Naturally small neutrino mass with asymptotic safety and gravitational-wave signatures

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Based on

JHEP 11(2023) 224 (arXiv:2308.1122)

with Kamila Kowalska, Enrico Maria Sessolo

National Center for Nuclear Research (NCBJ)
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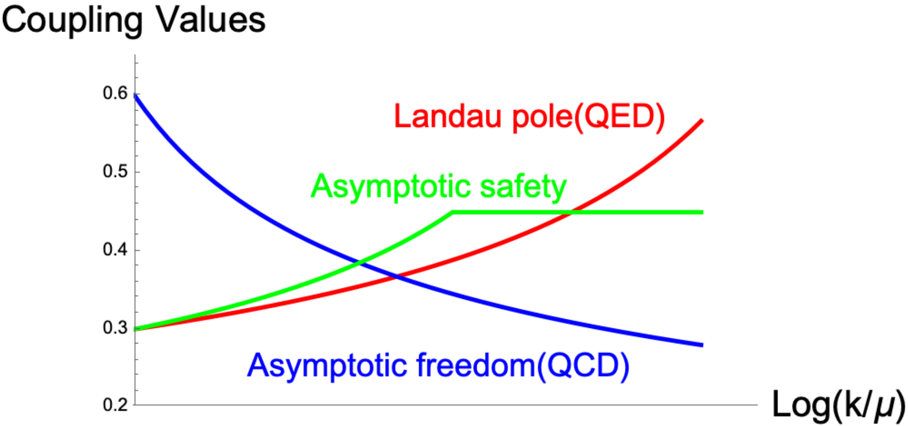
Asymptotic Safety meets Particle Physics and Friends Workshop

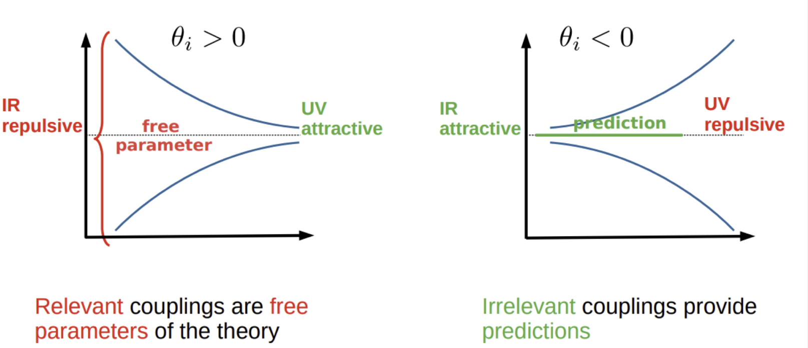
Asymptotic behaviour of the couplings



- UV complete theory: all the couplings approach a fixed point
  $\implies$ The theory can be extrapolated to infinitely large energy scales

Predictions and free parameters

- Fixed point: where all the couplings stay constant with the changing scale
  - $\beta_i(\{g_i\}) = 0$
- Linearized flow equation near the fixed point
  - Stability matrix: $M_{ij} \equiv \left.\frac{\partial \beta_i}{\partial g_j}\right|_{\{g_i^*\}} \longrightarrow \{\theta_i\}$ Critical exponents



- Choosing free-parameters at the UV boundary fixes the flow of all the couplings

Gravity corrections above the Planck scale

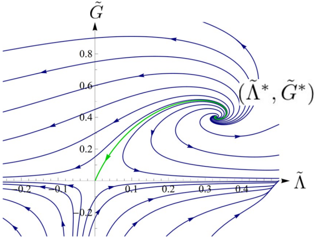
- Gravity correction to beta functions of gauge ($\{g\}$) and Yukawa couplings ($\{y\}$)

$$\beta_g = \beta_g^{SM+NP} - f_g(G^*, \Lambda^*)g$$

$$\beta_y = \beta_y^{SM+NP} - f_y(G^*, \Lambda^*)y$$

Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17

Reuter, Saueressig, hep-th/0110054



These corrections

- are universal: does not distinguish internal symmetries
- can cure UV divergencies
- can improve predictive power

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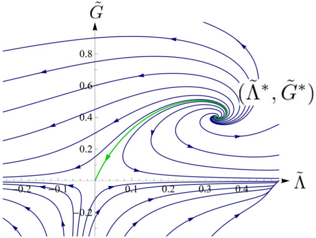

