Physics at a muon collider

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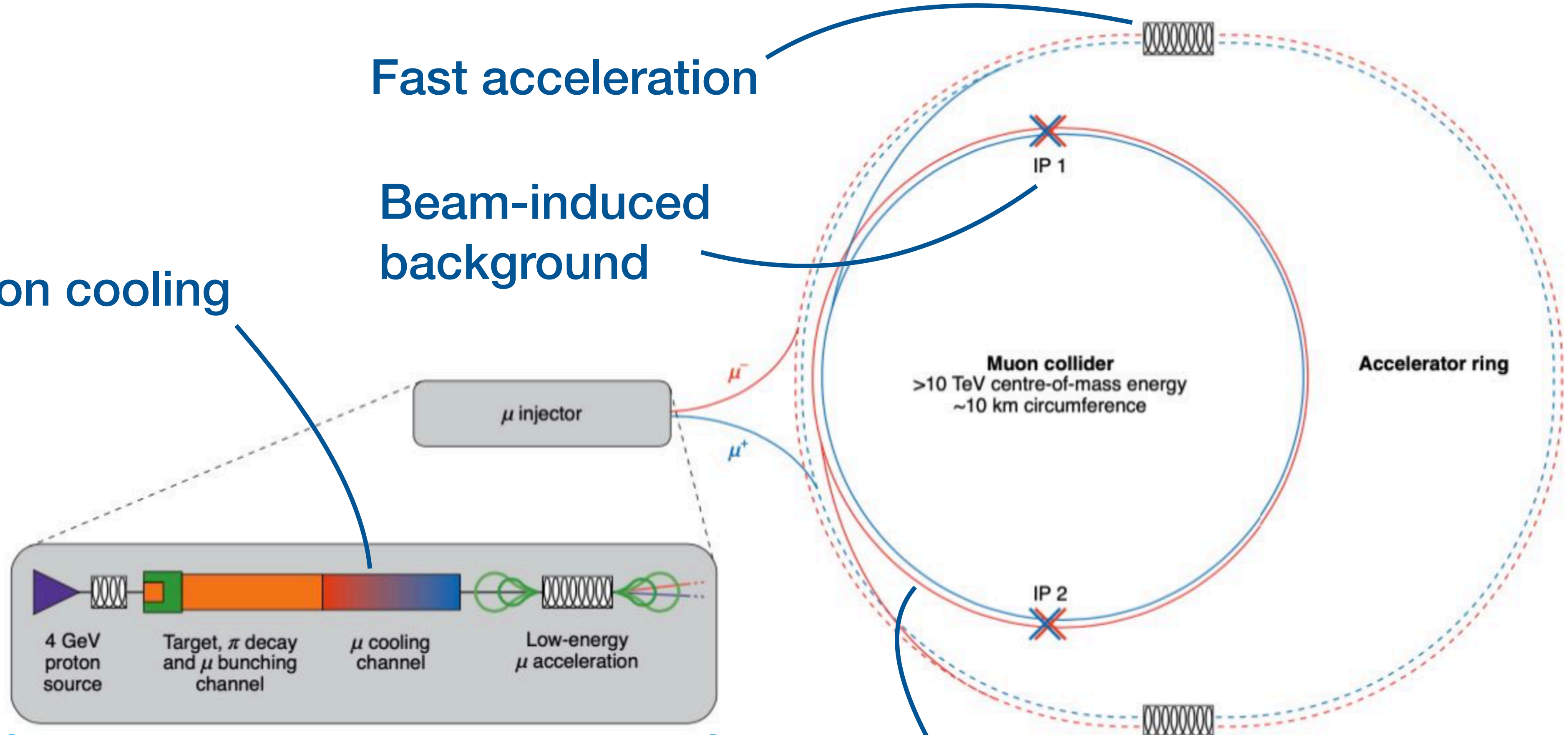
What is a muon collider?

- A muon collider is *not yet feasible* as of today!
- Several technical challenges that require major R&D effort



Fast acceleration

Beam-induced background

Muon cooling

Muon production

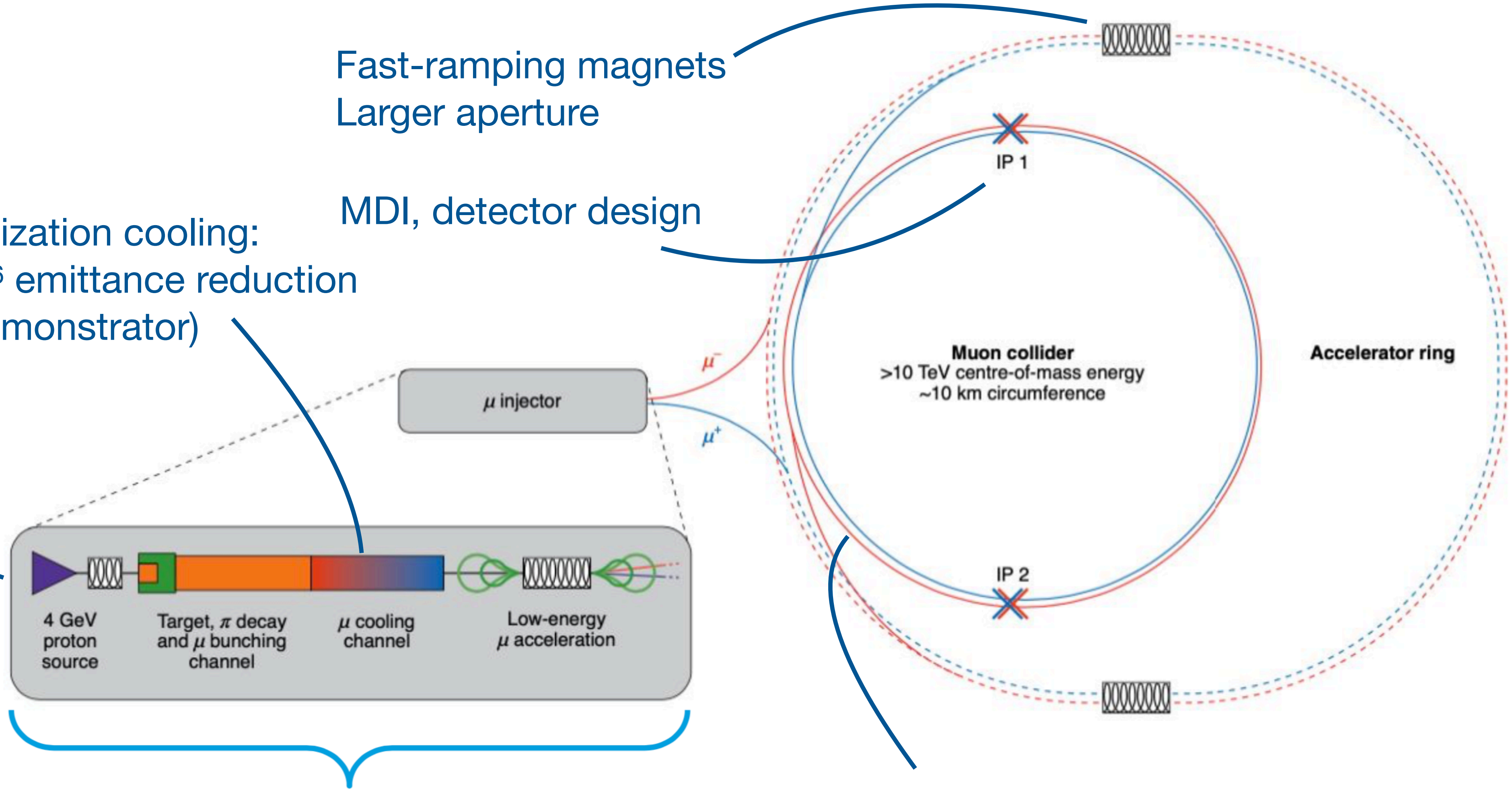
Neutrino flux

Proton-driven scheme

… it should not be compared with shovel-ready projects (like e+e- Higgs/EW factory)

What is a muon collider?

- A muon collider is *not science-fiction* either!
- Several technical challenges that require major R&D effort



Fast-ramping magnets
Larger aperture

MDI, detector design

Ionization cooling:
10^6 emittance reduction
(demonstrator)

Few-MW target
High-field solenoid

☛ talk by Nadia

Proton-driven scheme

Accelerator design,
mitigation systems

High energy lepton collider (10 TeV or more) is a dream for particle physics…

… dedicated R&D program crucial to establish feasibility in the next years!

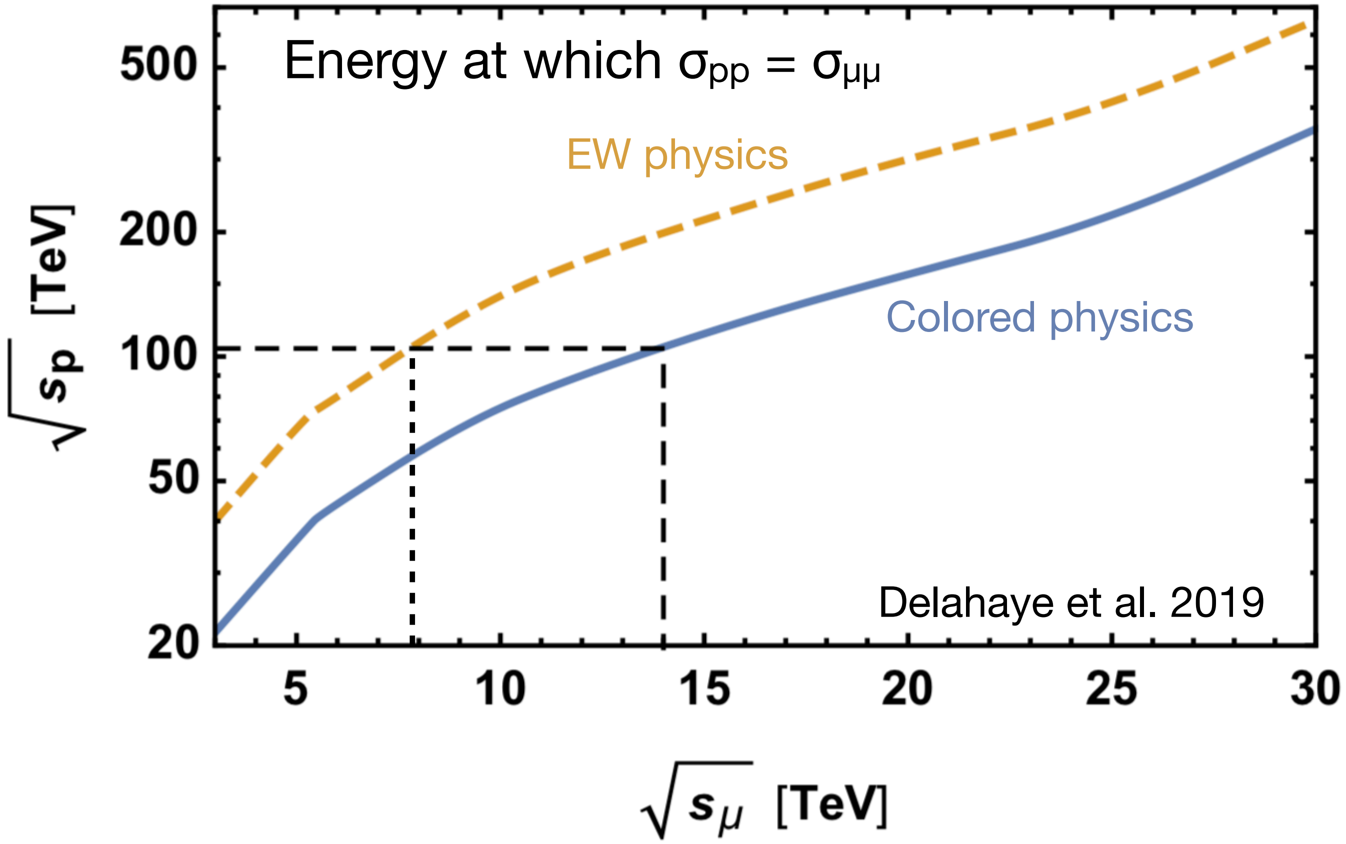
Why a muon collider?

- Lepton colliders are ideal probes of short-distance physics
  - elementary: no energy lost in PDFs,

    all beam energy is available for hard scattering

Colored particles:
14 TeV μμ ~ 100 TeV pp

EW particles:
14 TeV μμ ~ 200 TeV pp



Delahaye et al. 2019

- no strong interactions:

  no QCD background, high S/B

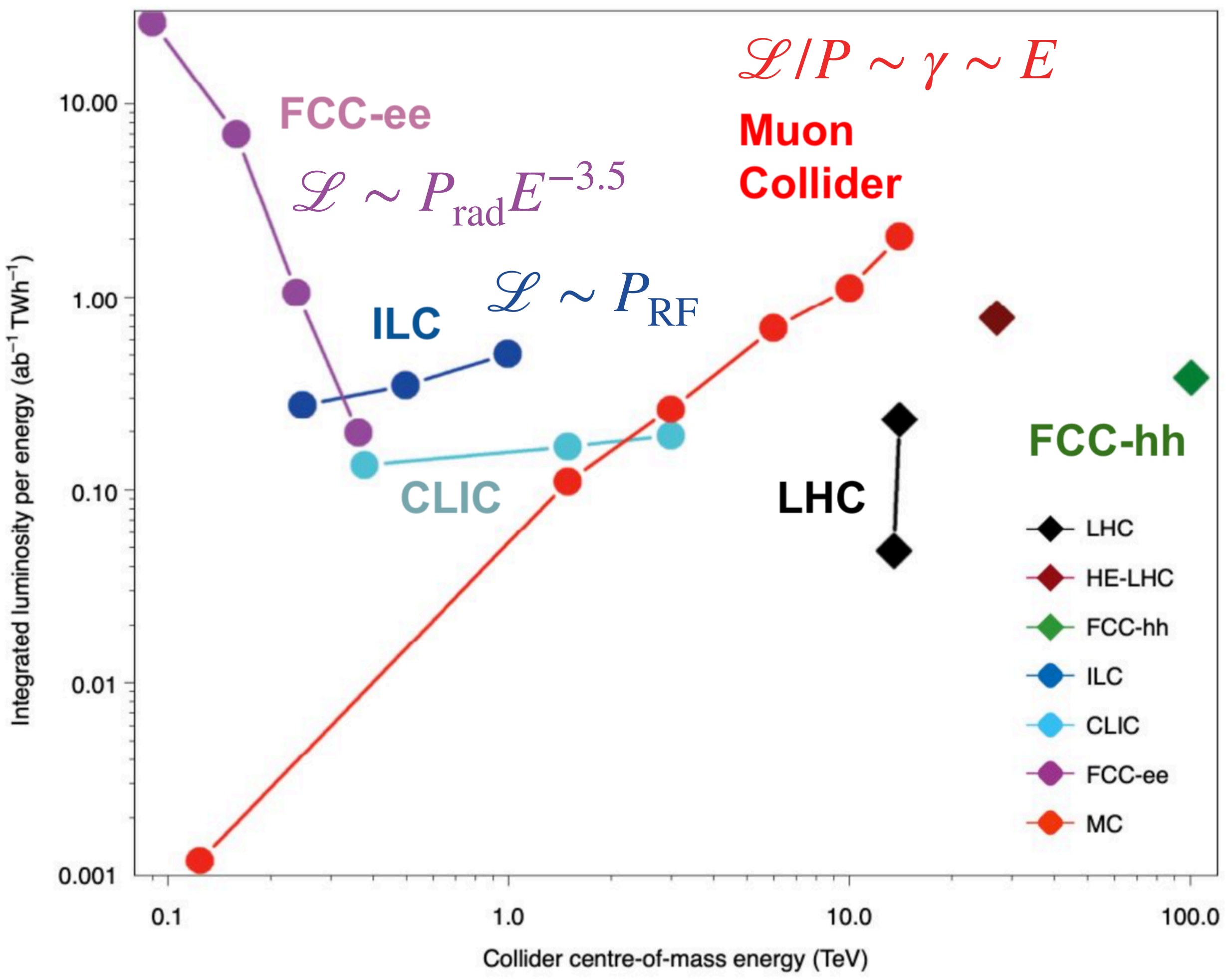
  collision rate 1000x smaller than LHC, but can produce 10^7-10^8 Higgs bosons

Why a muon collider?

- ✦ Lepton colliders are ideal probes of short-distance physics
- ✦ Muons are elementary and heavy (207 x electrons)
  - ▸ negligible energy loss in synchrotron radiation
  - ▸ negligible beamstrahlung

But they decay…

- ✦ Luminosity increases with the square of beam energy
  - ▸ muon lifetime increases
  - ▸ transverse emittance decreases



Why a muon collider?

- A muon collider has high energy AND precision

Direct searches	**High-rate SM measurements**	**High-energy SM measurements**
high energy to search for heavy new particles	high statistics for precise measurements	high energy to look for NP in SM processes

Goal: explore physics at least up to $M_{\rm NP} \approx 10\,{\rm TeV}$

☛ talk by Patrick

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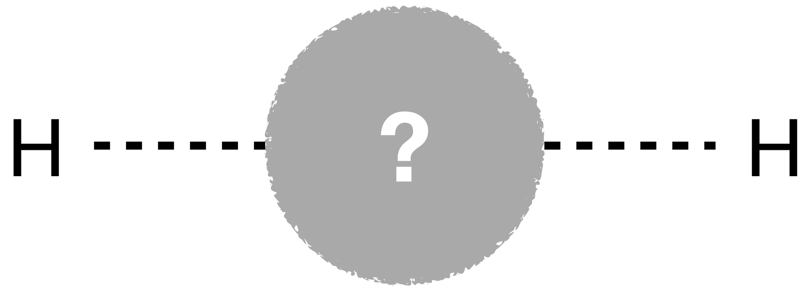
- What causes EWSB? i.e. does the SM hold up to few TeV?



→ $$M_{\mathrm{NP}} \lesssim 4\pi v \approx 3\,\mathrm{TeV}$$

rough estimate! there can easily be some O(1) factor

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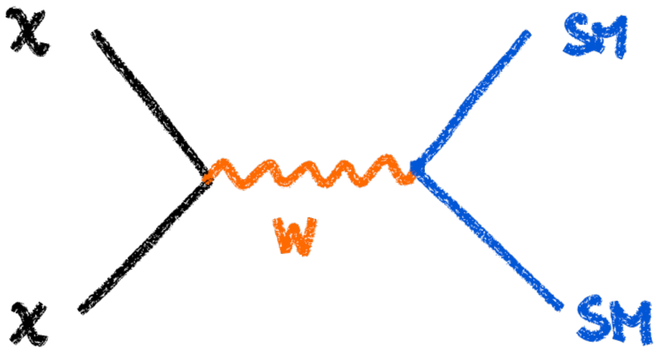
- What causes EWSB?

- What is dark matter? Is it a WIMP?



→ $M_{\mathrm{DM}} \approx 1-15\,\mathrm{TeV}$

Why a muon collider?

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Direct searches

high energy to search for heavy new particles

High-rate SM measurements

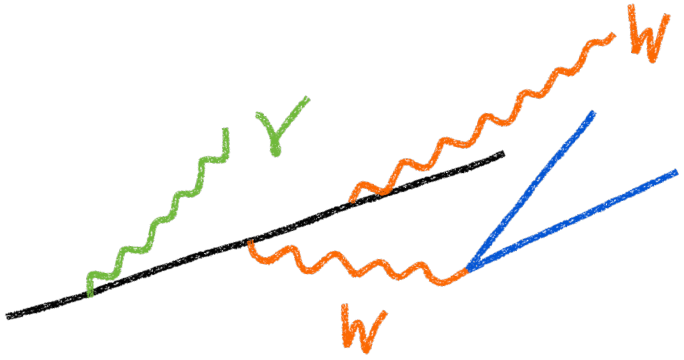
high statistics for precise measurements

High-energy SM measurements

high energy to look for NP in SM processes

Goal: explore physics at least up to $M_{\mathrm{NP}} \approx 10\,\mathrm{TeV}$

- What causes EWSB?
- What is dark matter? Is it a WIMP?
- Observe restoration of EW symmetry (EW radiation)



→ $E \approx 10\,\mathrm{TeV}$

Why a muon collider?

