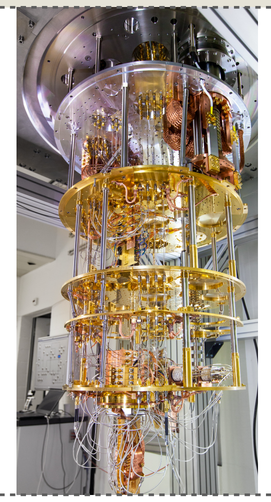
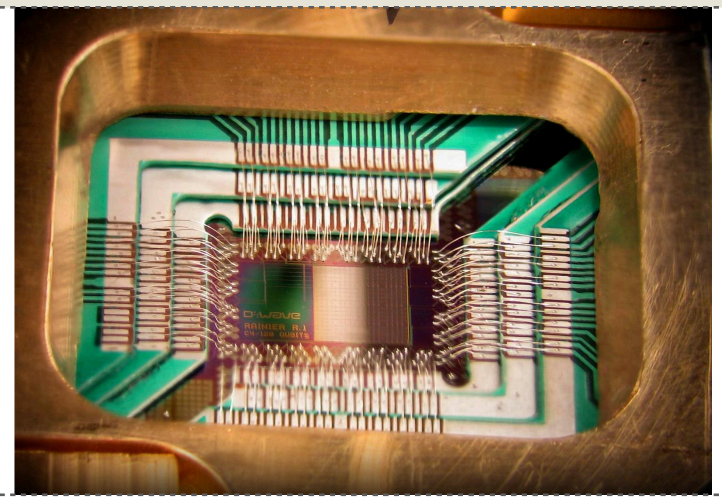
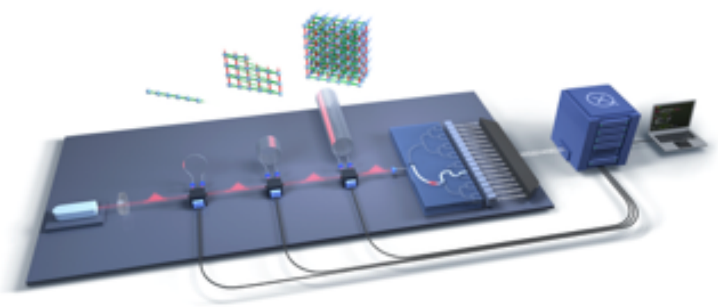
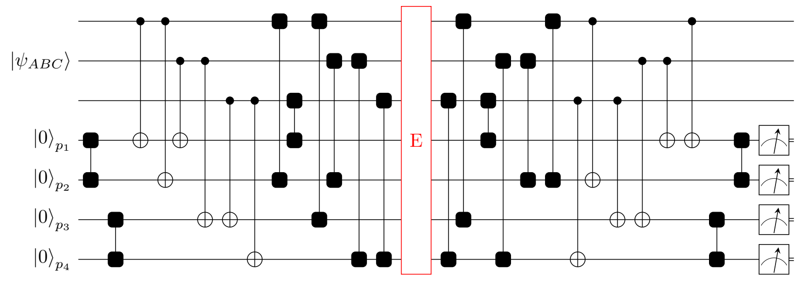
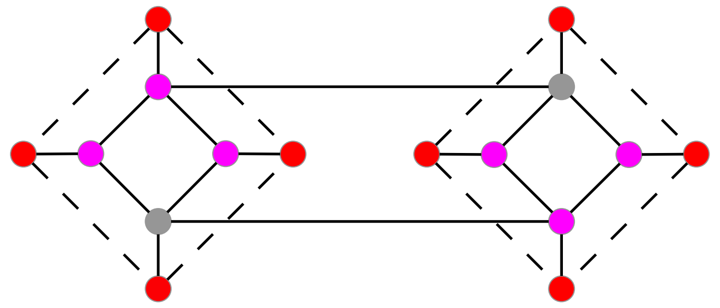
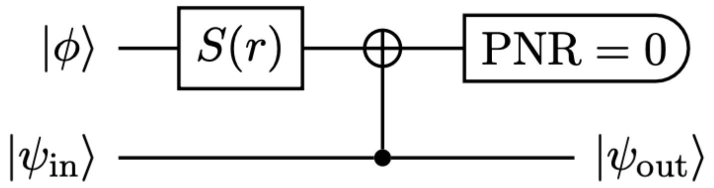
Novel methods for QFT on quantum computers

Steven Abel (IPPP, Durham)

w/ Spannowsky and Williams, arXiv:2403.10619
w/ Nutricati, *Fortschr. Phys.* 2022, 2200114
w/ Criado and Spannowsky, *Phys.Rev.A* 106 (2022) 2, 022601
w/ Blance and Spannowsky, *Phys.Rev.A* 106 (2022) 4, 042607
w/ Spannowsky arXiv:2006.06003, *PRX Quantum* 2, 010349 (2021)
w/ Chancellor and Spannowsky, arXiv:2003.07374, *Phys.Rev.D* 103, 016008 (2021)

Background: Quantum computing has a long and distinguished history but is only now becoming practicable. Three (at least) types of Quantum Computer:

Feynman '81, Zalka '96, Jordan, Lee, Preskill … see Preskill 1811.10085 for review.

Type	Discrete Gate	Quantum Annealer	Photonic devices
Property	Universal (any quantum algorithm can be expressed)	Not universal — certain quantum systems	Also universal — continuous variable model
Where?	IBM - Qiskit ~127 Qubits	DWave - LEAP ~7000 Qubits	Xanadu, ~8 Qumodes - but millions in principle
What?			
How?			

- In this talk I will argue that *photonic devices (*and related continuous variable quantum computers) are the natural devices for simulating QFT (i.e. performing full Hamiltonian evolution)

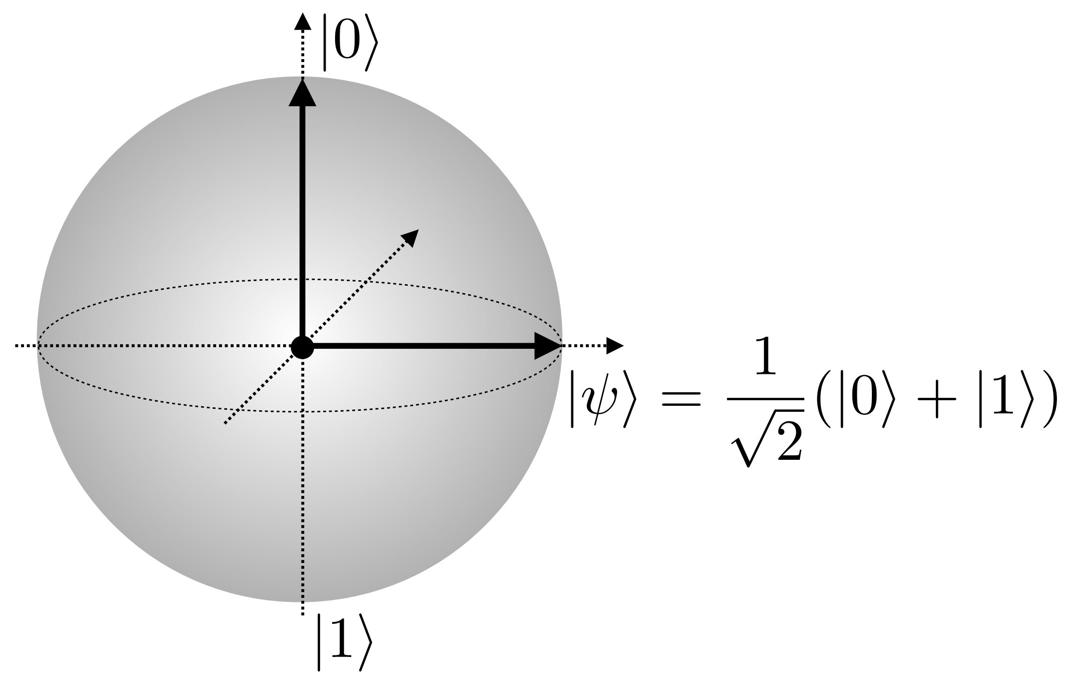
- I will also show how we can already solve QM on these devices very easily

Overview

1. Quantum mechanics with qubits: e.g. tunnelling
2. The trouble with qubits: How would we do QFT?
3. Photonic devices - and Quantum Mechanics
4. How would we do QFT on a photonic computer?

1. Quantum mechanics with qubits: e.g. tunnelling

Discrete qubit approach



- Operate on the Bloch sphere: basically measuring where $\sigma_i^Z|0\rangle = |0\rangle$, $\sigma_i^Z|1\rangle = -|0\rangle$

$$\sigma_i^Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

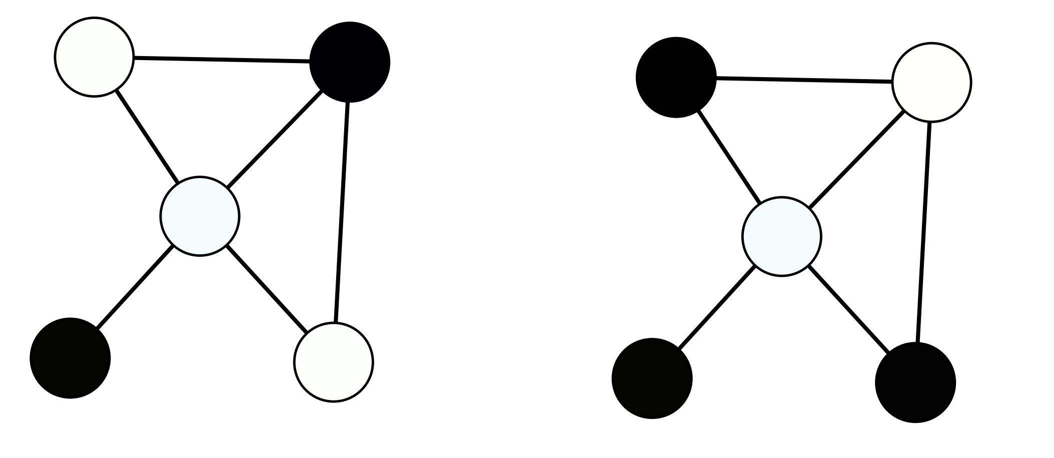
- Each i denotes a single qubit

- As an example consider tunnelling with a quantum annealers: based on the general transverse field Ising model making it natural for field theories (Kadowaki, Nishimori):

$$\mathcal{H}_{\mathrm{QA}}(t) = \sum_i \sum_j J_{ij}\sigma_i^Z \sigma_j^Z + \sum_i h_i \sigma_i^Z + \Delta(t) \sum_i \sigma_i^X$$

Commonly with annealers encode network problems in the general Ising model

- Example: how many vertices on a graph can we colour so that none touch? NP-hard problem.

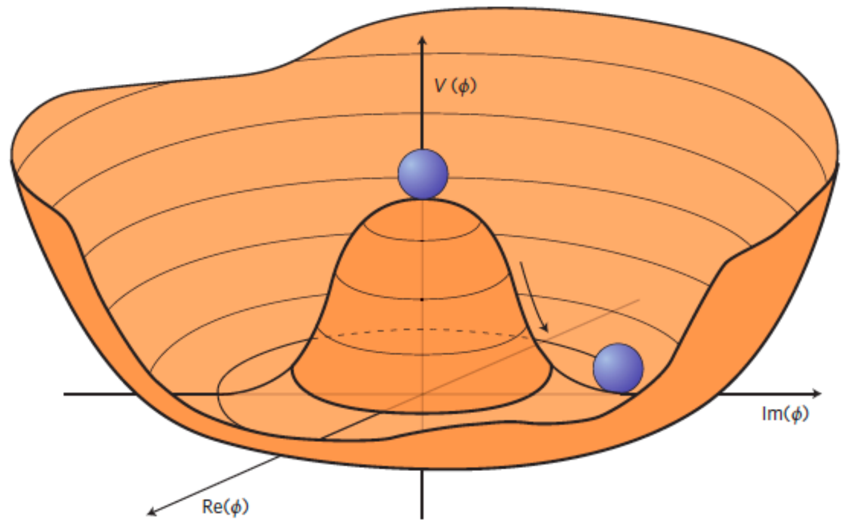


- Let non-coloured vertices have $\sigma_i^Z = -1$ and coloured ones have $\sigma_i^Z = +1$
- Add a reward for every coloured vertex, and for each link between vertices i, j we add a penalty if there are two +1 eigenvalues:

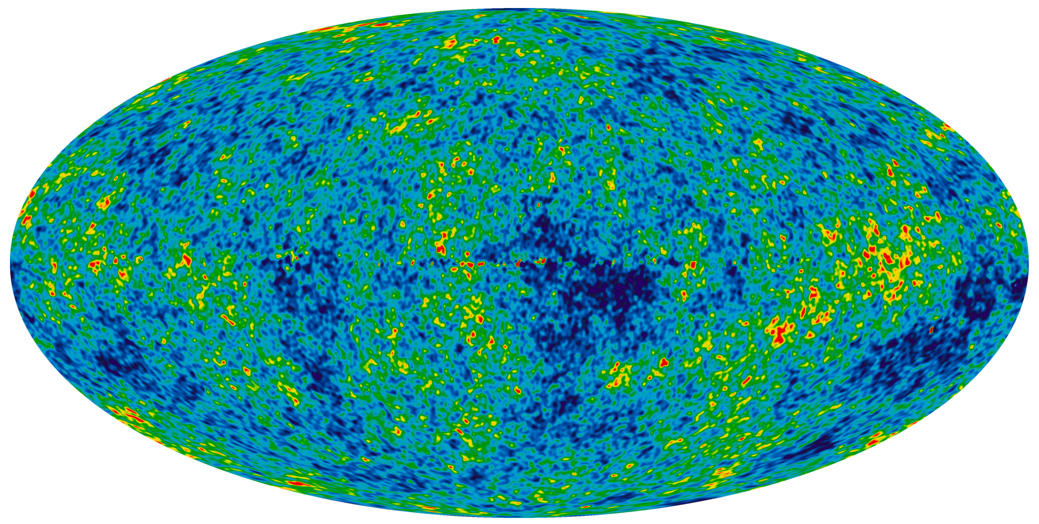
$$\mathcal{H} = -\Lambda \sum_i \sigma_i^Z + \sum_{\text{linked pairs } \{i,j\}} (1+\sigma_i^Z)(1+\sigma_j^Z)$$

