Survey of rare and exclusive few-body decays of the Higgs, Z, and W bosons, and the top quark

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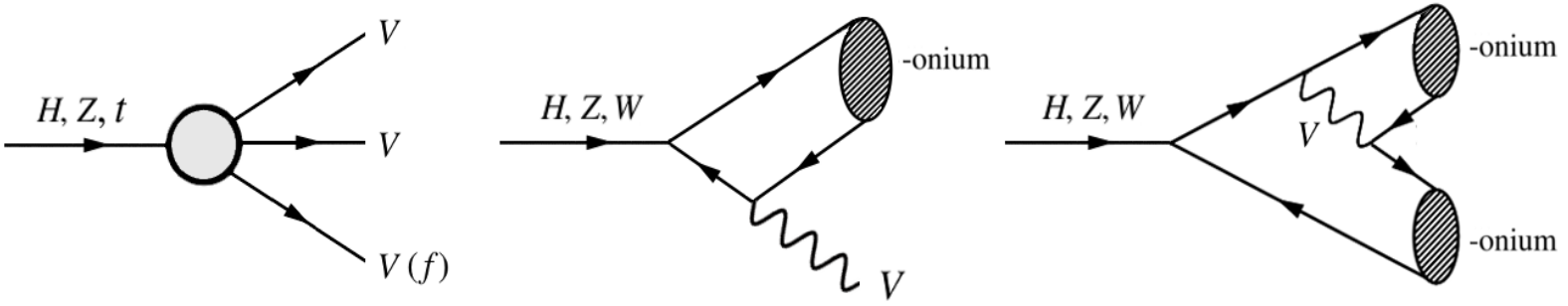
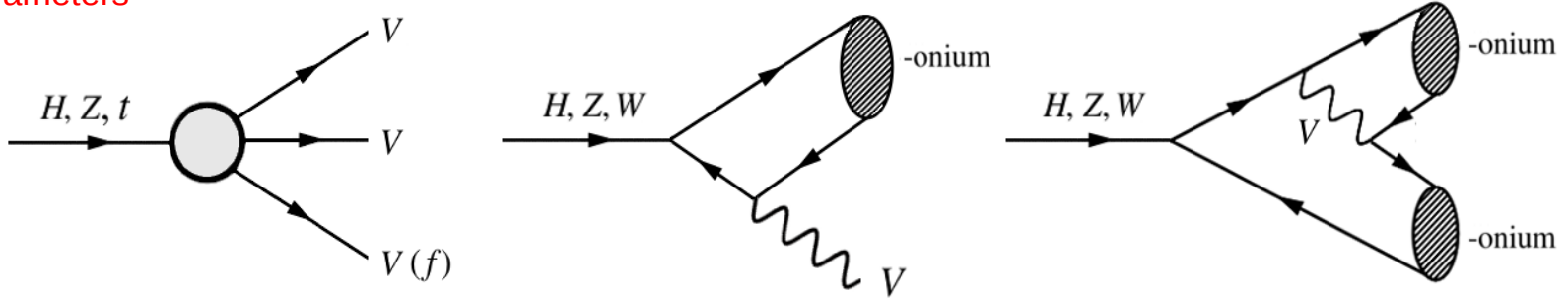


Introduction

- The work:
  - Comprehensive collection of all 2- & 3-body rare/exclusive decays branching ratios (BR<10^{-5}) of the 4 heaviest particles:

    ~150 unobserved channels (~50 upper limits today).
  - Identify rare decay channels missed so far and estimate their rates. Explicitly compute a few new decays (Z,H → leptonium+gamma, Higgs FCNC exclusive decays,...). Update some older theoretical BR results.
  - Make projections for HL-LHC/FCC-ee searches: Help guide and prioritize future experimental searches.
- Physics motivations:
  - Searches for BSM physics that might enhance very rare partial decay widths.
  - Precision tests of very suppressed processes in the SM : FCNC in Z, H, and top decays.
  - Probe lighter quark Yukawa couplings: Hcc, Hqq, via exclusive final states with mesons.
  - Stringent tests of the QCD factorization formalism, constraint poorly known nonperturbative hadronic bound-state parameters



Theoretical predictions

- For all rare decays collected, we indicate the BR of each channel and the theoretical framework used to compute them
  - For rare elementary decays: We indicate the perturbative QCD and/or EW accuracy
  - Exclusive hadronic channels are all based on pQCD factorization: cross-section = perturbative ⊗ non-perturbative. Models of QCD factorization:
    - Light cone (LC): nonperturbative objects described by LCDAs. Applied for light-quark mesons (uds)
    - Soft-Collinear Effective Theory (SCET): Resums multiple scales. Nonperturbative LCDAs. Mostly light mesons.
    - Heavy-Quark Effective Theory (HQET): LCDA describes mixed formation of light-heavy-quark mesons
    - Non-Relativistic QCD (NRQCD): For charmonium & bottomonium objects described by LDMEs
  - Leptonium channels: similar to hadronic ones, with much smaller BR. Never computed before. We have applied similar methods and derived the BR predictions.
- We have updated a few old results & computed a few new ones

using MadGraph5_aMC@NLO (virtual QCD & EW):

Example:

channels	updated		old results
$Br(Z \to \gamma+\gamma+\gamma)$	$= 6.58 \times 10^{-10}$	$\leftarrow$	5.4×10^{-10}
$Br(Z \to g+g+g)$	$= 1.75 \times 10^{-6}$	$\leftarrow$	1.9×10^{-6}
$Br(Z \to \hat{\nu}+\nu+\gamma)$	$= 1.19 \times 10^{-10}$	$\leftarrow$	7.2×10^{-10}
$Br(Z \to \gamma+g+g)$	$= 6.6 \times 10^{-7}$	$\leftarrow$	8.8×10^{-7}

Experimental limits: Present & projections

- For all rare decays collected, we:
  - Indicate all current limits (LEP, Tevatron, LHC), including most recent ones (not yet on PDG).
  - Provide extrapolation of limits for the HL-LHC either from
    - Existing dedicated CMS/ATLAS studies.
    - Our statistical projection from previous results
      - For LHC limits: scale the 13-TeV bounds down by $\sqrt{2 \times 3\ \mathrm{ab}^{-1}/\mathcal{L}_{\mathrm{int}}(13\,\mathrm{TeV})}$ ~ Improvement by ~6.5 factor

Example:

$H \to \gamma$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
	ρ^0	$(1.68 \pm 0.18) \times 10^{-5}$	SCET+LCDA [13]	$< 8.8\times10^{-4}$ [74]	$\lesssim 6.8 \times 10^{-5}$	✓	✓
	ω	$(1.48 \pm 0.17) \times 10^{-6}$	SCET+LCDA [13]	$< 1.5\times10^{-4}$ [76]	$\lesssim 2.2 \times 10^{-5}$	✓	✓
	ϕ	$(2.31 \pm 0.26) \times 10^{-6}$	SCET+LCDA [13]	$< 4.8\times10^{-4}$ [74]	$\lesssim 3.7 \times 10^{-5}$	✓	✓
	J/ψ	$(2.95 \pm 0.38) \times 10^{-6}$	SCET+LCDA [13]	$< 3.5\times10^{-4}$ [77]	$\lesssim 5.5 \times 10^{-5}$ [54]	✓	✓
		$(3.01 \pm 0.15) \times 10^{-6}$	NRQCD (NLL)+LDME [78]				
		$\left(3.0^{+0.2}_{-0.1}\right) \times 10^{-6}$	NRQCD+LCDA [79]				

Experimental limits: Present & projections

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      - For CDF limits, scale bounds down by $\sqrt{N_{\mathrm{X}}(\text{HL-LHC})/N_{\mathrm{X}}(\text{Tevatron})}$ ~ Improvement (W,Z) by ~70 factor

Collider	W± bosons		Z bosons		H bosons		top quarks	
	σ(W)	N(W)	σ(Z)	N(Z)	σ(H)	N(H)	$\sigma(t\bar{t})$	N(top)
LEP	4.0 pb	0.8×10^5	59 nb	2×10^7	~2, 1 fb	~5	–	–
FCC-ee	4.0 pb	5×10^8	59 nb	6×10^{12}	200, 30 fb	1.9×10^6	0.5 pb	3.8×10^6
Increase factor LEP ↦ FCC-ee	1	6250	1	300, 000	70, 30	400, 000	–	–
Tevatron (1.96 TeV, 10 fb^{-1})	25.3 nb	2.5×10^8	7.6 nb	7.6×10^7	1.1 pb	1.1×10^4	7.1 pb	1.4×10^5
HL-LHC (14 TeV, 2 × 3 ab^{-1})	200 nb	1.2×10^{12}	62.5 nb	3.8×10^{11}	58 pb	3.5×10^8	1 nb	1.2×10^{10}
FCC-hh (100 TeV, 30 ab^{-1})	1300 nb	4.1×10^{13}	415 nb	1.2×10^{13}	0.93 nb	2.8×10^{10}	35 nb	2.1×10^{12}
Increase factor Tevatron ↦ HL-LHC	8	4800	8.2	5000	52.7	31 800	141	86 000
Increase factor HL-LHC ↦ FCC-hh	6.5	34	6.7	32	16	80	35	175

Future limits: FCC-ee and FCC-hh reaches

- For all rare decays collected, we:
  - Indicate whether the decay will be producible at FCC-ee/FCC-hh by simply checking the relation $[BR(X)\times N(X)]>1\,?$

Number of H,W,Z,top produced

Collider	W± bosons		Z bosons		H bosons		top quarks	
	σ(W)	N(W)	σ(Z)	N(Z)	σ(H)	N(H)	$\sigma(t\bar{t})$	N(top)
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- For all rare decays collected, we:
  - Indicate whether the decay will be producible at FCC-ee/FCC-hh by simply checking the relation $[BR(X)\times N(X)]>1\,?$
  - Vertical (dashed) line for FCC-ee (FCC-hh) in bar-limits plots:

Example:

$H\to\gamma$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
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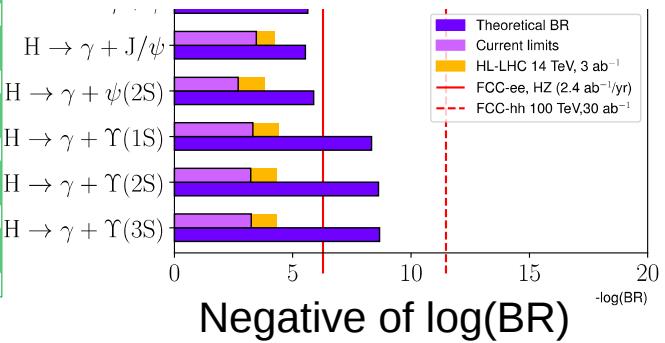
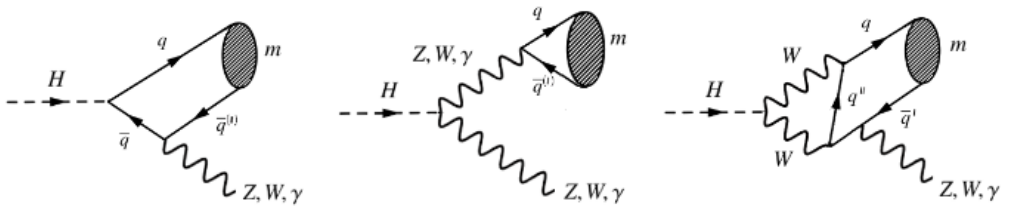
		$(3.01\pm0.15)\times10^{-6}$	NRQCD (NLL)+LDME [78]				
		$\left(3.0^{+0.2}_{-0.1}\right)\times10^{-6}$	NRQCD+LCDA [79]				
	$\psi(2S)$	$(1.3\pm0.1)\times10^{-6}$	SCET+LCDA [13]	$<2.0\times10^{-3}$ [80]	$\lesssim 1.6\times10^{-4}$	✓	✓
		$\left(4.6^{+3.9}_{-2.8}\right)\times10^{-9}$	SCET+LCDA [13]				
$H\to\gamma$ +							



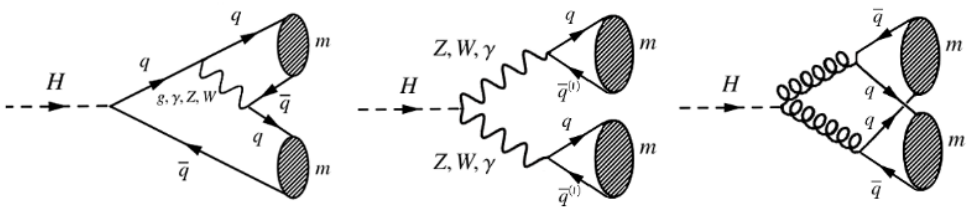
Negative of log(BR)

Exclusive Higgs decays: radiative + meson, meson + meson

- Due to the smallness of the H → cc,qq partial widths, it has been proposed to constrain quark Yukawa couplings via exclusive decays of Higgs into:
  - EW boson + 1 meson:
    - Contributions from 2 main mechanisms (direct, indirect) which interfere destructively.
    - Can be used to probe hZγ effective couplings



  - double meson:
    - Doubly suppressed → very small BR
      → Can't be produced until FCC-hh rates
    - Theoretical predictions have included more of these diagrams with time...