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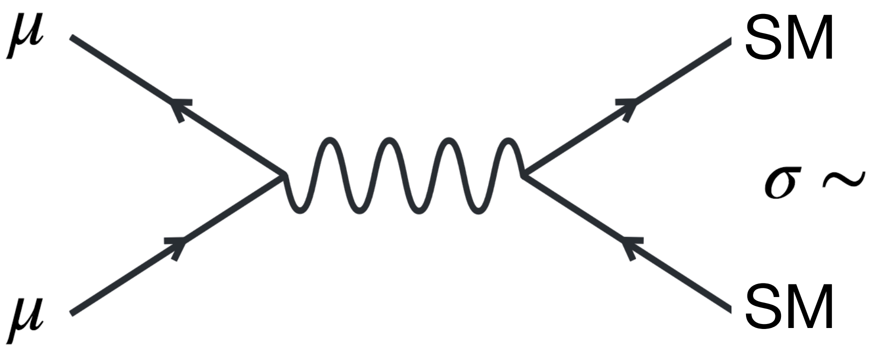
Direct searches	**High-rate SM measurements**	**High-energy SM measurements**
high energy to search for heavy new particles	high statistics for precise measurements	high energy to look for NP in SM processes



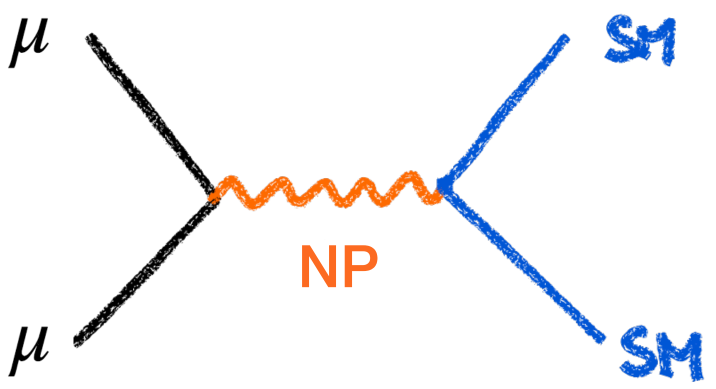
$$\sigma \sim \frac{1}{s} = \frac{1}{E^2}$$

Luminosity goal: $\mathscr{L} \gtrsim 10\,\mathrm{ab}^{-1} \times \left(\frac{E}{10\,\mathrm{TeV}}\right)^2$

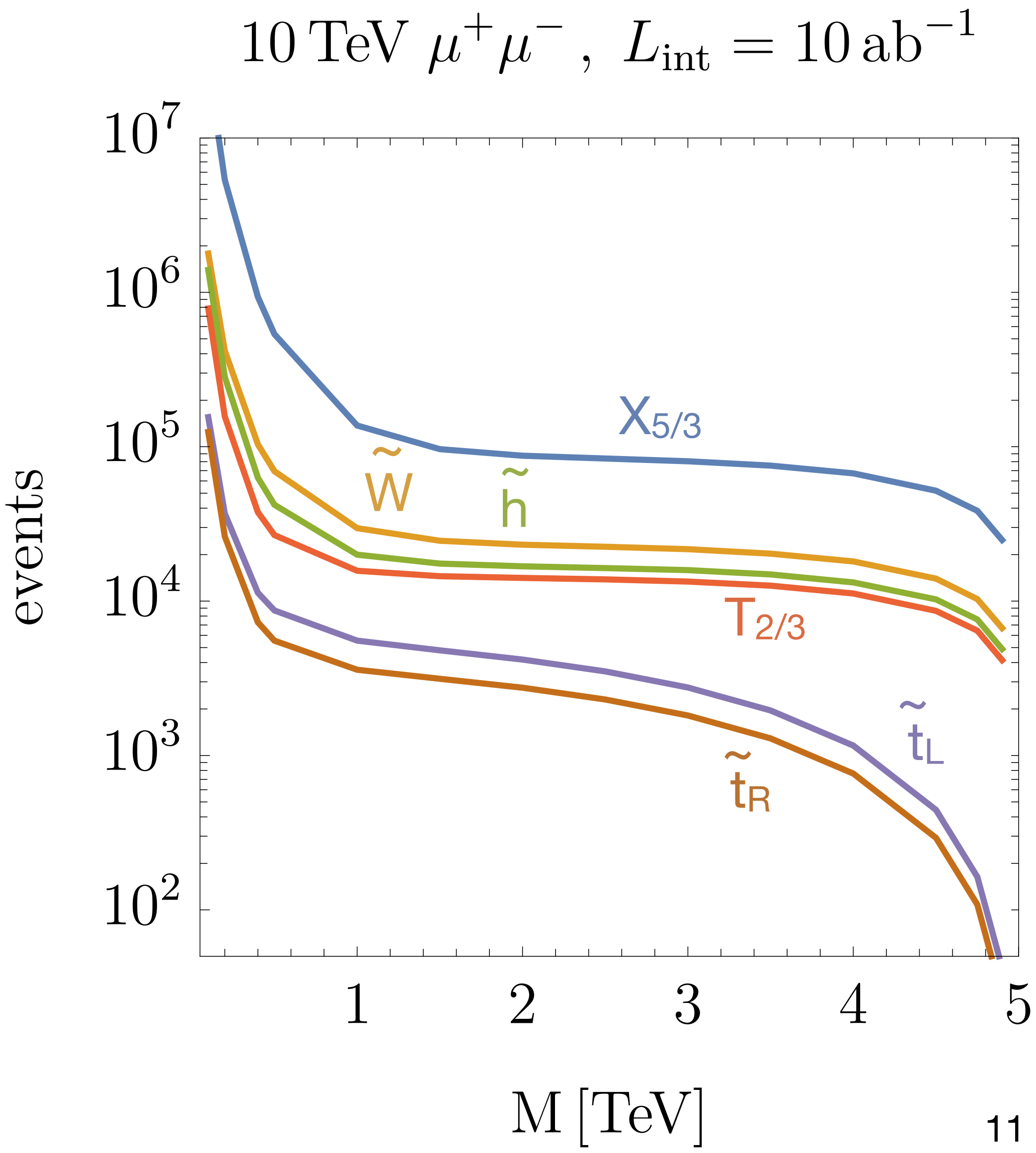
necessary to perform SM measurements with ~ % precision (10k events)

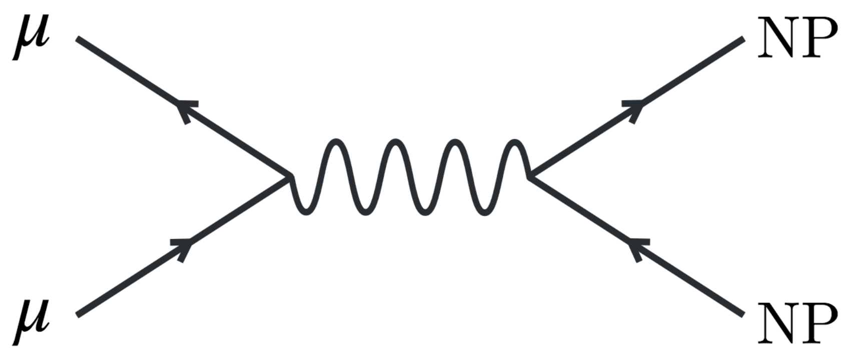
Direct searches

- Main motivation for a muon collider: ability to collide elementary particles at very high energies $\Longrightarrow$ **directly explore physics at 10+ TeV**



- Produce pairs of EW particles *up to kinematical threshold:* no loss of energy due to parton distribution functions!





Example: WIMP Dark Matter

- Weakly Interacting Massive Particle: most general EW multiplet with DM candidate that is
  - (a) stable,
  - (b) without coupling to γ & Z,
  - (c) calculable (perturbative).

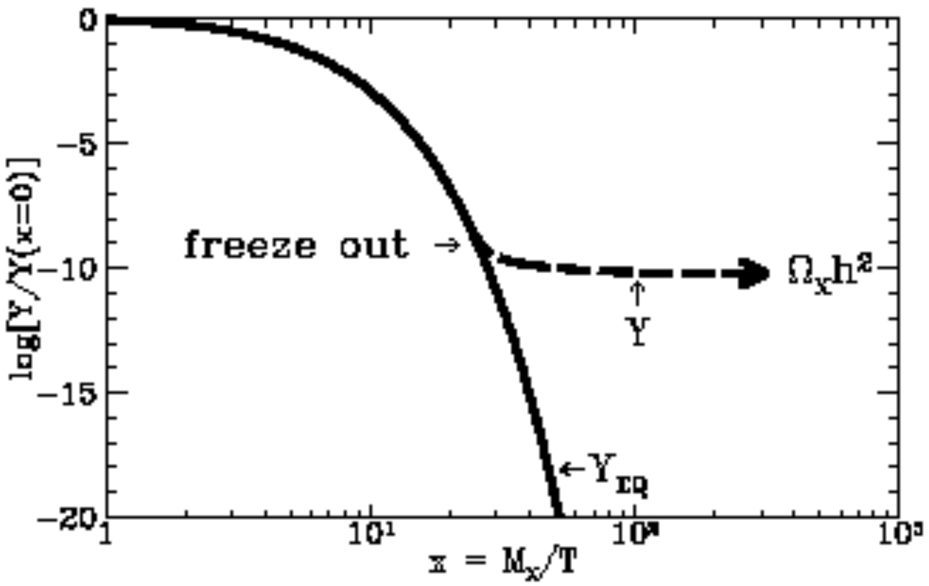
similar to Minimal DM:

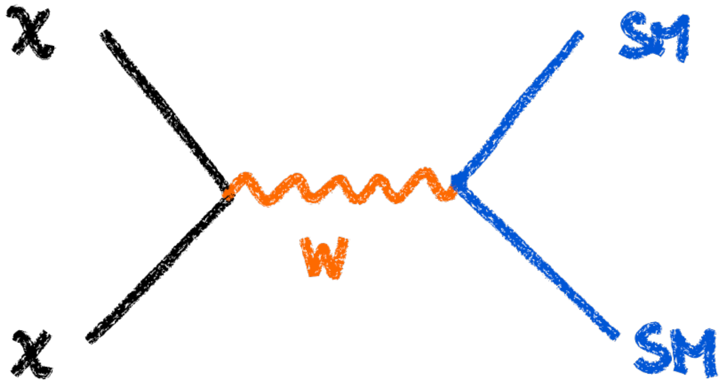
Cirelli, Fornengo, Strumia hep-ph/0512090

$$\chi_n = (\cdots, \chi^-, \boxed{\chi^0}, \chi^+, \cdots)$$

- Mass fixed by freeze-out DM abundance

Bottaro, DB, Costa, Franceschini, Panci, Redigolo, Vittorio 2107.09688, 2205.04486





EW n-plet	Mass [TeV]
$2_{1/2}$	1.08
3_0	2.86
$4_{1/2}$	4.8
5_0	13.6
5_1	9.9
$6_{1/2}$	31.8
7_0	48.8
9_0	113

☛ talks by Raki and Paolo

Energies of several TeV crucial to probe these WIMP candidates!

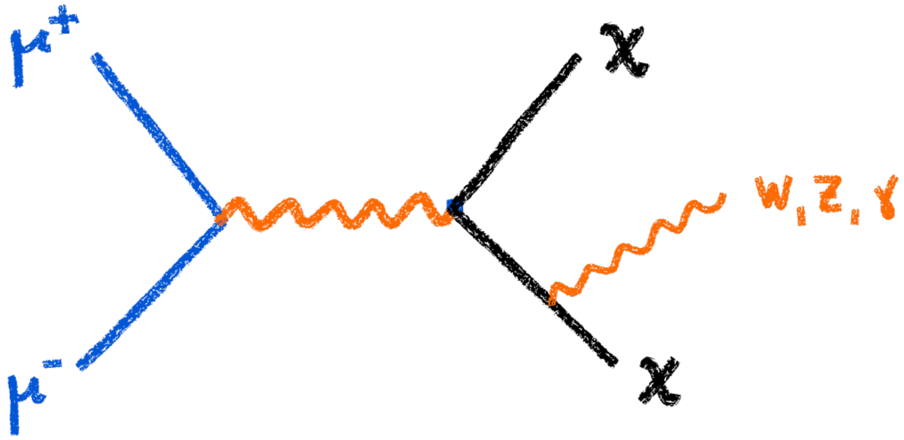
Example: WIMP Dark Matter

- Mono-γ/W/Z signals: $\mu\bar{\mu} \to \chi\bar{\chi} + X$

  DM pair production + EW radiation

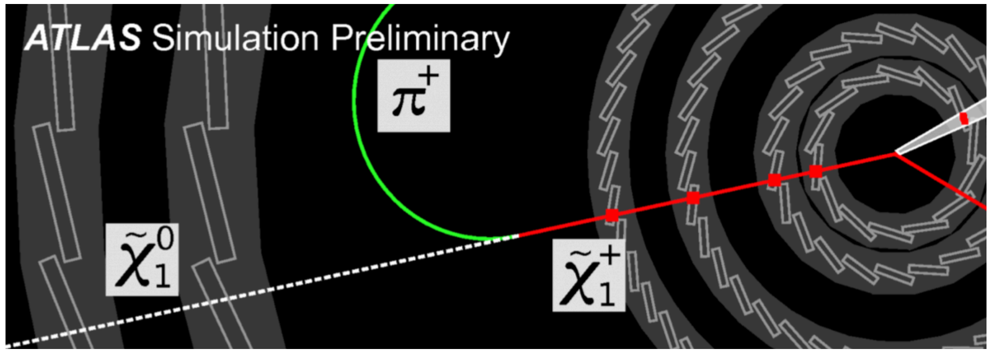
  Han et al. 2009.11287

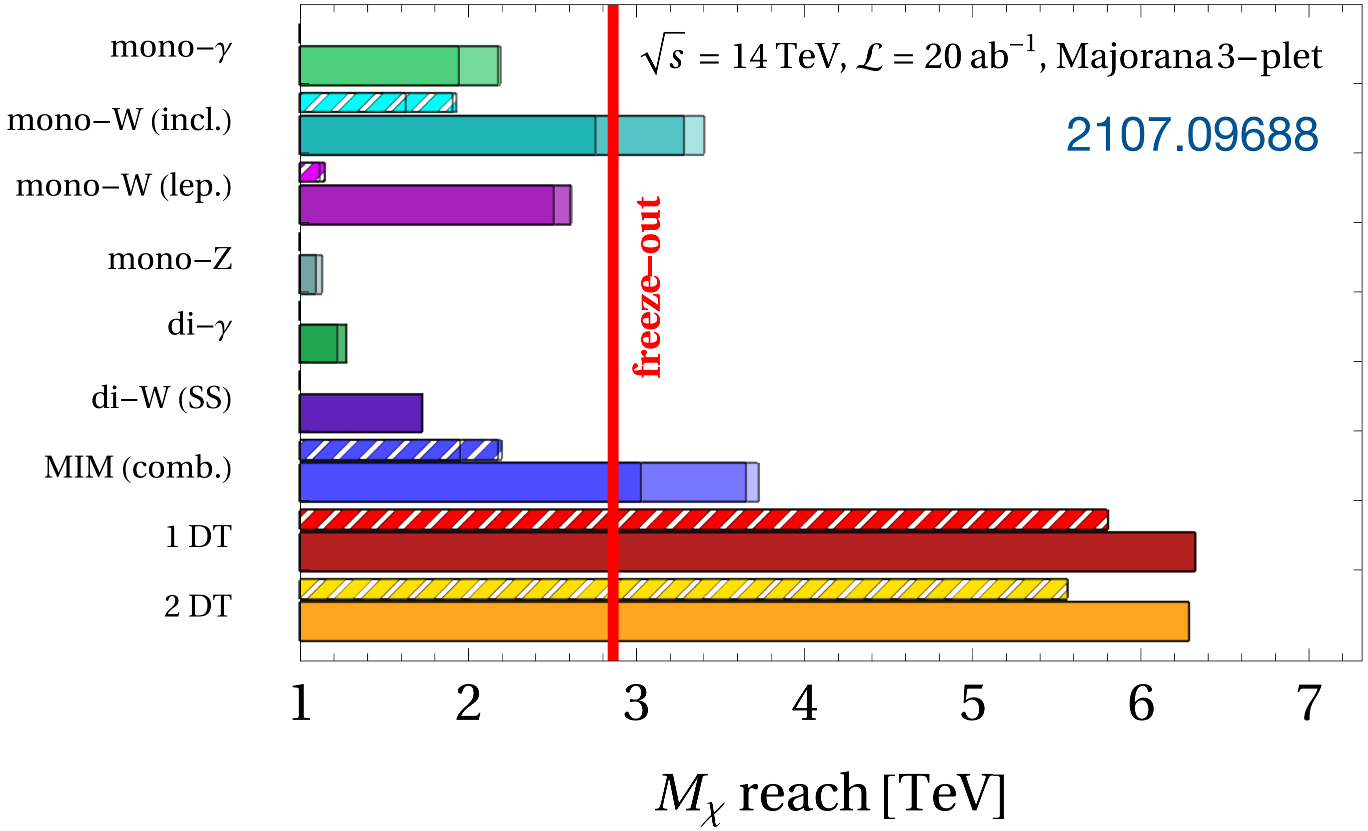
  Bottaro et al. 2107.09688, 2205.04486



- Disappearing tracks: charged components of χ can be long-lived $\chi^\pm \to \chi^0 \pi^\pm$

  Capdevilla et al. 2102.11292





μC can probe all relevant WIMP candidates!

