The path to gravitational scattering amplitudes:

form factors, field redefinitions and all that

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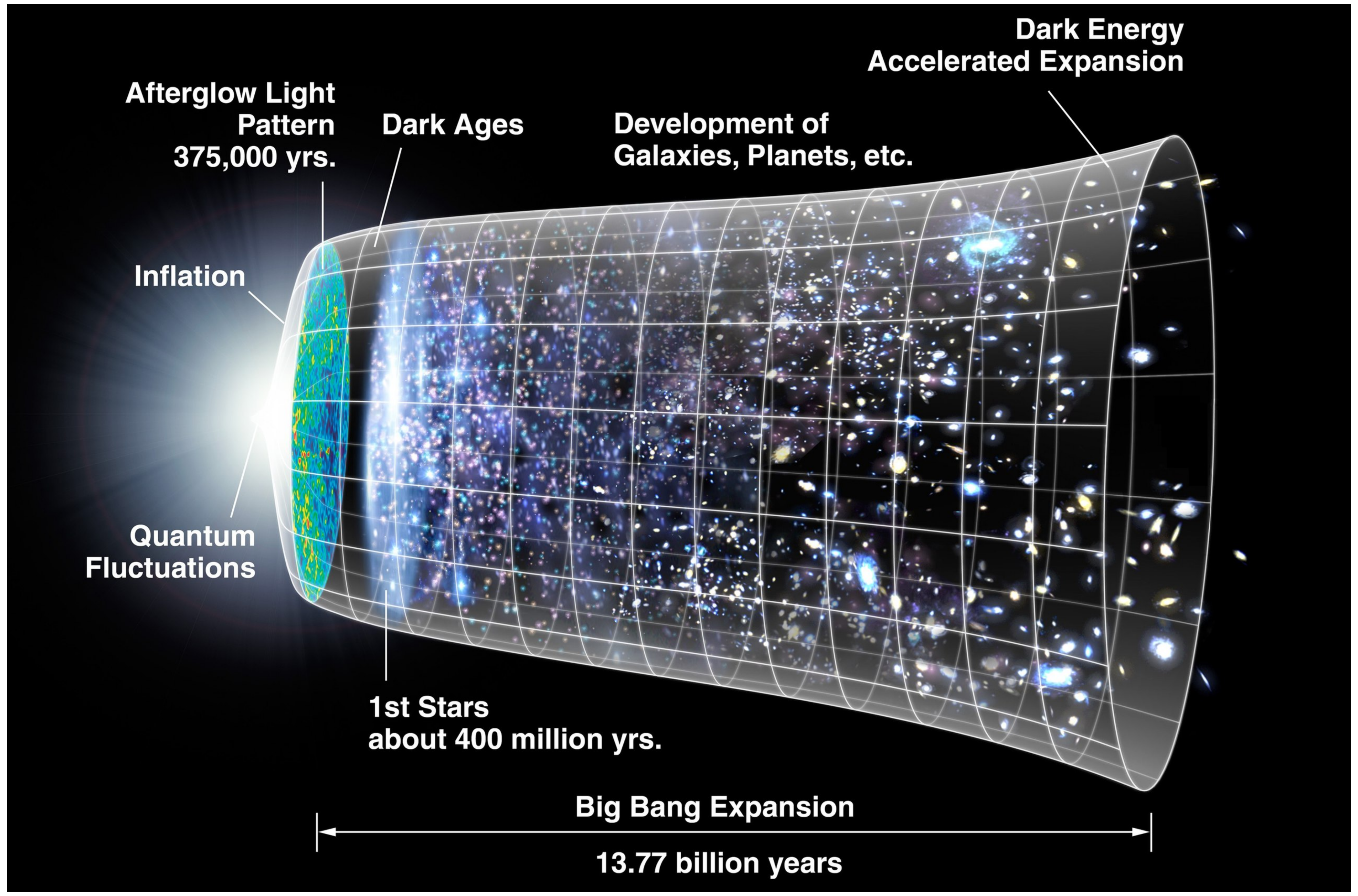
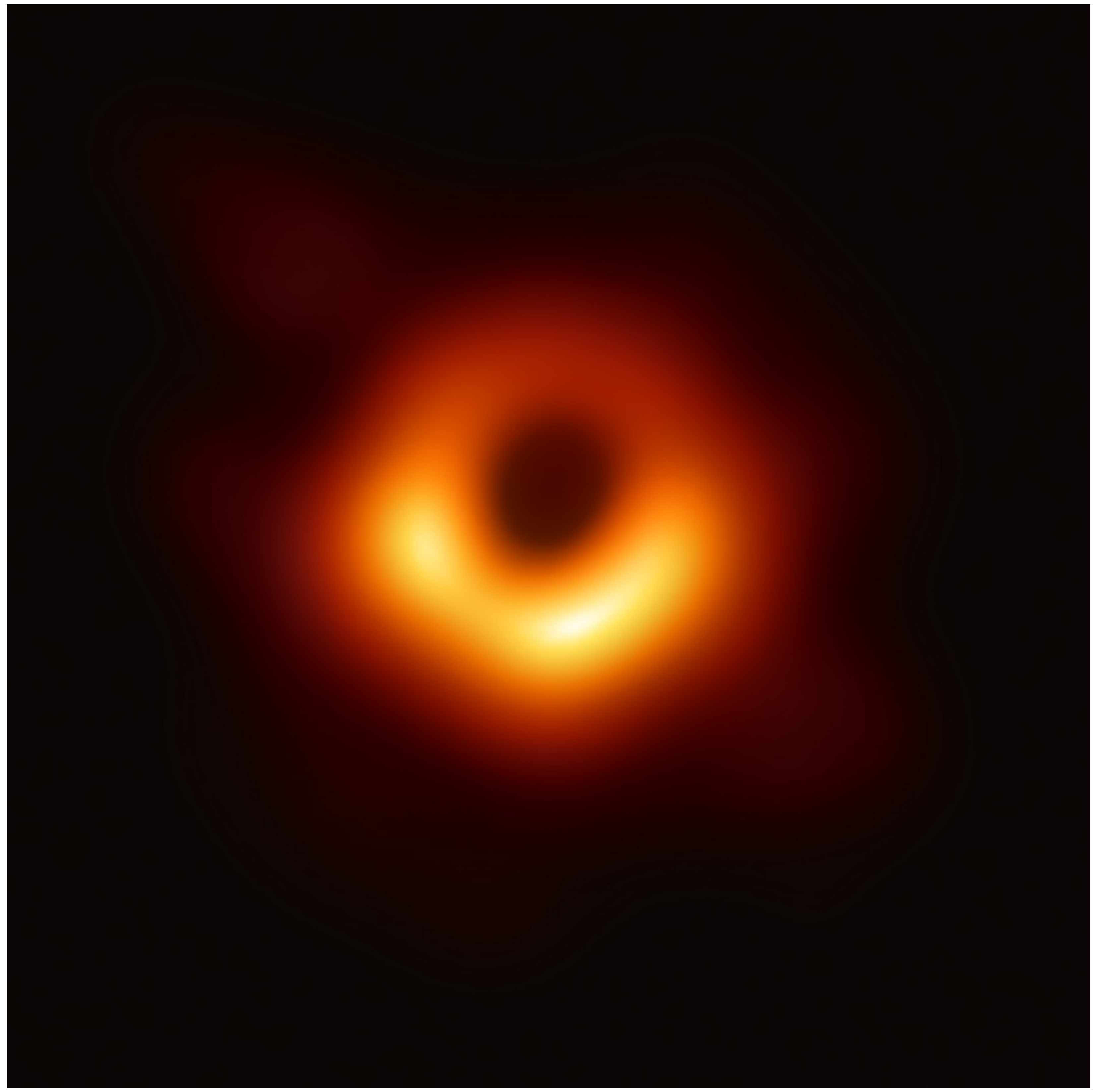
A Perspective on Quantum Gravity