Instead aim to model QFT processes: e.g. tunnelling

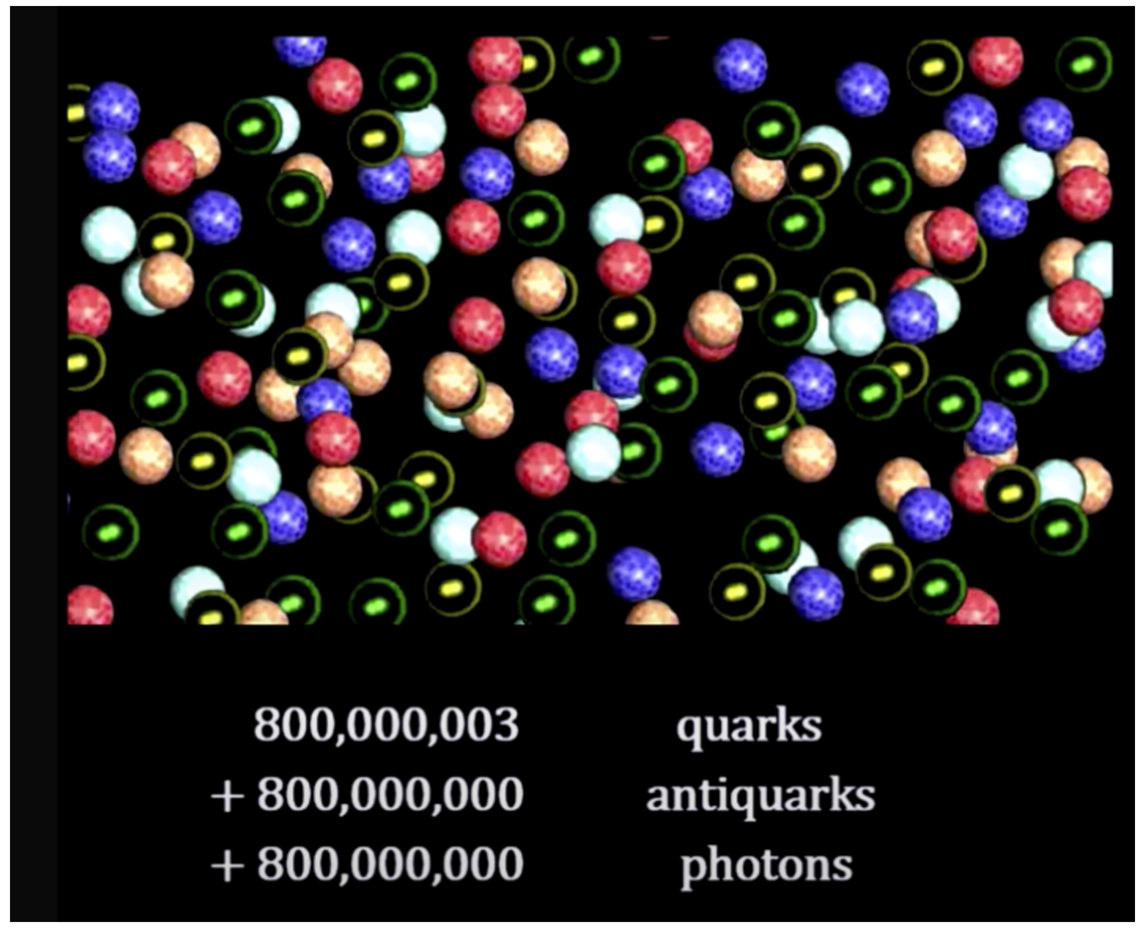
- Electroweak phase transition (Higgs mechanism)



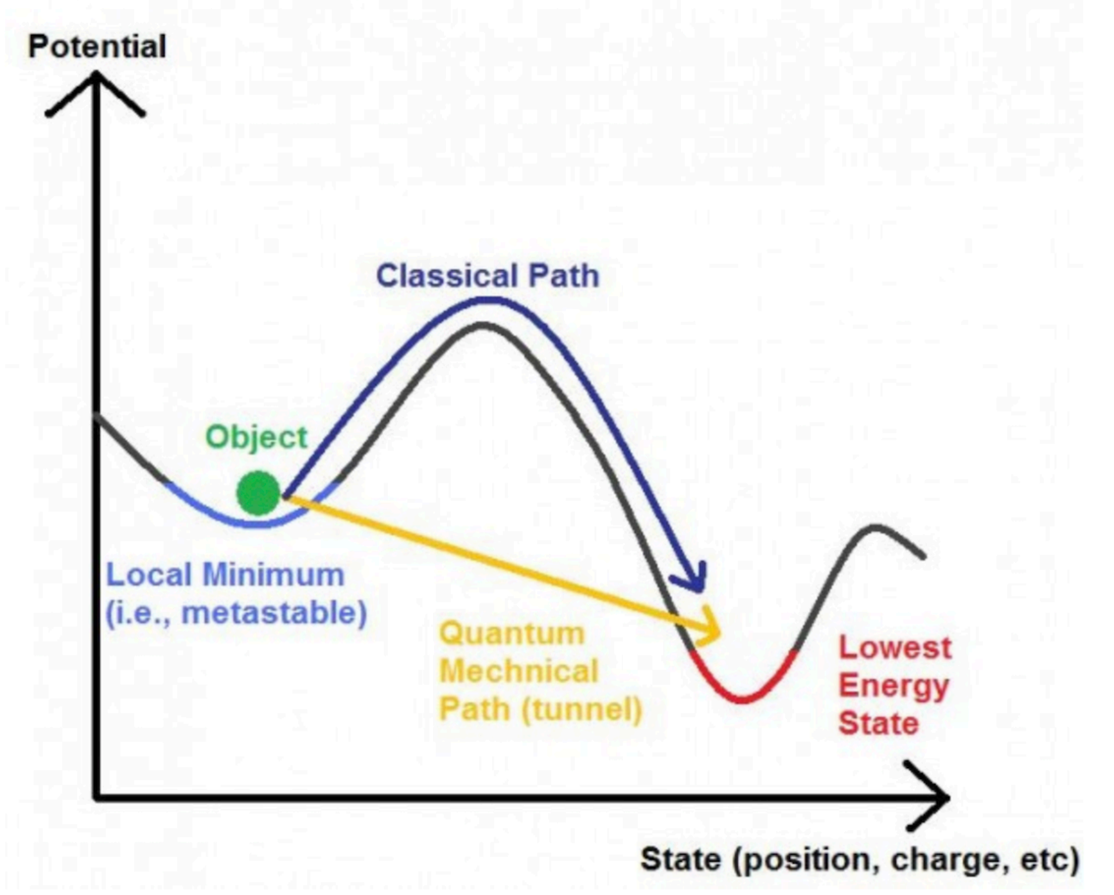
- Inflation



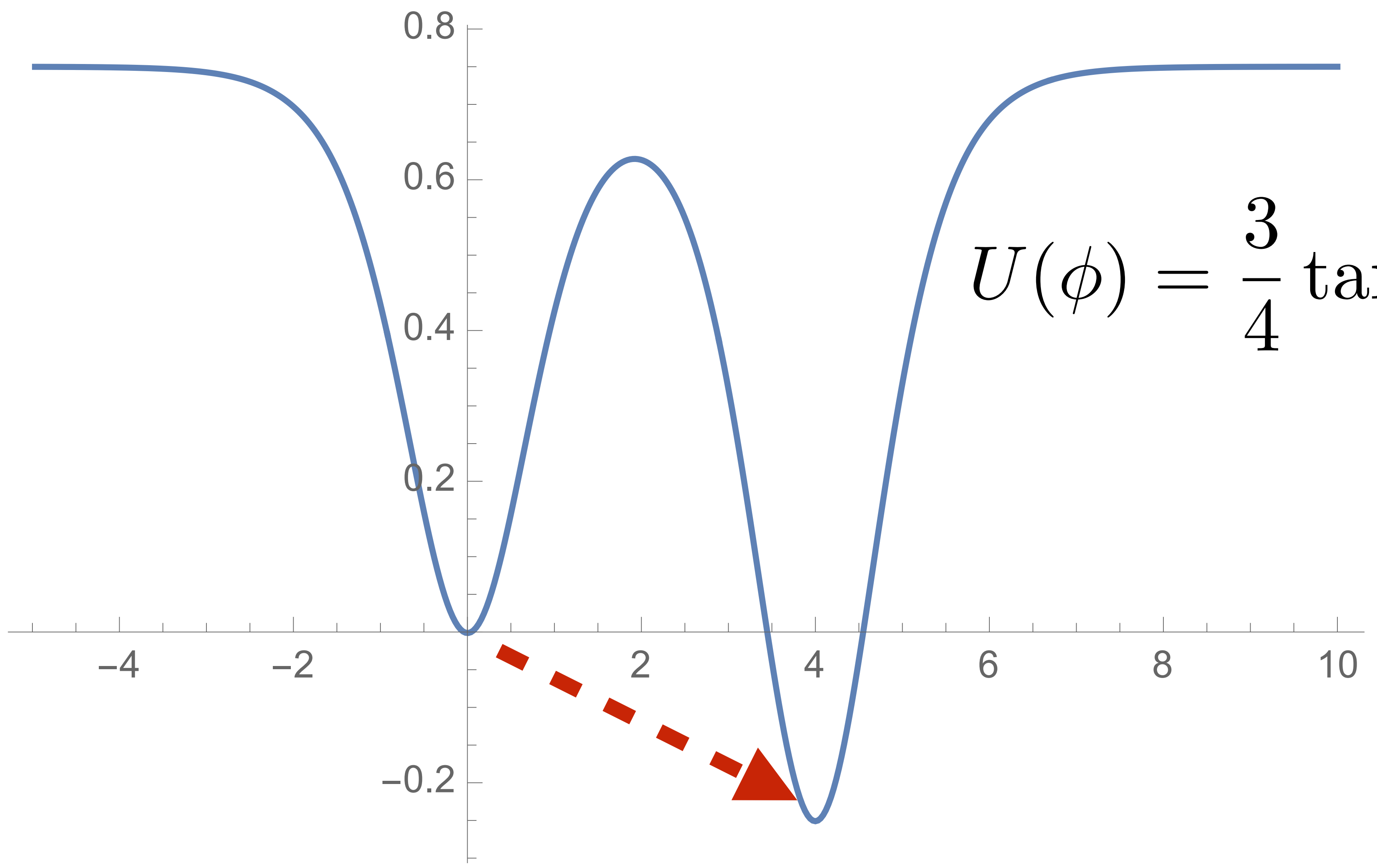
- Baryogenesis (creation of (anti)matter asymmetry)



- Instanton processes:



e.g. ***Tunnelling out of the false minimum of this potential (where ϕ is the single space coordinate):***



$$U(\phi) = \frac{3}{4}\tanh^2\phi + \tanh^2(\phi - v) - 1$$

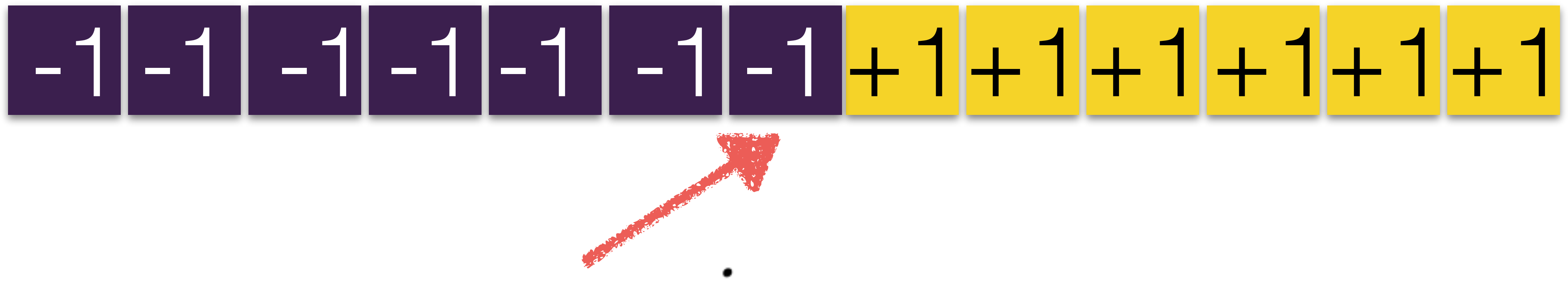
If we begin in the false minimum on the left, the system should be able to tunnel to the lower one on the right.

Encode ϕ by discretising its value using N qubits:

$$\phi = \phi_0 + j\,\xi \quad = \quad \phi_0 + \xi \;\ldots\; \phi_0 + N\xi$$

Represent it as a point on a spin chain $\Longrightarrow$ domain wall encoding:

