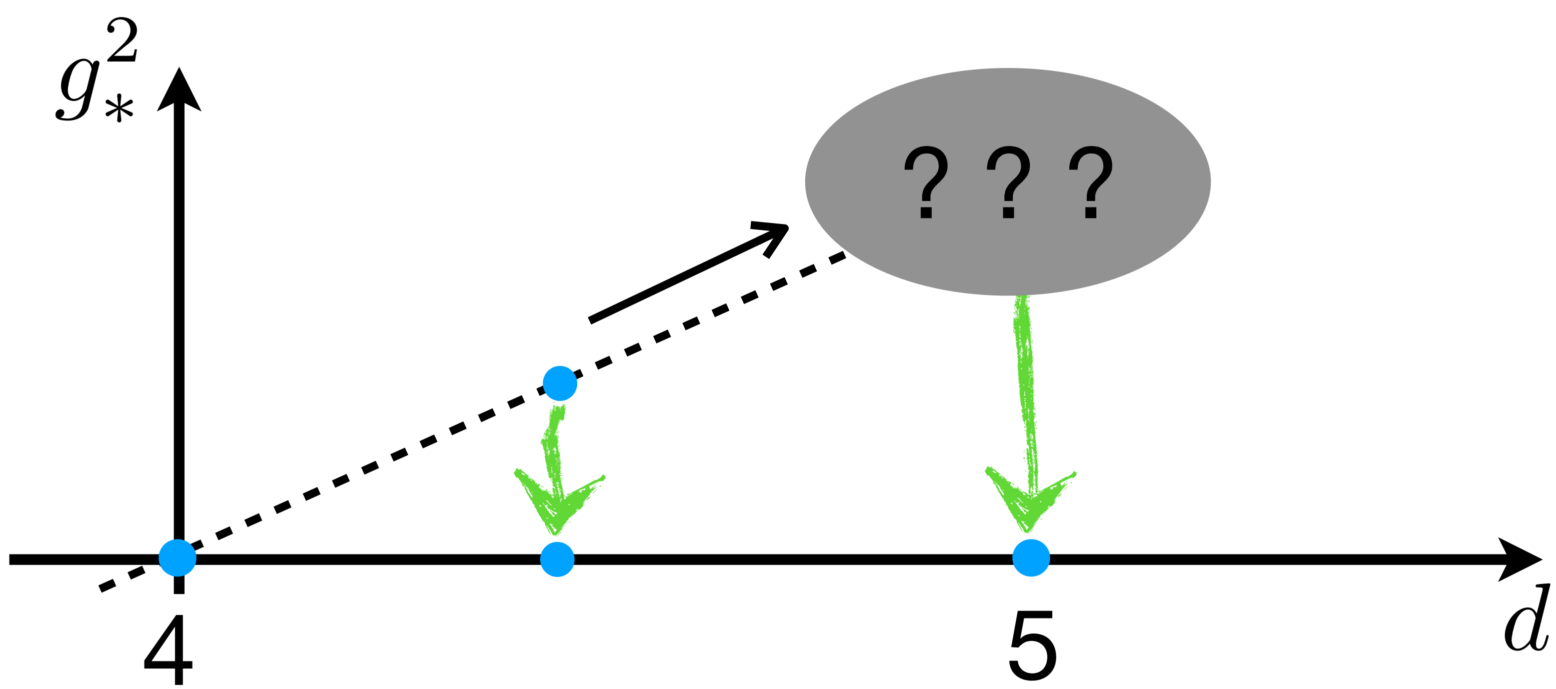
Epsilon-expansion

Dimensional continuation from 4d to d=4+2ϵ

$$\beta_g = \epsilon g + b_0\, g^3 + \mathcal{O}(g^5)\ , \quad b_0 < 0$$

weakly-coupled UV fixed point:

for $\epsilon \ll 1$: $g_*^2 \sim \epsilon$ [Peskin] (1980)



To reach 5d: resummation and extrapolation.

Most updated analysis previous to our work:

[Morris] (2004) used 4 loop result and "optimal truncation" to extrapolate.

Evidence for existence of fixed point up to 5d for pure YM.

In the meantime: 5 loop beta function and mass anomalous dimension for non-abelian gauge theory with fermions.

[Baykov, Chetyrkin, Kuhn, 1606.08659]
[Herzog et al, 1701.01404] [Luthe et al, 1709.07718]
[Chetyrkin et al, 1709.08541]

- Why reconsider it now?

- Update with 5 loops and Borel-Padé resummation

- New results from lattice

- New proposed construction from SCFT

- Can we trust this method? Should we worry about UV rather than IR fixed point?

- Analogy: Gross-Neveu and NLSM in $d = 2 + \epsilon$

- Applications?