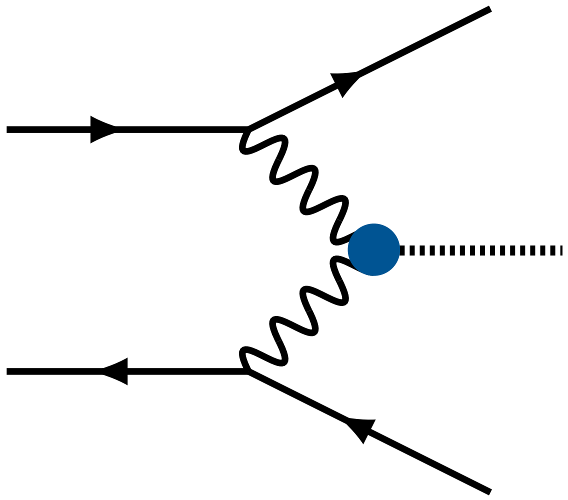
More difficult at hadron colliders, due to PDF suppression

FCC physics study

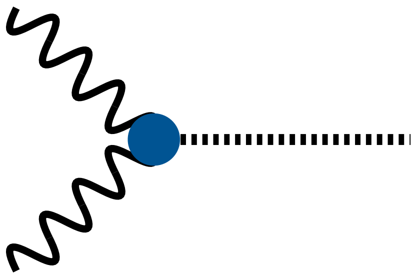
Cirelli, Sala, Taoso 1407.7058

Resonances in VBF

The μ-collider is a “vector boson collider”



$$\approx \frac{E^2}{m_\phi^2} \log \frac{E^2}{m_\phi^2}$$



enhanced if the
resonance is “light”
$m_\phi \ll E$

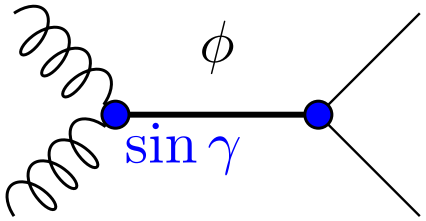
Dawson 1985

B, Redigolo, Sala, Tesi 1807.04743
Costantini et al. 2005.10289

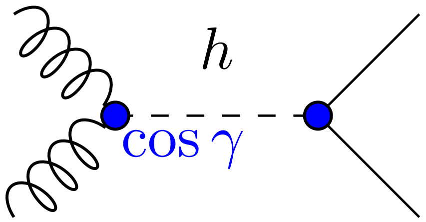
Al Ali, Arkani-Hamed et al. 2103.14043

- Example: singlet scalar, $\mathscr{L}_{\text{int}} \sim \phi |H|^2$ $\ell^+\ell^- \to \phi\nu\bar{\nu}$
  ϕ is like a heavy Higgs with narrow width + hh decay $\phi \to hh, WW, ZZ$

  cross-section grows at high energy due to longitudinal W-fusion



γ mixing angle between
SM Higgs *h* and singlet ϕ

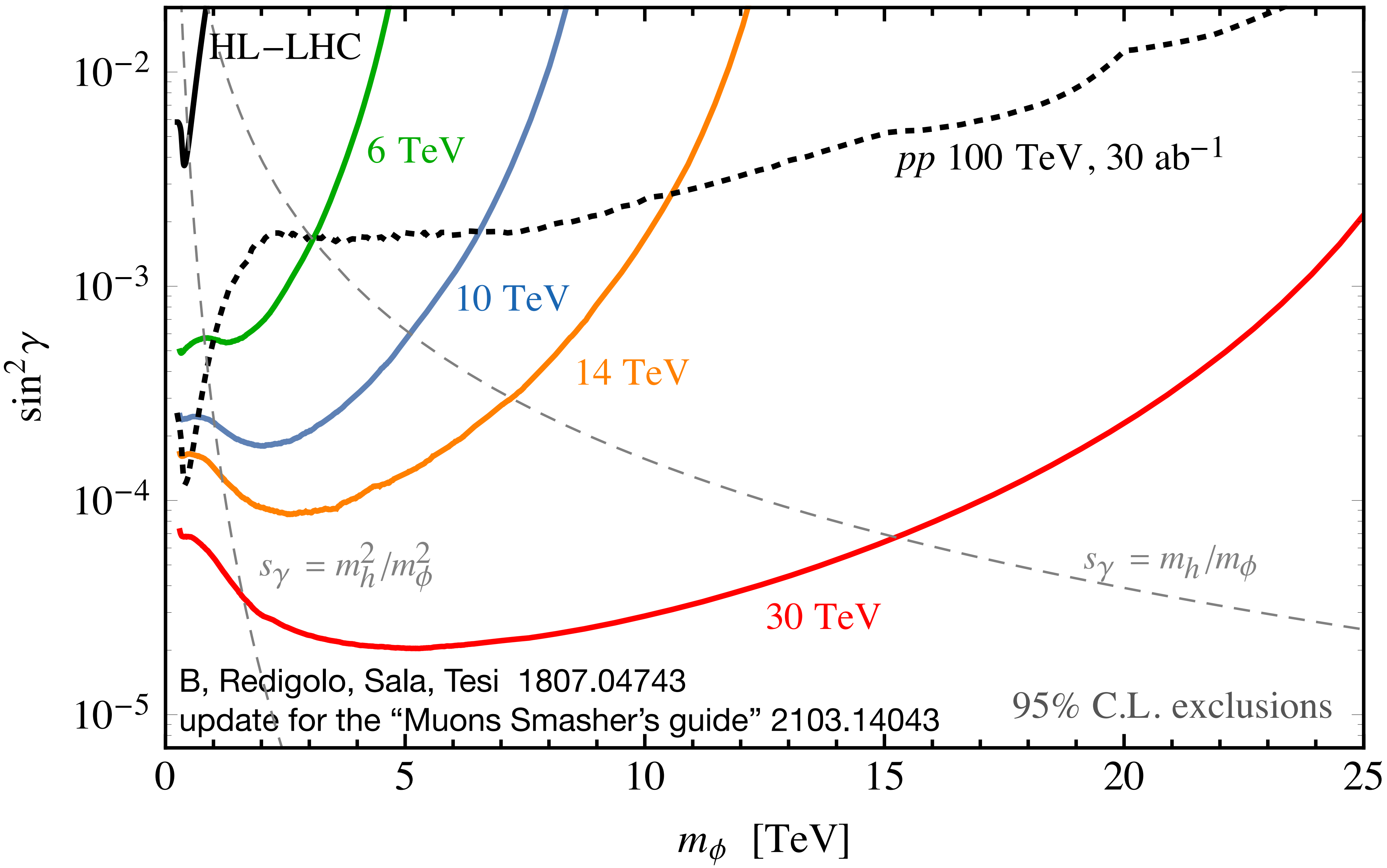


one single parameter controls resonance production, decay, & Higgs couplings

Example: scalar singlet

Compare direct and indirect reach of different colliders

$$\sin^2\gamma \approx \Delta\mu_h/\mu_h^{\rm SM} \approx \sigma_{VV\to\phi}/\sigma_{VV\to h}^{\rm SM}$$



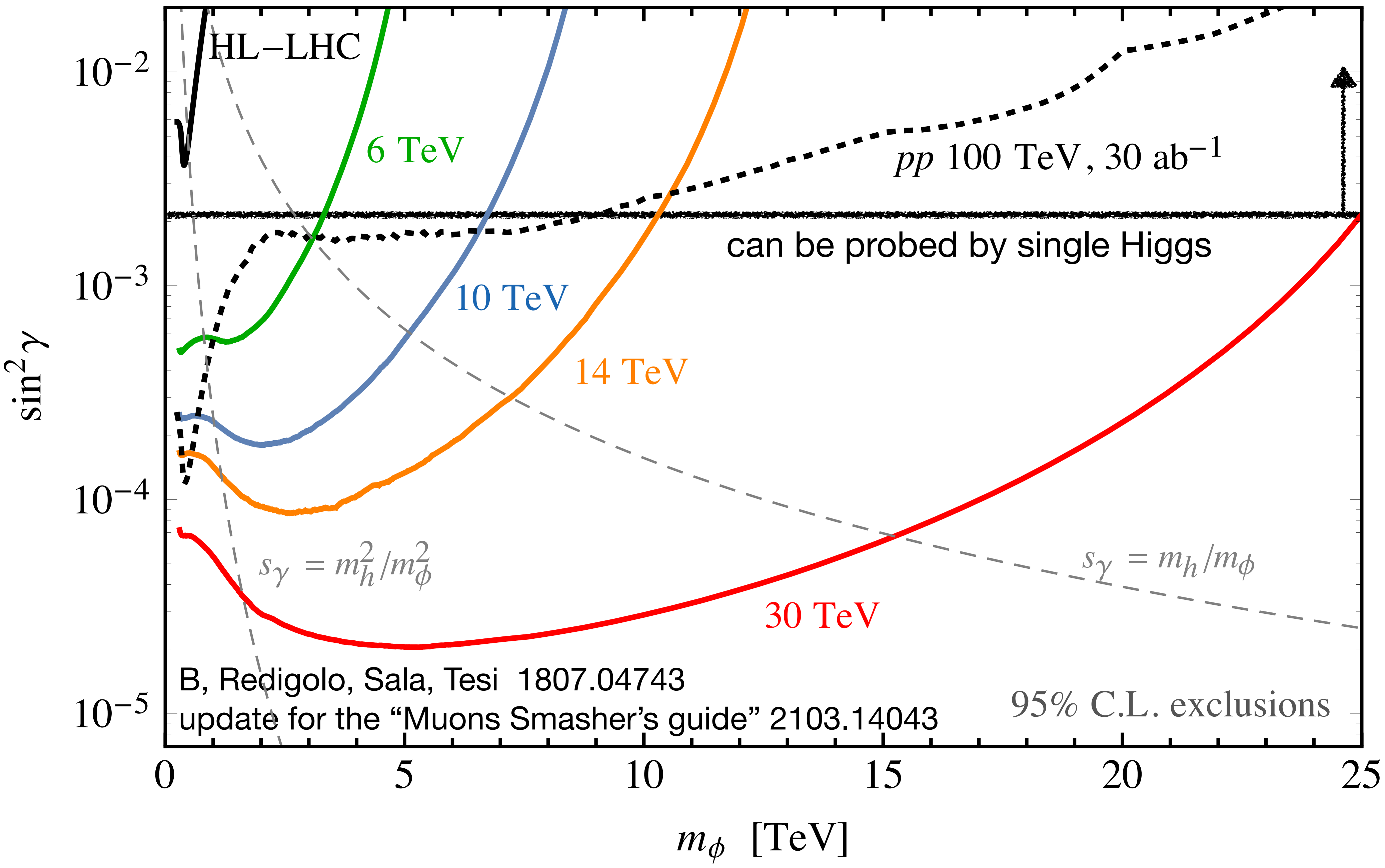
B, Redigolo, Sala, Tesi 1807.04743
update for the “Muons Smasher’s guide” 2103.14043

For this class of models, a high-energy $\mu^+\mu^-$ collider has an amazing reach if compared to single Higgs (or even direct searches at a 100 TeV pp collider)

Example: scalar singlet

Compare direct and indirect reach of different colliders

$$\sin^2\gamma \approx \Delta\mu_h/\mu_h^{\mathrm{SM}} \approx \sigma_{VV\to\phi}/\sigma_{VV\to h}^{\mathrm{SM}}$$



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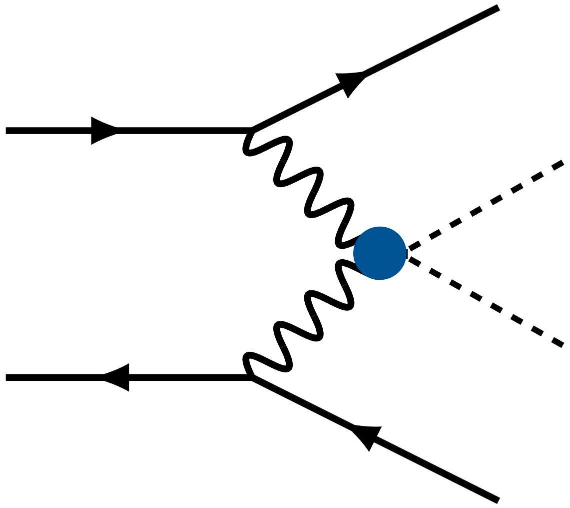
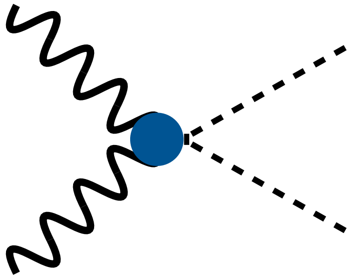
For this class of models, a high-energy μ⁺μ⁻ collider has an amazing reach if compared to single Higgs (or even direct searches at a 100 TeV pp collider)

High rate probes

- High rate: more events = better precision



$$\approx \frac{s}{\hat{s}} \log \frac{s}{\hat{s}}$$

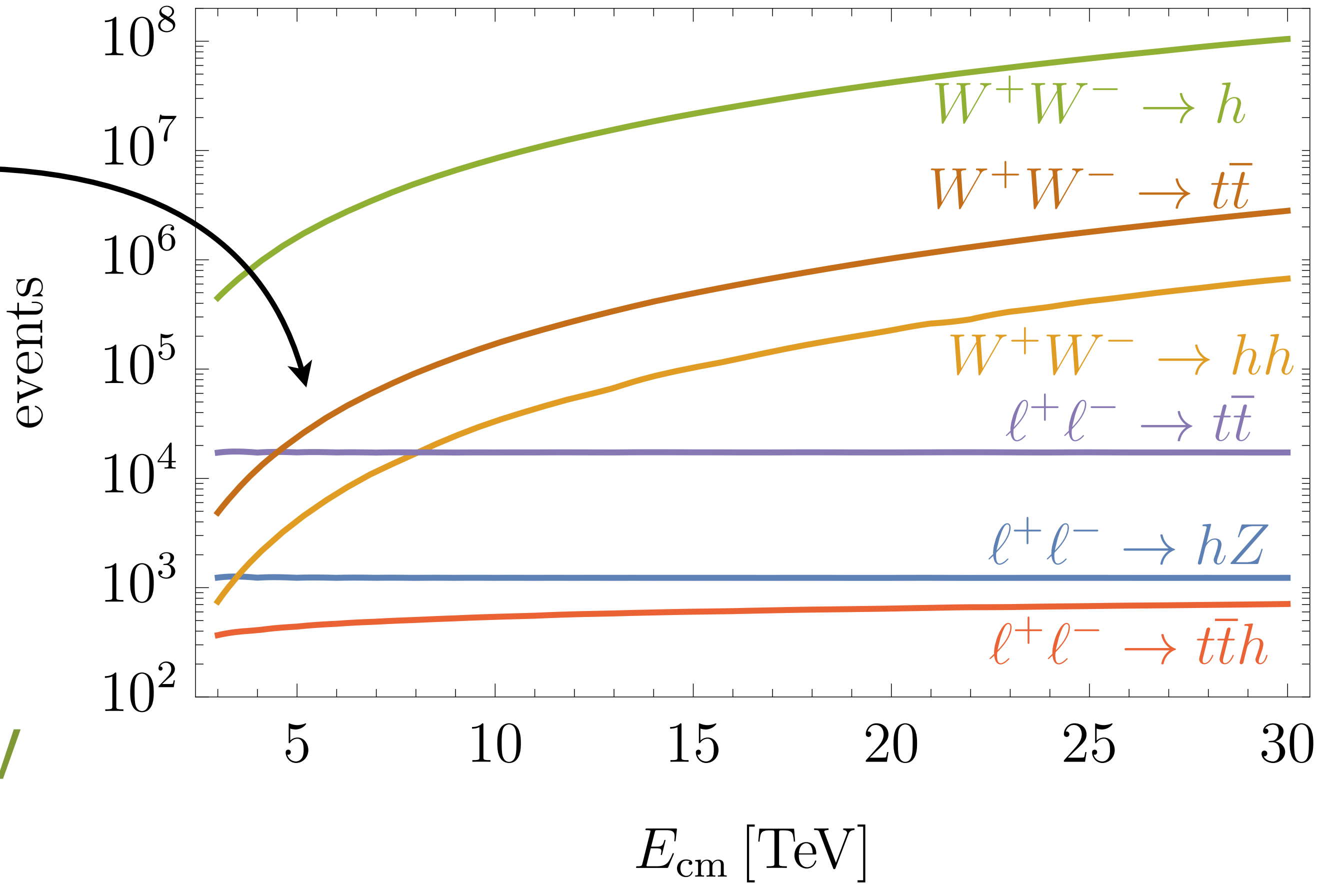


A High Energy Lepton Collider is a “vector boson collider”

For “soft” SM final state $\hat{s} \sim m_{\mathrm{EW}}^2$ cross-section is enhanced

Dawson 1985

Above few TeV the VBF cross-section dominates over the hard 2 → 2



- Huge single Higgs rate in vector-boson-fusion: 10^7-10^8 Higgs bosons at 10-30 TeV

High rate probes: Higgs physics

A 10+ TeV muon collider is a perfect Higgs factory!

- Signal-only estimate: ~ 10^7 Higgses at 10 TeV + efficiencies, BR
  - rough estimate: 10^{-3} for dominant decay channels @ 10 TeV

(as a comparison: 1.7 x 10^7 Z bosons @ LEP)

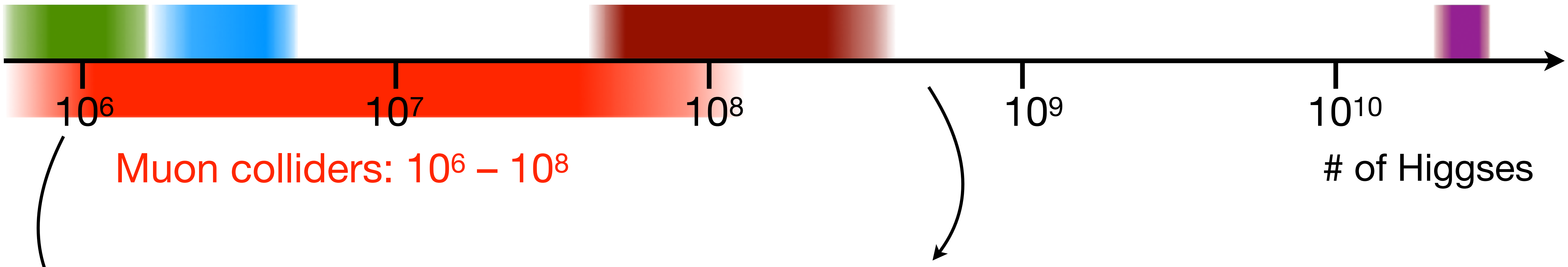
Low energy
e^+e^- factories
(FCC-ee, CEPC,
ILC, CLIC380)

TeV-scale
e^+e^- factories
(CLIC, ILC1000)

LHC: few x 10^7

HL-LHC: few x 10^8

FCC-hh:
few x 10^{10}



Muon colliders: 10^6 – 10^8

\# of Higgses

clean environment:
can measure "large" Higgs
BR w/ almost 10^{-3} precision

large QCD backgrounds:
only rare modes (BR < 10^{-3})
easily accessible

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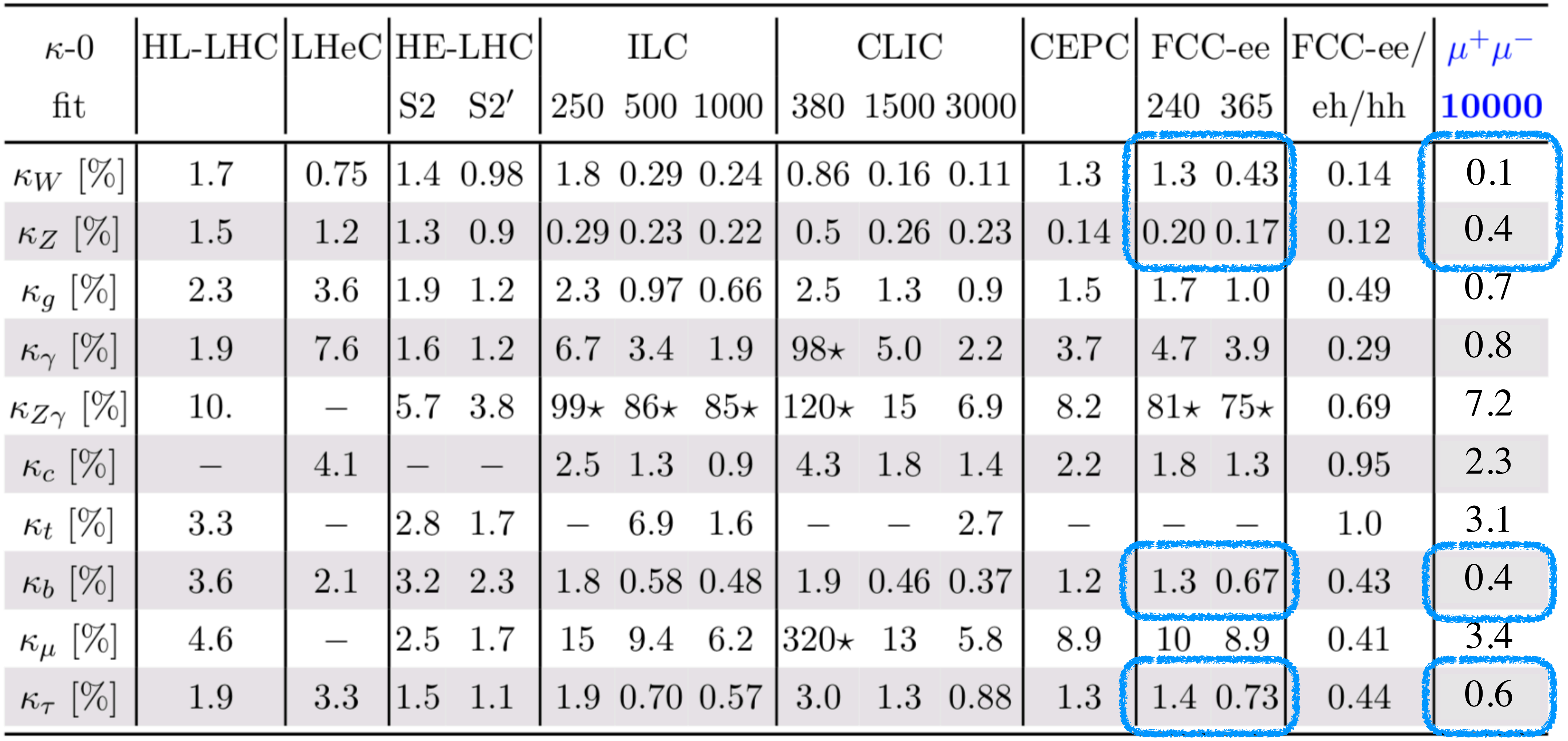