Chancellor;
SAA, Chancellor, Spannowsky

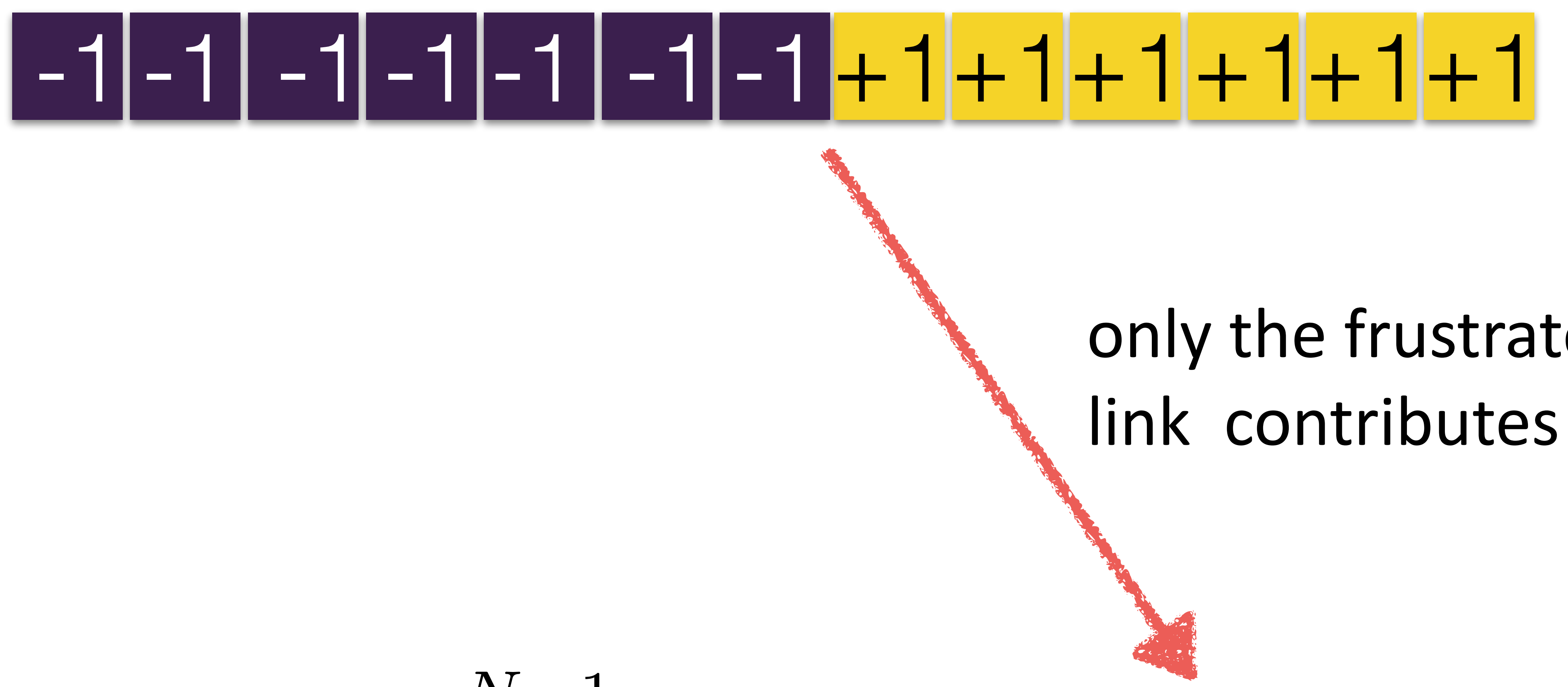


Position j

We can translate any spin chain back to the corresponding field value using

$$\phi = \phi_0 + \frac{\xi}{2}\sum_{i=1}^{N}(1 - \sigma_i^Z)$$

To add the potential U we then add a contribution to the linear h couplings

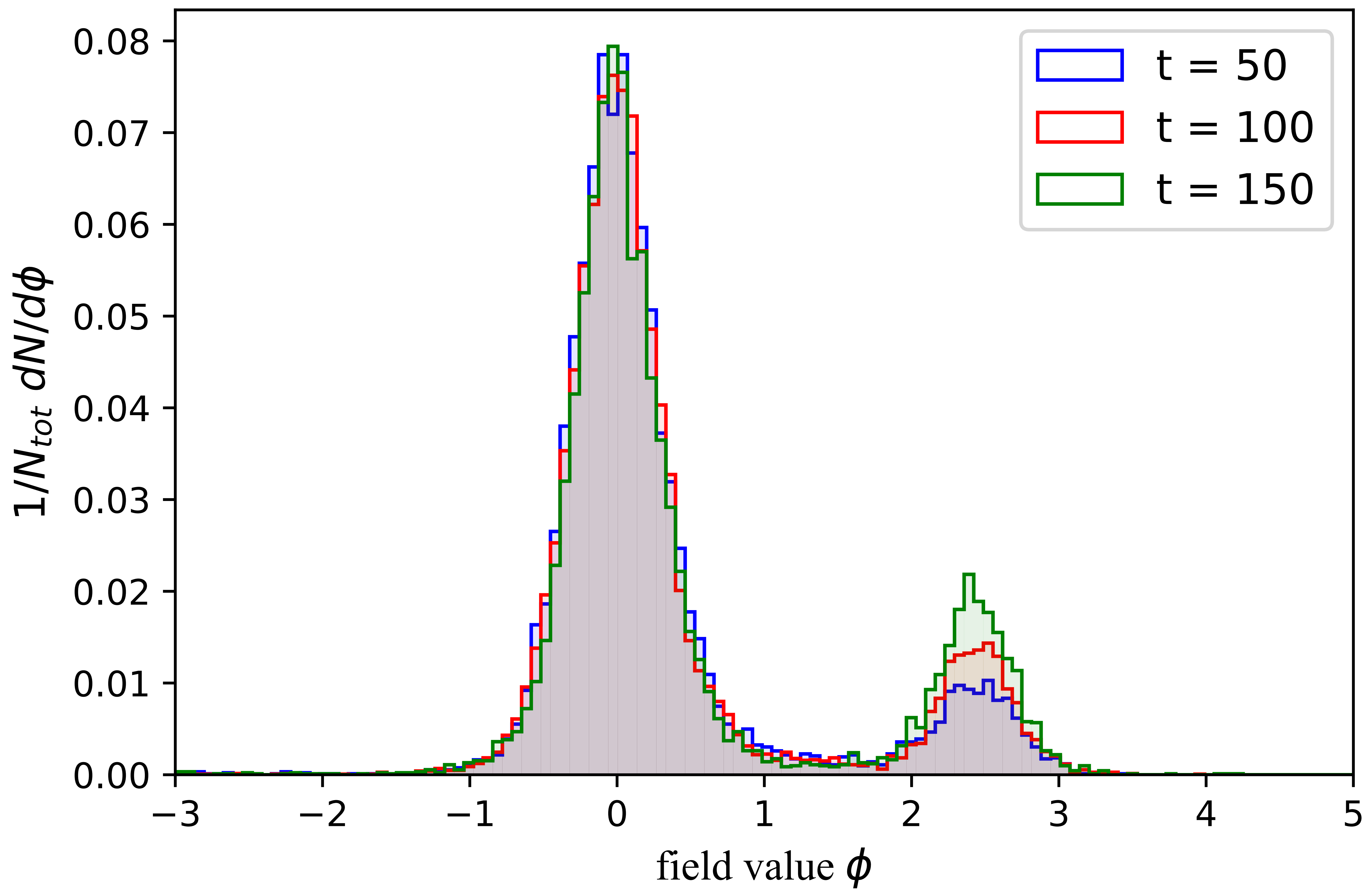


only the frustrated
link contributes

$$U(\phi) = \frac{1}{2} \sum_{i}^{N-1} U(\phi_0 + j\xi) \left(\sigma_{j+1}^{Z} - \sigma_{j}^{Z}\right)$$

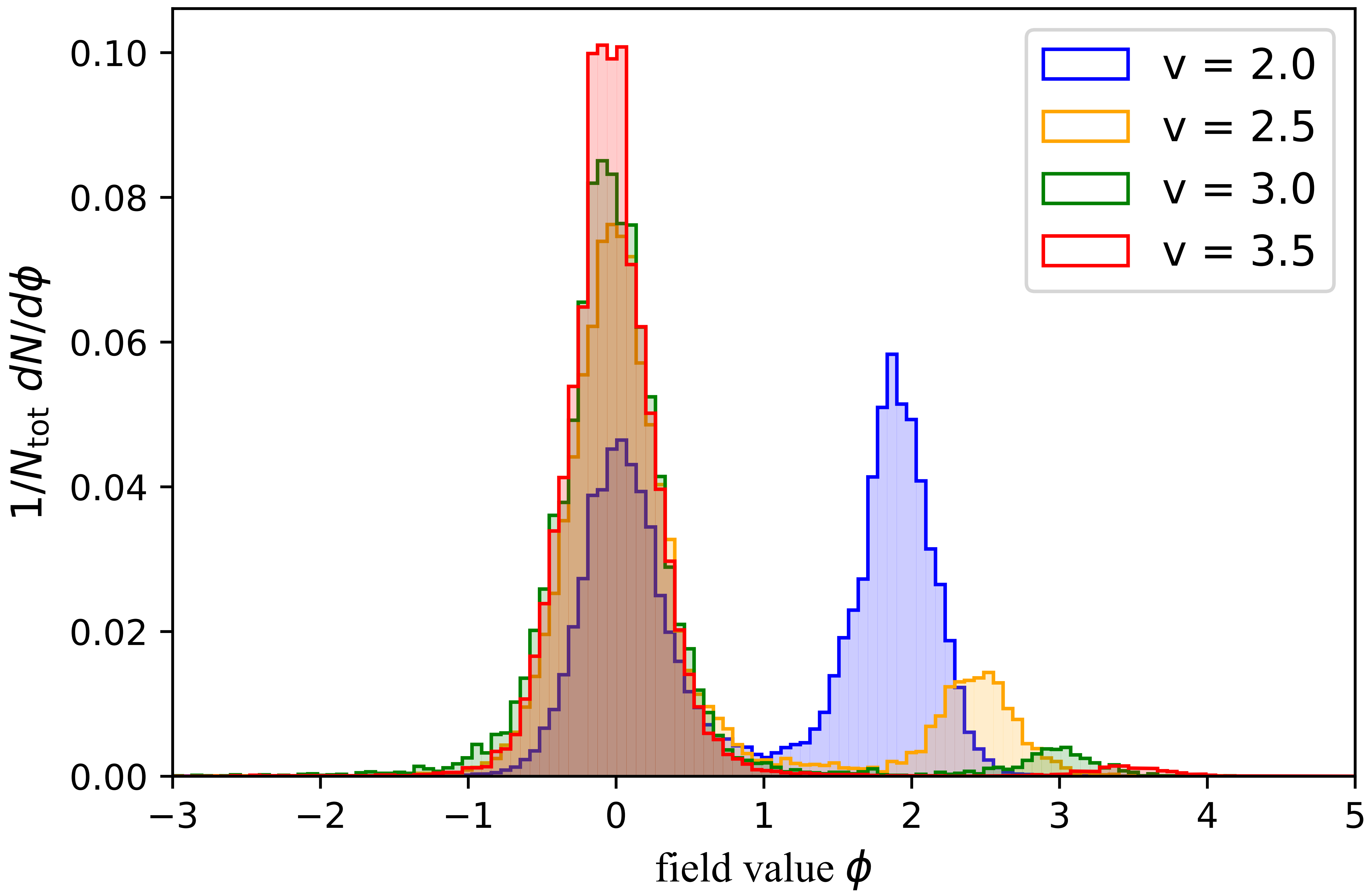
Results: ***(reverse anneal with 200 qubits) we see tunnelling — e.g. at*** $v = 2.5$

SAA, Spannowsky



It appears to decrease exponentially with v as expected (WKB approximation):

SAA, Spannowsky



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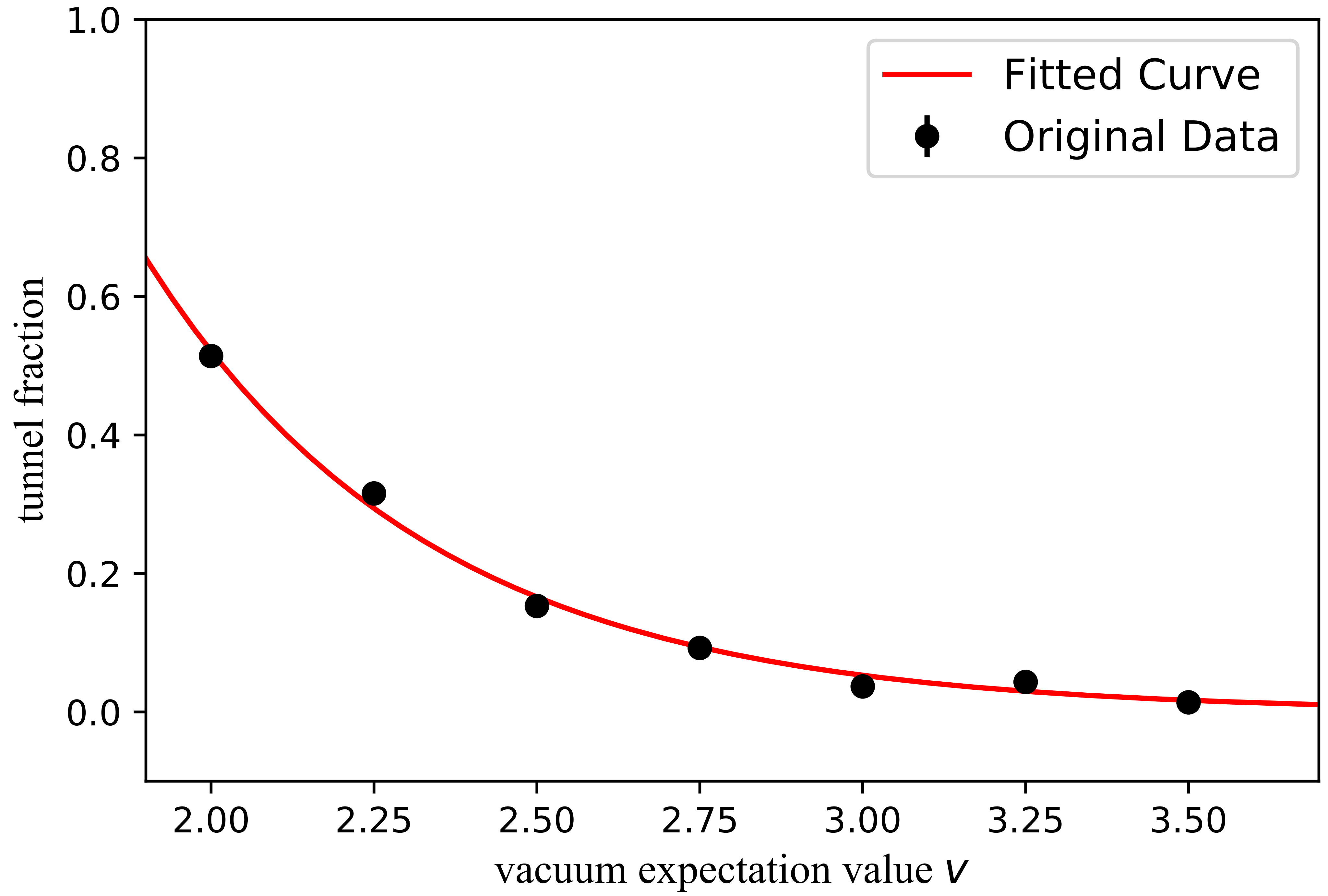
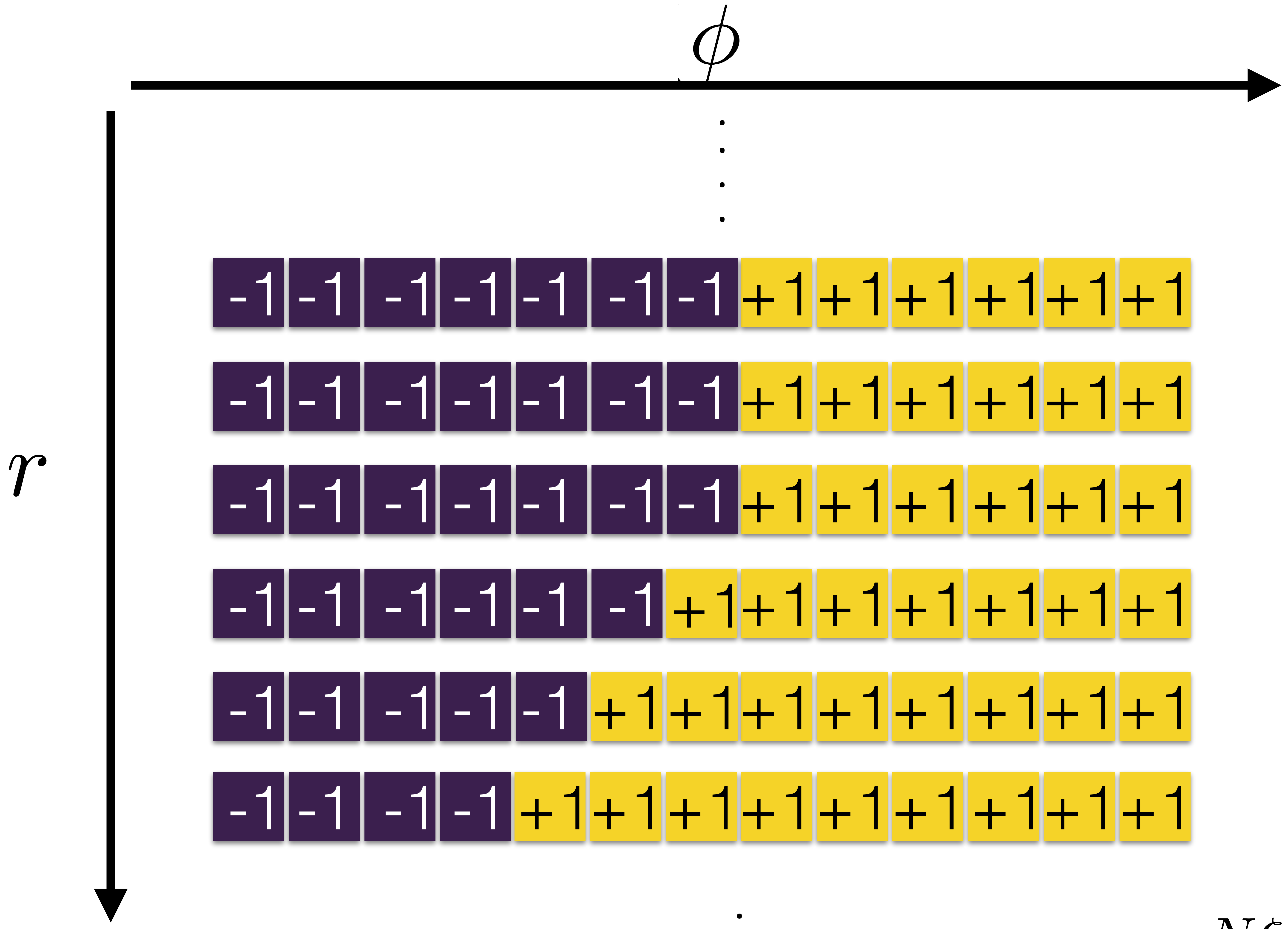


