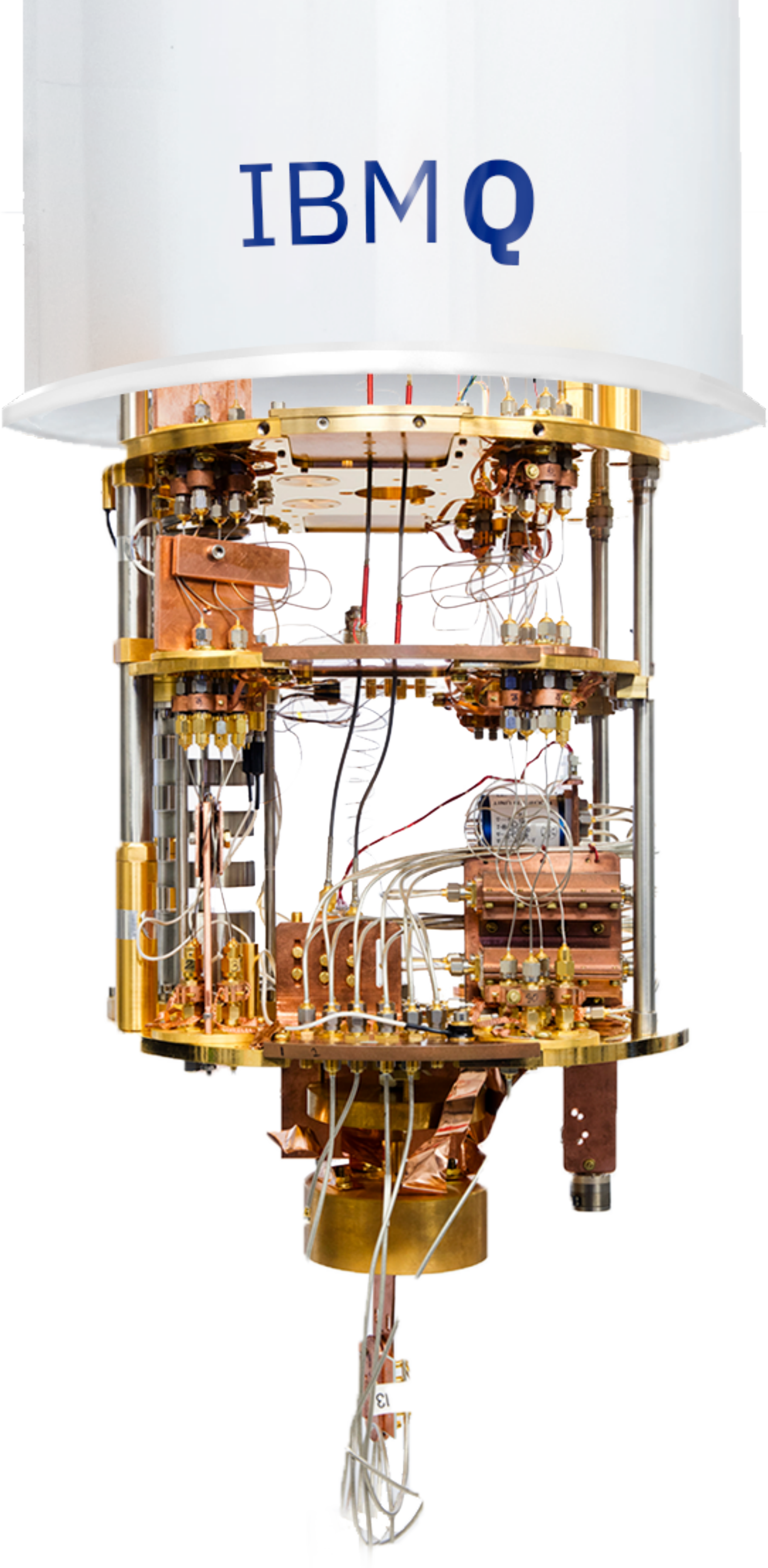

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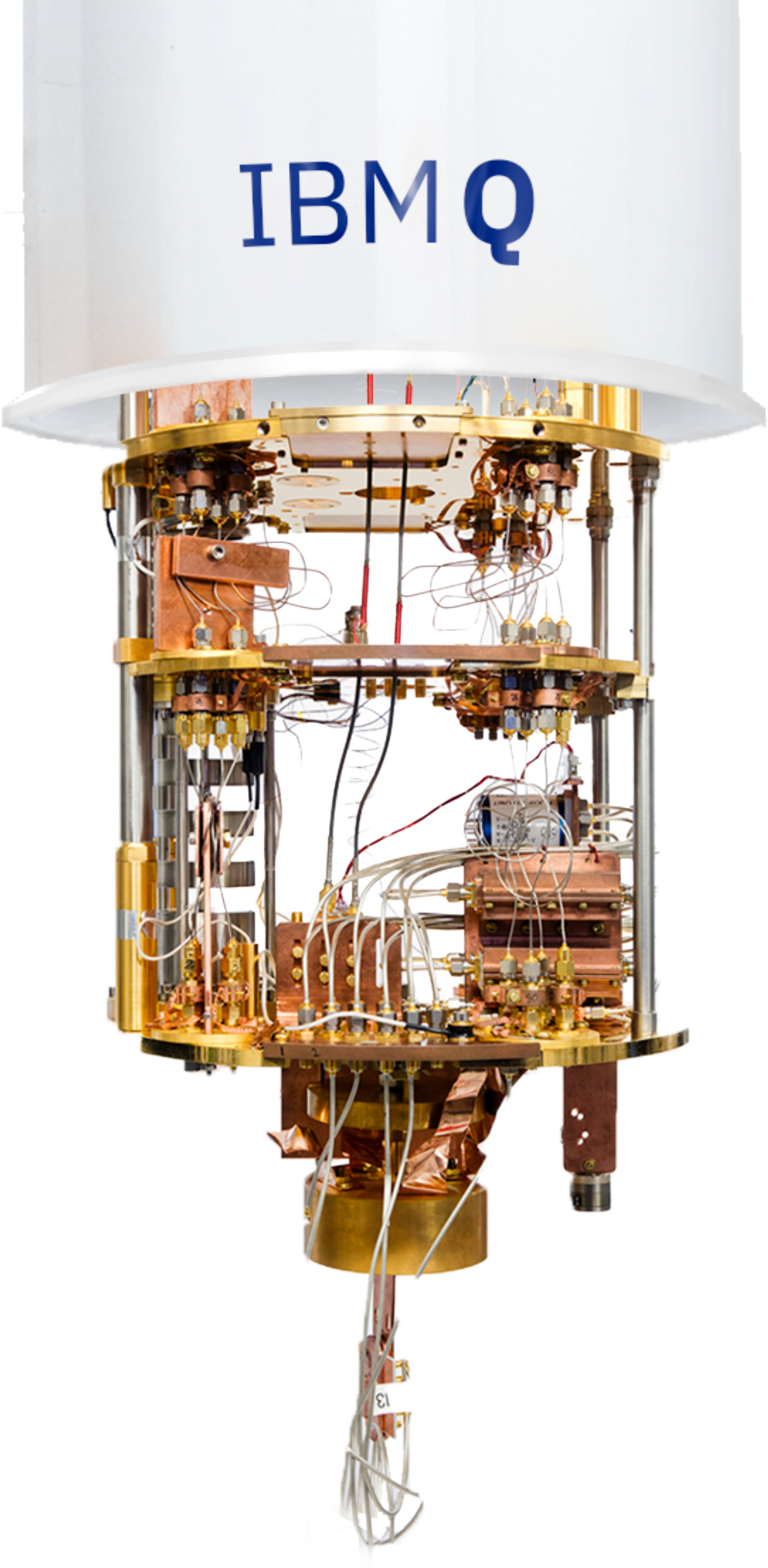
IP3

Simulating high-energy collision events with a quantum computer

Simon Williams

Future Colliders, Corfu Summer Institute,
24th May 2024



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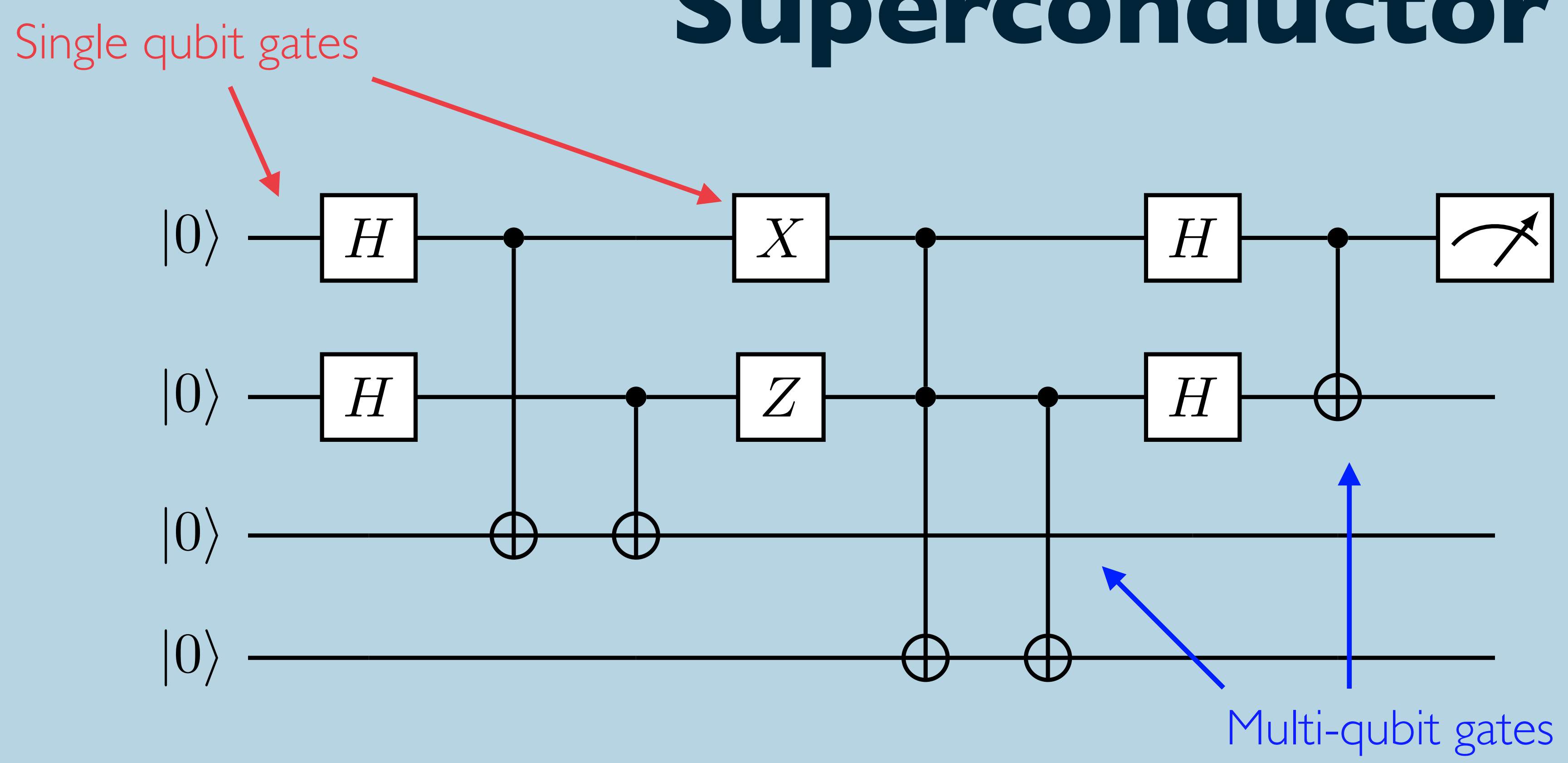
IP3

- Event Generation - What's the problem?
  - The Parton Shower
- Quantum Parton Shower
  - Discretising QCD
  - The Parton Shower as a Quantum Walk
- Quantum Charged Particle Track Finding

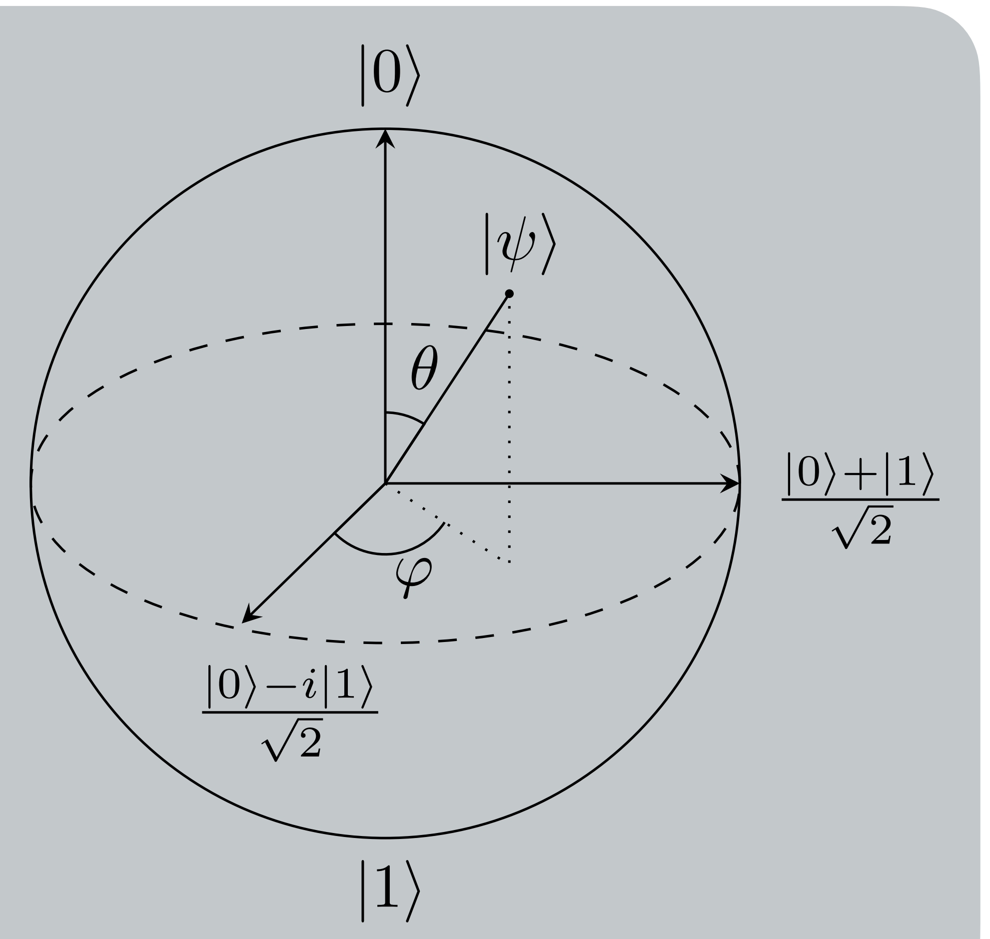
Discrete Gate Quantum Computing

Superconductor QCs



Qubit model:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



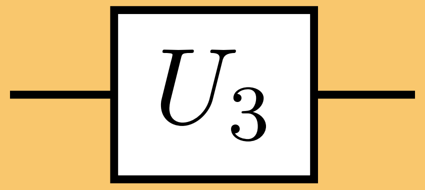
Advantages:

- Highly controllable qubits
- Universal computation

Disadvantages:

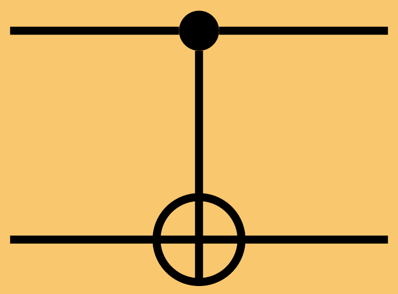
- Small number of qubits, not very fault tolerant

Single qubit gates:

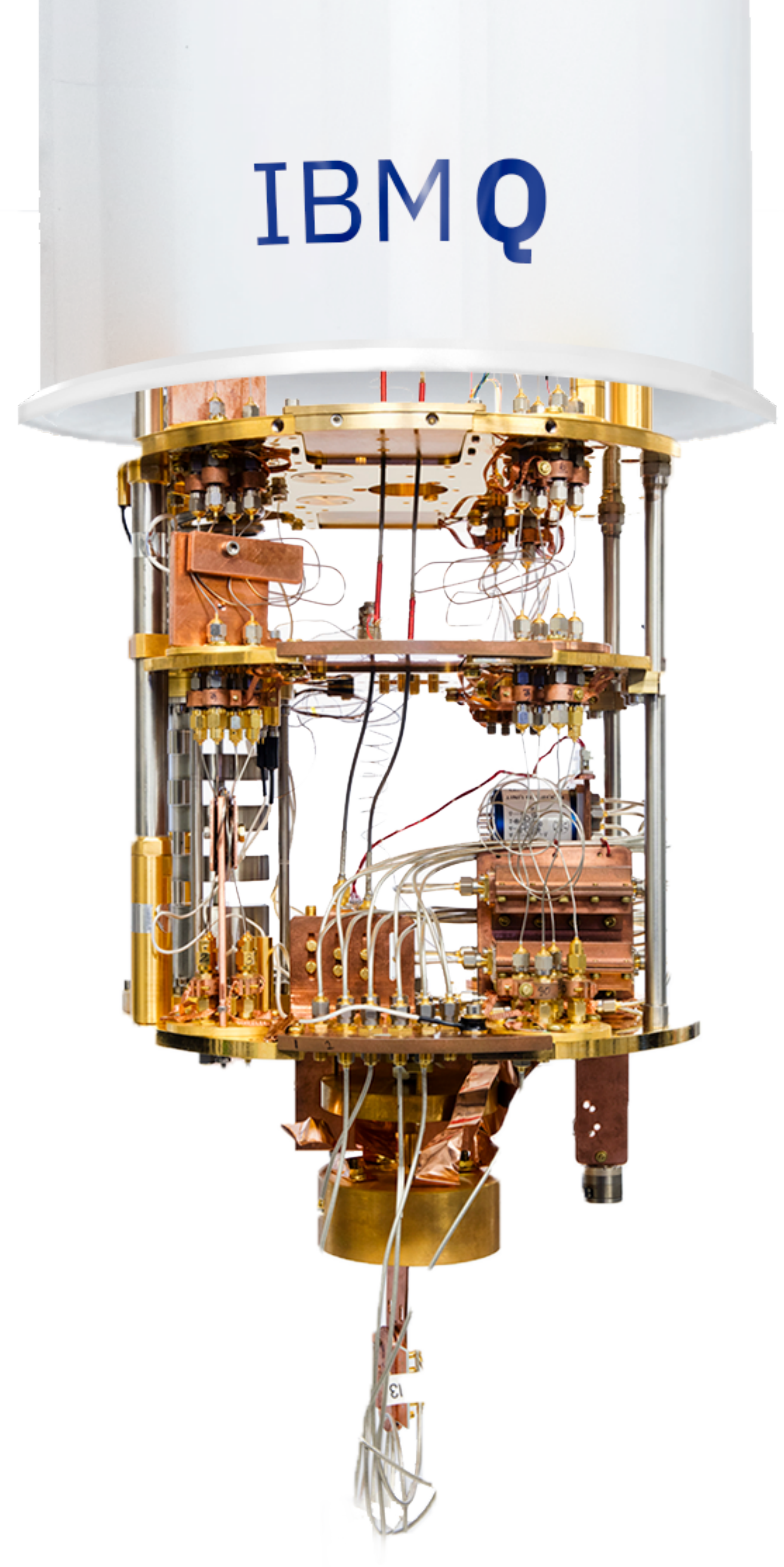


$$U_3|0\rangle \rightarrow \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Multi-qubit gates:



$$\text{CNOT}|00\rangle \rightarrow |00\rangle, \text{CNOT}|10\rangle \rightarrow |11\rangle,$$
$$\text{CNOT}|01\rangle \rightarrow |01\rangle, \text{CNOT}|11\rangle \rightarrow |10\rangle$$



Event Generation - What’s the problem?

Event Generation - What's the problem?

Typical hadron-hadron collisions are highly complex resulting in O(1000) particles

The theoretical description of collision events is **highly complex**

Monte Carlo Event Generators have been the most successful approach to simulating particle collisions

MC Event Generators exploit **factorisation theorems** in QCD -

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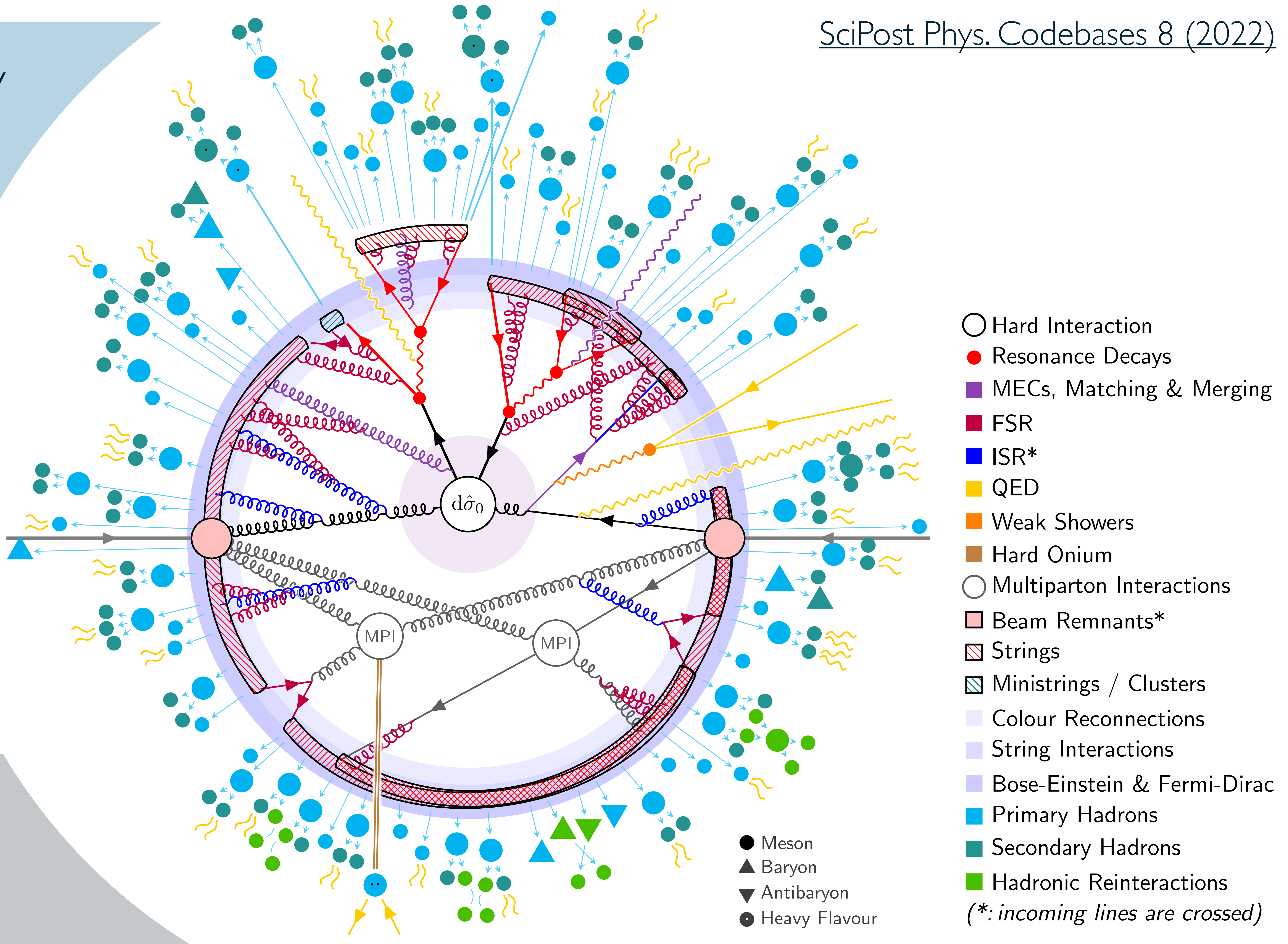
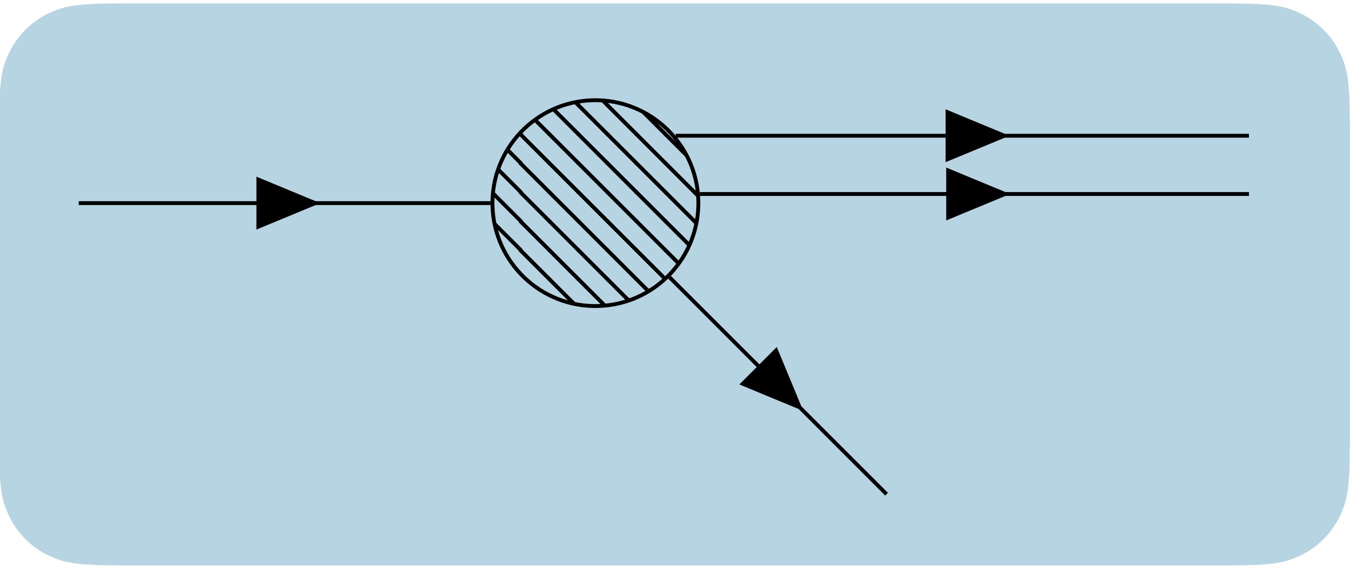
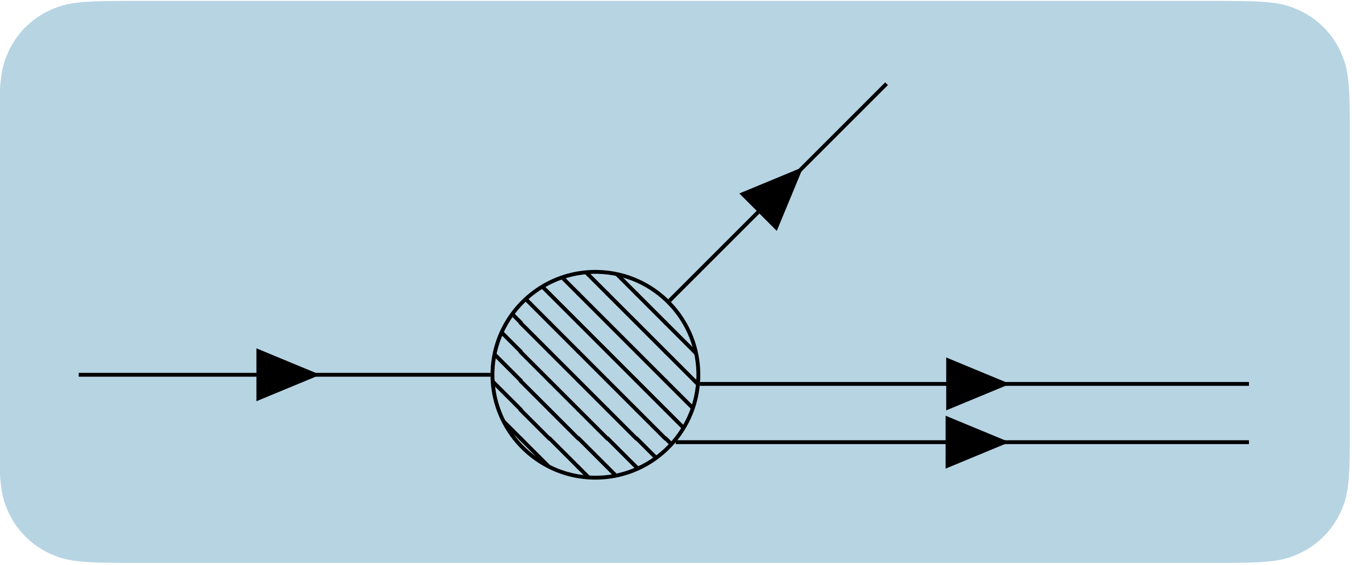


Event Generation - What's the problem?

Event Generation - What's the problem?

Parton Density Functions

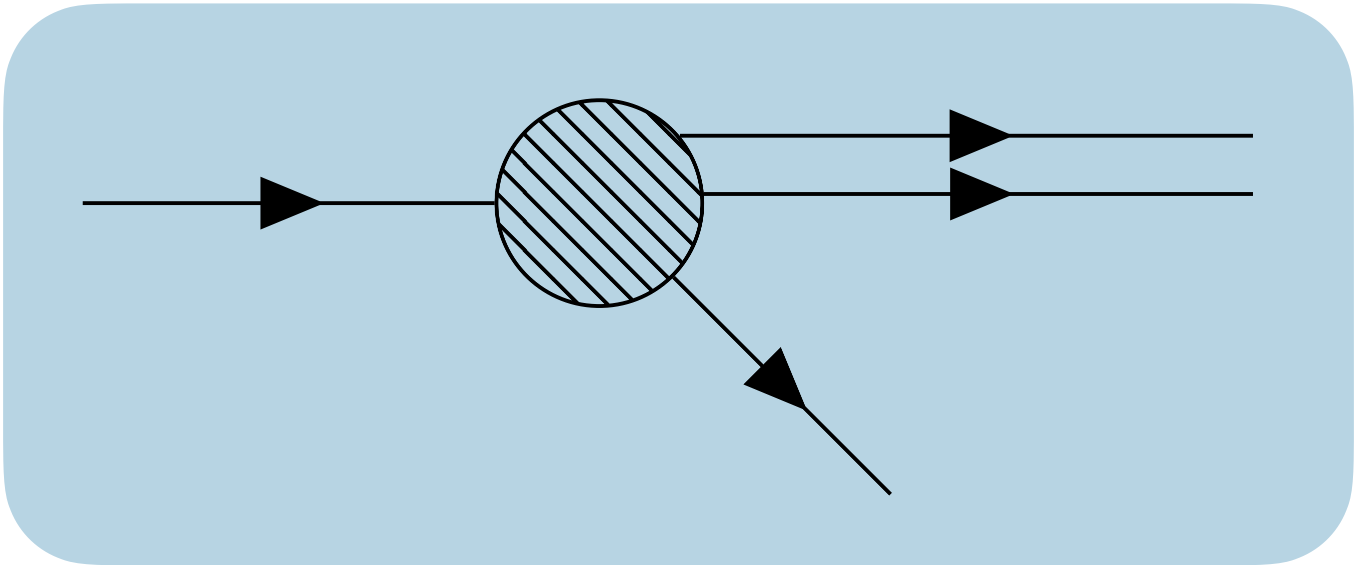




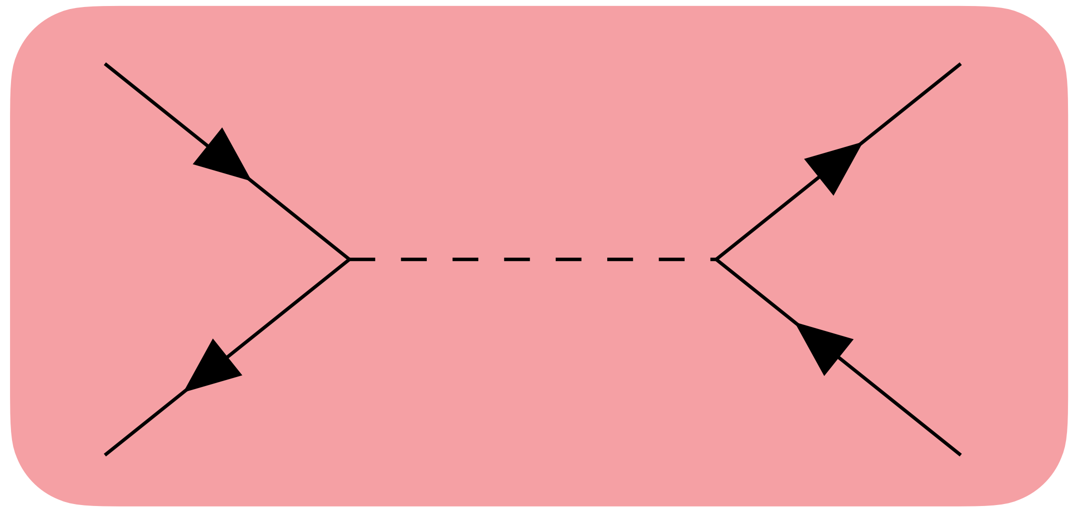
Phys. Rev. D 103, 034027

Event Generation - What's the problem?

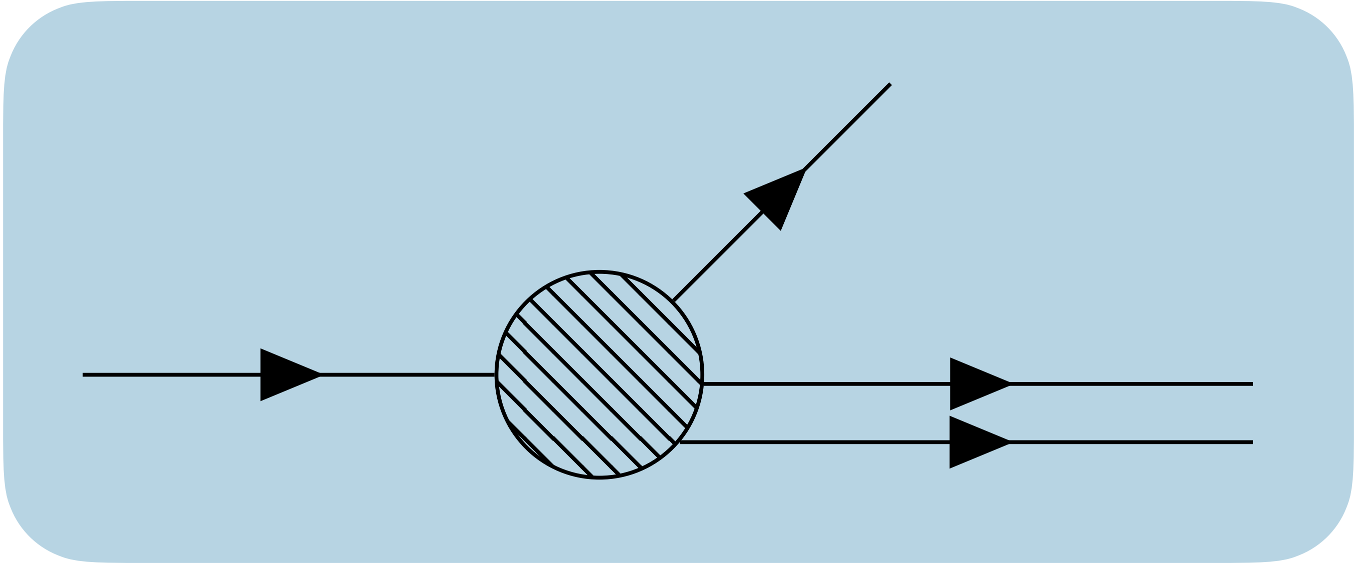
Parton Density Functions



Hard Process



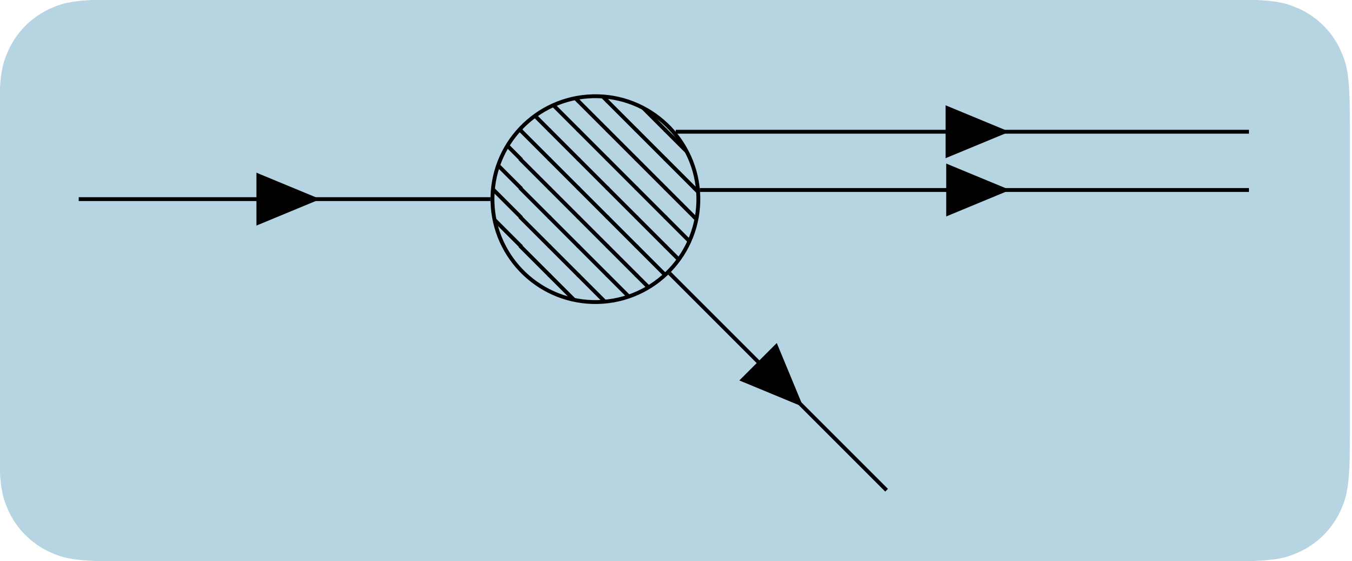
Phys. Rev. D 103, 076020



Phys. Rev. D 103, 034027

Event Generation - What's the problem?