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Example:

SM + U(1)$_X$

$$\begin{cases} \dfrac{dg_Y}{dt} = \dfrac{41}{6}\dfrac{g_Y^3}{16\pi^2} - f_g g_Y \\ \dfrac{dg_X}{dt} = 11\dfrac{g_X^3}{16\pi^2} - f_g g_X \end{cases}$$

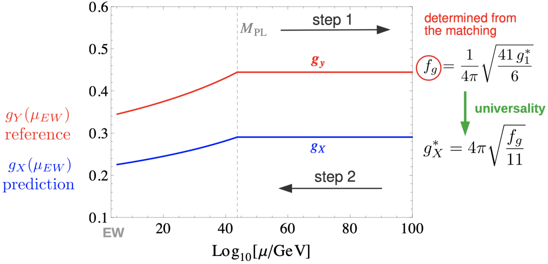


Fig from K. Kowalska

(Recalling last year's Kamila Kowalska's talk)

Dynamical mechanism of small neutrino mass generation from asymptotic safety

Dirac mass and Majorana mass

How can we explain the tiny mass of neutrinos?

Dirac mass: $m_D\, \bar{\nu}_R \nu_L$

$$y_\nu\, \bar{\nu}_R\, \phi\, \nu_L \xrightarrow[<\phi>=v_\phi]{SSB} y_\nu\, v_\phi\, \bar{\nu}_R\, \nu_L$$

$$\implies m_D = y_\nu v_\phi$$

Unnaturally small y_ν

Majorana mass: $m_M\, \bar{\nu}_R \nu_R^C$

$$\begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \xrightarrow{\text{diagonalize}} m_1\, m_2 \approx m_D^2$$

$$\implies m_1 = \frac{(y_f v_\phi)^2}{m_M}$$

see-saw mechanism

Case 1: The Majorana mass and the seesaw scale

K.Kowalska, S.Pramanick, E.M. Sessolo, 2204.00866

- Relevant beta functions:

$$\beta_{g_Y} = \frac{1}{16\,\pi^2}\frac{41}{6}g_y^3 - f_g\,g_Y$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - \frac{17}{12}g_y^2 + y_v^2\right) - f_y\,y_t$$

$$\beta_{y_\nu} = \frac{y_\nu}{16\pi^2}\left(3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right) - f_y\,y_\nu$$

- Fixed-point analysis:

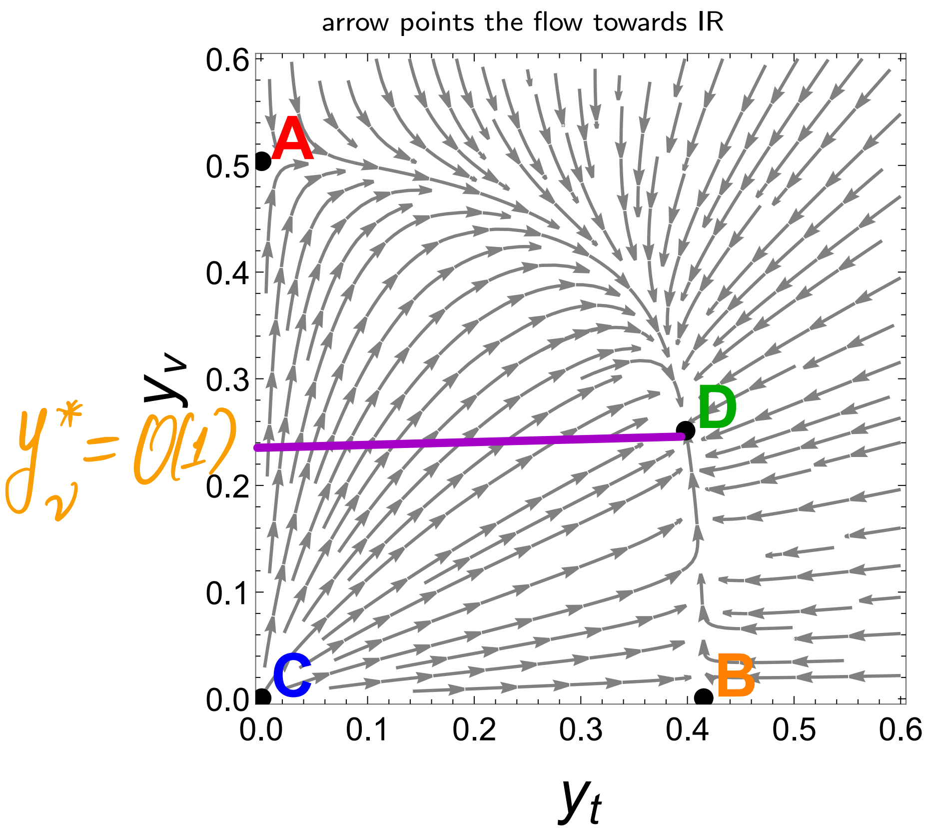
  If $f_y > f_{y,Crit}$

  $\implies$ IR-attractive fixed-point is at:

  $y_\nu^* \neq 0$, $y_t^* \neq 0$

  i.e $\theta_{y_\nu} < 0$ (at $y_\nu^* \neq 0$)

- Majorana mass: $M_\nu \approx \frac{(y_\nu\,v_h)^2}{m_\nu}$



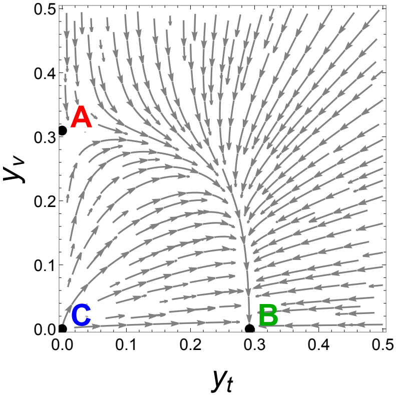
Majorana neutrinos, still prediction of the seesaw scale

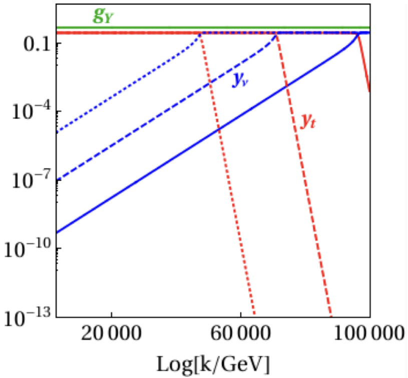
- Fixed point analysis:

  If $f_y < f_{y,Crit} \approx 8 \times 10^{-4}$

  $\Longrightarrow$ IR attractive fixed-point at $y_\nu^* = 0$ i.e. $\theta_{y_\nu} < 0$ at $y_\nu^* = 0$

K.Kowalska, S.Pramanick, E.M. Sessolo, 2204.00866



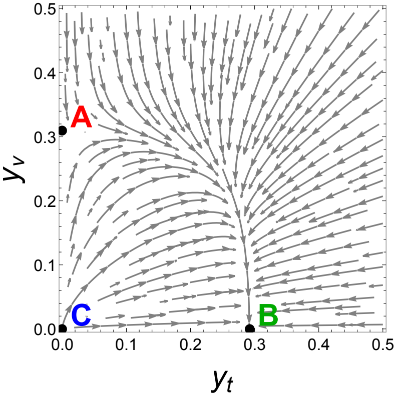


Case 2: Dynamical mechanism of small neutrino Yukawa

- Fixed point analysis:

  If $f_y < f_{y,Crit} \approx 8 \times 10^{-4}$

  $\Longrightarrow$ IR attractive fixed-point at $y_\nu^* = 0$ i.e. $\theta_{y_\nu} < 0$ at $y_\nu^* = 0$



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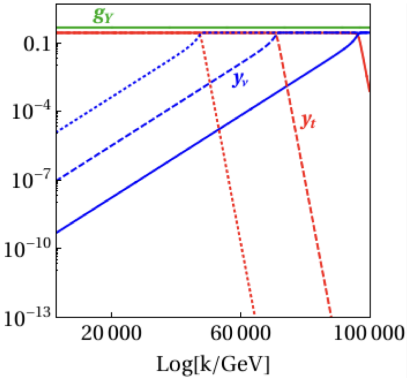
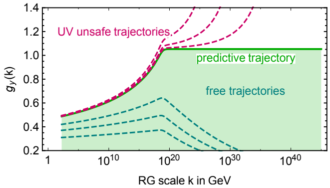