Theory: $\log\Gamma \;=\; 3.0 \times (1.66 - v)$

Exp: $\log\Gamma \;=\; 2.29 \times (1.71 - v)$

2.How would we go about doing QFT rather than QM?

Back to the domain wall encoding : to make this a QFT add a discretised spacetime coordinate, *r:*



$$r_\ell = \ell\nu \in \{\nu, 2\nu, \dots M\nu\}$$

$$\phi(r_\ell) = \phi_0 + \frac{N\xi}{2} - \frac{\xi}{2}\sum_{j=1}^{N} \sigma^Z_{\ell N + j}$$

Everything done for QM is then trivially extended in the ℓ spacetime index …

except kinetic space-derivative terms which are as follows:

$$H_{\mathrm{KE}} = \int_0^{\Delta r = M\nu} dr \frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2 = \lim_{M\to\infty} \sum_{\ell=1}^{M-1} \frac{1}{2\nu}\left(\phi(\rho_{\ell+1}) - \phi(\rho_\ell)\right)^2$$

$$= \sum_{\ell=1}^{M-1} \sum_{ij}^{N} \frac{\xi^2}{8\nu}\left[\sigma^Z_{(\ell+1)N+i} - \sigma^Z_{\ell N+i}\right] \times \left[\sigma^Z_{(\ell+1)N+j} - \sigma^Z_{\ell N+j}\right]$$

But now we find a huge number of couplings, or equivalently a huge number of gates on a discrete gate device

Taking stock …

Advantages:

Can encode many field theories using similar discretisation e.g. Jordan, Lee, Preskill; Jordan, Krovi, Lee, Preskill; Kclo, Savage

Can observe `vacuum decay' processes — requires a coherent quantum tunnelling

Can also perform this on gate quantum computers

Disadvantages:

Decoherence becomes critical after few nanoseconds on annealer (such short times are now becoming possible)

For QFT number of qubits becomes huge due to discretisation of fields

Gate depth becomes huge (on any discrete gate system) due to all the kinetic cross terms (billions to do a 3d lattice with disc'n of 10)

(basically every qubit describing field at x is connected to every qubit of the neighbouring space points)

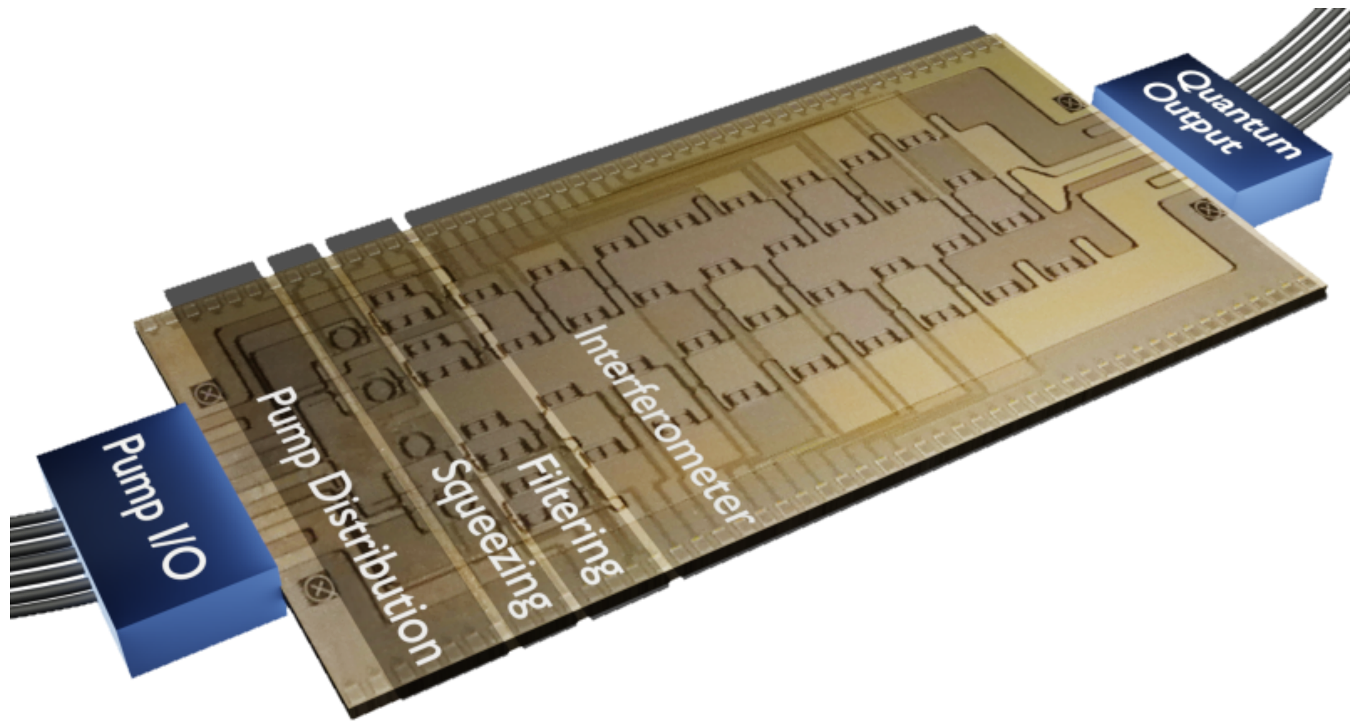
Is there a better way?? …

3.Photonics for Quantum Mechanics

Photonics work at room temperature. It relies on the manipulation of optical circuits using optical equipment such as interferometers. States can be stored using optical fibres (c.f. RAM)

The quantum circuits are defined by the continuous variables (CV) that is the x, p of quantum harmonic oscillators.

e.g. Borealis and X8 chips of Xanadu …

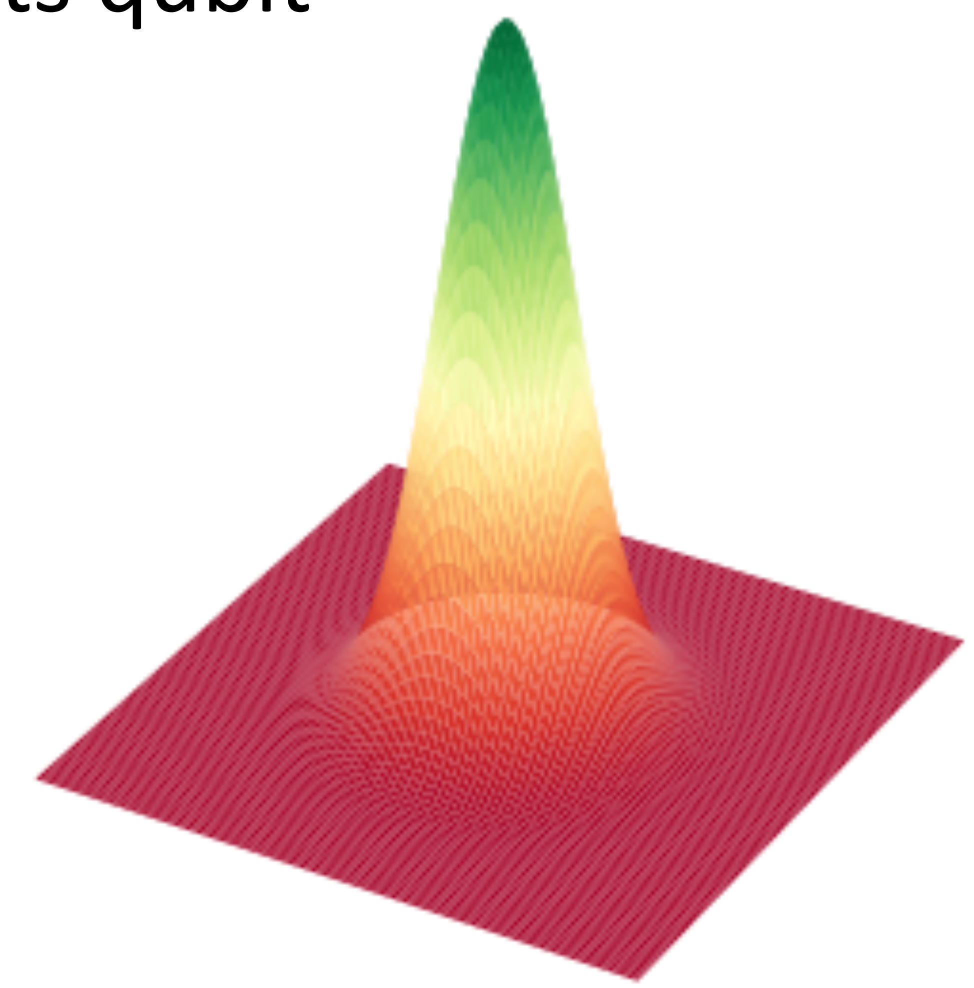


Qubit-based computations can be embedded into the CV picture (e.g., by using the Gottesman-Kitaev-Preskill (GKP) embedding), so the CV model is as (at least as) computationally powerful as its qubit counterparts.

qubit: $$|\phi\rangle = \phi_0 |0\rangle + \phi_1 |1\rangle \, ,$$

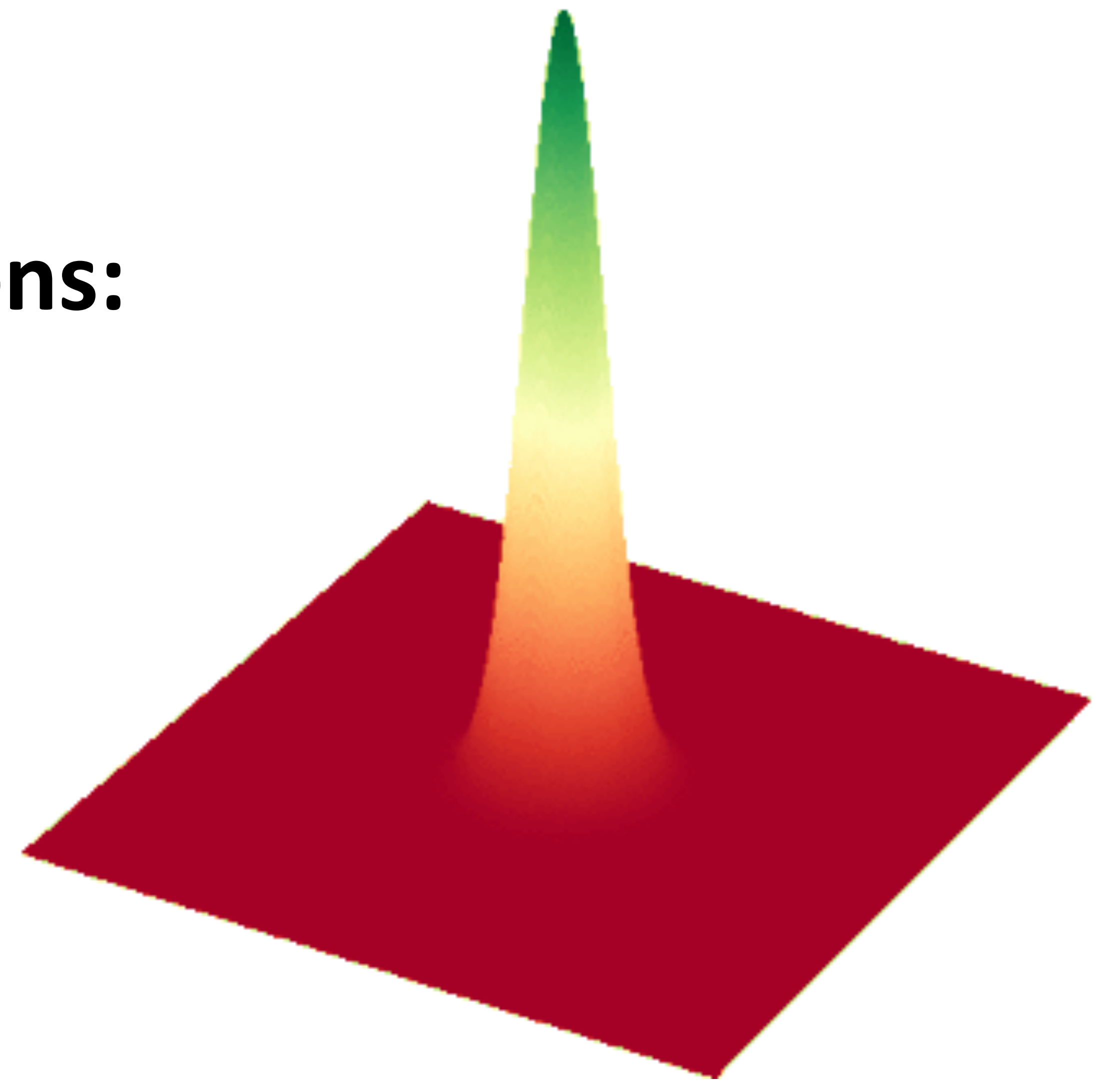
qumode: $$|\psi\rangle = \int dx \, \psi(x) \, |x\rangle \, .$$