Parton Density Functions



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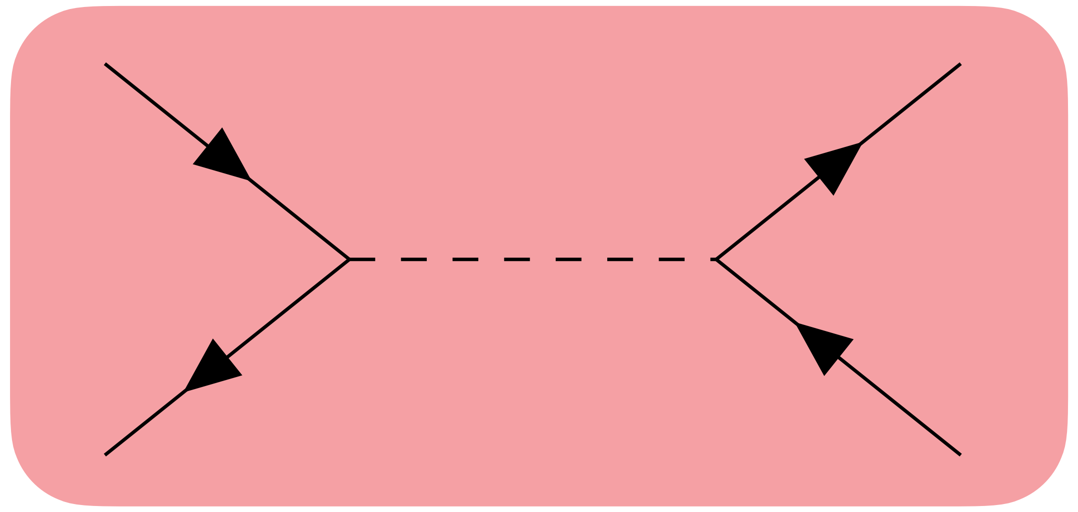

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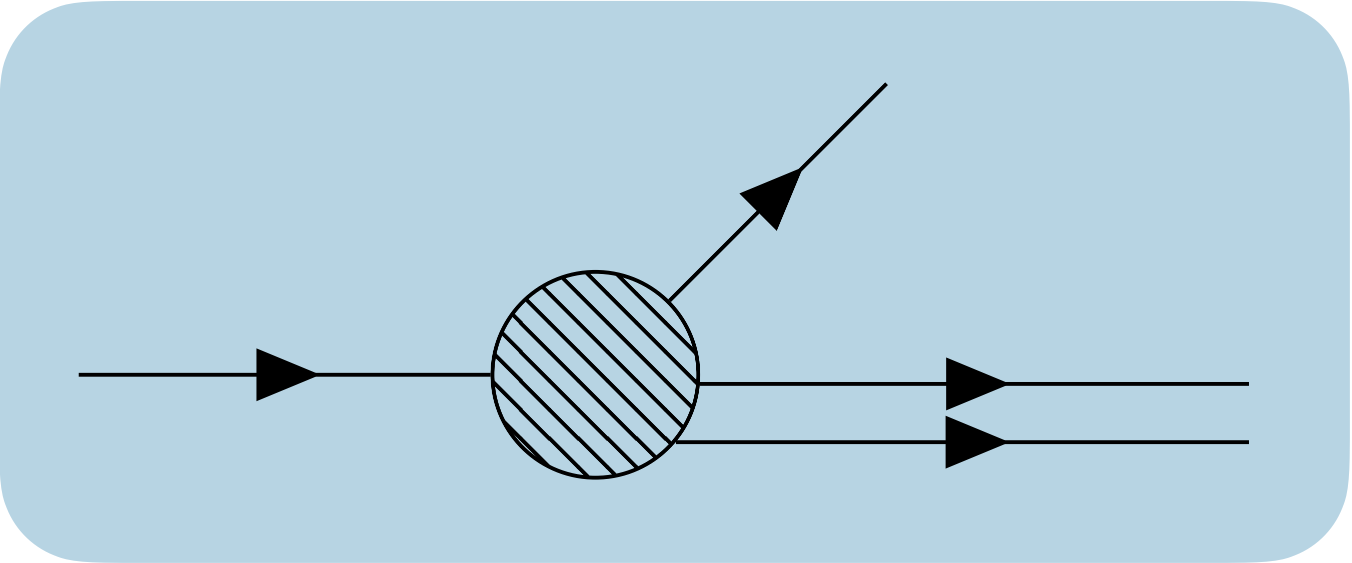

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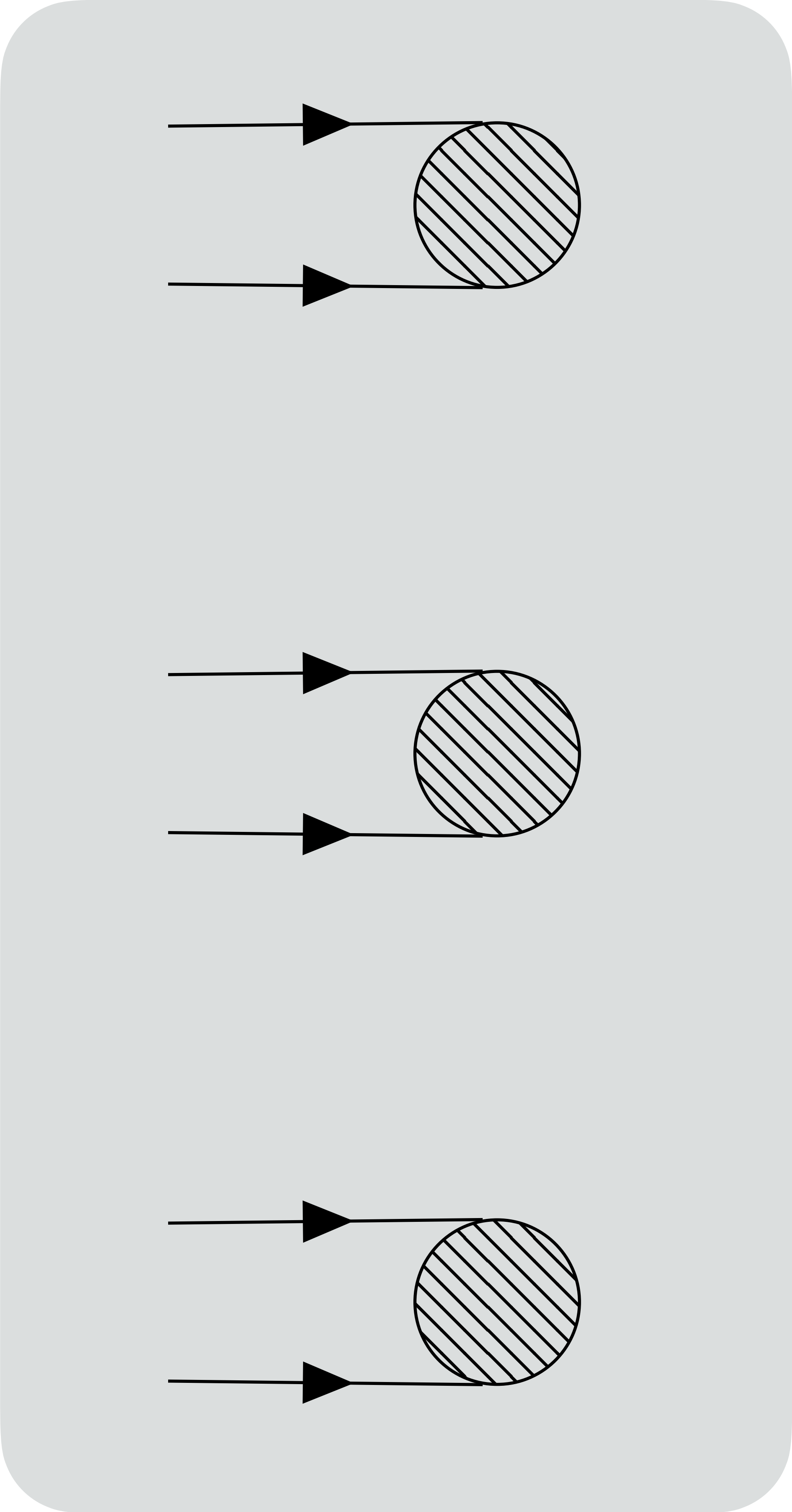
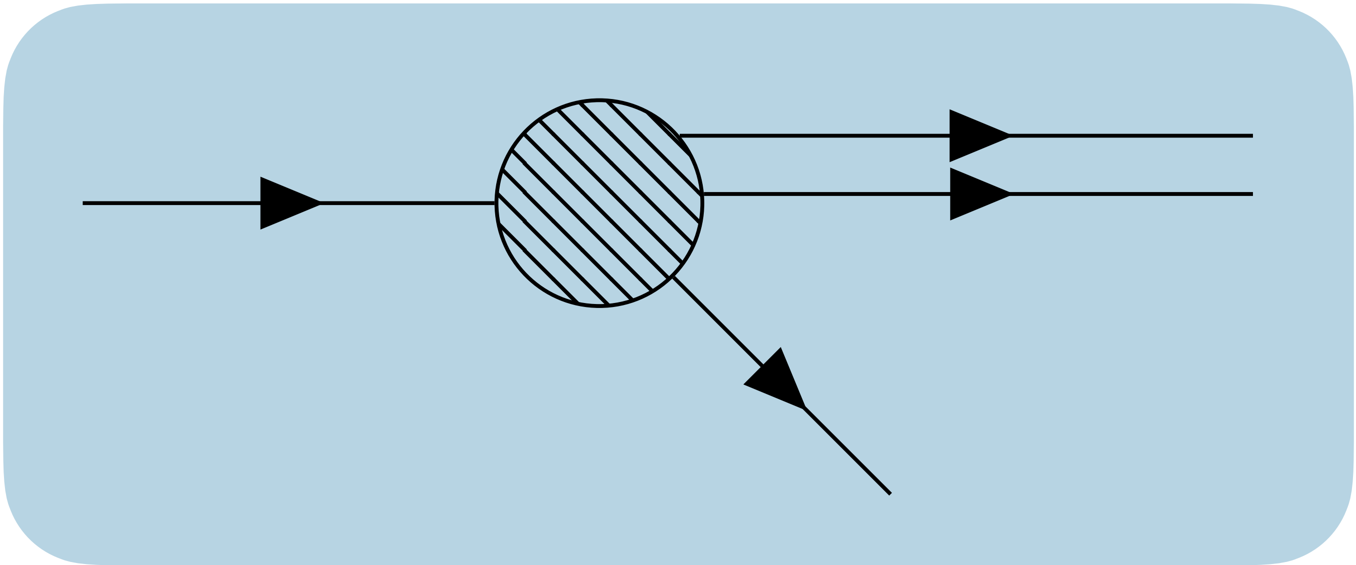
Hadronisation



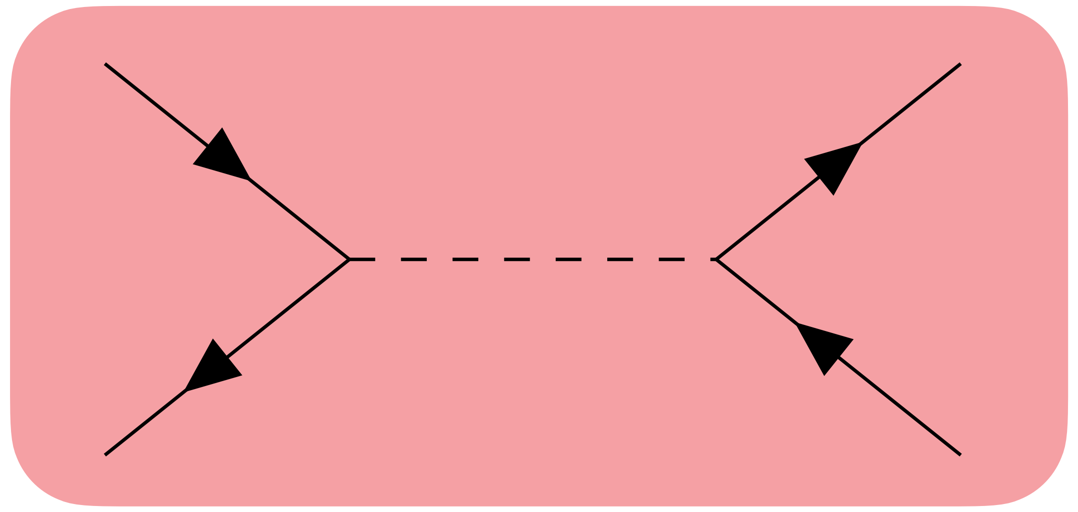
JHEP 11 (2022) 035

Event Generation - What's the problem?

Parton Density Functions



Hard Process



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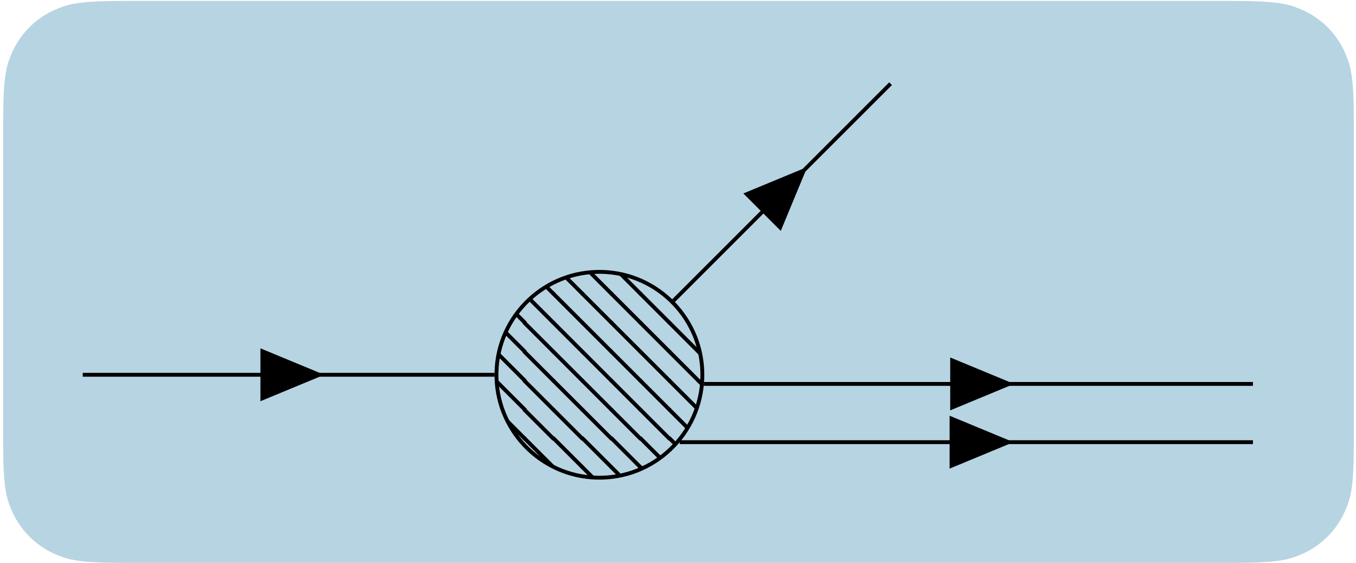
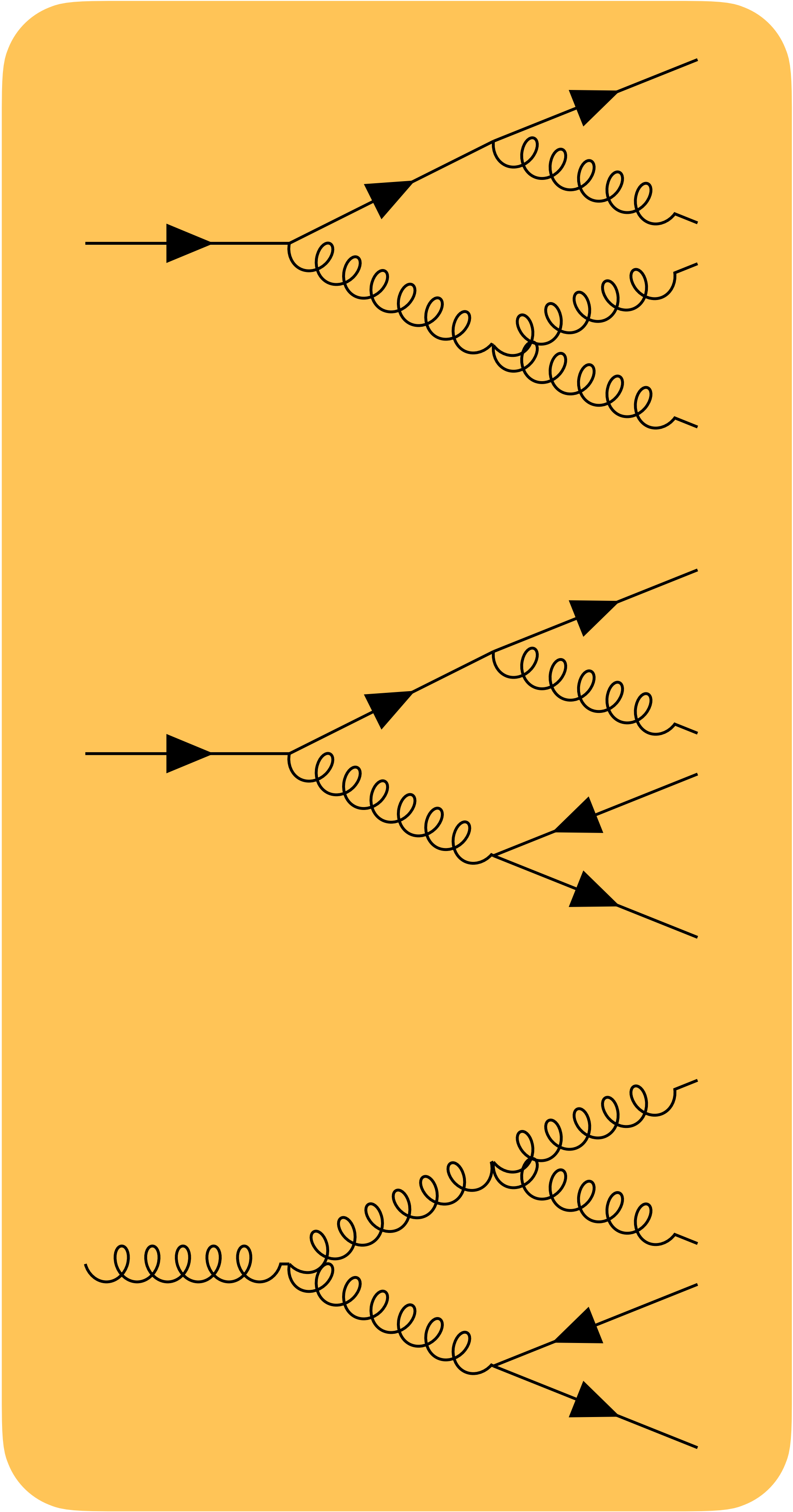


Phys. Rev. D 106, 056002

Phys. Rev. D 103, 034027

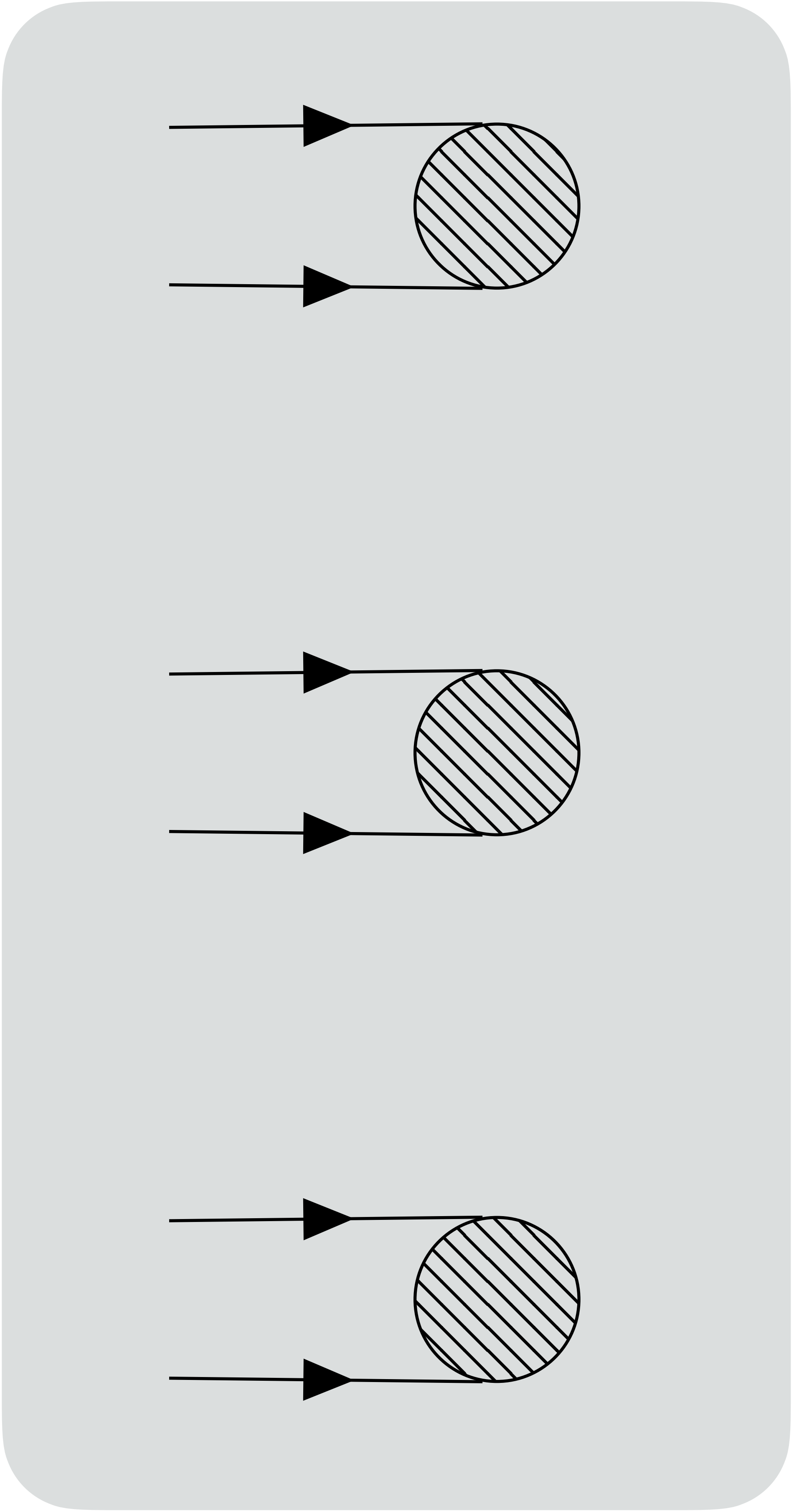
Phys. Rev. Lett. 126, 062001

Parton Shower



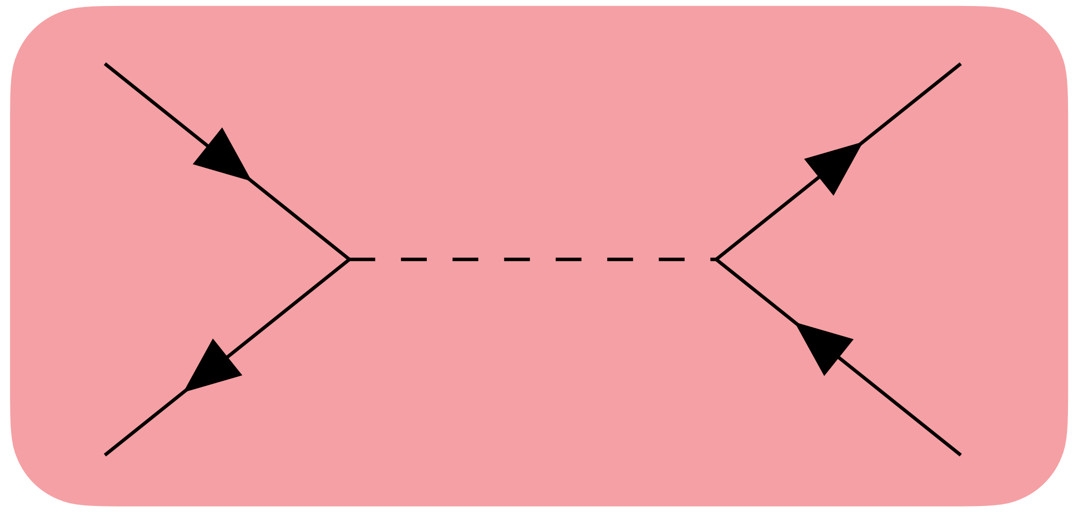
JHEP 11 (2022) 035

Hadronisation

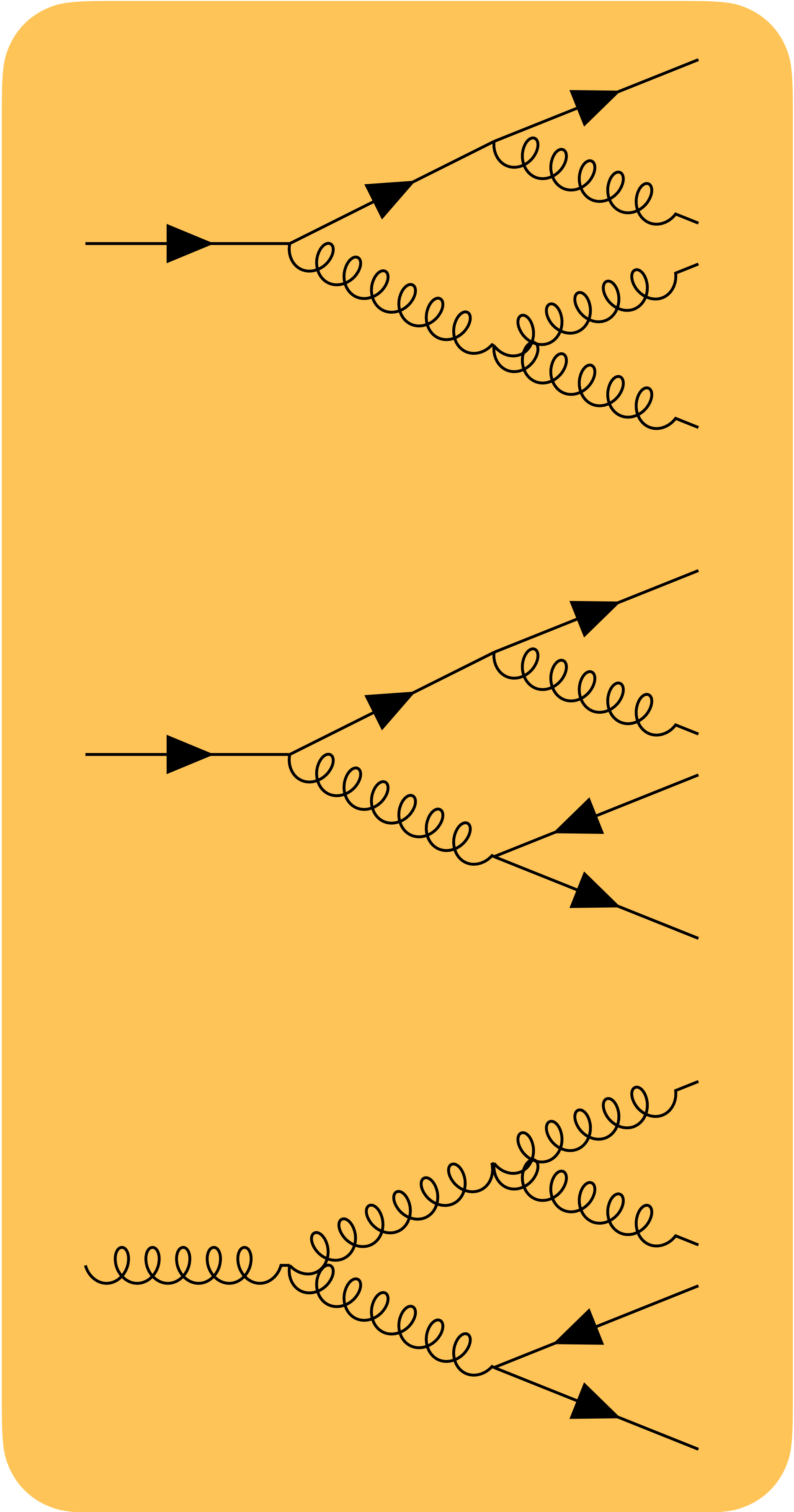


Event Generation - What's the problem?

Hard Process



Parton Shower



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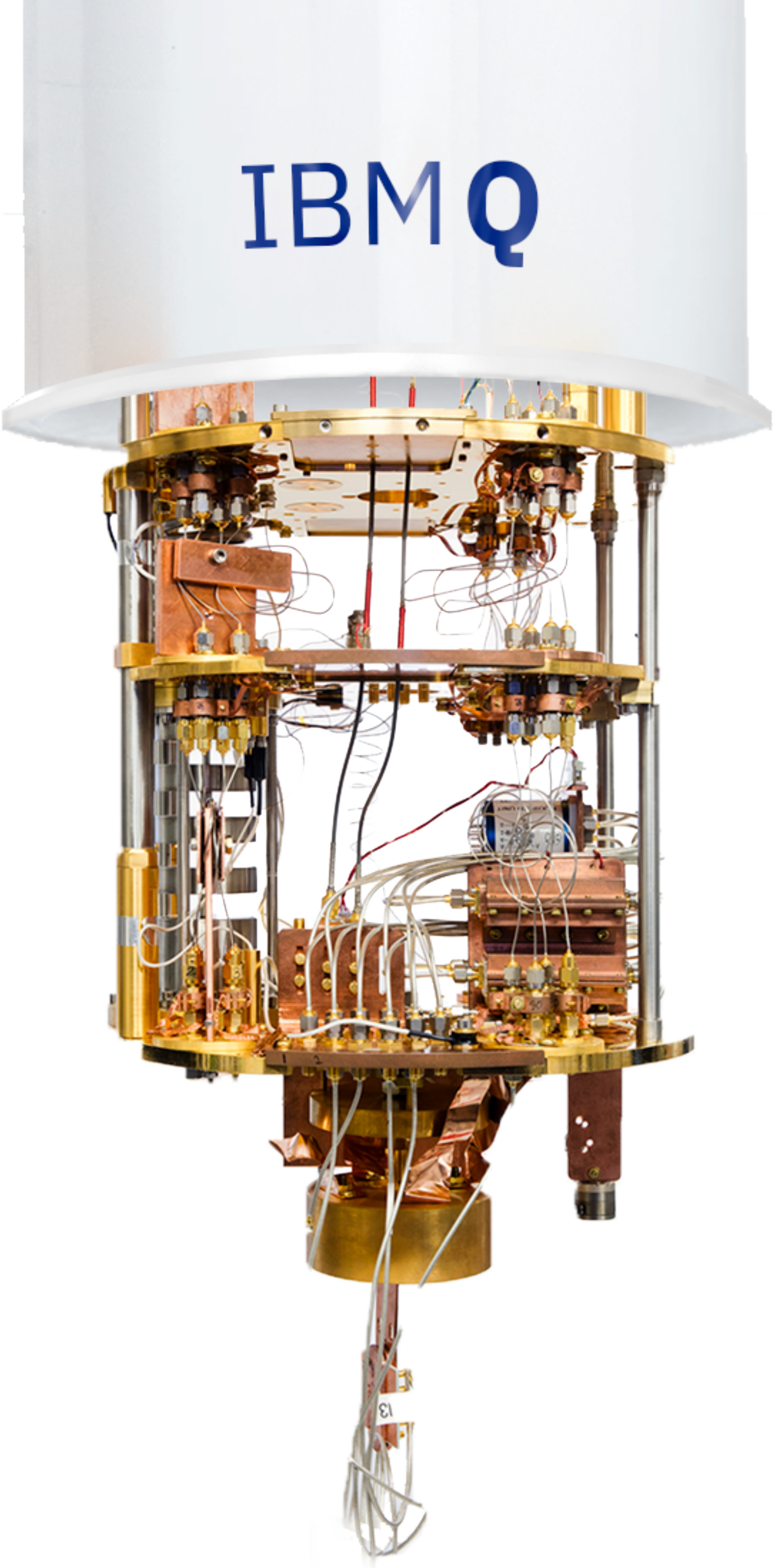
JHEP 11 (2022) 035



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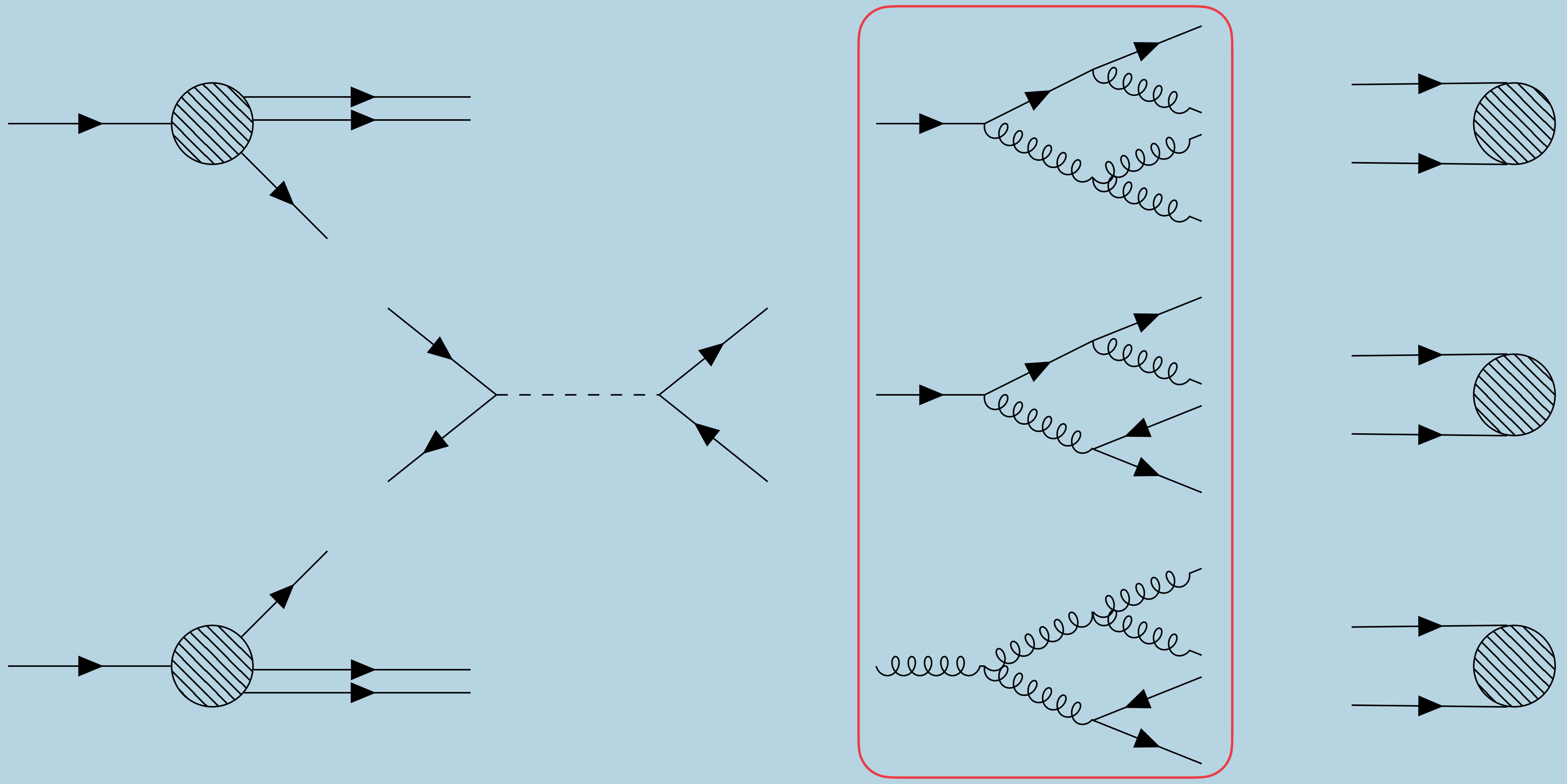
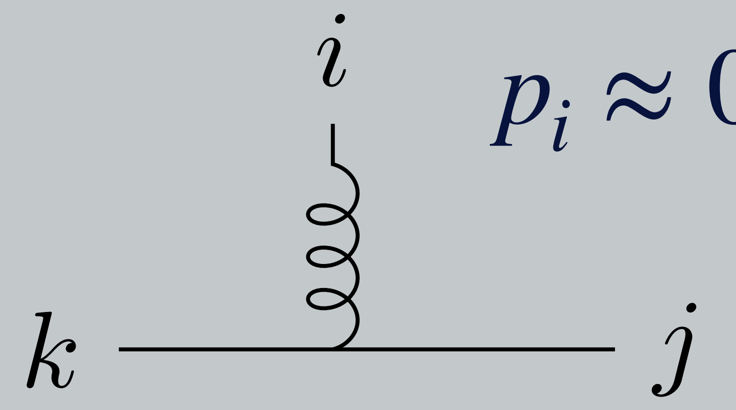
Event Generation - What's the problem?