κ-0	HL-LHC	LHeC	HE-LHC		ILC			CLIC			CEPC	FCC-ee		FCC-ee/	$\mu^+\mu^-$
fit			S2	S2′	250	500	1000	380	1500	3000		240	365	eh/hh	**10000**
κ_W [%]	1.7	0.75	1.4	0.98	1.8	0.29	0.24	0.86	0.16	0.11	1.3	1.3	0.43	0.14	0.1
κ_Z [%]	1.5	1.2	1.3	0.9	0.29	0.23	0.22	0.5	0.26	0.23	0.14	0.20	0.17	0.12	0.4
κ_g [%]	2.3	3.6	1.9	1.2	2.3	0.97	0.66	2.5	1.3	0.9	1.5	1.7	1.0	0.49	0.7
κ_γ [%]	1.9	7.6	1.6	1.2	6.7	3.4	1.9	98⋆	5.0	2.2	3.7	4.7	3.9	0.29	0.8
$\kappa_{Z\gamma}$ [%]	10.	–	5.7	3.8	99⋆	86⋆	85⋆	120⋆	15	6.9	8.2	81⋆	75⋆	0.69	7.2
κ_c [%]	–	4.1	–	–	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3	0.95	2.3
κ_t [%]	3.3	–	2.8	1.7	–	6.9	1.6	–	–	2.7	–	–	–	1.0	3.1
κ_b [%]	3.6	2.1	3.2	2.3	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67	0.43	0.4
κ_μ [%]	4.6	–	2.5	1.7	15	9.4	6.2	320⋆	13	5.8	8.9	10	8.9	0.41	3.4
κ_τ [%]	1.9	3.3	1.5	1.1	1.9	0.70	0.57	3.0	1.3	0.88	1.3	1.4	0.73	0.44	0.6

dominant channels ~ other Higgs factories

rare modes better (~ hadron collider)

2103.14043

☛ talk by Patrick

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A 10+ TeV muon collider is a perfect Higgs factory!

- ✦ Signal-only estimate: ~ 10^7 Higgses at 10 TeV + efficiencies, BR
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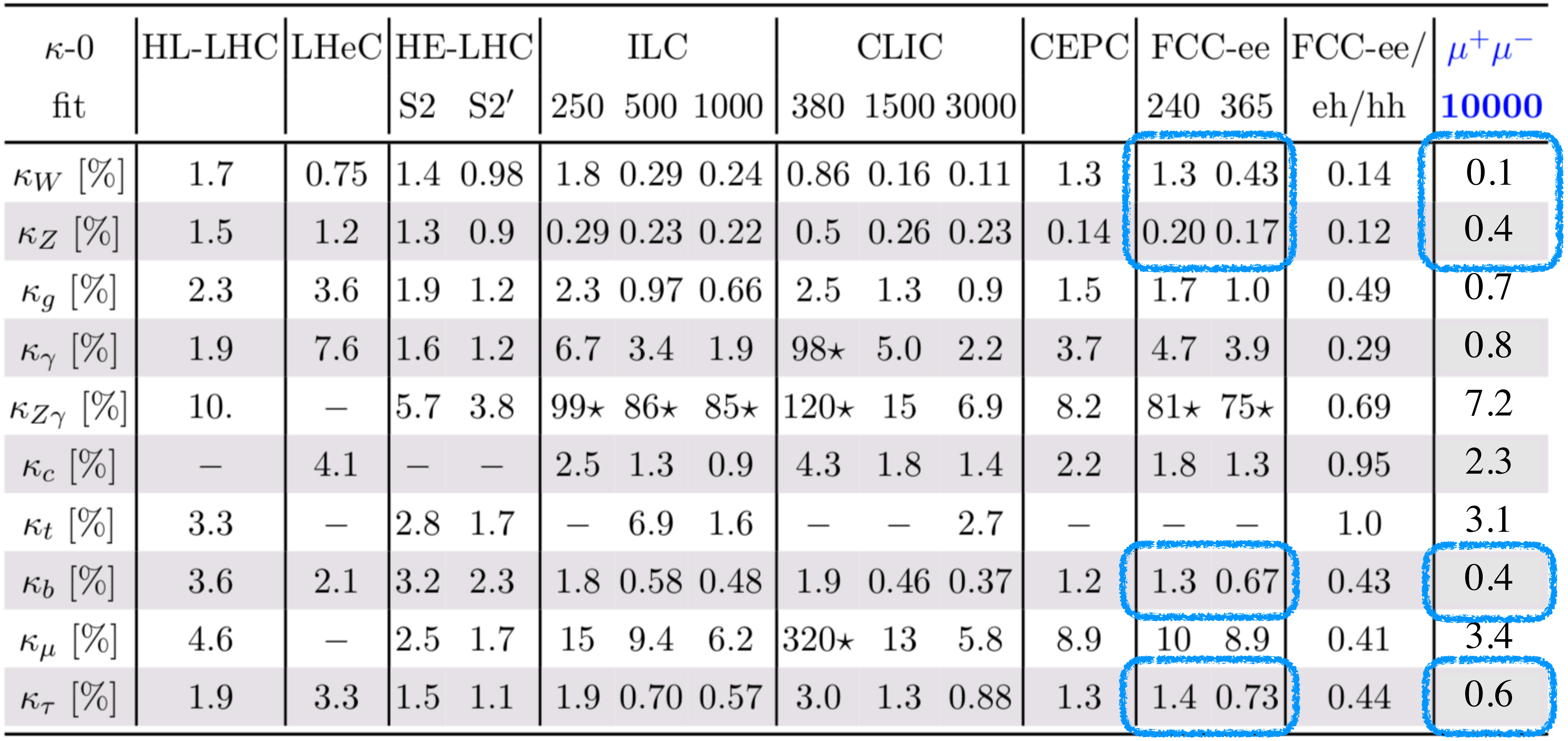

κ-0 fit	HL-LHC	LHeC	HE-LHC S2	HE-LHC S2′	ILC 250	ILC 500	ILC 1000	CLIC 380	CLIC 1500	CLIC 3000	CEPC	FCC-ee 240	FCC-ee 365	FCC-ee/eh/hh	$\mu^+\mu^-$ **10000**
κ_W [%]	1.7	0.75	1.4	0.98	1.8	0.29	0.24	0.86	0.16	0.11	1.3	1.3	0.43	0.14	0.1
κ_Z [%]	1.5	1.2	1.3	0.9	0.29	0.23	0.22	0.5	0.26	0.23	0.14	0.20	0.17	0.12	0.4
κ_g [%]	2.3	3.6	1.9	1.2	2.3	0.97	0.66	2.5	1.3	0.9	1.5	1.7	1.0	0.49	0.7
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$\kappa_{Z\gamma}$ [%]	10.	–	5.7	3.8	99⋆	86⋆	85⋆	120⋆	15	6.9	8.2	81⋆	75⋆	0.69	7.2
κ_c [%]	–	4.1	–	–	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3	0.95	2.3
κ_t [%]	3.3	–	2.8	1.7	–	6.9	1.6	–	–	2.7	–	–	–	1.0	3.1
κ_b [%]	3.6	2.1	3.2	2.3	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67	0.43	0.4
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dominant channels ~ other Higgs factories

rare modes better (~ hadron collider)

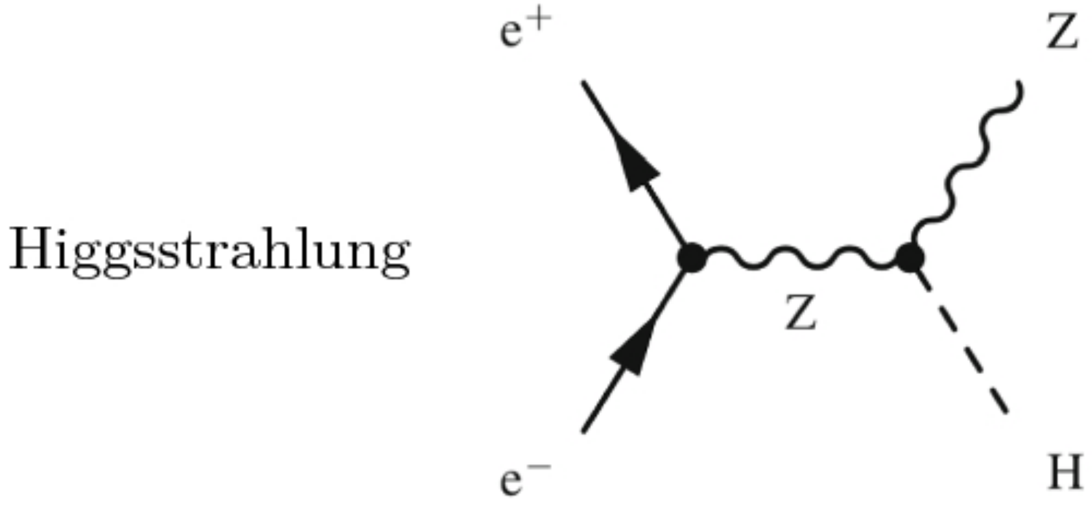
2103.14043

☛ talk by Patrick

Inclusive Higgs search

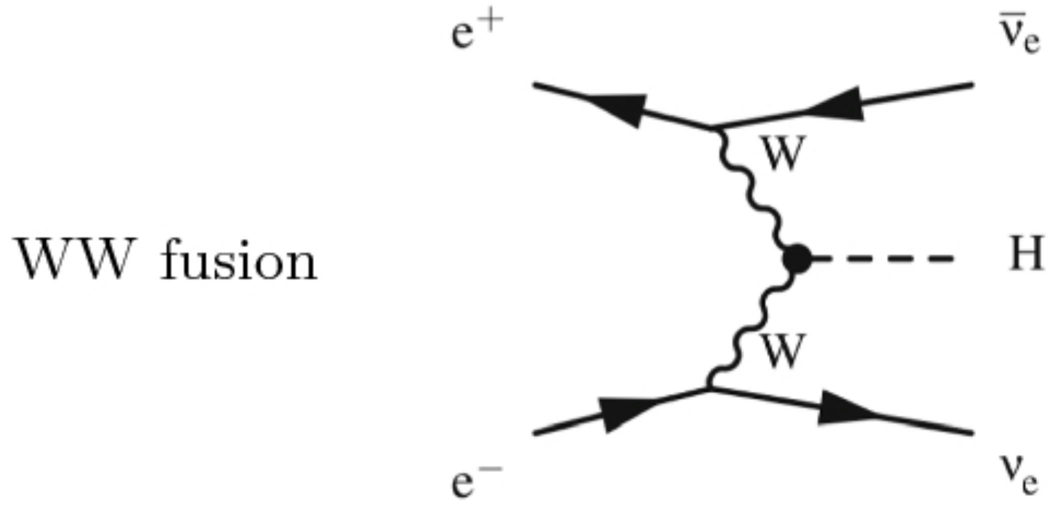
- Caveat: single Higgs at μC can access only

$$\mu_f = \sigma_h \times \mathrm{BR}_{h \to f} \sim \frac{g_W^2 \times g_f^2}{\Gamma_h} \quad \text{(similar to LHC)}$$



$$s = (p_h + p_Z)^2$$

Inclusive measurement, $\sigma_h \sim g_Z^2$



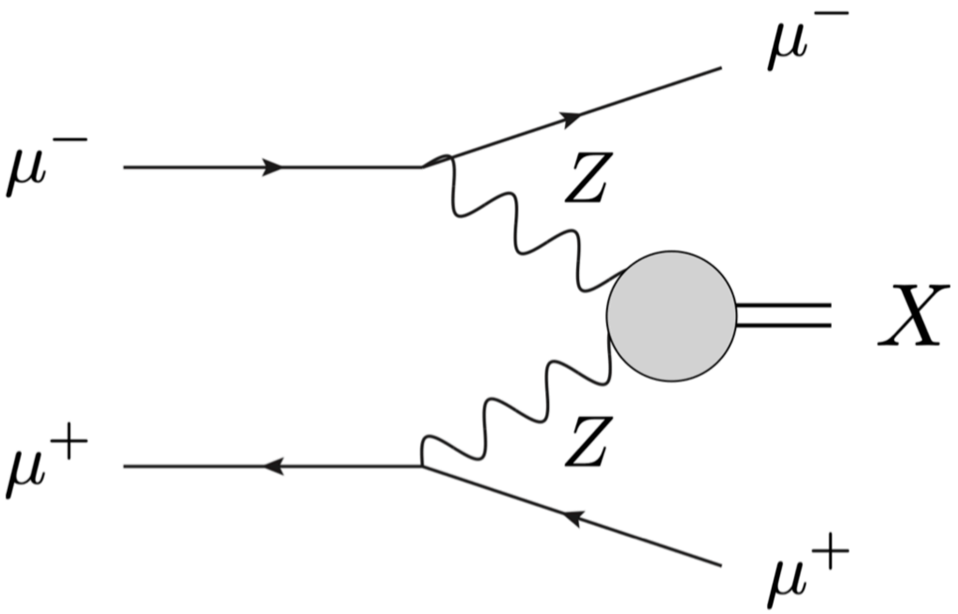
Hard neutrinos not seen,

WW → h → WW depends on g_W and Γ

cannot disentangle deviations in the couplings from modifications of total width

Inclusive Higgs search

- Try to do an inclusive single Higgs measurement with ZZ → h



- cross-section ~ 10x lower than WW
- **needs forward muon detection!**

$$s = (p_h + p_{\mu 1} + p_{\mu 2})^2$$

- Untagged: % sensitivity

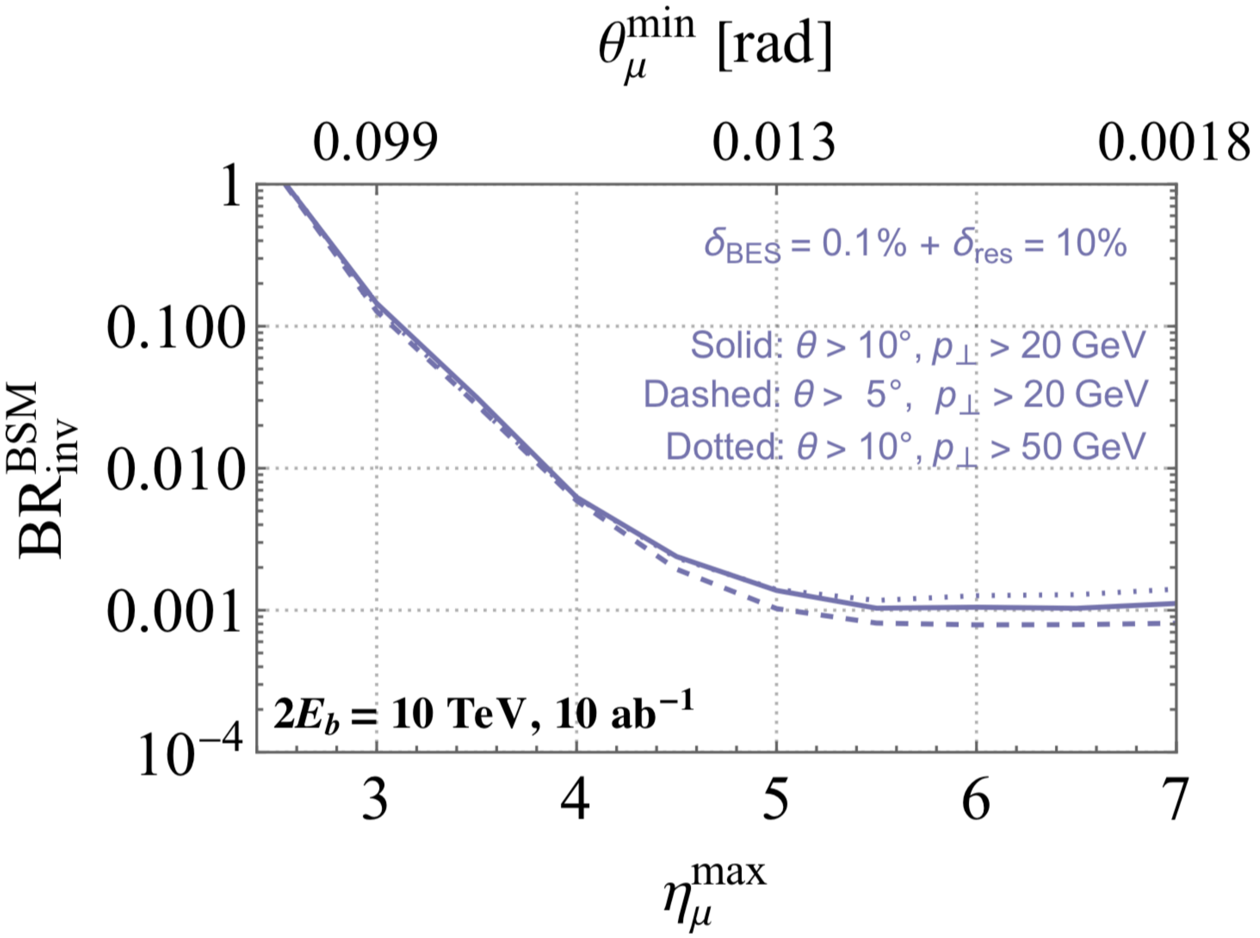
  if muons detected at $\eta \gtrsim 6$

  P. Li, Z. Liu, K. Lyu 2401.08756

- Invisible: 10^{-3} sensitivity

  if muons detected at $\eta \gtrsim 5$

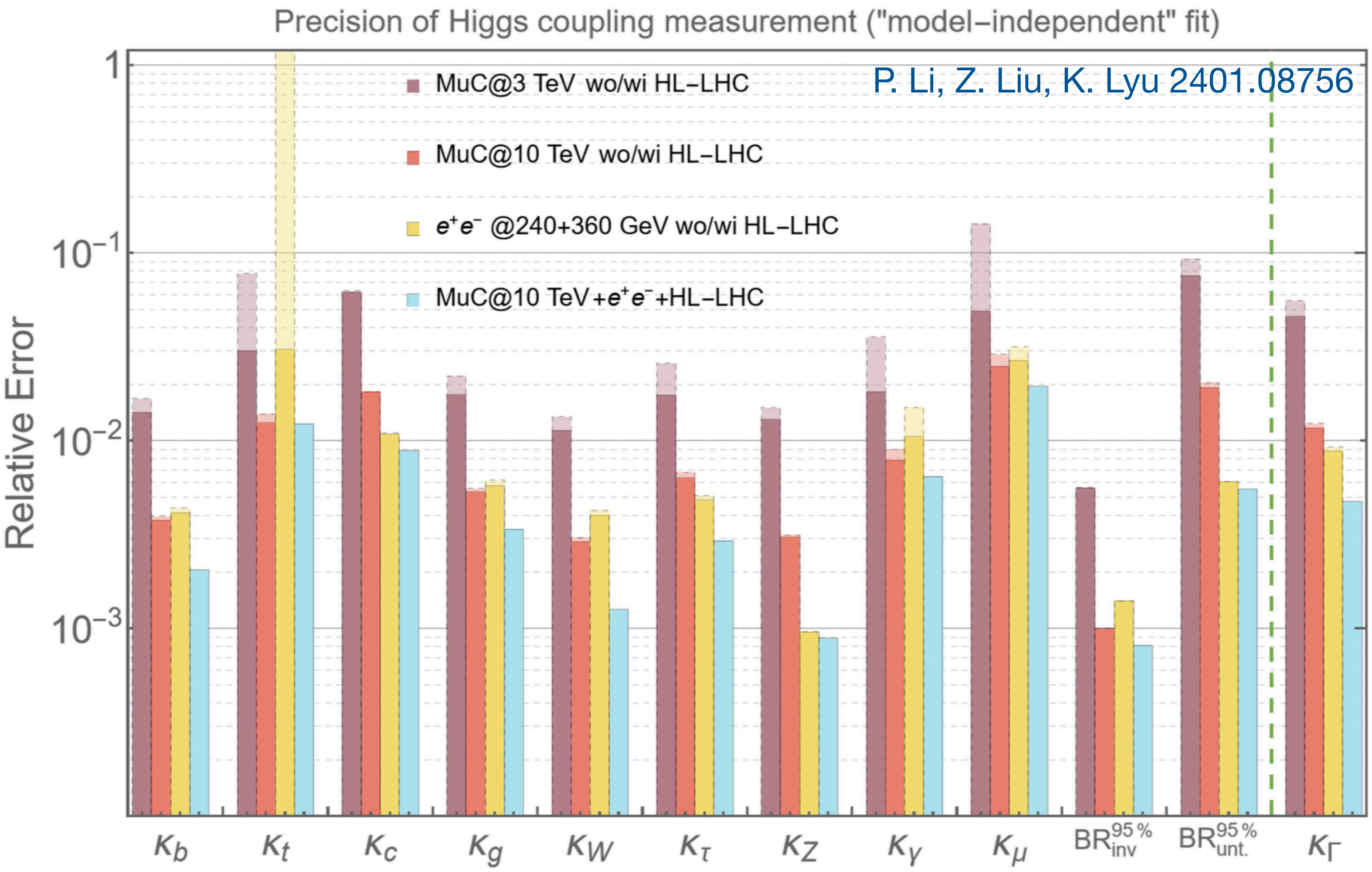
  Ruhdorfer, Salvioni, Wulzer 2303.14202
  Forslund, Meade 2308.02633