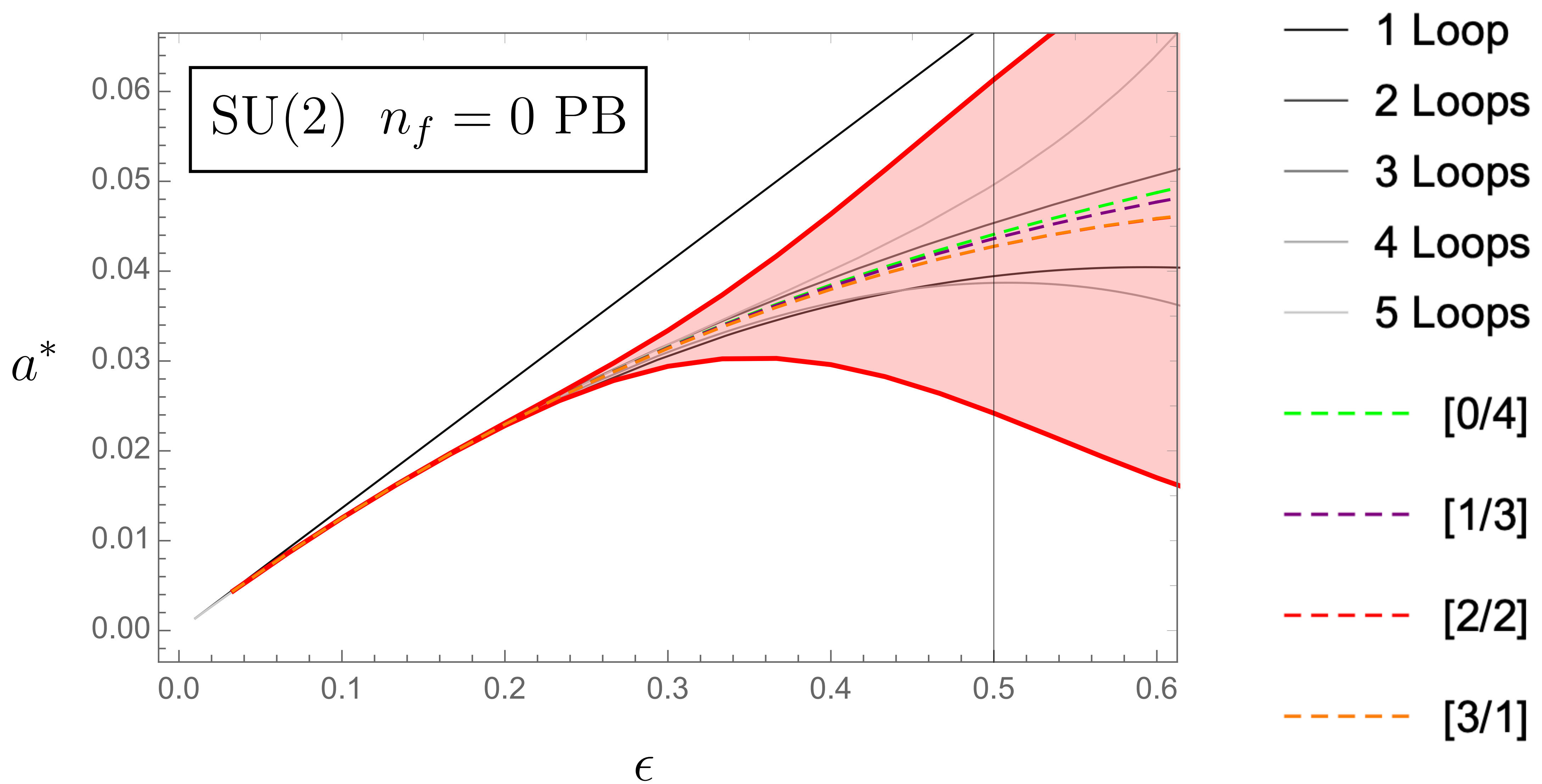
- A case study for asymptotic safety: simpler than gravity, cross-checks between functional RG and other methods

- Extra-dimensional models: cutoff can be higher than g^{-2} if CFT exists

- Knowledge of the phase diagrams of the higher dimensional theory can lead to constraints on lower dimensional theories, e.g. living on domain walls, or by dimensional reduction
[Gaiotto, Komargodski, Seiberg]

We considered $SU(n_c)$ with n_f fundamental Dirac fermions. **Results**:

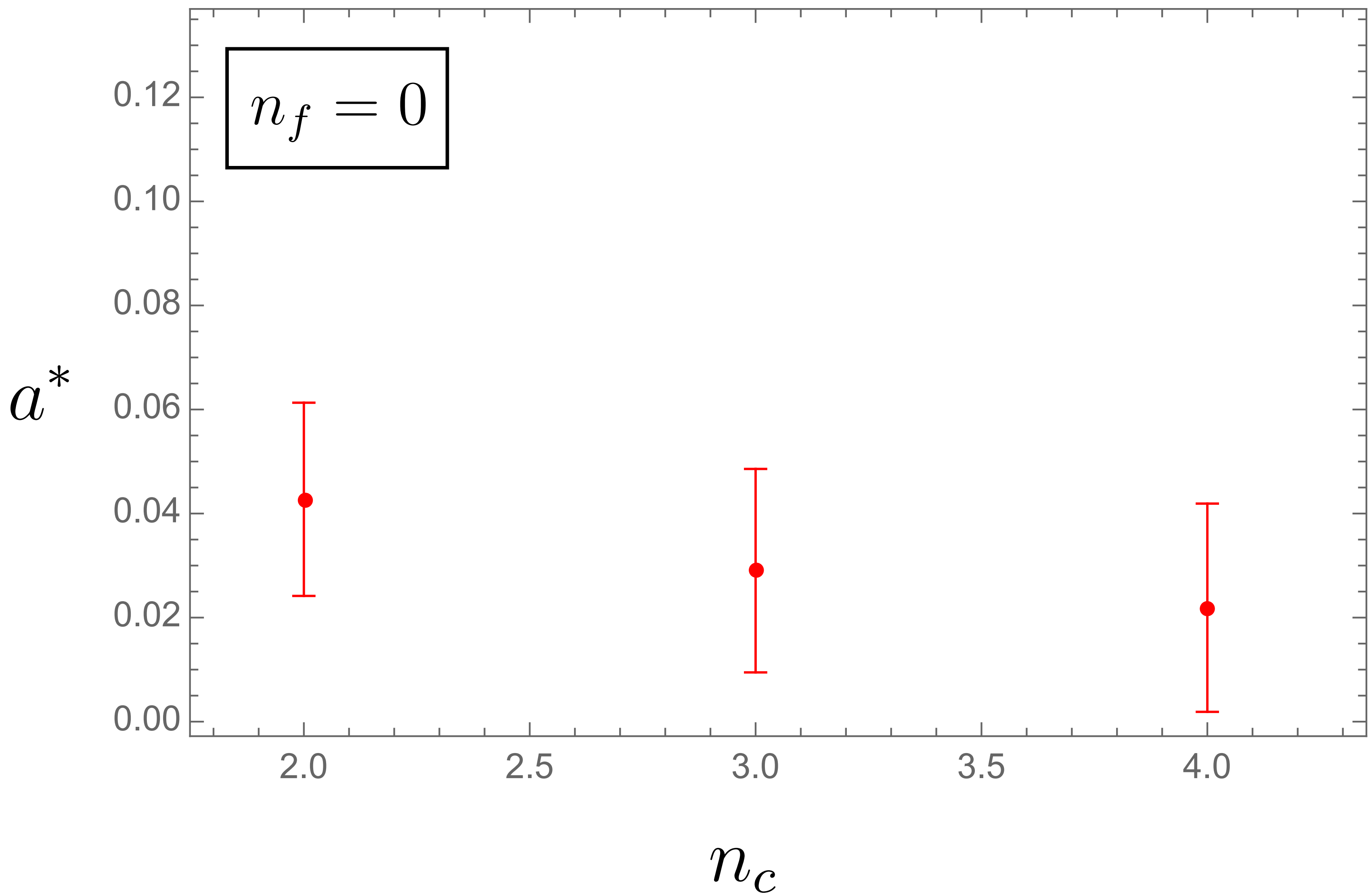
- Evidence for fixed point for $n_c = 2\,,\, n_f = 0$



Error bars are estimates, not rigorous.

