Exploring the conformal window and infrared fixed points using lattice field theory

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Asymptotic Safety meets Particle Physics & Friends
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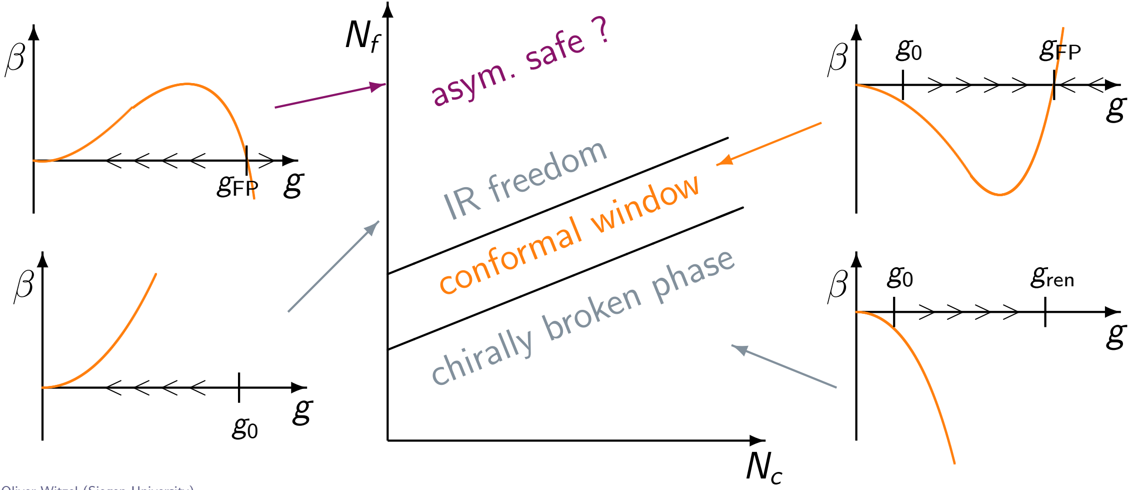
Gauge-fermion systems

- Gauge-fermion system with $N_c \geq 2$ colors and N_f flavors in some (fundamental) representation



Renormalization Group β function

$$\beta(g^2) = \mu^2 \frac{dg^2}{d\mu^2}$$

- Encodes dependence of coupling g^2 on the energy scale μ^2
- β has no explicit dependence on μ^2, only implicit through $g^2(\mu)$
- Known perturbatively up to 5-loop order in the $\overline{\text{MS}}$ scheme (1- and 2-loop are universal)
  [Baikov, Chetyrkin, Kühn PRL118(2017)082002] [Ryttov and Shrock PRD94(2016)105015]
- Known perturbatively at 3-loop order in the gradient flow scheme [Harlander, Neumann JHEP06(2016)161]
- Perturbative predictions reliable at weak coupling,
  nonperturbative methods needed for strong coupling

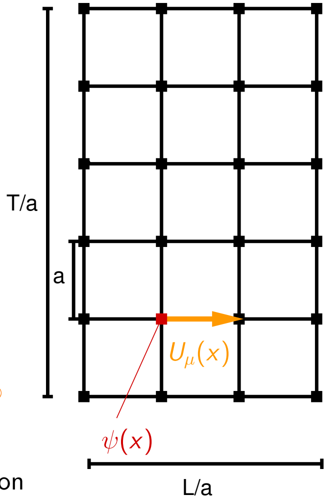
Lattice calculations

- Wick-rotate to Euclidean time $t \to i\tau$
- Discretize space-time and set up a hypercube of finite extent $(L/a)^3 \times T/a$ and spacing a
- Use path integral formalism

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

$\Rightarrow$ Large but finite dimensional path integral

- Finite lattice spacing $a \to$ UV regulator
- Finite volume of length $L \to$ IR regulator
  - $\to$ Study physics in a finite box of volume $(aL)^3$ plus limit $L \to \infty$
- Different discretizations for gauge and fermion actions possible
  - $\to$ Wilson, Symanzik gauge; Wilson, staggered, domain-wall fermion
  - $\to$ Discretization effects disappear after taking $a \to 0$ continuum limit



Gradient flow

[Narayanan and Neuberger JHEP 0603 (2006) 064] [Lüscher CMP 293 (2010) 899][JHEP 1008 (2010) 071]

- Add flow time coordinate t with dimension [-2] and define gauge field $B_\mu(x,t)$

$$\frac{\partial}{\partial t}B_\mu(t) = \mathcal{D}_\nu(t)G_{\nu\mu}(t) \qquad \text{with} \qquad B_\mu(t=0) = A_\mu$$

  - → Ordinary differential equation (ODE) ⇝ solve numerically using Runge-Kutta
  - → Covariant derivative defined in terms of the flow field $B_\mu(t)$ and the Yang-Mills action $S_{YM}(B)$
- Gradient flow is a smoothing/averaging transformation
  - → Can be related to RG flow ⇝ perfect to define RG β function, ...
- Energy density $\langle E(t)\rangle = -\frac{1}{2}Tr\langle G_{\mu\nu}(x,t)G_{\mu\nu}(x,t)\rangle$
  - → Renormalized coupling, scale setting ($\sqrt{8t_0}$, w_0)
- Different gradient flows: Wilson flow, Symanzik flow, Zeuthen flow, ...