Exclusive Higgs decays
H → γ + meson

H → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → γ +	ρ^0	$(1.68 \pm 0.18) \times 10^{-5}$	SCET+LCDA [13]	$< 8.8\times10^{-4}$ [74]	$\lesssim 6.8 \times 10^{-5}$	✓	✓
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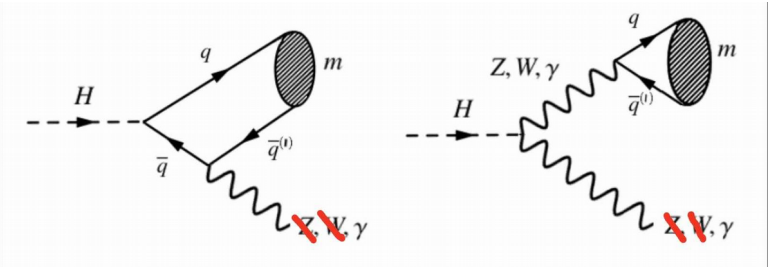
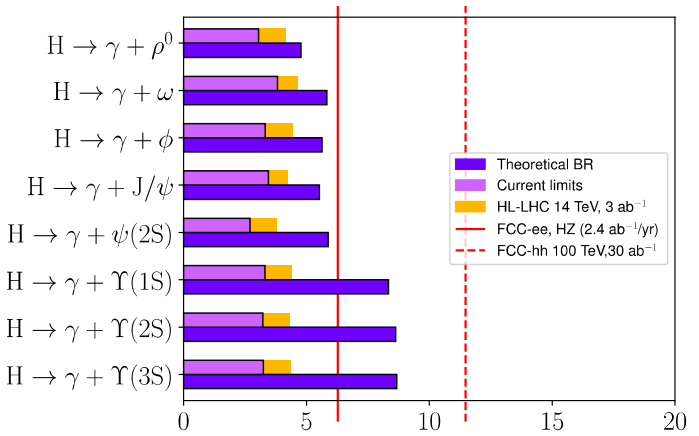
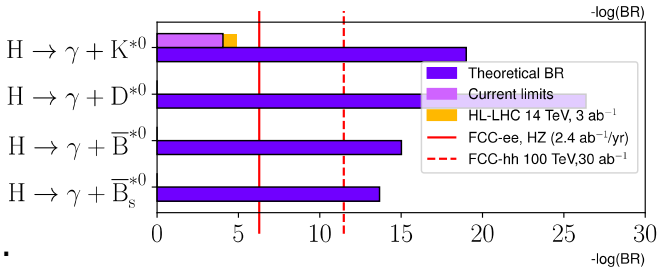
		$(3.01 \pm 0.15) \times 10^{-6}$	NRQCD (NLL)+LDME [78]				
		$(3.0^{+0.2}_{-0.1}) \times 10^{-6}$	NRQCD+LCDA [79]				
	$\psi(2S)$	$(1.3 \pm 0.1) \times 10^{-6}$	SCET+LCDA [13]	$< 2.0\times10^{-3}$ [80]	$\lesssim 1.6 \times 10^{-4}$	✓	✓
	$\Upsilon(1S)$	$(4.6^{+3.9}_{-2.8}) \times 10^{-9}$	SCET+LCDA [13]	$< 4.9\times10^{-4}$ [80]	$\lesssim 3.8 \times 10^{-5}$	✘	✓
		$(10.0^{+4.0}_{-3.0}) \times 10^{-9}$	NRQCD (NLL)+LDME [78]				
		$(5.2^{+2.0}_{-1.7}) \times 10^{-9}$	NRQCD+LCDA [79]				
	$\Upsilon(2S)$	$(2.3^{+1.7}_{-2.2}) \times 10^{-9}$	SCET+LCDA [13]	$< 5.9\times10^{-4}$ [80]	$\lesssim 4.6 \times 10^{-5}$	✘	✓
		$(2.6^{+1.4}_{-0.9}) \times 10^{-9}$	NRQCD (NLL)+LDME [78]				
		$(1.4^{+0.7}_{-0.6}) \times 10^{-9}$	NRQCD+LCDA [79]				
	$\Upsilon(3S)$	$(2.1^{+1.7}_{-2.5}) \times 10^{-9}$	SCET+LCDA [13]	$< 5.7\times10^{-4}$ [80]	$\lesssim 4.4 \times 10^{-5}$	✘	✓
		$(1.9^{+1.1}_{-0.7}) \times 10^{-9}$	NRQCD (NLL)+LDME [78]				
		$(9.1^{+4.8}_{-3.8}) \times 10^{-10}$	NRQCD+LCDA [79]				
H → γ +	K^{*0}	1.0×10^{-19}	EFT+LCDA This work	$< 8.9\times10^{-5}$ [76]	$\lesssim 1.3 \times 10^{-5}$	✘	✘
	D^{*0}	4.4×10^{-27}	EFT+LCDA This work	–	–	✘	✘
	$\overline{B}^{*0}$	9.7×10^{-16}	EFT+LCDA This work	–	–	✘	✘
	$\overline{B}_s^{*0}$	2.1×10^{-14}	EFT+LCDA This work	–	–	✘	✘



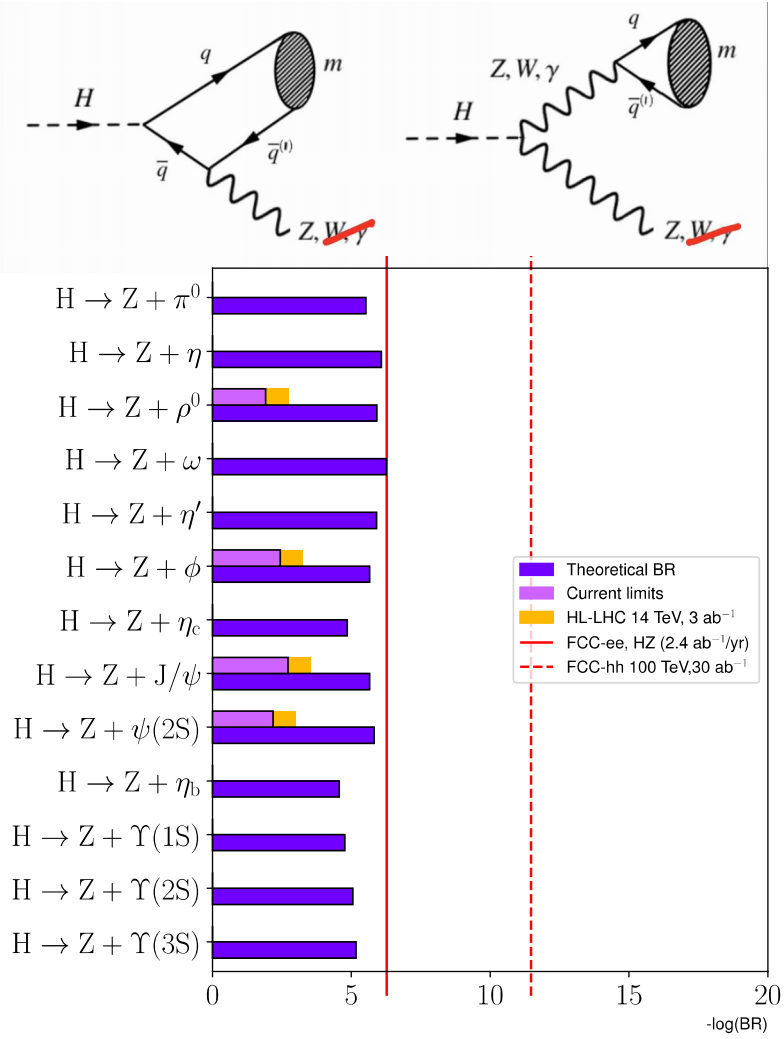




- Theory BRs: O(10^{-5} – 10^{-10}). Exp. limits: O(10^{-3} – 10^{-4})
- 9 channels studied . 5 (8) producible channels at FCC-ee (FCC-hh).
- H→γ+ρ maybe observed at HL-LHC
- Higgs FCNC γ + flavoured-meson decays estimated by us for the 1st time: BR~10^{-14} – 10^{-27} (in the absence of BSM)

Exclusive Higgs decays
H → Z + meson

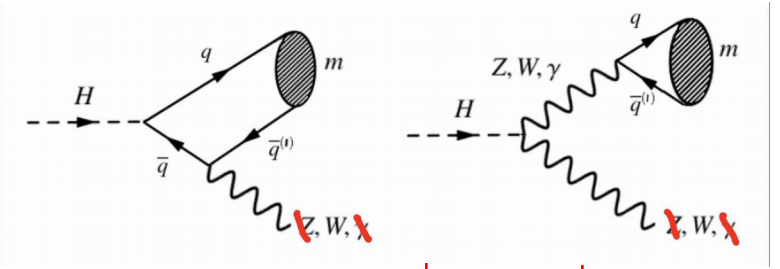
H → Z +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → Z +	π^0	$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		3.0×10^{-6}	EFT+NRQM [11]				
	η	$(8.3 \pm 0.9) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
	ρ^0	$(7.19 \pm 0.29) \times 10^{-6}$	EFT+LCDA [86]	$< 1.2 \times 10^{-2}$ [87]	$\lesssim 1.8 \times 10^{-3}$	✓	✓
		1.2×10^{-6}	EFT+NRQM [11]				
	ω	$(5.6 \pm 0.2) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
	η'	$(1.24 \pm 0.13) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
	ϕ	$(2.42 \pm 0.10) \times 10^{-6}$	EFT+LCDA [86]	$< 3.6 \times 10^{-3}$ [87]	$\lesssim 5.4 \times 10^{-4}$	✓	✓
		2.2×10^{-6}	EFT+NRQM [11]				
	η_c	$(1.00 \pm 0.01) \times 10^{-5}$	EFT+LCDA [88]	–	–	✓	✓
		1.4×10^{-5}	EFT+NRQM [11]				
	J/ψ	3.4×10^{-6}	NRQCD (NLO)+LMDE [89]	$< 1.9 \times 10^{-3}$ [90]	$\lesssim 2.9 \times 10^{-4}$ [56]	✓	✓
		$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [86]				
		2.2×10^{-6}	EFT+NRQM [11]				
		3.2×10^{-6}	EFT+NRQM [91]				
	$\psi(2S)$	1.5×10^{-6}	EFT+NRQM [91]	$< 6.6 \times 10^{-3}$ [90]	$\lesssim 1.0 \times 10^{-3}$	✓	✓
	η_b	$(2.69 \pm 0.05) \times 10^{-5}$	EFT+LCDA [88]	–	–	✓	✓
		$\left(4.7^{+0.3}_{-0.2}\right) \times 10^{-5}$	EFT (NLO)+LCDA [92]				
	$\Upsilon(1S)$	1.7×10^{-5}	NRQCD (NLO)+LMDE [89]	–	–	✓	✓
		$(1.54 \pm 0.06) \times 10^{-5}$	EFT+LCDA [86]				
		1.7×10^{-5}	EFT+NRQM [91]				
	$\Upsilon(2S)$	$(7.5 \pm 0.3) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		8.9×10^{-6}	EFT+NRQM [91]				
	$\Upsilon(3S)$	$(5.63 \pm 0.24) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		6.7×10^{-6}	EFT+NRQM [91]				

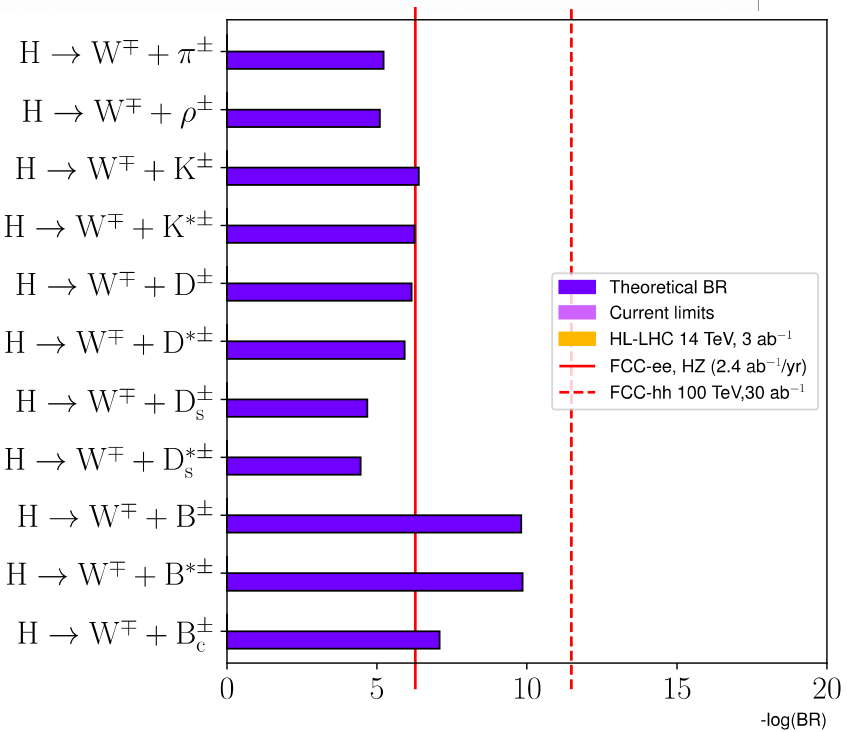


- Theory BRs: O(10^{-5} – 10^{-7}). Exp. limits: O(10^{-2} – 10^{-3})
- 4 channels searched for . All channels are producible at FCC-ee
- Botomonia have largest BRs, but no bound set so far.
- No observable channel at HL-LHC