We considered $SU(n_c)$ with n_f fundamental Dirac fermions. **Results**:

- Similar evidence for $n_f = 0$, n_c up to 4 and $n_c = 2$, n_f up to 4



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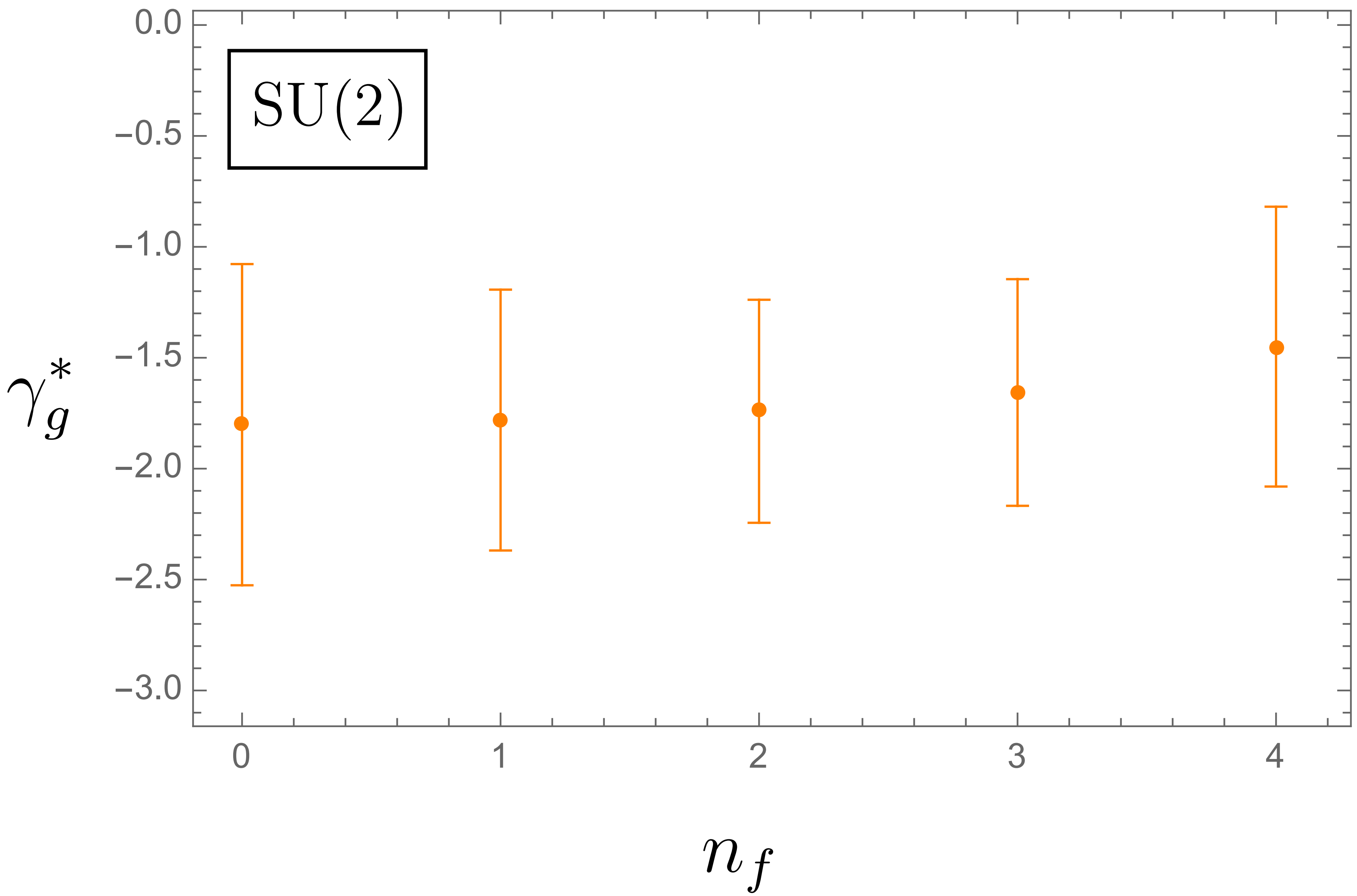
We considered $SU(n_c)$ with n_f fundamental Dirac fermions. **Results**:

- Scaling dimensions $\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}]$



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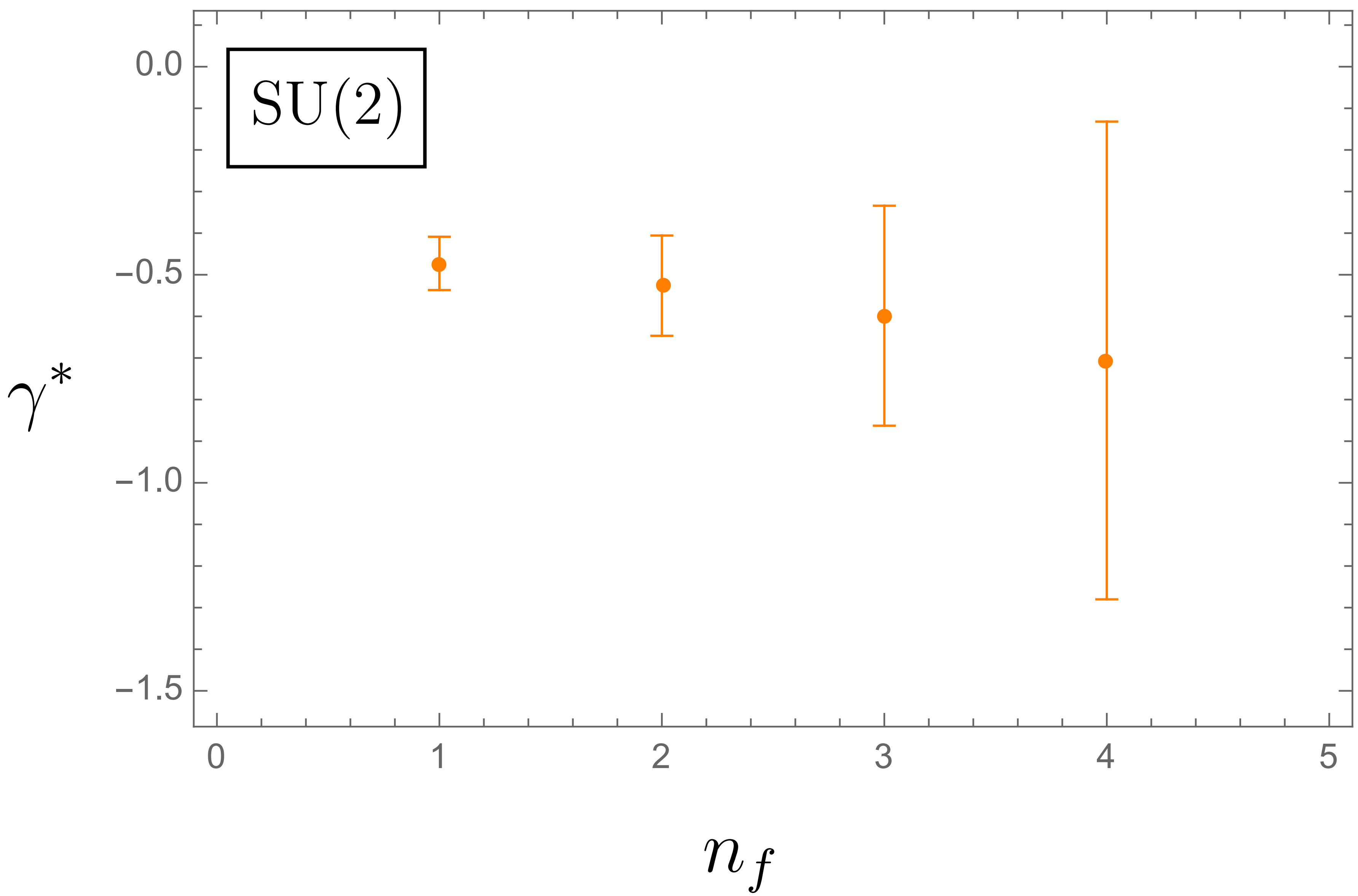