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lack of smoking gun
quantum gravity experiments

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many big ideas
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**why trust any
approach in particular?**

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  1. set up quantum theory of gravity and matter (at least SM)

  2. **simultaneously** confront the theory with as much available theory constraints (unitarity, causality, …) and experimental data (cosmological evolution, particle masses, GWs…) as possible

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- tool of choice: **gravitational scattering amplitudes**

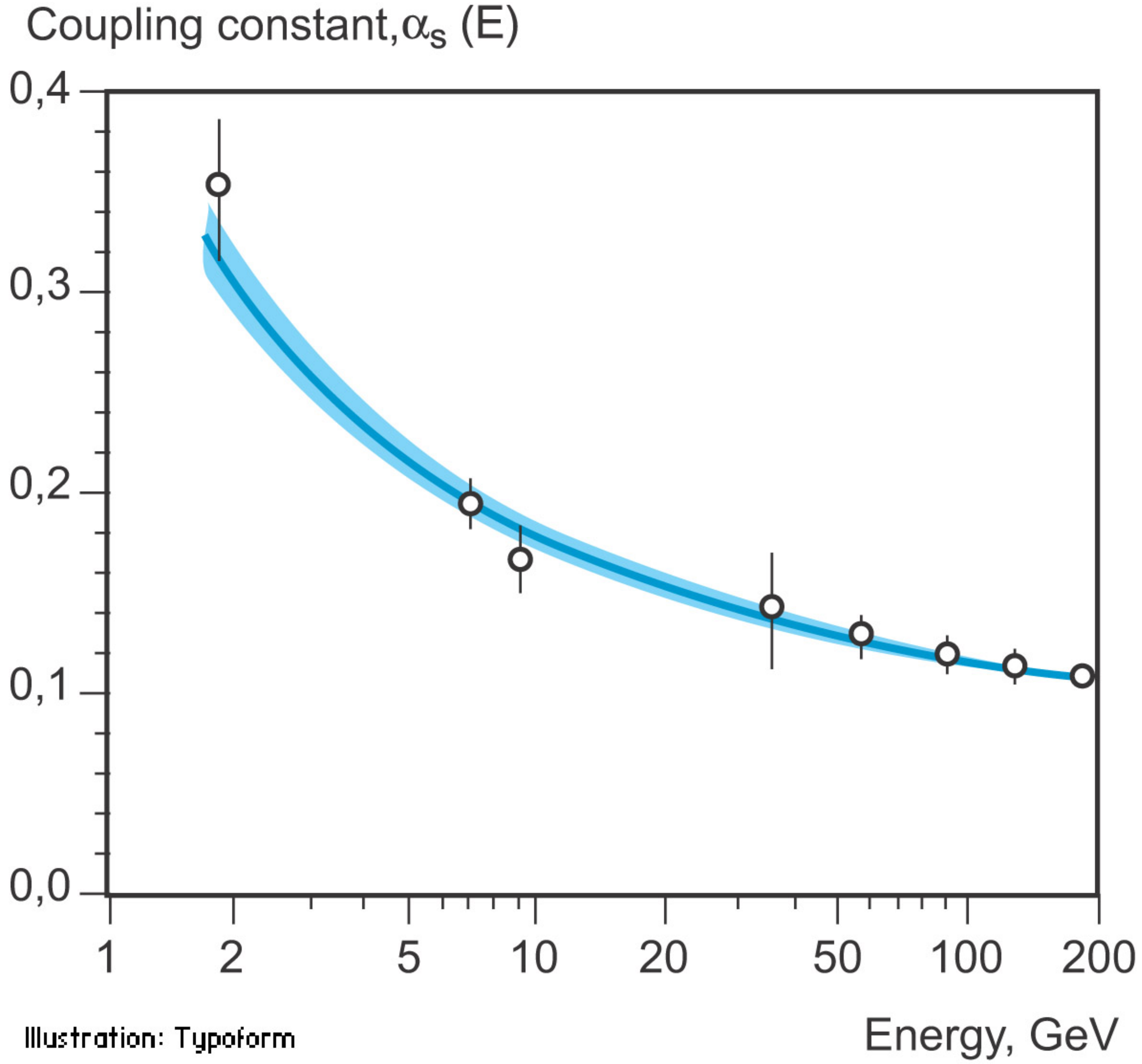
Outline

- Running couplings in a covariant theory - form factors
- Gravity-mediated scattering amplitudes
- Field redefinitions and the minimal essential scheme
- Momentum-dependent field redefinitions in Asymptotic Safety