Exclusive Higgs decays
H → W + meson

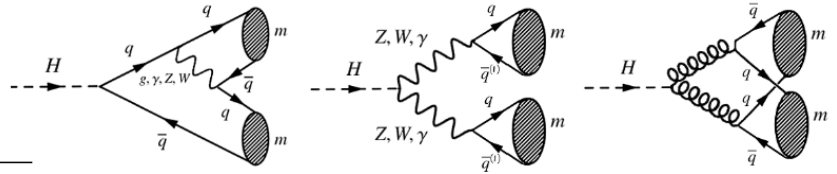
H → W∓ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → W∓ +	$\pi^{\pm}$	$(4.3 \pm 0.2) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		6.0×10^{-6}	EFT+NRQM [11]				
	$\rho^{\pm}$	$(1.09 \pm 0.05) \times 10^{-5}$	EFT+LCDA [86]	–	–	✓	✓
		8.0×10^{-6}	EFT+NRQM [11]				
	$K^{\pm}$	$(3.3 \pm 0.1) \times 10^{-7}$	EFT+LCDA [86]	–	–	✘	✓
		4.0×10^{-7}	EFT+NRQM [11]				
	$K^{*\pm}$	$(5.6 \pm 0.4) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
	$D^{\pm}$	$(5.6 \pm 0.5) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
		7.0×10^{-7}	EFT+NRQM [11]				
	$D^{*\pm}$	$(1.04 \pm 0.14) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		1.2×10^{-6}	EFT+NRQM [11]				
	$D_s^{\pm}$	$(1.71 \pm 0.11) \times 10^{-5}$	EFT+LCDA [86]	–	–	✓	✓
		2.1×10^{-5}	EFT+NRQM [11]				
	$D_s^{*\pm}$	$(2.51 \pm 0.19) \times 10^{-5}$	EFT+LCDA [86]	–	–	✓	✓
		3.5×10^{-5}	EFT+NRQM [11]				
	$B^{\pm}$	$(1.54 \pm 0.40) \times 10^{-10}$	EFT+LCDA [86]	–	–	✘	✓
	$B^{*\pm}$	$(1.41 \pm 0.36) \times 10^{-10}$	EFT+LCDA [86]	–	–	✘	✓
	$B_c^{\pm}$	$(8.21 \pm 0.83) \times 10^{-8}$	EFT+LCDA [86]	–	–	✘	✓



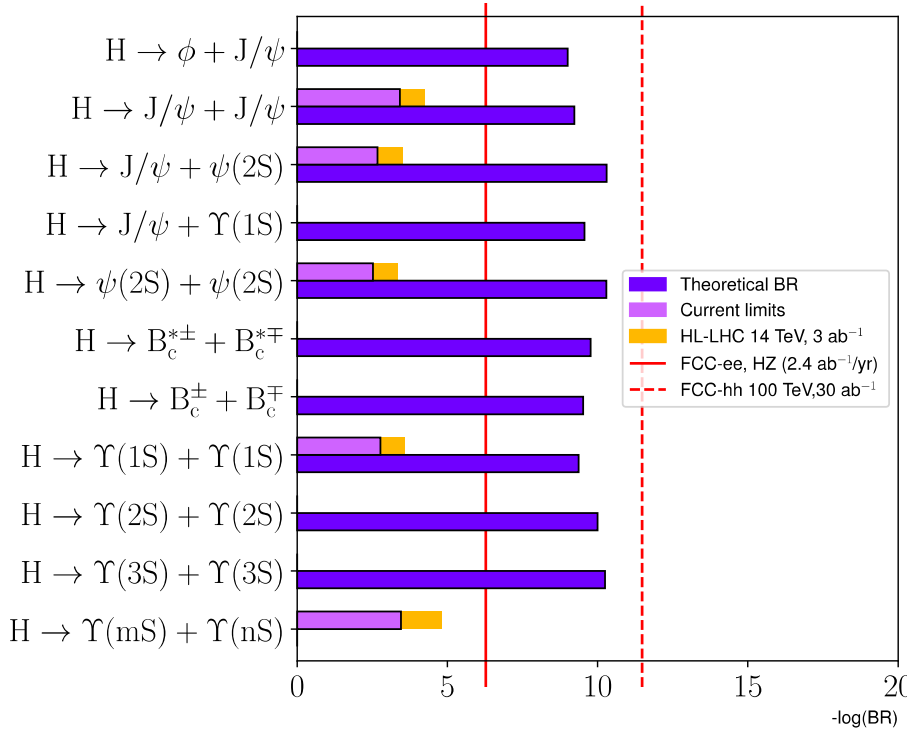


- Theory BRs: O(10^{-5} – 10^{-10}). No Exp. Limits.
- No search-performed so far. 7 (11) producible channels at FCC-ee (FCC-hh)

Exclusive Higgs decays
H → meson + meson

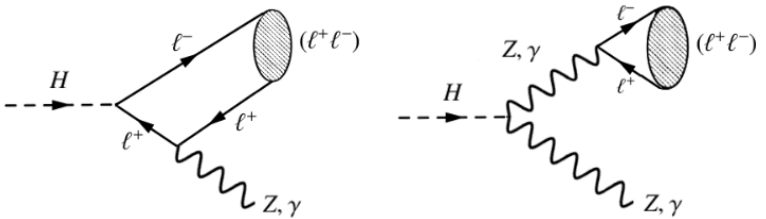


H →	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H→	ϕ	+	J/ψ	1.0×10^{-9}	LC+LCDA [96]	–	–	✘	✓
	J/ψ	+	J/ψ	$(5.8 – 6.0) \times 10^{-9}$	NRQCD+LDME [101]	$< 3.8\times10^{-4}$ [90]	$\lesssim 5.8 \times 10^{-5}$	✘	✓
				1.7×10^{-10}	RQM [98]				
				2.1×10^{-10}	RQM [100]				
				$(5.9 \pm 2.3) \times 10^{-10}$	NRQCD/NRCSM [99]				
				1.5×10^{-10}	LC+LCDA [96]				
	$\psi(2S)$	+	J/ψ	5.0×10^{-11}	–	$< 2.1\times10^{-3}$ [90]	$\lesssim 3.2 \times 10^{-4}$	✘	✓
			$\psi(2S)$	$(5.1 \pm 2.0) \times 10^{-11}$	NRQCD/NRCSM [99]	$< 3.0\times10^{-3}$ [90]	$\lesssim 4.5 \times 10^{-4}$	✘	✓
	$B_c^{*\mp}$	+	$B_c^{*\pm}$	$(1.4 – 1.7) \times 10^{-10}$	RQM [97]	–	–	✘	✓
	$B_c^{\mp}$	+	$B_c^{\pm}$	$(2.0 – 3.0) \times 10^{-10}$	RQM [97]	–	–	✘	✓
	$\Upsilon(1S)$	+	J/ψ	$(2.7 – 3.6) \times 10^{-10}$	NRQCD+LDME [101]	–	–	✘	✓
				1.6×10^{-11}	LC+LCDA [96]				
			$\Upsilon(1S)$	$(8.5 – 9.2) \times 10^{-10}$	NRQCD+LDME [101]	$< 1.7\times10^{-3}$ [90]	$\lesssim 2.6 \times 10^{-4}$	✘	✓
				1.8×10^{-10}	RQM [98]				
				2.3×10^{-9}	RQM [100]				
				$(4.3 \pm 0.9) \times 10^{-10}$	NRQCD/NRCSM [99]				
				2.3×10^{-9}	LC+LCDA [96]				
	$\Upsilon(2S)$	+	$\Upsilon(2S)$	$(1.0 \pm 0.2) \times 10^{-10}$	NRQCD/NRCSM [99]	–	–	✘	✓
	$\Upsilon(3S)$	+	$\Upsilon(3S)$	$(5.7 \pm 1.2) \times 10^{-11}$	NRQCD/NRCSM [99]	–	–	✘	✓
	$\Upsilon(mS)$	+	$\Upsilon(nS)$		–	$< 3.5\times10^{-4}$ [90]	$\lesssim 1.5 \times 10^{-5}$ [56]	✘	✘

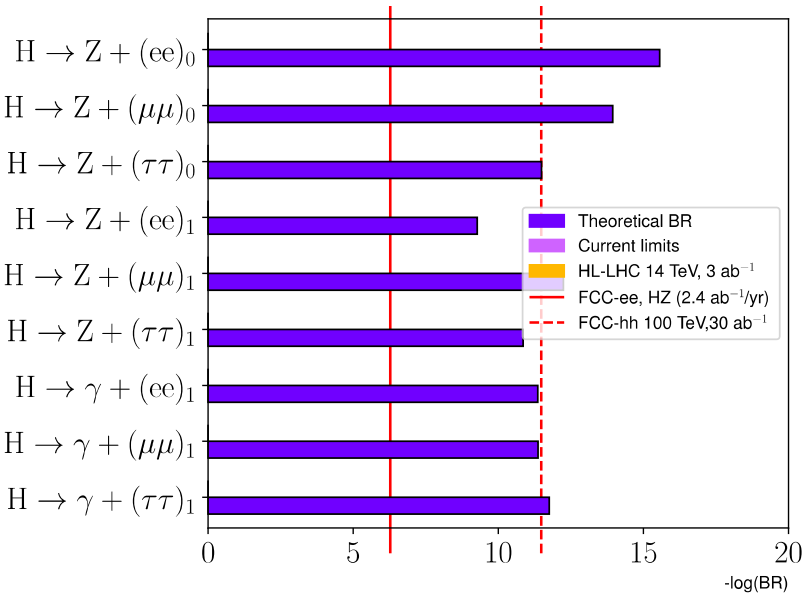


- Theory BRs: O(10^{-9}–10^{-11}). Exp. limits: O(10^{-3} – 10^{-4}).
- 5 channels searched-for. No (all) producible channels at FCC-ee (FCC-hh)
- Many predictions for double-QQbar from adding more contributing diagrams.

Exclusive Higgs decays
H → γ + leptonium



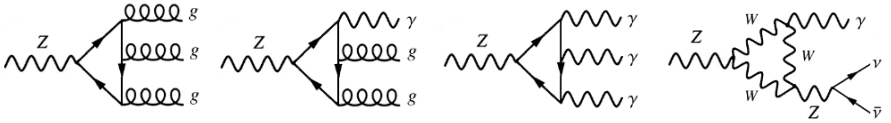
				Exp. limits		Producible at	
H → γ +	X	Branching fraction	Framework	2023	HL-LHC	FCC-ee	FCC-hh
H → γ +	$(ee)_1$	4.4×10^{-12}	This work	–	–	✘	✓
	$(\mu\mu)_1$	4.3×10^{-12}	This work	–	–	✘	✓
	$(\tau\tau)_1$	1.8×10^{-12}	This work	–	–	✘	✘
H → Z +	$(ee)_0$	2.7×10^{-16}	This work	–	–	✘	✘
	$(\mu\mu)_0$	1.1×10^{-14}	This work	–	–	✘	✘
	$(\tau\tau)_0$	3.2×10^{-12}	This work	–	–	✘	✘
	$(ee)_1$	5.4×10^{-10}	This work	–	–	✘	✓
	$(\mu\mu)_1$	5.7×10^{-13}	This work	–	–	✘	✘
	$(\tau\tau)_1$	1.4×10^{-11}	This work	–	–	✘	✓



- Tiny BRs O(10^{-11} – 10^{-16}). First time computed here.
- No channel searched for. No (4) producible channels at FCC-ee (FCC-hh)
- Note: Leptonia are long-lived = LLP signature (displaced γ, e, μ vertices)

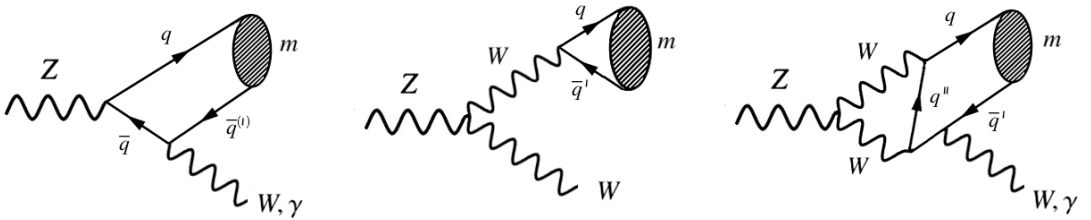
Rare and exclusive Z decays

- Rare Z decays: Old BRs (<year 2000) have been updated

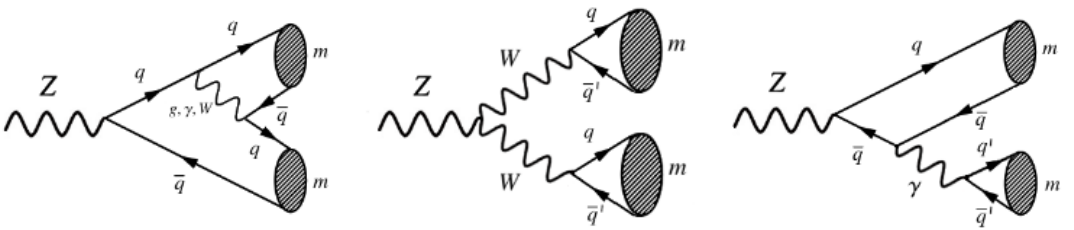


- Exclusive Z decays:
  - Similar mechanism to the exclusive Higgs radiative decays.
  - Large Z boson yields at colliders.