- The Parton Shower

The Parton Shower



Soft mode:

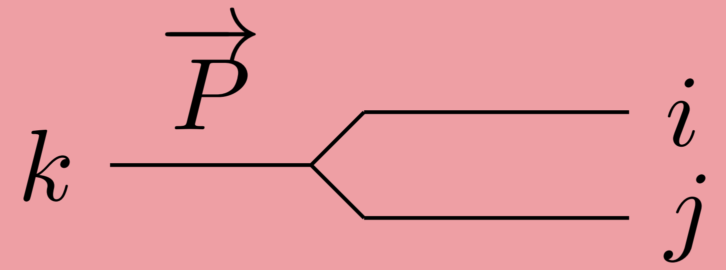


$p_i \approx 0$

Interference effects only allow for partial factorisation

Leading contributions to the decay rate in the collinear limit are included in the soft limit

Collinear mode:



$p_i = zP, \quad p_j = (1-z)P$

Successive decay steps factorise into independent quasi-classical steps

In this limit, the decay from high energy to low energy proceeds as a **colour-dipole cascade**.

This interpretation allows for straightforward interference patterns and momentum conservation

The Parton Shower - The Veto Algorithm

The choice of the variables ξ and t is known as the **phase space parameterisation**

Non-Emission Probability

$$\Delta(t_n, t) = \exp\left(-\int_t^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2s_{ik}(t,\xi)}{s_{ij}(t,\xi)s_{jk}(t,\xi)}\right)$$

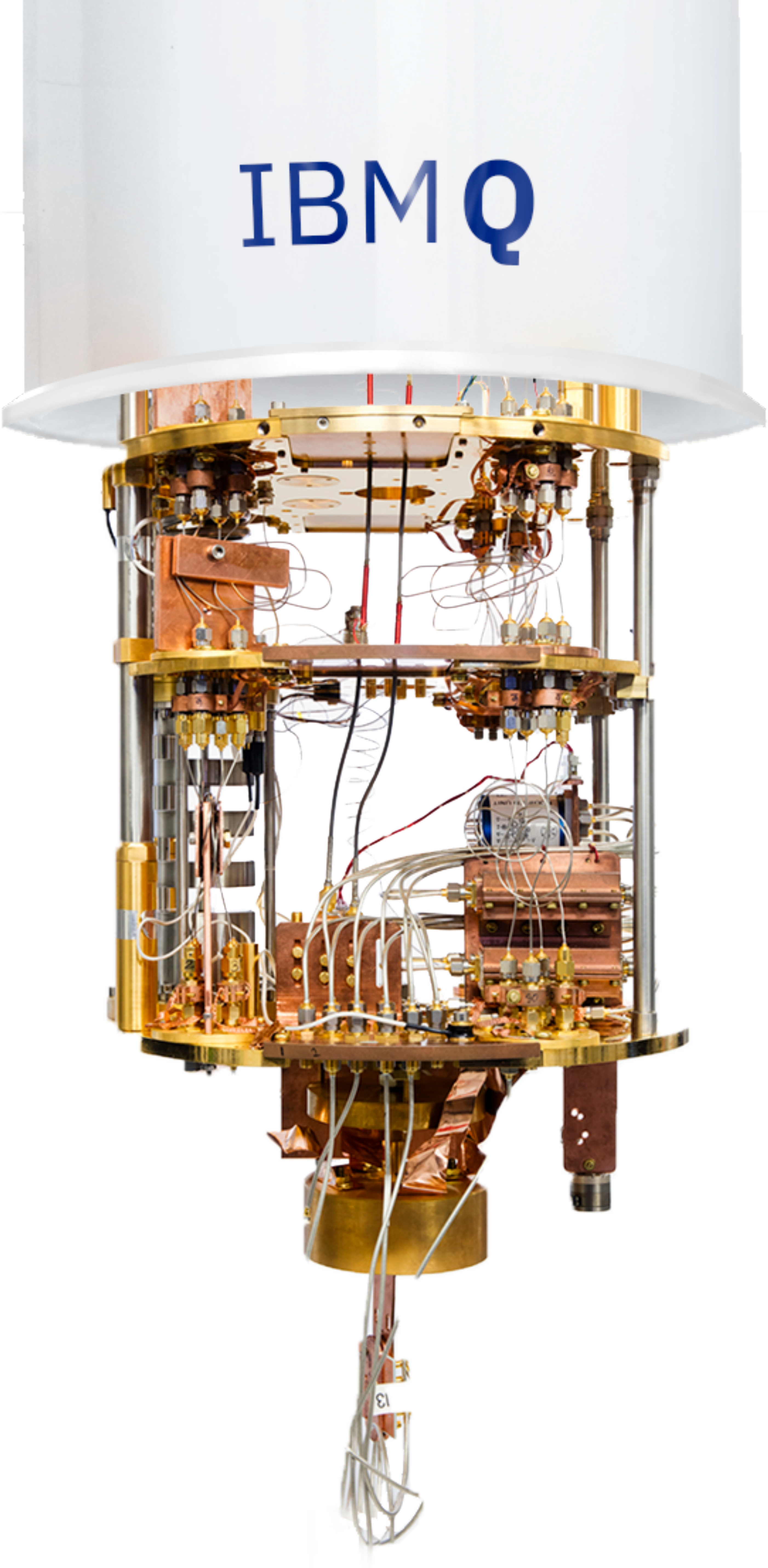
Master Equation

$$\mathcal{F}_n(\Phi_n, t_n, t_c; O) = \Delta(t_n, t_c) O(\Phi_n) + \int_{t_c}^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2s_{ik}(t,\xi)}{s_{ij}(t,\xi)s_{jk}(t,\xi)} \Delta(t_n, t) \mathcal{F}_n(\Phi_{n+1}, t, t_c; O)$$

Inclusive Decay Probability

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_i)g(p_j)\bar{q}(p_k)\right) \simeq \frac{ds_{ij}}{s_{\mathrm{IK}}} \frac{ds_{jk}}{s_{\mathrm{IK}}} C \frac{\alpha_s}{2\pi} \frac{2s_{\mathrm{IK}}}{s_{ij}s_{jk}}$$

Current interpretations of the veto algorithm treat the phase space variables ξ and t as **continuous**



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Quantum Parton Shower

G. Gustafson, S. Prestel, M. Spannowsky and S. Williams, Collider Events on a Quantum Computer, *JHEP* 11 (2022) 035, arXiv:2207.10694

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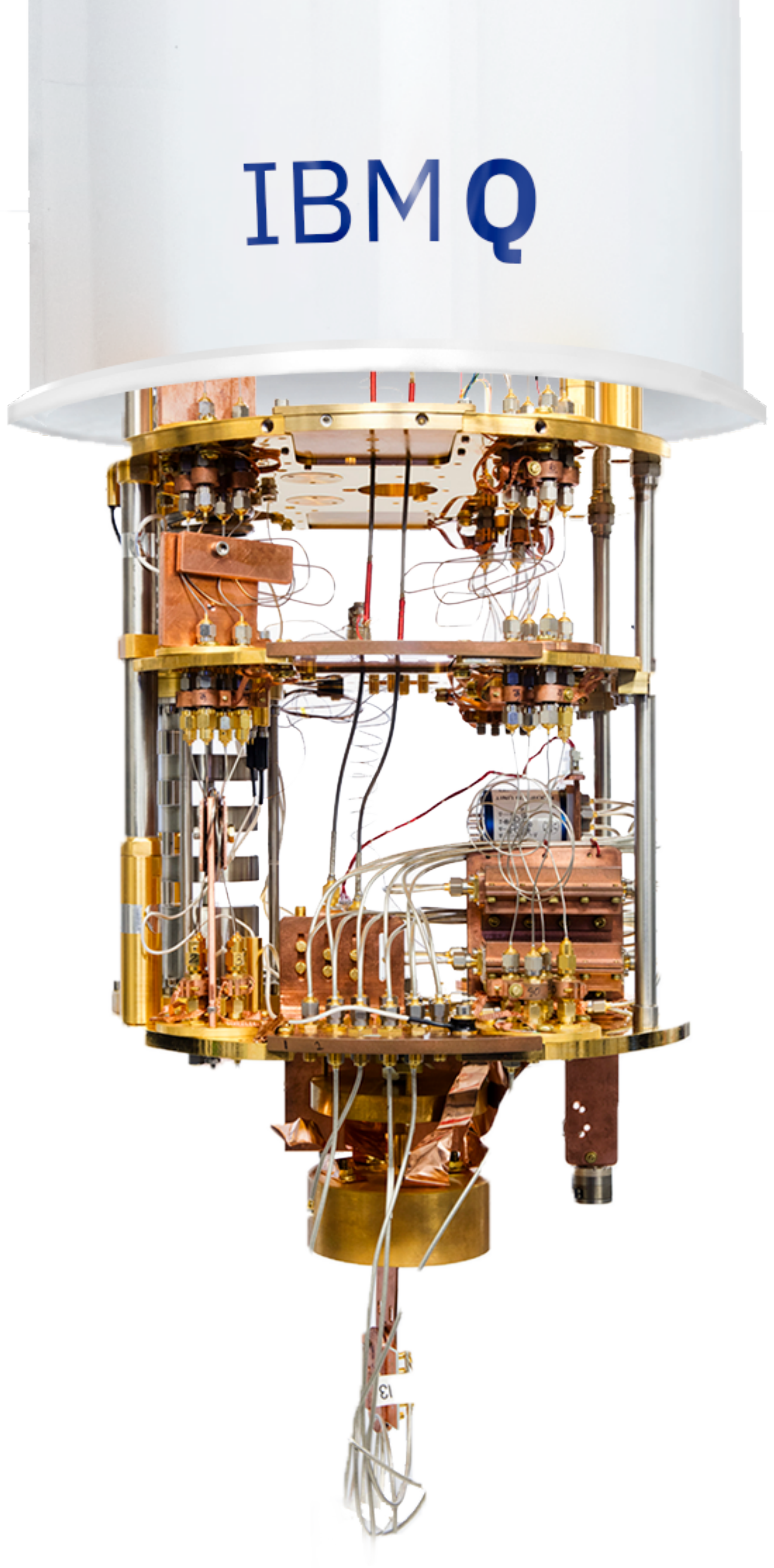

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Quantum Parton Shower

- Discretising QCD

G. Gustafson, S. Prestel, M. Spannowsky and S. Williams, Collider Events on a Quantum Computer, *JHEP* 11 (2022) 035, arXiv:2207.10694

Lund University

Imperial College London

Discrete QCD - Abstracting the Parton Shower Method

1. Parameterise phase space in terms of gluon transverse momentum and rapidity:

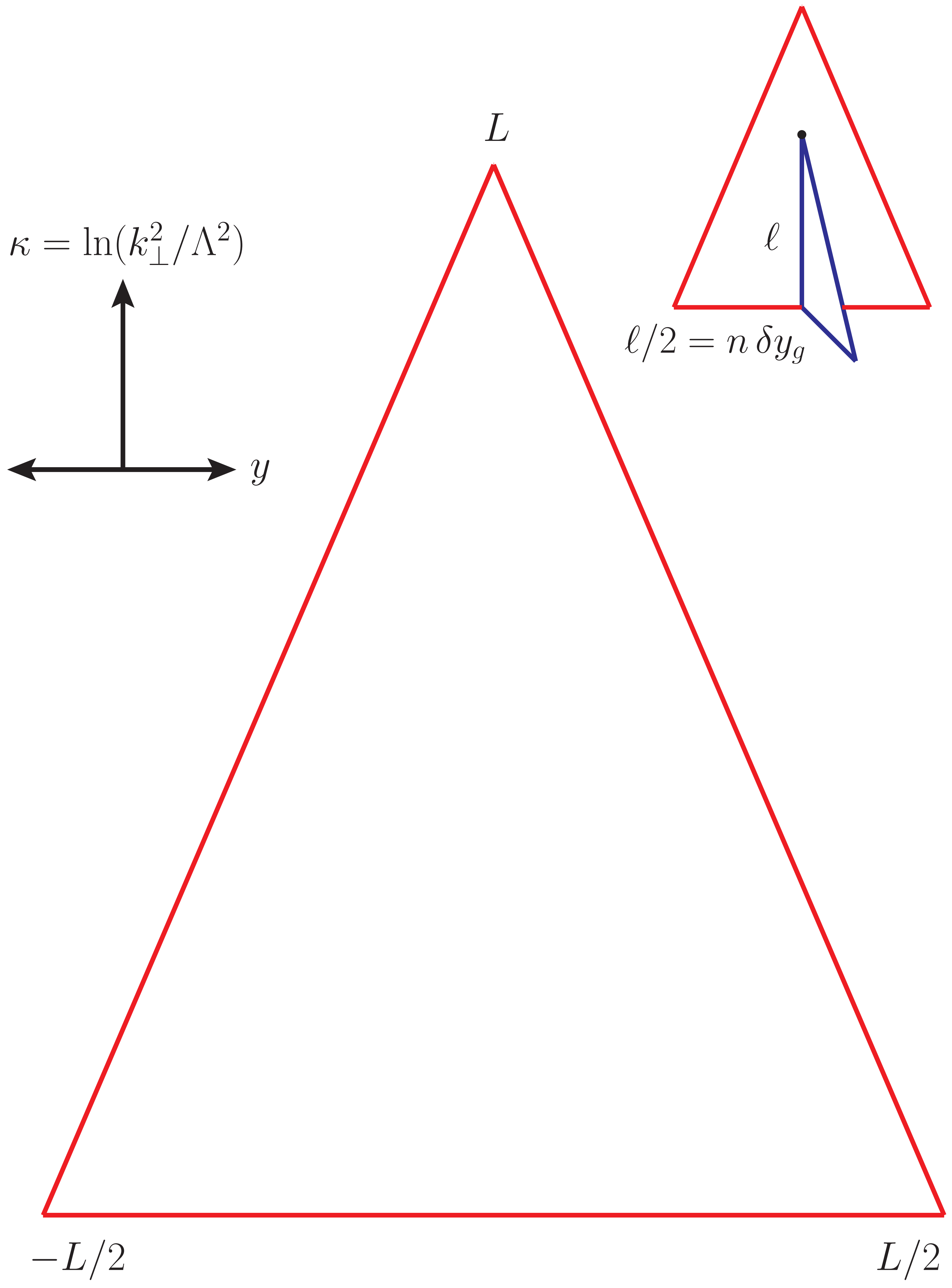
$$k_\perp^2 = \frac{s_{ij}s_{jk}}{s_{\mathrm{IK}}} \qquad \text{and} \qquad y = \frac{1}{2}\ln\left(\frac{s_{ij}}{s_{jk}}\right)$$

which leads to the inclusive probability:

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_i)g(p_j)\bar{q}(p_k)\right) \simeq= \frac{C\alpha_s}{\pi}d\kappa dy$$

where $\kappa = \ln\left(\frac{k_\perp^2}{\Lambda^2}\right)$ and $\boldsymbol{\Lambda}$ is an arbitrary mass scale

Due to the colour charge of emitted gluons, the rapidity span for subsequent dipole decays is increased. This is interpreted as **"folding out"**



Discrete QCD - Abstracting the Parton Shower Method

2. Neglect $g \to q\bar{q}$ splittings and examine transverse-momentum-dependent running coupling

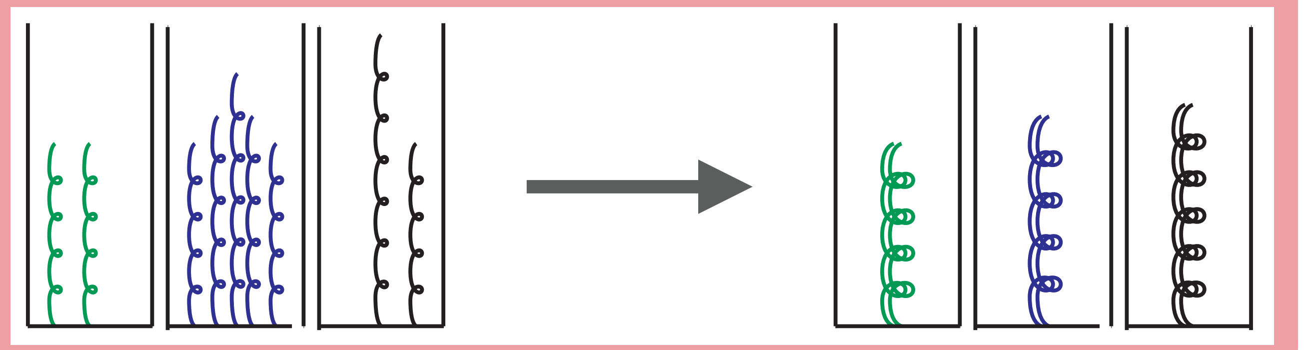
$$\alpha_s(k_\perp^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_\perp^2/\Lambda_{\rm QCD}^2)}$$

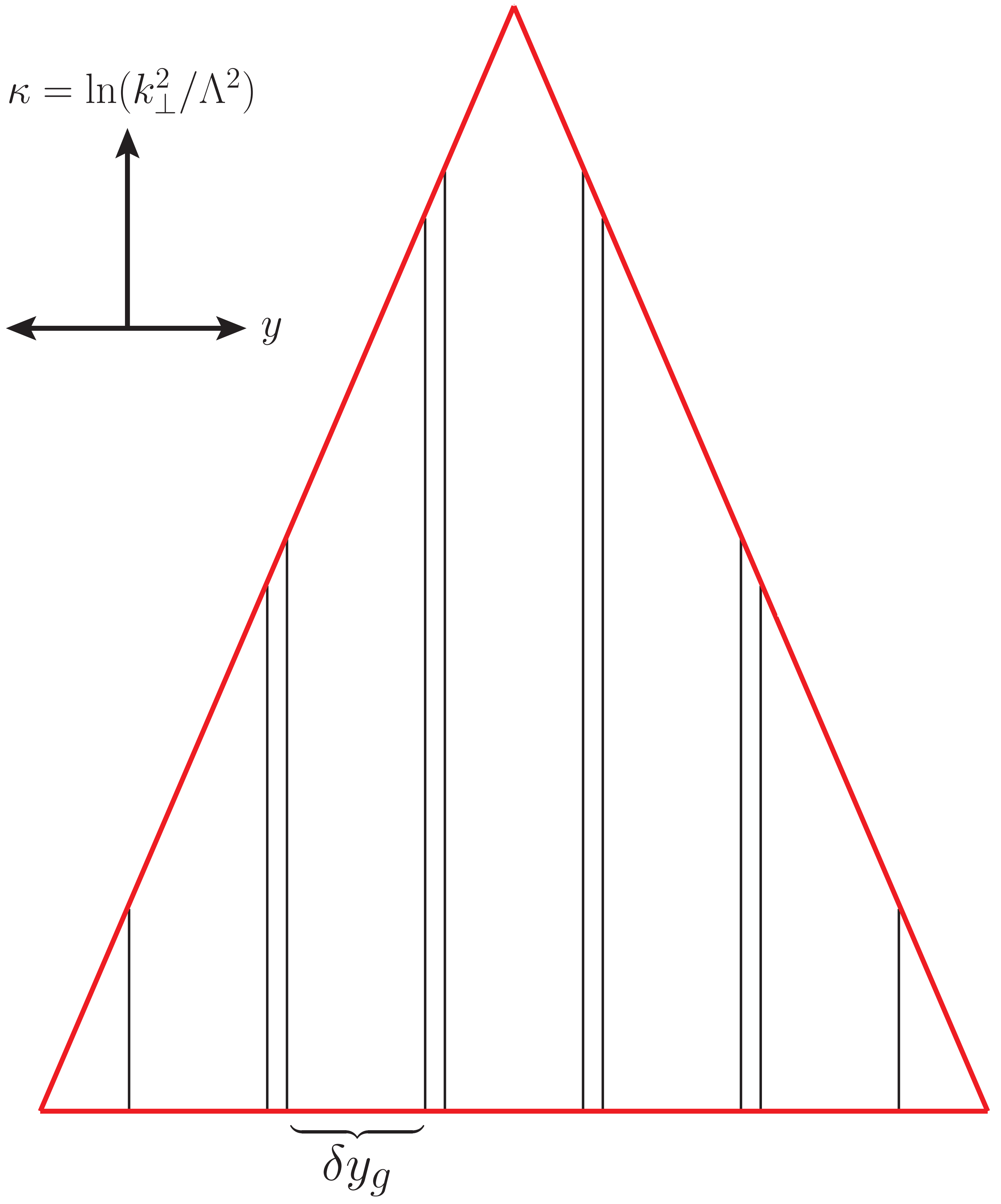
leads to the inclusive probability

$$d\mathcal{P}\left(q(p_{\rm I})\bar{q}(p_{\rm K}) \to q(p_i)g(p_j)\bar{q}(p_k)\right) \simeq= \frac{d\kappa}{\kappa}\frac{dy}{\delta y_g} \quad \text{with} \quad \delta y_g = \frac{11}{6}$$

Interpreting the running coupling renormalisation group as a gain-loss equation:

Gluons within δy_g act coherently as one effective gluon





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$$\alpha_s(k_\perp^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_\perp^2/\Lambda_{\mathrm{QCD}}^2)} = \frac{\mathrm{const.}}{\kappa}$$

leads to the inclusive probability

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_i)g(p_j)\bar{q}(p_k)\right) \simeq = \frac{d\kappa}{\kappa}\frac{dy}{\delta y_g} \quad \text{with} \quad \delta y_g = \frac{11}{6}$$

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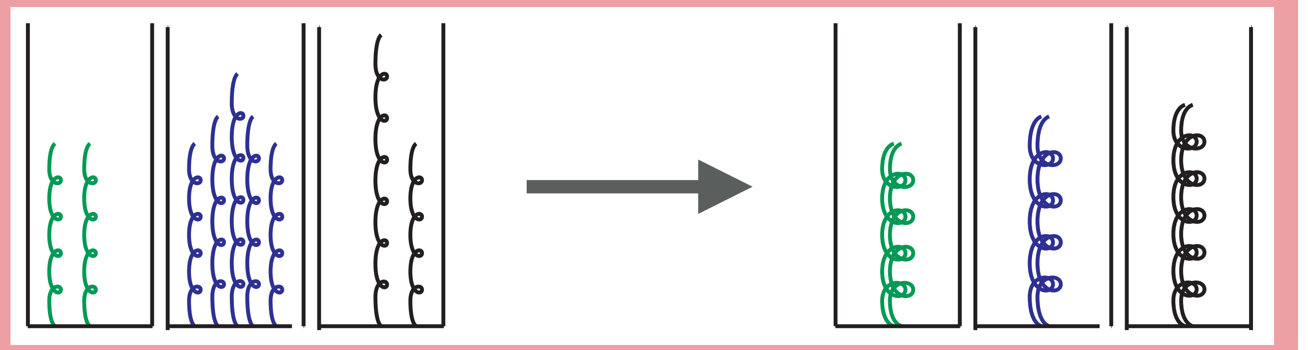
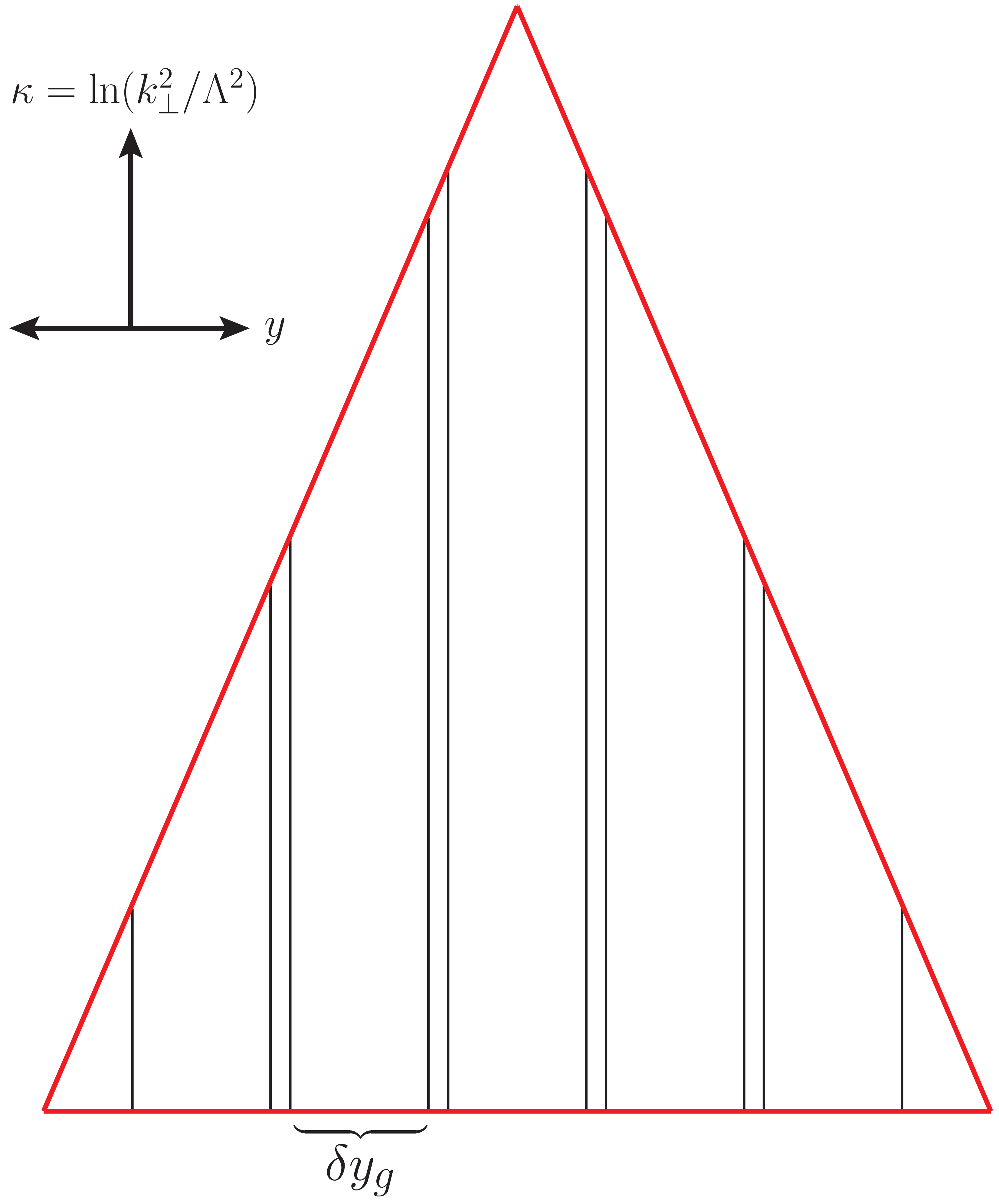
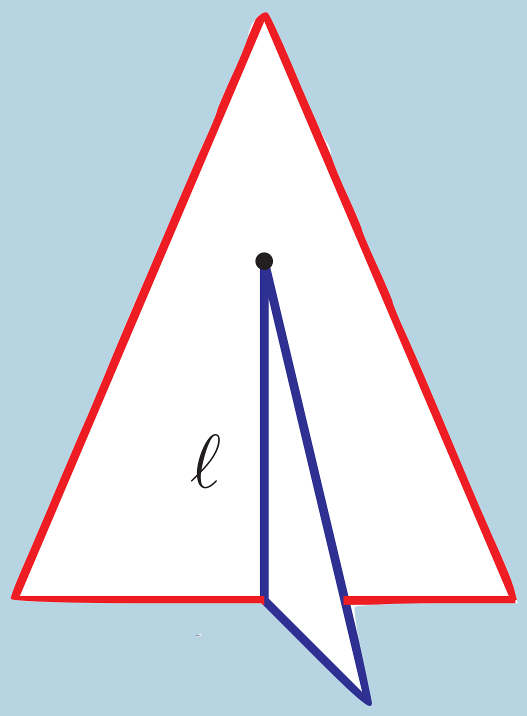




Discrete QCD - Abstracting the Parton Shower Method

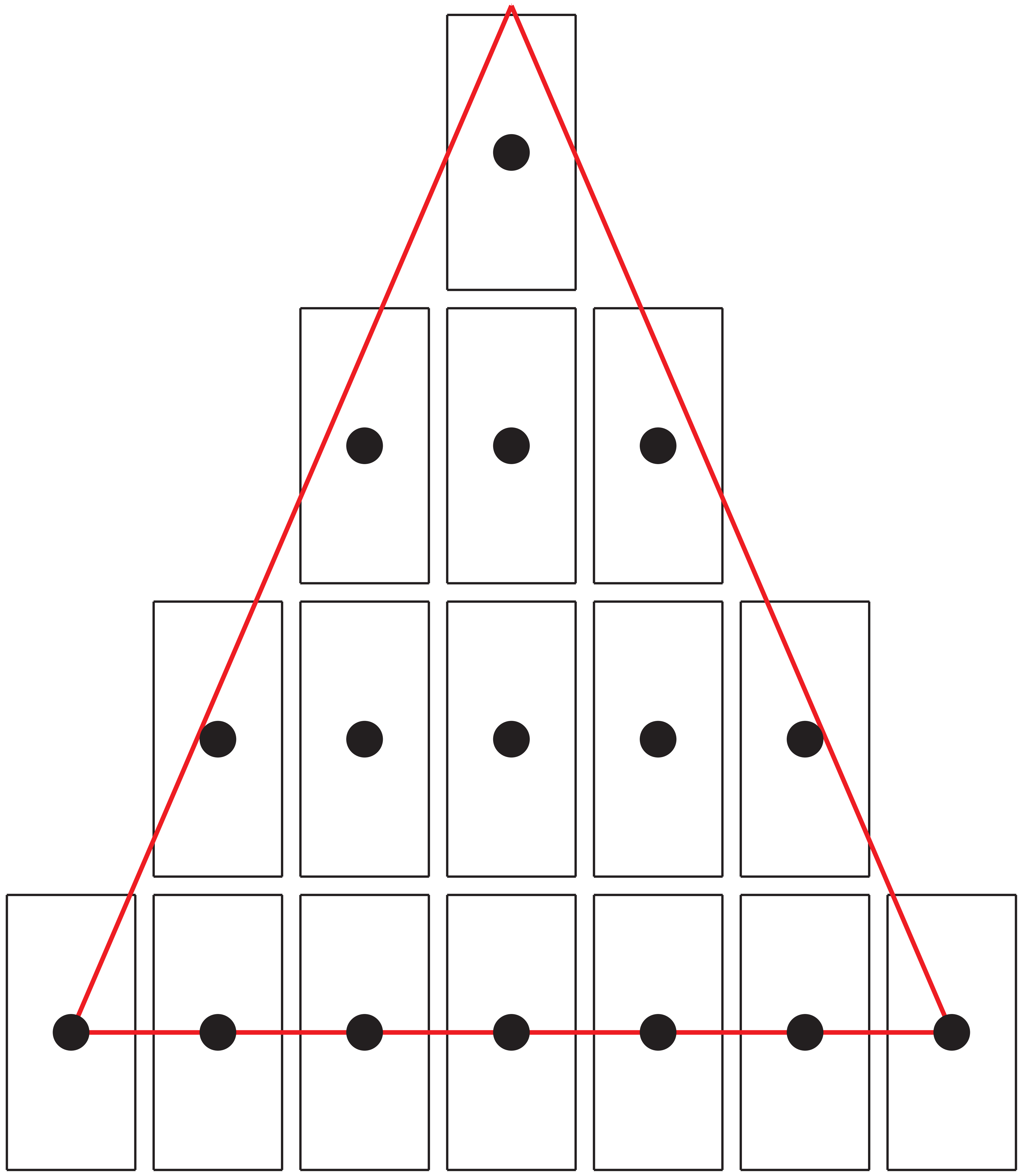
Folding out extends the baseline of the triangle to positive y by $\frac{l}{2}$, where l is the height at which to emit effective gluons



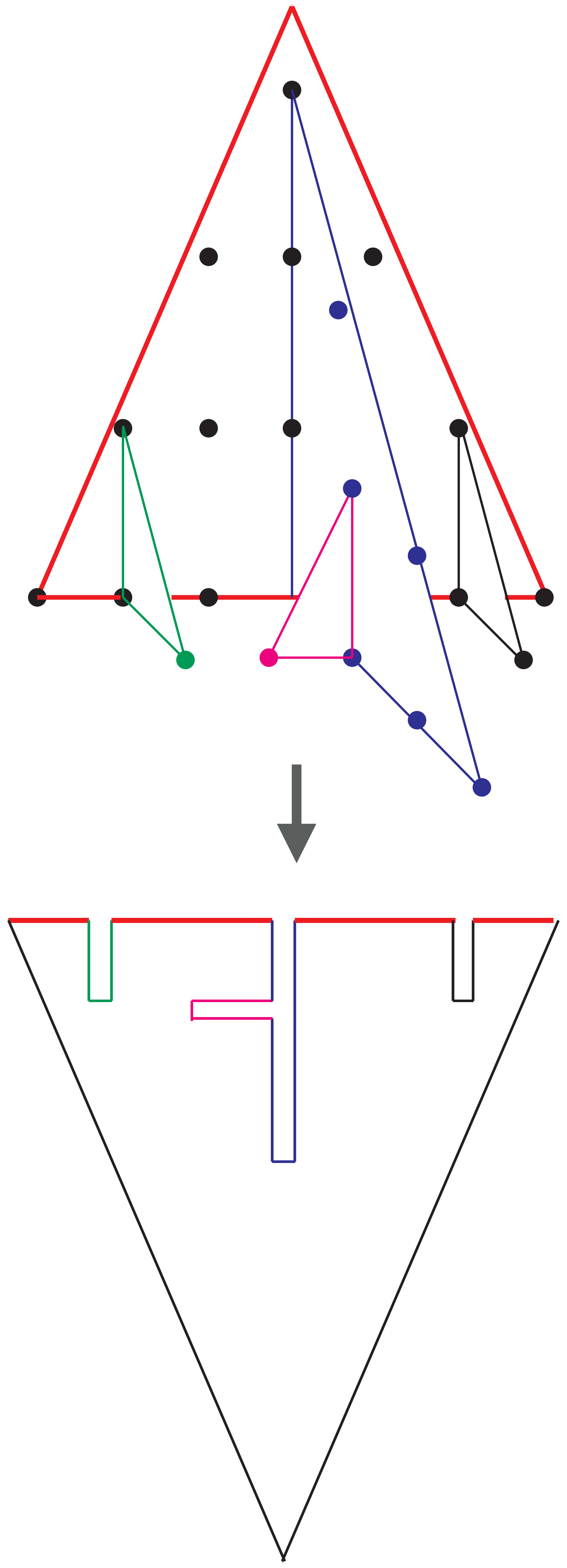
A consequence of folding is that the κ axis is quantised into multiples of $2\delta y_g$

Each rapidity slice can be treated independently of any other slice. The exclusive rate probability takes the simple form:

$$\frac{d\kappa}{\kappa} \exp\left(- \int_{\kappa}^{\kappa_{max}} \frac{d\bar{\kappa}}{\bar{\kappa}} \right) = \frac{d\kappa}{\kappa_{max}}$$