Basic object is the SHO vacuum state and it's excitations … Wigner function looks like:

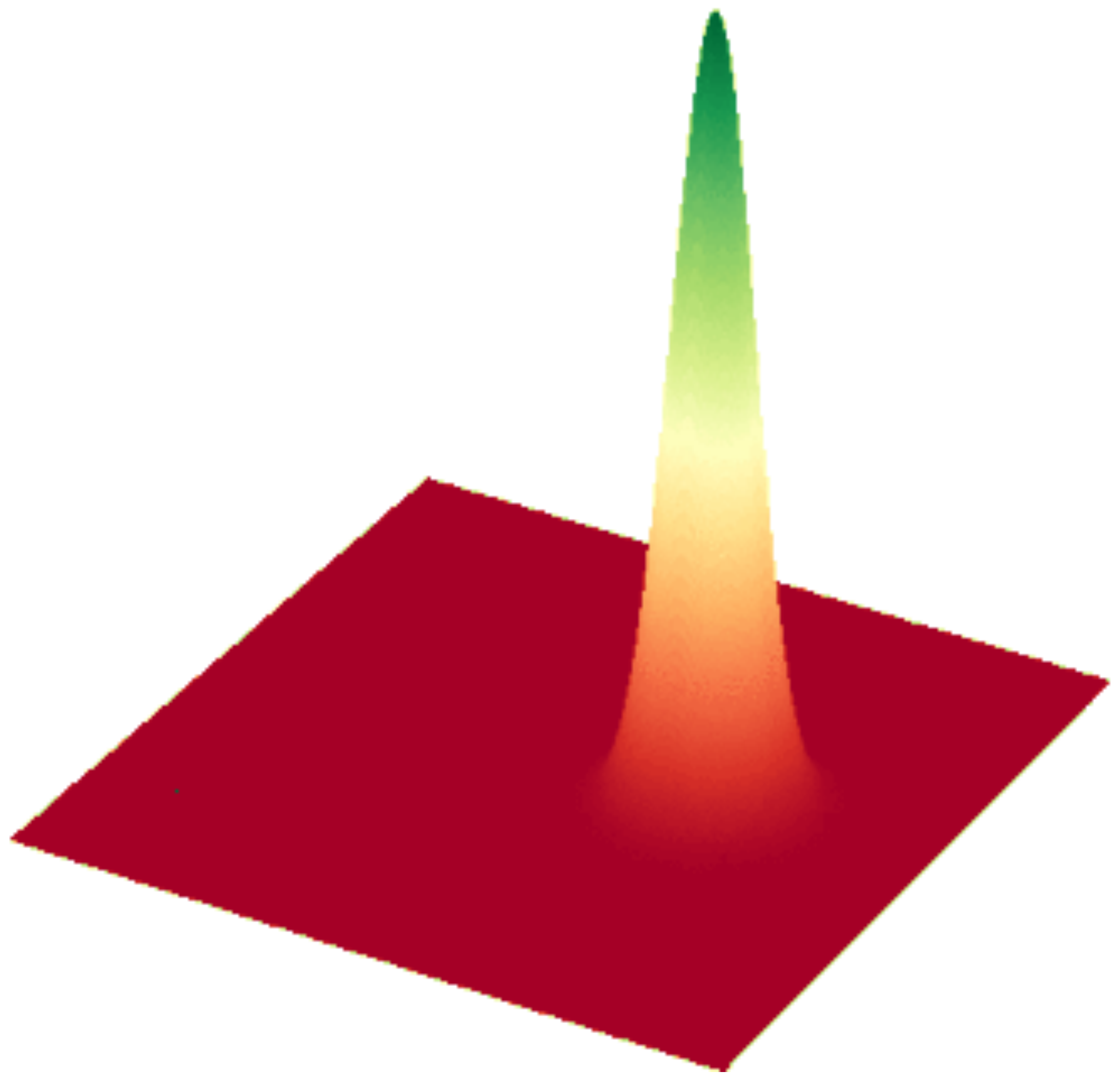


Some simple operations:

Squeezing Gate S:



Rotation Gate R:

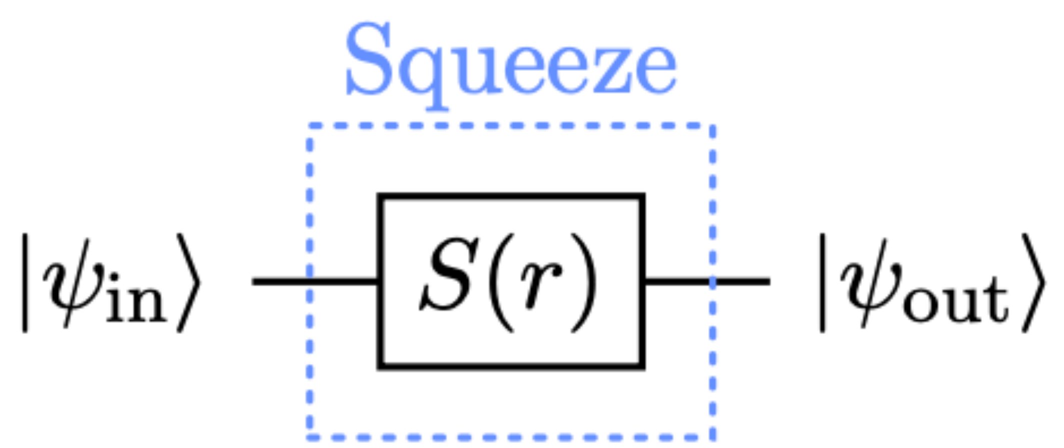


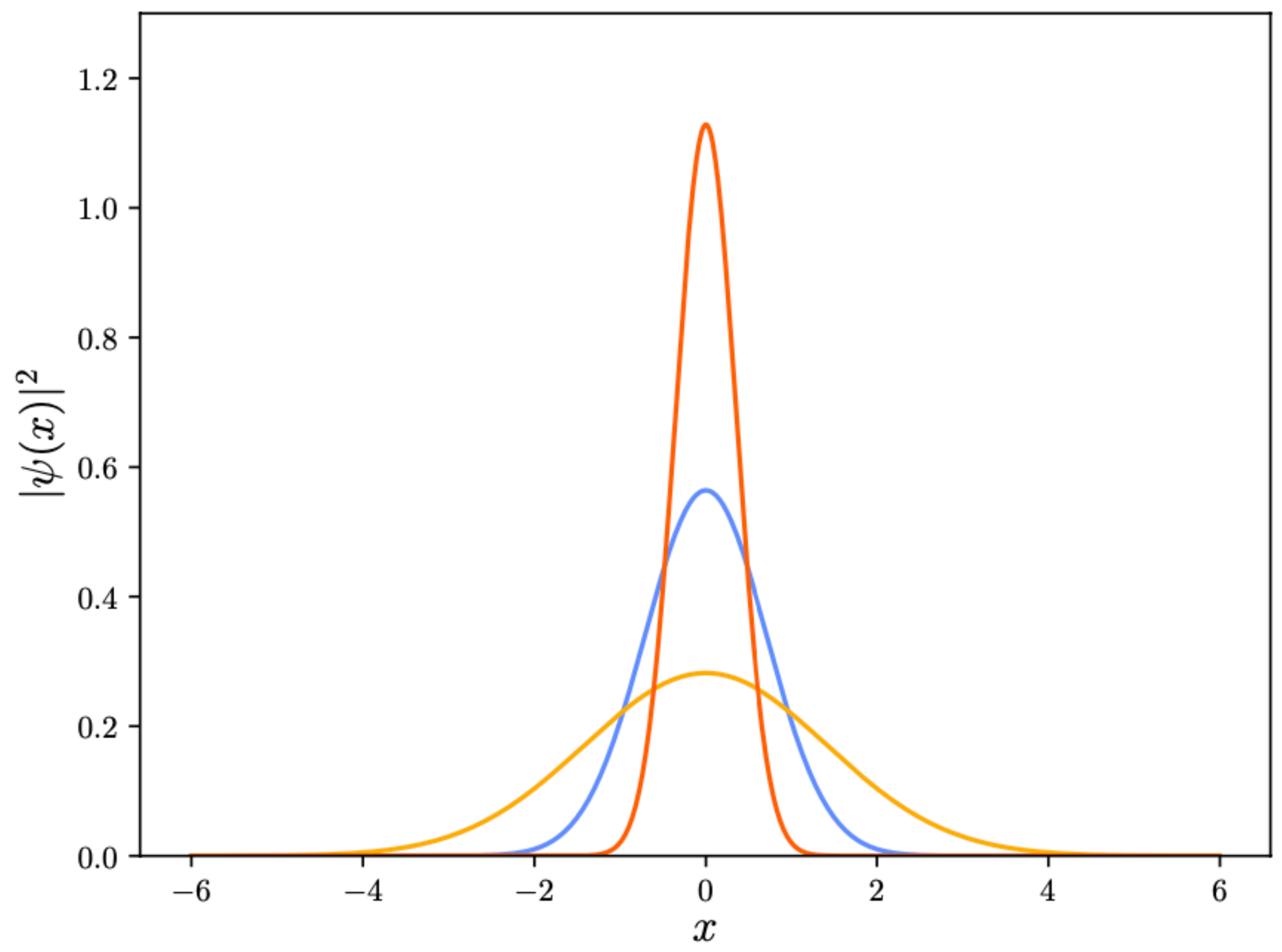
$$S^\dagger(z)\hat{x}S(z) = e^{-r}\hat{x} \; ; \qquad S^\dagger(z)\hat{p}S(z) = e^{r}\hat{p} \; .$$

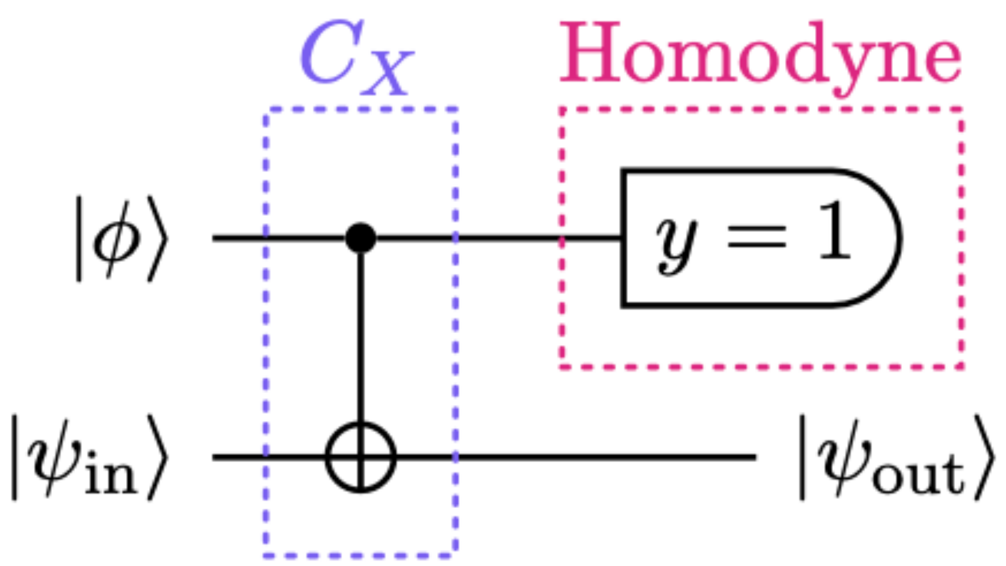
$$R(\theta) = e^{i\theta\hat{a}^\dagger\hat{a}} = e^{i\theta\left(\frac{1}{2}\hat{p}^2+\frac{1}{2}\hat{x}^2-\frac{1}{2}\right)}$$