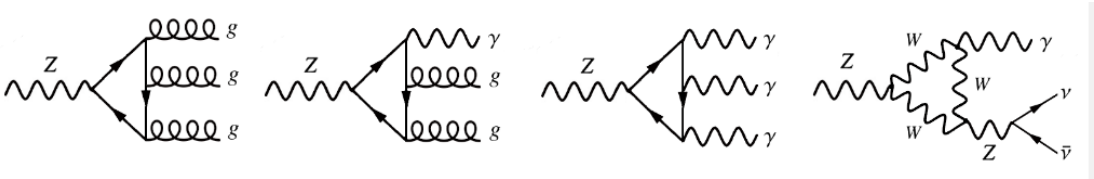
    Provides valuable information both theoretical (SCET & NRQCD validation, and LCDAs/LDMEs' params) and experimental (optimization of search techniques to study exclusive Higgs boson decays).
  - Exclusive Z decays into:
    - γ,W + 1 meson:



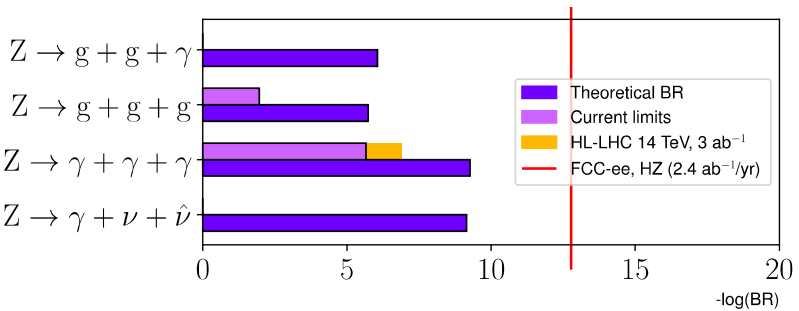
    - Double mesons:



Rare Z decays

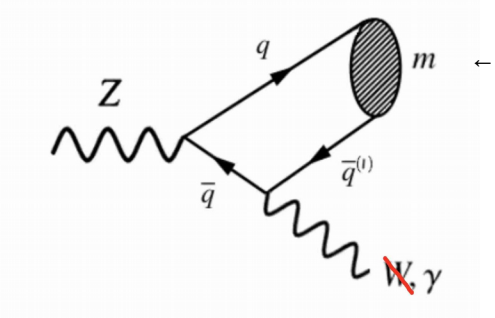


Z→ X+Y+Z	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
$Z \to g+g+g$	1.9×10^{-6}	NLO QCD [107]	$< 1.1\times10^{-2}$ [109]	?	✓
	$(1.752^{+0.003}_{-0.003}) \times 10^{-6}$	NLO QCD This work			
$Z \to \gamma+g+g$	8.8×10^{-7}	NLO QCD [105]	–	–	✓
	$(6.62 \pm 0.01) \times 10^{-7}$	NLO QCD+EW This work			
$Z \to \gamma+\gamma+\gamma$	5.4×10^{-10}	NLO QCD+EW [106]	$< 2.2\times10^{-6}$ [110]	$\lesssim 1.3 \times 10^{-7}$	✓
	$(6.58 \pm 0.01) \times 10^{-10}$	NLO EW This work			
$Z \to \gamma+\nu+\hat{\nu}$	7.2×10^{-10}	NLO EW [108]	–	–	✓
	$(1.195^{+0.003}_{-0.003}) \times 10^{-10}$	NLO QCD+EW This work			



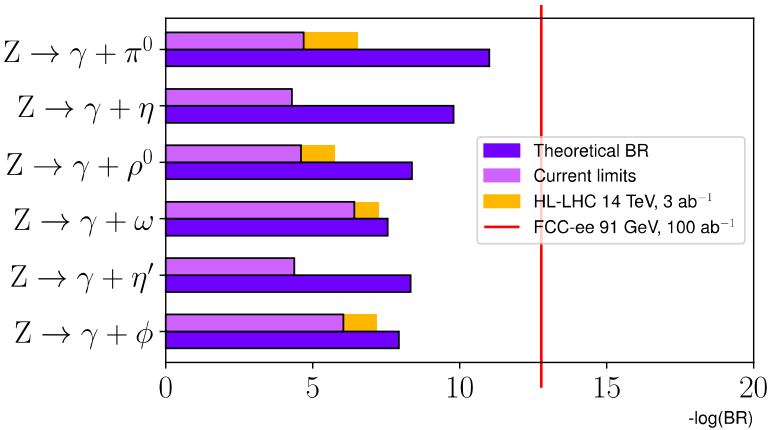
- Theory BRs: O(10^{-5} – 10^{-10}). Exp. limits: O(10^{-2} – 10^{-6}).
- 2 channels searched for. 4 producible channels at FCC-ee.
- Recomputed/Updated with MG5@NLO here.
- All SM channels are unobservable at HL-LHC, but will be cleanly visible at FCC-ee

Exclusive decays:
Z → γ + light meson



←light quarks (u, d ,s)

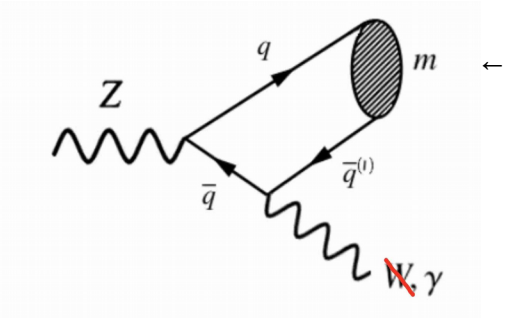
Z → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z → γ +	π^0	$(9.8 \pm 1.0) \times 10^{-12}$	SCET+LCDA [58]	$< 2.0\times10^{-5}$ [116]	$\lesssim 2.8 \times 10^{-7}$	✓
	η	$(1.0 - 17.0) \times 10^{-10}$	SCET+LCDA [117]	$< 5.1\times10^{-5}$ [118]	?	✓
	ρ^0	$(4.19 \pm 0.47) \times 10^{-9}$	SCET+LCDA [58]	$< 2.5\times10^{-5}$ [74]	$\lesssim 1.8 \times 10^{-6}$	✓
	ω	$(2.82 \pm 0.41) \times 10^{-8}$	SCET+LCDA [58]	$< 3.8\times10^{-7}$ [76]	$\lesssim 5.7 \times 10^{-8}$	✓
	η'	$(3.1 - 4.8) \times 10^{-9}$	SCET+LCDA [117]	$< 4.2\times10^{-5}$ [118]	?	✓
	ϕ	$(1.17 \pm 0.08) \times 10^{-8}$	LC+LCDA [119]	$< 9.0\times10^{-7}$ [74]	$\lesssim 6.6 \times 10^{-8}$	✓
		$(1.04 \pm 0.12) \times 10^{-8}$	SCET+LCDA [58]			



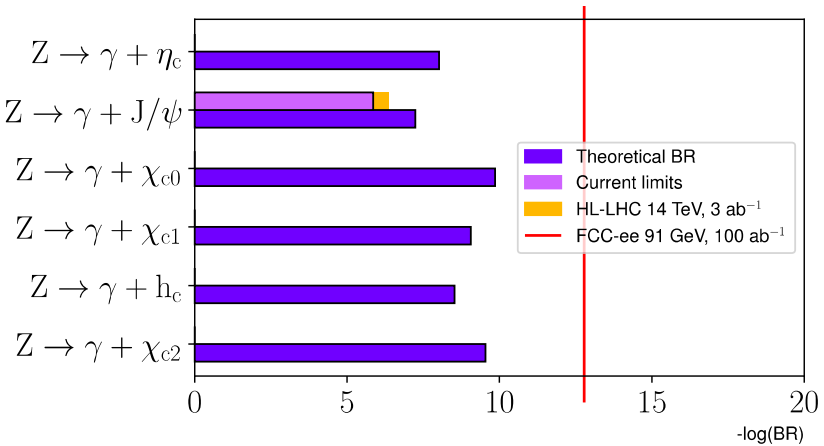
- Theory BRs: O(10^{-8} – 10^{-11}). Exp. Limits: O(10^{-5} – 10^{-7})
- 6 channels searched for. 6 producible channels at FCC-ee.
- Z→γ+ω is very close to be detected at HL-LHC (BR = ½ of projected limit)
- All channels will be visible at FCC-ee

Exclusive Z decays:
Z → γ + charm meson

Z → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z → γ +	η_c	$(9.5 \pm 0.2) \times 10^{-9}$	NRQCD (NNLO+NLL) [120]	–	–	✓
		$(7.42 \pm 0.61) \times 10^{-9}$	NRQCD (NLO+NLL) [121]			
		6.6×10^{-9}	NRQCD+LDME [122]			
		$(9.4 \pm 1.0) \times 10^{-9}$	LC+LCDA [122]			
	J/ψ	$\left(5.75^{+0.08}_{-0.09}\right) \times 10^{-8}$	NRQCD (NNLO+NLL) [120]	$< 1.4\times10^{-6}$ [114]	$\lesssim 4.4 \times 10^{-7}$ [54]	✓
		$\left(9.0^{+1.5}_{-1.4}\right) \times 10^{-8}$	LC+LCDA [123]			
		4.5×10^{-8}	NRQCD+LDME [122]			
		$(8.8 \pm 0.9) \times 10^{-8}$	LC+LCDA [122]			
		$(9.96 \pm 1.86) \times 10^{-8}$	NRQCD+LDME [119]			
		$(8.02 \pm 0.45) \times 10^{-8}$	SCET+LCDA [58]			
	χ_{c0}	$(3.74 \pm 0.05) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
		1.4×10^{-10}	NRQCD+LDME [122]			
		$(5.0 \pm 2.0) \times 10^{-10}$	LC+LCDA [122]			
	χ_{c1}	$\left(2.38^{+0.01}_{-0.02}\right) \times 10^{-9}$	NRQCD+LDME [124]	–	–	✓
		8.7×10^{-10}	NRQCD+LDME [122]			
		$(5.6 \pm 2.0) \times 10^{-9}$	LC+LCDA [122]			
	h_c	$\left(3.49^{+0.21}_{-0.23}\right) \times 10^{-9}$	NRQCD+LDME [124]	–	–	✓
		3.0×10^{-9}	NRQCD+LDME [122]			
		$(1.0 \pm 0.4) \times 10^{-8}$	LC+LCDA [122]			
	χ_{c2}	$\left(3.38^{+0.19}_{-0.22}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
		2.9×10^{-10}	NRQCD+LDME [122]			
		$(1.0 \pm 0.4) \times 10^{-9}$	LC+LCDA [122]			

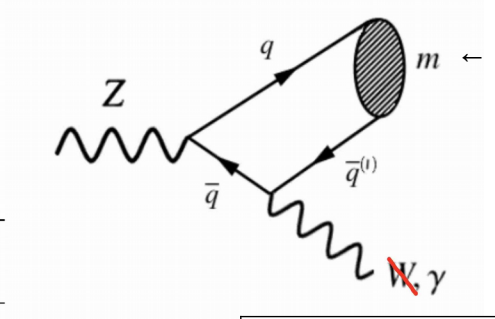


← c quarks



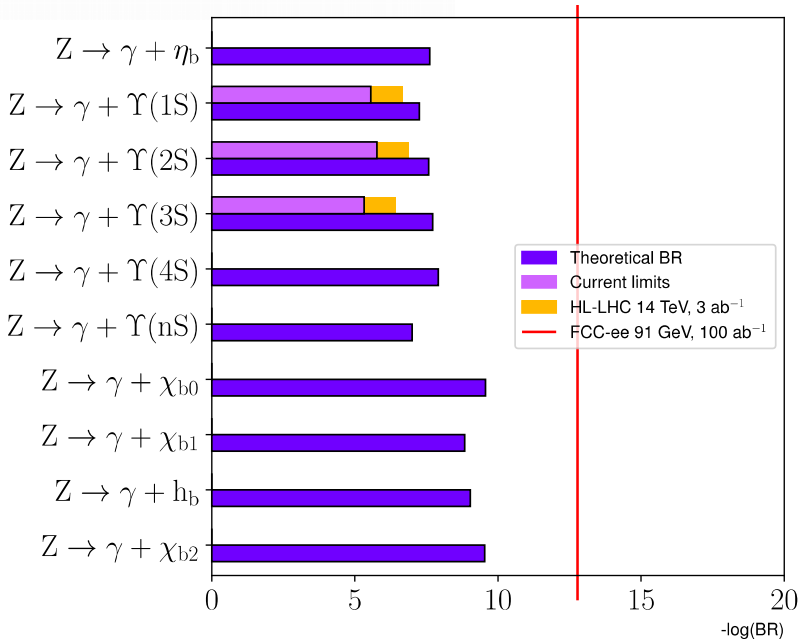
- Theory BRs: O(10^{-8} – 10^{-10}). Multiple calculations (LC, SCET, NRQCD). Exp. limits: O(10^{-6}).
- 1 channel searched for. 6 producible channel at FCC-ee
- Z → γ+ J/ψ maybe visible at HL-LHC

Exclusive Z decays:
Z → γ + bottom meson



← b quarks

Z → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z → γ +	η_b	$(2.43 \pm 0.01) \times 10^{-8}$	NRQCD (NNLO+NLL) [120]	–	–	✓
		$(2.8 \pm 0.5) \times 10^{-8}$	NRQCD (NLO+NLL) [121]			
	$\Upsilon(1S)$	$(4.63 \pm 0.02) \times 10^{-8}$	NRQCD (NNLO+NLL) [120]	$< 2.8\times10^{-6}$ [80]	$\lesssim 2.2 \times 10^{-7}$	✓
		$(5.61 \pm 0.29) \times 10^{-8}$	NRQCD+LDME [125]			
		$\left(4.8^{+0.3}_{-0.2}\right) \times 10^{-8}$	LC+LCDA [123]			
		$(4.93 \pm 0.51) \times 10^{-8}$	NRQCD+LDME [119]			
		$(5.39 \pm 0.16) \times 10^{-8}$	SCET+LCDA [58]			
	$\Upsilon(2S)$	$(2.66 \pm 0.31) \times 10^{-8}$	NRQCD+LDME [125]	$< 1.7\times10^{-6}$ [80]	$\lesssim 1.3 \times 10^{-7}$	✓
		$\left(2.44^{+0.14}_{-0.13}\right) \times 10^{-8}$	LC+LCDA [123]			
	$\Upsilon(3S)$	$(1.93 \pm 0.25) \times 10^{-8}$	NRQCD+LDME [125]	$< 4.8\times10^{-6}$ [80]	$\lesssim 3.7 \times 10^{-7}$	✓
		$\left(1.88^{+0.11}_{-0.10}\right) \times 10^{-8}$	LC+LCDA [123]			
	$\Upsilon(4S)$	$(1.22 \pm 0.13) \times 10^{-8}$	SCET+LCDA [58]	–	–	✓
	$\Upsilon(nS)$	$\left(9.96^{+0.28}_{-0.26}\right) \times 10^{-8}$	SCET+LCDA [58]	–	–	✓
	χ_{b0}	$\left(2.7^{+0.1}_{-0.0}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
	χ_{b1}	$\left(1.473^{+0.010}_{-0.011}\right) \times 10^{-9}$	NRQCD+LDME [124]	–	–	✓
	h_b	$\left(9.27^{+0.36}_{-0.41}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
	χ_{b2}	$\left(2.92^{+0.12}_{-0.14}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓

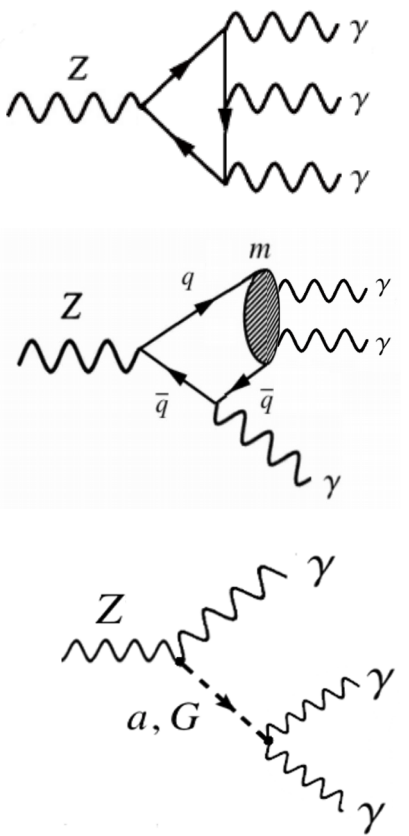


- Theory BRs: O(10^{-8} – 10^{-10}). Multiple calculations (LC, SCET, NRQCD). Exp. limits: O(10^{-6}).
- 3 channels searched-for (ATLAS). 10 producible channel at FCC-ee
- Z → y+Y(1S), might be visible at HL-LHC, (BR = 1/4 projected limit)

BSM example in rare decays Z → 3 γ

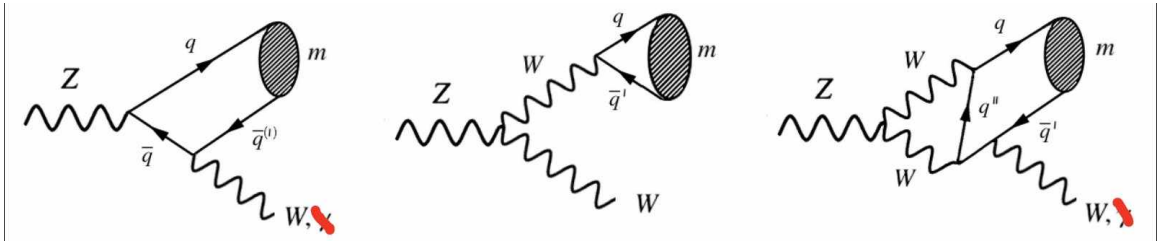
TABLE XI. CAPTION

Z → γ	+	X(γγ)	Branching fraction	Visible at FCC-ee
Z → γ	+	$\pi^0(\gamma\gamma)$	9.7×10^{-12}	✓
		$\eta(\gamma\gamma)$	6.3×10^{-11}	✓
		$\eta'(\gamma\gamma)$	1.1×10^{-10}	✓
		$\eta_c(\gamma\gamma)$	$1.06–1.53 \times 10^{-12}$	✘
		$\chi_{c0}(\gamma\gamma)$	$2.86–10.20 \times 10^{-14}$	✘
		$\chi_{c1}(\gamma\gamma)$	$5.48–35.28 \times 10^{-15}$	✘
		$\chi_{c2}(\gamma\gamma)$	$8.26–28.50 \times 10^{-14}$	✘
		$\chi_{b0}(\gamma\gamma)$	1.6×10^{-12}	✘
		$\chi_{b2}(\gamma\gamma)$	1.6×10^{-12}	✘
Total			$1.85–1.86 \times 10^{-10}$	
Z	⟶	γγγ	5.4×10^{-10} $(6.58 \pm 0.01) \times 10^{-10}$	✓

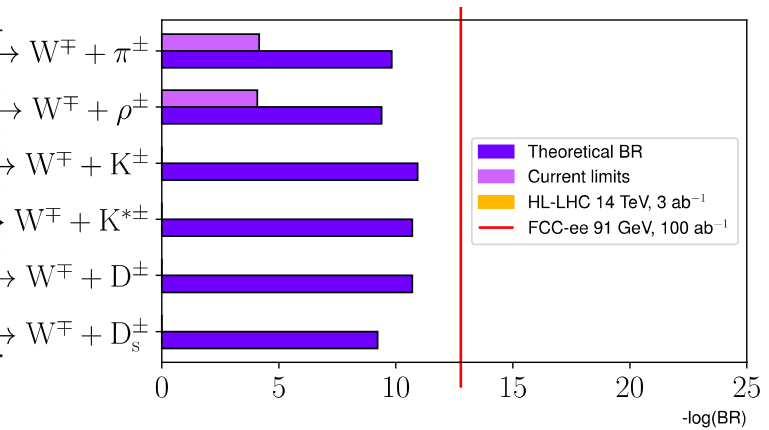


- Z → 3γ decay is very suppressed in the SM (6.5 10^{-10}).
- Z → γ + a(γγ) is a typical ALP/graviton search channel. 10 mesonic channels share same final state
- Z → γ + meson(γγ) provides about 30% extra contributions to the SM BR[Z→3γ]

Exclusive Z decays
Z → W + meson

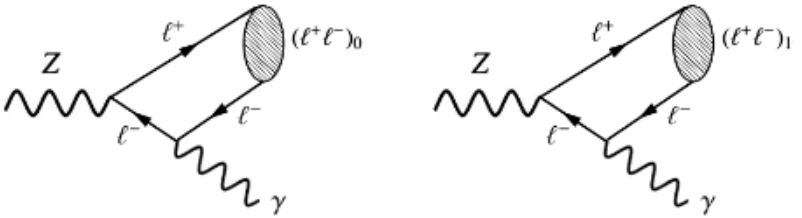


$Z \to W^{\mp}$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
$Z \to W^{\mp}$ +	$\pi^{\pm}$	$(1.51 \pm 0.01) \times 10^{-10}$	SCET+LCDA [58]	$< 7.0 \times 10^{-5}$ [118]	?	✓
	$\rho^{\pm}$	$(4.0 \pm 0.1) \times 10^{-10}$	SCET+LCDA [58]	$< 8.3 \times 10^{-5}$ [118]	?	✓
	$K^{\pm}$	$(1.16 \pm 0.01) \times 10^{-11}$	SCET+LCDA [58]	–	–	✓
	$K^{*\pm}$	$(1.96 \pm 0.12) \times 10^{-11}$	SCET+LCDA [58]	–	–	✓
	$D^{\pm}$	$(1.99 \pm 0.17) \times 10^{-11}$	SCET+LCDA [58]	–	–	✓
	$D_s^{\pm}$	$(6.04 \pm 0.30) \times 10^{-10}$	SCET+LCDA [58]	–	–	✓



- Theory BRs: O(10^{-10} – 10^{-11}). Exp. limits: O(10^{-5})
- 2 channels searched for. 6 producible channels at FCC-ee

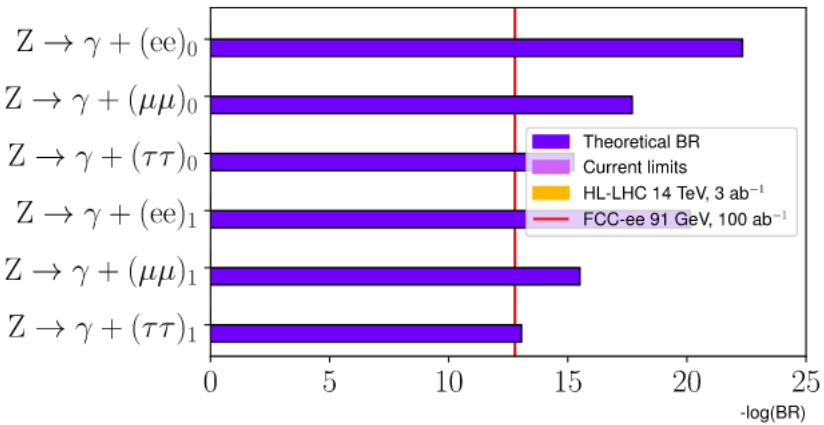
Exclusive Z decays
$Z \to \gamma$ + leptonium



$$\mathcal{B}(Z \to (\ell^+\ell^-)_0 + \gamma) = \frac{\alpha(0)^4\alpha(m_Z)^2}{9 \cdot 256\, n^3} \frac{m^2_{\ell^+\ell^-}}{m^2_Z} \frac{\left(1-4s_w^2\right)^2\left(8s_w^4-4s_w^2+1\right)\left(m_Z^2-m^2_{\ell^+\ell^-}\right)}{\Gamma_{ee}\Gamma_Z s_w^4 c_w^4}$$

$$\mathcal{B}(Z \to (\ell^+\ell^-)_1 + \gamma) = \frac{\alpha(0)^4\alpha(m_Z)^2}{9 \cdot 256\, n^3} \frac{m^2_{\ell^+\ell^-}}{m^4_Z} \frac{\left(8s_w^4-4s_w^2+1\right)\left(m_Z^4-m^4_{\ell^+\ell^-}\right)}{\Gamma_{ee}\Gamma_Z s_w^4 c_w^4}$$

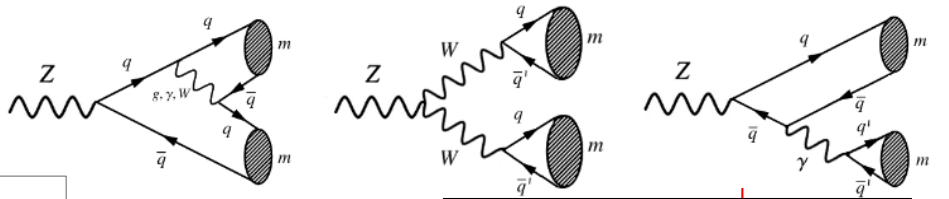
				Exp. limits		Producible at
$Z \to \gamma$ +	X	Branching fraction	Framework	2023	HL-LHC	FCC-ee
$Z \to \gamma$ +	$(ee)_0$	4.7×10^{-23}	This work	–	–	✘
	$(\mu\mu)_0$	2.0×10^{-18}	This work	–	–	✘
	$(\tau\tau)_0$	5.7×10^{-16}	This work	–	–	✘
	$(ee)_1$	7.3×10^{-21}	This work	–	–	✘
	$(\mu\mu)_1$	3.1×10^{-16}	This work	–	–	✘
	$(\tau\tau)_1$	8.9×10^{-14}	This work	–	–	✘

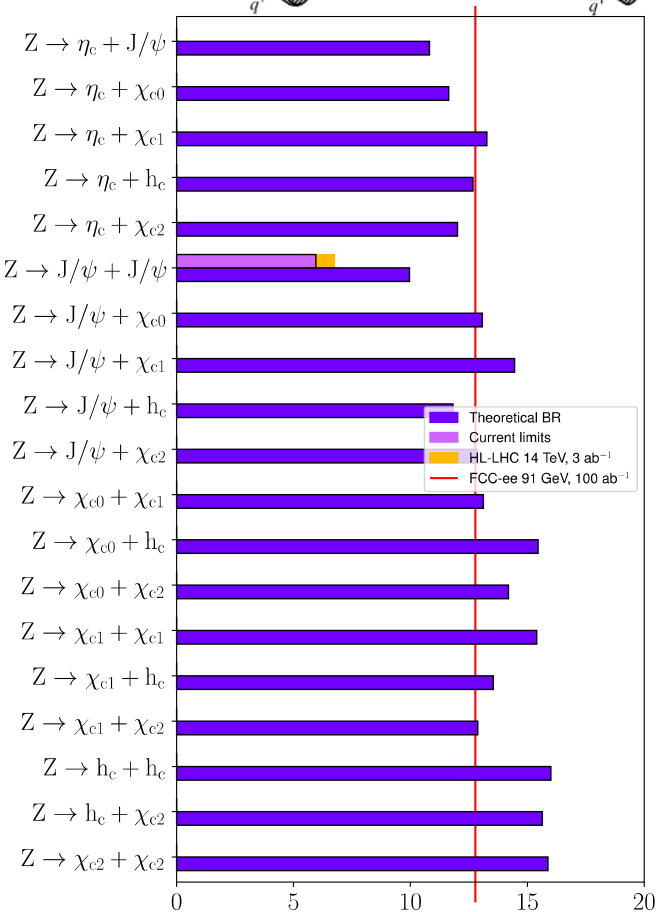


- Tiny BRs: O(10^{-14} – 10^{-23}). First time computed here.
- No channel searched-for. No producible channel at FCC-ee.
- Note: Leptonia are long-lived = LLP signature (displaced γ, e, μ vertices)