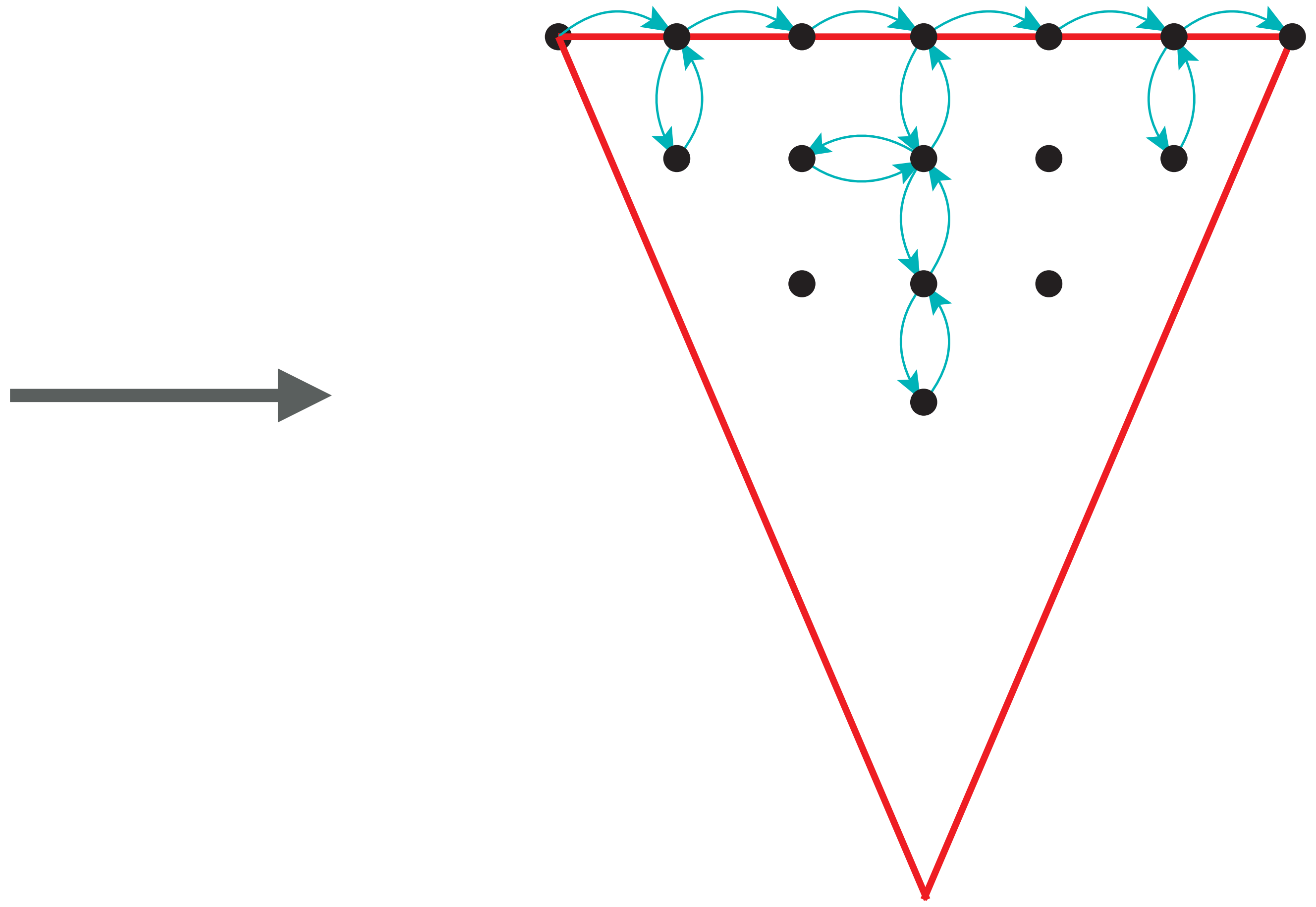
Discrete QCD as a Quantum Walk

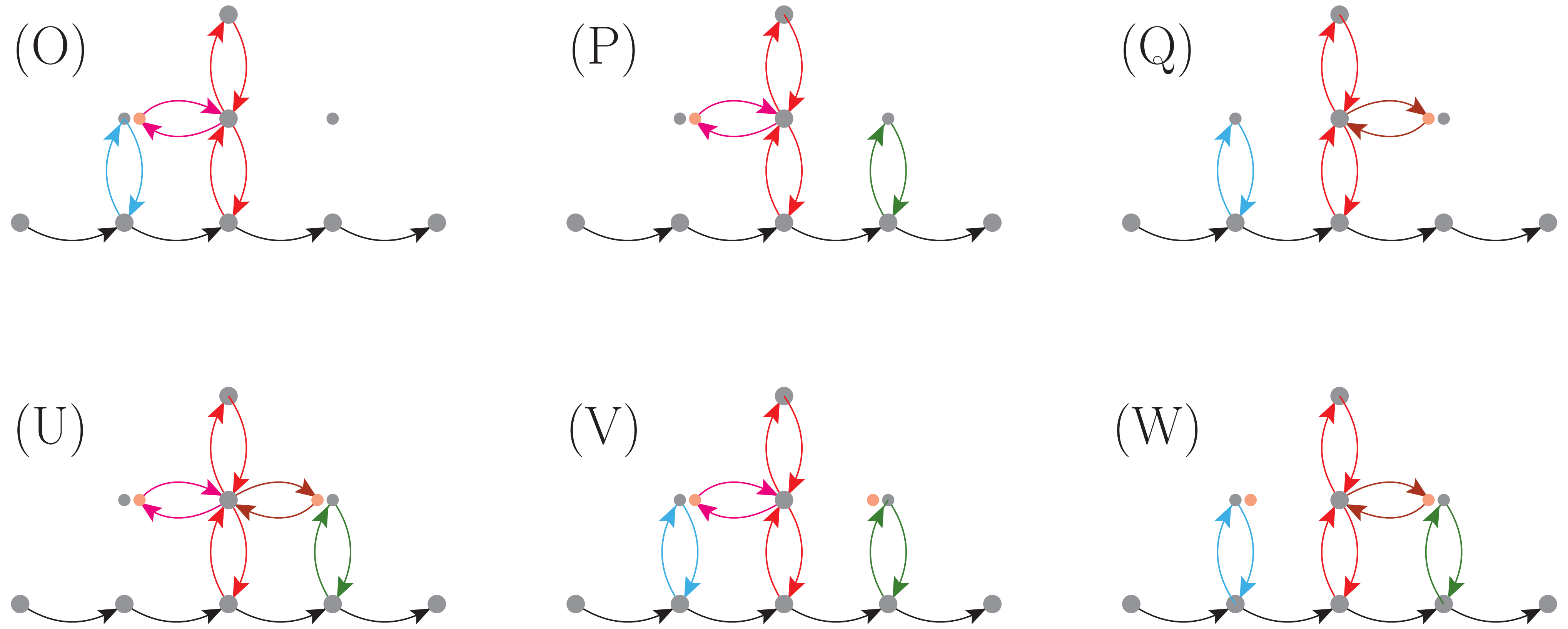


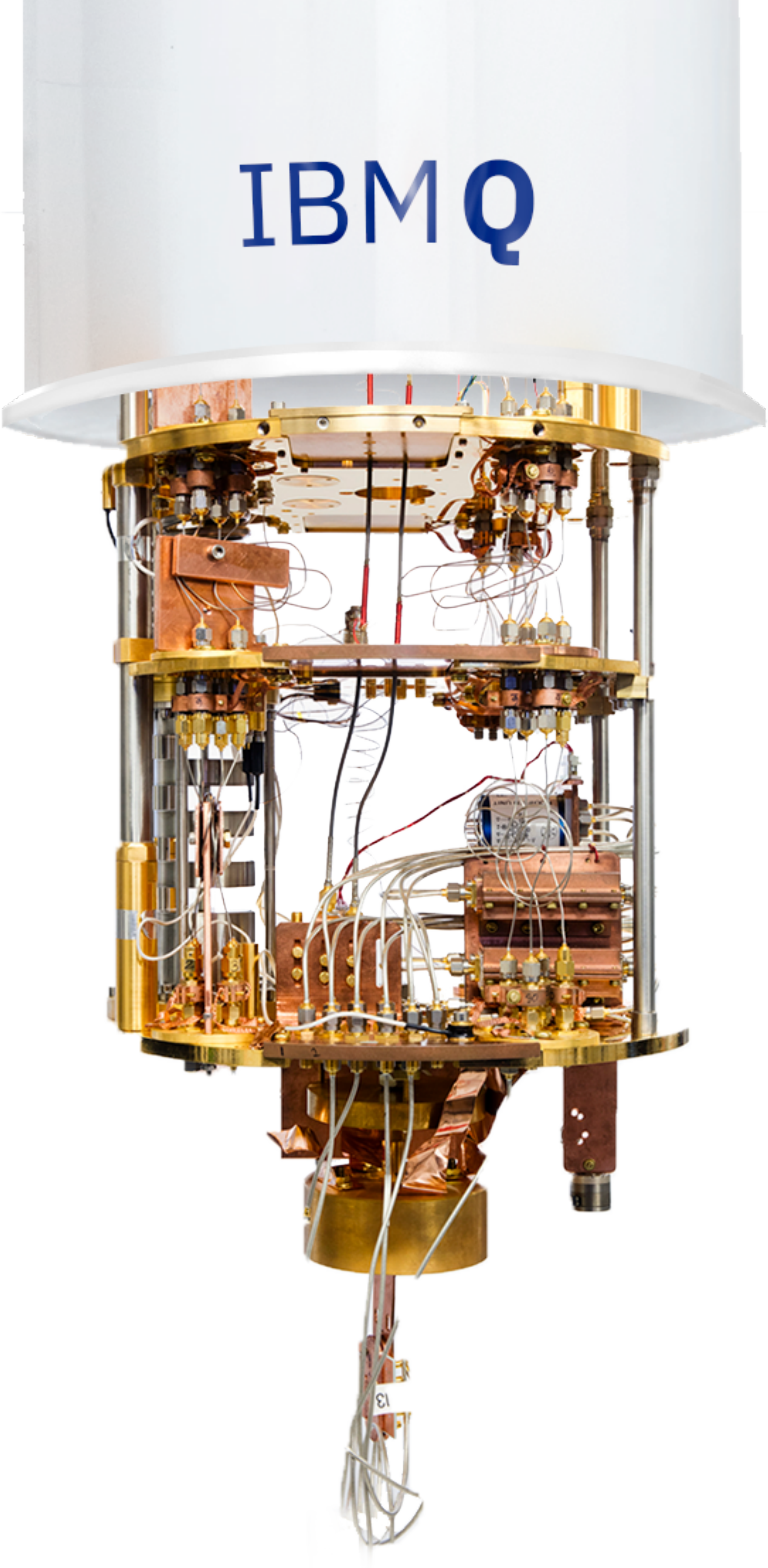
The **baseline** of the grove structure contains all kinematics information

For LEP data there are **24 unique grove structures** for $\Lambda_{QCD} \in [0.1, 1]$ GeV

The groves can be constructed **and enumerated** to achieve an **efficient** sampling algorithm for the dipole shower







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IP3

Quantum Parton Shower

- Discretising QCD

- **Parton Shower as a Quantum Walk**

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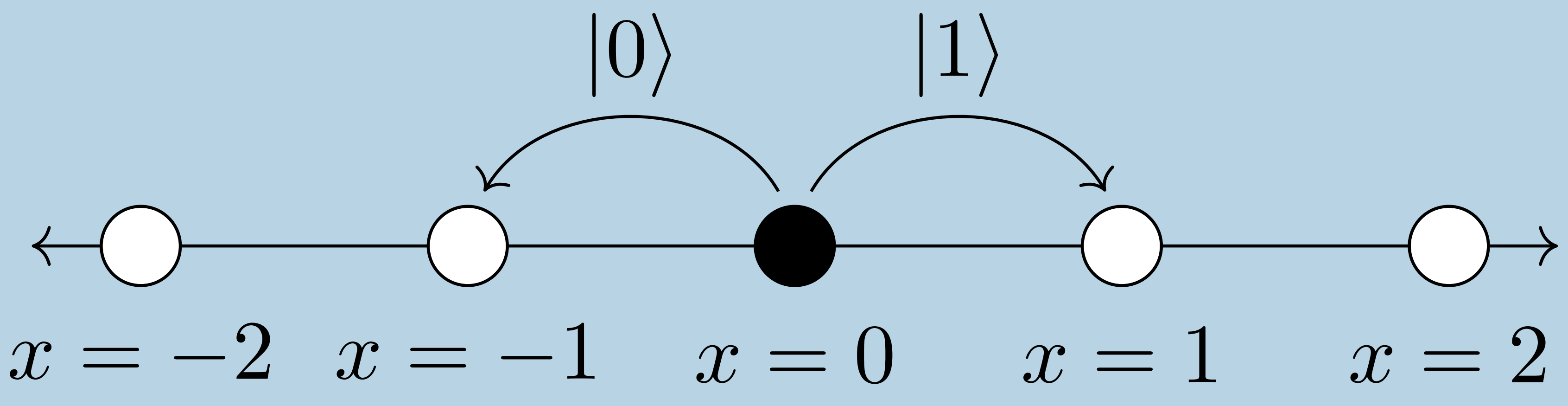
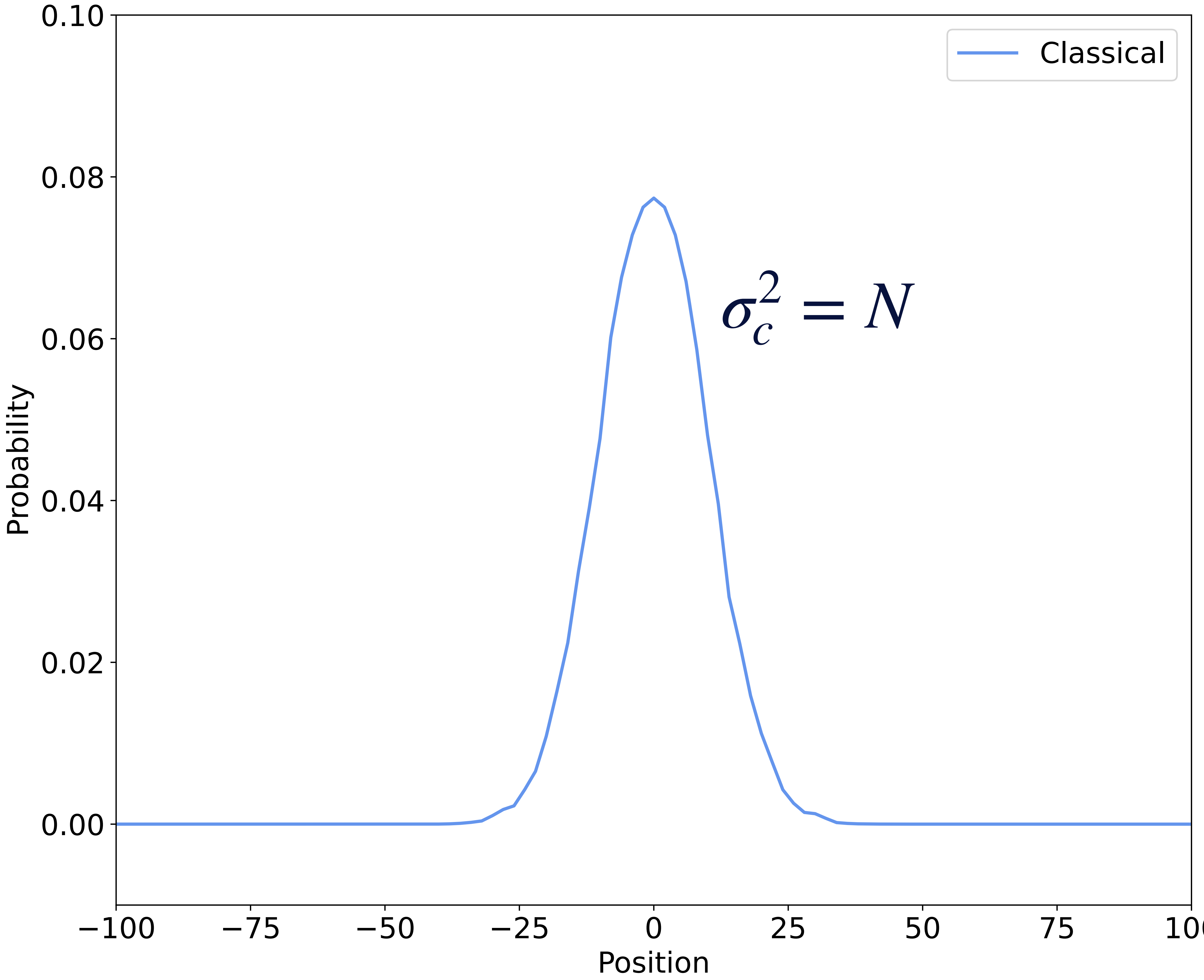
The Quantum Walk

The Quantum Walk





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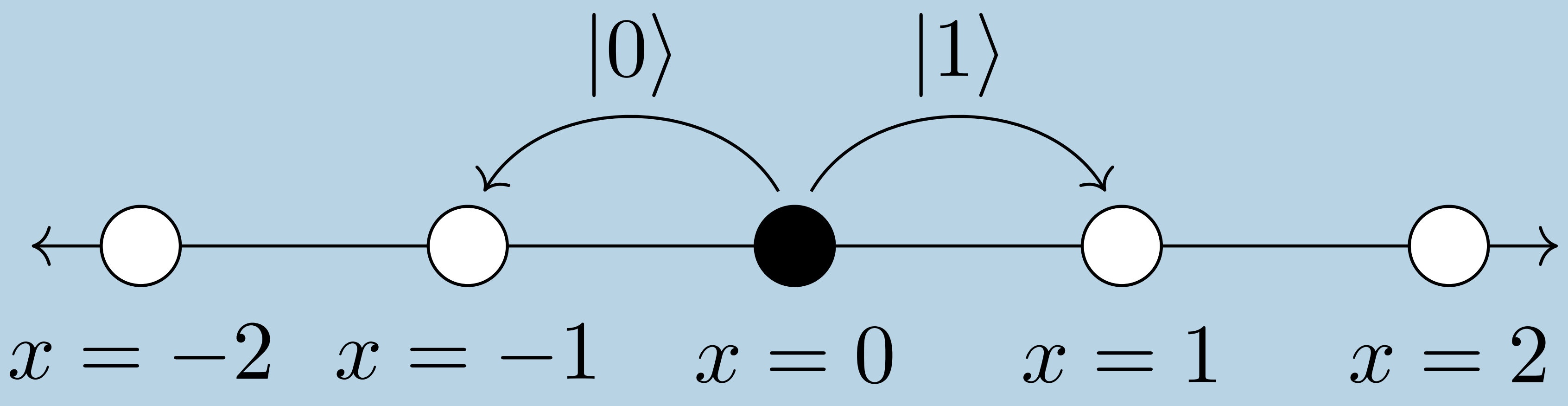
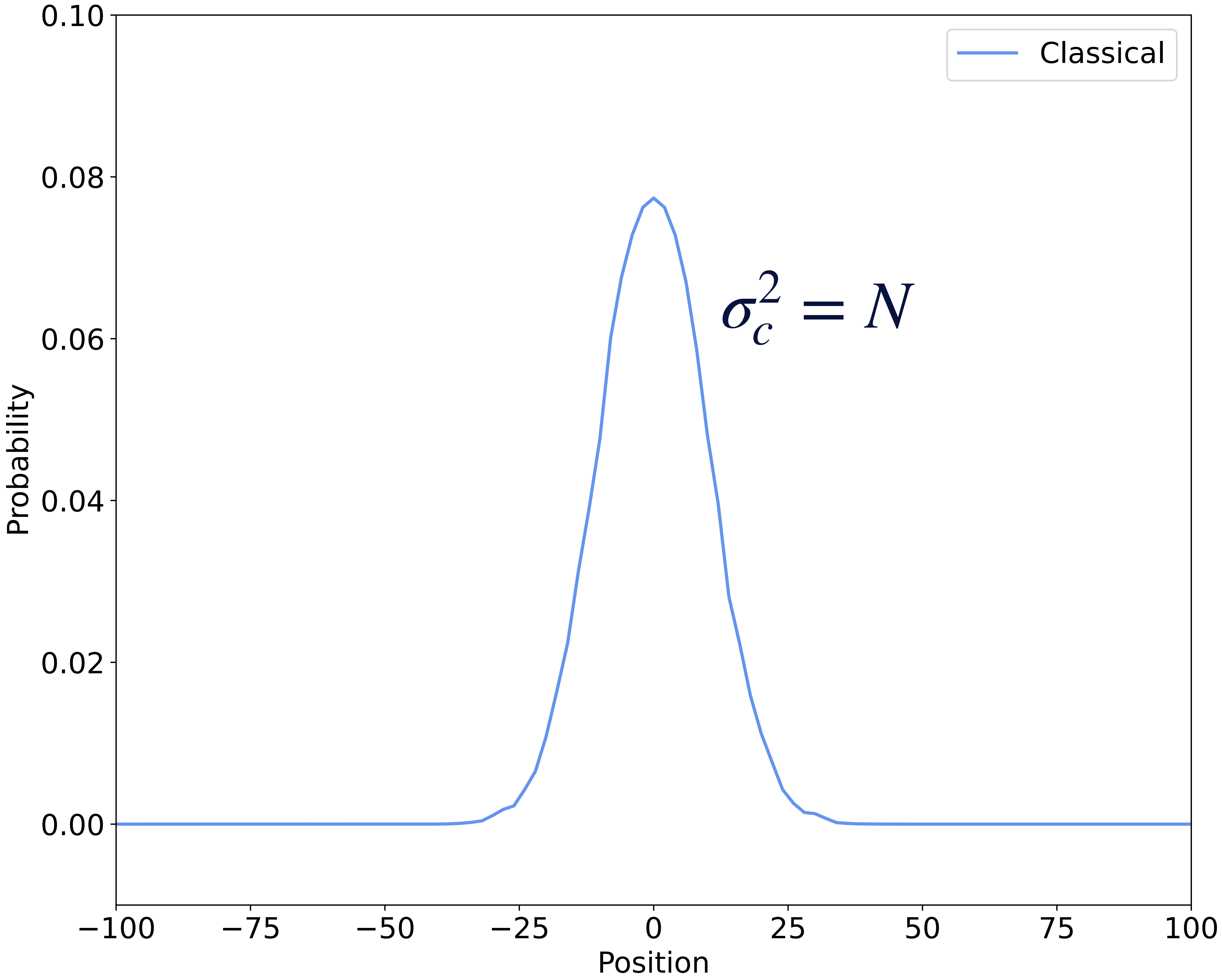


Coin

Operation:

$$C|0\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$$



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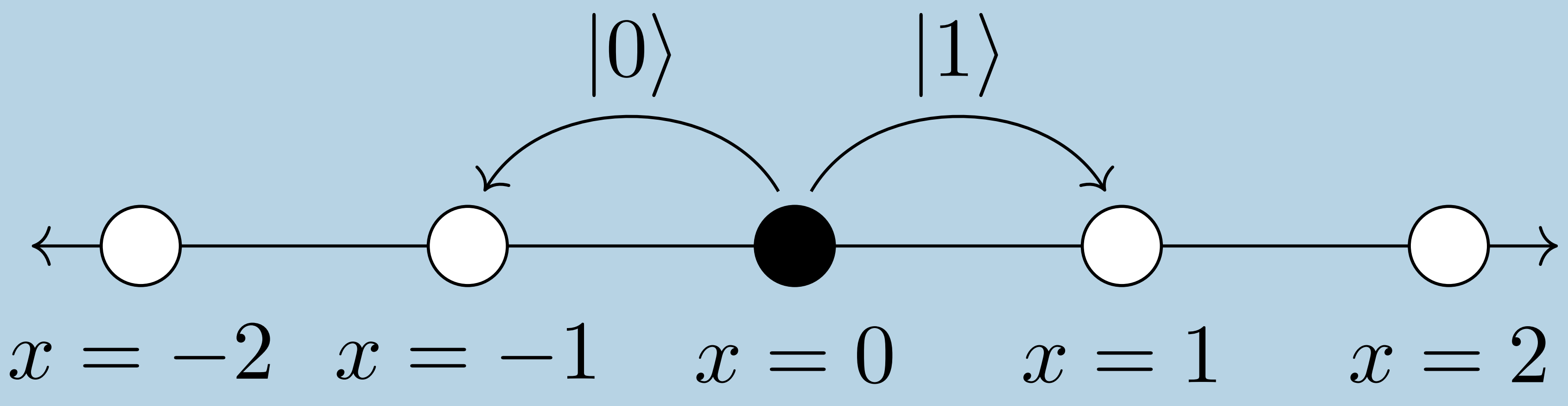

$$\left.\begin{array}{l}\mathscr{H}_P = \{\,|i\rangle : i \in \mathbb{Z}\}\\ \mathscr{H}_C = \{\,|0\rangle, |1\rangle\}\end{array}\right\}\mathscr{H} = \mathscr{H}_C \otimes \mathscr{H}_P$$

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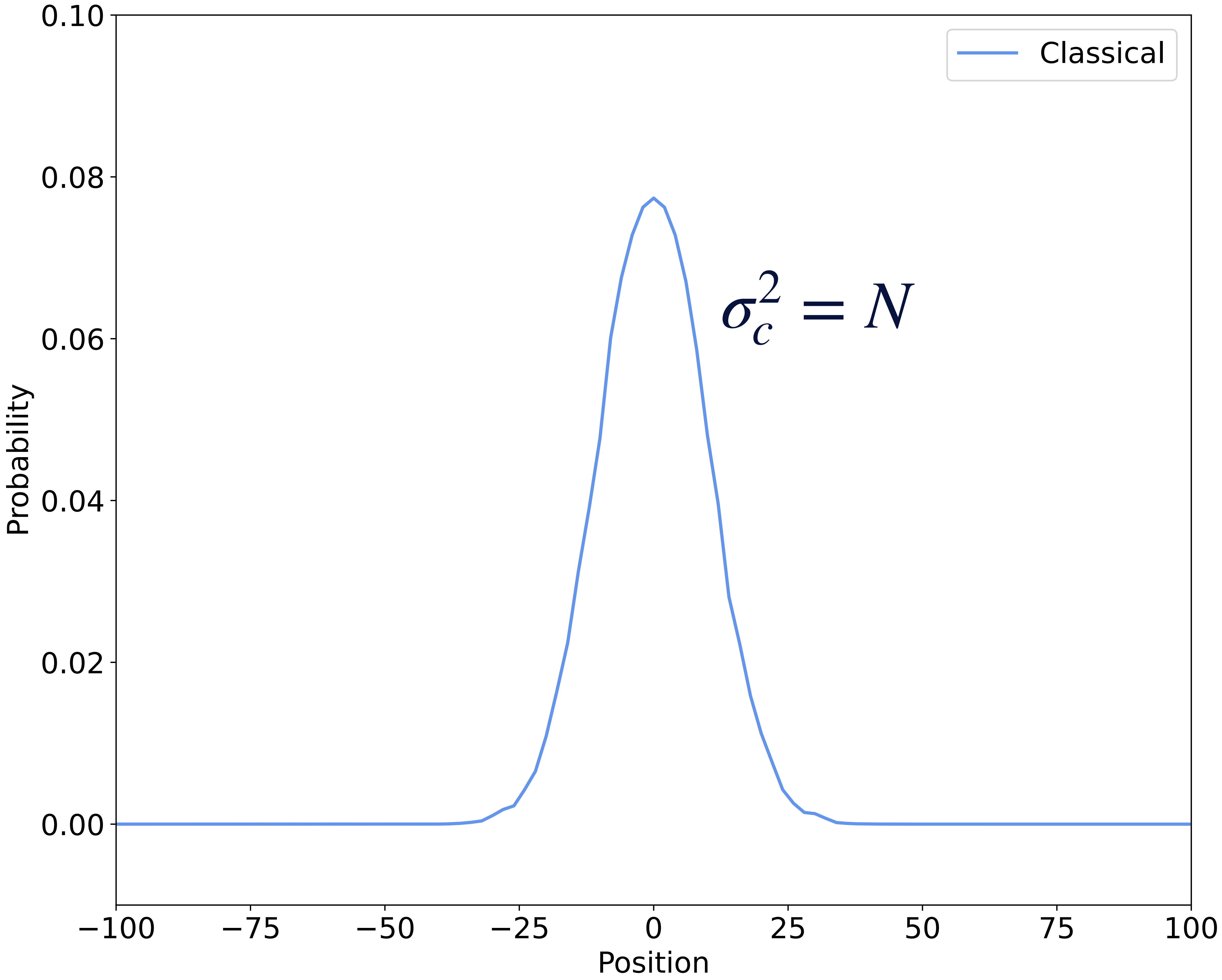

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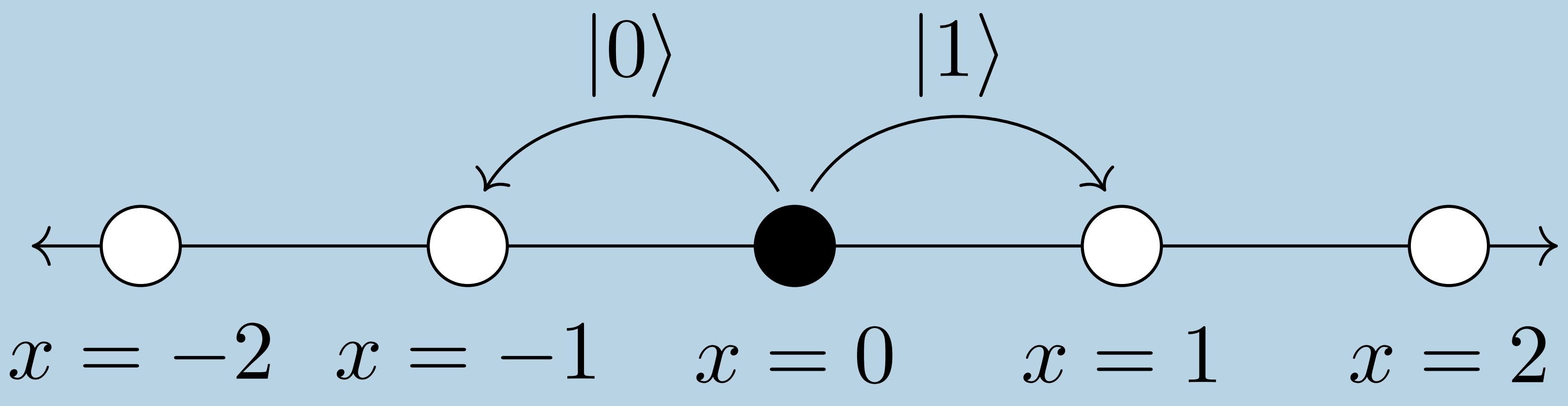

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Unitary Transformation:

$$U = S\cdot(C \otimes I)$$

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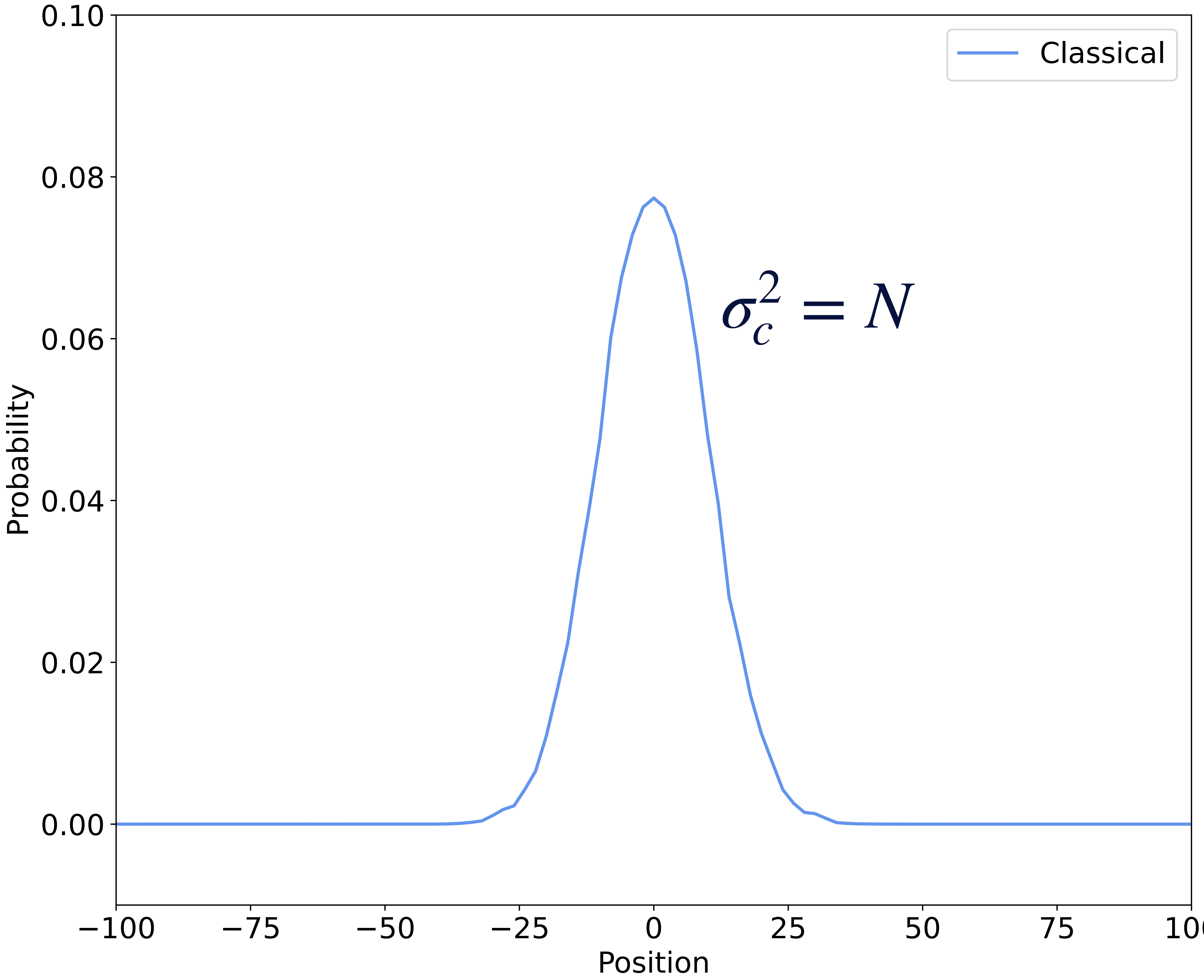

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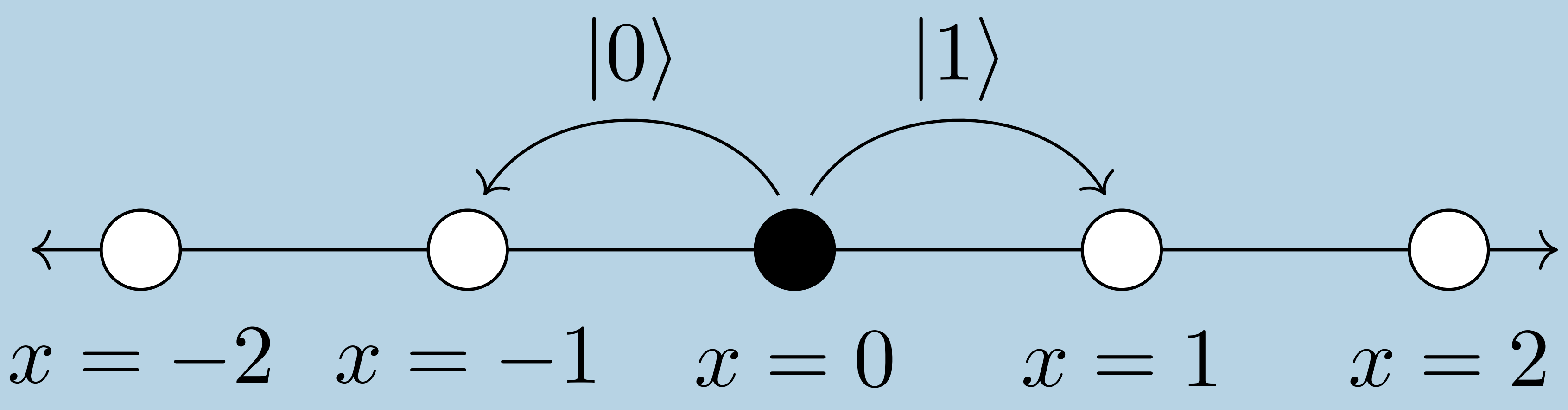