Running couplings in a covariant theory - form factors

Running coupling constants

- established experimental fact: coupling constants “run with energy”



Nobel prize in Physics 2004
(Gross, Politzer, Wilczek)
“for the discovery of asymptotic freedom in the theory of the strong interaction”

nobelprize.org

Running coupling constants

- established experimental fact: coupling constants “run with energy”
- measure scattering cross sections and compare them to theoretical predictions - coupling “constants” depend on energy scale dictated by their beta functions

$$\beta_{\alpha_s} = -\left(11 - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

Running coupling constants

- What is the fundamental meaning of “running coupling constants”?
  - “fundamental”: discuss in terms of QFT concepts using the language of the effective action Γ
- How do we generalise this notion to a curved spacetime?

Form Factors

- RG running = dependence of a coupling in the effective action on covariant derivatives

$$\text{EM/YM:} \qquad \Gamma = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{4}\mathcal{F}^{\mu\nu}\frac{1}{\alpha_s(\Box)}\mathcal{F}_{\mu\nu} + \mathcal{O}(\mathcal{F}^3)\right]$$

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$$\text{gravity:} \qquad \Gamma = \frac{1}{16\pi G_N}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[2\Lambda - R + R\,f_{RR}(\Box)\,R + S^{\mu\nu}\,f_{SS}(\Box)\,S_{\mu\nu} + \mathcal{O}(\mathcal{R}^3)\right]$$

Form Factors

- interaction terms are more complicated, e.g. three-point function:

$$\Gamma^{(3)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^3}(\Box_1, \Box_2, \Box_3)\, R\, R\, R$$

- four-point function and higher: operator ordering needs convention (difference is of higher order)

$$\Gamma^{(4)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^4}\left(\{-D_i \cdot D_j\}\right)\, R\, R\, R\, R$$

Form Factors

- RG running of couplings generically depends on several momentum scales - there is no unique scale in many processes

see also discussion in 2307.00055
(Buccio, Donoghue, Percacci)

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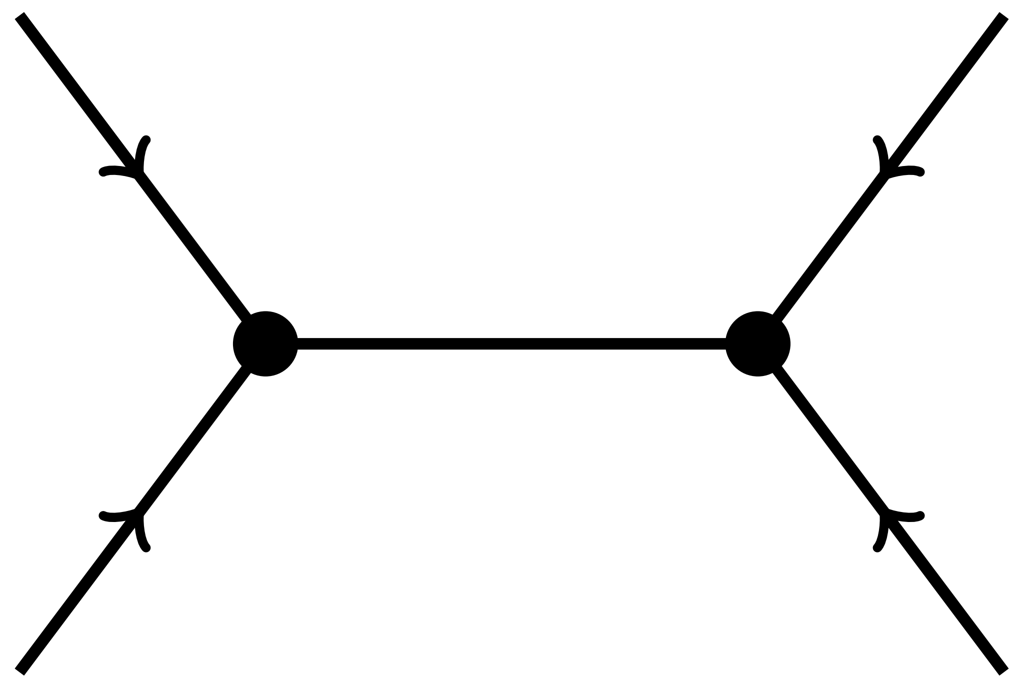
- based on curvature/field strength expansion - can access momentum dependence by considering n-point function at vanishing gauge field/flat metric