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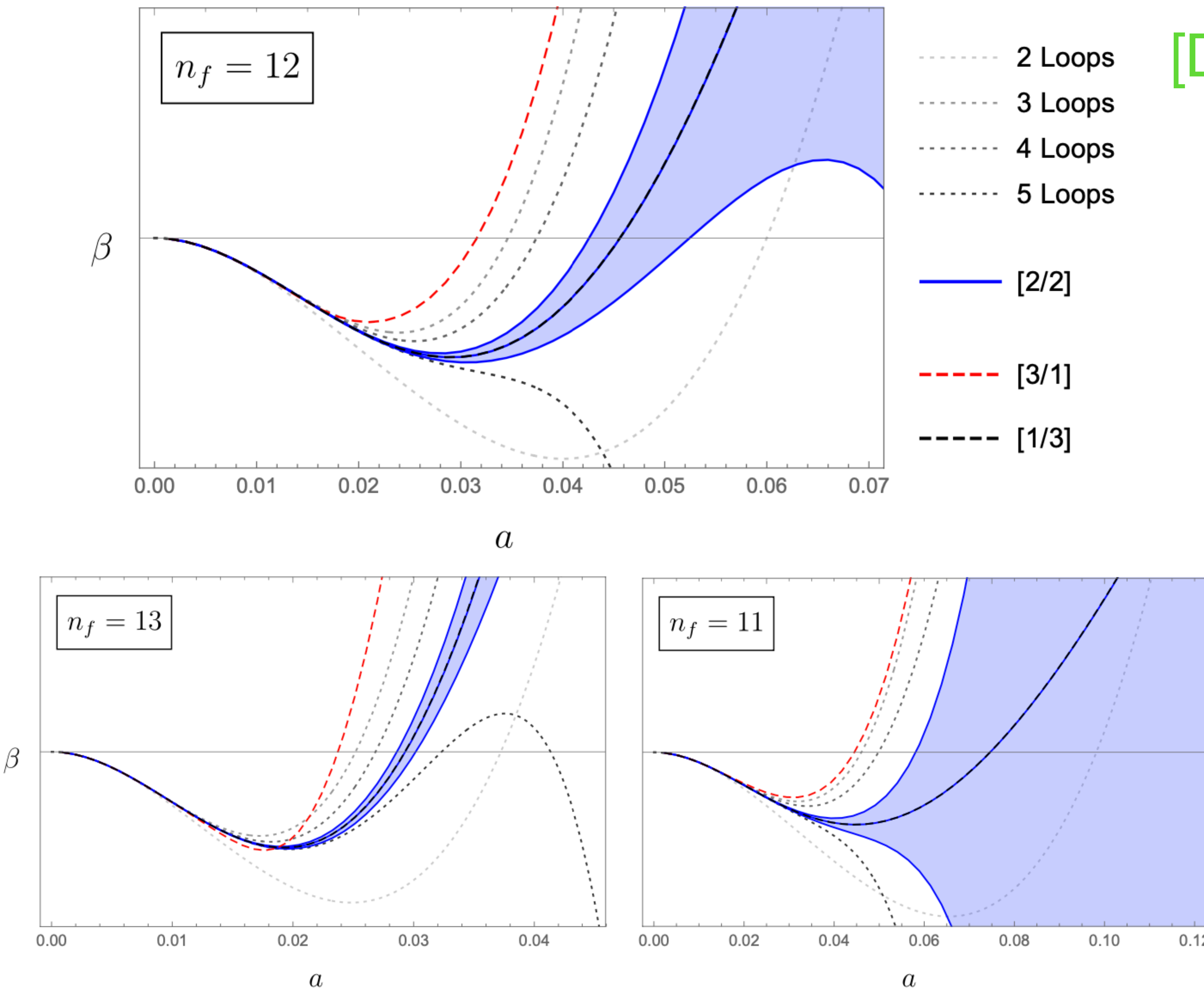
Similar approach for conformal window of QCD:

[DP, Serone]