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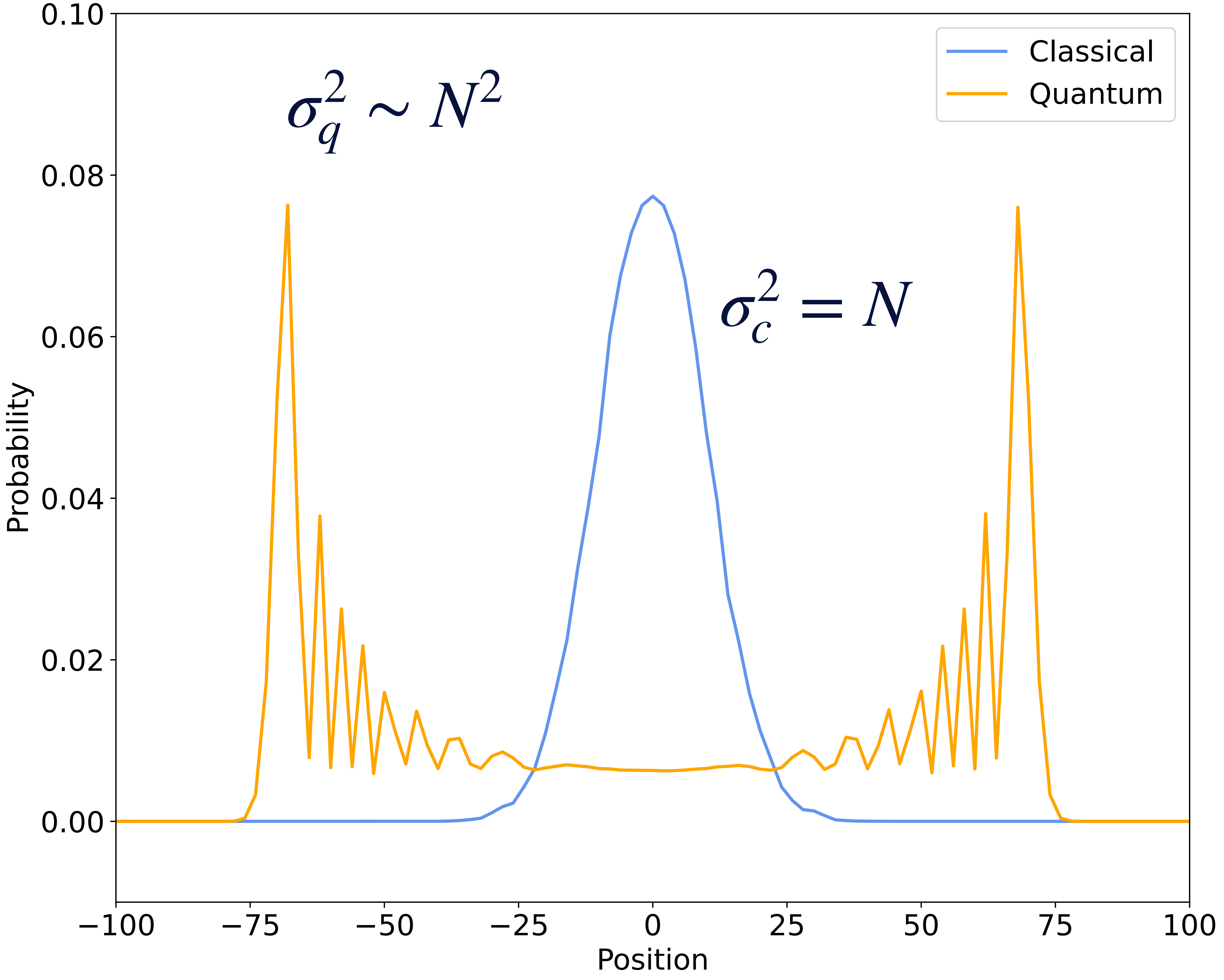
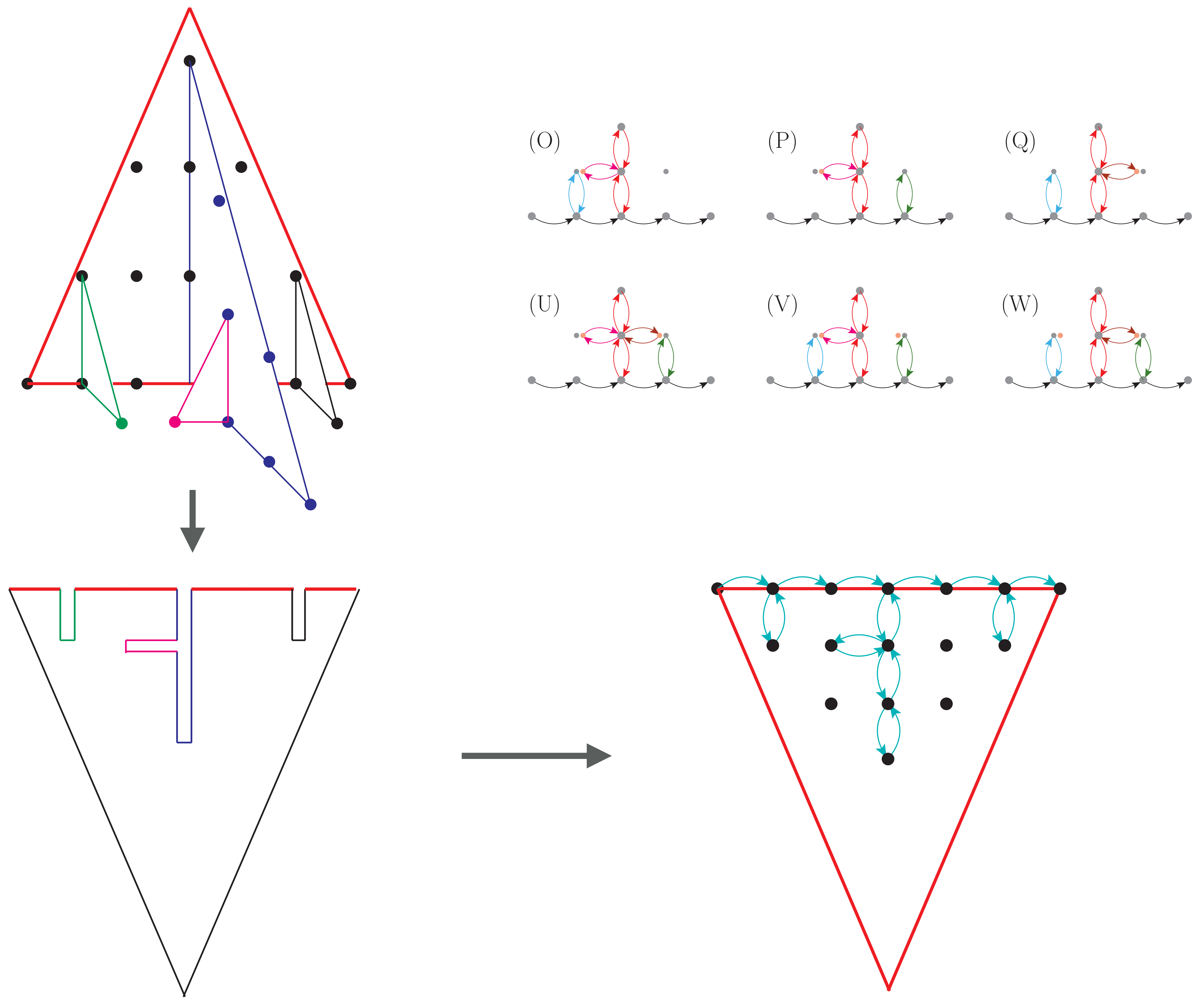
Coin Operation:

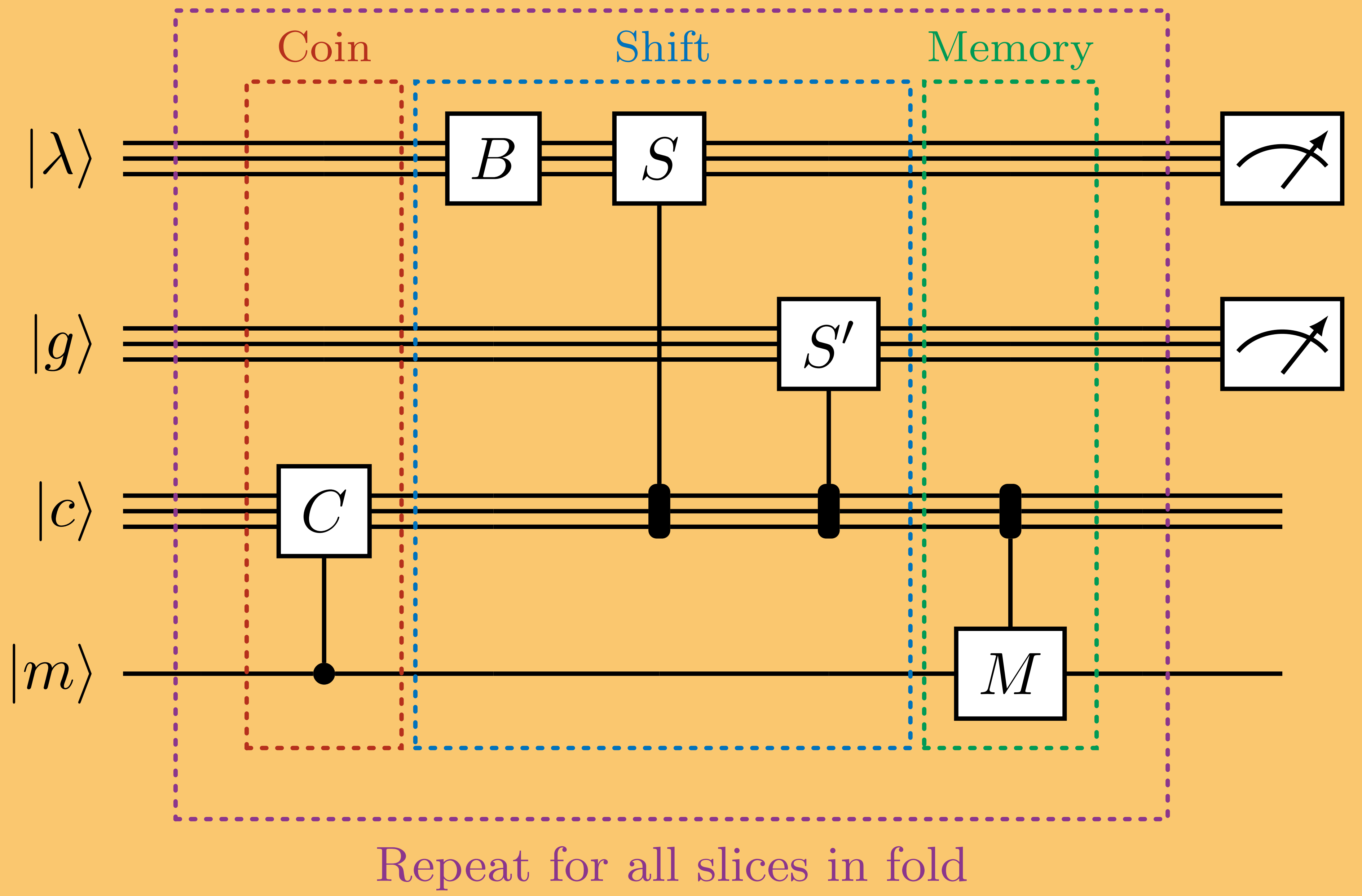
$$C|0\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$$

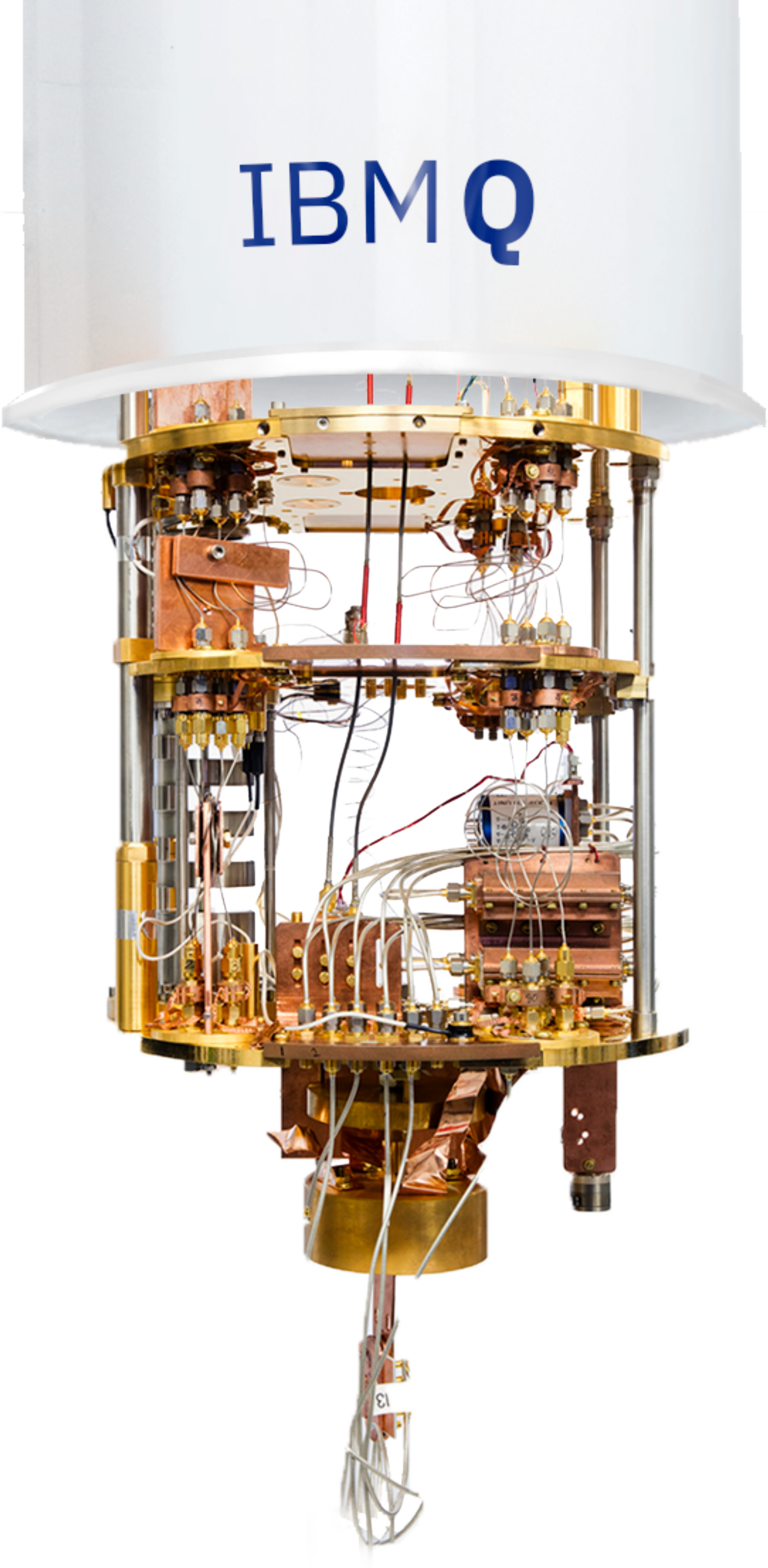


Discrete QCD as a Quantum Walk



The Discrete-QCD dipole cascade can therefore be implemented as a simple **Quantum Walk**





Durham University

IP3

Quantum Parton Shower

- Discretising QCD
- Parton Shower as a Quantum Walk
- **Generate Scattering Events**

G. Gustafson, S. Prestel, M. Spannowsky and S. Williams, Collider Events on a Quantum Computer, *JHEP* 11 (2022) 035, arXiv:2207.10694

Lund University

Imperial College London

Generating Scattering Events from Groves

Once the grove structure has been selected, event data can be synthesised in the following steps using the baseline:

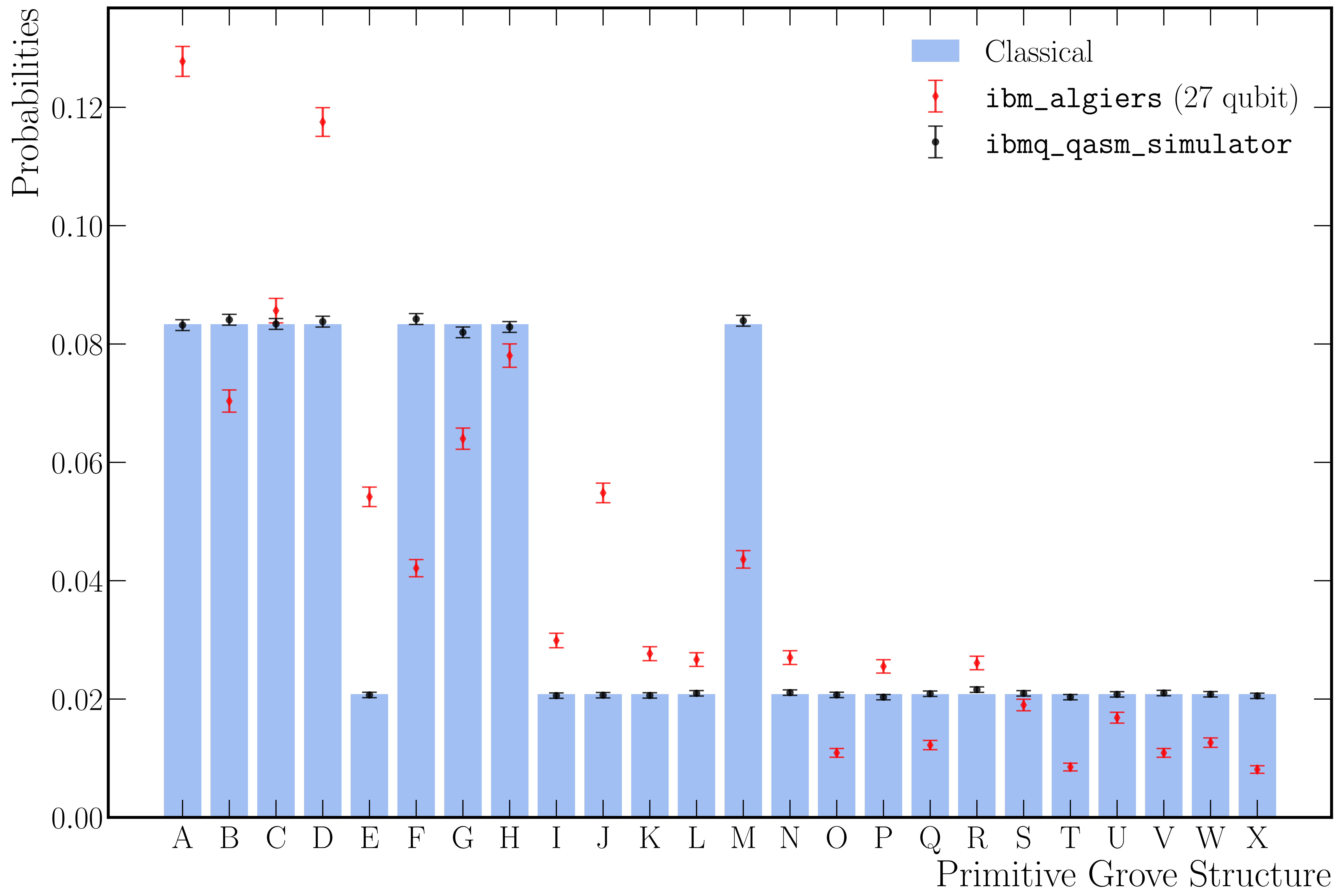
1. Create the highest κ effective gluons first (i.e. go from top to bottom in phase space)

2. For each effective gluon j that has been emitted from a dipole IK, read off the values s_{ij}, s_{jk} and s_{IK} from the grove

3. Generate a uniformly distributed azimuthal decay angle ϕ, and then employ momentum mapping (here we have used Phys. Rev. D 85, 014013 (2012), 1108.6172) to produce post-branching momenta

The algorithm has been run on both the ibm_qasm_simulator and the ibm_algiers 27 qubit device.
A like-for-like classical implementation has been used as a comparison.

Discrete QCD as a Quantum Walk - Raw Grove Simulation



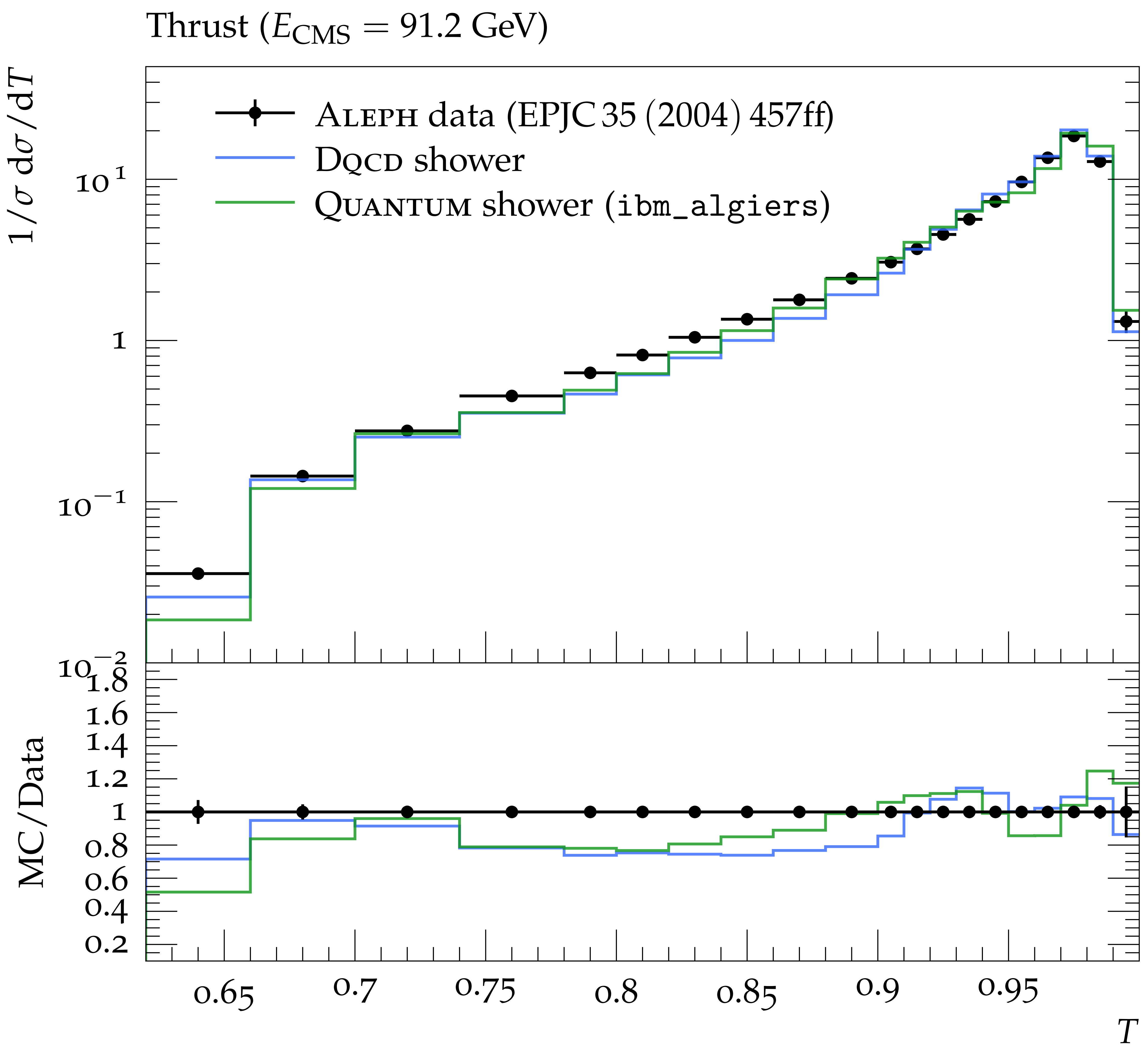
The algorithm has been run on the **IBM Falcon 5.11r chip**

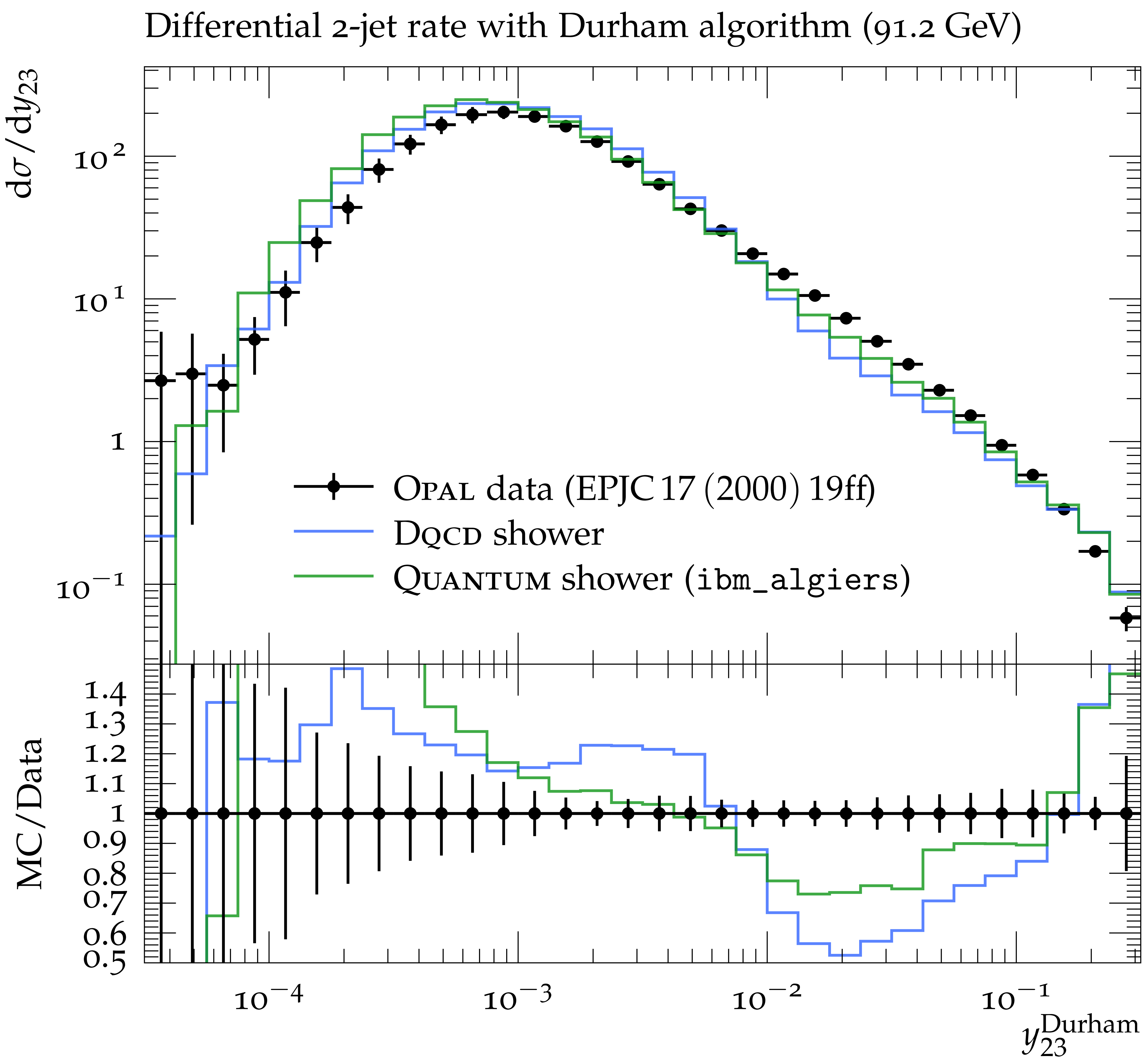
The figure shows the uncorrected performance of the **ibm_algiers** device compared to a simulator

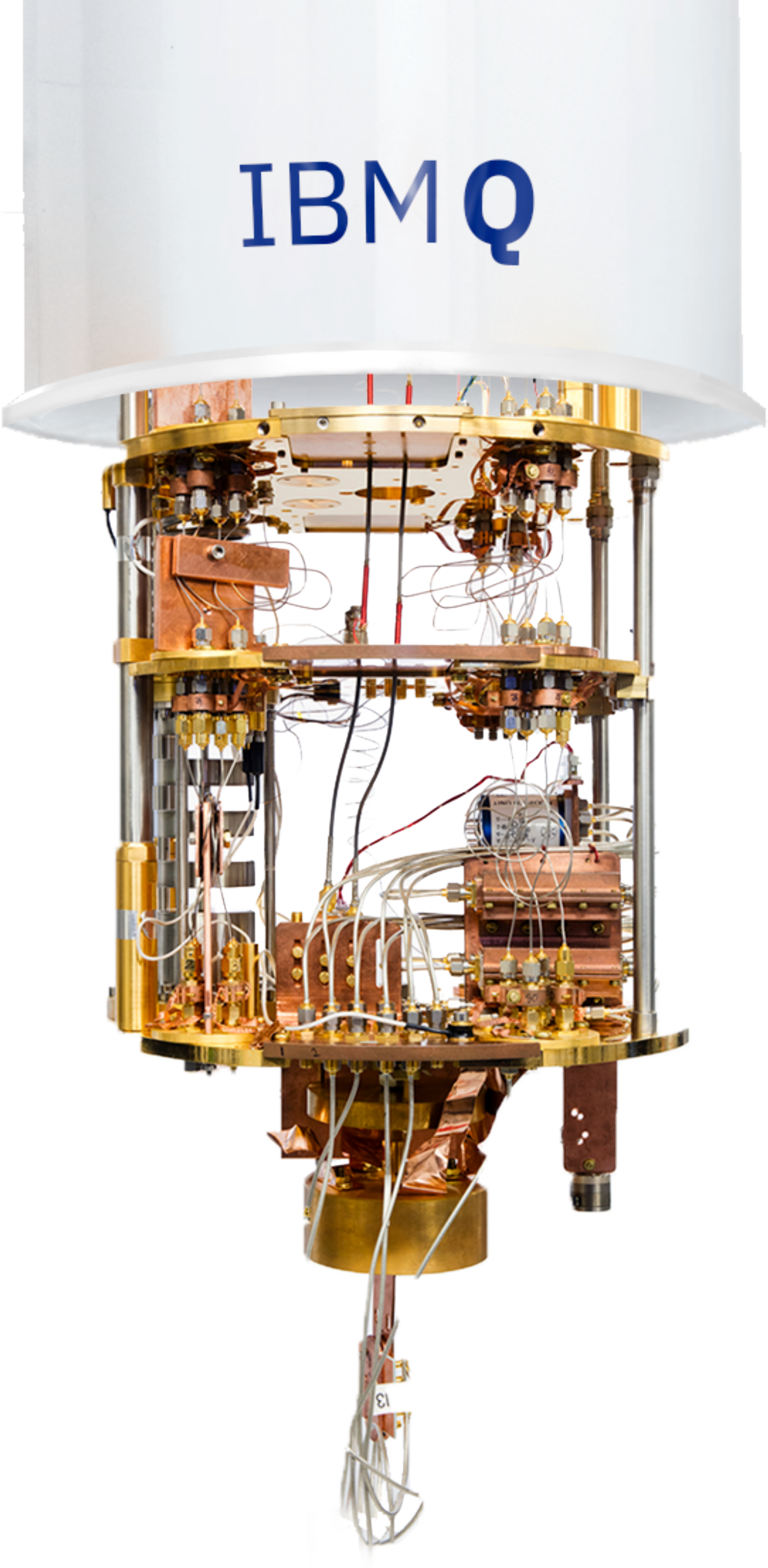
The 24 grove structures are generated for a $E_{CM} = 91.2$ GeV, corresponding to typical collisions at LEP.

Main source of error from CNOT errors from large amount of SWAPs

Collider Events on a Quantum Computer







Durham University

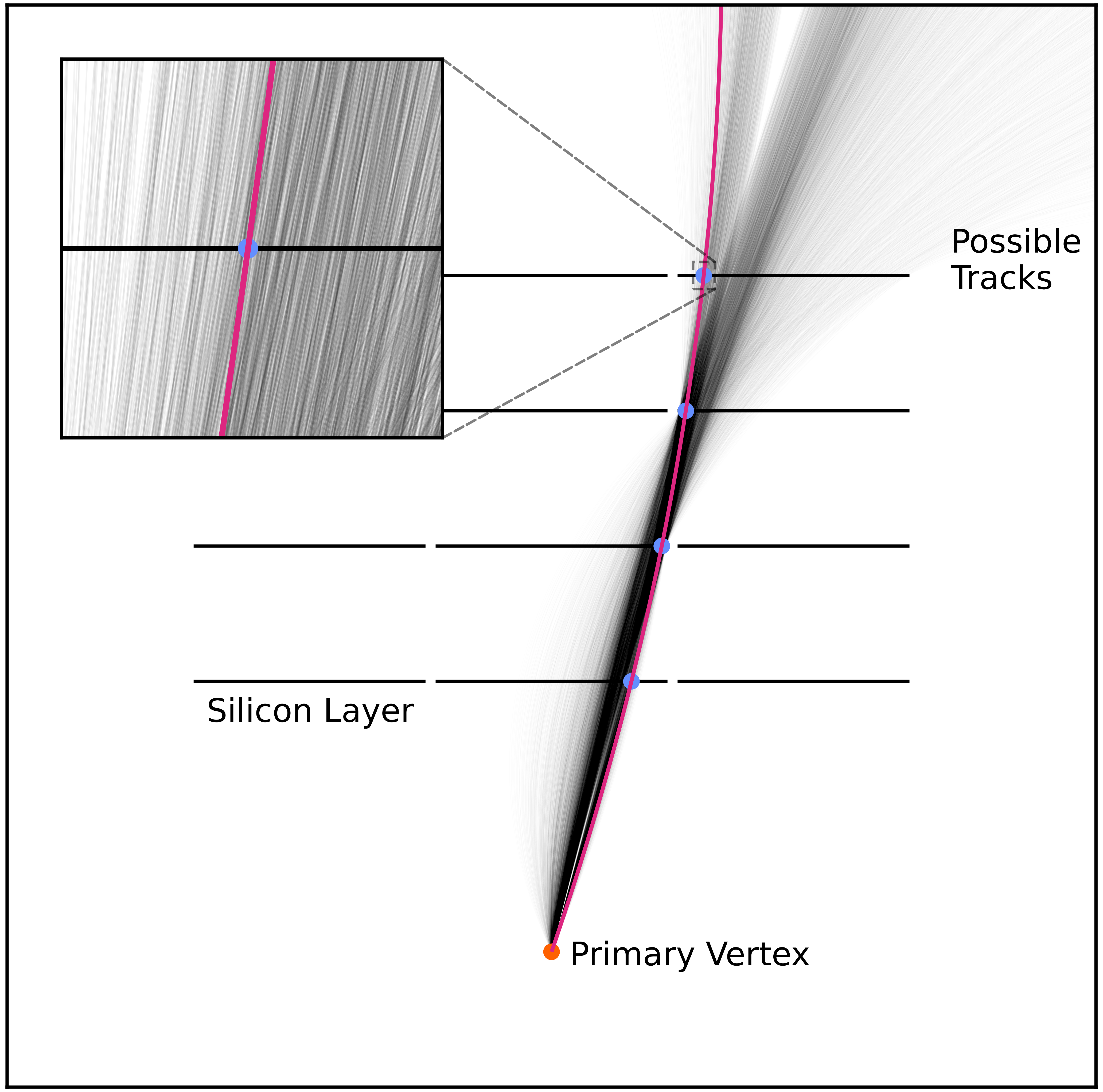
IP3

Quantum Charged Track Finding

C. Brown, M. Spannowsky, A. Tapper, S. Williams and I. Xiotidis (2024) Quantum pathways for charged track finding in high-energy collisions. Front. Artif. Intell. 7:1339785. arXiv:2311.00766