Step-Scaling β function

- Discretized β function determined using numerical lattice field theory calculations [Lüscher et al. NPB359(1991)221]
  - → Choose symmetric L^4 setup where the size L of the lattice is the only scale
  - → Determine β function by calculating scale change $L \to s \cdot L$
  - → Conventionally differs by a minus sign compared to the continuum definition

- Use gradient flow to define a renormalized coupling

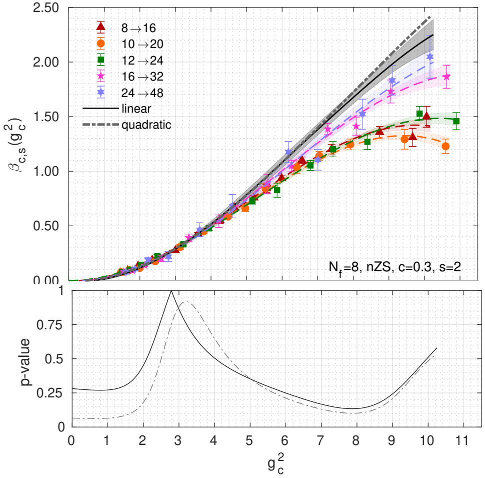
$$g_c^2(L) = \frac{128\pi^2}{3(N_c^2 - 1)} \frac{1}{C(c, L)} t^2 \langle E(t) \rangle$$

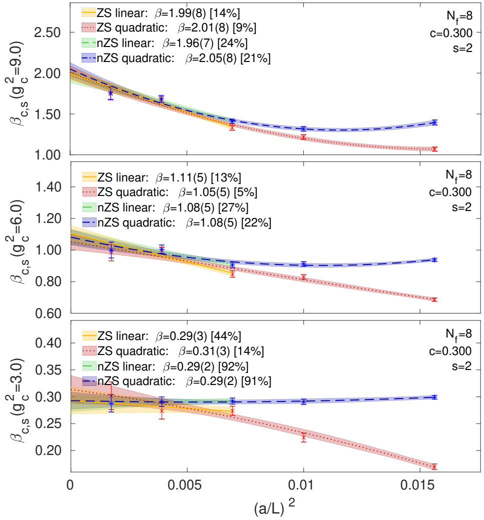
  - → Relate flow time t to scale L: $\sqrt{8t} = c \cdot L$ [Fodor et al. JHEP11(2012)007][JHEP09(2014)018]
  - → Calculate scale difference

$$\beta_s^c(g_c^2; L) = \frac{g_c^2(sL) - g_c^2(L)}{\log(s^2)}$$

  - → Extrapolate $L/a \to \infty$ to remove discretization effects and take combined continuum and infinite volume limit

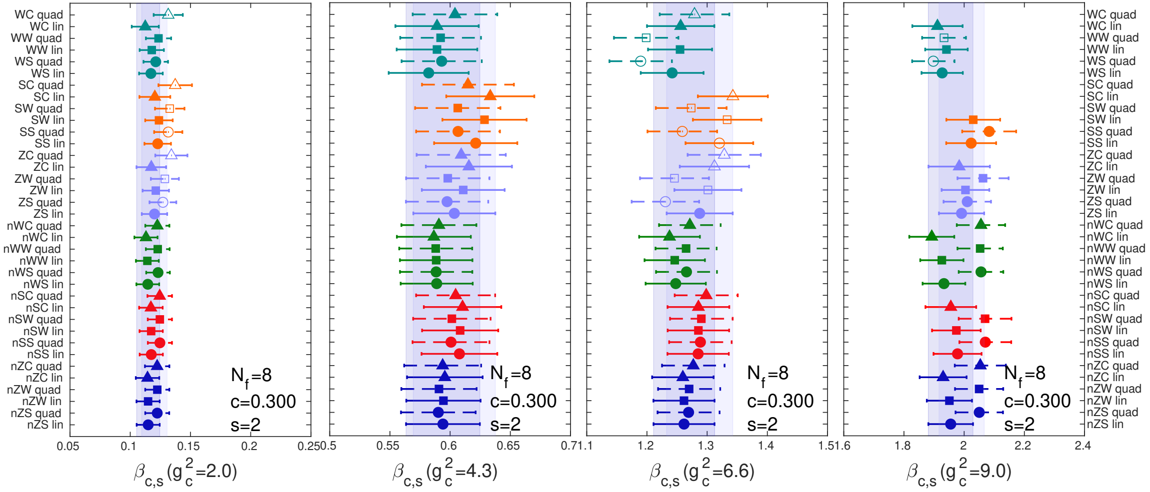
Analysis SU(3) with $N_f = 8$ fundamental flavors [Hasenfratz, Rebbi, OW PRD 107(2023)114508]



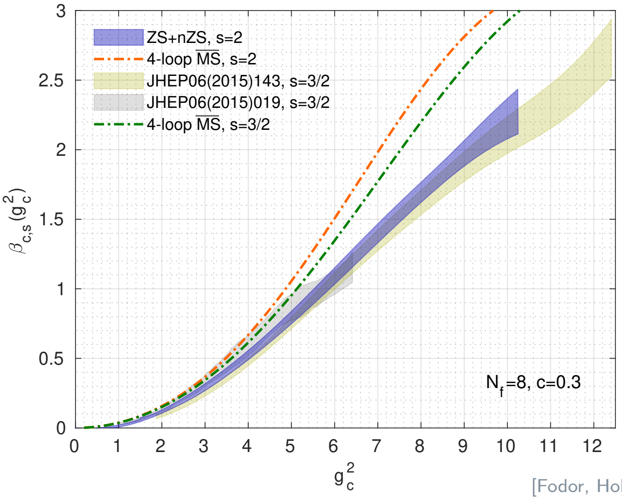


Systematic effects SU(3) with $N_f = 8$ fundamental flavors

[Hasenfratz, Rebbi, OW PRD 107(2023)114508]