Evidence for conformal window down to $n_f = 12$

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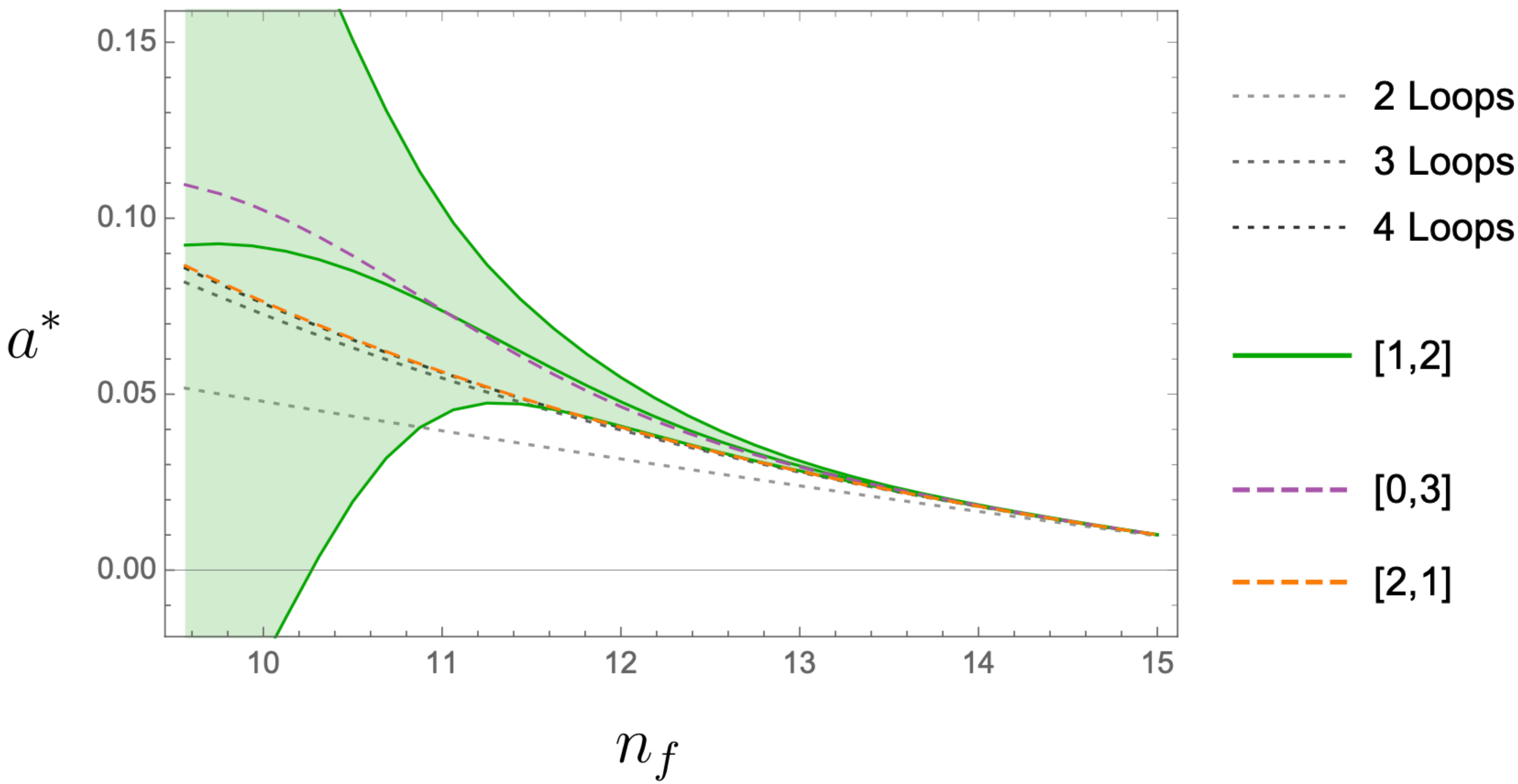


Evidence for conformal window down to $n_f = 12$

Some remarks about resummation of perturbation theory:

$$f(\lambda) \sim \sum_{n=0}^{\infty} c_n \lambda^n$$ is <u>**divergent asymptotic**</u>, i.e.

$$f(\lambda) - \sum_{n=0}^{N} c_n \lambda^n = \mathcal{O}(\lambda^{N+1})$$ however the series diverges.

$f(\lambda)$ non analytic in $\lambda = 0$.

Typical growth of coefficients: $c_n \sim n! a^n$ for large n

<u>**Optimal truncation:**</u> $$N \approx \frac{1}{|a|\lambda}$$

Minimizes the error: $\sim e^{\frac{1}{|a|\lambda}}$ adding more terms the approximation gets worse

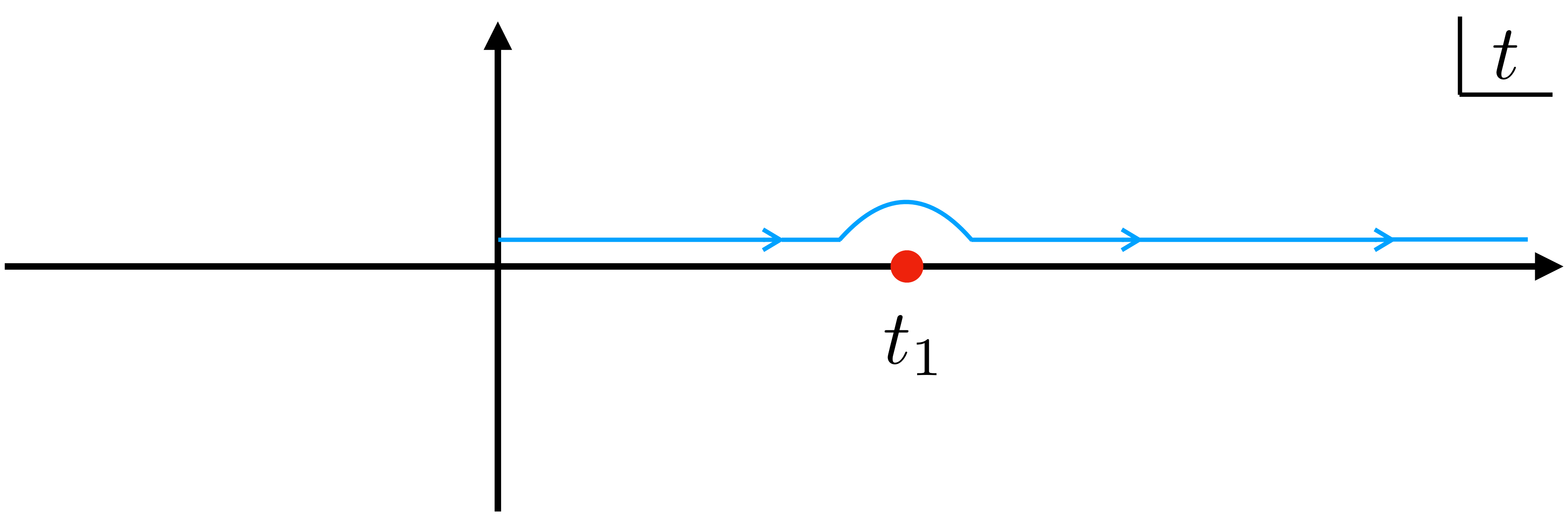
Our resummation technique: **Borel-Padé**