Imperial College London

Track Finding via Associative Memory

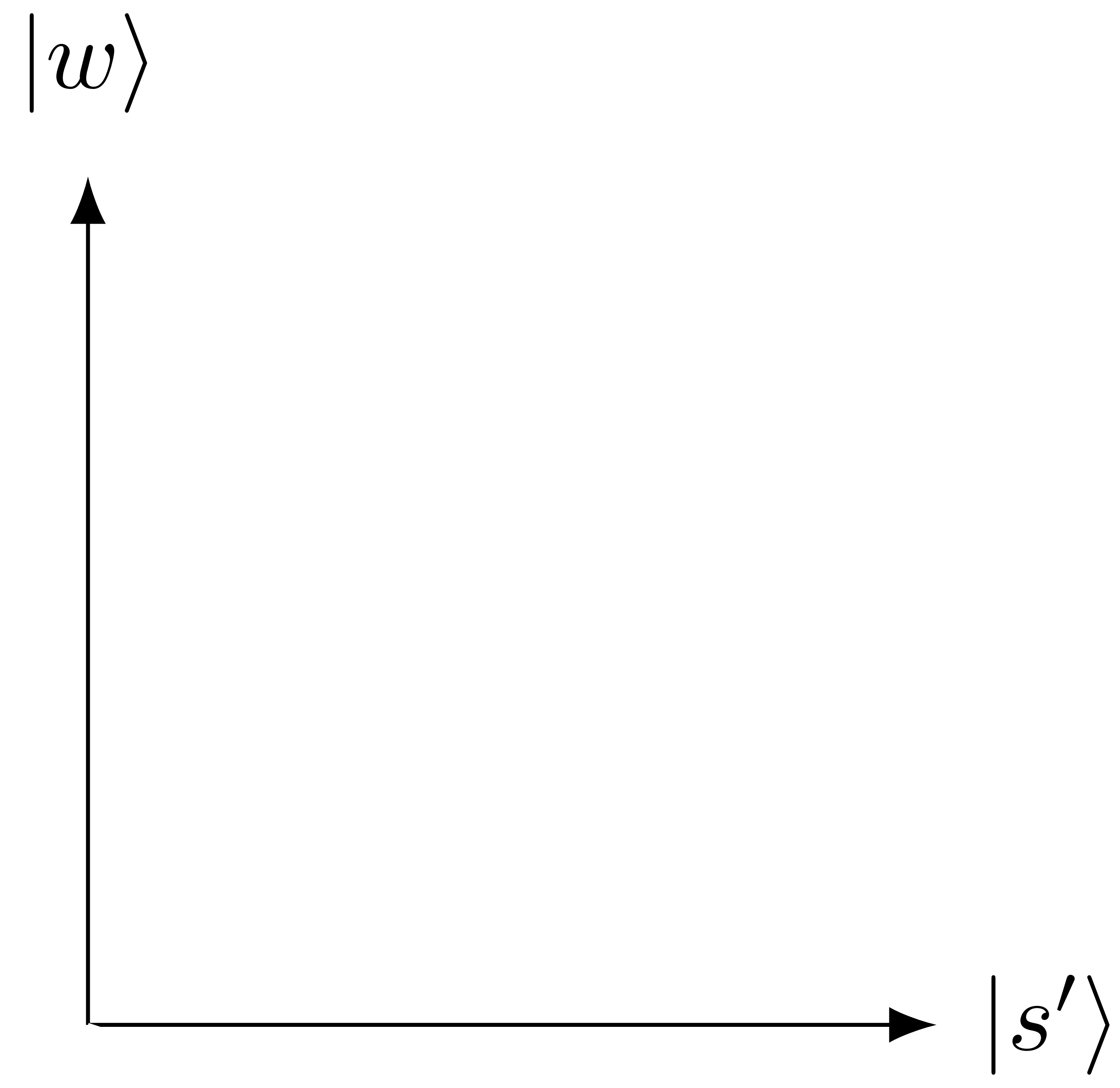


A critical stage of event reconstruction and classification in modern colliders is the identification of **charged particle trajectories**

Highly **granular** detectors are used to efficiently measure the **position** of **charged particles** as they move through the detector

Classical techniques like **Associative Memory** have been shown to be **highly effective**, but **new approaches** are required as collider **energy and luminosity increase** to handle the growing number of **tracks and combinatorics**

Quantum Amplitude Amplification



The aim is to **identify** interesting states in a database $X = \{x_0, x_1, \ldots, x_N\}$ with **interesting states** m_i encoded on a quantum device as $|s\rangle = \mathscr{A}|0\rangle^{\otimes n}$

Marking interesting states, $|m\rangle$ using the **oracle**

$$f(x) = \begin{cases} 1 & \text{if } x = m, \\ 0 & \text{otherwise}. \end{cases} \longrightarrow S_f|x\rangle = (-1)^{f(x)}|x\rangle$$

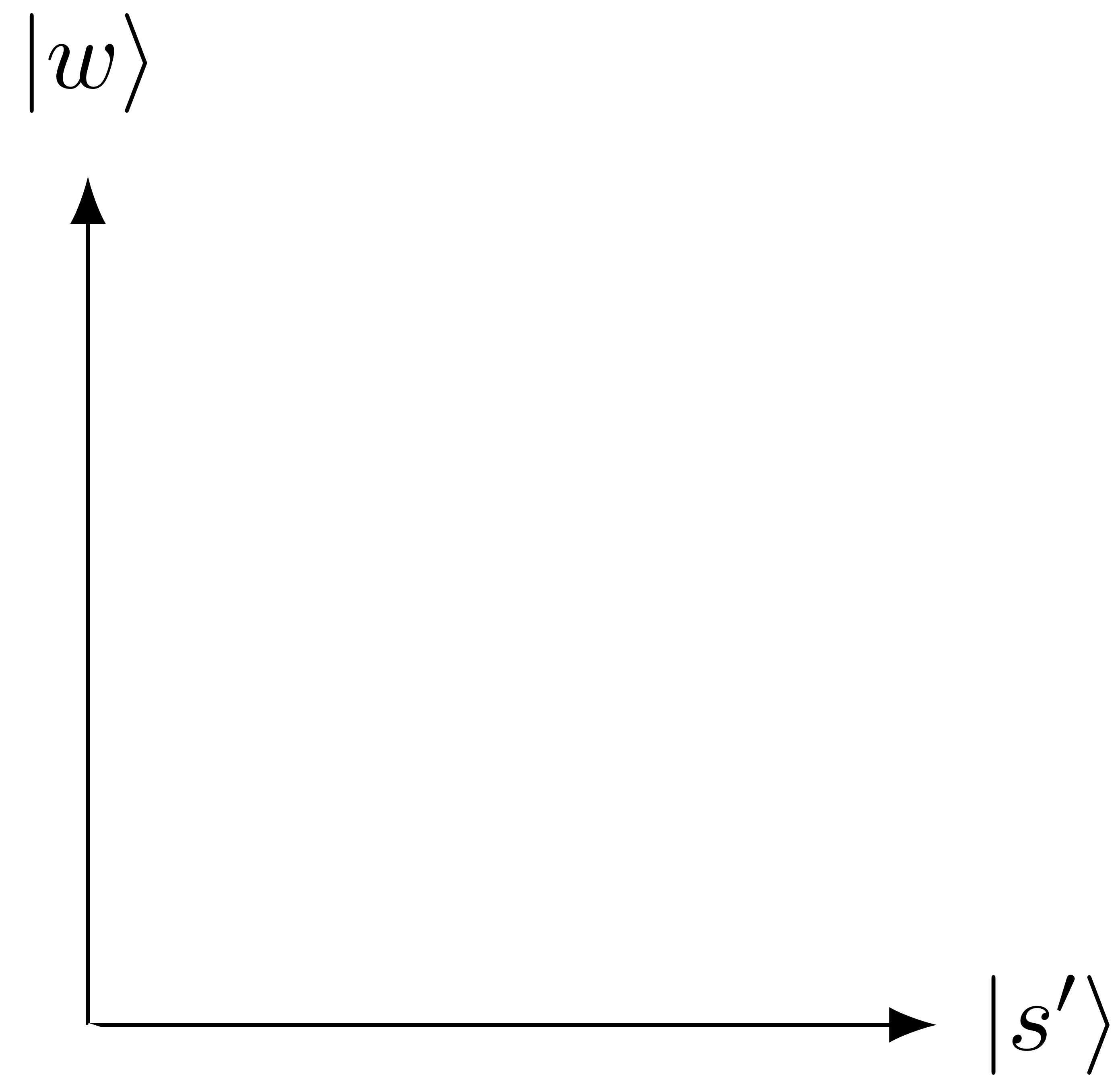
Amplify marked states using the diffusion operation:

$$D = \mathscr{A}^\dagger S_0 \mathscr{A}$$

Therefore, can iteratively apply the **Grover Iterator**:

$$\mathscr{Q} = \mathscr{A}^\dagger S_0 \mathscr{A} S_f$$

Quantum Amplitude Amplification



$$|s'\rangle = \frac{1}{\sqrt{N-1}} \sum_{n-1} |n-1\rangle$$

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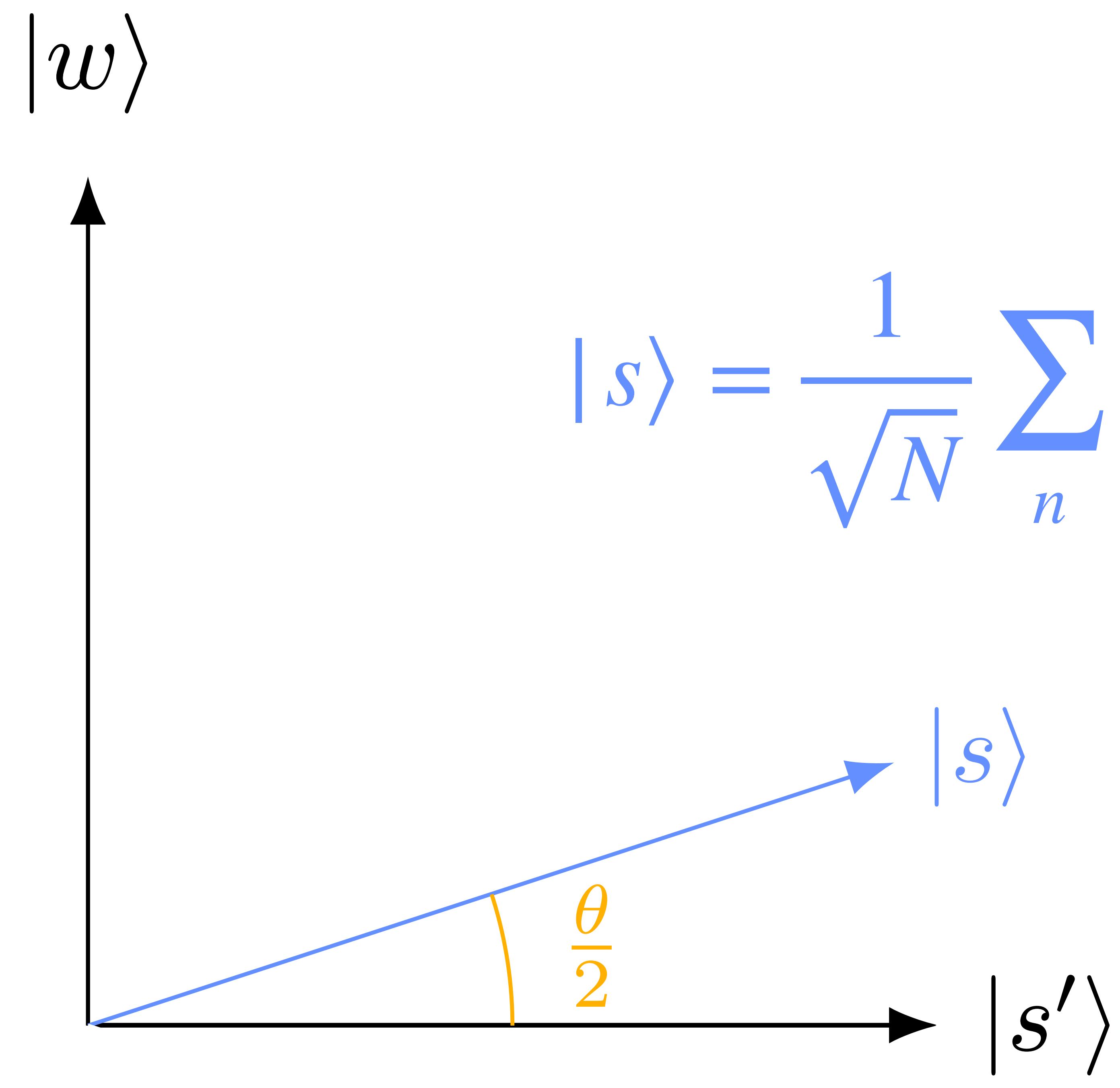

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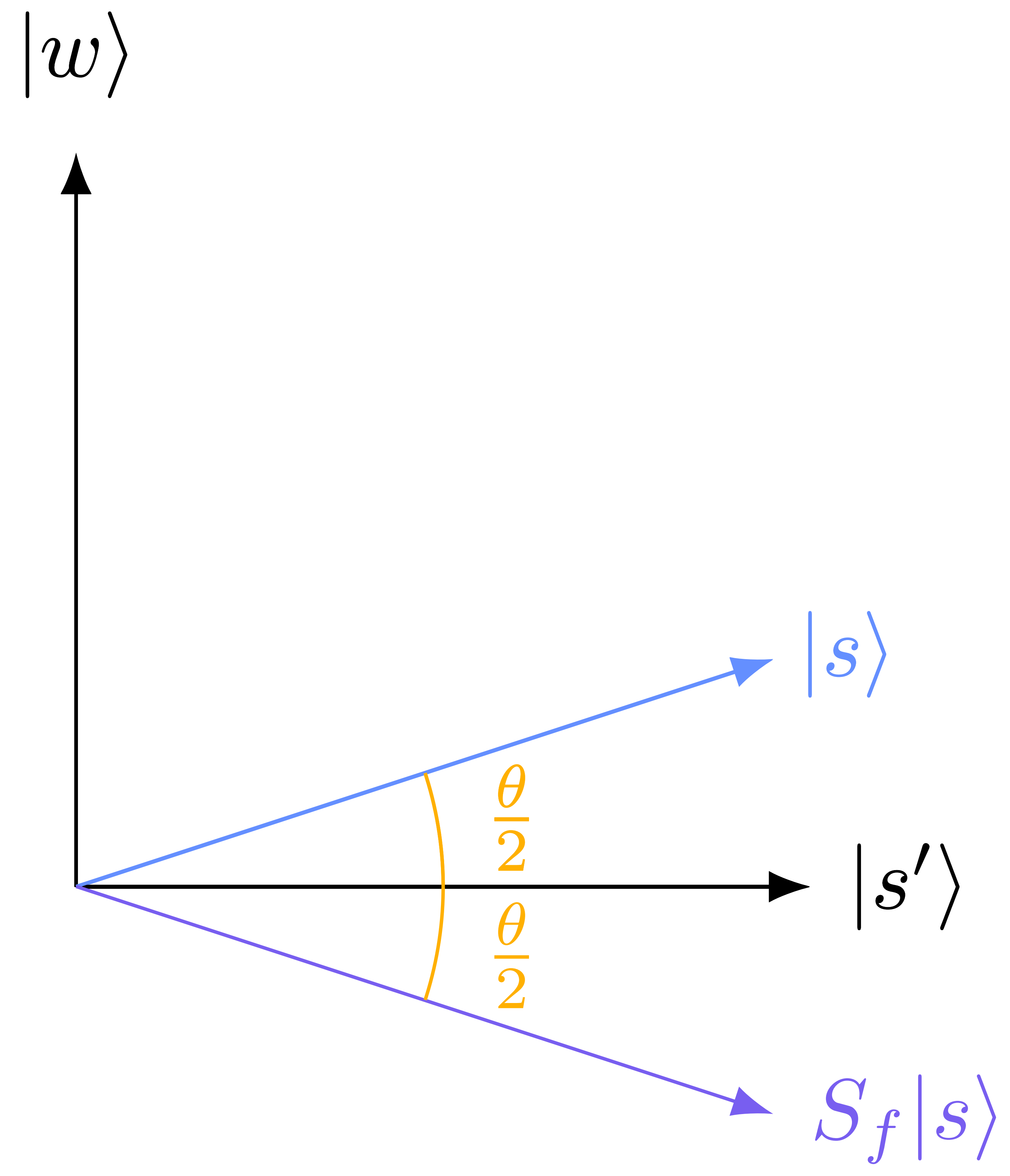

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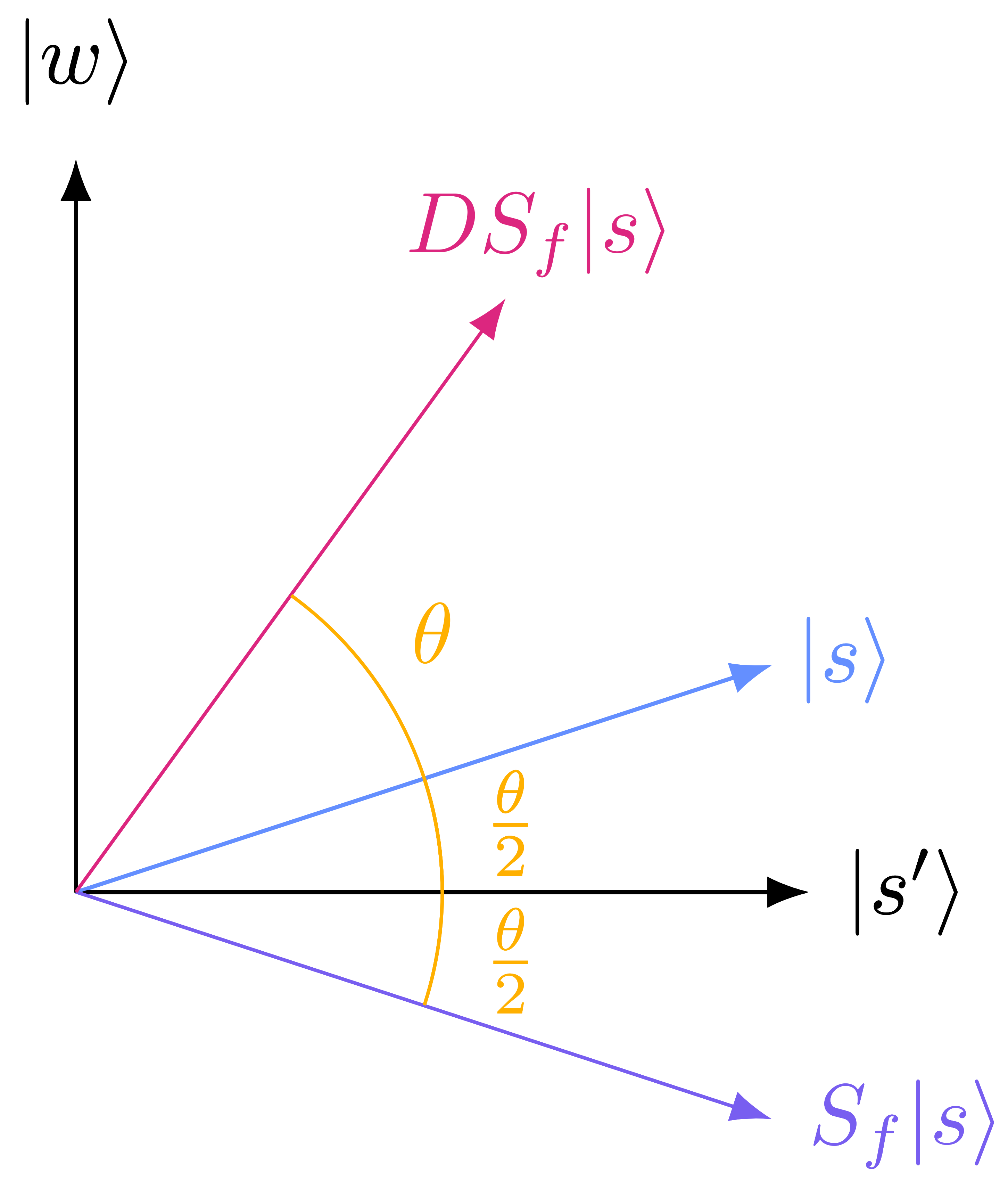

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Therefore, can iteratively apply the **Grover Iterator**:

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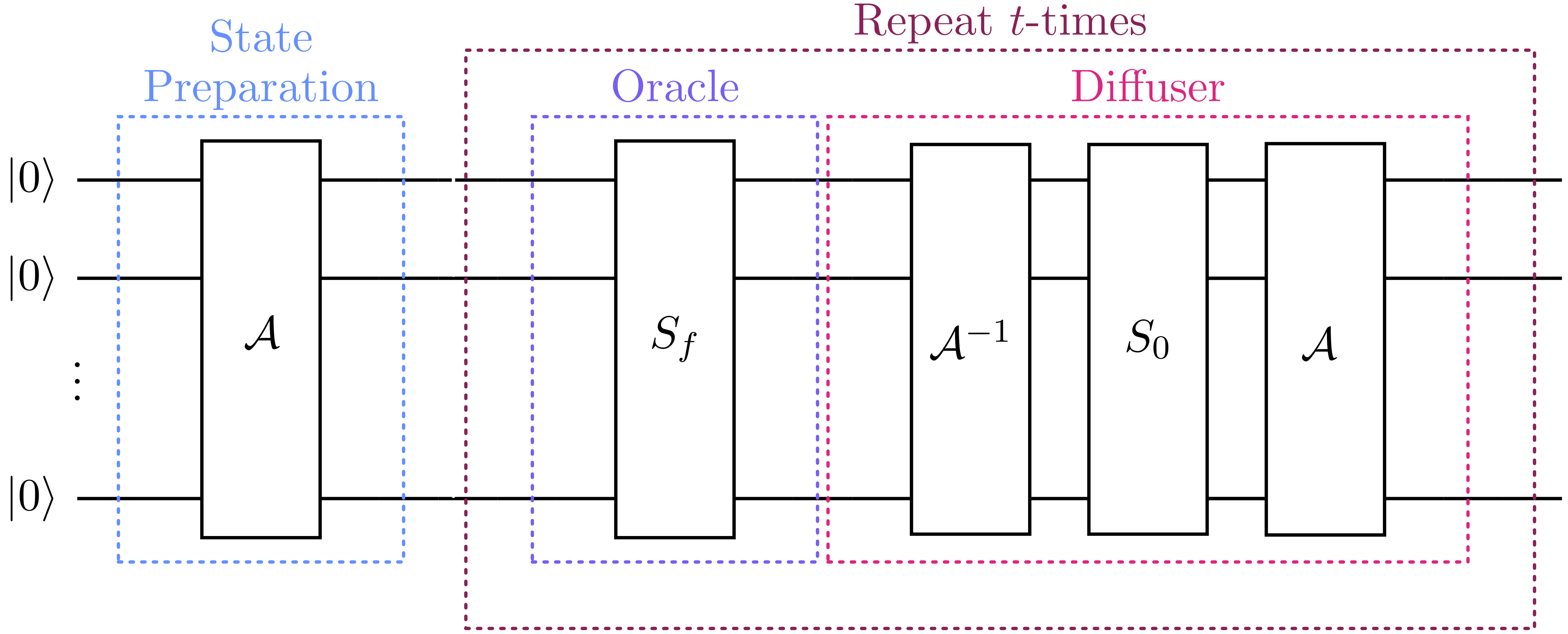
Quantum Amplitude Amplification

The optimal number of iterations of the QAA routine $\mathcal{Q}$ is given by

$$t = \left\lfloor \frac{\pi}{4}\sqrt{\frac{N}{m}} \right\rfloor$$

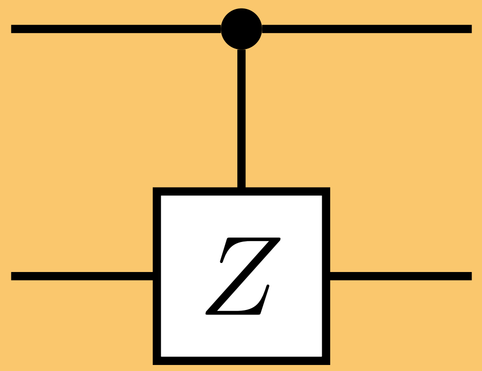
After t iterations of $\mathcal{Q}$, measurement will return a marked state with high probability



QAA therefore scales as $\mathcal{O}(\sqrt{N})$, thus achieving a **polynomial speedup** over classical search algorithms, which scale as $\mathcal{O}(N)$

Oracle Construction

Consider a two qubit example where $|11\rangle$ is the marked state



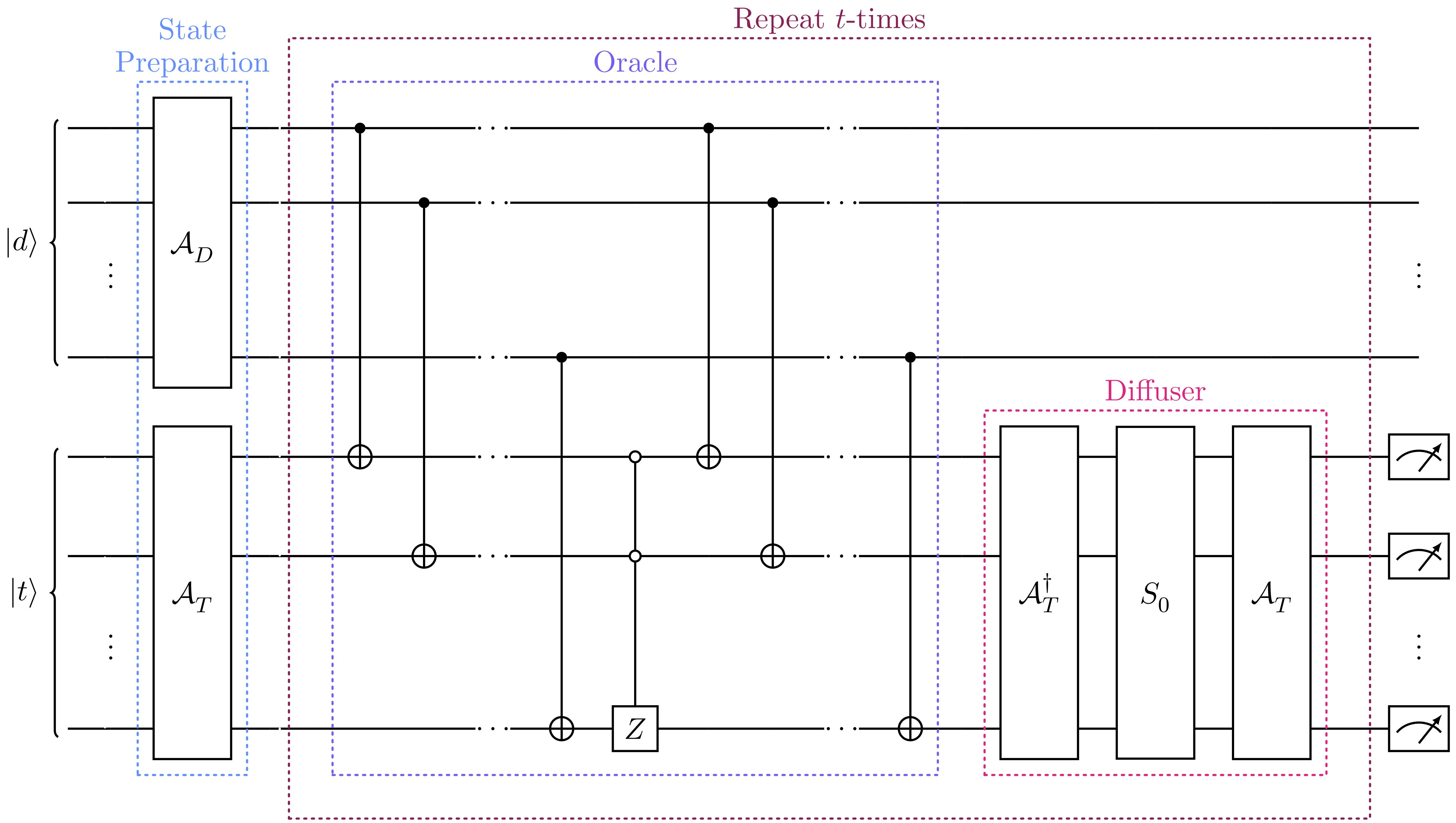
$$S_f : I \otimes |0\rangle\langle 0| + Z \otimes |1\rangle\langle 1|$$

Quantum Template Matching

The perform template matching, we must **abstract** the QAA routine by constructing a new **oracle**

Introducing a new **data register** and acting the oracle across **two registers** allows for **data** to be **parsed directly** to the algorithm

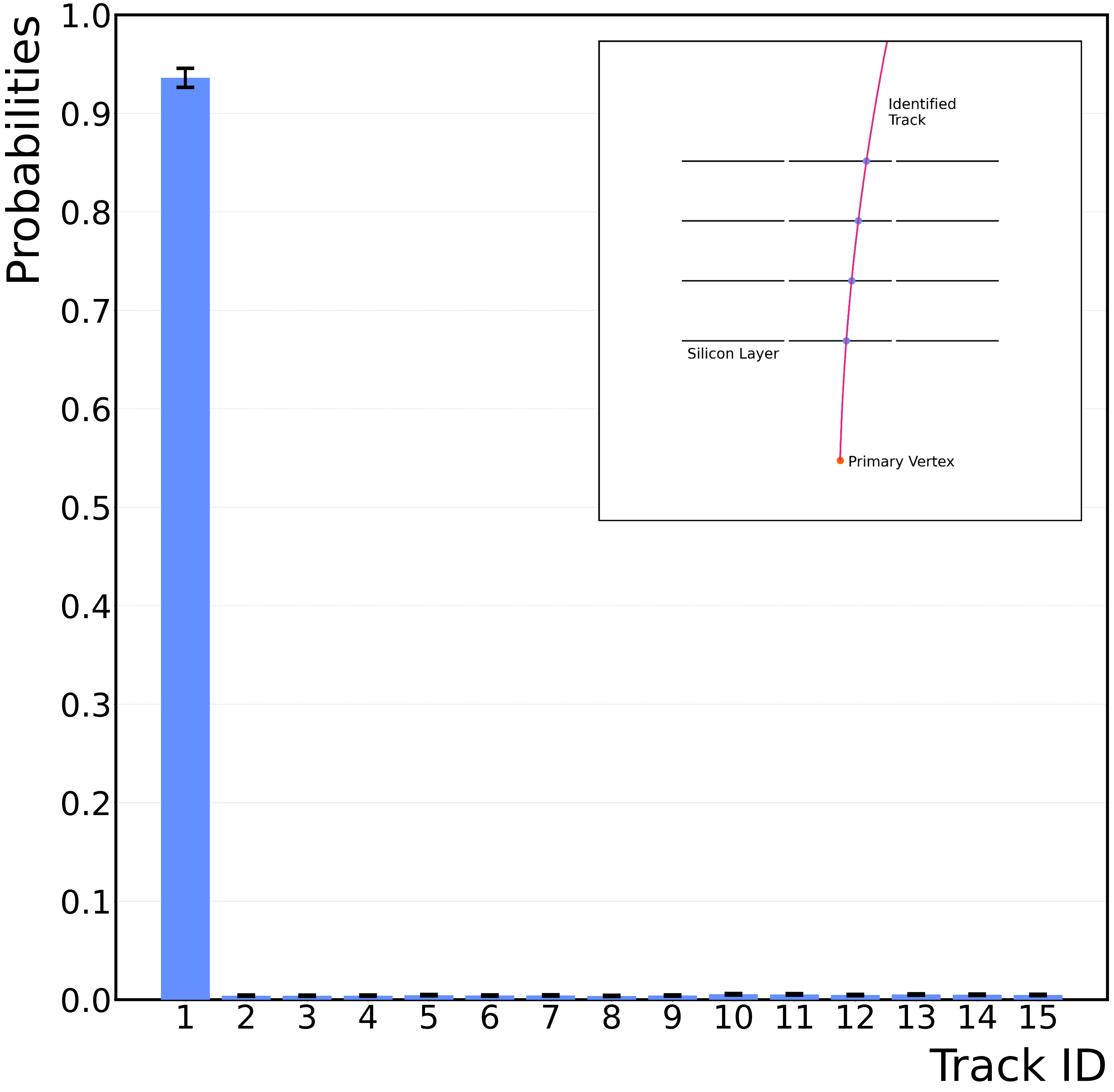
The oracle is constructed from a series of **CNOT** gates and a phase inversion about the zero state on the template register

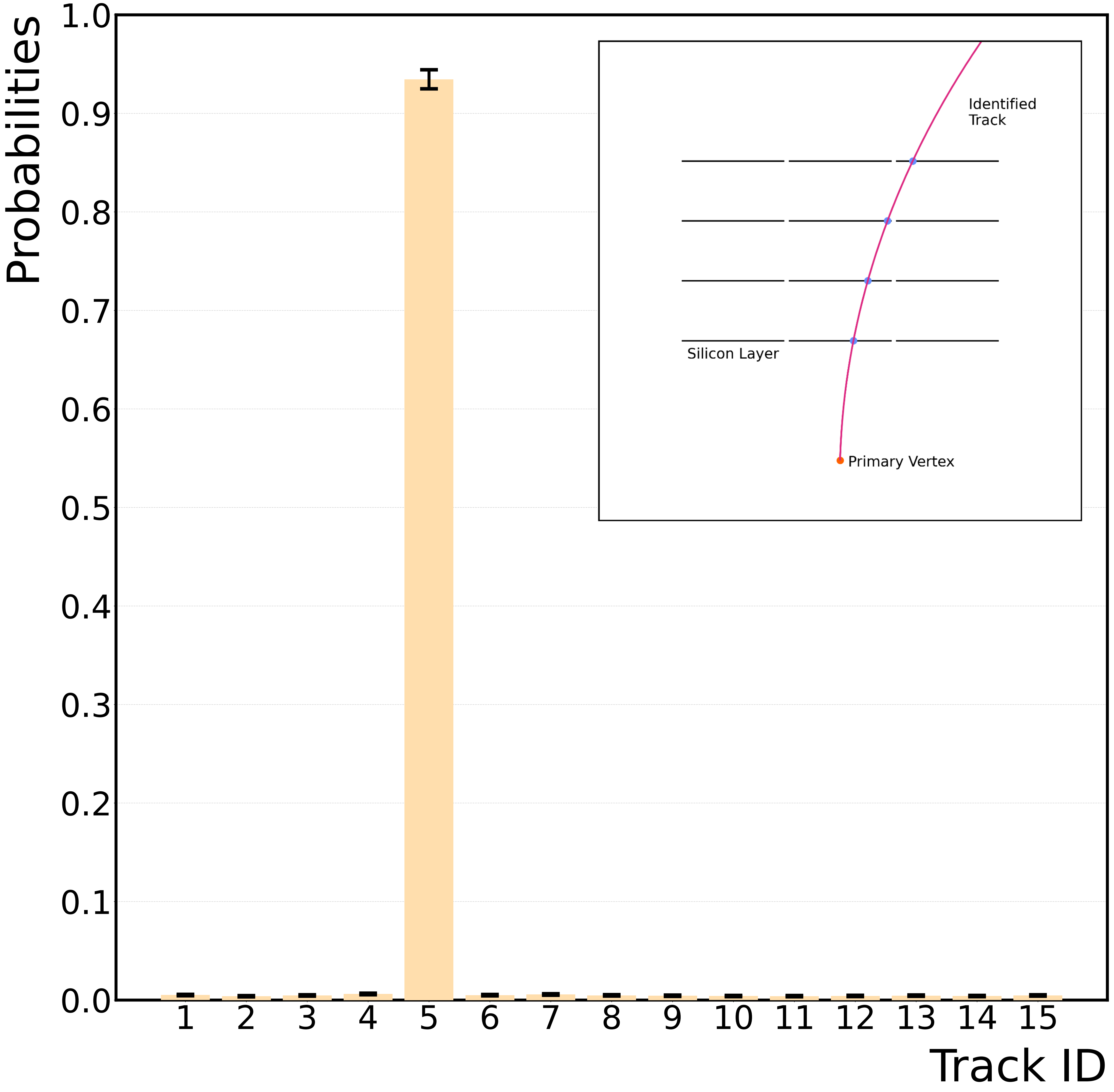


The **diffusion operation** then has the same form as the regular QAA routine

$$\mathcal{Q} = \mathcal{A}^\dagger S_0 \mathcal{A} S_f'$$

Quantum Template Matching for Track Finding



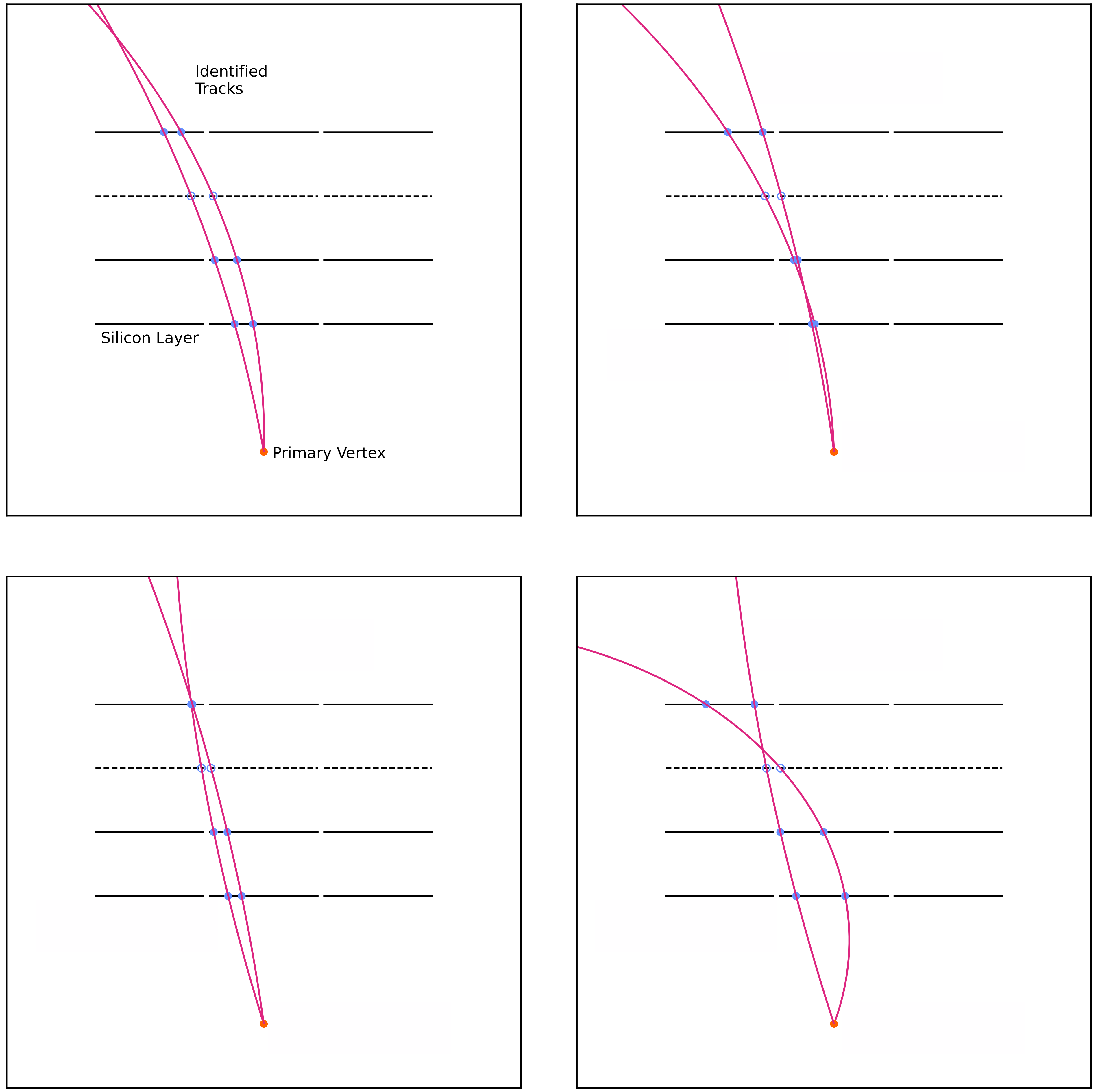


Quantum Track Finding with Missing Hits

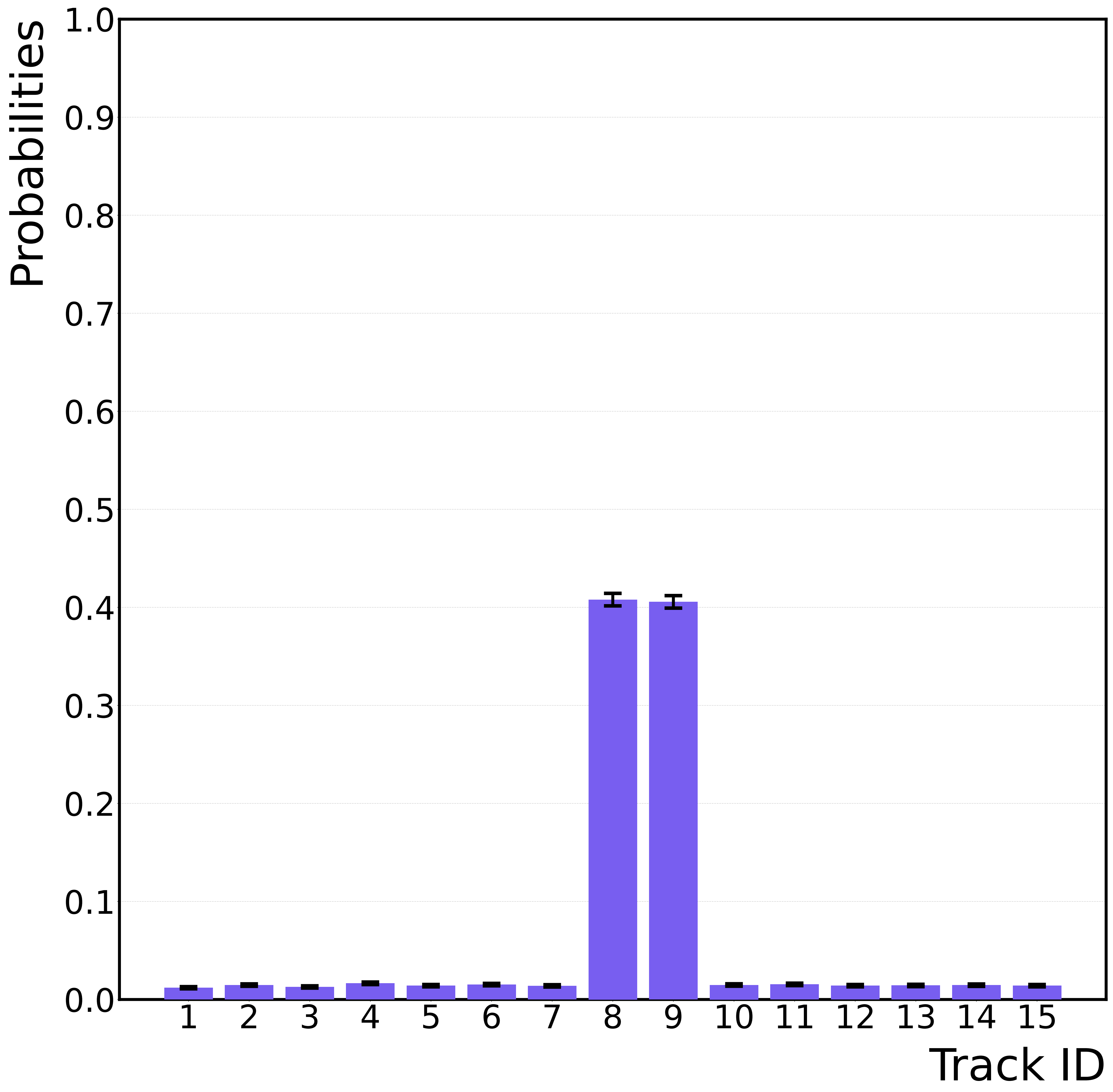
A primary challenge for track finding algorithms is when a particle traverses a detector without registering a hit in one or more detector module