Comparison SU(3) with $N_f = 8$ fundamental flavors



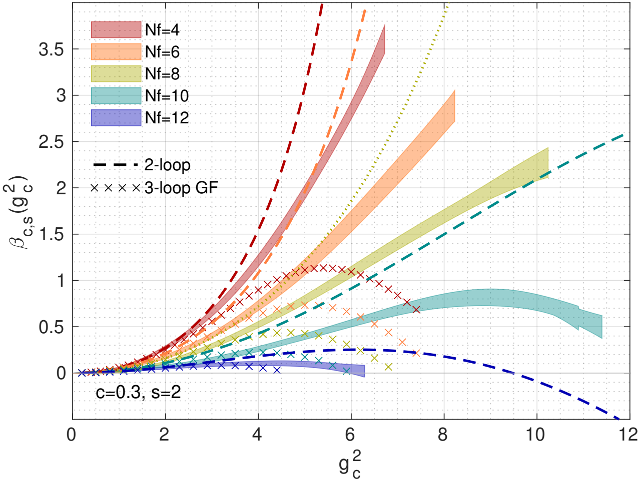
[Hasenfratz, Rebbi, OW PRD 107(2023)114508]

- Small effect due to different scale change s
- GF $c = 0.3$ scheme $\neq$ 4-loop $\overline{\text{MS}}$

[Hasenfratz, Schaich, Veernala JHEP06(2015)143]

[Fodor, Holland, Kuti, Mondal, Nogradi, Wong JHEP06(2015)019]

Step-Scaling β function for $N_f = 4, 6, 8, 10, 12$



- 3-loop GF
  [Harlander, Neumann JHEP06(2016)161]
- $N_f = 12$: indication of an IRFP at $g_c^2 \approx 6.3$
- $N_f = 10$: hints for an IRFP at $g_c^2 \gtrsim 13$

[Hasenfratz, Rebbi, OW PLB 798(2019)134937]
[Hasenfratz, Rebbi, OW PRD 100(2019)114508]
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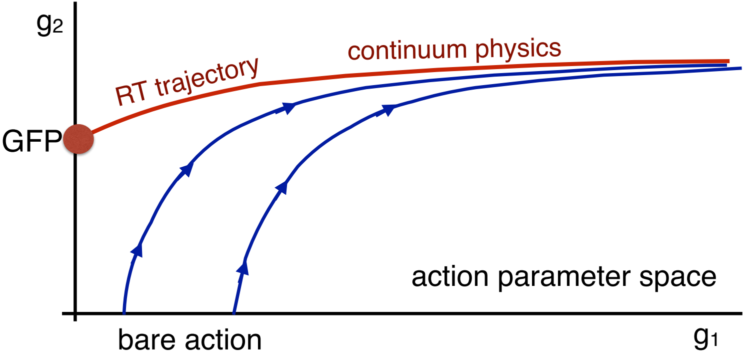
Gradient flow and real-space renormalization Group (RG) flow



Gradient flow and real-space renormalization Group (RG) flow

- RG flow: change of (bare) parameters and coarse graining (blocking)
- Gradient flow is a continuous transformation
  - → Define real-space RG blocked quantities by incorporating coarse graining as part of calculating expectation values [Carosso, Hasenfratz, Neil PRL 121 (2018) 201601]
- Relate GF time t/a^2 to RG scale change $b \propto \sqrt{t/a^2}$
  - → Quantities at flow time t/a^2 describe physical quantities at energy scale $\mu \propto 1/\sqrt{t}$
  - → Local operator with non-vanishing expectation value can be used to define running coupling
    - ⇝ Simplest choice: $t^2\langle E(t)\rangle$ [Lüscher JHEP 1008 (2010) 071]
- Continuous RG β function

$$\beta_{GF}(g^2_{GF}) = \mu^2 \frac{dg^2_{GF}}{d\mu^2} = -t\frac{dg^2_{GF}}{dt}$$

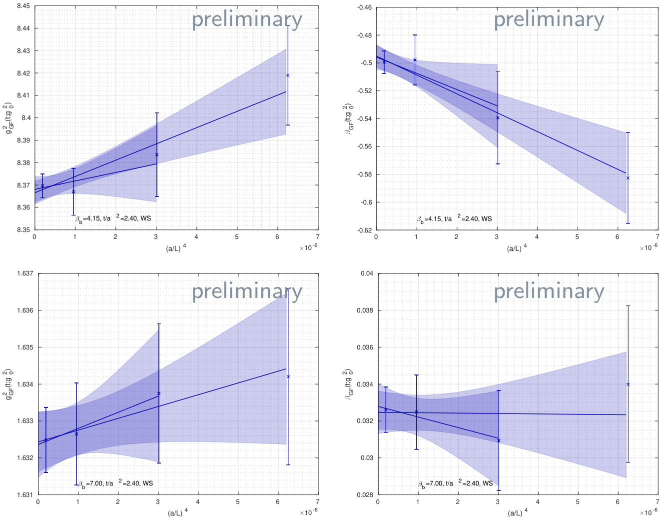
Continuous RG β function

[Fodor et al. EPJ Web Conf. 175 (2018) 08027]
[Hasenfratz, OW PRD 101 (2020) 034514] [Hasenfratz, OW PoS LATTICE2019 (2019) 094]
[Wong et al. PoS LATTICE2022 (2023) 043] [Hasenfratz, Peterson, Van Sickle, OW PRD108 (2023) 014502]