Borel resummation:

$$\mathcal{B}f(t) = \sum_{n=0}^{\infty} \frac{c_n}{n!} t^n$$

$$f_{\mathcal{B}}(\lambda) = \int_0^{\infty} dt\, e^{-t} \mathcal{B}f(t\lambda)$$

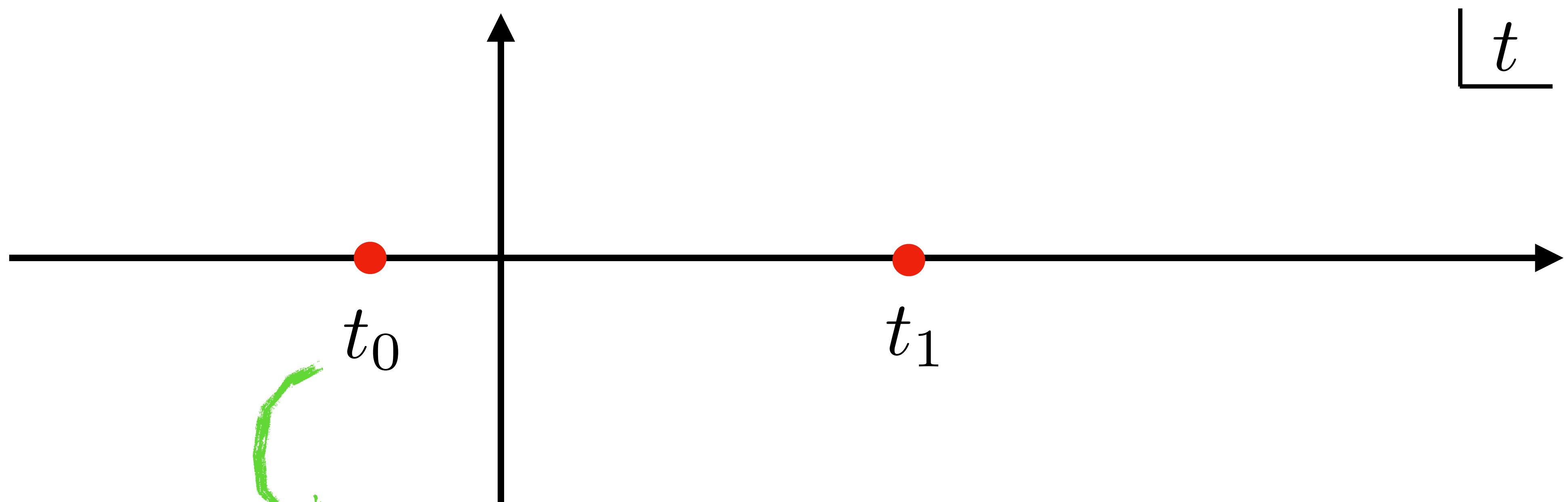
Singularities expected: Instantons and Renormalons.



ambiguity signals that we are missing $\propto e^{-t_1/\lambda}$

Then why Borel resummation for something not Borel resummable?

Because the error is anyway **better than optimal truncation**



large-order behavior controlled by the singularity **closest to the origin**

$$t_0 = 1/a \Rightarrow c_n \sim n! a^n$$

Therefore: $$e^{-|t_0|/\lambda} > e^{-t_1/\lambda}$$

So far, we imagined to know the full asymptotic series.

In practice we have a finite number of terms.

We use **Padé approximants:**

$$[\mathcal{B}f(t)]_N = \sum_{n=0}^{N} \frac{c_n}{n!} t^n$$

$$\Rightarrow \quad [\mathcal{B}f(t)]_{[m,k]} = \frac{\sum_{n=0}^{m} a_n t^n}{1 + \sum_{n=1}^{k} b_n t^n} \; , \; m + k = N$$

Estimate of the error:

- Convergence (1): variation of arbitrary parameter in the Borel transform

$$\mathcal{B}_b f(t) = \sum_{n=0}^{\infty} \frac{c_n}{\Gamma(n+b+1)} t^n \;,\; f_{\mathcal{B}_b}(\lambda) = \int_0^{\infty} dt\, t^b\, e^{-t}\, \mathcal{B}_b f(t\lambda)$$

- Convergence (2): diff. with lower order Padé

- Non-perturbative corrections: $\propto e^{-\frac{2}{\beta_0 a}}$

We sum these contributions.

Summary so far:

Evidence from resummation of epsilon-expansion that the fixed point survives up to 5d, for:

SU(2) QCD with $n_f \leq 4$

pure YM with $n_c \leq 4$

Observables:

YM

n_c	2	3	4
γ_g^*	-1.76(72)	-1.76(62)	-1.76(18)