- Dirac mass: $m_\nu \sim y_\nu \nu$

Predicts small Dirac mass without fine-tuning

QG perspective on naturalness of this mechanism in SMRHN

Eichhorn et.al. 1709.07252



- IR attractive fixed-point at $y_\nu^* = 0$ is a crucial condition for this mechanism i.e. $\theta_{y_\nu} \approx \frac{-2}{3} g_Y^{*2} + \frac{3}{2} y_t^{*2} < 0 \implies g_Y^* \neq 0$

$$f_g \approx 0.0097$$

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- Values of f_g and f_y from asymptotically safe Quantum Gravity (QG)

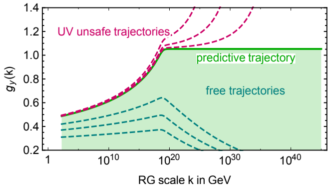
A. Eichhorn et. al. 1707.01107, 1604.02041

$$f_g(\tilde{G}_N^*, \tilde{\Lambda}^*) \approx \frac{\tilde{G}_N^*(1-4\tilde{\Lambda}^*)}{4\pi(1-2\tilde{\Lambda}^*)^2}$$

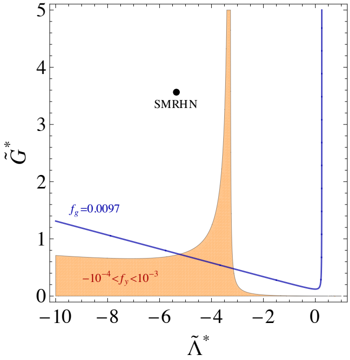
$$f_y(\tilde{G}_N^*, \tilde{\Lambda}^*) \approx \left(\frac{\tilde{G}_N^*\left(-56\tilde{\Lambda}^{*3} - 103\tilde{\Lambda}^{*2} + 235\tilde{\Lambda}^* - 96\right)}{12\pi\left(8\tilde{\Lambda}^* - 10\tilde{\Lambda}^* + 3\right)^2}\right)$$

- $\tilde{G}_N^*$ and $\tilde{\Lambda}^*$ depend on the number of Dirac fermions, gauge fields, and scalar fields

Eichhorn et.al. 1709.07252



AC, K. Kowalska, E.M. Sessolo 2308.06114



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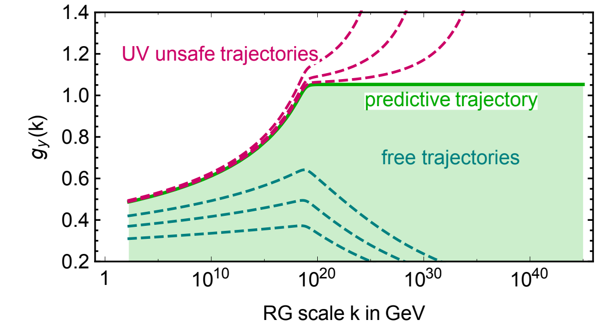

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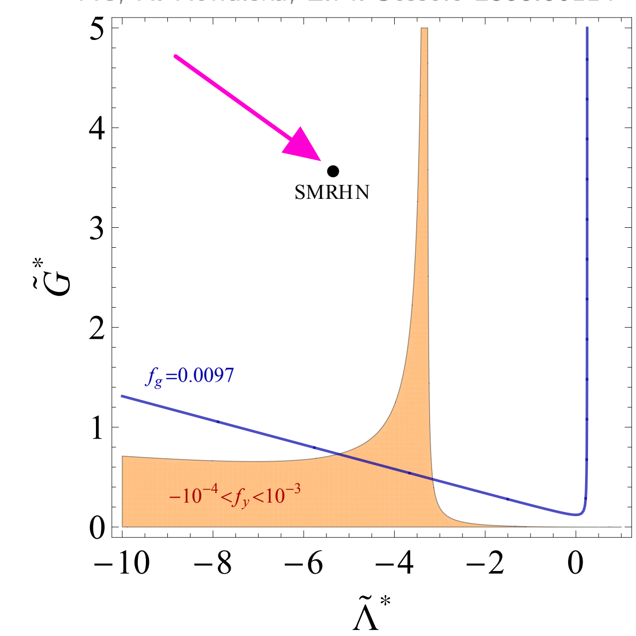