Higgs couplings at muon collider

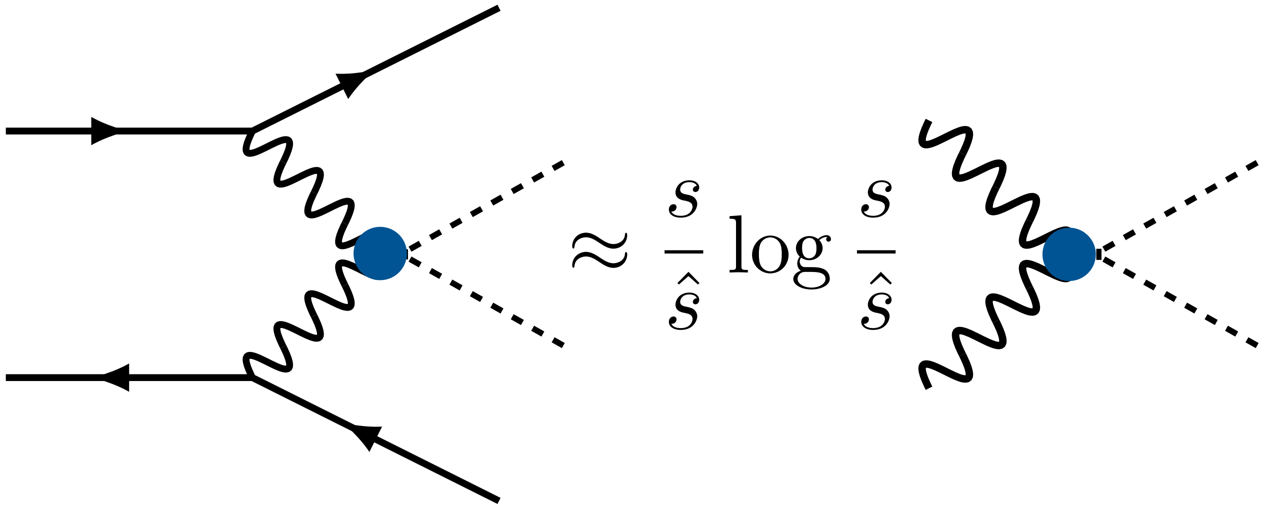
- A full-fledged Higgs-physics program is possible at a μC

- Single Higgs couplings can more easily be studied at e^+e^- factory! *(most likely before a μC!)*



High rate probes

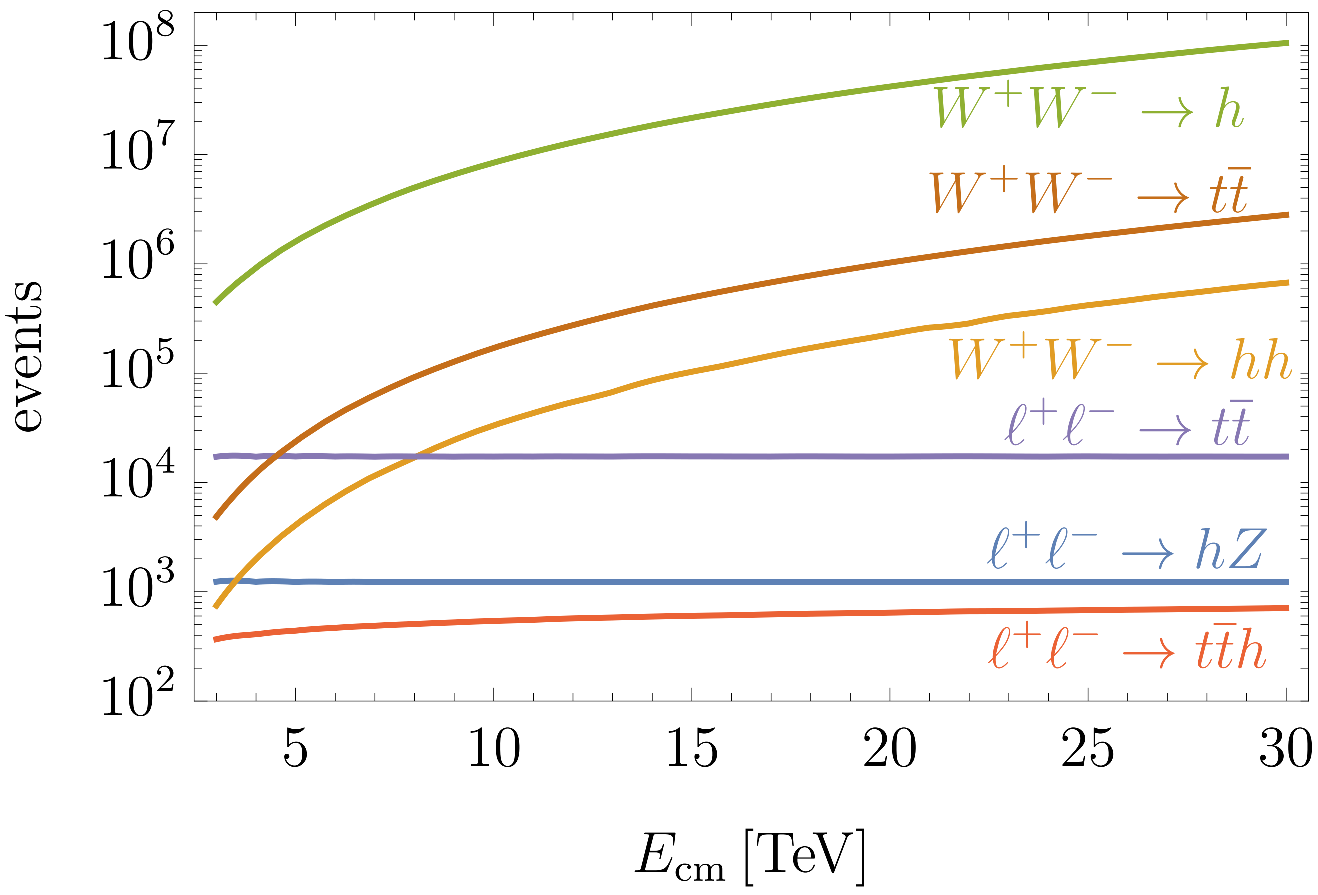
- High rate: more events = better precision



$$\approx \frac{s}{\hat{s}} \log \frac{s}{\hat{s}}$$

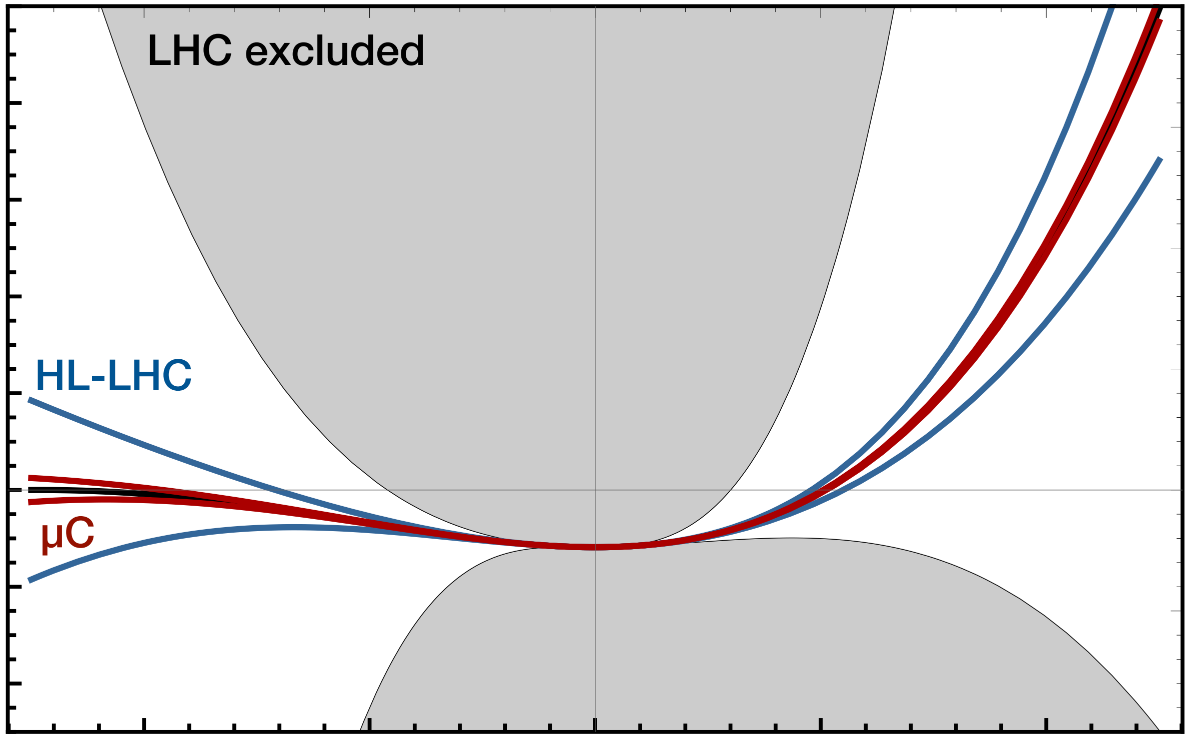
A High Energy Lepton Collider is a “vector boson collider”

- Huge single Higgs rate in vector-boson-fusion: 10^7 Higgs bosons at 10 TeV
- Large double Higgs VBF rate
    - Higgs 3-linear coupling
- Triple Higgs production accessible
    - Higgs 4-linear coupling (dim. 8 operator, suppressed)



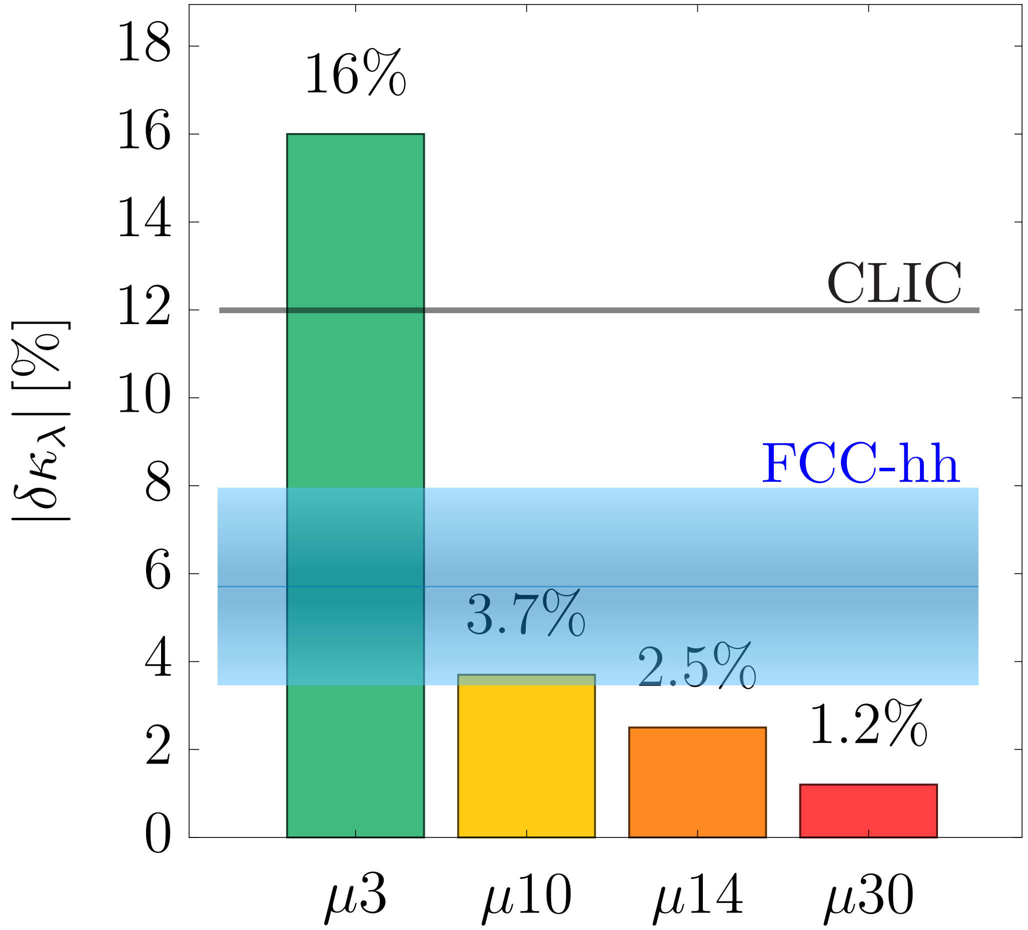
Double Higgs production

- Measurement of trilinear coupling: access to the Higgs potential



credits: Craig, Petrossian-Byrne

- very poorly known today!
- HL-LHC will only reach 50% precision on SM value



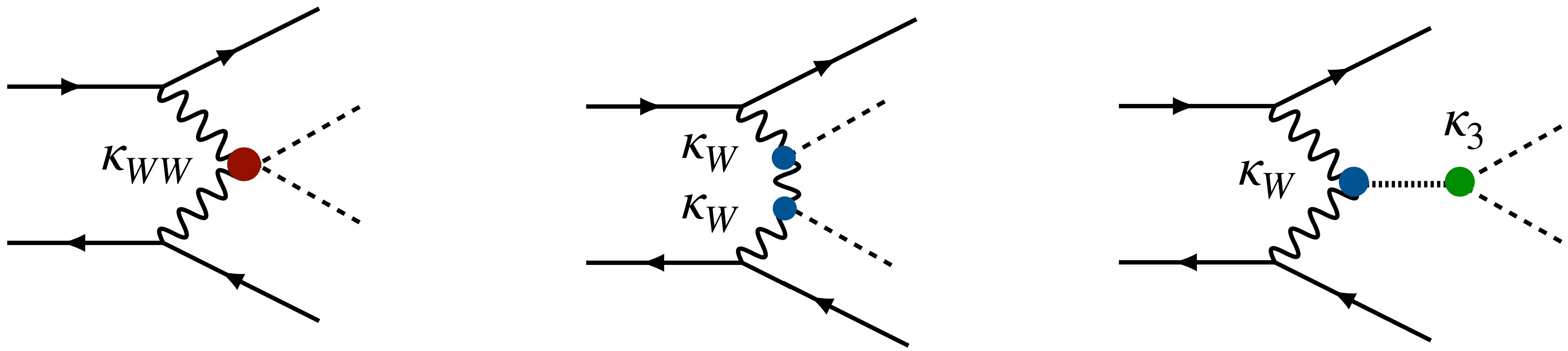
- Precise determination *only* possible at high-energy machines: FCC-hh or multi-TeV lepton collider

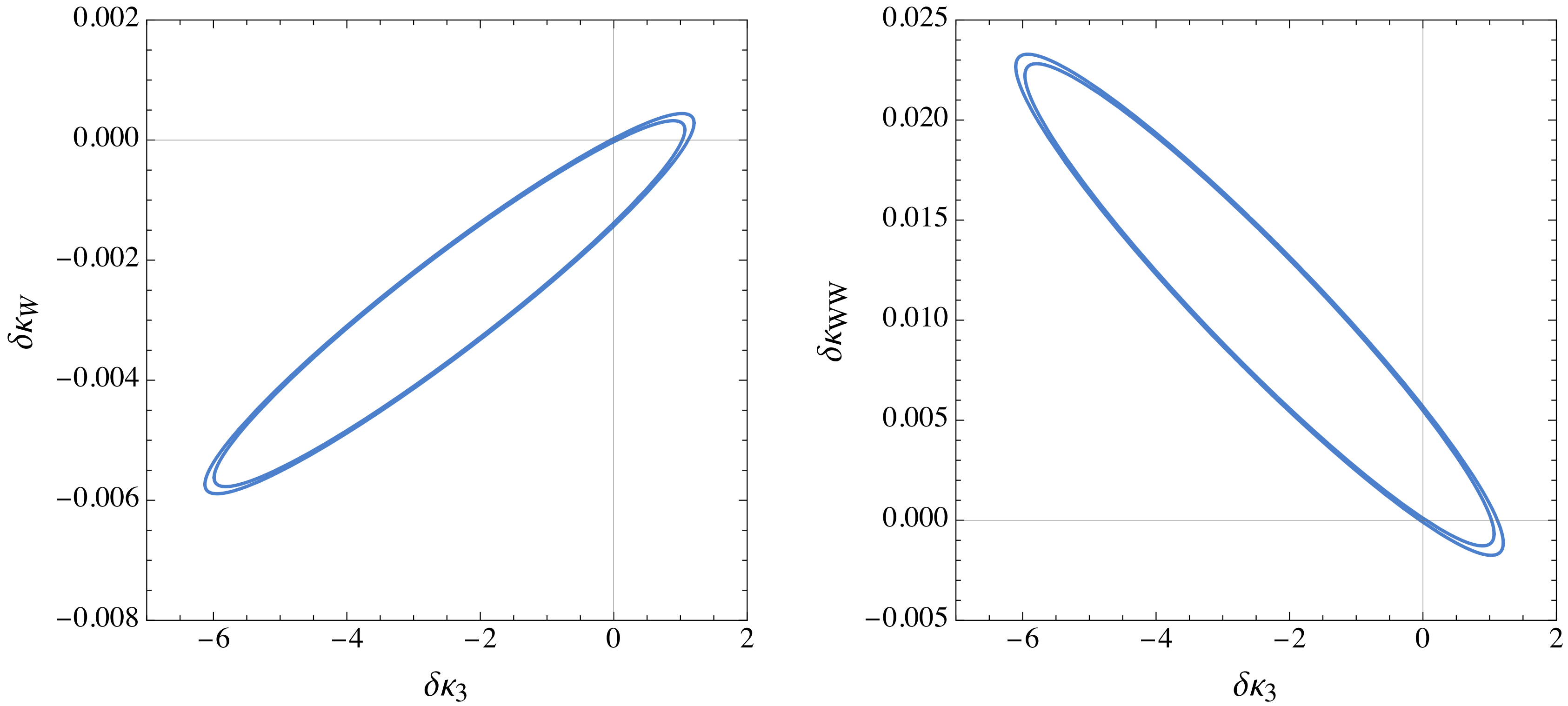
Mangano et al. 2004.03505
B, Franceschini, Wulzer 2012.11555
Costantini et al. 2005.10289