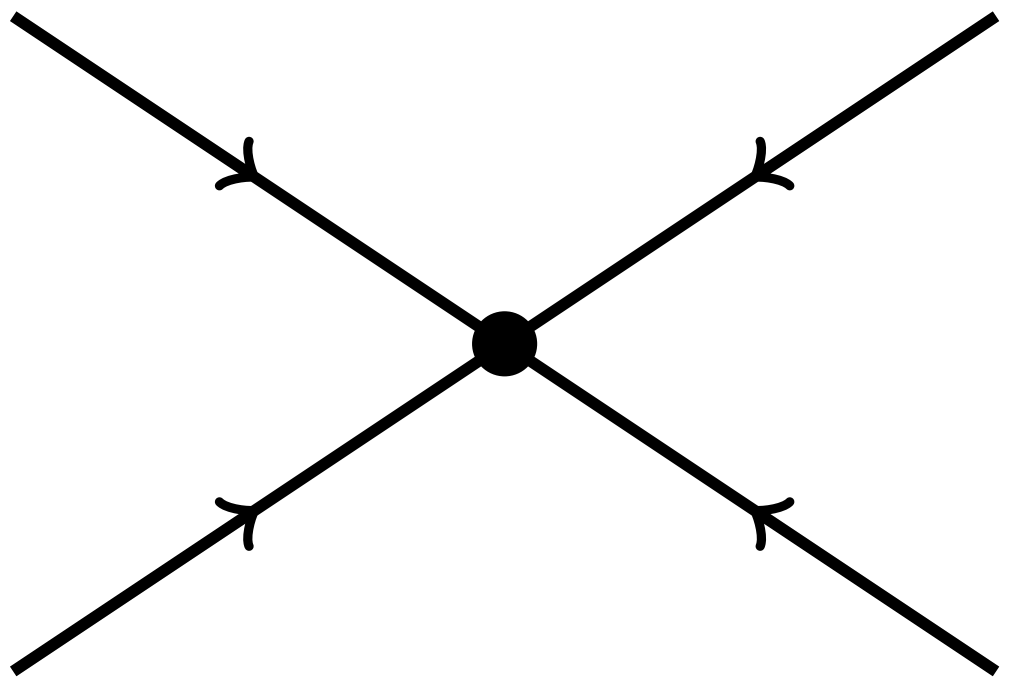
BK-Ripken-Saueressig collaboration: 1907.02903, 2111.12365, 2210.16072

Gravity-mediated scattering amplitudes

Graviton-mediated scattering amplitudes

- easiest non-trivial example: compute $2 \to 2$ gravitational scattering amplitudes





Graviton-mediated scattering amplitudes

- easiest non-trivial example: compute $2 \to 2$ gravitational scattering amplitudes
- benefits:
  - probe quantum gravity effects
  - direct link to observables
  - independent of arbitrary choices
  - use effective action = tree-level diagrams encode “everything”

Graviton-mediated scattering amplitudes

- strategy for a given scattering amplitude:

    1. parameterise all possible terms in the effective action that contribute to the scattering event

    2. compute ingredients from first principles

    3. confront with experimental data and theoretical constraints (finiteness, unitarity, causality, …)

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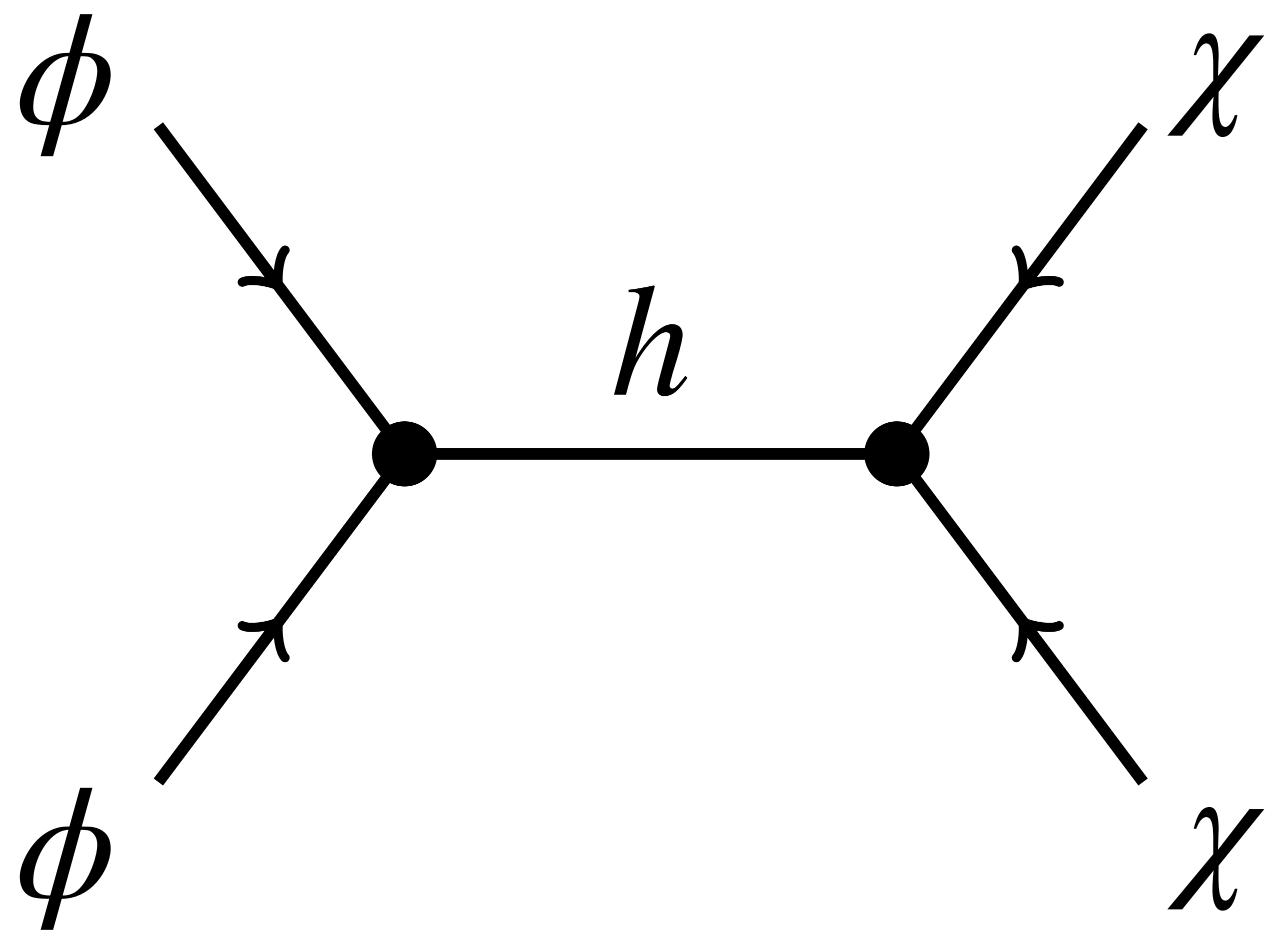
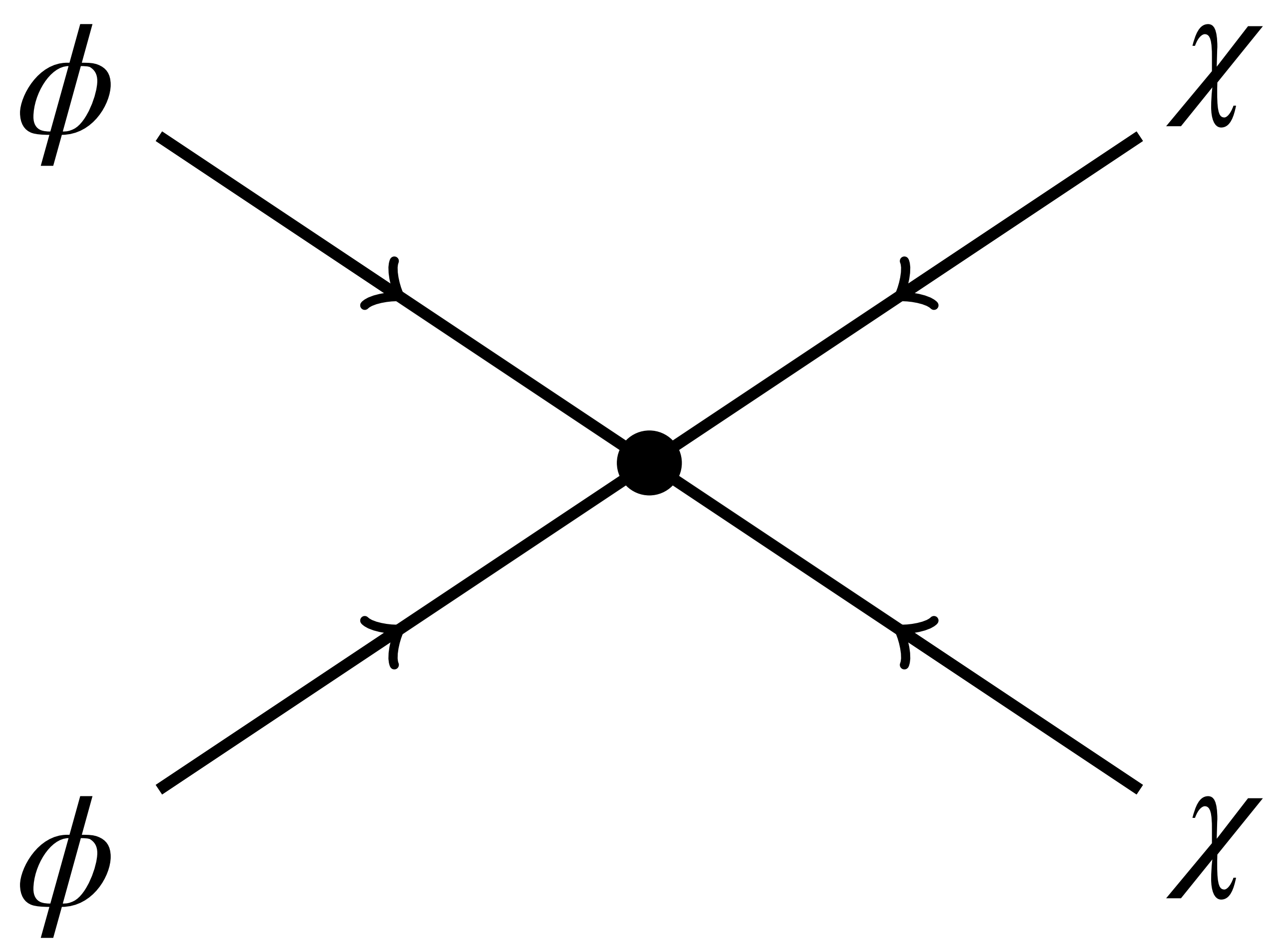
2007.00733, 2007.04396, 2111.12365, 2205.13558, 2210.16072