- Gauged $U(1)_{B-L}$ model:

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu} + i\bar{f}\left(\partial^{\mu} - ig_Y Q_Y \tilde{B}^{\mu} - ig_{B-L}Q_{B-L}\tilde{X}^{\mu}\right)\gamma_{\mu} f$$

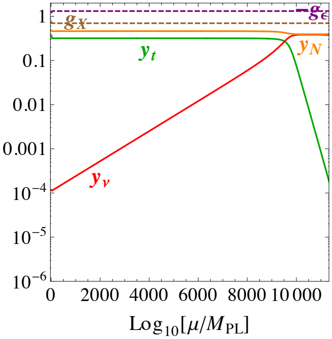
$$g_X = \frac{g_{B-L}}{\sqrt{1-\epsilon^2}}$$

$$g_{\epsilon} = -\frac{\epsilon\, g_Y}{\sqrt{1-\epsilon^2}}$$

- IR-attractive fixed-point at $y_{\nu}^* = 0$ is possible even if $g_Y^* = 0$ i.e. $f_g \neq 0.0097$

f_g determines g_X and g_{ϵ}

How? If $g_X^* \neq 0$ and $g_{\epsilon}^* \neq 0$



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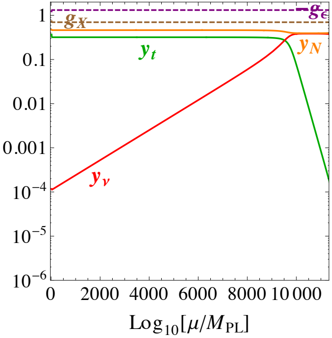

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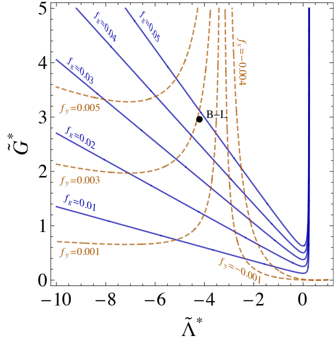
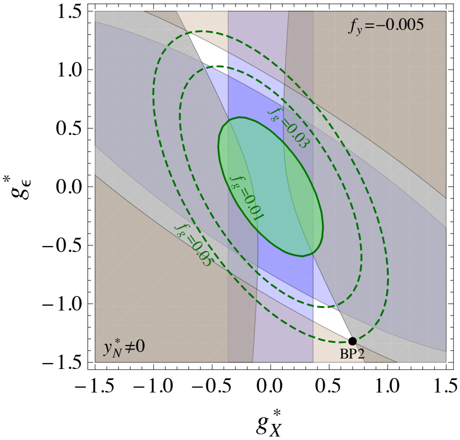


Fixed points of the gauge-Yukawa system

Conditions on the choice of fixed points:

- Feasible dynamical mechanism: $y_\nu^* = 0$, irrelevant
- $g_Y^* = 0$, relevant $\Longrightarrow f_g > 0.0097$
- Matching top-quark mass



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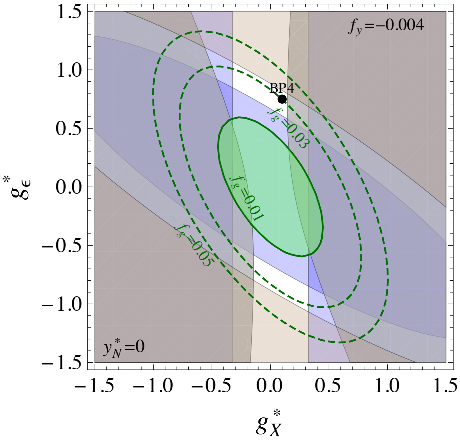