Han et al. 2008.12204
CLIC 1901.05897

Double Higgs production

- Double Higgs production depends on trilinear coupling κ_3 but also on W-boson couplings κ_W, κ_{WW} that enter the production cross-section



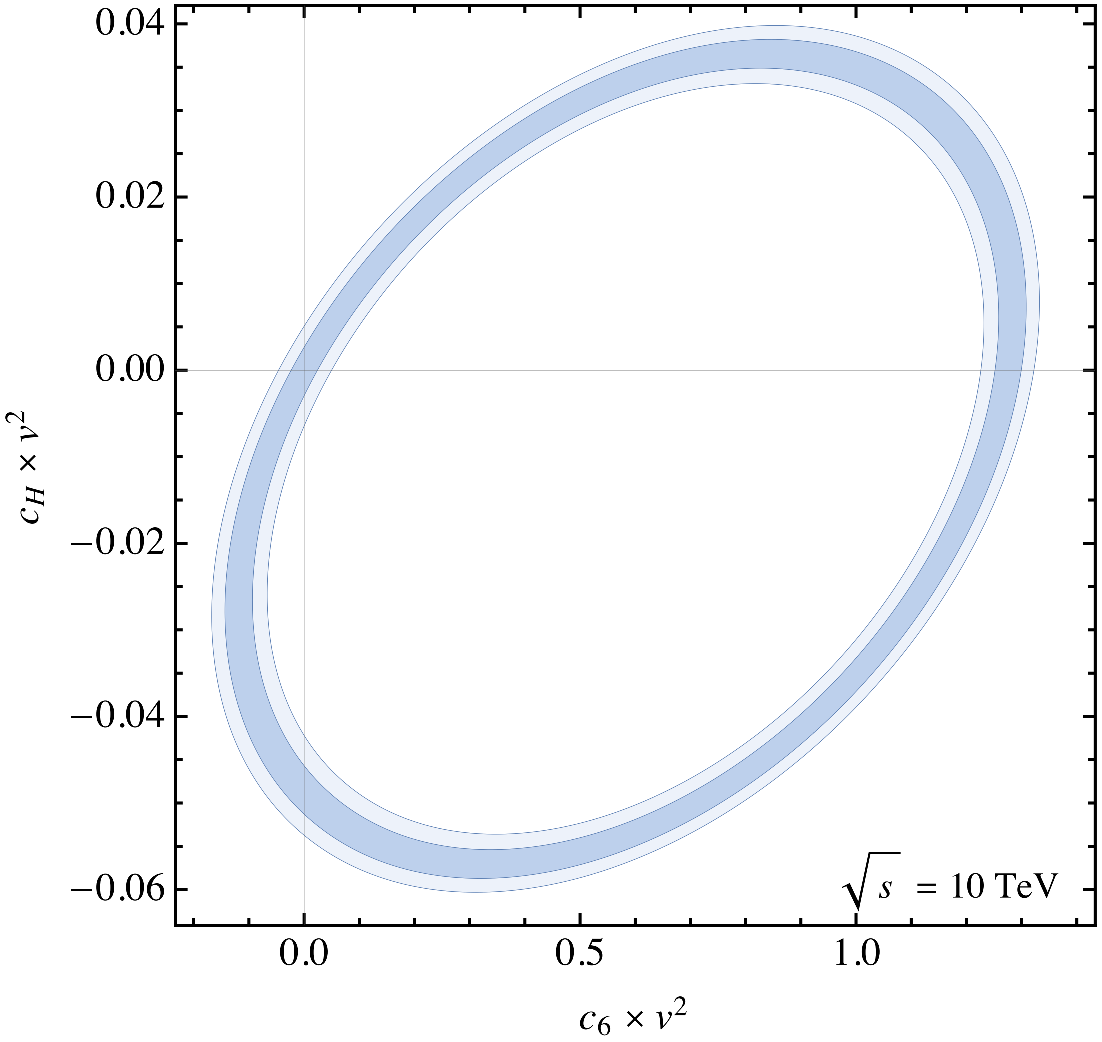


large degeneracy in total cross-section: coefficients not determined from *hh* production alone

Double Higgs production

- Double Higgs production depends on trilinear coupling κ_3 but also on W-boson couplings κ_W, κ_{WW} that enter the production cross-section
- Two dim. 6 operators: $\mathcal{O}_6 = -\lambda |H|^6$ $\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |H|^2\right)^2$

$$\kappa_3 = 1 + v^2\left(C_6 - \frac{3}{2}C_H\right) \qquad \kappa_W = 1 - v^2 C_H/2 \qquad \kappa_{WW} = 1 - 2v^2 C_H$$



large degeneracy in total cross-section: coefficients not determined in general

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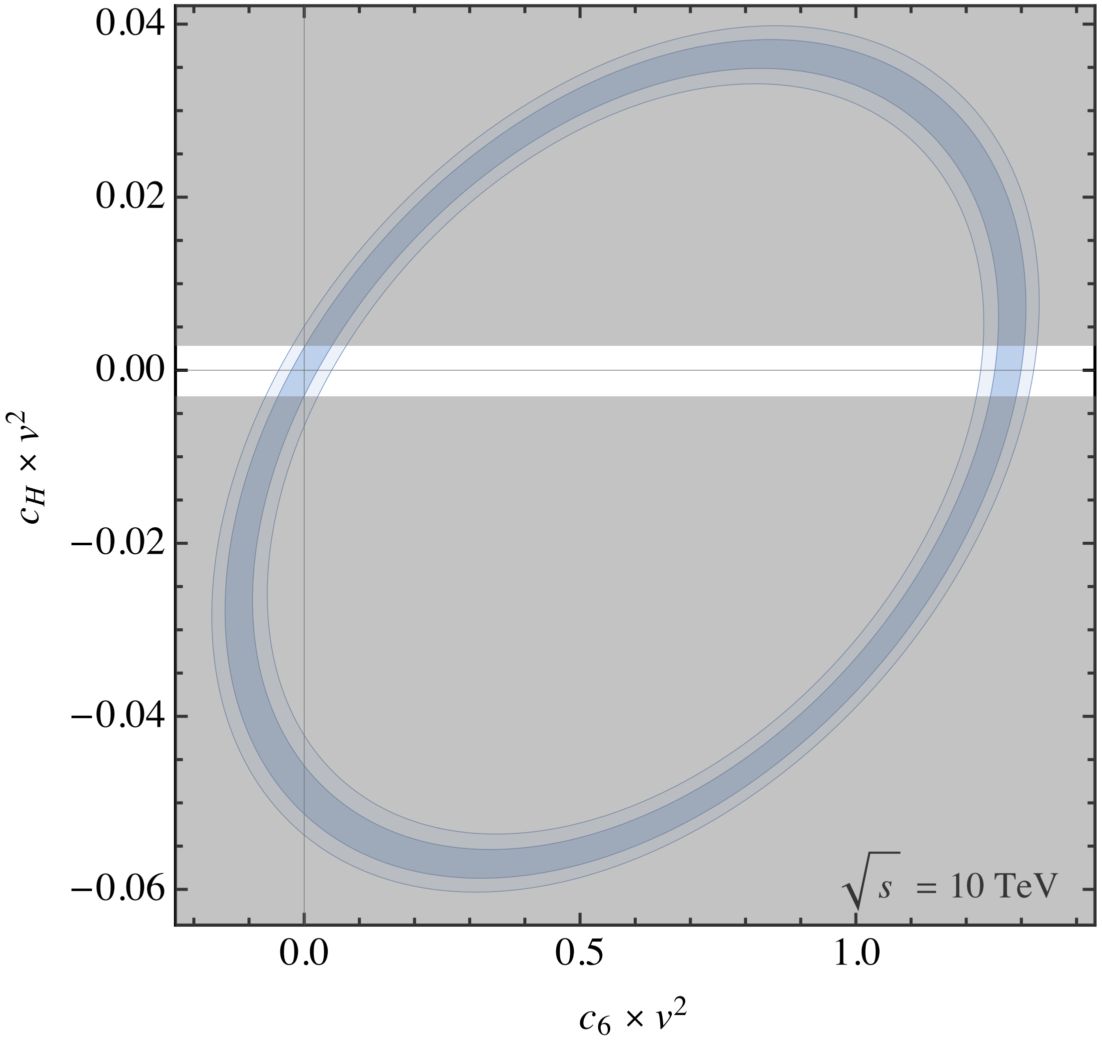


large degeneracy in total cross-section: coefficients not determined in general

O_H also affects all single Higgs couplings universally:

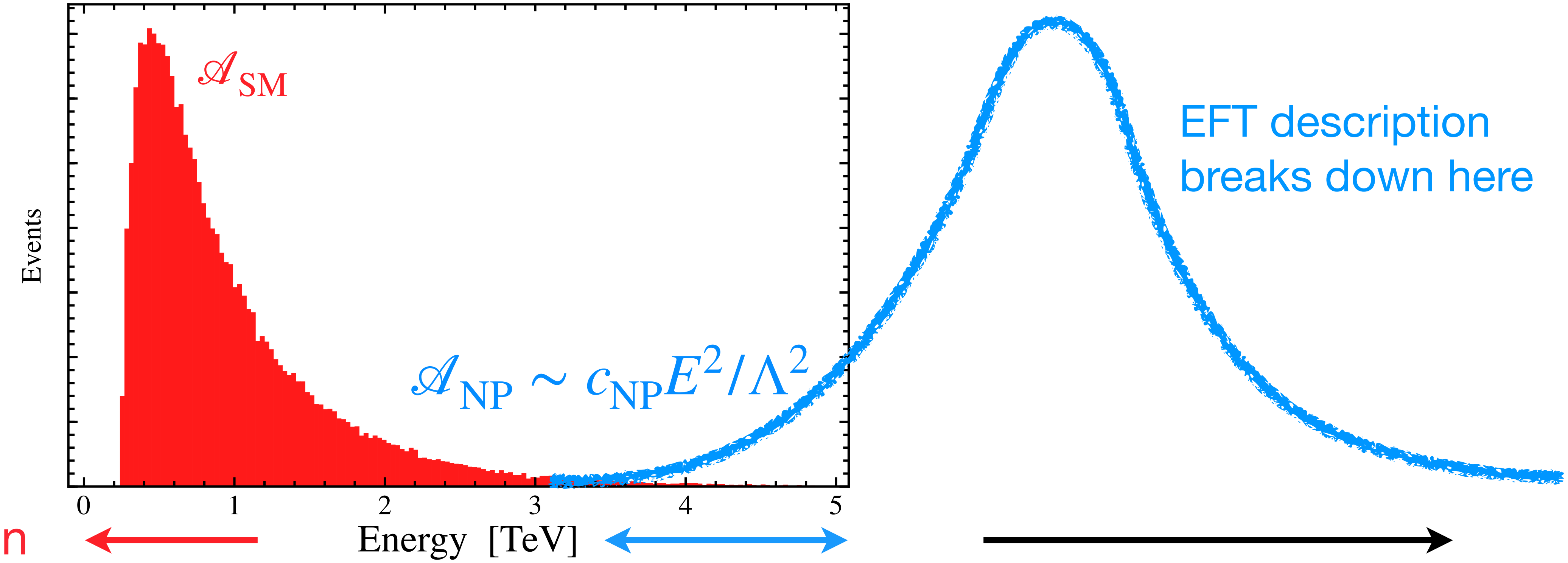
$$\kappa_{V,f} = 1 - v^2 C_H/2$$

C_H can be constrained from Higgs couplings $\Delta\kappa_V \sim C_H v^2 \lesssim \text{few} \times 10^{-3}$

Higgs at high-energy

- Higgs physics doesn't mean just couplings. There's much more information in the energy dependence of the interactions! (form factors)

- NP effects are more important at high energies (≈ high-pT tails at LHC)



Precision SM measurements

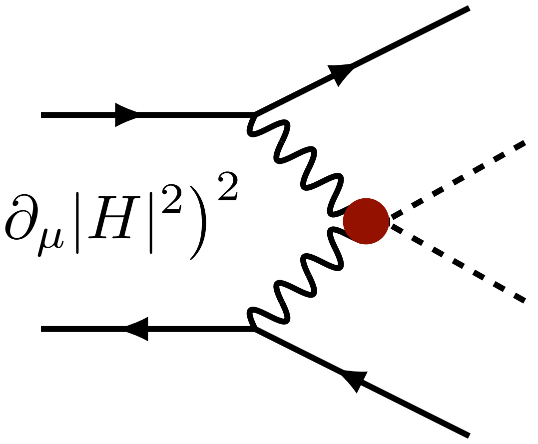
High energy (indirect) probes

direct searches

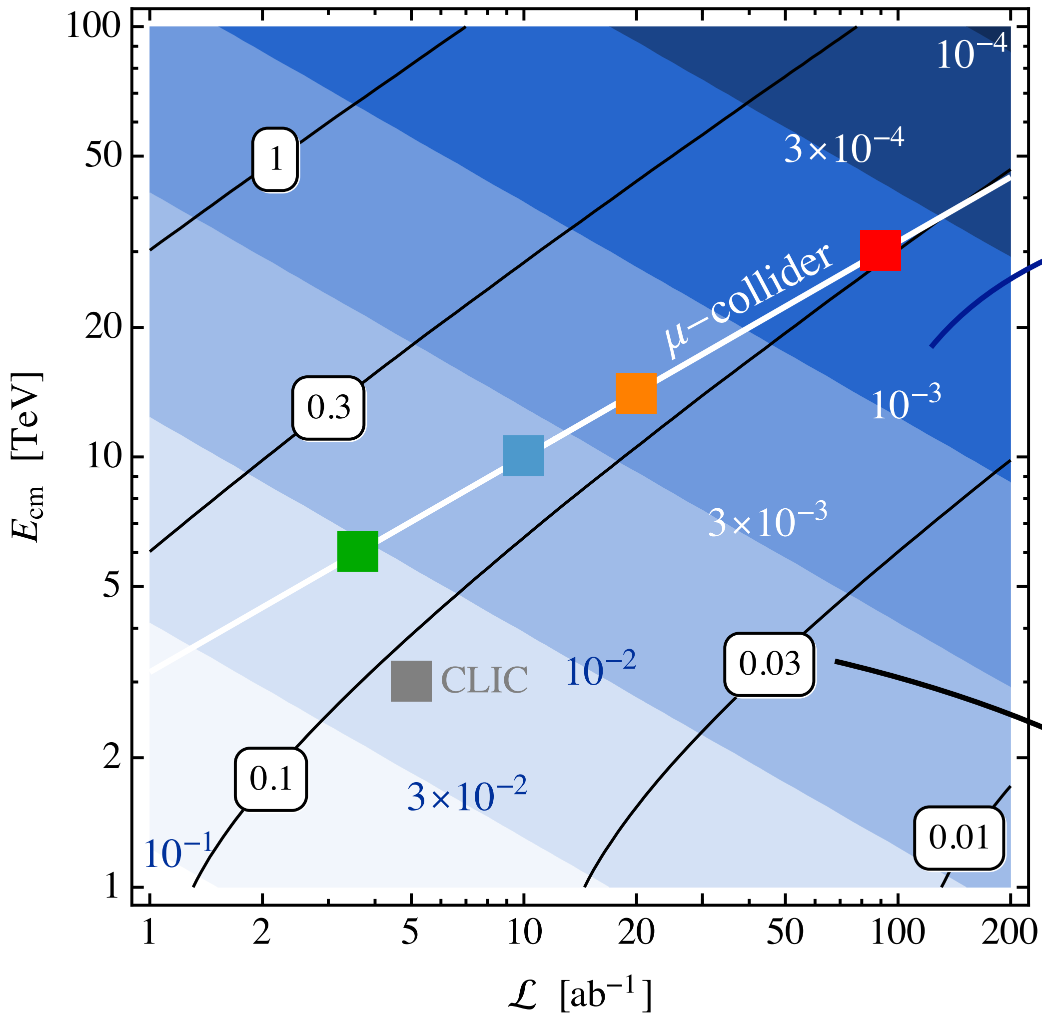
Double Higgs at high mass

- NP contribution from $\mathcal{O}_H$ (equivalently κ_W, κ_{WW}) grows as E²: high mass tail gives a *direct* measurement of C_H

$$\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |H|^2\right)^2$$



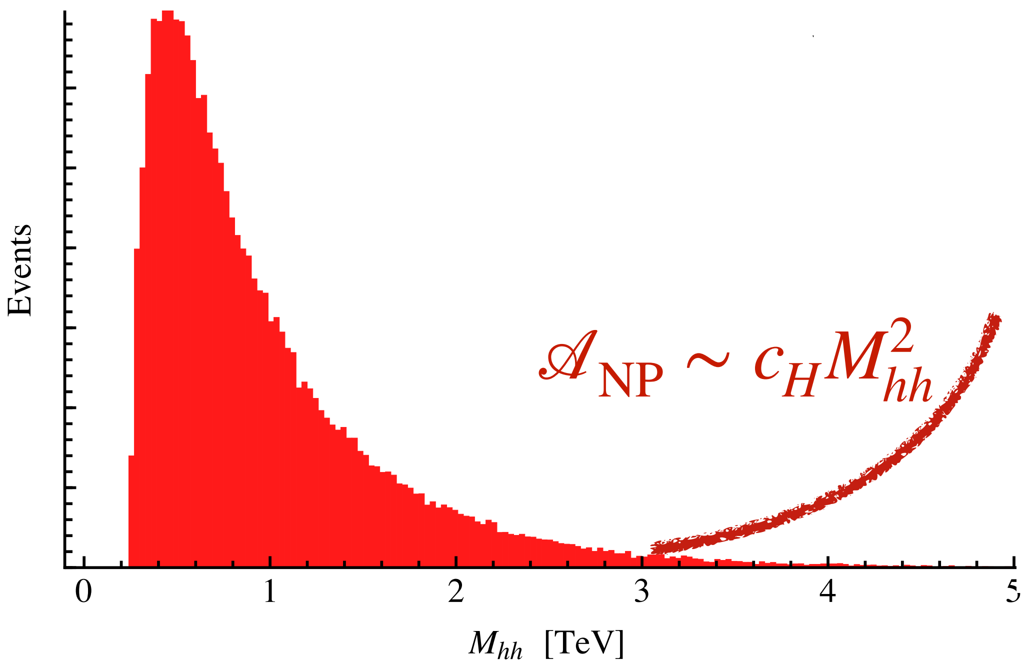
High-energy WW → *hh* more sensitive than Higgs pole physics at energies ≳ 10 TeV