Exclusive Z decays
Z → c-meson + c-meson

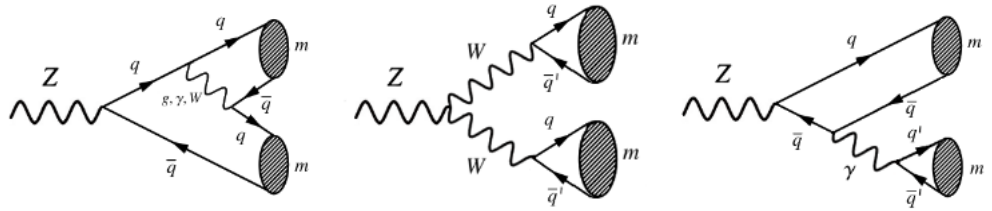
Z →	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z →	η_c	+	J/ψ	$(1.5 \pm 0.4) \times 10^{-11}$	NRQCD/NRCSM [130]	–	–	✓
				$(1.8 - 2.7) \times 10^{-11}$	NRQCD+LDME [128]			
				2.7×10^{-14}	NRQCD+LDME [131]			
			χ_{c0}	2.3×10^{-12}	NRQCD+LDME [131]	–	–	✓
				$(2.3 \pm 1.0) \times 10^{-12}$	LC+LCDA [131]			
			χ_{c1}	5.4×10^{-14}	NRQCD+LDME [131]	–	–	✘
			h_c	2.1×10^{-13}	NRQCD+LDME [131]	–	–	✓
				$(1.0 \pm 0.5) \times 10^{-12}$				
			χ_{c2}	9.7×10^{-13}	NRQCD+LDME [131]	–	–	✓
				$(4.6 \pm 2.0) \times 10^{-12}$				
	J/ψ	+	J/ψ	$(1.1 \pm 0.3) \times 10^{-10}$	NRQCD/NRCSM [130]	–	–	✓
				$(1.1^{+0.3}_{-0.2}) \times 10^{-10}$	NRQCD+LDME [128]			
				$(1.1 - 1.3) \times 10^{-10}$	NRQCD+LDME [131]			
				2.3×10^{-14}	NRQCD+LDME [131]			
				2.7×10^{-11}	LC+LCDA [131]			
			χ_{c0}	$(1.1 - 4.1) \times 10^{-12}$	NRQCD+LDME [131]	–	–	✓
				8.3×10^{-14}	LC+LCDA [131]			
			χ_{c1}	$(4.7 \pm 2.0) \times 10^{-13}$	NRQCD+LDME [131]	–	–	✘
				$(3.5 - 4.4) \times 10^{-12}$	NRQCD+LDME [131]	–	–	✘
				3.5×10^{-15}	LC+LCDA [131]			
			h_c	1.5×10^{-12}	NRQCD+LDME [131]	–	–	✓
				$(9.5 \pm 5.0) \times 10^{-12}$	LC+LCDA [131]			
			χ_{c2}	$(9.6 - 24.8) \times 10^{-13}$	NRQCD+LDME [128]	–	–	✓
				1.4×10^{-13}	NRQCD+LDME [131]			
				$(9.3 \pm 4.0) \times 10^{-13}$	LC+LCDA [131]			
	χ_{c0}	+	χ_{c1}	7.6×10^{-14}	NRQCD+LDME [131]	–	–	✘
				$(1.4 \pm 1.0) \times 10^{-12}$	LC+LCDA [131]			
			h_c	3.5×10^{-16}	NRQCD+LDME [131]	–	–	✘
			χ_{c2}	6.4×10^{-15}	NRQCD+LDME [131]	–	–	✘
	χ_{c1}	+	χ_{c1}	3.9×10^{-16}	NRQCD+LDME [131]	–	–	✘
			h_c	2.9×10^{-14}	NRQCD+LDME [131]	–	–	✘
				$(6.1 \pm 5.0) \times 10^{-13}$	LC+LCDA [131]			
			χ_{c2}	1.3×10^{-13}	NRQCD+LDME [131]	–	–	✘
				$(2.8 \pm 2.0) \times 10^{-12}$	LC+LCDA [131]			
	h_c	+	h_c	9.9×10^{-17}	NRQCD+LDME [131]	–	–	✘
			χ_{c2}	2.3×10^{-16}	NRQCD+LDME [131]	–	–	✘
	χ_{c2}	+	χ_{c2}	1.3×10^{-16}	NRQCD+LDME [131]	–	–	✘



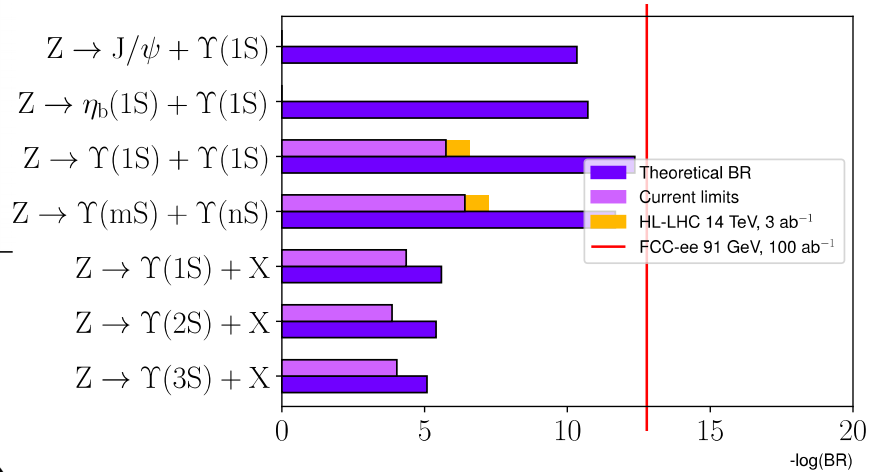


- Theory BRs: O(10^{-10} – 10^{-17}). Exp. limits: O(10^{-6})
- 1 channel searched for. 10 producible channels at FCC-ee
- Large uncertainty, variation between theoretical predictions.
- Most promising place to study double-charmonia decay.

Exclusive Z decays
Z → b-meson + b-meson



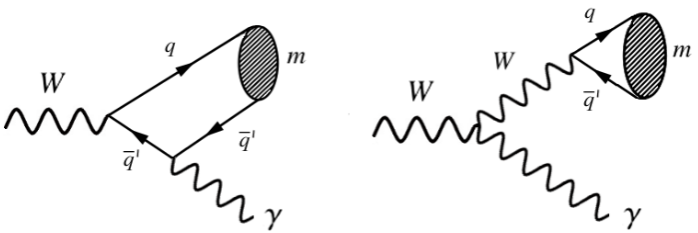
$Z\to$	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at at FCC-ee
$Z\to$	J/ψ	+	$\Upsilon(1S)$	$4.6\ \times10^{-11}$	NRQCD [127]	–	–	✓
	$\eta_b(1S)$	+	$\Upsilon(1S)$	$(1.9\pm0.2)\ \times10^{-11}$	NRQCD/NRCSM [130]	–	–	✓
	$\Upsilon(1S)$	+	$\Upsilon(1S)$	$\left(4.4^{+0.6}_{-0.3}\right)\ \times10^{-13}$	NRQCD/NRCSM [130]	$<1.8\times10^{-6}$ [90]	$\lesssim 2.7\times10^{-7}$	✓
	$\Upsilon(mS)$	+	$\Upsilon(nS)$	$2.1\ \times10^{-12}$	NRQCD [127]	$<3.9\times10^{-7}$ [90]	$\lesssim 5.9\times10^{-8}$	✓
	$\Upsilon(1S)$	+	X	$(2.6-2.9)\ \times10^{-6}$	NRQCD+LDME (NLO) [132]	$<4.4\times10^{-5}$ [133]	?	✓
	$\Upsilon(2S)$	+	X	$(3.7-4.3)\ \times10^{-6}$	NRQCD+LDME (NLO) [132]	$<1.4\times10^{-4}$ [134]	?	✓
	$\Upsilon(3S)$	+	X	$(7.6-8.3)\ \times10^{-6}$	NRQCD+LDME (NLO) [132]	$<9.4\times10^{-5}$ [134]	?	✓
	$\Upsilon(nS)$	+	X	$(1.4-1.5)\ \times10^{-5}$	NRQCD+LDME (NLO) [132]	$(1.0\pm0.5)\times10^{-4}$ [135]	?	✓



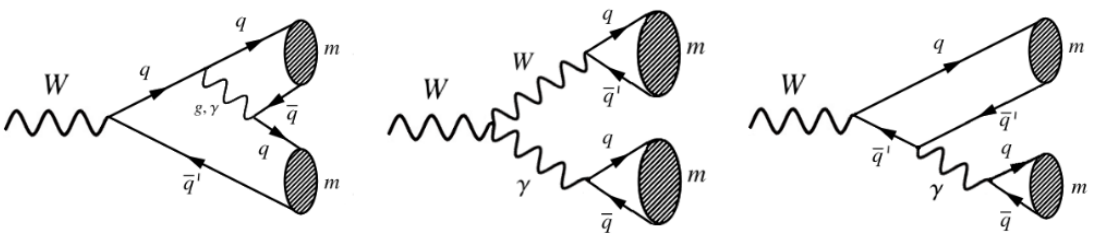
- Theory BRs: O(10^{-10} – 10^{-17}). Exp. limits: O(10^{-4} – 10^{-7})
- 5 channels searched for. 7 producible channels at FCC-ee
- Most promising place to study double-bottomonium decays.

Exclusive W decays

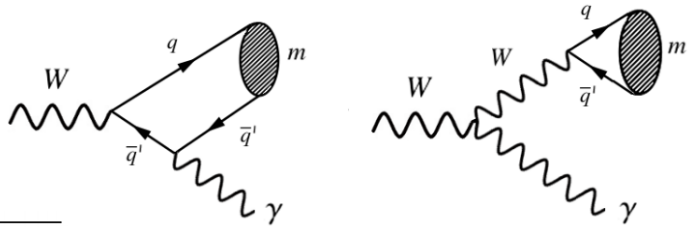
- Exclusive W decays:
  - Similar mechanism to exclusive Z and H decays.
  - Provides cross-check of pQCD factorization models and info on open-flavour meson form factors.
  - Exclusive Z decays into:
    - γ + 1 charged meson



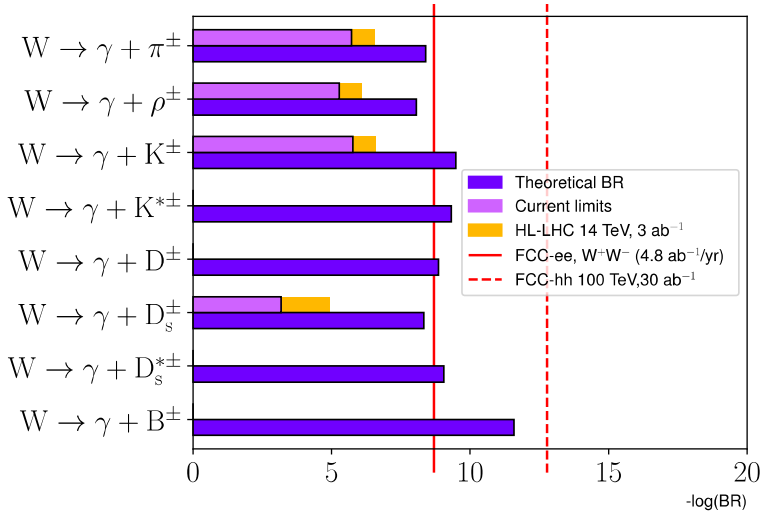
    - Meson + charged meson



Exclusive W decays
W→ γ + meson

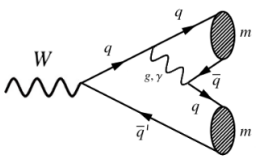


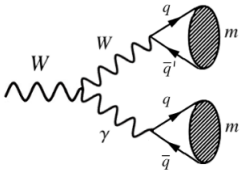
$W^{\mp} \to \gamma$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
	$\pi^{\pm}$	$(4.0 \pm 0.8) \times 10^{-9}$	SCET+LCDA [58]	$< 1.9\times10^{-6}$ [136, 137]	$\lesssim 2.9 \times 10^{-7}$	✓	✓
	$\rho^{\pm}$	$(8.74 \pm 1.91) \times 10^{-9}$	SCET+LCDA [58]	$< 5.2\times10^{-6}$ [136]	$\lesssim 7.9 \times 10^{-7}$	✓	✓
	$K^{\pm}$	$(3.25 \pm 0.69) \times 10^{-10}$	SCET+LCDA [58]	$< 1.7\times10^{-6}$ [136]	$\lesssim 2.6 \times 10^{-7}$	✘	✓
	$K^{*\pm}$	$(4.78 \pm 1.15) \times 10^{-10}$	SCET+LCDA [58]	–	–	✘	✓
	$D^{\pm}$	$\left(1.4^{+0.5}_{-0.3}\right) \times 10^{-9}$	SCET+LCDA [58]	–	–	✘	✓
	$D_s^{\pm}$	$\left(3.7^{+1.5}_{-0.8}\right) \times 10^{-8}$	SCET+LCDA [58]	$< 6.5\times10^{-4}$ [115]	$\lesssim 1.2 \times 10^{-5}$	✓	✓
		4.7×10^{-9}	NRQCD+LDME [138]				
		3.4×10^{-9}	LC+LCDA [138]				
	$D_s^{*\pm}$	8.9×10^{-10}	NRQCD+LDME [138]	–	–	✘	✓
		3.4×10^{-9}	LC+LCDA [138]				
	$B^{\pm}$	$\left(1.6^{+0.8}_{-0.6}\right) \times 10^{-12}$	SCET+LCDA [58]	–	–	✘	✓
		$\left(2.6^{+3.1}_{-1.3}\right) \times 10^{-12}$	HQET+LCDA [139]				
		$\left(2.0^{+2.5}_{-0.8}\right) \times 10^{-12}$	SCET+LCDA[a] [139]				

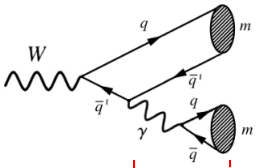


- Theory BRs: O(10^{-8} – 10^{-12}). Exp. limits: O(10^{-4} – 10^{-6})
- 4 channels searched-for.
- 3 (5) channels producible at FCC-ee (FCC-hh)