- Extract $g^2_{GF}(t;\beta_b,L/a)$ its derivative $\beta_{GF}(t;\beta_b,L/a)$ for a range of GF times on each ensembles
  → Different bare coupling β_b on different volumes $(L/a)^4$ or $(L/a)^3\times T/a$
- Perform infinite volume extrapolation at fixed bare coupling β_b and GF time t
  → Obtain $g^2_{GF}(t;\beta_b)$ and $\beta_{GF}(t;\beta_b)$
- Interpolate discrete infinite volume values to get continuous values at fixed flow time
  → $g^2_{GF}(t)$ and $\beta_{GF}(t;g^2_{GF})$
- Take continuum limit $(a^2/t\to 0)$ for fixed g^2_{GF} and obtain $\beta_{GF}(g^2_{GF})$

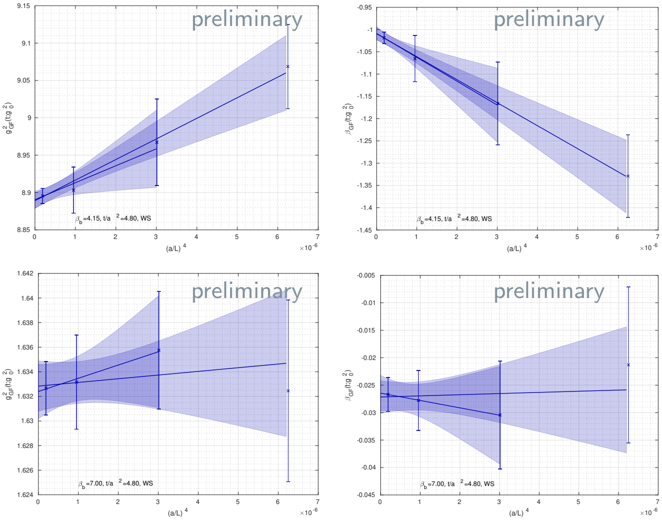
⇒ Reanalyze data of the step-scaling calculations

$N_f = 8$: Infinite volume extrapolation



- For each bare coupling β_b and at all GF times
- Relevant GF times typically $t/a^2 \in [2.20, 5.00]$
- Strong ($\beta_b = 4.15$) and weak ($\beta_b = 7.00$) coupling at $t/a^2 = 2.4$

$N_f = 8$: Infinite volume extrapolation



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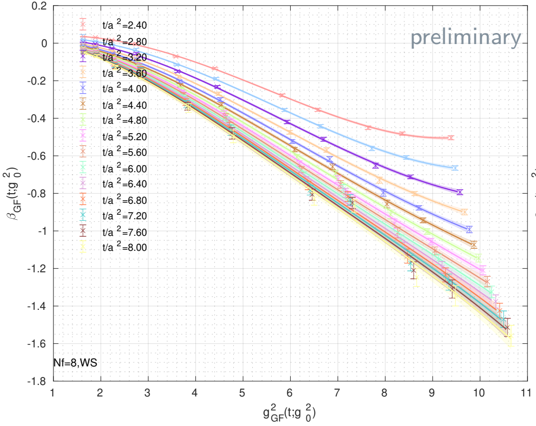
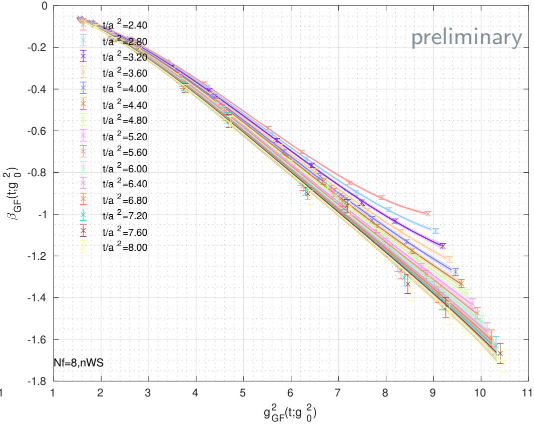
- Relevant GF times typically $t/a^2 \in [2.20, 5.00]$
- Strong ($\beta_b = 4.15$) and weak ($\beta_b = 7.00$) coupling at $t/a^2 = 4.8$

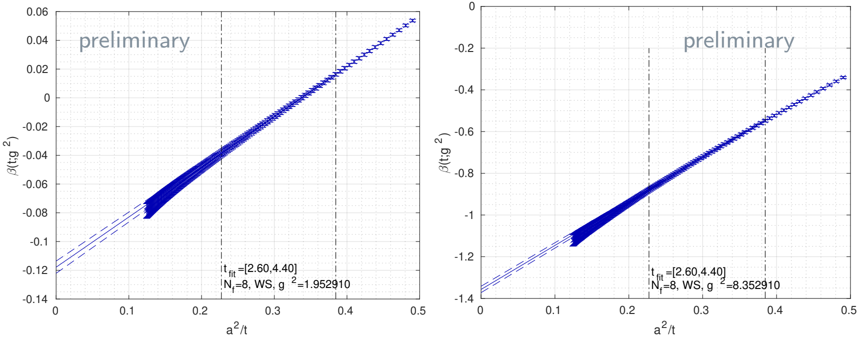
$N_f = 8$: Polynomial interpolation in g^2_{GF}