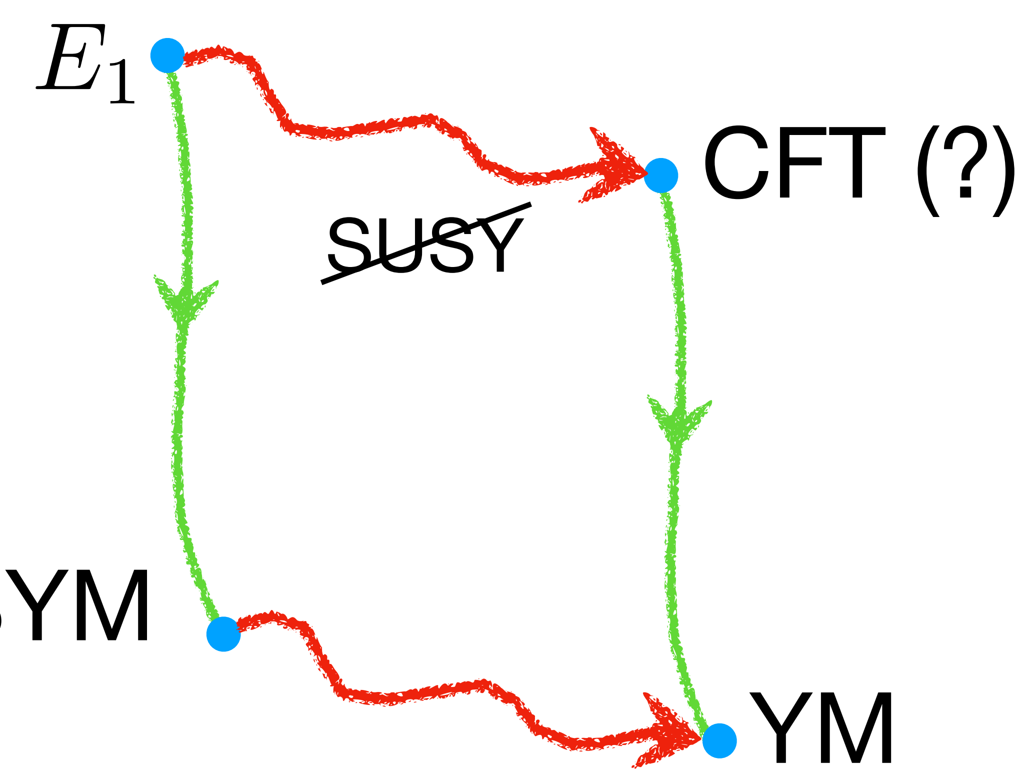
SU(2) QCD

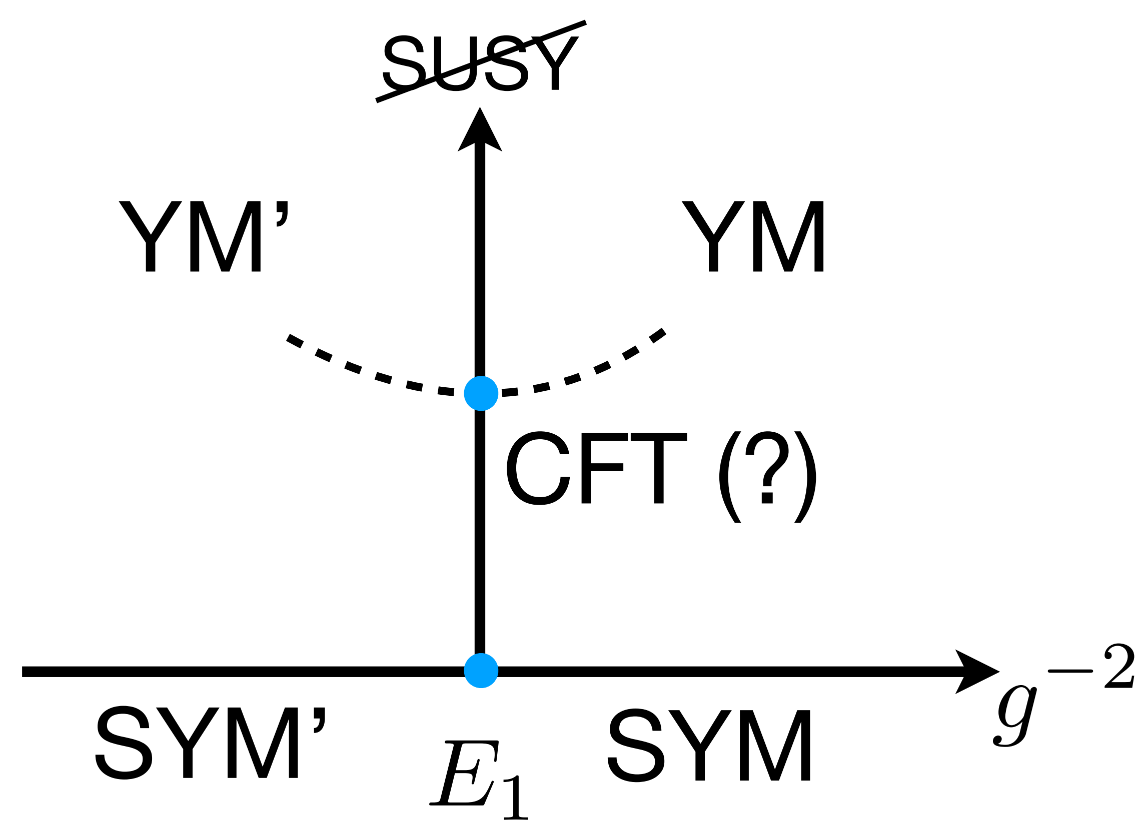
n_f	0	1	2	3	4
γ_g^*	-1.76(72)	-1.78(58)	-1.74(50)	-1.66(51)	-1.45(63)
γ^*	—	-0.47(12)	-0.52(21)	-0.57(41)	-0.65(88)

Deformation of the SCFT

$SU(2)$ SQCD with N_f flavors has UV completion for $0 \leq N_f \leq 7$: E_{N_f+1} SCFT [Seiberg]

[Benetti-Genolini, Honda, Kim, Tong, Vafa] : SUSY-breaking relevant deformation of E_1 as UV completion of $SU(2)$ YM.

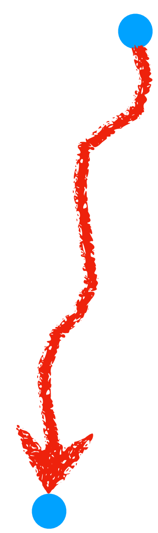




How can we test this scenario?

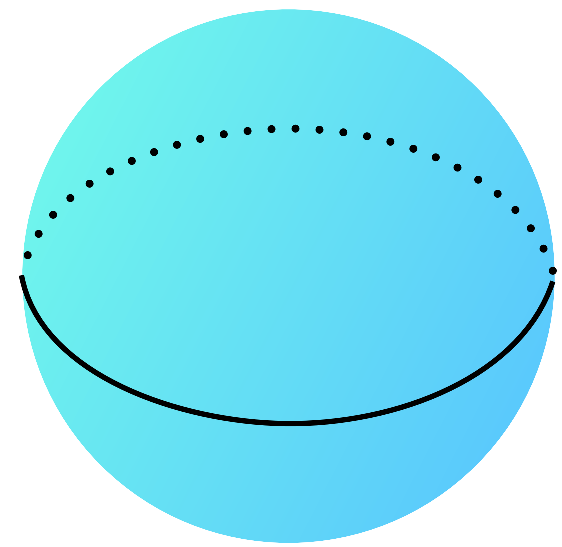
It predicts the flow:

E_1



CFT (?)