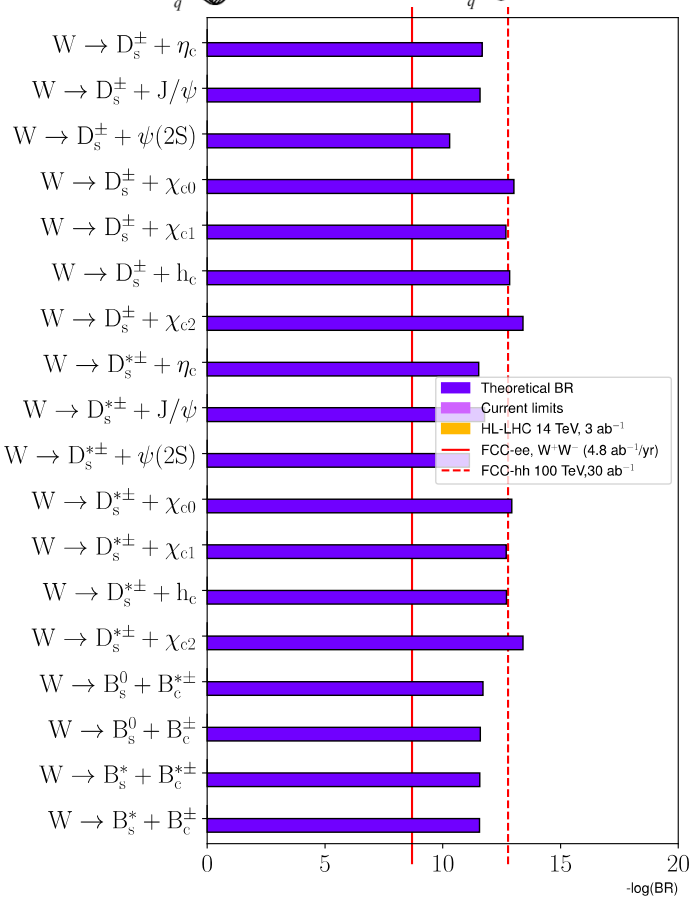
Exclusive W decays
W→ meson + meson







W± →	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
W± →	η_c	+	$D_s^{\pm}$	2.1×10^{-12}	NRCSM+LCDA [140]	–	–	✘	✓
				$(1.3^{+0.3}_{-0.2}) \times 10^{-11}$	LC+LCDA [140]				
			$D_s^{*\pm}$	3.0×10^{-12}	NRCSM+LCDA [140]	–	–	✘	✓
				$(1.5^{+0.4}_{-0.2}) \times 10^{-11}$	LC+LCDA [140]				
	J/ψ	+	$D_s^{\pm}$	2.6×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
				2.1×10^{-12}	NRCSM+LCDA [140]				
				$(1.8^{+0.4}_{-0.2}) \times 10^{-11}$	LC+LCDA [140]				
			$D_s^{*\pm}$	1.7×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
				3.0×10^{-12}	NRCSM+LCDA [140]				
				$(2.0^{+0.5}_{-0.2}) \times 10^{-11}$	LC+LCDA [140]				
	$\psi(2S)$	+	$D_s^{\pm}$	5.1×10^{-11}	NRQCD+LDME [138]	–	–	✘	✓
			$D_s^{*\pm}$	7.4×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
	χ_{c0}	+	$D_s^{\pm}$	9.4×10^{-14}	NRQCD+LDME [138]	–	–	✘	✘
				4.7×10^{-14}	NRCS				
				$(7.1^{+3.5}_{-3.1}) \times 10^{-13}$	LC+L				
			$D_s^{*\pm}$	1.2×10^{-13}	NRQ	–	–	✘	✓
				8.1×10^{-14}	NRCS				
				$(8.0^{+3.7}_{-3.1}) \times 10^{-13}$	LC+L				
	χ_{c1}	+	$D_s^{\pm}$	2.0×10^{-13}	NRQ	–	–	✘	✘
				2.9×10^{-13}	NRCS				
				$(7.8^{+3.4}_{-3.0}) \times 10^{-12}$	LC+L				
			$D_s^{*\pm}$	2.0×10^{-13}	NRQ	–	–	✘	✓
				4.0×10^{-13}	NRCS				
				$(8.8^{+3.5}_{-3.1}) \times 10^{-12}$	LC+L				
			$D_s^{\pm}$	1.4×10^{-13}	NRCS	–	–	✘	✘
				$(2.1^{+1.0}_{-0.8}) \times 10^{-12}$	LC+LCDA [140]				
	h_c	+	$D_s^{*\pm}$	2.0×10^{-13}	NRCSM+LCDA [140]	–	–	✘	✓
				$(2.4^{+1.1}_{-0.9}) \times 10^{-12}$	LC+LCDA [140]				
			$D_s^{\pm}$	3.9×10^{-14}	NRQCD+LDME [138]	–	–	✘	✘
				9.6×10^{-14}	NRCSM+LCDA [140]				
				$(1.4^{+0.6}_{-0.5}) \times 10^{-12}$	LC+LCDA [140]				
	χ_{c2}	+	$D_s^{*\pm}$	3.9×10^{-14}	NRQCD+LDME [138]	–	–	✘	✘
				1.4×10^{-13}	NRCSM+LCDA [140]				
				$(1.6^{+0.7}_{-0.6}) \times 10^{-12}$	LC+LCDA [140]				
	B_s^0	+	$B_c^{*\pm}$	2.0×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
			$B_c^{\pm}$	2.5×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
	B_s^*	+	$B_c^{*\pm}$	2.7×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
			$B_c^{\pm}$	2.7×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓

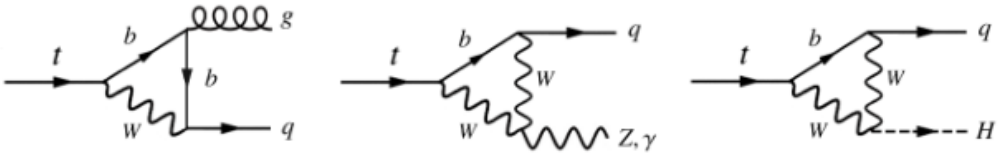


- Theory BRs: O(10^{-11} – 10^{-14}). No existing exp. limit
- No channels searched-for anywhere.
- No (13) channels producible at FCC-ee (FCC-hh)

Rare top decays

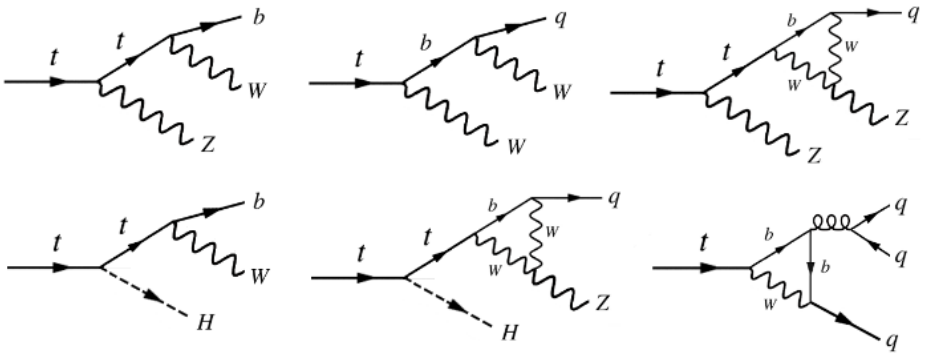
- The FCNC top-quark decays:

  t → Zq, t → cγ, and t → cg;

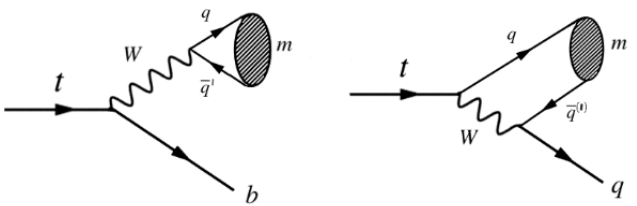


  Highly suppressed in the SM (loops, GIM), but significantly enhanced in BSM models

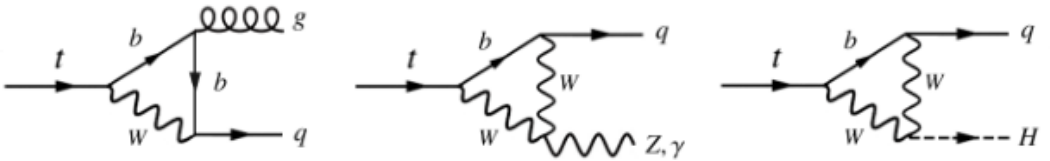
- Three-body top decays:
  possible thanks to the
  large top mass
  → multiple onshell heavy boson
  decays kinematically accessible



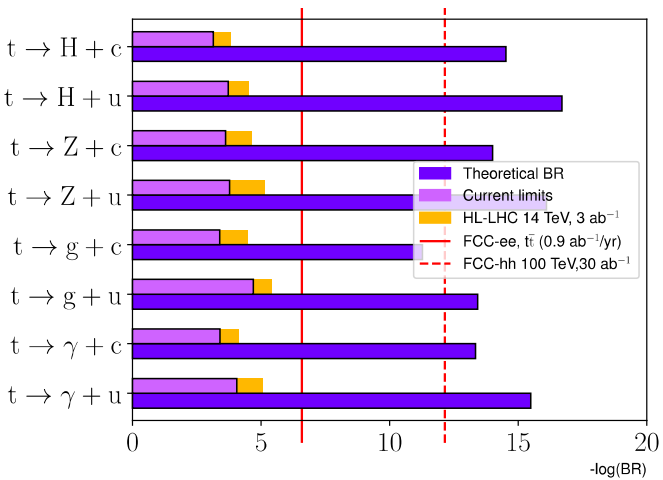
- Semi-exclusive top quark decays:
  t → q+meson: alternative m_{top} determination?



Flavor changing neutral current (FCNC) rare top-quark decays

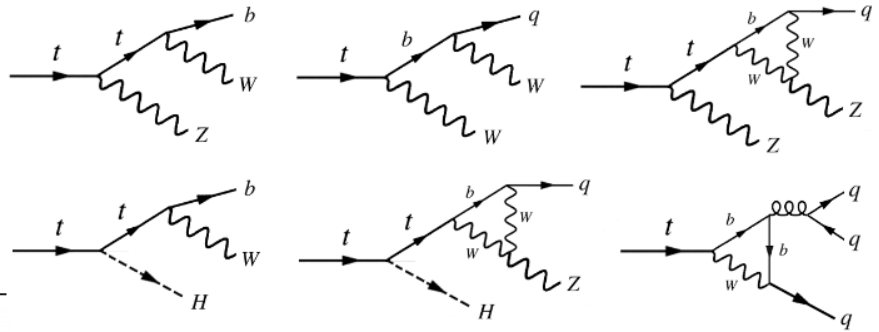


t →	X	+	Y	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
t →	H	+	c	3.0×10^{-15}	1-loop SM [27]	$< 7.3\times10^{-4}$ [143]	$\lesssim 1.5 \times 10^{-4}$ [53]	✘	✘
			u	2.0×10^{-17}	1-loop SM [27]	$< 1.9\times10^{-4}$ [143]	$\lesssim 2.9 \times 10^{-5}$	✘	✘
	Z	+	c	1.0×10^{-14}	1-loop SM [27]	$< 2.4\times10^{-4}$ [112]	$\lesssim 2.3 \times 10^{-5}$ [57]	✘	✘
			u	8.0×10^{-17}	1-loop SM [27]	$< 1.7\times10^{-4}$ [112]	$\lesssim 7.3 \times 10^{-6}$ [57]	✘	✘
	g	+	c	$(5.31 \pm 0.27) \times 10^{-12}$ $(4.6^{+3.0}_{-1.0}) \times 10^{-12}$ 5.7×10^{-12}	1-loop SM [144] 1-loop SM [27] 1-loop SM [146]	$< 4.1\times10^{-4}$ [145]	$\lesssim 3.2 \times 10^{-5}$ [56]	✘	✓
			u	$(3.81 \pm 0.34) \times 10^{-14}$ 3.7×10^{-14}	1-loop SM [144] 1-loop SM [27]	$< 2.0\times10^{-5}$ [145]	$\lesssim 3.8 \times 10^{-6}$ [56]	✘	✘
	γ	+	c	$(4.55 \pm 0.23) \times 10^{-14}$ $(4.6^{+2.0}_{-1.0}) \times 10^{-14}$	1-loop SM [144] 1-loop SM [27]	$< 4.0\times10^{-4}$ [147]	$\lesssim 7.4 \times 10^{-5}$ [56]	✘	✘
			u	$(3.26 \pm 0.34) \times 10^{-16}$ 3.7×10^{-16}	1-loop SM [144] 1-loop SM [27]	$< 8.9\times10^{-5}$ [147]	$\lesssim 8.6 \times 10^{-6}$ [56]	✘	✘

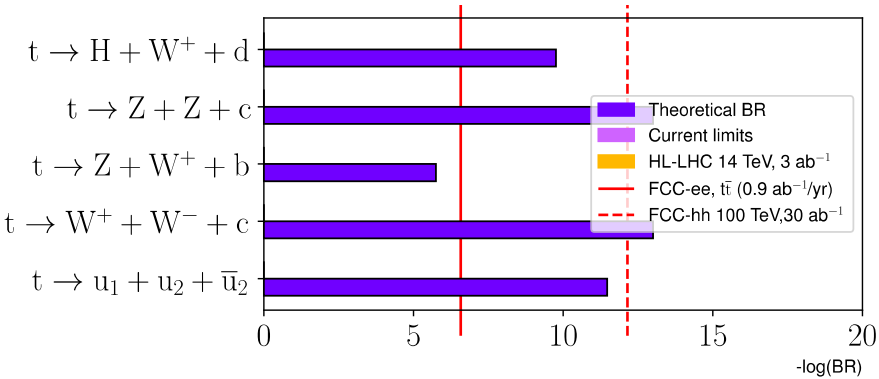


- Theory BRs: O(10^{-3}–10^{-17}). Exp. limits: O(10^{-4} – 10^{-5})
- All 8 channels searched-for. No (1) channels producible at FCC-ee (FCC-hh), in the absence of BSM
- Interesting channels as FCNC is a “hot BSM” topic.