Scalar-scalar scattering

- gravity-mediated scalar scattering:





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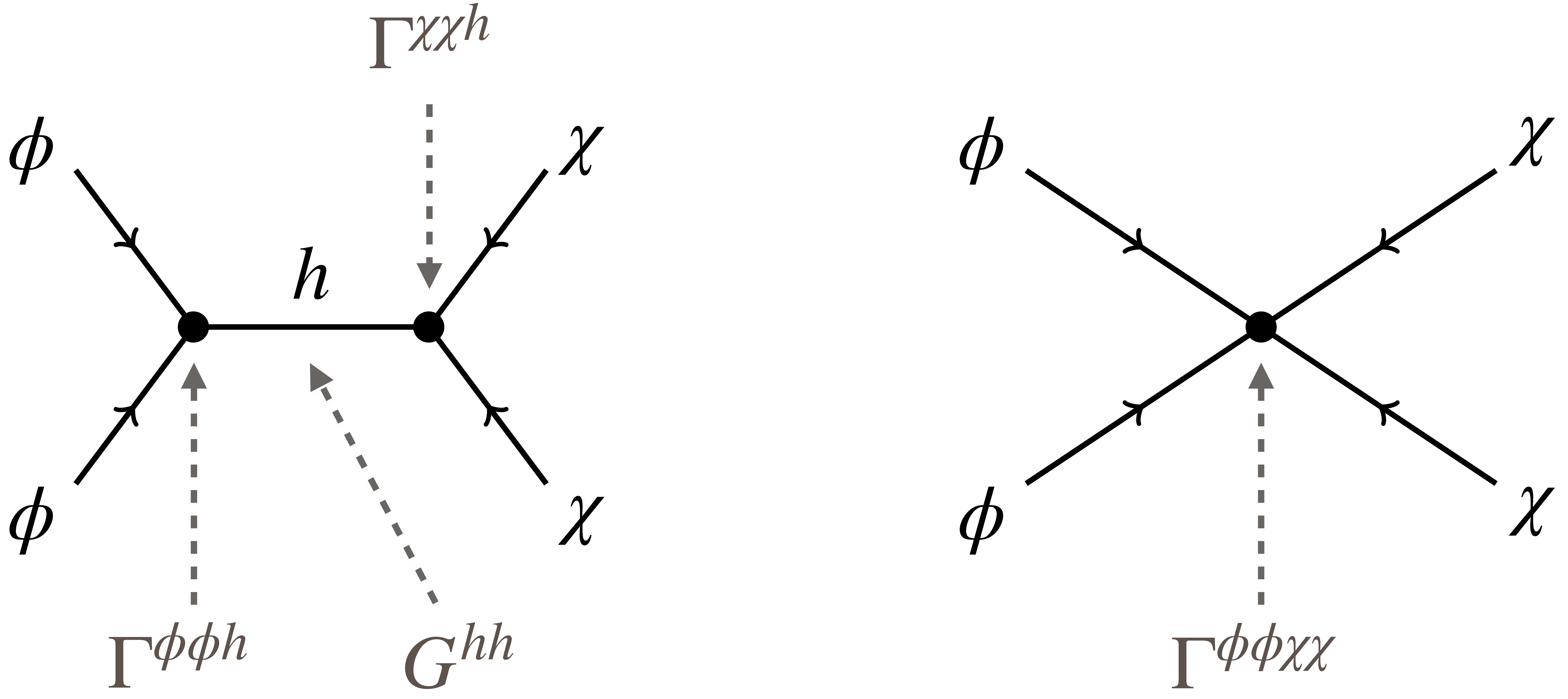