Predictions in the $B - L$ model

- Benchmark points for different f_g and f_y such that
  - IR-attractive fixed-point at $y_\nu^* = 0$
  - Predictions for the New Physics couplings (g_X, g_ϵ, y_N)

$$\mathcal{L}_M = -y_N^{ij} S\, \nu_{R,i}\, \nu_{R,j} + \text{H.c.}$$

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	$g_X\ (10^{5,7,9}\text{GeV})$	$g_\epsilon\ (10^{5,7,9}\text{GeV})$	$y_N\ (10^{5,7,9}\text{GeV})$
BP1	0.01	0.0005	0.10	-0.55	0.12	0.29, 0.29, 0.30	-0.26, -0.27, -0.28	0.16, 0.16, 0.16
BP2	0.05	-0.005	0.70	-1.32	0.47	0.40, 0.41, 0.44	-0.52, -0.56, -0.61	0.42, 0.44, 0.45
BP3	0.02	-0.0015	0.10	-0.75	0.0	0.12, 0.12, 0.12	-0.33, -0.35, -0.37	0.0
BP4	0.03	-0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0

- RGE flow ensures $y_N = 0$; not some global symmetry

Dirac $(y_N = 0)$: BP3, BP4

Majorana $(y_N \neq 0)$: BP1, BP2

- Experimental constrains on kinetic mixing and direct coupling of Z'

$v_S > 10$ TeV $>> v_H$

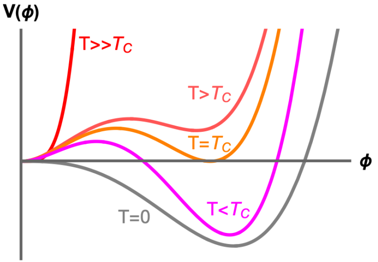
$$\epsilon = \frac{g_\epsilon}{\sqrt{g_Y^2 + g_\epsilon^2}} \approx 0.5 - 0.8$$

Possible signatures?

Possible signatures?
Gravitational Waves!

SSB through Coleman-Weinberg mechanism

- Since $v_H << v_S$, H and S effectively decouple from each other
- S_{min} through Coleman-Weinberg mechanism
  $$V_{tot}(\phi) = V_{CW}(\phi) + V_{thermal}(\phi)\ , \qquad \phi \equiv Re(S)$$
- The Yukawa coupling effect is also included



$$V_{CW}(\phi) = \frac{1}{2} m_S^2(t)\phi^2 + \frac{1}{4}\lambda_2(t)\,\phi^4 + \frac{1}{128\,\pi^2}\left[20\lambda_2^2(t) + 96\,g_X^4(t) - 48\,y_N^4(t)\right]\phi^4\left(-\frac{25}{6} + \ln\frac{\phi^2}{\mu^2}\right)$$

$$V_{\rm thermal}(\phi, T) = \frac{T^4}{2\pi^2}\sum n_i J_i\left(\frac{m_i^2(\phi)}{T^2}\right)$$

$$m_{Z'}^2(\phi) = 4\,g_X^2\,\phi^2$$
$$\mathrm{m}_{\nu_R}^2(\phi) = 2\,y_N^2\,\phi^2$$
$$\mathrm{m}_{\phi}^2(\phi) = 3\,\lambda_2\,\phi^2 + m_S^2$$
$$\mathrm{m}_{G}^2(\phi) = \lambda_2\,\phi^2 + m_S^2$$

Gravitational waves from FOPT

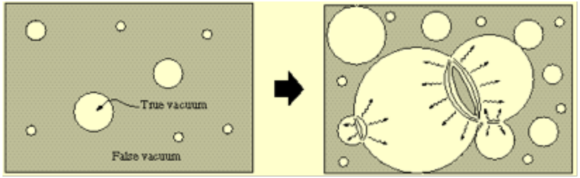
$$V_{eff} \longrightarrow \text{Bubble nucleation}(\Gamma(T)) \longrightarrow \text{Thermal parameters}(\alpha,\, T_{rh},\, \beta) \longrightarrow h^2\Omega(f)$$

Bubble nucleation rate

$\Updownarrow$

depth of potential, barrier between vacua

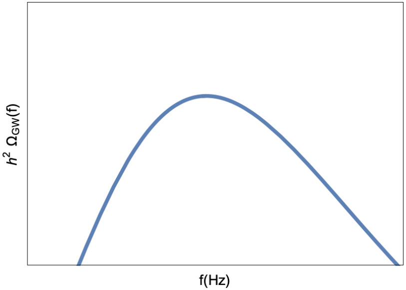
CTC Cambridge Website



The strength of the signal ($h^2\Omega(f)$)