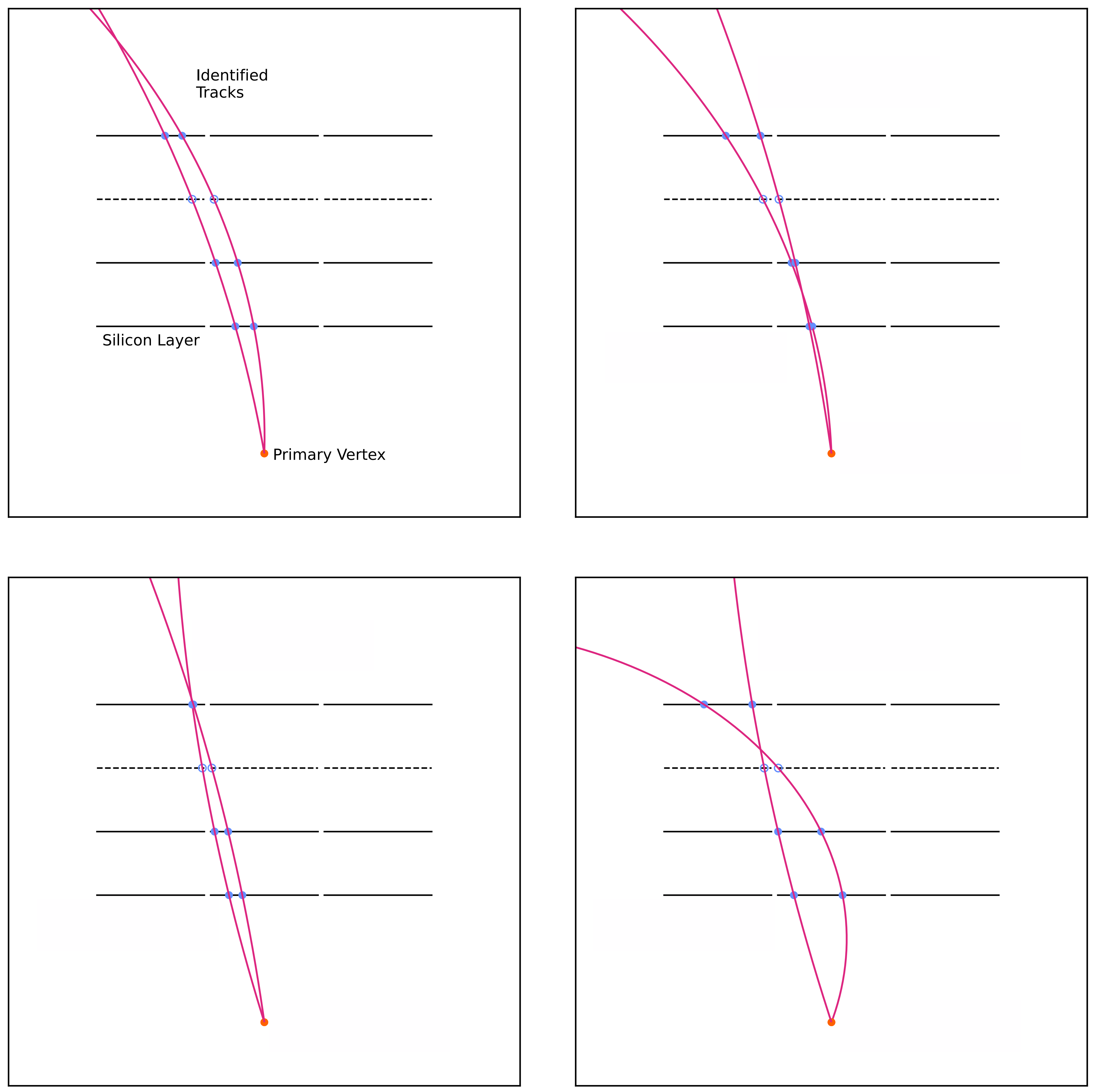
An Associative Memory approach to track finding cannot manage **missing hit data**

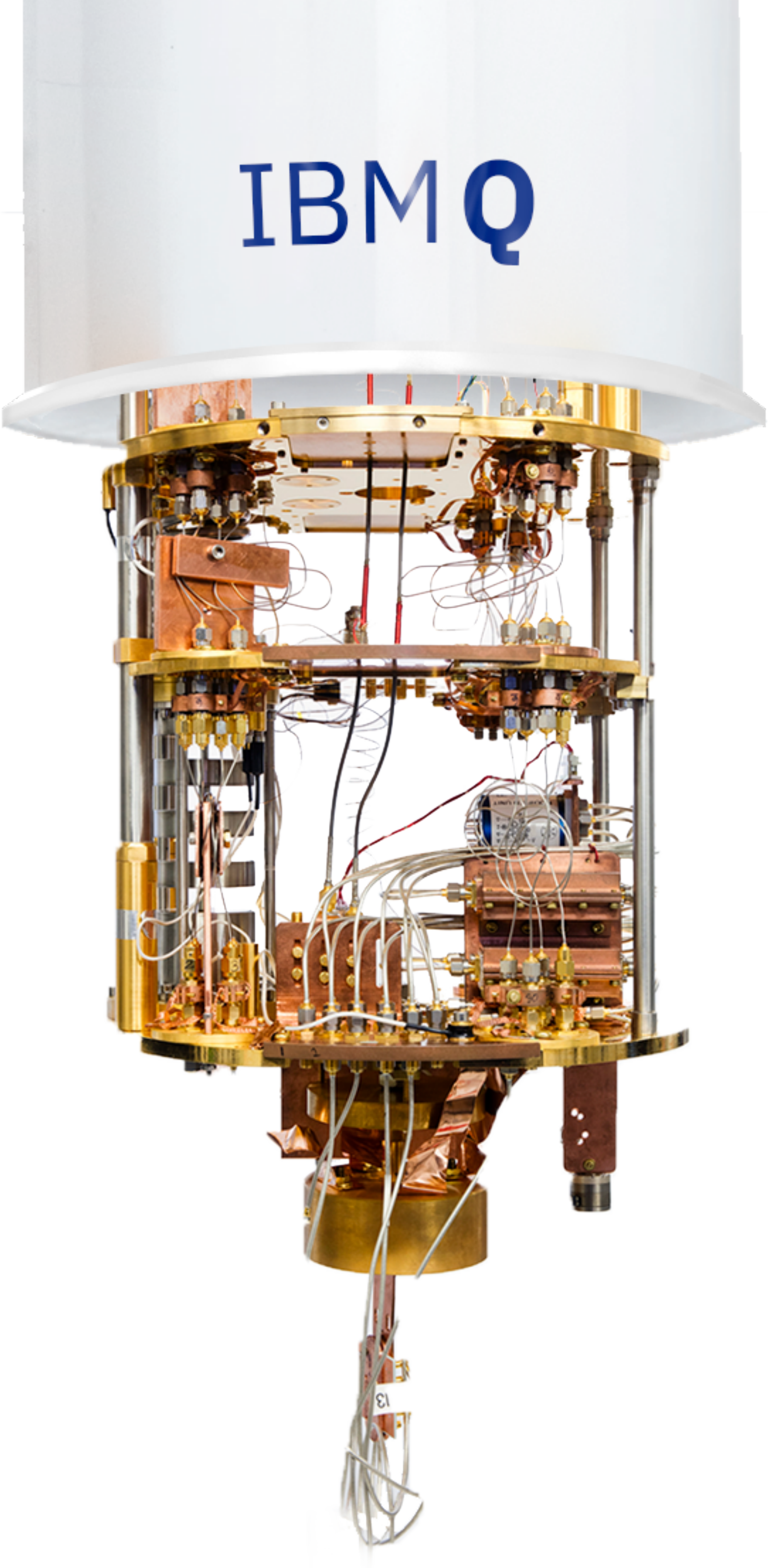
Modifying the oracle allows for the quantum template algorithm to efficiently search on missing hit data, **without an increase in resources** and retaining the **high accuracy** and **speedup**



Quantum Track Finding with Missing Hits







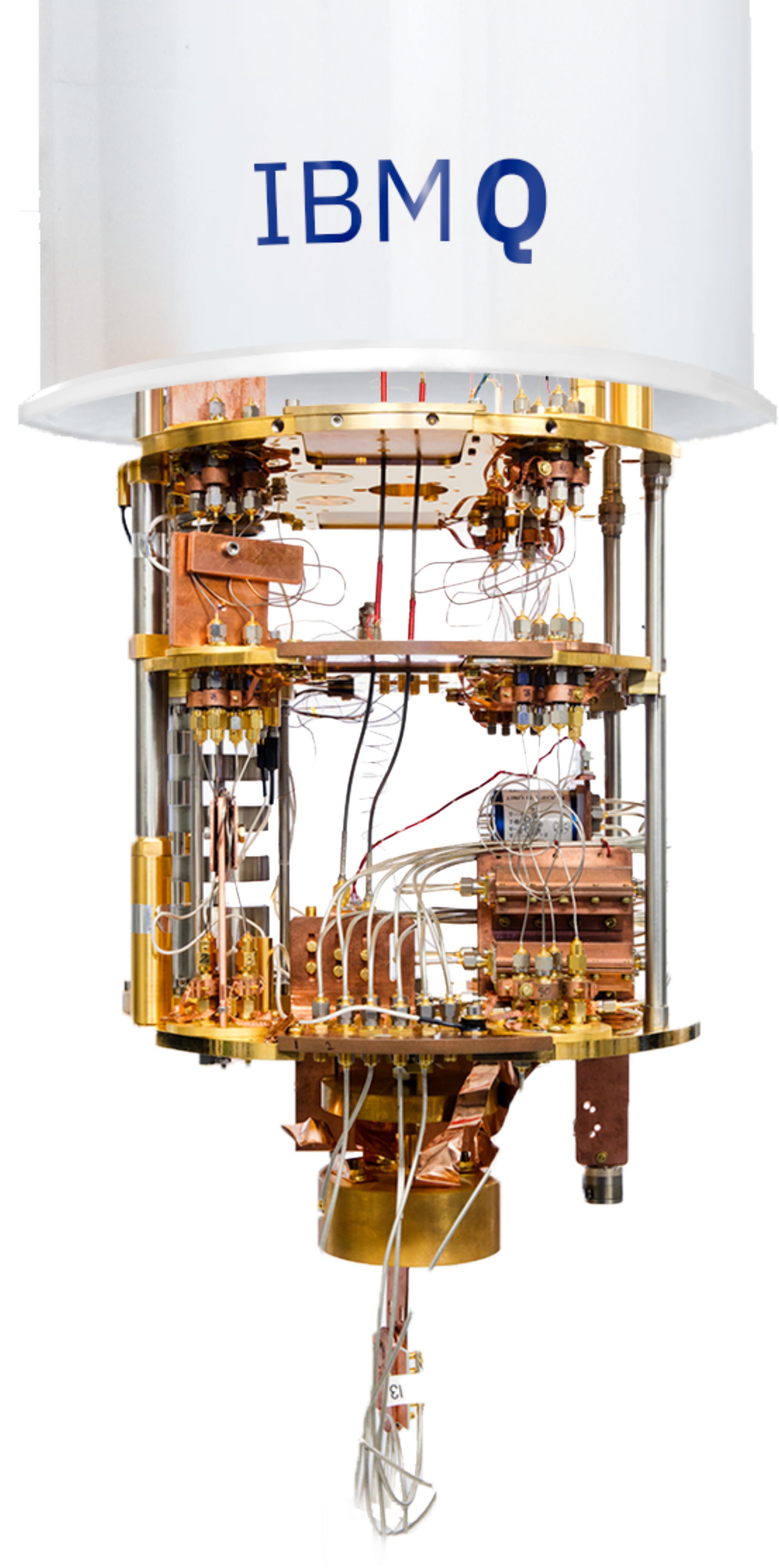
Summary

High Energy Physics is on the edge of a **computational frontier,** the High Luminosity Large Hadron Collider and FCC will provide **unprecedented amounts of data**

Quantum Computing offers an impressive and powerful tool to **combat computational bottlenecks**, both for theoretical and experimental purposes

The **first realistic simulation** of a **high energy collision** has been presented using a compact **quantum walk** implementation, allowing for the algorithm to be run on a **NISQ device**

We present an **efficient** approach to track finding using quantum computers by exploiting the **QAA** routine and employing a **novel oracle** paving the way for **practical quantum track finding**



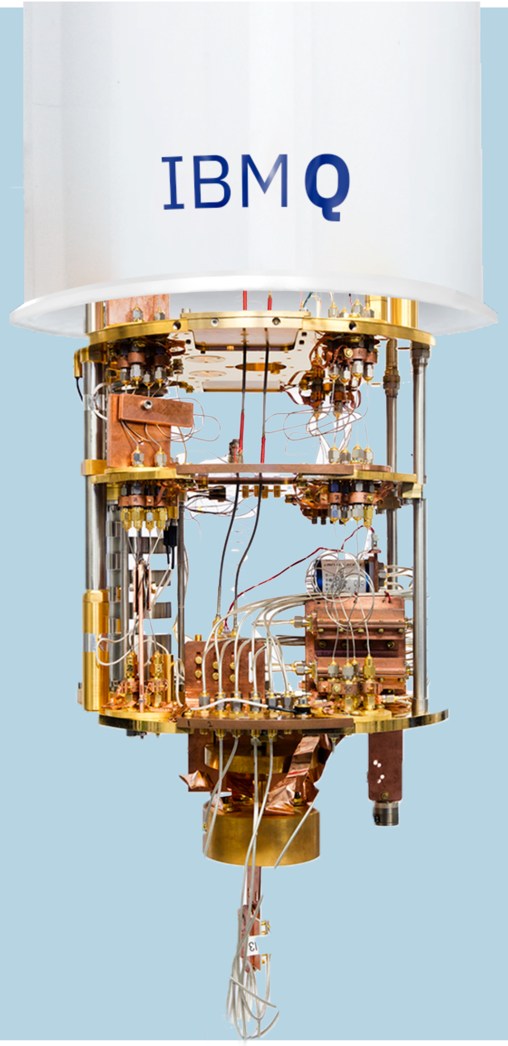
Backup Slides

Simon Williams

Future Colliders, Corfu Summer Institute,
24th May 2024

Noisy Intermediate-Scale Quantum Devices

NISQ devices:



No continuous quantum error correction, prone to large noise effects from environment.

Transpilation:

Loading the circuit onto the backend, transpilation can be used to optimise the circuit: **qubit and coupling mapping, noise models, etc.**

Quantum errors:

Mutliqubit qubit gates: CNOT gates have higher associated errors than single qubit gates.

SWAP errors: SWAP operations require 3 CNOT gates

T1 times: The time it takes for an excited qubit to decay back to the ground state.

Circuit depth! - Compact circuits needed!

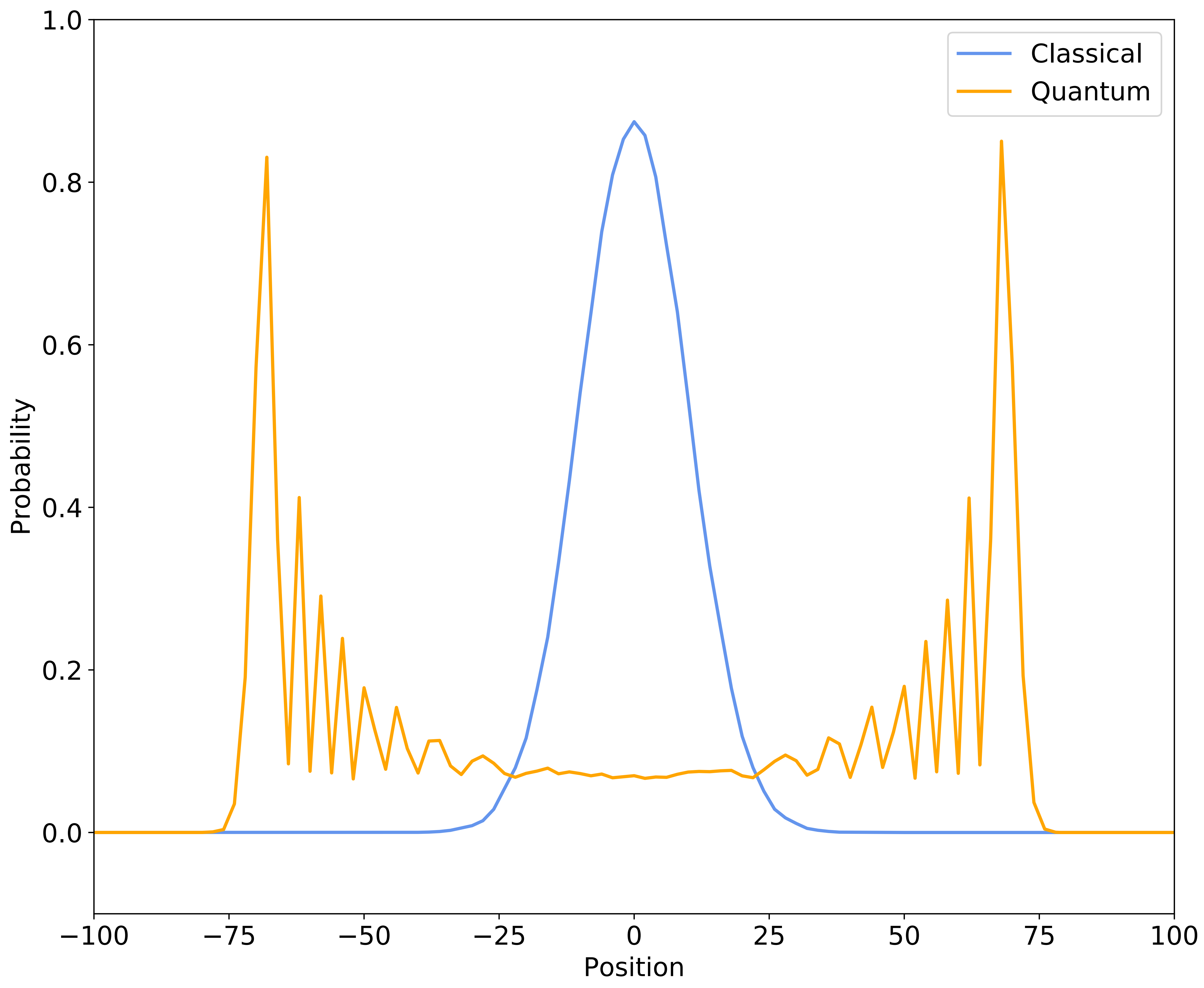
Speed up via Quantum Walks

Quantum Walks have long be conjectured to achieved at least **quadratic speed up**

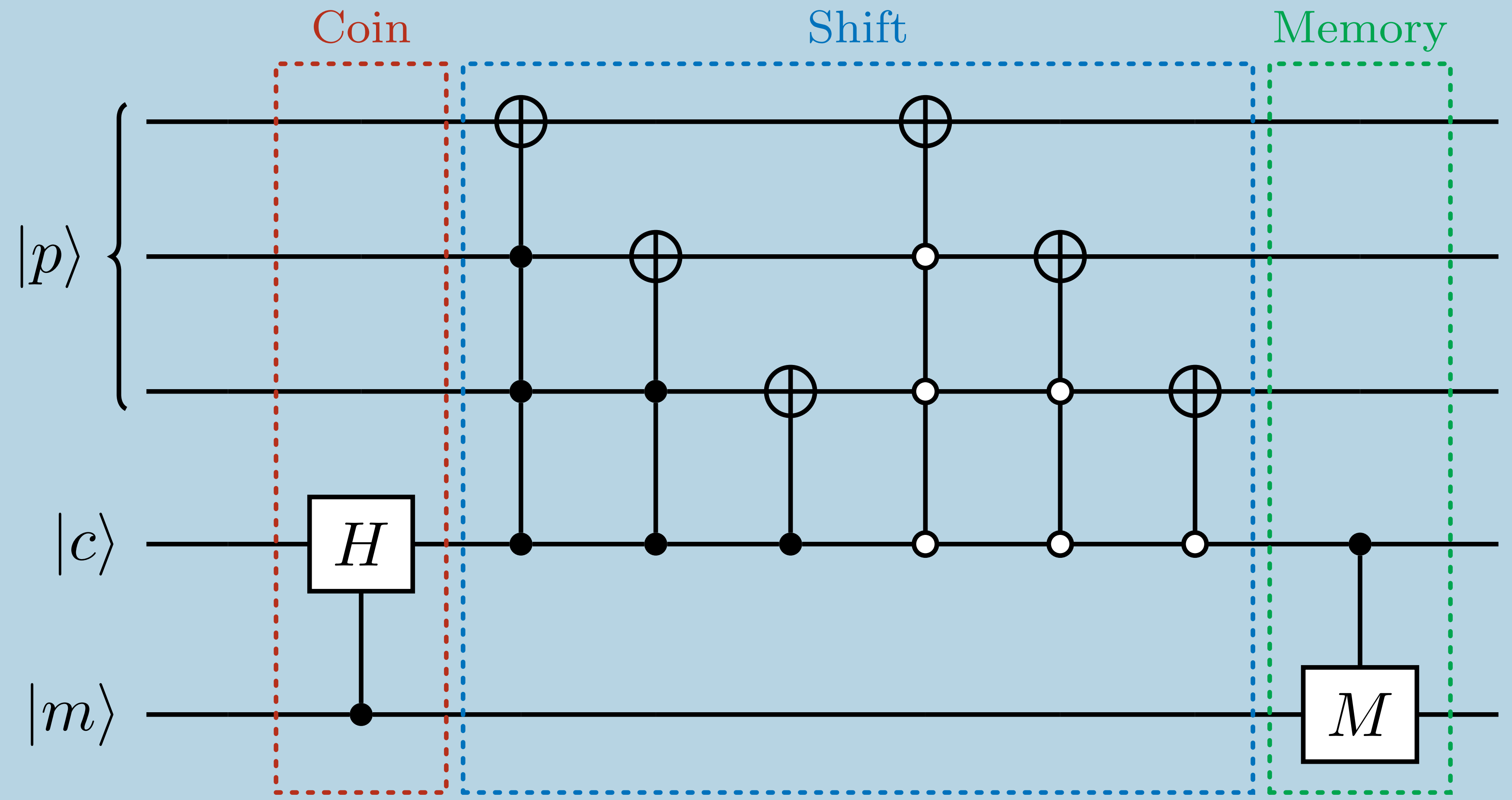
Szegedy Quantum Walks have been proven to achieve quadratic speed up for **Markov Chain Monte Carlo**

This has been proven under the condition that the MCMC algorithm is **reversible and ergodic**

Work is ongoing to prove this is true for all QWs, but latest upper limits are on par with classical RW



Quantum Walks with Memory



Qubit model:

Augment system further by adding an additional memory space

$$\mathscr{H} = \mathscr{H}_P \otimes \mathscr{H}_C \otimes \mathscr{H}_M$$

Advantages:

- Arbitrary dynamics
- Classical dynamics in unitary evolution

Disadvantages:

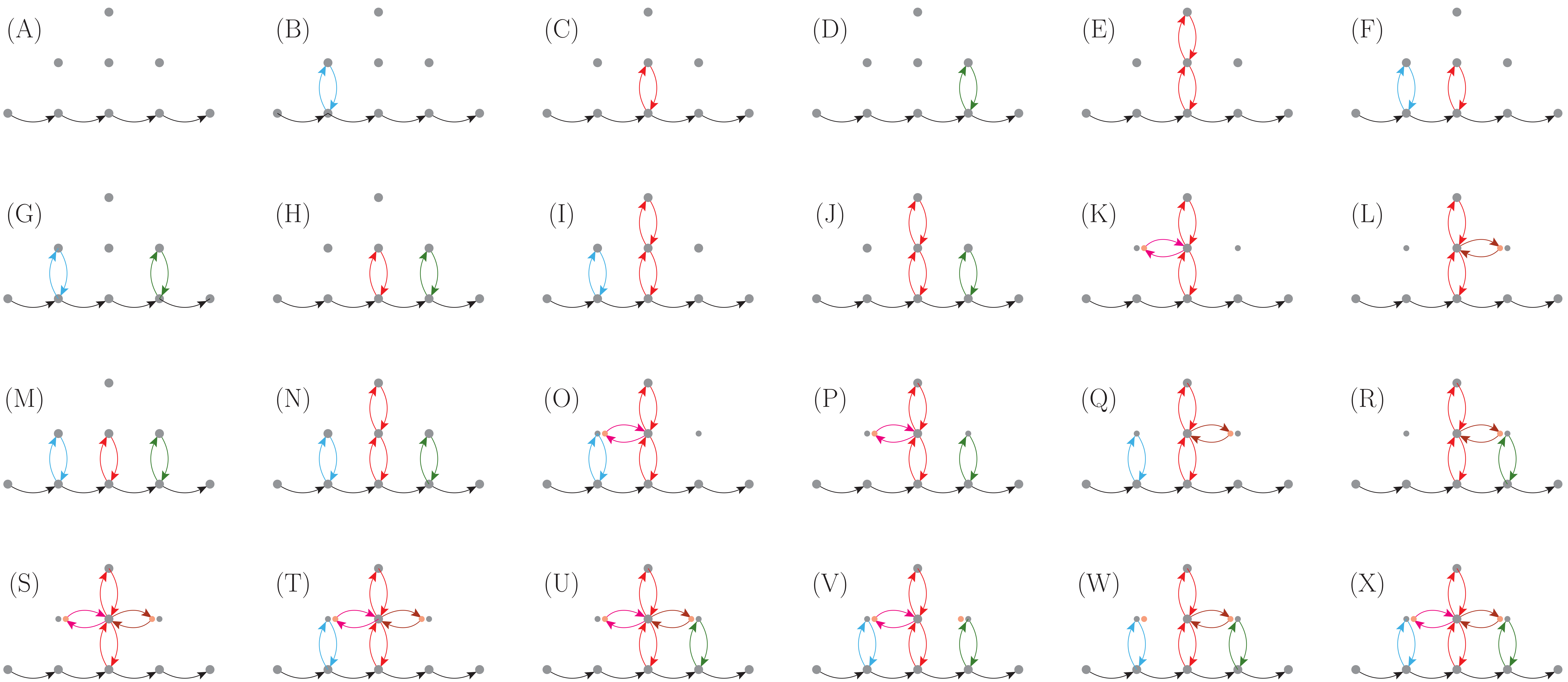
- Tight conditions on quantum advantage

Quantum Parton Showers:

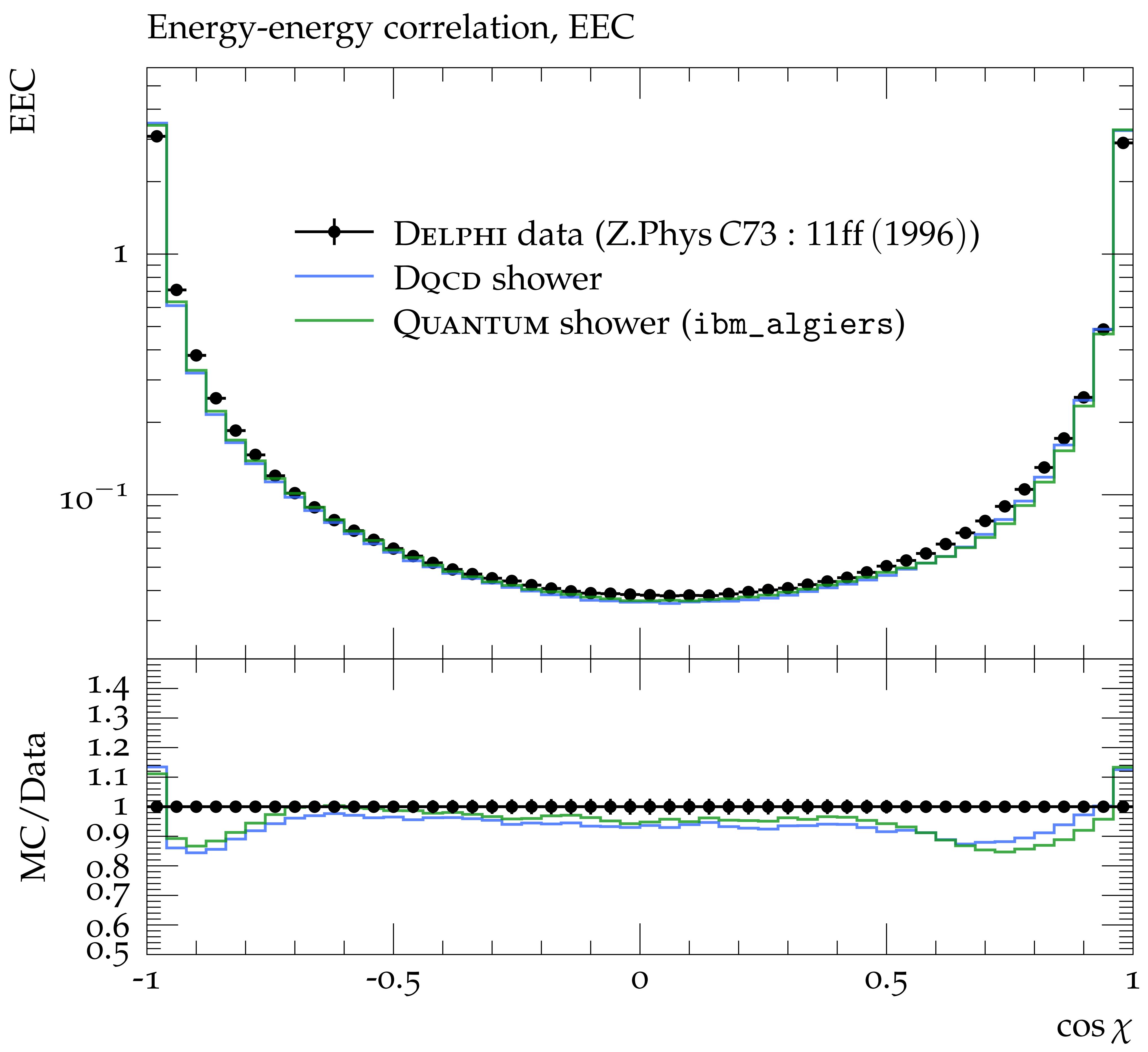
Quantum Walks with memory have proven to be very useful for quantum parton showers.