Scalar-scalar scattering

- necessary ingredients in the effective action:

G^{hh}

$$\Gamma \simeq \frac{1}{16\pi G_N} \int \mathrm{d}^4x \, \sqrt{-g} \, [-R + R \, f_R(\Box) \, R + C^{\mu\nu\rho\sigma} f_C(\Box) C_{\mu\nu\rho\sigma}]$$

$$+ \int \mathrm{d}^4x \, \sqrt{-g} \left[\frac{1}{2} \phi \, f_\phi(\Box) \, \phi + f_{R\phi\phi}(\Box_1, \Box_2, \Box_3) R \, \phi \, \phi + f_{Ric\phi\phi}(\Box_1, \Box_2, \Box_3) R^{\mu\nu} \, (D_\mu D_\nu \phi) \, \phi \right] + (\phi \to \chi)$$

$\Gamma^{\phi\phi h}$ $\Gamma^{\chi\chi h}$

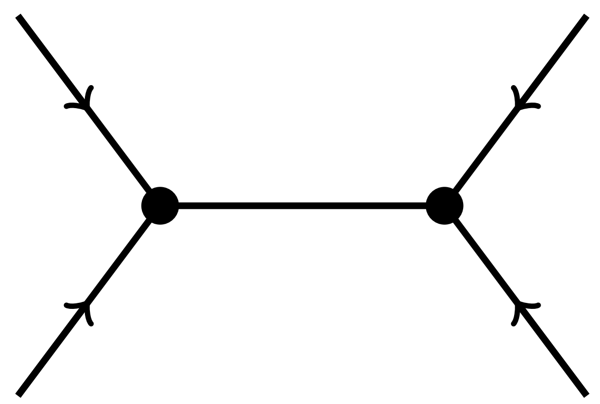
$$+ \frac{1}{(2!)^2} \int \mathrm{d}^4x \, \sqrt{-g} \, f_{\phi\chi}(\{-D_i \cdot D_j\}) \, \phi \, \phi \, \chi \, \chi$$

$\Gamma^{\phi\phi\chi\chi}$

full momentum dependence is key

form factor toolbox:
BK, Ripken, Saueressig
1907.02903

Scalar-scalar scattering



$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Bigg[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$
$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$
$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Bigg]$$

$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1+p_2)^2$$
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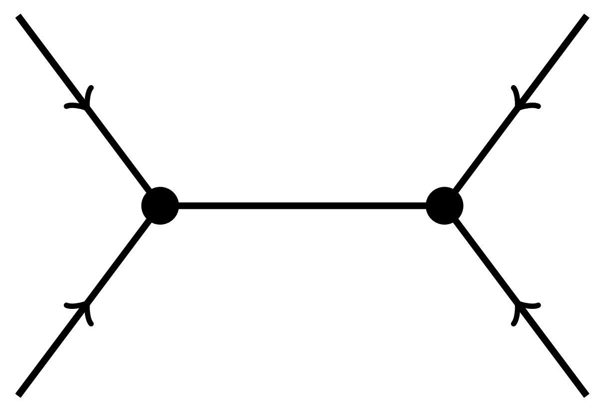
Draper, BK, Ripken, Saueressig
2007.00733, 2007.04396

Scalar-scalar scattering



vertex factors

graviton propagator

contraction factor

spin 2

$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Big[-\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s},m_\phi^2,m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s},m_\chi^2,m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$

$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s},m_\phi^2,m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s},m_\phi^2,m_\phi^2)\right)$$

$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s},m_\chi^2,m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s},m_\chi^2,m_\chi^2)\right) G_R(\mathfrak{s})\Big]$$

spin 0

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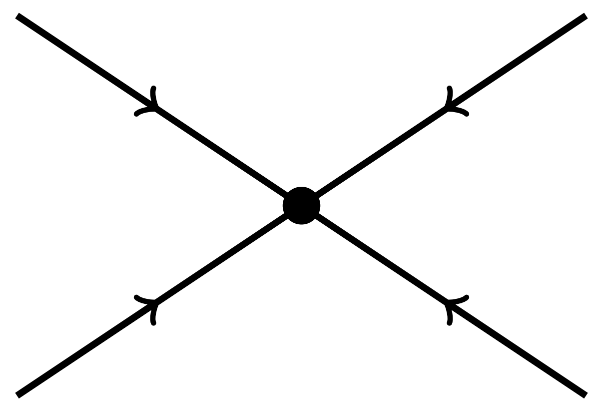

$$\mathcal{A}_4^{\phi\chi} = f_{\phi\chi}\left(\frac{\mathfrak{s}-2m_\phi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{s}-2m_\chi^2}{2}\right)$$