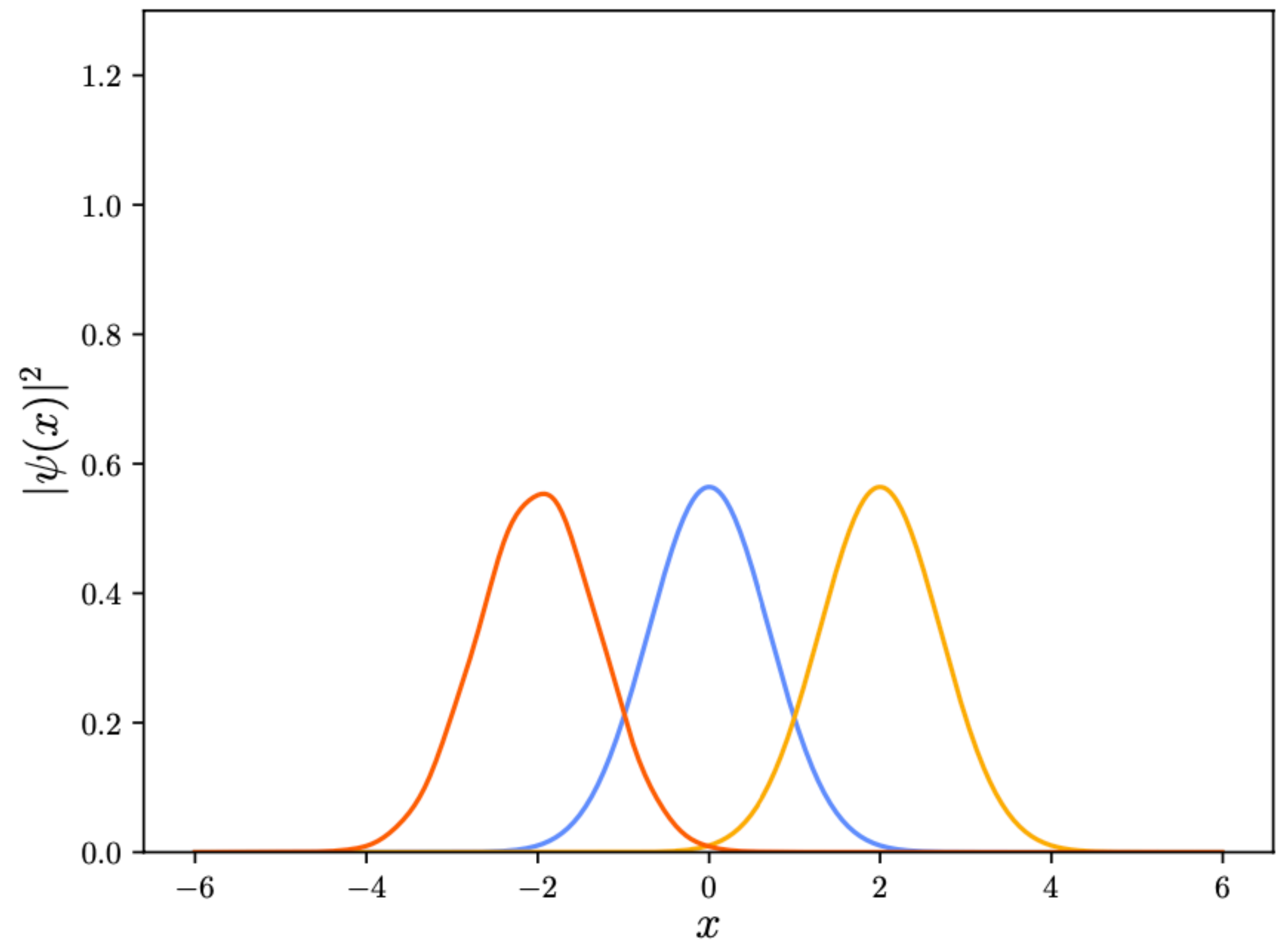
Controlled-X: $C_X(s;\hat{y},\hat{p}_x) = e^{-is\hat{y}\hat{p}_x}$ ==> $\hat{x} \rightarrow \hat{x} + s\hat{y}$.

Try some simple examples









Measurement based approach for Schrödinger evolution: the principle of evolver-states

Ultimately we would like to be able to do this to an arbitrary in state …

$$|\psi_{\rm out}\rangle \;=\; e^{-i\mathcal{H}t}|\psi_{\rm in}\rangle$$

for any in-state. But unless the hamiltonian is trivial (i.e. quadratic) this will require *non-Gaussian* gates.

An *evolver-state* is a resource state that we `factor' onto the in-state to make it evolve in time. It is an ancilla qumode with coordinate y, which looks like this:

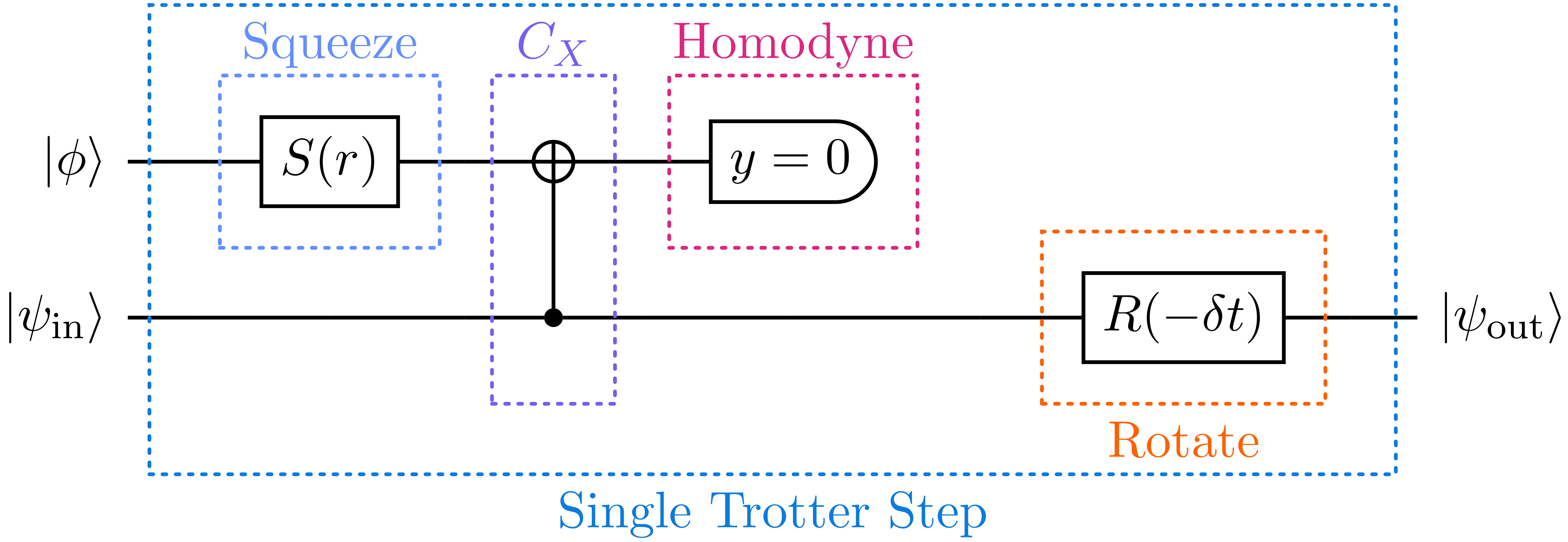
$$\langle y|\phi\rangle = \langle y|e^{-iH_1(\hat{y}/q)\delta t}|\phi_0\rangle$$

where $H_1(x)$ is the part of the Hamiltonian that is non-quadratic - (e.g. quartic potential), and $\langle y\,|\,\phi_0\rangle$ is a wavefunction that looks like a top-hat (for reasons that will become clear).

Using homodyne measurement for Schrödinger evolution: the principle of evolver-states

SAA, Spannowsky, Williams

Evolver - “Gadget”:



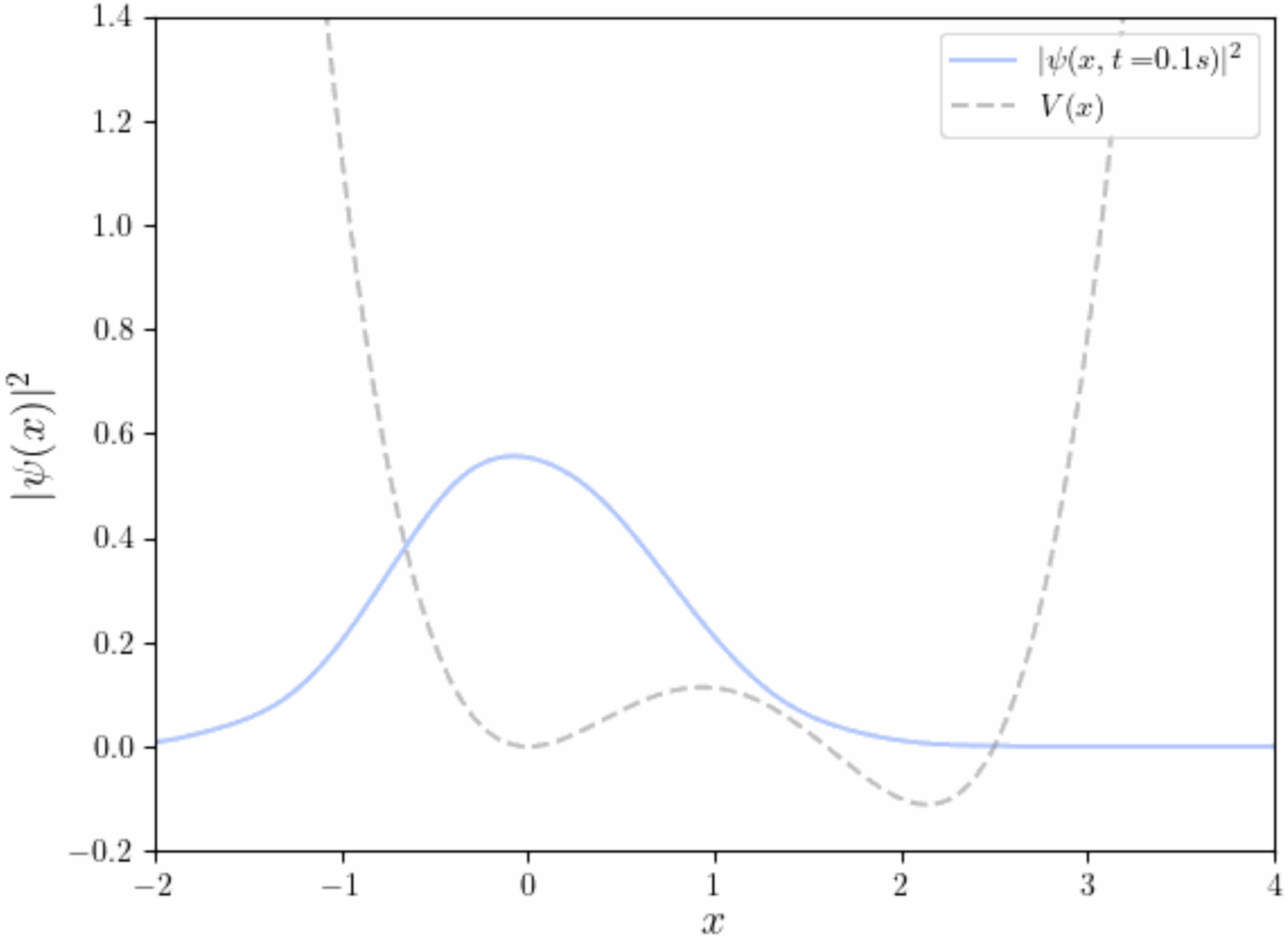
$$\langle x|\psi_{\text{out}}\rangle = \exp\left(-\frac{i}{2}(\hat{p}^2 + \hat{x}^2)\delta t\right) e^{-iH_1(\hat{x})\delta t} \langle qx|\phi_0\rangle\langle x|\psi_{\text{in}}\rangle$$

`noise’ function which is roughly constant if we choose the top-hat

Evolving QM using an evolver state: example

SAA, Spannowsky, Williams

$$V(x) = \frac{1}{8}(x^2 - 2x)^2 - \frac{\varepsilon}{8}x^3$$



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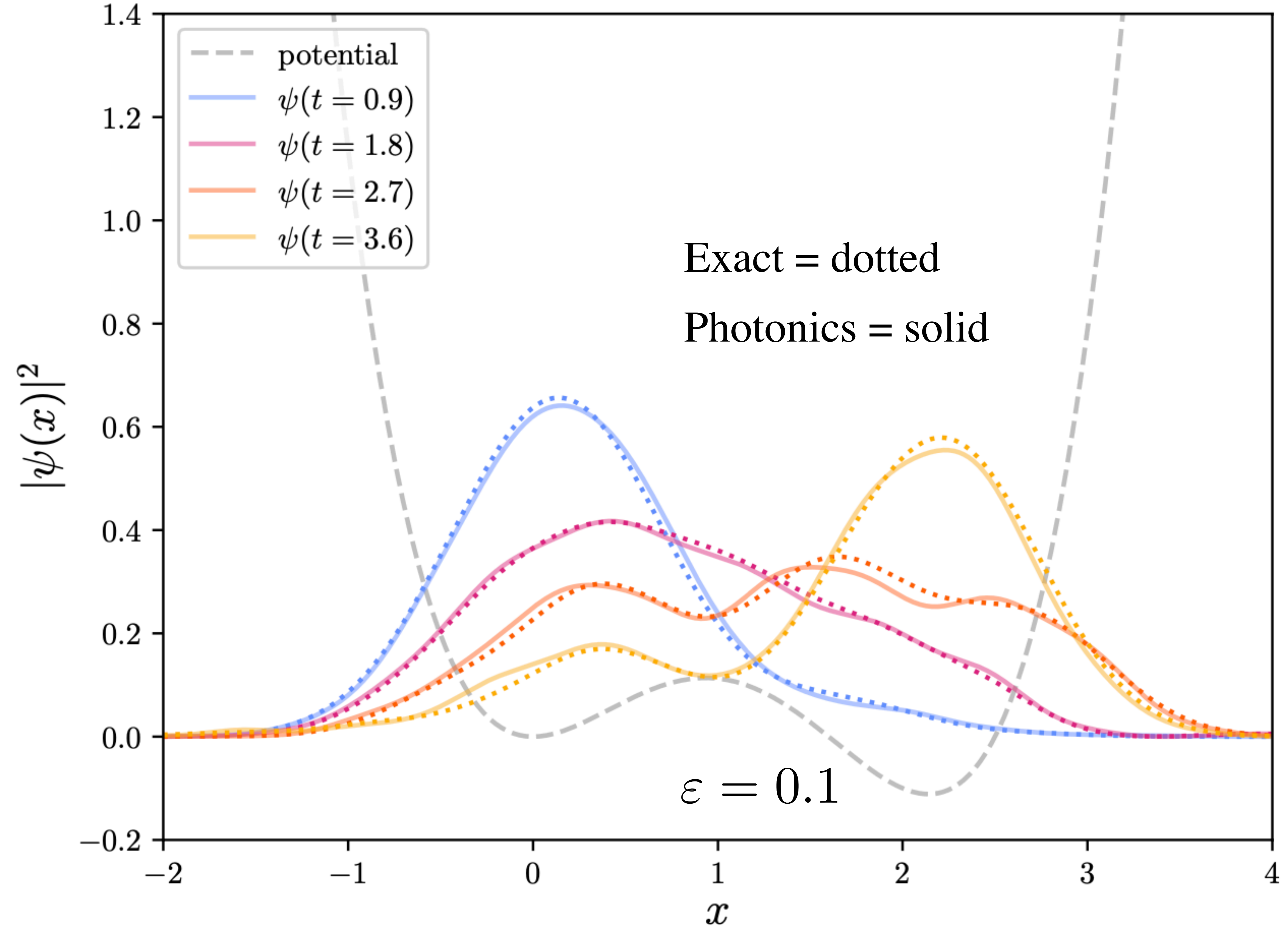
SAA, Spannowsky, Williams