<u>Monotonic quantity:</u>



$$-F = \log Z_{S^5}$$

$$-F_{\mathrm{UV}} > -F_{\mathrm{IR}}$$

[Klebanov, Pufu, Safdi]

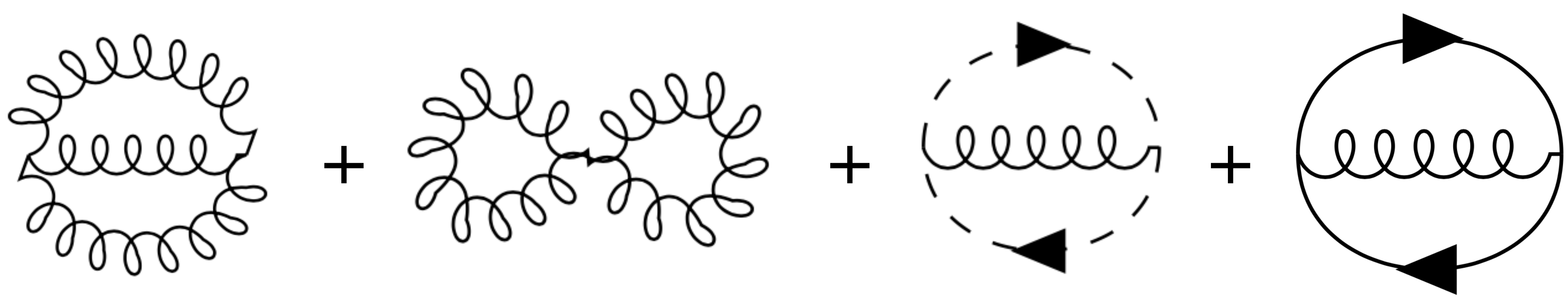
Hard to compute in interacting CFT.
Possible in E_1 thanks to SUSY.
[Chang, Fluder, Lin, Wang]

We can compute $-F_{\mathrm{IR}}$ using epsilon expansion:

$$\tilde{F} = \sin\left(\frac{\pi d}{2}\right)\log Z_{S^d} \quad \text{with} \quad d = 4 + 2\epsilon$$

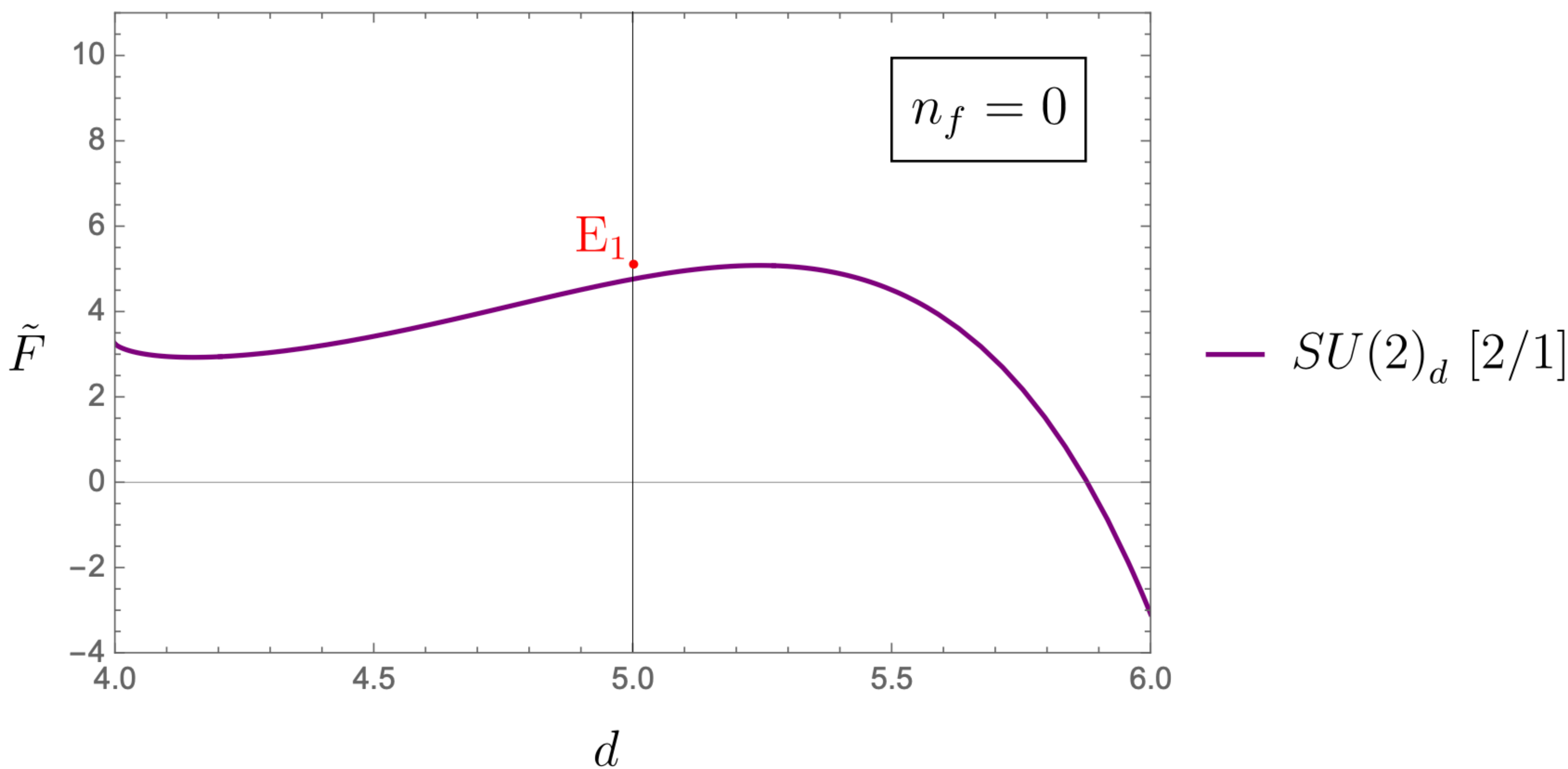
[Klebanov, Giombi]

Up to two loops:



$$\delta\widetilde{F}(\epsilon) = (n_c^2-1)\frac{31\pi}{90} + \left((n_c^2-1)4.696 - \pi\log\left(\frac{\mathrm{vol}(SU(n_c))}{(2\pi)^{n_c^2-1}}\right)\right)\epsilon$$
$$+ (n_c^2-1)\left(\frac{n_f\pi(584n_fn_c - 1089 - 737n_c^2)}{484n_c(11n_c-2n_f)^2} + \frac{386\pi + 363\pi(\gamma+\log(4\pi))}{726} - 10.098\right)\epsilon^2 + \mathcal{O}(\epsilon^3)\,.$$

Extrapolation:



The flow is allowed! Remarkably close to E_1 value.

Thank You

What, finally, is the importance of the set of observations we have discussed here? I must say frankly that I do not know.

from [Peskin] (1980)