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Field redefinitions and the minimal essential scheme

BK 2311.12097

Field redefinitions

- observables like scattering amplitudes cannot depend on our technical choices

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- different choices of field redefinitions give rise to different schemes, moves momentum dependence in scattering amplitude between different diagrams

BK 2311.12097

Minimal essential scheme

- minimal essential scheme (MES): set everything to zero that you can set to zero by suitable field redefinition

Baldazzi, Ben Alì Zinati, Falls
2105.11482
Baldazzi, Falls
2107.00671

BK 2311.12097

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- have to make assumptions on spectrum of theory:
  - ***[GR]***: propagator only has massless pole
  - ***[Stelle]***: propagator has spectrum of Stelle gravity

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- have to make assumptions on spectrum of theory:
  - ***[GR]***: propagator only has massless pole
  - ***[Stelle]***: propagator has spectrum of Stelle gravity
- in theory with given spectrum, can put propagator into tree-level form

BK 2311.12097

Minimal essential scheme

- ***[GR]***:

$$g_{\mu\nu} \mapsto g_{\mu\nu} + a_R(\Box)\, R\, g_{\mu\nu} + a_S(\Box)\, S_{\mu\nu}$$

BK 2311.12097

Minimal essential scheme

- ***[GR]***:

$$g_{\mu\nu} \mapsto g_{\mu\nu} + a_R(\Box)\, R\, g_{\mu\nu} + a_S(\Box)\, S_{\mu\nu}$$

- can remove quadratic curvature form factors from action (f_{RR},f_{SS}):

$$a_R(\Box) = -\frac{1}{2\Box}\left(1 - \frac{1}{3}\frac{1}{\sqrt{1+\Box f_{SS}(\Box)}} - \frac{2}{3}\frac{1}{\sqrt{1 - 6\Box f_{RR}(\Box) - \frac{1}{2}\Box f_{SS}(\Box)}}\right)$$

$$a_S(\Box) = \frac{2}{\Box}\left(\frac{1}{\sqrt{1+\Box f_{SS}(\Box)}} - 1\right)$$

BK 2311.12097

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- can remove quadratic curvature form factors from action (f_{RR}, f_{SS})

- similarly:

    - can remove almost all non-trivial momentum dependence from cubic curvature form factors, except **local** Goroff-Sagnotti term

    - can remove most of the non-trivial momentum dependence of the quartic curvature form factors

BK 2311.12097

Minimal essential scheme

- path forward: use MES to simplify computations of scattering amplitudes, e.g. in **Asymptotic Safety**
- first step: implement running field redefinitions so that propagator is tree-level at every RG step

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Momentum-dependent field redefinitions in Asymptotic Safety

MES with form factors in Asymptotic Safety

- curvature expansion:

$$\Gamma_k = \frac{1}{16\pi G_{N,k}} \int \mathrm{d}^4x \sqrt{g}\, [2\Lambda_k - R]$$

$$\Psi_{k,\mu\nu} = \gamma_g g_{\mu\nu} + \gamma_R(\Delta)\, R\, g_{\mu\nu} + \gamma_S(\Delta_2)\, S_{\mu\nu}$$

- use FRG - can derive RG equations by hand!

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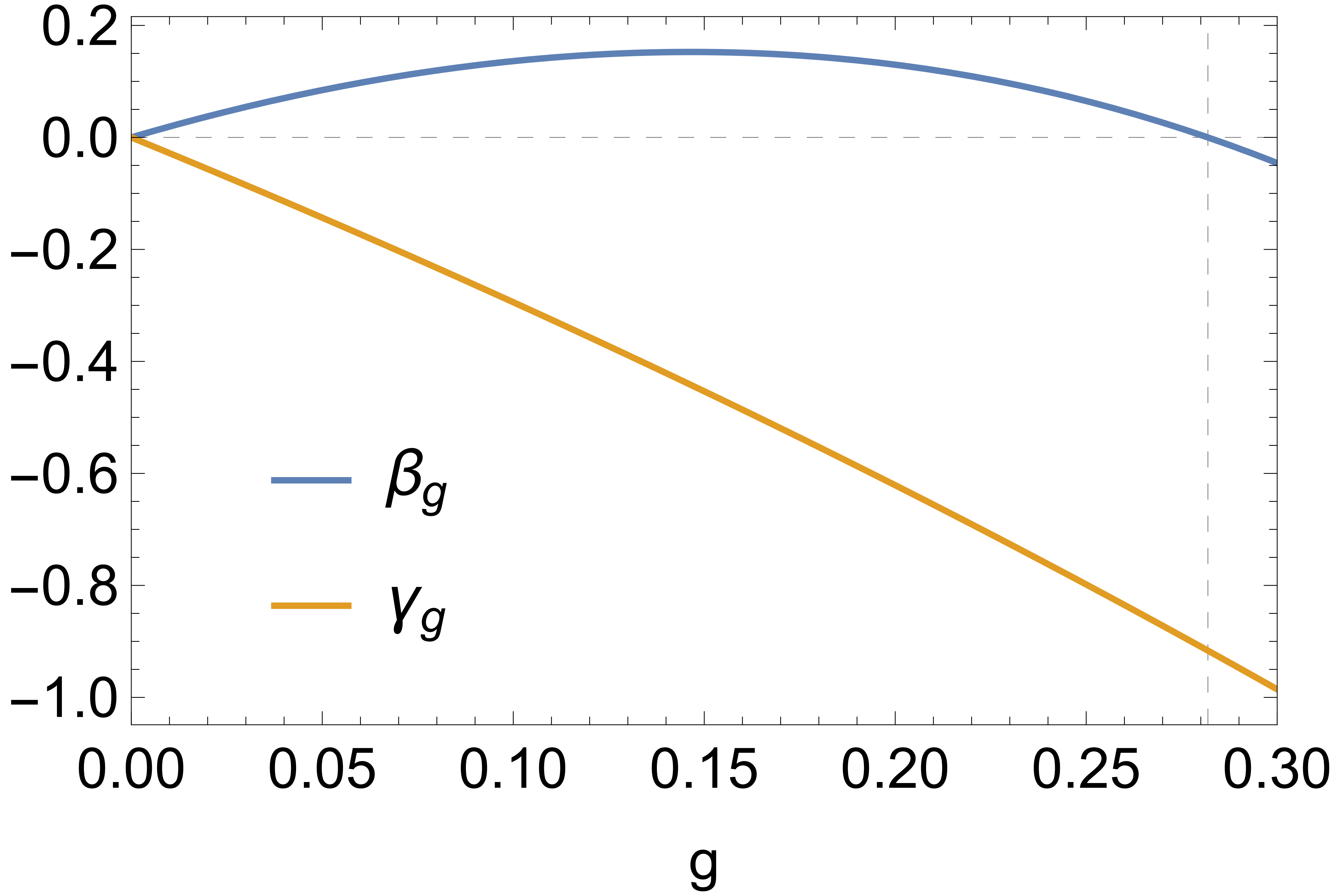
$$k\partial_k \Gamma_k + \Psi_k \circ \Gamma_k^{(1)} = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathfrak{R}_k\right)^{-1}\left\{k\partial_k + 2\Psi_k^{(1)}\right\}\mathfrak{R}_k\right]$$