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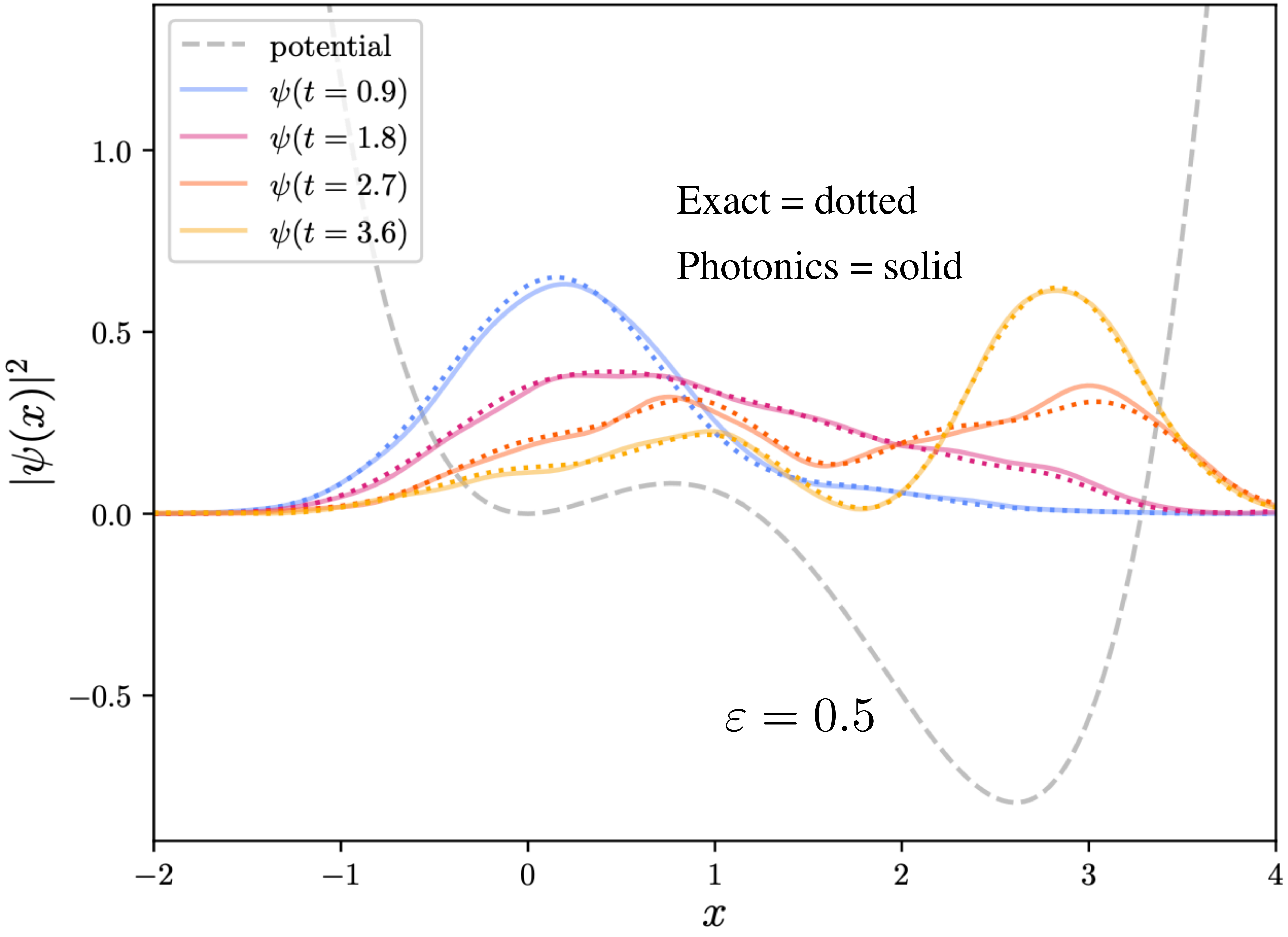
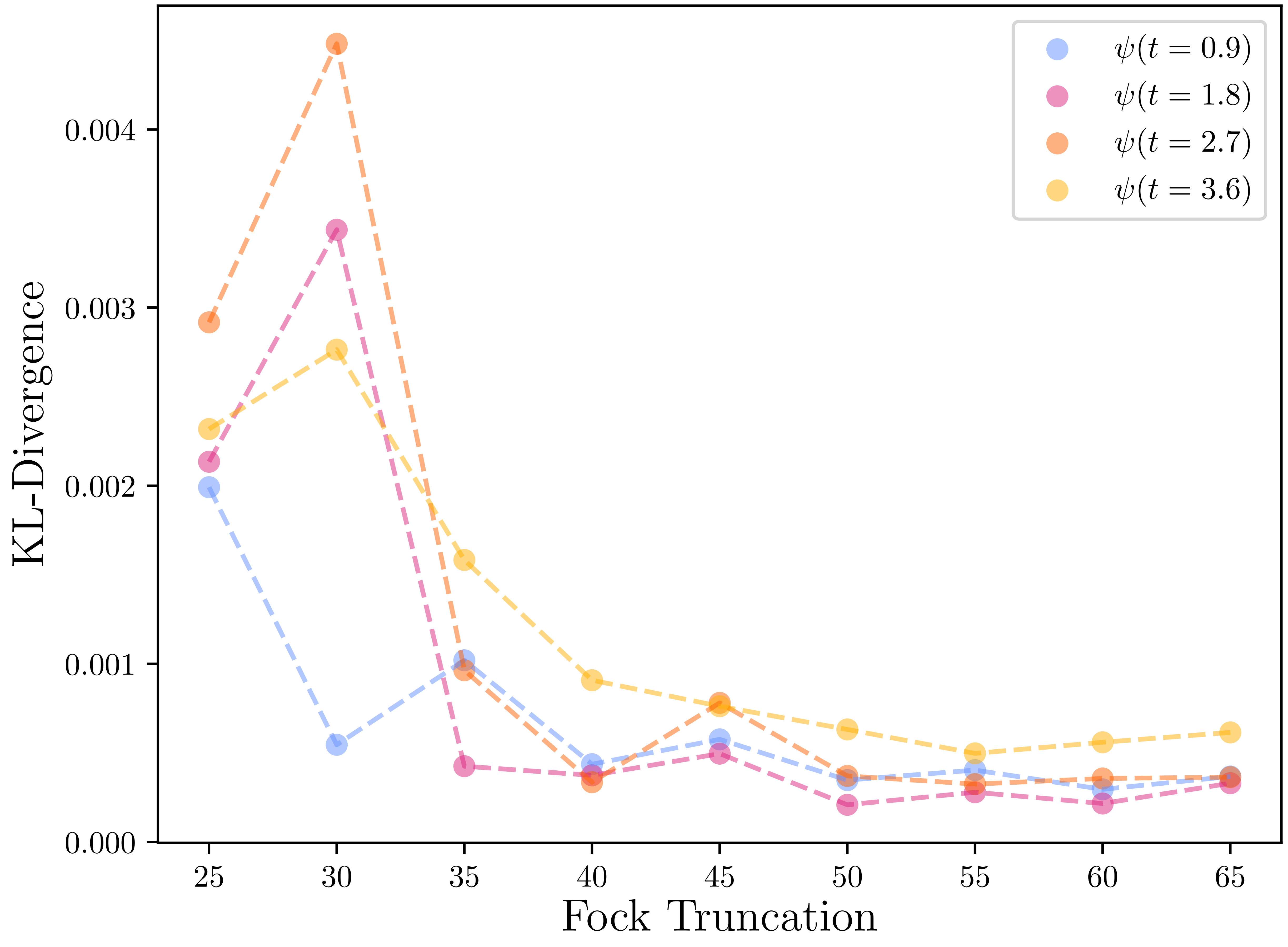


Evolving QM using an evolver-state: example

SAA, Spannowsky, Williams

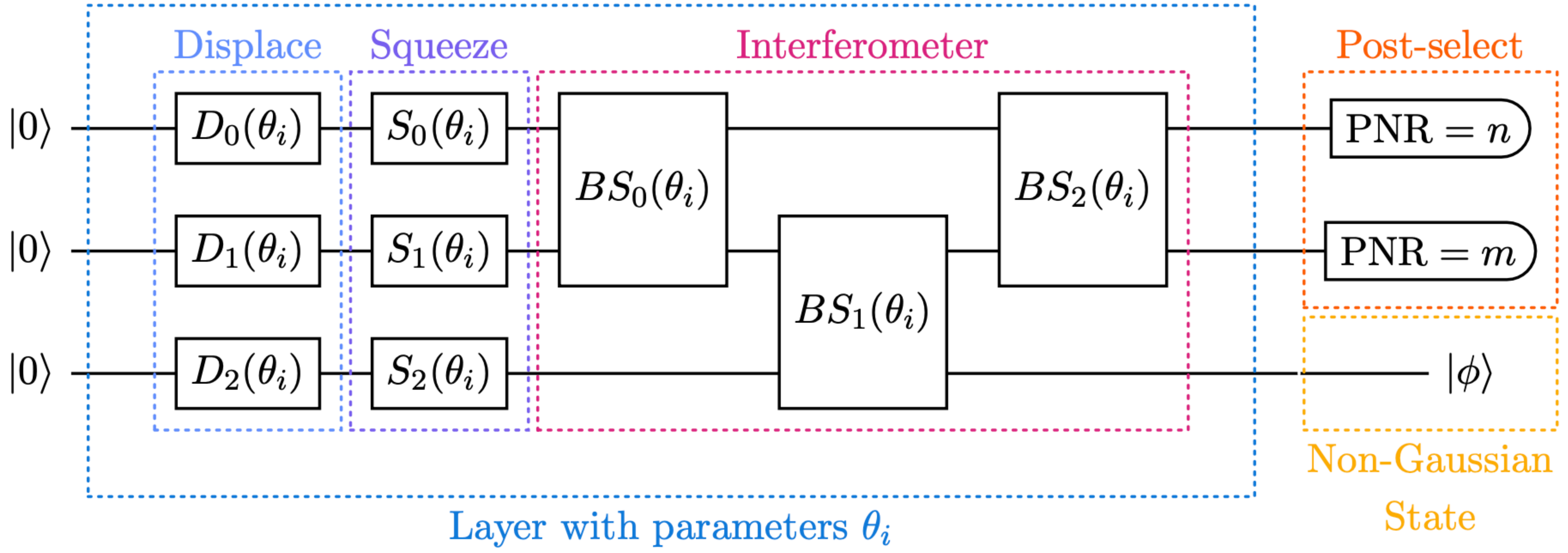
KL-divergence for different Fock truncations:



SAA, Spannowsky, Williams

Machine learning the evolver-state:

In these studies we set the initial ancilla evolver-state using the Fock back-end of StrawberryFields - (`Ket` command). To use on entirely photonic device can use photon measurement and ML to tune a circuit that gives desired non-Gaussian state on the ancilla qumode … (Izaac, Myers, Sabapathy, Su, Weedbrook)



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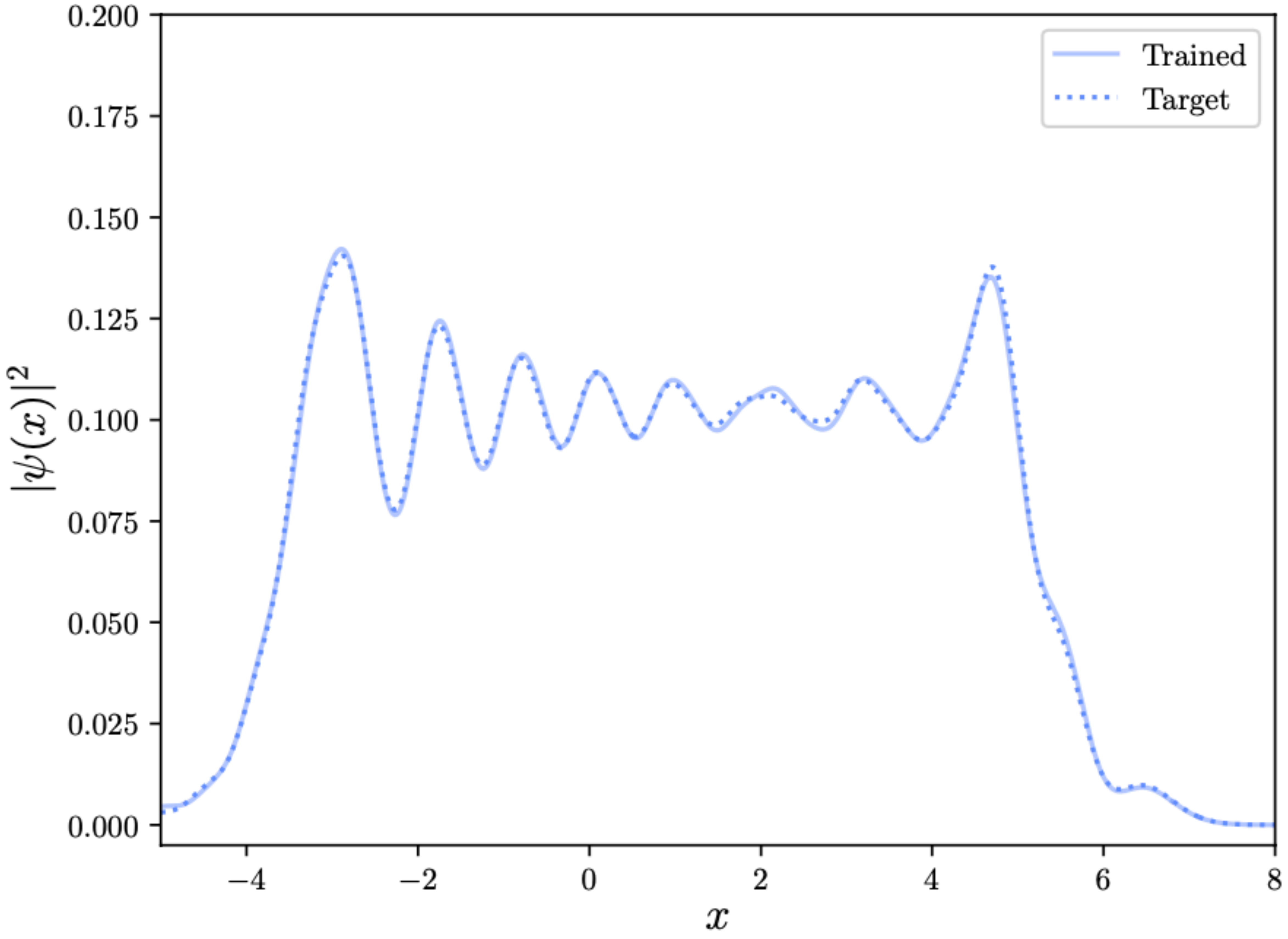


4.How can we do QFT on a photonic device?