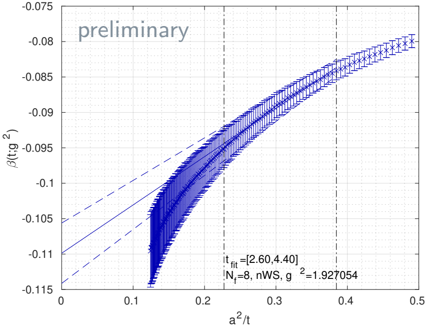
- Without tree-level improvement
- With tree-level improvement

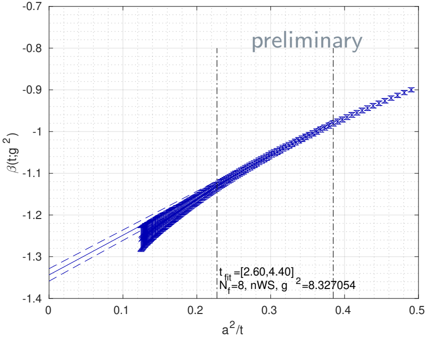
$N_f = 8$: Continuum limit ($a^2/t \to 0$)



- Simple continuum limit for range of flow times
- Without tree-level improvement

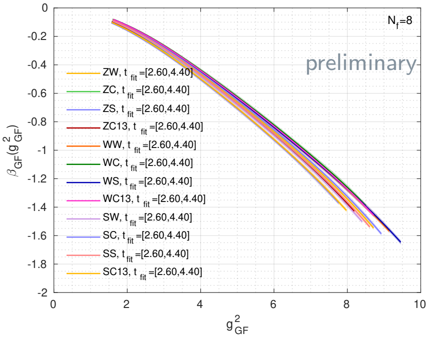
$N_f = 8$: Continuum limit $(a^2/t \to 0)$

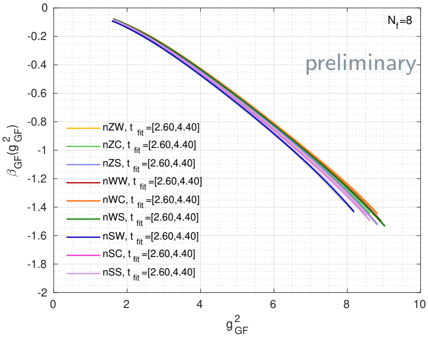




- Simple continuum limit for range of flow times
- With tree-level improvement

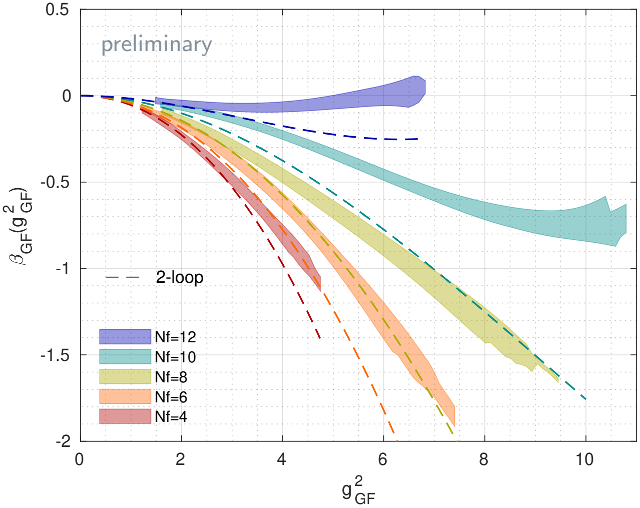
$N_f = 8$: Continuous β function





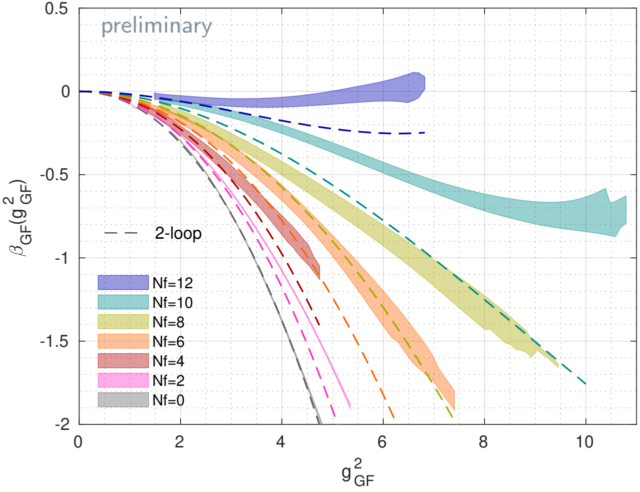
- Different flow-operator combinations with and without tree-level improvement

Summary continuous β function for $N_f = 4, 6, 8, 10, 12$



- Systematic effects for $N_f = 10$ likely underestimated
- Reach in g^2 limited by 1st order bulk phase transition (lattice artifact)
- Qualitative behavior captured by 2-loop PT prediction for $N_f \leq 8$
- IRFP for $N_f = 12$ not (well) resolved