$\xi \equiv C_H v^2$

S/B low-precision measurement

$\mu^+\mu^- \to hh\nu\bar{\nu}$



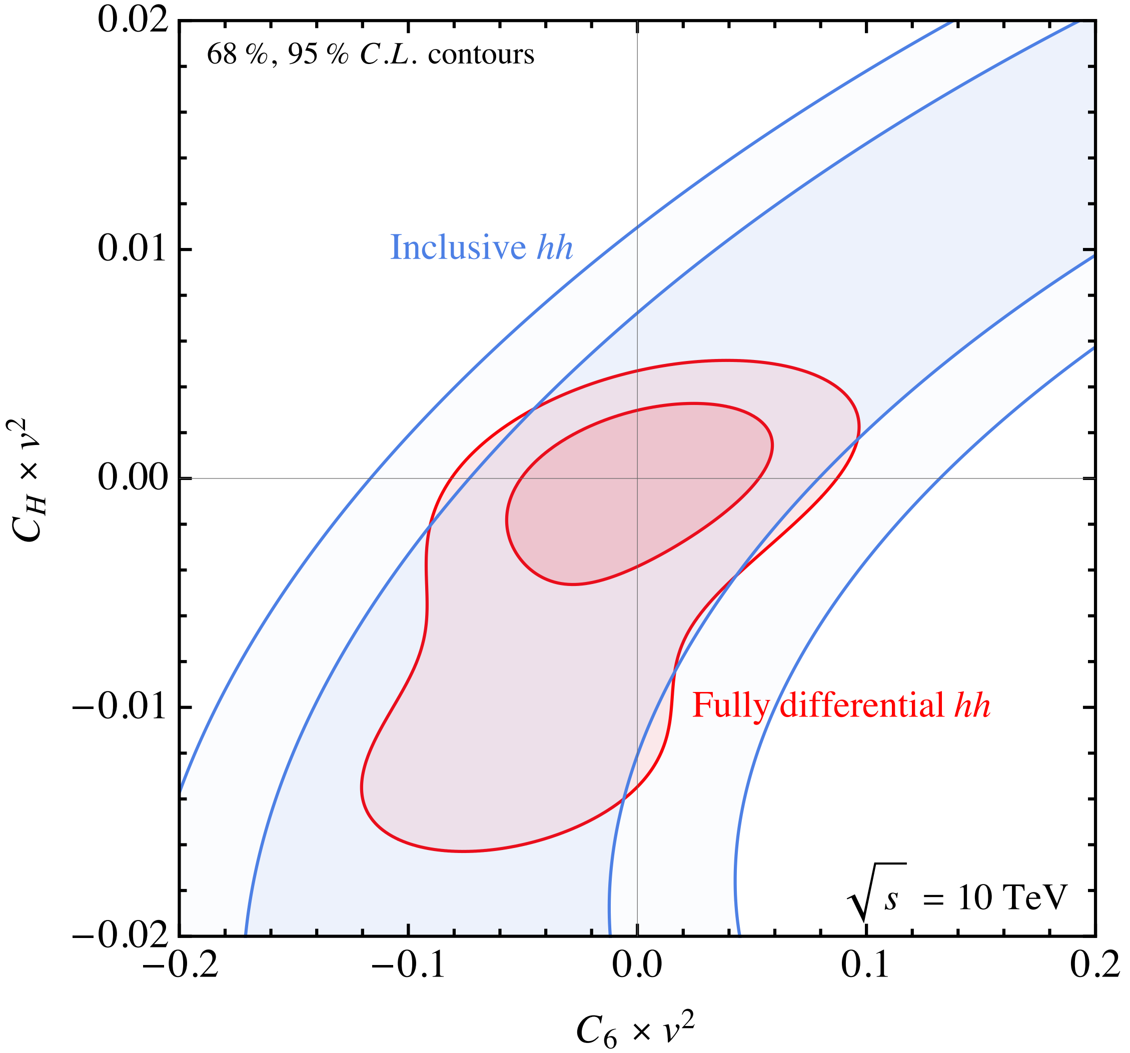
(see also Contino et al. 1309.7038)

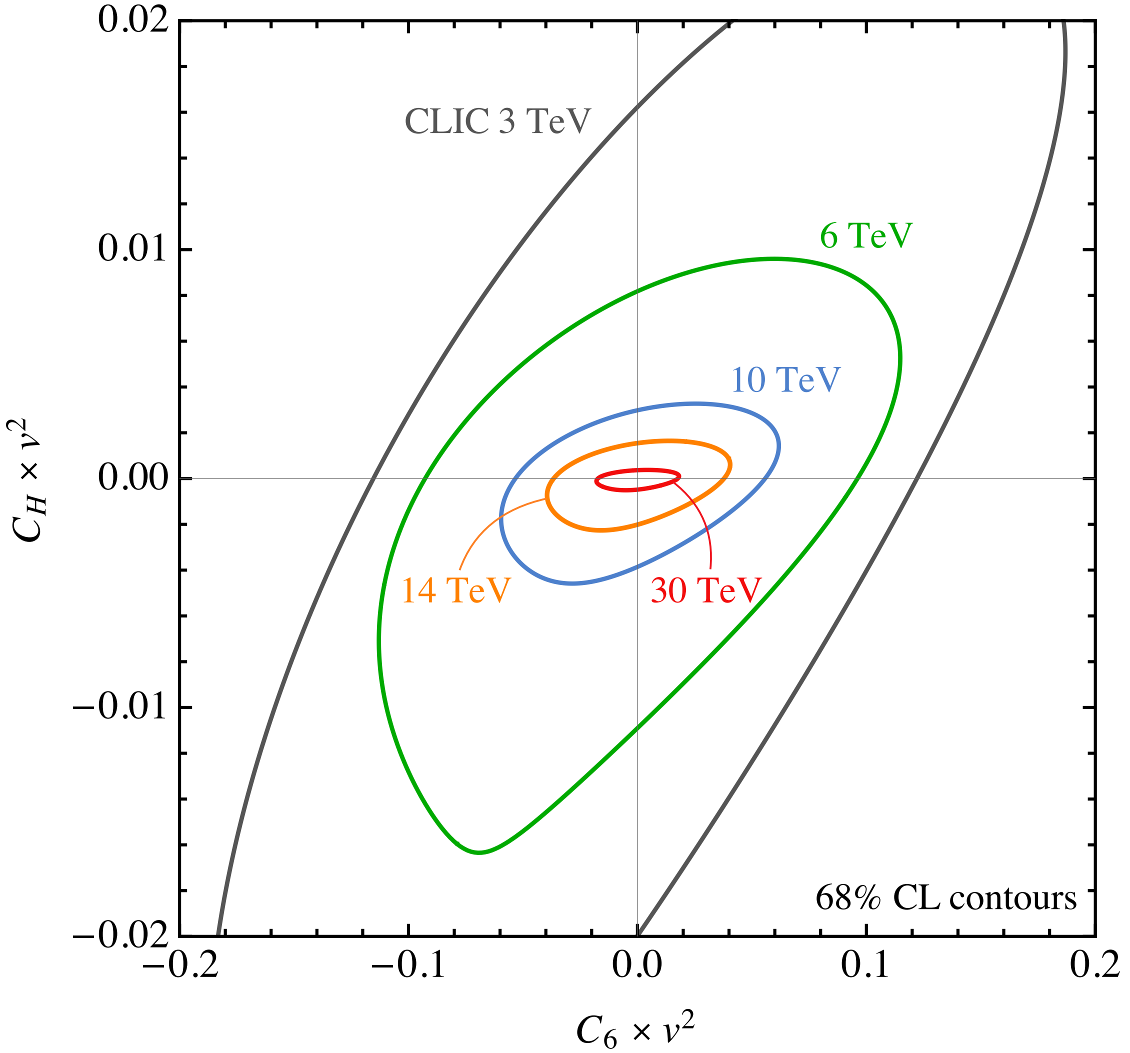
Double Higgs at high mass

- SM Effective Theory: $\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_i C_i \mathcal{O}_i^{(6)} + \cdots$
- Trilinear coupling is affected by two operators: $\kappa_3 = 1 + v^2\left(C_6 - \frac{3}{2}C_H\right)$

$$\mathcal{O}_6 = -\lambda |H|^6 \qquad \mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |H|^2\right)^2$$

Differential analysis in p_T and M_{hh}:



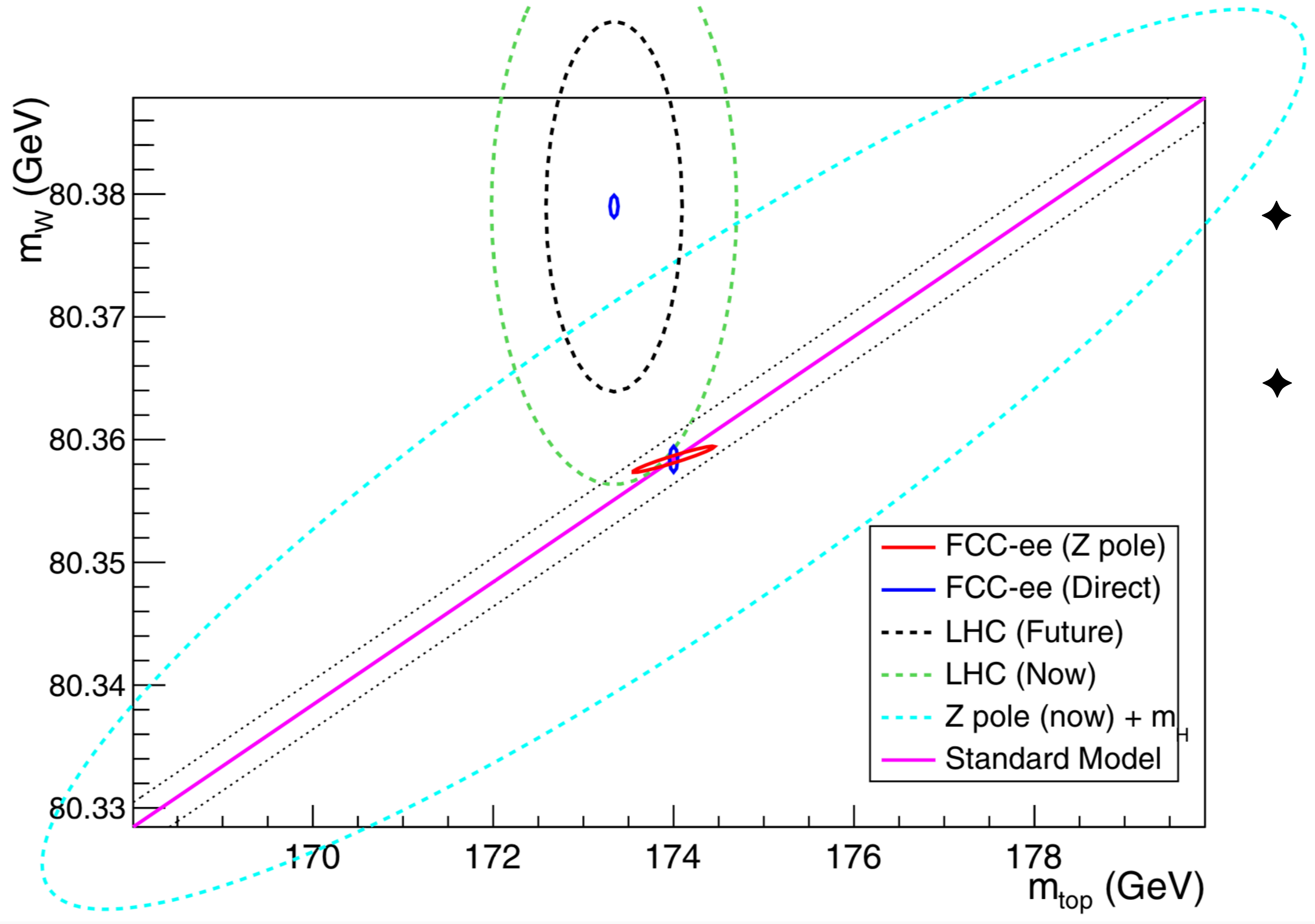


EW precision

- Higgs & EWSB physics ⟷ Ew precision measurements

$$\boxed{\mathcal{O}_T = \left(H^\dagger D^\mu H\right)^2}$$ $\Delta\rho$

$$\boxed{\begin{aligned}\mathcal{O}_W &= \left(H^\dagger \sigma^a D^\mu H\right) D^\nu W^a_{\mu\nu} \\ \mathcal{O}_B &= \left(H^\dagger D^\mu H\right) \partial^\nu B_{\mu\nu}\end{aligned}}$$ $\sin^2\theta_{\text{eff}}$

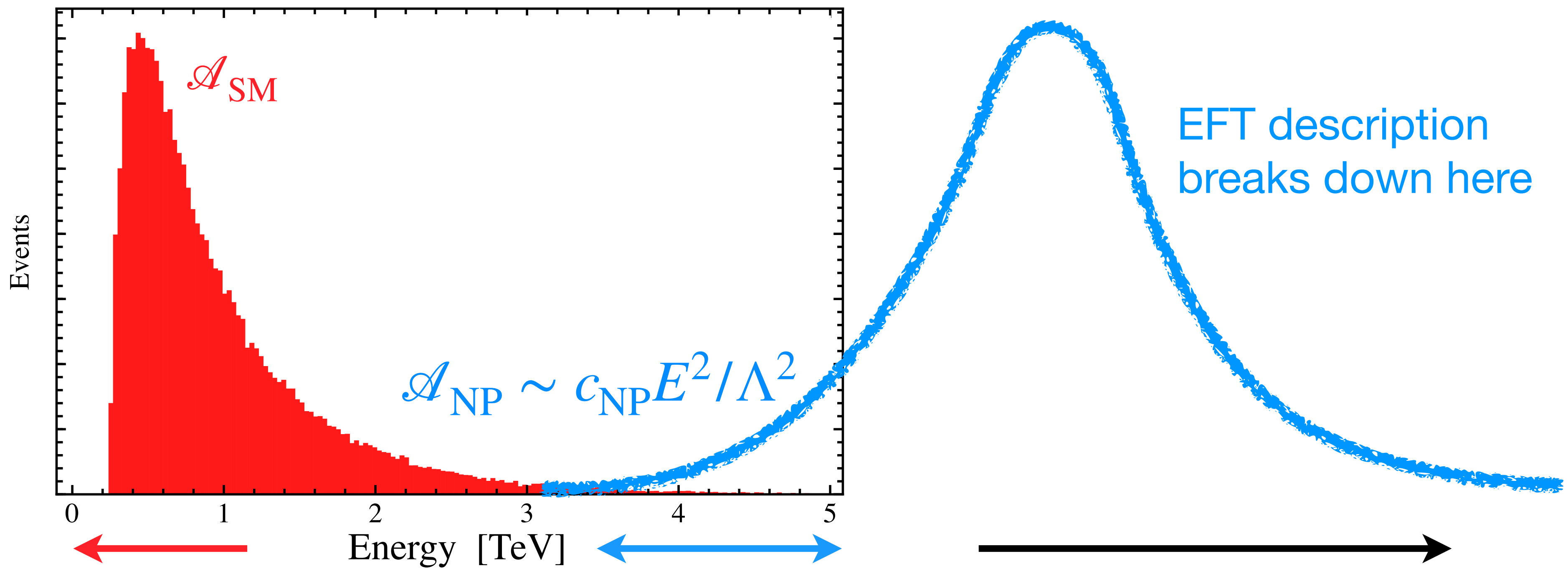


- LEP: 10^7 Z bosons, $\Delta\hat{S} \lesssim 10^{-3}$
- FCC-ee: 6 x 10^{12} Z bosons ultimate precision at the Z pole, limited by syst. and th. errors

$$\Delta\hat{S} \sim \frac{m_W^2}{M_{\text{NP}}^2} \lesssim \text{few} \times 10^{-5}$$

EW precision at high-energy

- NP effects are more important at high energies $\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda^2} \sum C_i \mathcal{O}_i$



EFT description breaks down here

Precision SM measurements

High energy (indirect) probes

direct searches

$$\frac{\Delta\sigma(E)}{\sigma_{\rm SM}(E)} \propto \frac{E^2}{\Lambda^2_{\rm BSM}} \approx \begin{cases} 10^{-6}, & E \sim 100\,{\rm GeV} \\ 10^{-2}, & E \sim 10\,{\rm TeV} \end{cases}$$

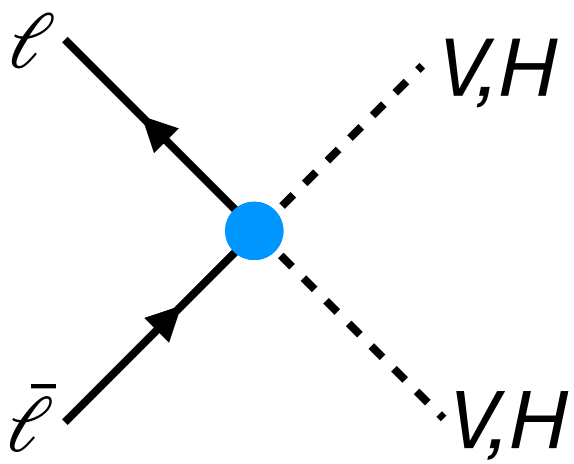
- Effective at LHC, FCC-hh, CLIC: “energy helps accuracy”…

Farina et al. 1609.08157, Franceschini et al. 1712.01310, …

… taken to the extreme at a μ-collider with 10’s of TeV!