Rare 3-body top-quark decays

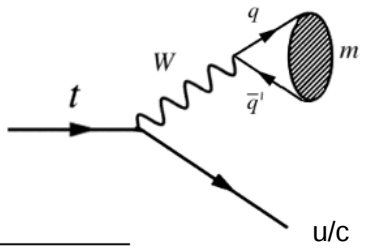


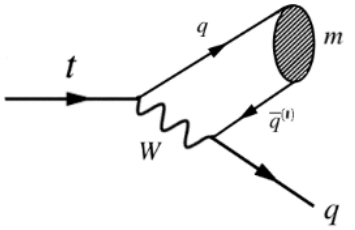
							Exp. limits		Producible at	
t →	X	+	Y	+	Z	Branching fraction	Framework	2023	HL-LHC	FCC-ee	FCC-hh
t	H	+	W^+	+	d	1.8×10^{-10}	NLO QCD+EW This work	–	–	✘	✓
			Z	+	c	$< 1.0 \times 10^{-13}$	LO SM [151]	–	–	✘	✘
	Z	+	W^+	+	b	1.8×10^{-6}	LO SM [152]	–	–	✓	✓
						$(5.4^{+4.7}_{-2.0}) \times 10^{-7}$	LO SM [148]				
						2.0×10^{-6}	LO SM [149]				
	W^+	+	W^-	+	c	1.0×10^{-13}	LO [26]	–	–	✘	✘
						2.0×10^{-13}	LO SM [151]				
	u_2	+	u_1	+	$\bar{u}_2$	3.4×10^{-12}	1-loop SM [150]	–	–	✘	✓



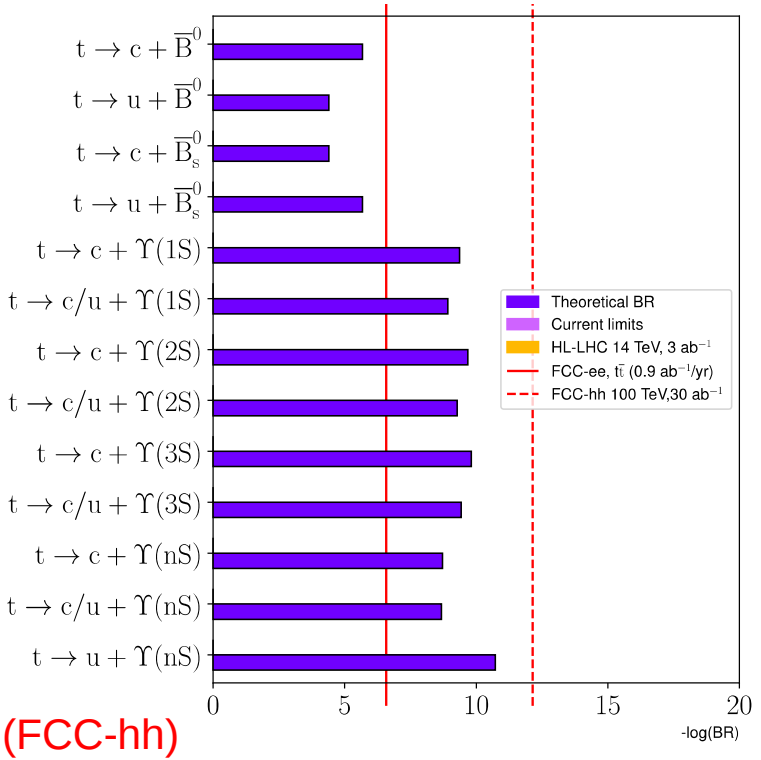
- Theory BRs: O(10^{-6} – 10^{-13}). No existing exp. limits
- 4 channels searched-for. 1 (3) channels producible at FCC-ee (FCC-hh)
- Note: t→ Z+W+b (91.2+80.4+4. GeV ≈ m_{top}) has "large" BR: $2 \cdot 10^{-6}$

Semi-exclusive decays
t → meson + c/u quark



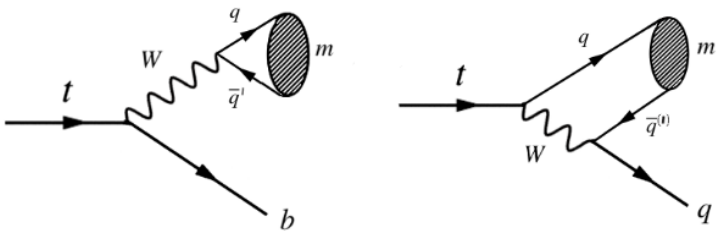


t →	X	+	Y	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
t →	$\overline{B}^0$	+	c	$(2.1^{+2.1}_{-1.1}) \times 10^{-6}$	NRQCD+LDME [25]	–	–	✓	✓
			u	$(4.0^{+4.0}_{-2.0}) \times 10^{-5}$	NRQCD+LDME [25]	–	–	✓	✓
	$\overline{B}^0_s$	+	c	$(4.0^{+4.0}_{-2.0}) \times 10^{-5}$	NRQCD+LDME [25]	–	–	✓	✓
			u	$(2.1^{+2.1}_{-1.1}) \times 10^{-6}$	NRQCD+LDME [25]	–	–	✓	✓
	$\Upsilon(1S)$	+	c	4.3×10^{-10}	NRQCD+CSM [155]	–	–	✗	✓
				$(6.4 \pm 1.3) \times 10^{-10}$	NRQCD+COM [154]				
			c/u	$(1.0 - 1.5) \times 10^{-9}$	NRQCD+LDME [25]	–	–	✗	✓
	$\Upsilon(2S)$	+	c	2.1×10^{-10}	NRQCD+CSM [155]	–	–	✗	✓
			c/u	$(1.7 - 5.3) \times 10^{-10}$	NRQCD+LDME [25]	–	–	✗	✓
	$\Upsilon(3S)$	+	c	1.6×10^{-10}	NRQCD+CSM [155]	–	–	✗	✓
			c/u	$(2.7 - 3.8) \times 10^{-10}$	NRQCD+LDME [25]	–	–	✗	✓
	$\Upsilon(nS)$	+	c	$(1.9^{+0.2}_{-0.1}) \times 10^{-9}$	NRQCD+LDME [25]	–	–	✗	✓
			c/u	$(1.5 - 2.1) \times 10^{-9}$	NRQCD+LDME [25]	–	–	✗	✓
			u	$(1.9^{+0.2}_{-0.1}) \times 10^{-11}$	NRQCD+LDME [25]	–	–	✗	✓

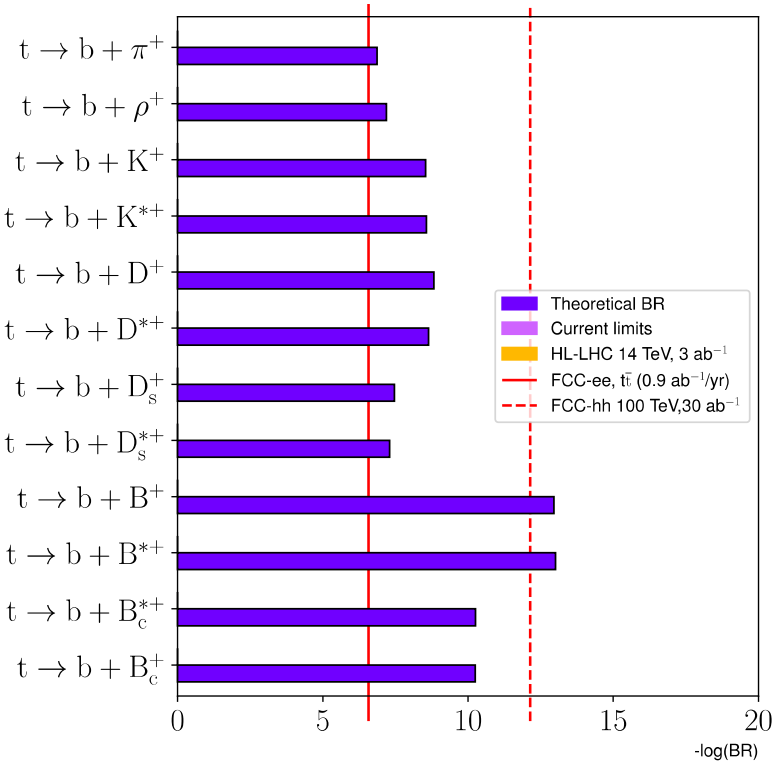


- Theory BRs: O(10^{-5} – 10^{-11}). No existing exp. limits
- No channel searched for. 4 (9) visible channels at FCC-ee (FCC-hh)
- Note: t→ B+c/u-quark have “large” BR: $4 \cdot 10^{-5}$

Semi-exclusive decays
t → meson + b quark



t →	X	+	Y	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
t →	π^+	+	b	1.3×10^{-7}	EFT+LCDA This work	–	–	✘	✓
	ρ^+	+	b	6.4×10^{-8}	EFT+LCDA This work	–	–	✘	✓
	K^+	+	b	2.9×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	K^{*+}	+	b	2.7×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	D^+	+	b	1.5×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	D^{*+}	+	b	2.3×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	D_s^+	+	b	3.4×10^{-8}	EFT+LCDA This work	–	–	✘	✓
	D_s^{*+}	+	b	5.1×10^{-8}	EFT+LCDA This work	–	–	✘	✓
	B^+	+	b	1.1×10^{-13}	EFT+LCDA This work	–	–	✘	✘
	B^{*+}	+	b	9.8×10^{-14}	EFT+LCDA This work	–	–	✘	✘
	B_c^{*+}	+	b	5.5×10^{-11}	EFT+LCDA This work	–	–	✘	✓
	B_c^+	+	b	5.7×10^{-11}	EFT+LCDA This work	–	–	✘	✓



- Theory BRs: O(10^{-7} – 10^{-14}). No existing exp. limits
- No channel searched for. No (10) visible channels at FCC-ee(FCC-hh)
- First time those decays have been estimated.

Summary

- Comprehensive survey of the theoretical & experimental status of more than 150 rare and exclusive few-body decays of the 4 heaviest SM particles (H,Z,W,t): BR≈10^{-5} – 10^{-20}
  - Sensitive to BSM physics scenarios (FCNC), backgrounds to many BSM decays (H,Z → ALPs, gravitons, dark γ, ...), and study of pQCD factorization/meson formation.
- Up-to-date collection of TH BRs and EXP limits from the literature.
  - Current LHC limits for 44 decays.
- Calculation of new rare decay channels: radiative leptonium, exclusive Higgs FCNC decays, Z boson 3-body decays, semiexclusive t → b-quark +m, ...
  - H, Z → leptonium + γ decays: Tiny. Very hard to measure.
- Estimation of reachabilities of HL-LHC, FCC-ee/FCC-hh observations (if not BSM-enhanced):
  - HL-LHC can potentially observe a few of them: H→γ+ρ, Z→γ+ω, Z→γ+J/ψ
  - FCC-ee can discover about 50% of such experimentally unobserved decays
  - FCC-hh can produce most all those decays channels

Outlook

- Paper (~50 pages) in preparation…

We hope it motivates people (LHC, FCC-ee) to perform new BSM searches...
(and find new physics ;-))

Rare and exclusive few-body decays of the Higgs, Z, W bosons, and the top quark

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(Dated: October 17, 2023)

We perform an extensive survey of rare and exclusive few-body decays —defined as those with two or three final-state particles, and branching fractions $\mathcal{B} \lesssim 10^{-5}$— of the Higgs, Z, and W bosons, and the top quark. Such rare decays can probe physics beyond the Standard Model (BSM), may constitute a background for possible decays into new BSM particles, and/or provide precise information on quantum chromodynamics factorization with small nonperturbative corrections. First, we collect and tabulate the $\mathcal{B}$ values calculated for more than 150 decay channels of the four heaviest elementary particles, indicating the current experimental limits in their observation. Second, we compute for the first time H and Z boson decays into leptonium-plus-photon, very rare H boson decays to photons and/or neutrinos, and radiative H and Z quark-flavour-changing exclusive decays, while revisiting and updating predictions for a few other rare Z boson and top quark partial widths. Third, the feasibility of measuring each of these unobserved decays is estimated for proton-proton collisions at the high-luminosity Large Hadron Collider (HL-LHC), and for e^+e^- and p-p collisions at the future circular collider (FCC).

I. INTRODUCTION

With the discovery of the Higgs boson at the CERN Large Hadron Collider (LHC) ten years ago [1, 2], the full particle content of the Standard Model (SM) of particle physics has become fully fixed. Among the 17 existing elementary particles (6 quarks, 6 leptons, 4 gauge bosons, and the scalar boson), the top quark, the Higgs and the electroweak (W, Z) bosons are the most massive ones. Studying in detail the properties of the four heaviest elementary particles, with masses around the electroweak scale $\Lambda_{\rm EW} \approx O(100$ GeV), is an important priority in precision SM studies and in searches for new physics beyond it (BSM). At the LHC, the large center-of-mass (c.m.) energies and integrated luminosities (up to $\mathcal{L}_{\rm int} = 3$ ab^{-1}expected at the end of the high-luminosity, HL-LHC, phase) [3] available in