Summary continuous β function for $N_f = 0, 2, 4, 6, 8, 10, 12$



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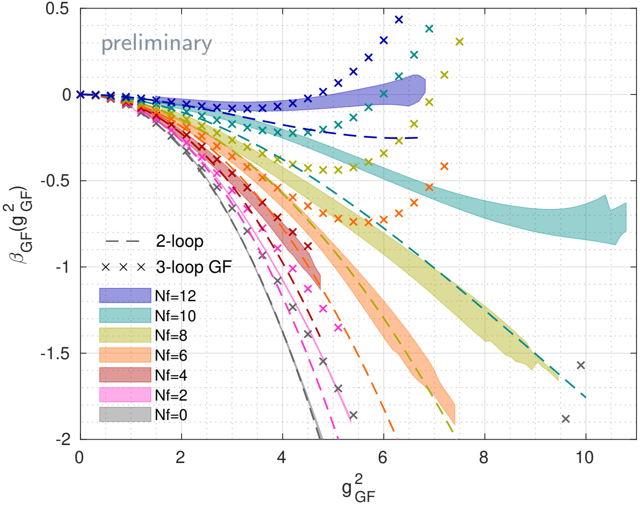
- Including $N_f = 2$ and $N_f = 0$

[Hasenfratz, OW PRD 101(2020)034514]

[Hasenfratz, Peterson, Van Sickle, OW PRD108(2023)014502]

Summary continuous β function for $N_f = 0, 2, 4, 6, 8, 10, 12$



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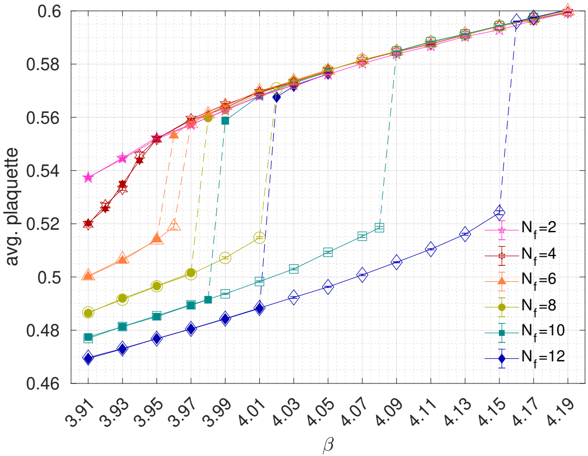
  [Hasenfratz, OW PRD 101(2020)034514]

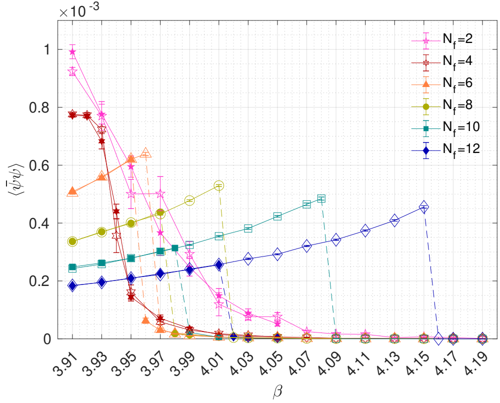
  [Hasenfratz, Peterson, Van Sickle, OW PRD108(2023)014502]
- 3-loop GF prediction tracks nonperturbative result longer, but than turns away showing different qualitative behavior

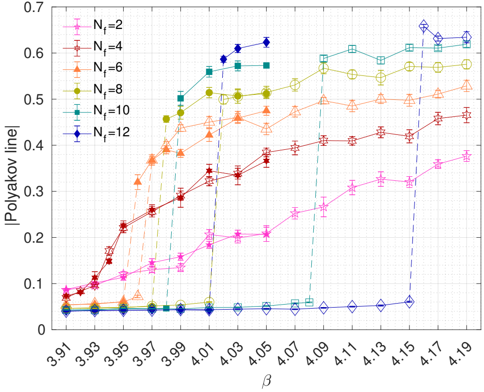
Reach in g^2_{GF} [Hasenfratz, Rebbi, OW PRD 107(2023)114508]



- $N_f = 2, 4, 6$: zero mass simulations limited by confinement transition
  - → Perform finite mass simulations plus chiral extrapolation
- $N_f = 8, 10, 12$: 1st order bulk phase transition limits reach in g^2
  - → Lattice artifact due to choice of actions (3× stout-smeared MDWF+Symanzik)
  - → Wide hysteresis
    - ⇝ Artifacts may affect strongest coupling
  - → Even larger volumes will not allow to overcome these issues
    - ⇝ add e.g. Pauli-Villars field

Reach in g^2_{GF} [Hasenfratz, Rebbi, OW PRD 107(2023)114508]





Adding extra Pauli-Villars fields

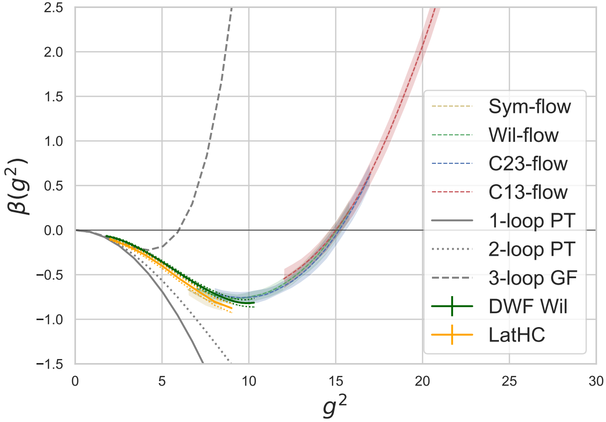
- Pauli-Villars (PV) are introduced when constructing 5-dim. domain-wall fermions (DWF)
  → Chiral fermions obeying the Ginsparg-Wilson relation
  bypassing the Nielsen-Niyomiya no-go theorem
- Bosonic fields with mass $am_{PV} = 1$
- Automatically integrated out, when taking the continuum limit