Pawlowski hep-th/0512261; Baldazzi, Ben Alì Zinati, Falls 2105.11482

BK 2311.12097

MES with form factors in Asymptotic Safety



$\theta = 2.347$

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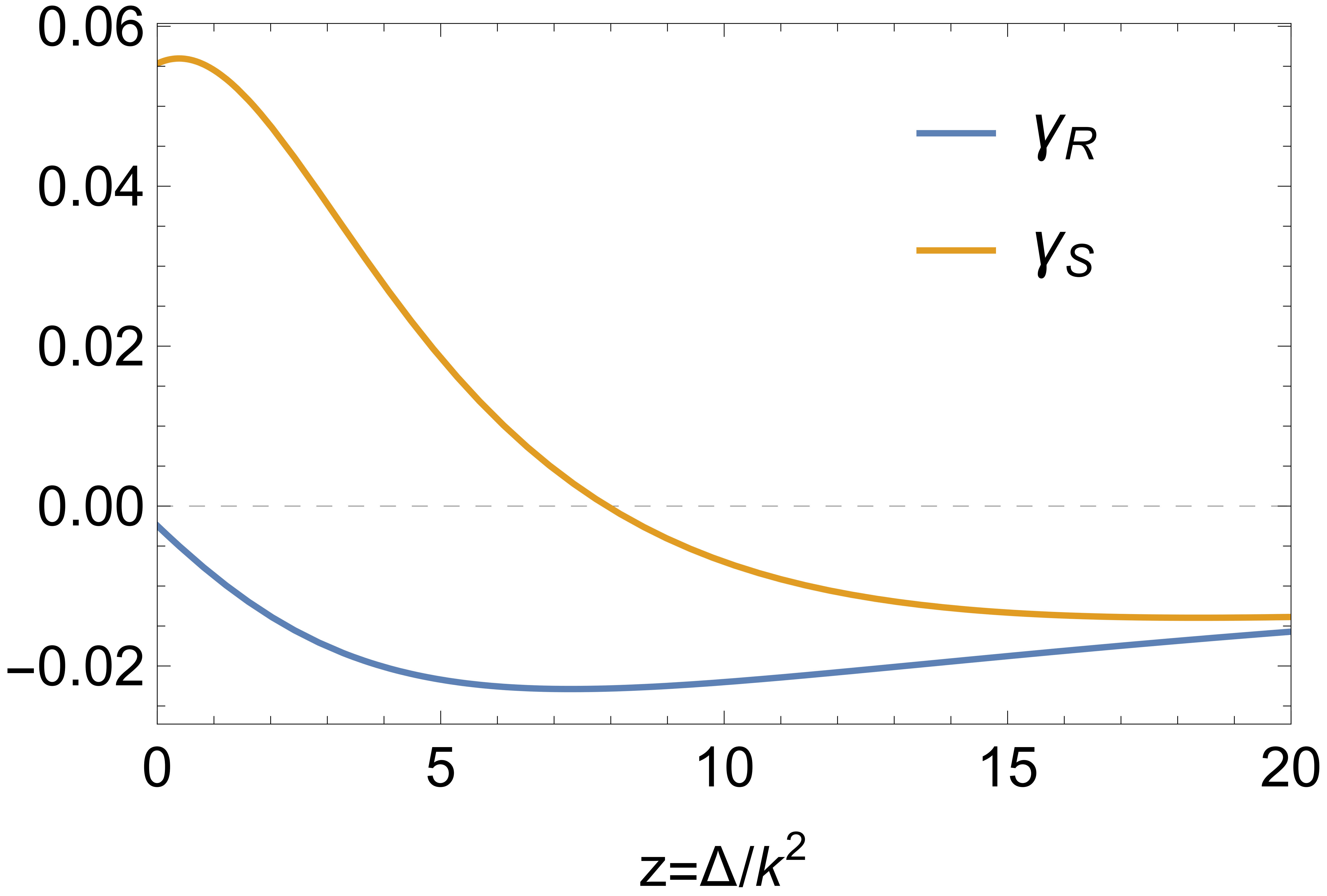
MES with form factors in Asymptotic Safety



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MES with form factors in Asymptotic Safety

- properties of fixed point:
  - standard Reuter FP - strong indication that this FP is in ***[GR]***, no additional degrees of freedom

Platania, Wetterich 2009.06637
Platania 2206.04072

BK 2311.12097

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Platania, Wetterich 2009.06637
Platania 2206.04072

  - running momentum-dependent field redefinitions at this order quantitatively unimportant

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Summary

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- ingredients can be computed ab initio, no need to guess
- field redefinitions allow for significant simplifications

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- TODO: compute three- and four-point function (homework)