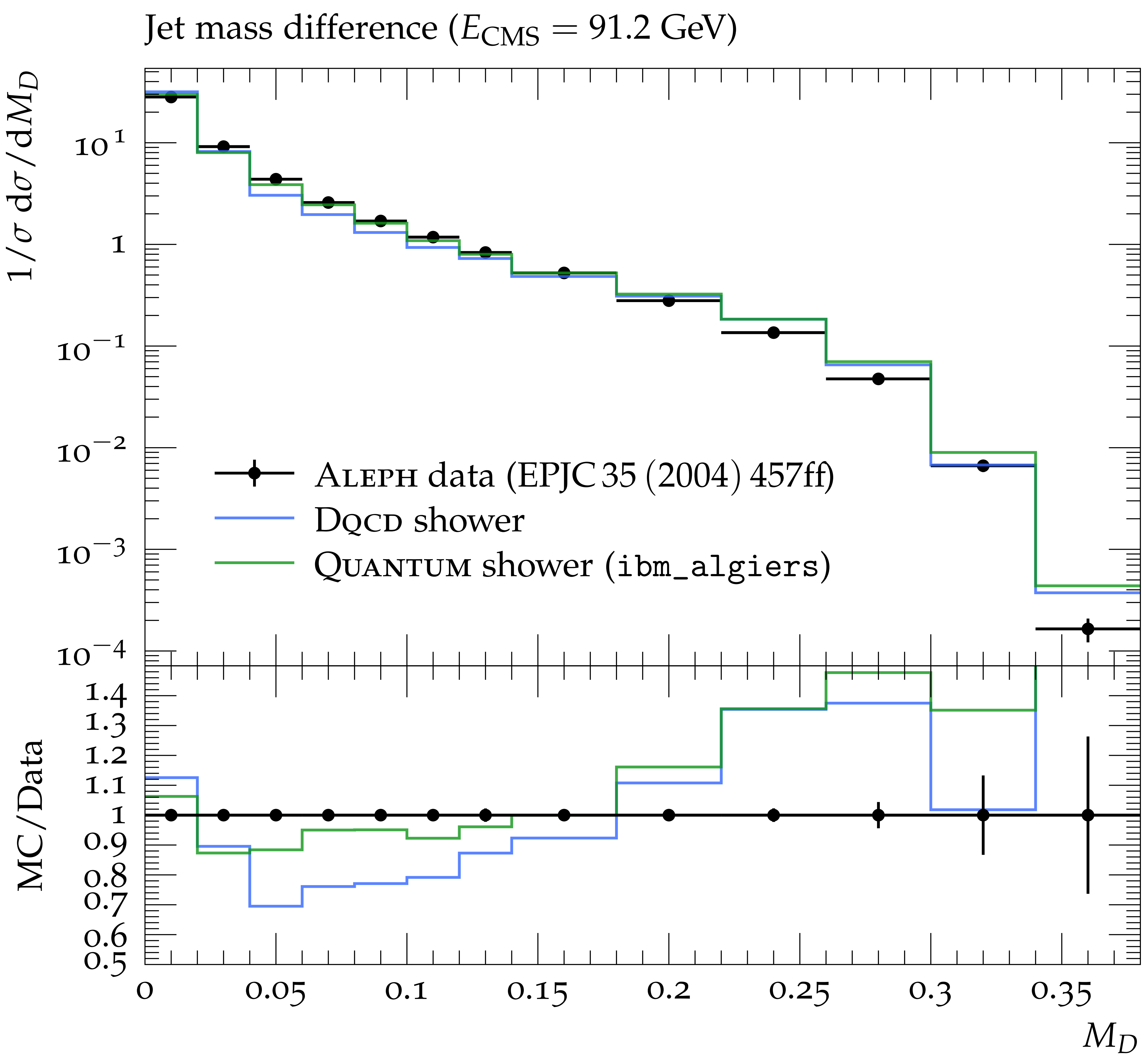
K. Bepari, S. Malik, M. Spannowsky and SW, Phys. Rev. D 106 (2022) 5, 056002

Discrete QCD - Grove Structures

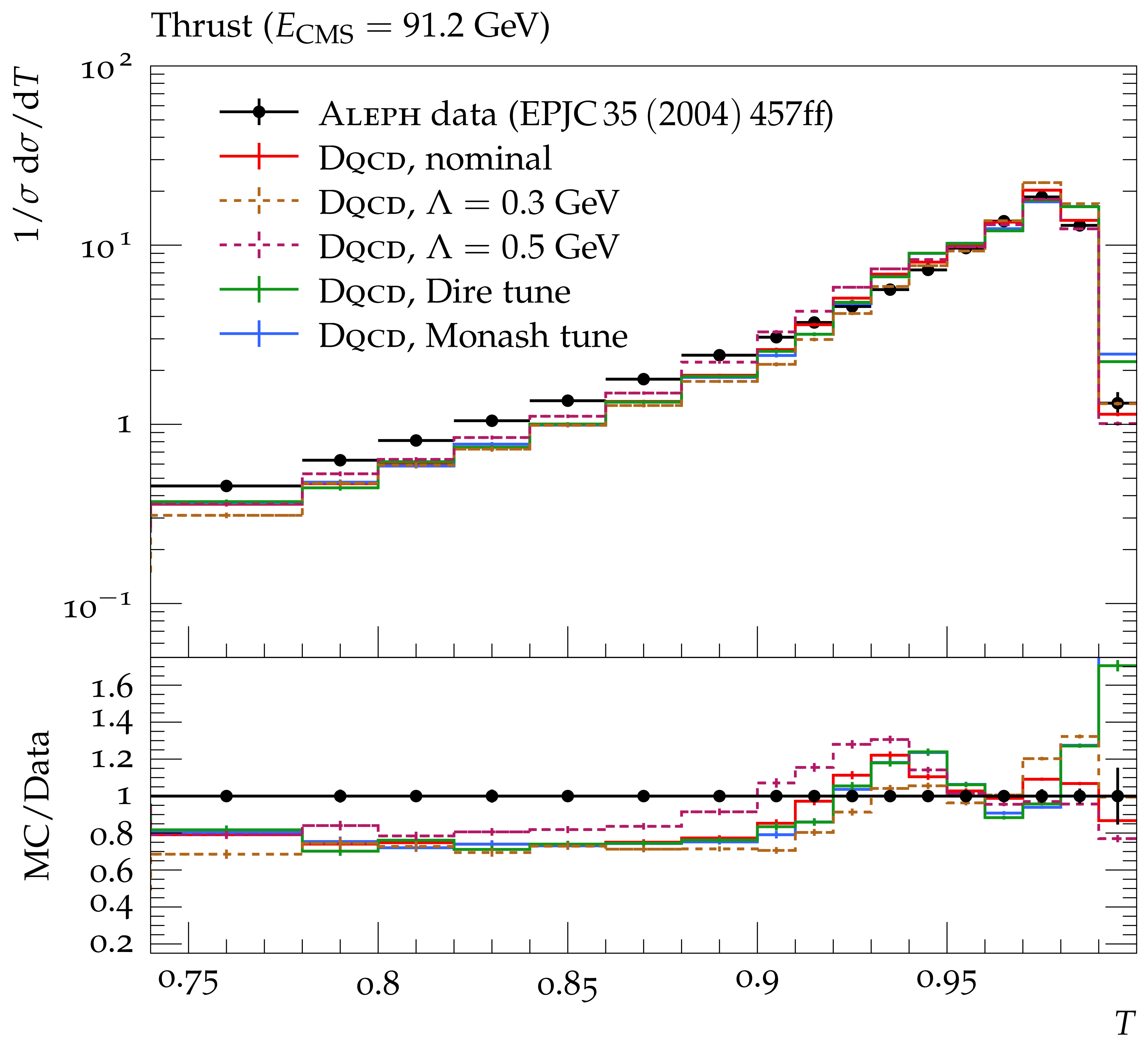


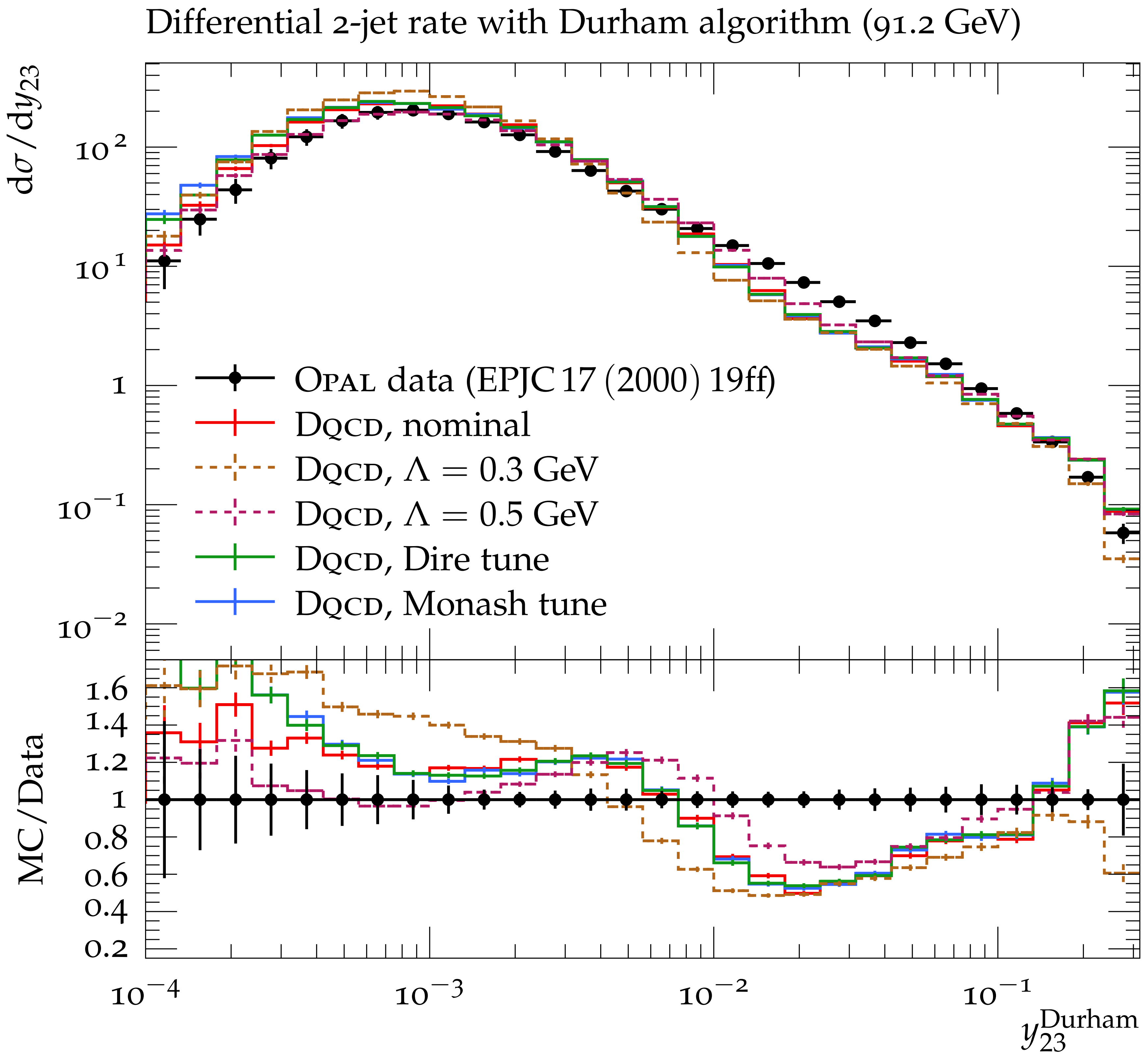
Collider Events on a Quantum Computer





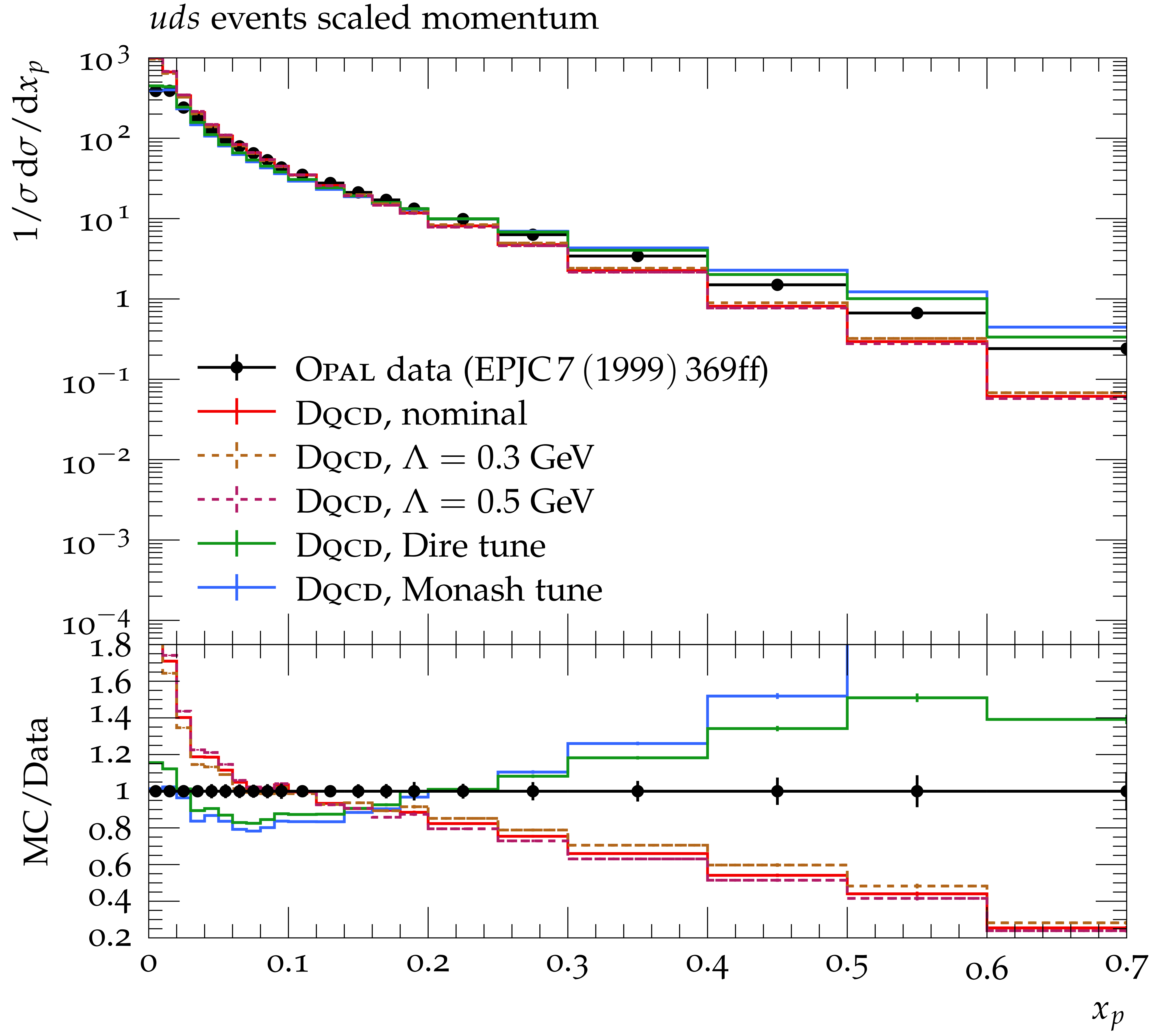
Collider Events on a Quantum Computer - Varying Λ

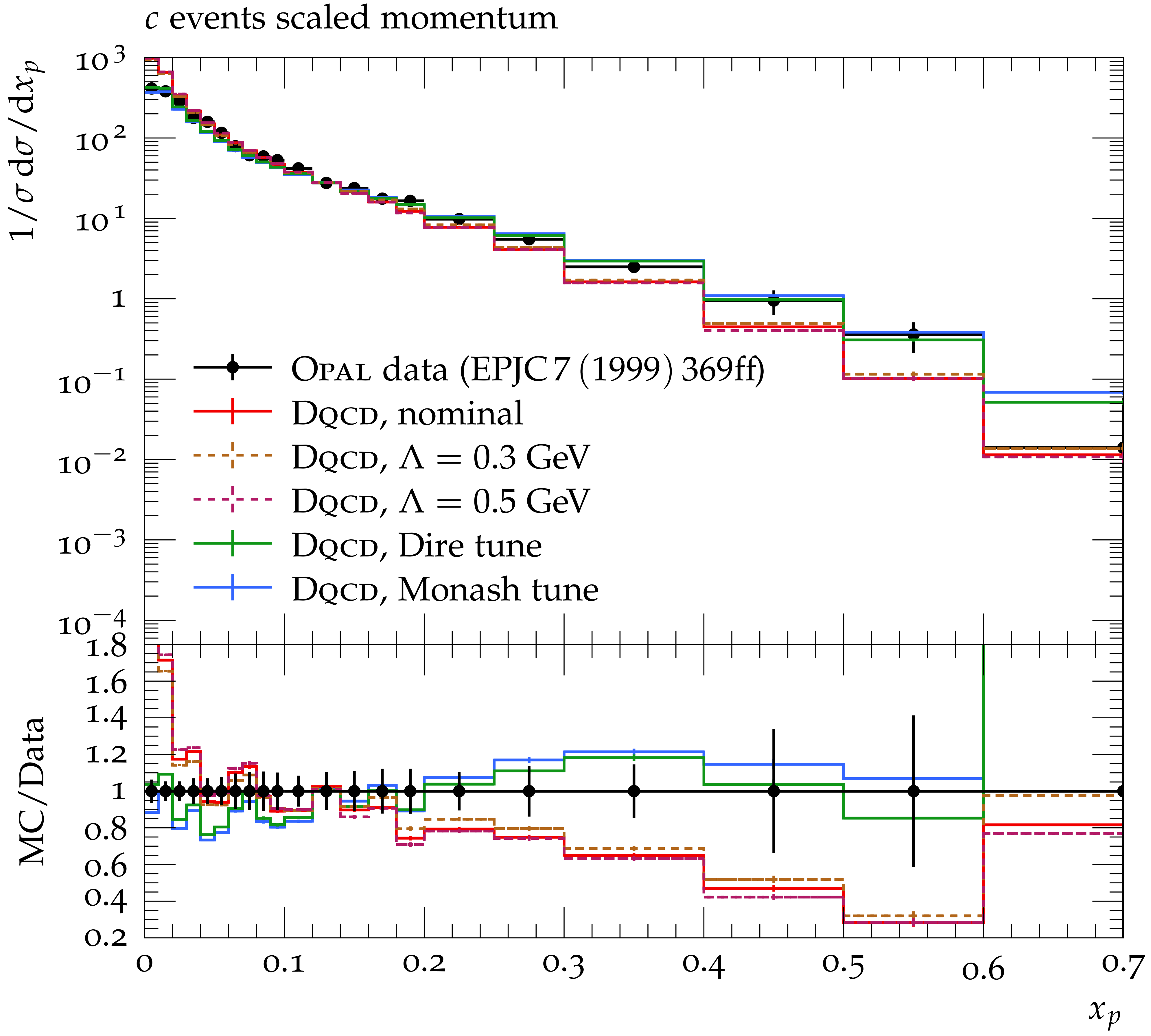




Varying values for the mass scale Λ. This leads to non-negligible uncertainties, however this is expected from a leading logarithm model.

Collider Events on a Quantum Computer - Varying Λ





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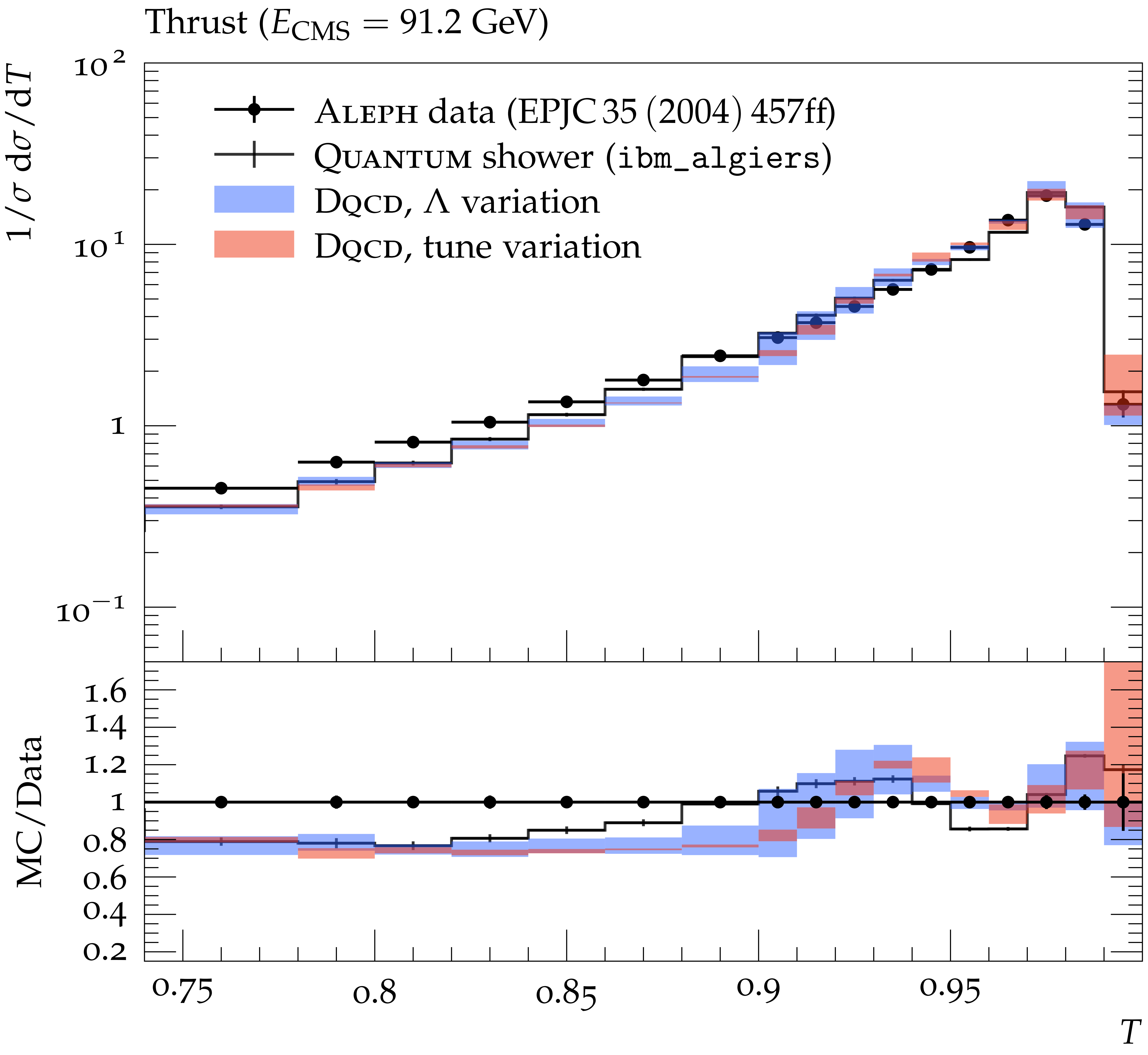
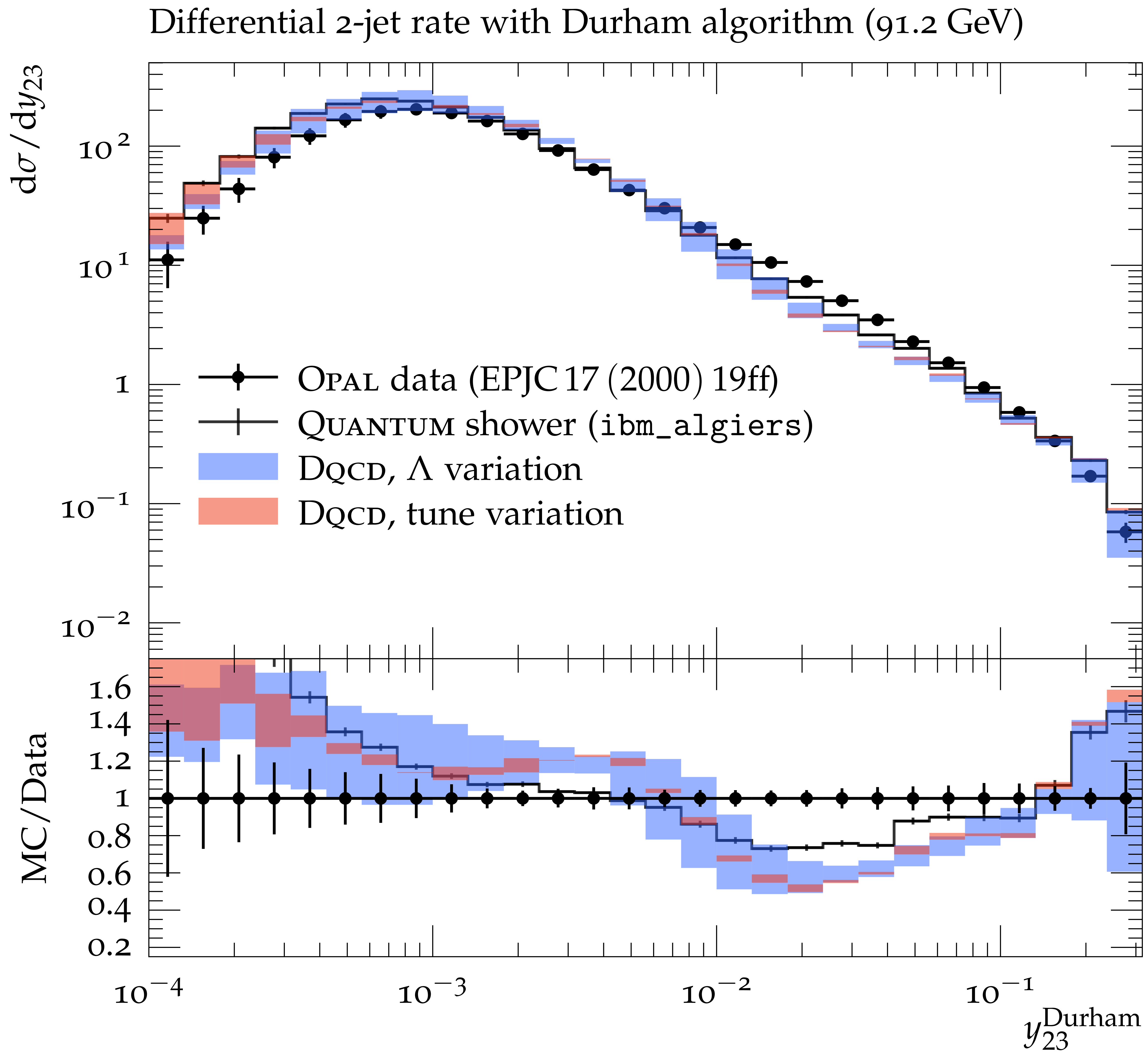
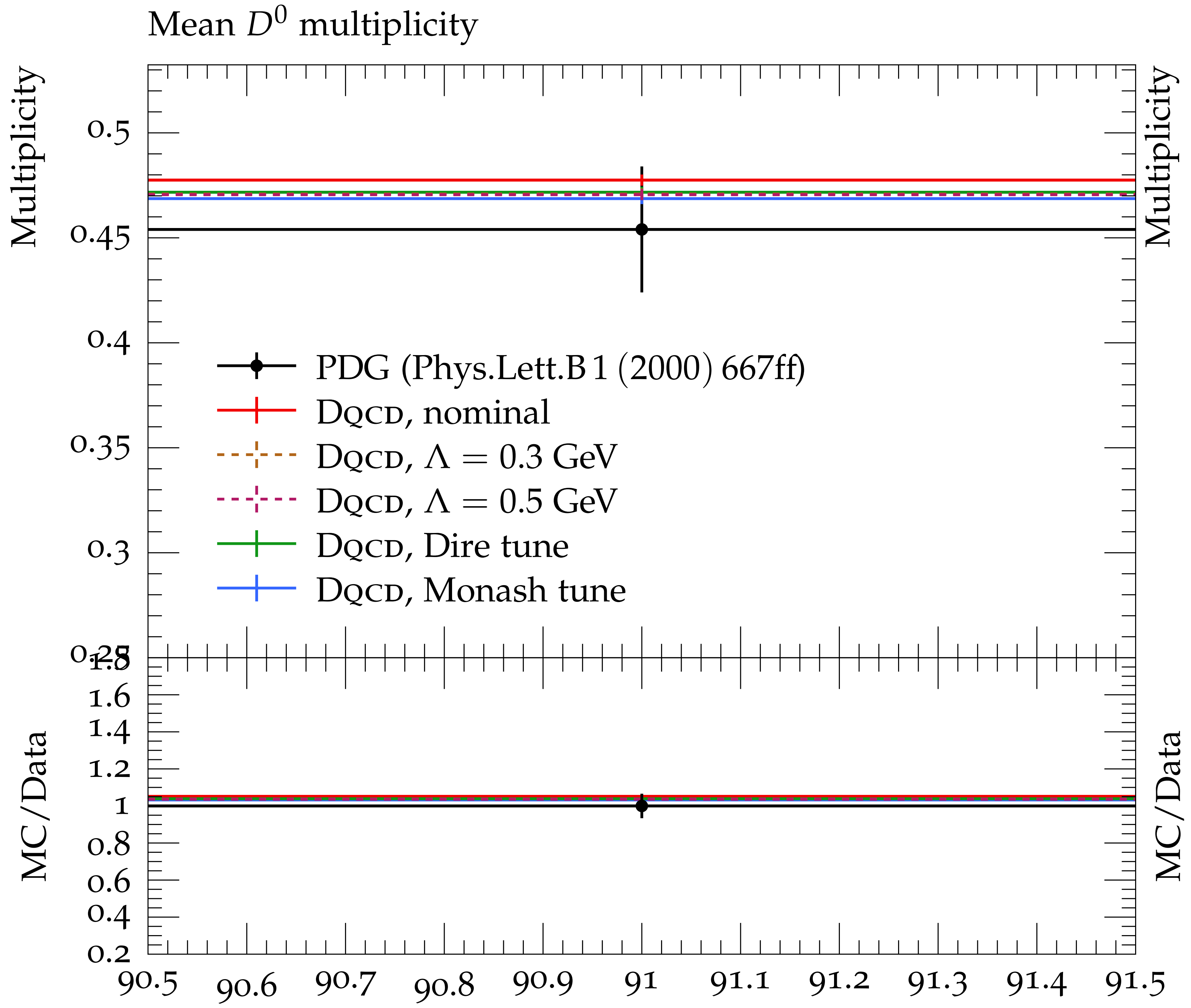
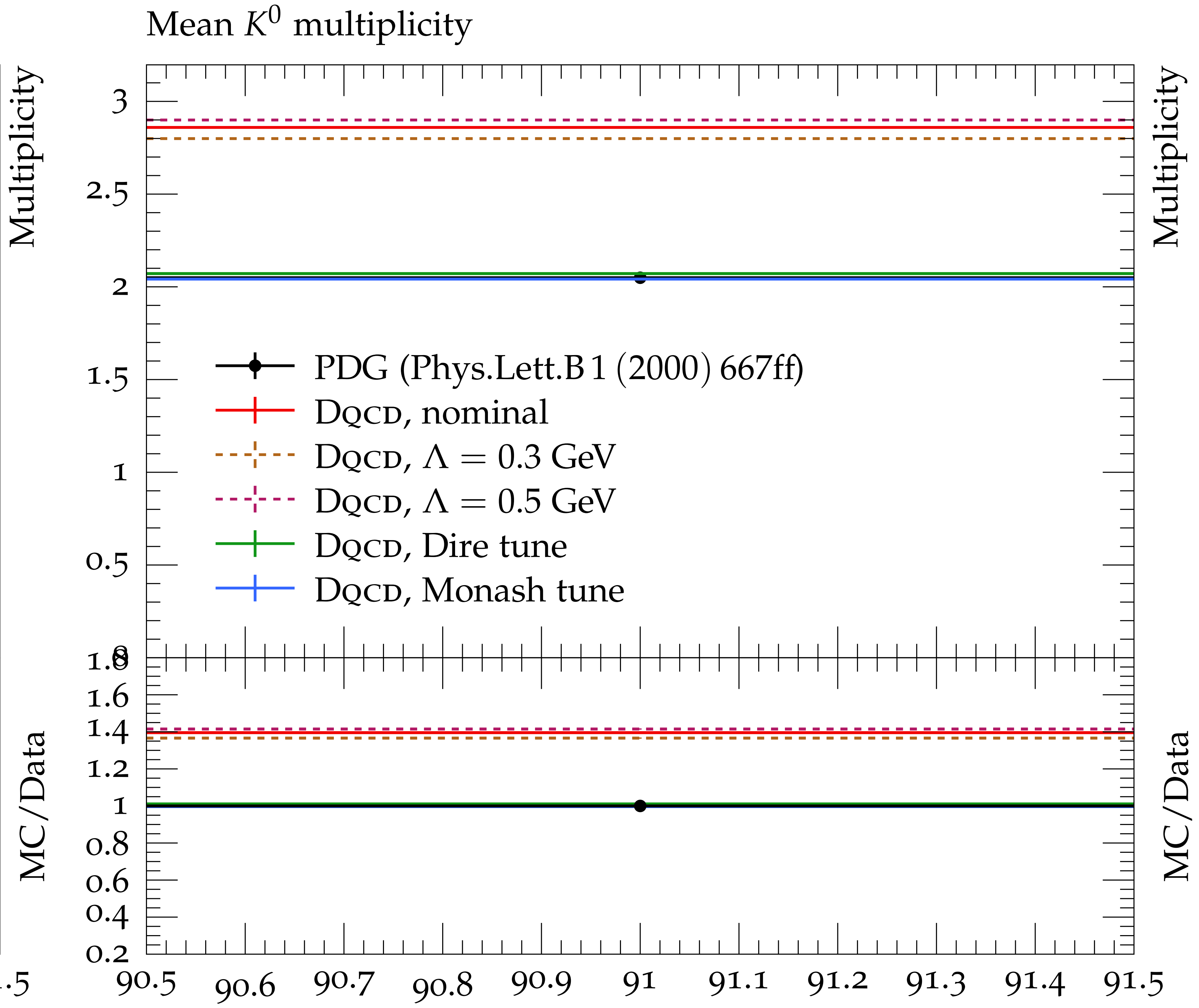
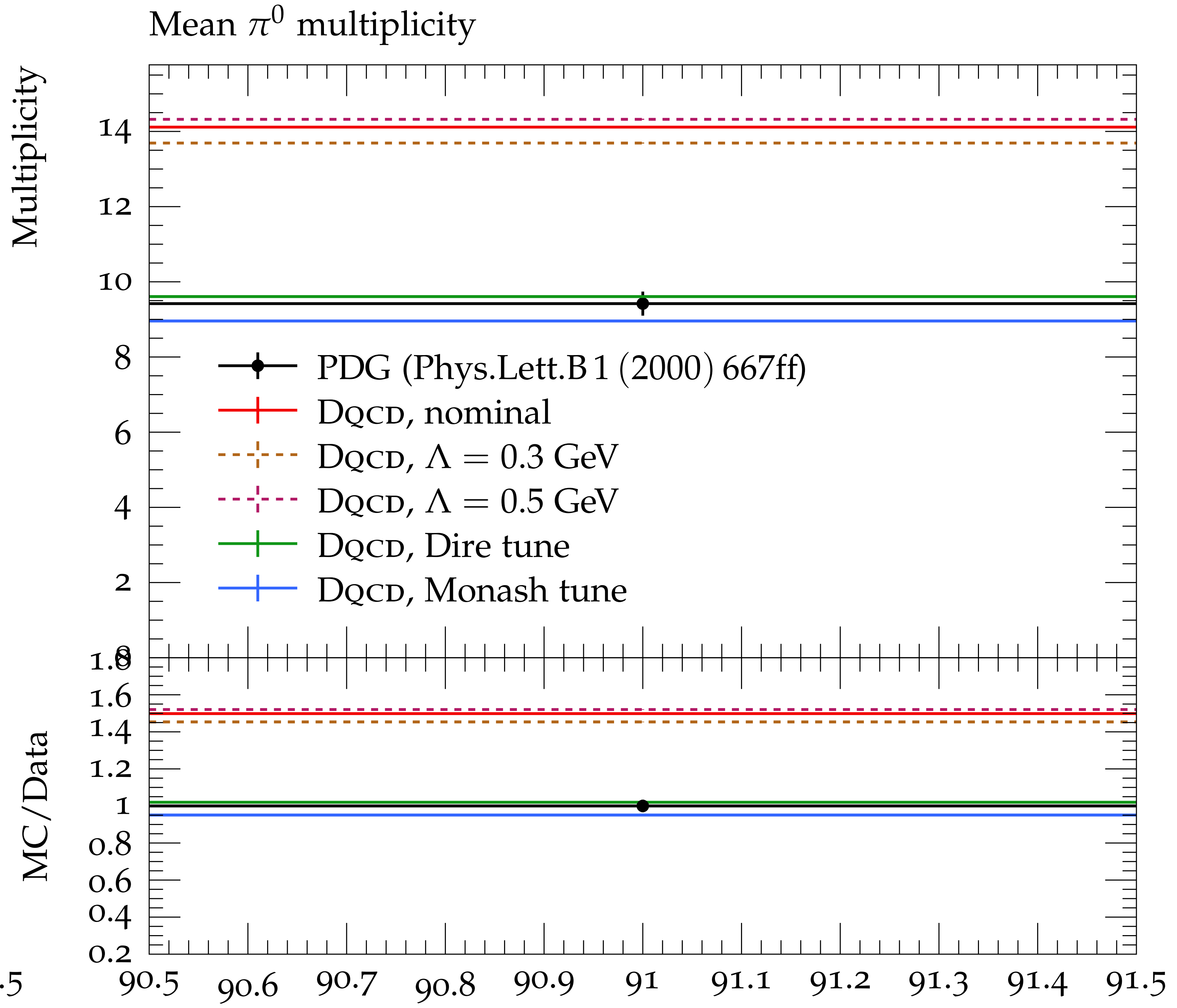




Collider Events on a Quantum Computer - Changing tune







Observables dominated by non-perturbative dynamics show mild dependence on the mass scale Λ, but are highly sensitive to changes in the tune.