$\Updownarrow$

the latent heat, β, T_{rh}



$$\Omega^{peak}(\alpha, \beta, T_{rh})\,,\; f^{peak}(\alpha, \beta, T_{rh})$$

$$\longrightarrow \mathrm{h}^2\Omega(f) = h^2\Omega^{peak} \times \mathcal{F}(f/f^{peak})$$

GW signals with scale-invariance ($m_S^2 = 0$)

- If $g_X \lesssim 0.25$, the percolation temperature (T_P) is below QCD phase-transition

	g_X ($10^{5,7,9}$GeV)	y_N ($10^{5,7,9}$GeV)
BP1	0.29, 0.29, 0.30	0.16, 0.16, 0.16
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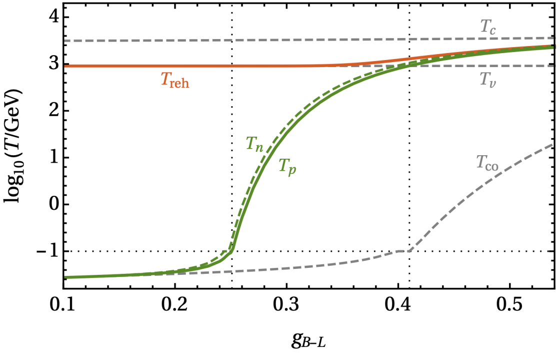
C. Marzo, L.Marzola, V.Vaskonen; 1811.11169



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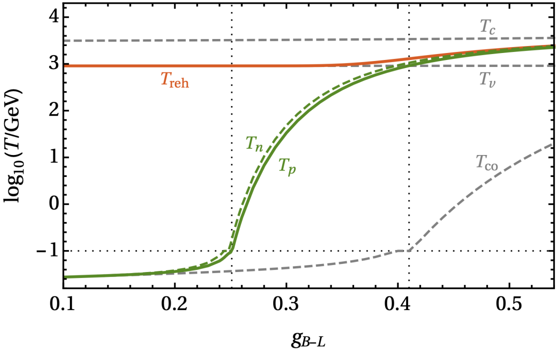

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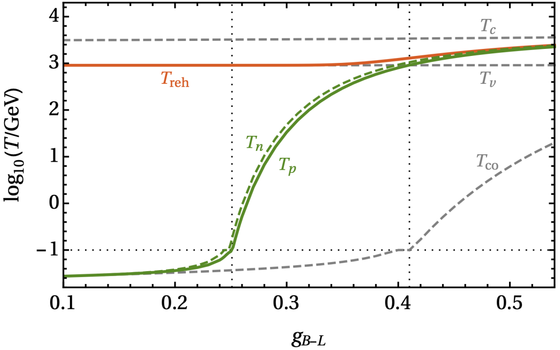

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No GW signal with $m_S^2 = 0$

QG perspective on scale-invariance

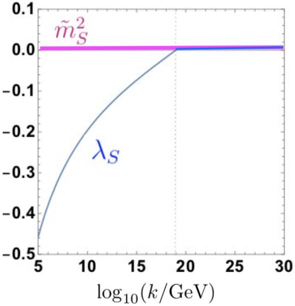
$$\frac{d\tilde{m}_S^2}{dt} \approx (-2 - f_\lambda)\tilde{m}_S^2$$

Gravity corrections to (m_S, λ_S). Similar to gauge and Yukawa

$$\frac{d\lambda_S}{dt} \approx -f_\lambda \lambda_S + \frac{6}{\pi^2} g_X^{*4}$$

$f_\lambda << -2 \implies m_S^{*2} = 0$, irrelevant

(destabilizes the vacuum)



$f_\lambda > 0 \implies \lambda_S$ relevant

m_S^2 is also relevant

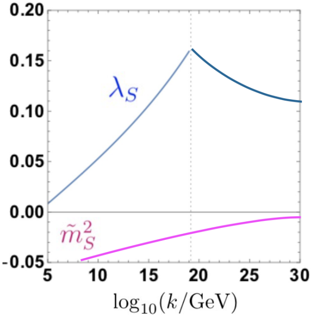


Fig from E.M. Sessolo

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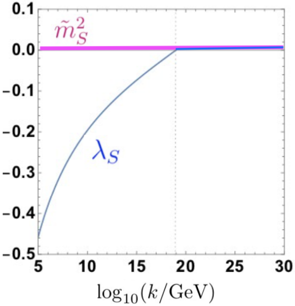
$$\frac{d\lambda_S}{dt} \approx -f_\lambda \lambda_S + \frac{6}{\pi^2} g_X^{*4}$$