QFT? Considerable simplification !...

Consider *1-d field theory* with space-dimension labelled r ... discrete lattice of oscillators at

$$r_k \;=\; r_0 + k\,a \quad ; \qquad k = 1\ldots M$$

with Hamiltonian density given by

$$\mathcal{H}a^{-1} \;=\; \sum_{k=1}^{M}\left(\frac{1}{2}\pi_k^2 + \frac{1}{2}(\partial_r\varphi_k)^2 + V(\varphi_k)\right)$$

The oscillators are connected only by the cross-terms in the kinetic piece which connect neighbouring points. Finite difference ...

$$(\partial_r\varphi_k)^2(r) \;=\; \frac{(\varphi(r_k + a) - \varphi(r_k))^2}{a^2}$$

Suppose we encode field values and their conjugate momenta at each point as a qumode variable. Their commutation relations are correct if we identify ...

$$\boxed{\begin{aligned} \varphi(r_k) \;&=\; \hat{x}_k \\ \pi(r_k) \;&=\; a^{-1}\hat{p}_k \end{aligned}}$$

$\Longrightarrow$

$$\mathcal{H}a = \sum_{k=1}^{M}\left(\frac{1}{2}\hat{p}_k^2 + \frac{1}{2}(\hat{x}_{\overline{k+1}} - \hat{x}_k)^2 + a^2 V(\hat{x}_k)\right) \qquad \overline{k+1} = k+1 \mod(M)$$

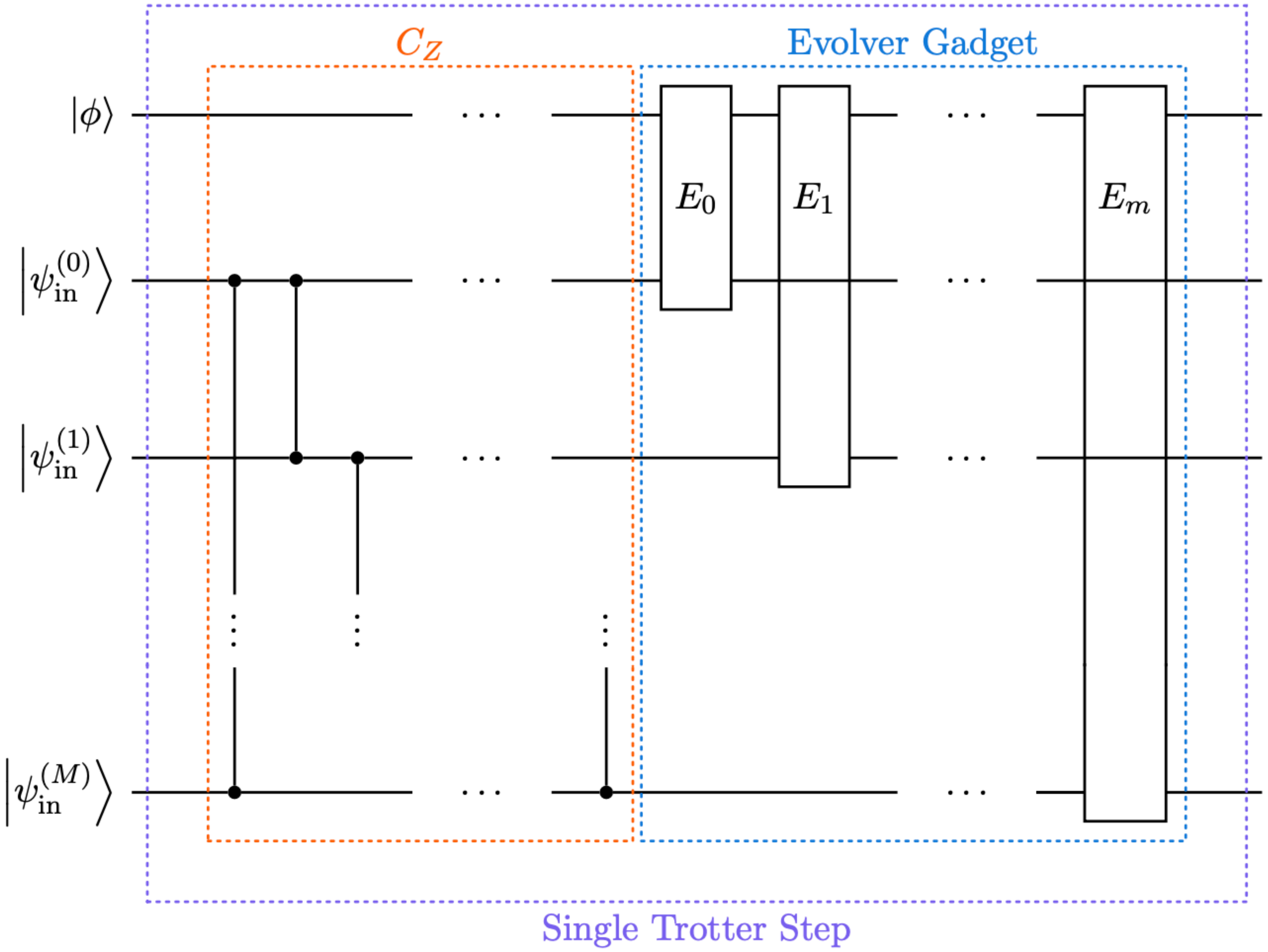
$$= \sum_{k=1}^{M}\left(\frac{1}{2}\hat{p}_k^2 + \frac{1}{2}\hat{x}_k^2 + H_1(\hat{x}_k)\right) - \sum_{k=1}^{M}\hat{x}_{\overline{k+1}}\hat{x}_k$$

where $H_1(x) = \frac{1}{2}x^2 + a^2V(x)$ can be treated as the effective potential of each local node.

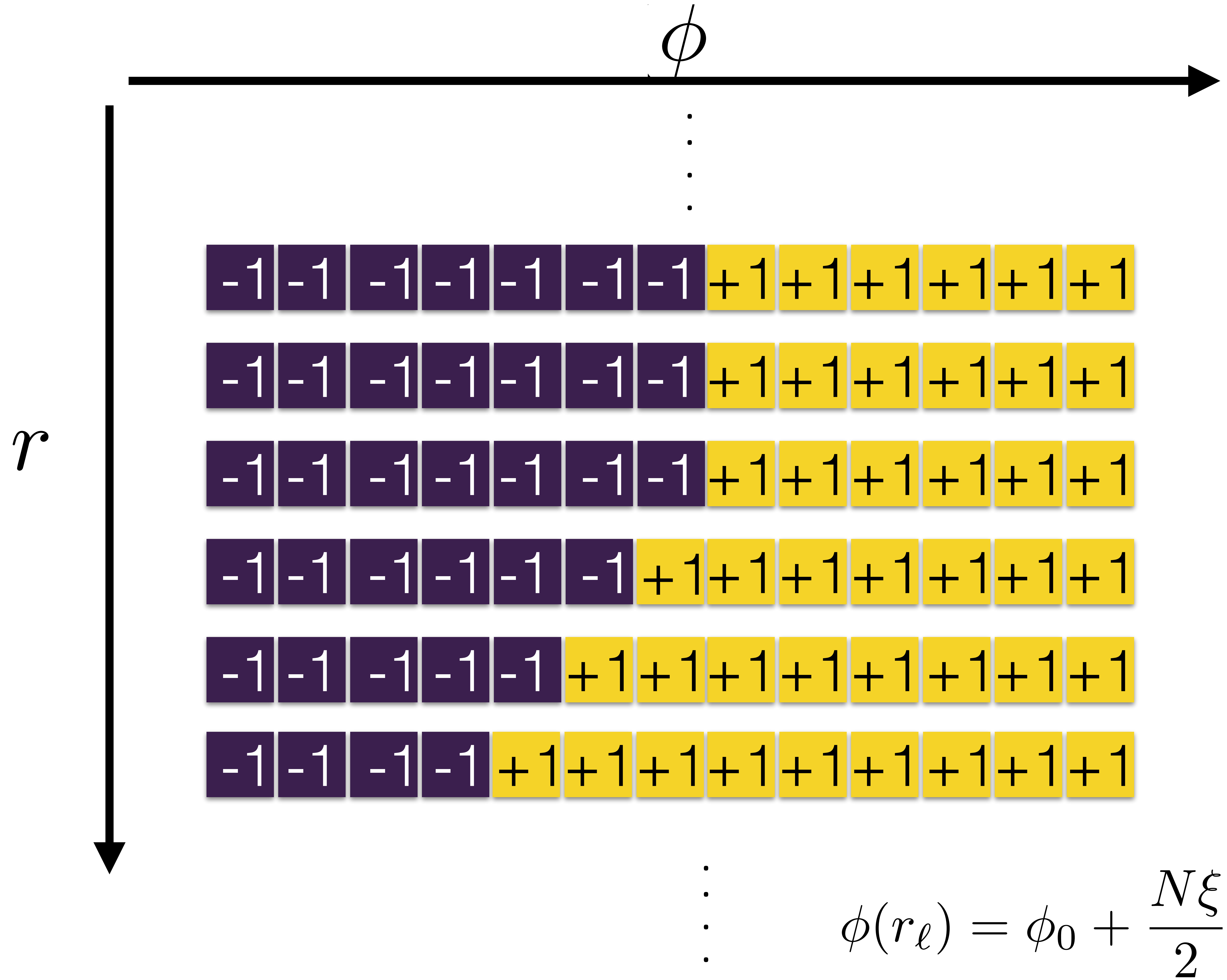
This is essentially just M of the Q.M. problem that we have already solved, coupled together with hopping couplings which correspond to a single ***Controlled-Z*** gate between adjacent space points.

Circuit diagram for M space-points looks like this:

SAA, Spannowsky, Williams



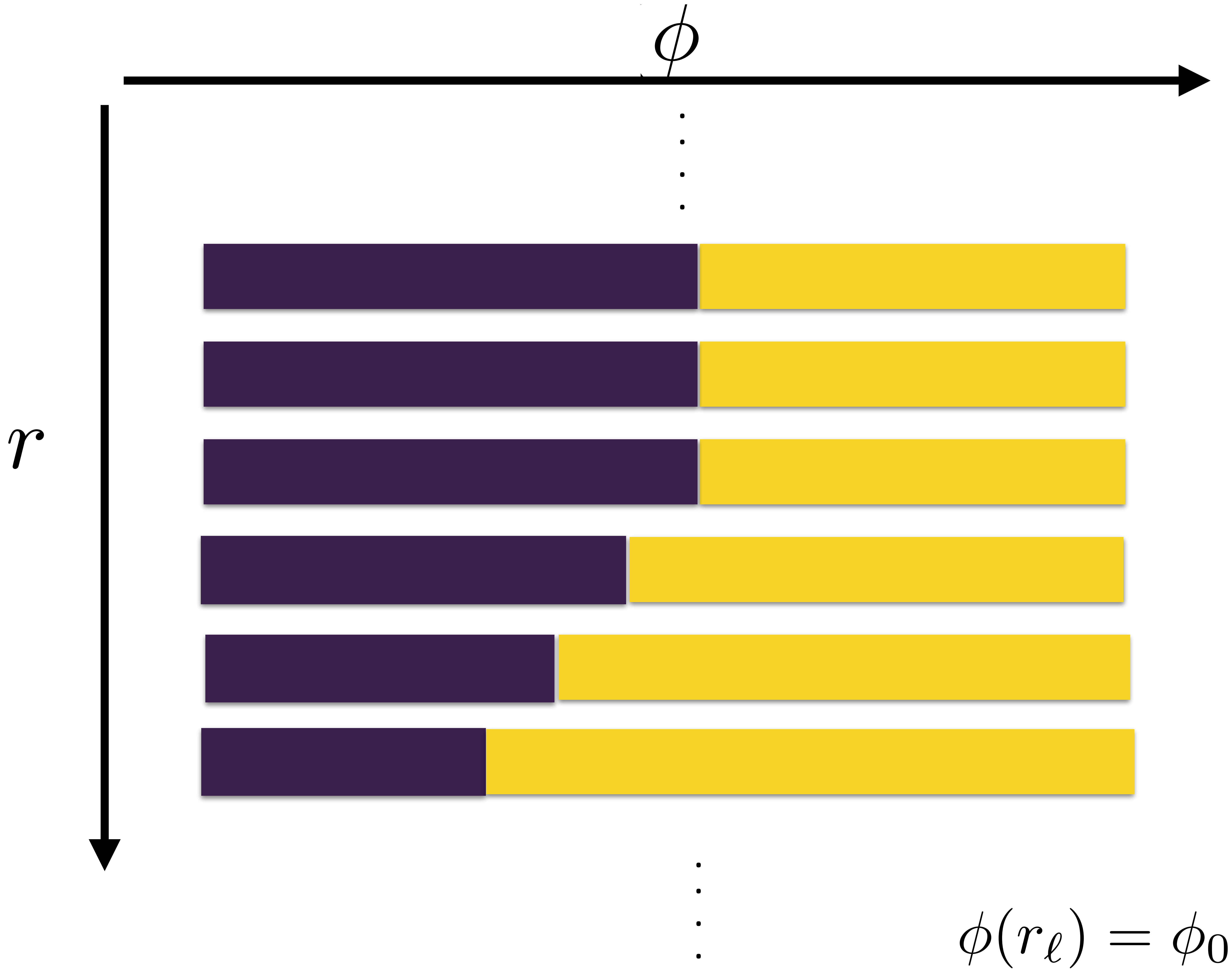
For 1-d field theory **before** we had this … (this small 1D example would need 1000 gates)



$$r_\ell = \ell\nu \in \{\nu, 2\nu, \ldots M\nu\}$$

$$\phi(r_\ell) = \phi_0 + \frac{N\xi}{2} - \frac{\xi}{2}\sum_{j=1}^{N} \sigma^Z_{\ell N+j}$$

Now we have this … (which in this example has 5 CZ gates and 5 evolver gadgets)



$$r_\ell = \ell\nu \in \{\nu, 2\nu, \ldots M\nu\}$$

$$\phi(r_\ell) = \phi_0 + x_\ell$$

Conclusions

- Able to build continuous quantum theories by hand in order to produce tunnelling processes
- Observe and measure tunnelling out of false vacuum
- Moving to QFT is difficult in any discrete quantum field encoding
- Continuous Variable Quantum Computing has great promise
- Can solve QM with arbitrary potentials on a single qumode with very little loss of coherence using Gaussian boson sampling
- Continuous variables could be essential for circumventing problems with simulating a QFT