- Side-effect: PV fields largely absorb effective shift in bare coupling due to (many) fermions
  → At weak coupling avg. plaquette does essentially not vary with N_f for DWF

- Idea: add PV fields also to Wilson or Staggered fermions for studying many flavor systems at strong coupling [Hasenfratz, Shamir, Svetitsky PRD 104 (2021) 074509]

$N_f = 10$: Wilson fermion with Pauli-Villars fields

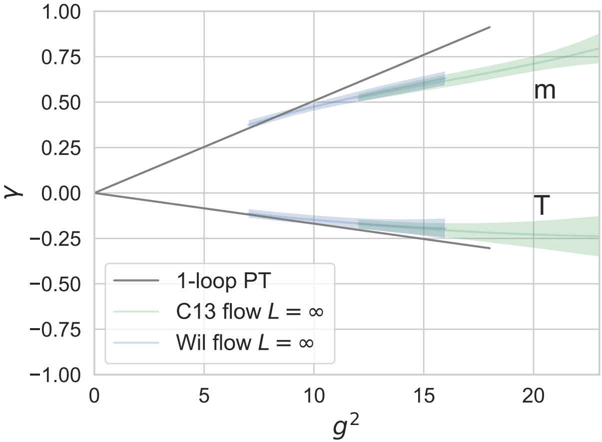
[Hasenfratz, Neil, Shamir, Svetitsky, OW PRD 108 (2023) L071503]



- Infrared fixed-point around $g^2 \approx 15$
  $\Rightarrow$ SU(3) with $N_f = 10$ is conformal
- Consistent result for different gradient flows
- Weak coupling overlaps with our DWF and LatHC's staggered result
  [Fodor et al. PoS Lattice2018 (2018) 199]
  [Fodor et al. PoS Lattice2019 (2019) 121]
  [Kuti et al .PoS Lattice2021 (2022) 321)]
- Reaching couplings $g^2 > 20$

$N_f = 10$: Anomalous dimension

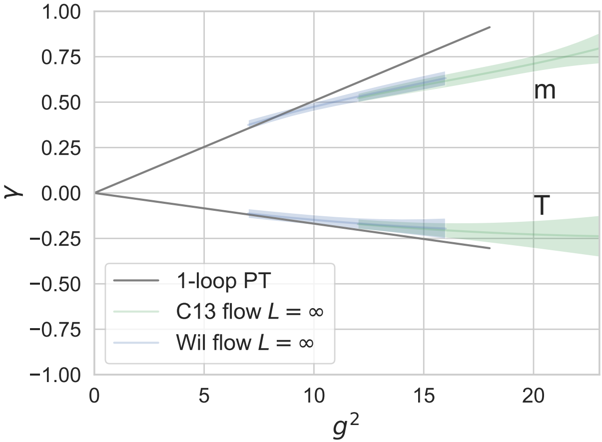
[Hasenfratz, Neil, Shamir, Svetitsky, OW PRD 108 (2023) L071503]



- Calculate 2-pt function of flowed mesonic source X' with unflowed source X
  $\langle X(0)X'(t)\rangle \sim t^{-(d+\eta+\gamma)/2}$
  $\rightarrow \gamma$ anomalous dimension of operator
  $\rightarrow \eta$ anomalous dimension of fermion field
- Cancel η by defining ratio over conserved vector current
  $$R(t) = \frac{X(0)X'(t)}{V(0)V'(t)} \sim t^{-\gamma/2}$$
- Logarithmic derivative
  $$\gamma = -2\frac{t}{R}\frac{\partial R}{\partial t}$$
  with $t \gg x_4$ (Euclidean time separation)

$N_f = 10$: Anomalous dimension

[Hasenfratz, Neil, Shamir, Svetitsky, OW PRD 108 (2023) L071503]



- PT at 1-loop
  - → Mass anomalous dimension (scalar)

    $\gamma_m = \frac{6g^2 C_2}{16\pi^2}$

    $C_2 = \frac{4}{3}$ quadratic Casimir operator
  - → Anomalous dimension of the tensor density

    $\gamma_T = -\frac{1}{3}\gamma_m$

- At the IRFP $g^2 \approx 15$

  $\gamma_m \simeq 0.6$

  → Consistent with [LSD PRD 103 (2021) 014504]