$f_\lambda << -2 \implies m_S^{*2} = 0$, irrelevant

(destabilizes the vacuum)



J. M. Pawlowski, M. Reichert, C. Wetterich, and M. Yamada; 1811.11706

A. Pastor-Gutierrez, J. M. Pawlowski, and M. Reichert; 2207.09817

$f_\lambda > 0 \implies \lambda_S$ relevant

m_S^2 is also relevant

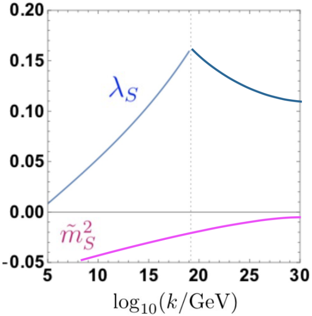


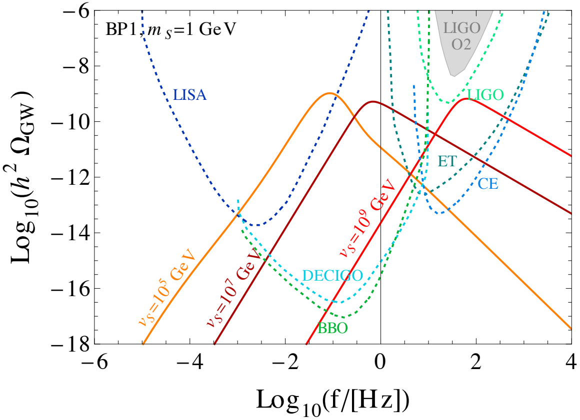
Fig from E.M. Sessolo

FRG based calculations suggests $f_\lambda < 0$. However, including higher dimensional operators of scalar field and curvature terms could alter this conclusion.

GW with different v_S and $m_S^2 \neq 0$

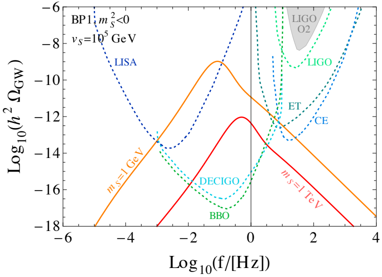
We do have observable GW signals!

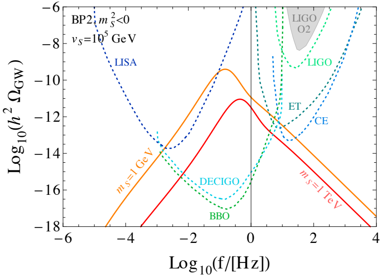
AC, K.Kowalska, E.M. Sessolo, 2308.06114

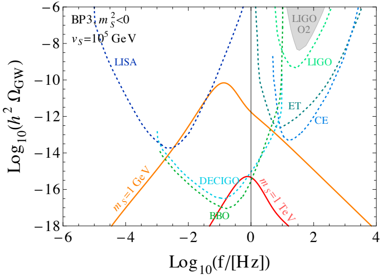


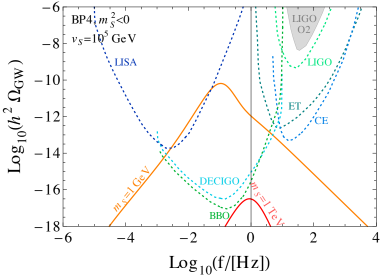
GW signals with $m_S^2 \neq 0$

AC, K.Kowalska, E.M. Sessolo, 2308.06114









But discriminating features are washed out by the strong dependence on mS^2

Conclusions

- Asymptotically safe gravity could induce IR-attractive Gaussian fixed point in y_ν $\Longrightarrow$ dynamical mechanism to generate arbitrarily small Dirac mass for neutrinos
- Existing FRG estimates implies that small Dirac mass arises more naturally in gauged $B-L$ compared to SMRHN
- Observable gravitational wave signal in new-generation interferometers. But discriminating features are obscured due to strong dependence on the mass parameter

Thank you for your attention!