  $\gamma_m = 0.47(5)$ at $g^2 \sim 10$

  $\gamma_T \simeq -0.2$

Outlook

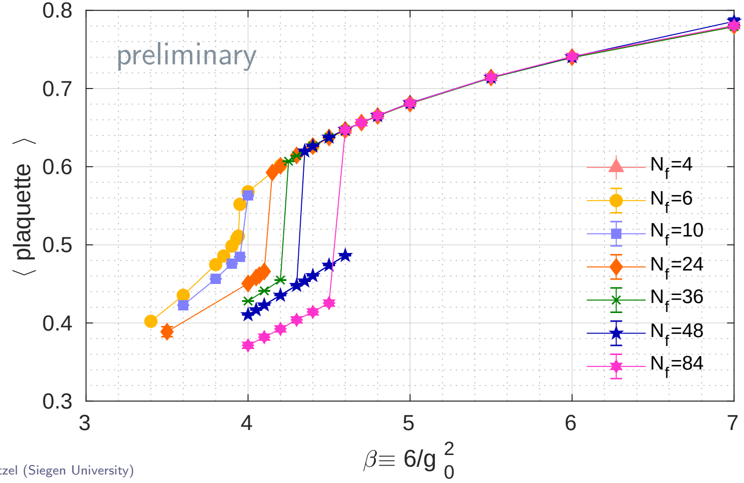
- SU(3) with $N_f = 12$ is conformal, IRFP around $g^2 \sim 6$
- SU(3) with $N_f = 10$ is conformal, IRFP around $g^2 \sim 15$

- What is the nature of $N_f = 8$?
  - → Is $N_f = 8$ the onset of the conformal window?
  - → Does $N_f = 8$ exhibit a new phase (symmetric mass generation)? [Hasenfratz PRD 106 (2022) 014513]
  - → Ongoing and new investigations by the LSD collaboration incl. simulations with PV fields

- What happens for larger N_f? Asymptotic safety?

SU(3) with many (more) flavors

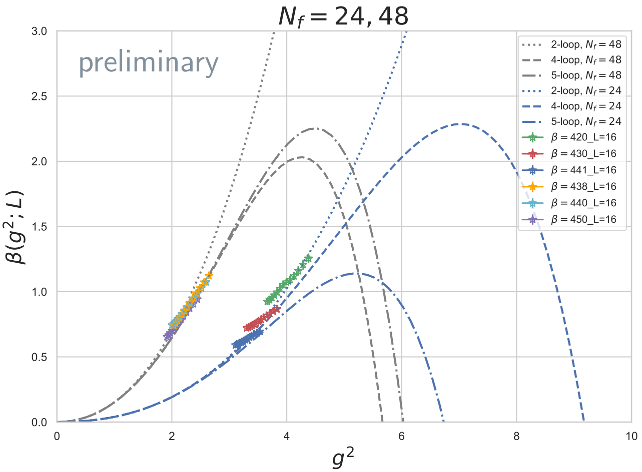
- Exploring bare parameter space for SU(3) with many fundamental flavors (stout-smeared MDWF and Symanzik gauge action)



- Seeking 1st order transition e.g. jump in average plaquette
- Cheap 8^4 lattices
- Transition moves to weaker for more flavors

→ Verification of bulk phase transition vs. finite T to be completed

SU(3) with many (more) flavors



- Only $L/a = 16$
- β function runs backward!
- Run into bulk phase transition before observing a sign of an UVFP
- Add four-fermion interaction
  $\rightarrow$ N-JL type models
- Repeat with a better action

extra

DWF setup

- Symanzik gauge action
- Möbius domain-wall fermions with three levels of stout smearing ($\varrho = 0.1$)
- Input quark mass $am_q = 0$, $L_s = 12$ or 16 such that $am_{res} < 10^{-5}$
- Fermions with anti-periodic boundary conditions in space and time

$N_f = 4$	$N_f = 6$	$N_f = 8$
7–9 bare couplings	7–11 bare couplings	11–16 bare couplings
$\beta = 8.50 - 4.60(4.20)$	$\beta = 8.50 - 4.30(4.05)$	$\beta = 7.00 - 4.10(4.02)$

- 5 volume pairs with $s = 2$

$40^4, 32^4, 24^4, 20^4, 16^4, 12^4, 10^4, 8^4$ $\quad$ $48^4, 32^4, 24^4, 20^4, 16^4, 12^4, 10^4, 8^4$

- Simulations performed using `Grid` [Boyle et al. PoS Lattice2015 023]
- Measuring Zeuthen flow, Symanzik flow, and Wilson flow in `Qlua` [Pochinsky PoS Lattice2008 040]
- Apply tree-level normalization to reduce cutoff effects [Fodor et al. JHEP09(2014)018]

DWF setup

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- Möbius domain-wall fermions with three levels of stout smearing ($\varrho = 0.1$)
- Input quark mass $am_q = 0$, $L_s = 12$ or 16 such that $am_{res} < 10^{-5}$
- Fermions with anti-periodic boundary conditions in space and time

$N_f = 10$	$N_f = 12$
17 bare couplings	17 bare couplings
$\beta = 7.00 - 4.02$	$\beta = 7.00 - 4.10$

- 5 volume pairs with $s = 2$

$$32^4,\ 28^4,\ 24^4,\ 20^4,\ 16^4,\ 14^4,\ 12^4,\ 10^4,\ 8^4$$

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Wilson setup

- Wilson gauge action with nHYP dislocation suppression (NDS)
- nHYP-smeared Wilson-clover fermions with $(\alpha_1, \alpha_2, \alpha_3) = (0.75,\ 0.6,\ 0.3))$
  - → Mass at zero (κ tuned to κ_{critical})
  - → Fermions with periodic (anti-periodic) boundary conditions in space (time)
- 30 additional Pauli-Villars fields with $am_{PV} = 1$
- 6 bare couplings $\beta_b = 7.0,\ 6.7,\ 6.5,\ 6.3,\ 6.2,\ 6.0$
- $24^3 \times 48$ and $28^3 \times 56$ volumes
- Measuring Symanzik flow, Wilson, C13 and C23 flows