Example: high-energy di-bosons

- Longitudinal 2 → 2 scattering amplitudes at high energy:



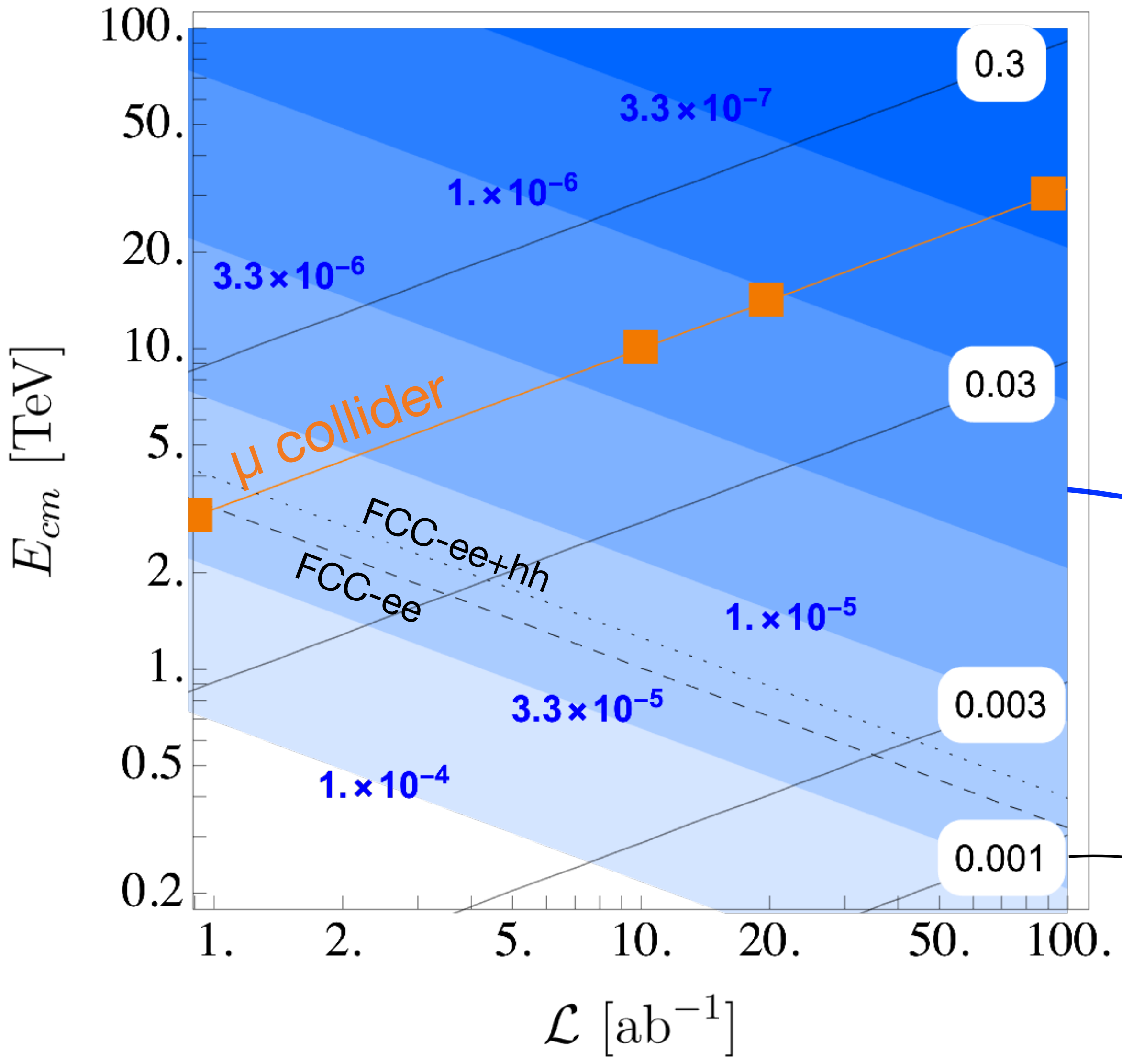
$$\ell^+\ell^- \to W_L^+ W_L^-$$

$$\ell^+\ell^- \to Z_L H$$

Determined by the same two operators that affect also EWPT (in flavor-universal theories):

$$\mathcal{O}_W = \left(H^\dagger \sigma^a D^\mu H\right) D^\nu W^a_{\mu\nu}$$

$$\mathcal{O}_B = \left(H^\dagger D^\mu H\right) \partial^\nu B_{\mu\nu}$$



related with Z-pole observables

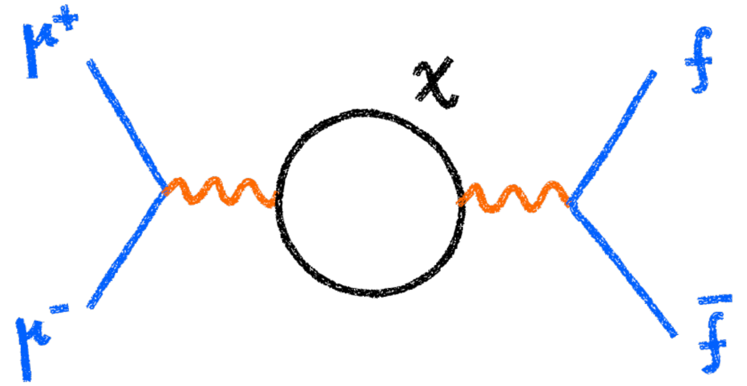
$$\hat{S} = m_W^2 (C_W + C_B)$$

LEP: 10^{-3}, FCC: few 10^{-5} **MuC:** 10^{-6}

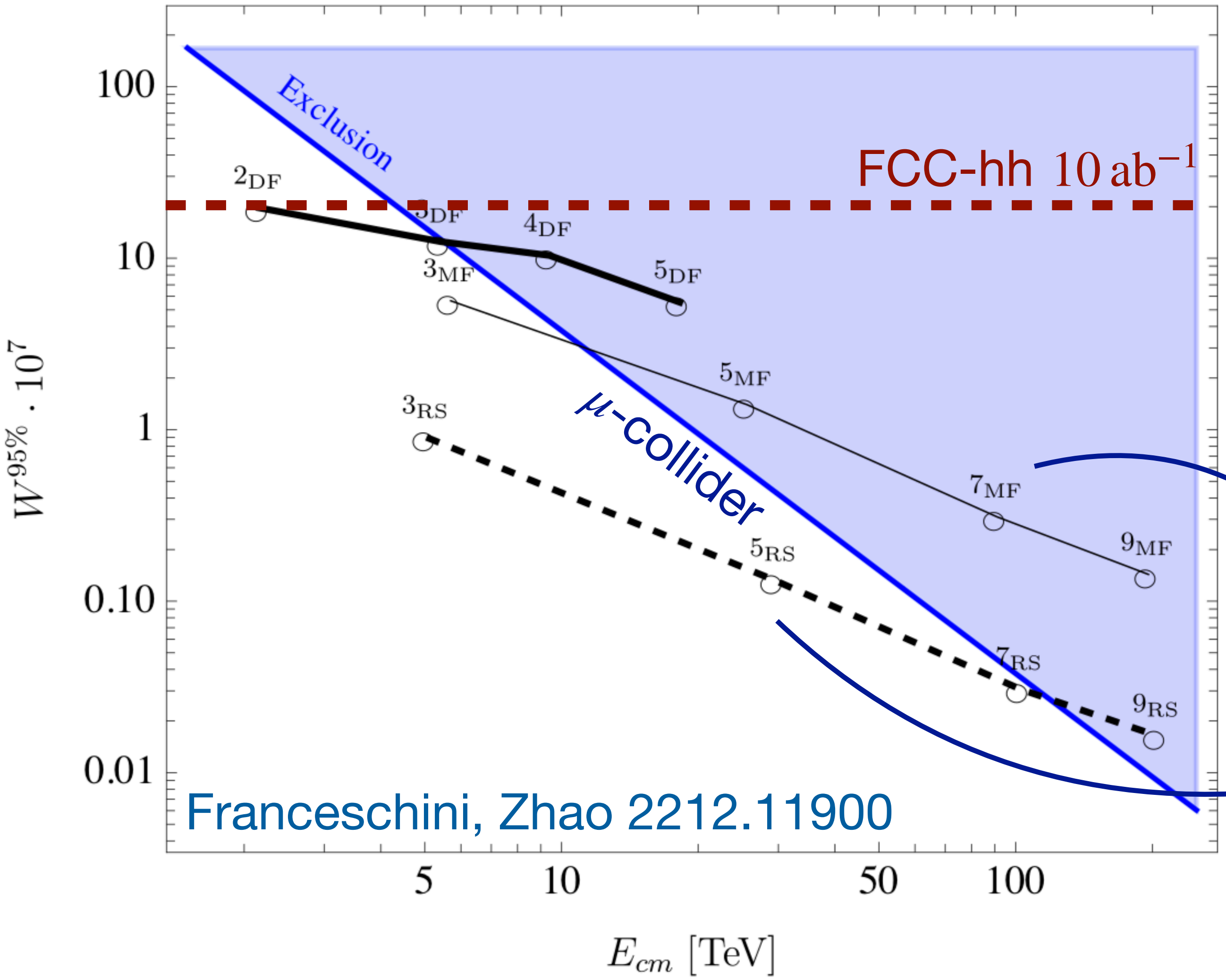
precision of measurement

EW-charged matter

- All EW multiplets contribute to high-energy 2 → 2 fermion scattering: effects that grow with energy, can be tested at μ collider



can be WIMP dark matter if M ~ few TeV



Franceschini, Zhao 2212.11900

$$\hat{W} \approx 10^{-7} \times \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{DM}}}\right)^2 n^3 \propto 1/n^2$$

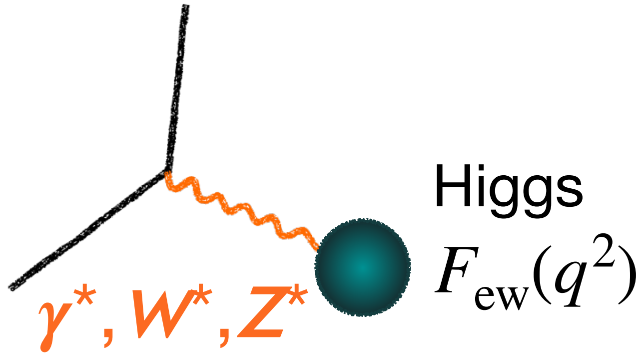
$$\hat{Y} \approx 10^{-7} \times \left(\frac{1\,\mathrm{TeV}}{M_{\mathrm{DM}}}\right)^2 Y^2 n \propto 1/n^4$$

right of blue line: can be tested indirectly

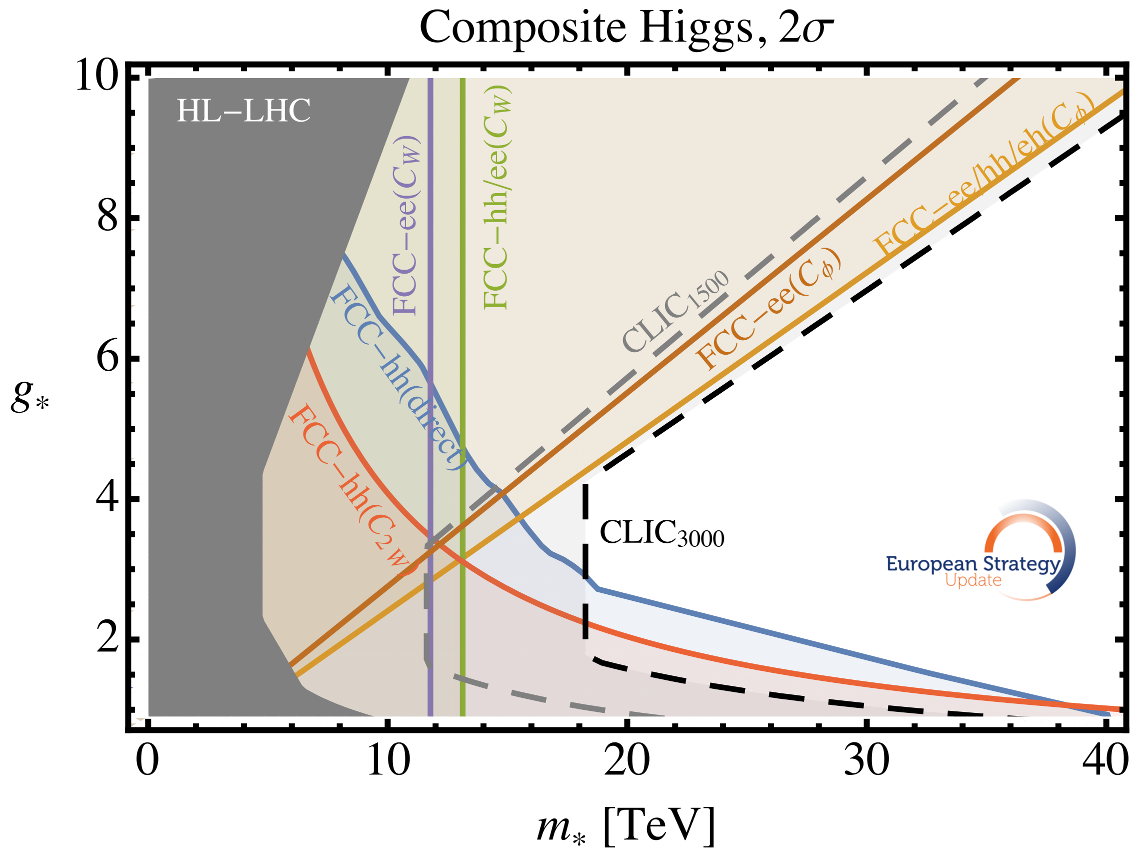
left of blue line: can be tested directly

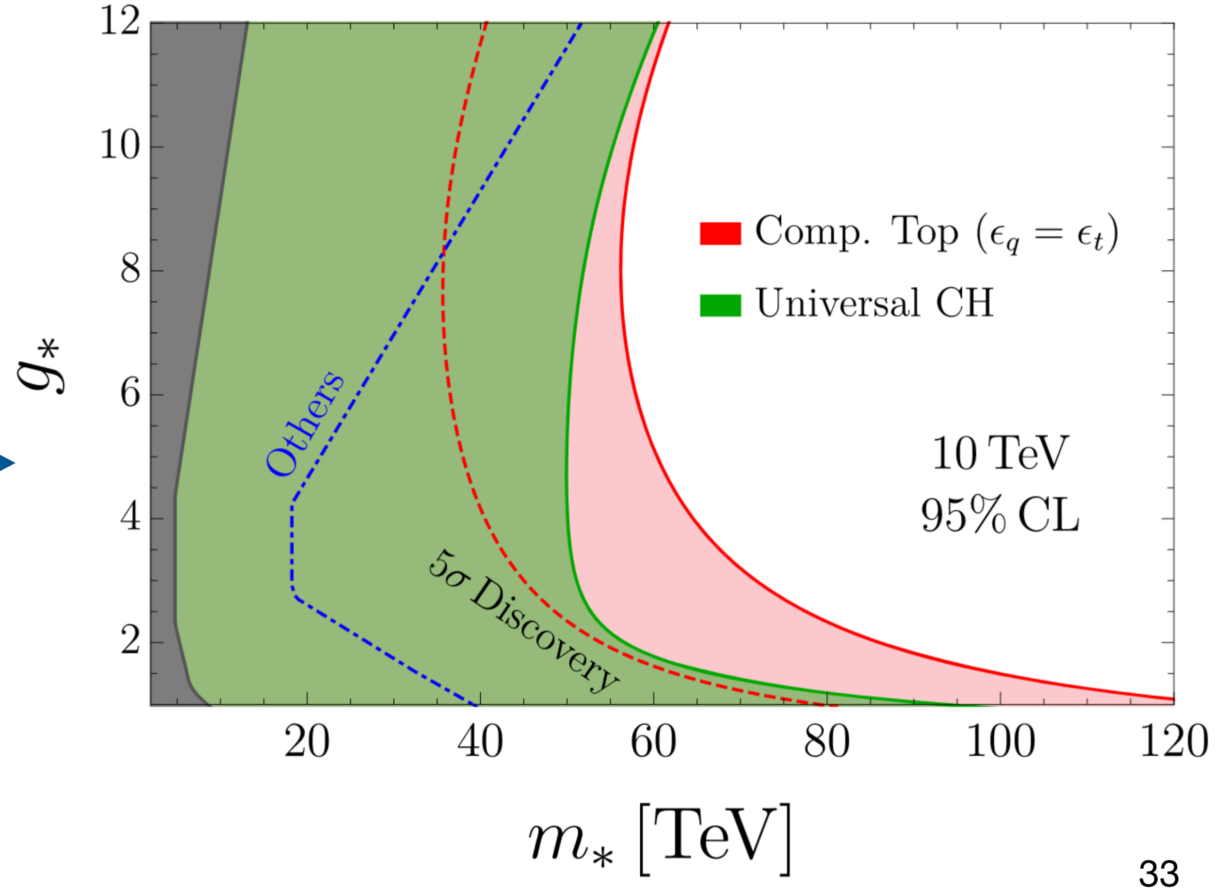
High-energy probes: EW & Higgs physics

- High-energy processes at a 10–30 TeV lepton collider are able to probe EW new physics scales of 100 TeV or more.
  - 10x higher than ultimate precision at Z pole



- Example: new physics with mass $m_\star$ and coupling $g_\star$ to Higgs





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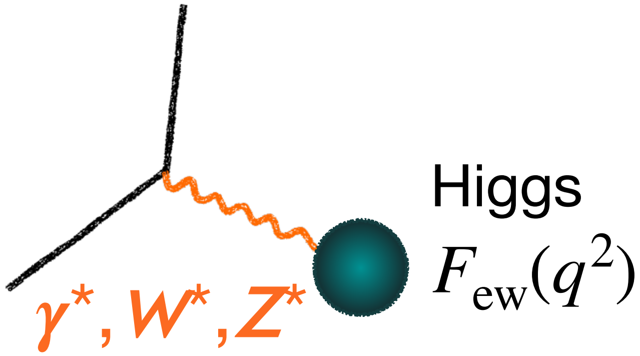
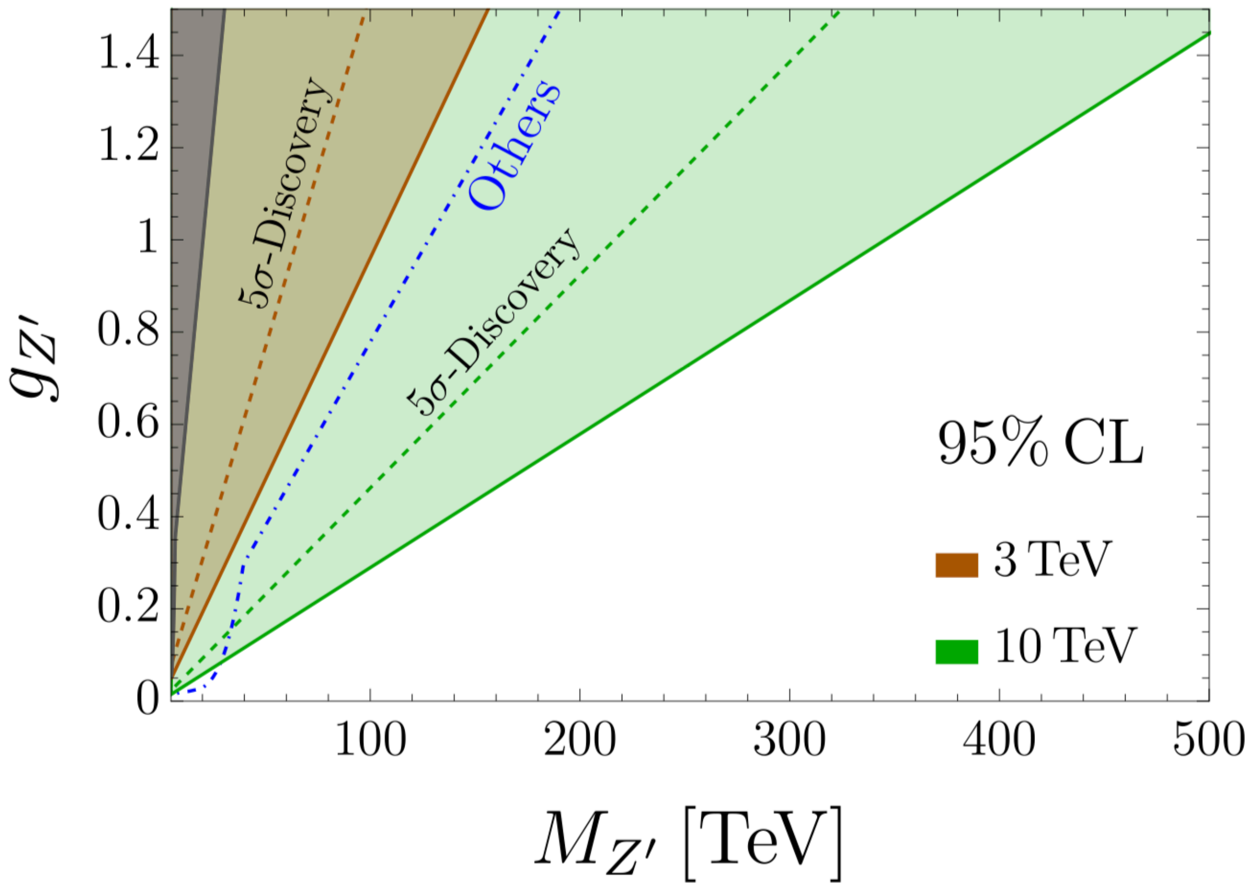
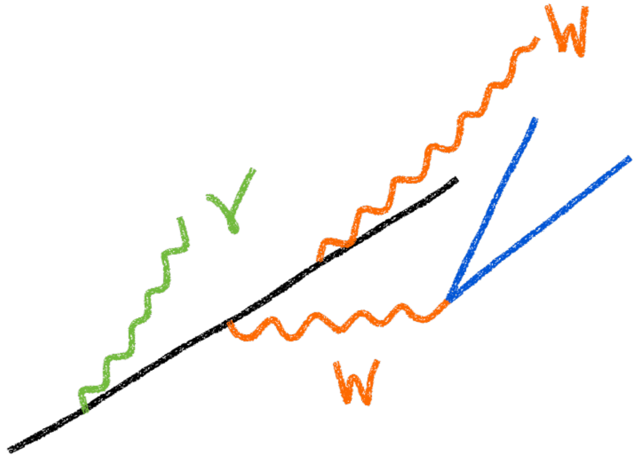


- Example: heavy resonance with mass $m_{Z'}$ and coupling $g_{Z'}$ to fermions



EW radiation



EW radiation becomes important at multi-TeV energies!

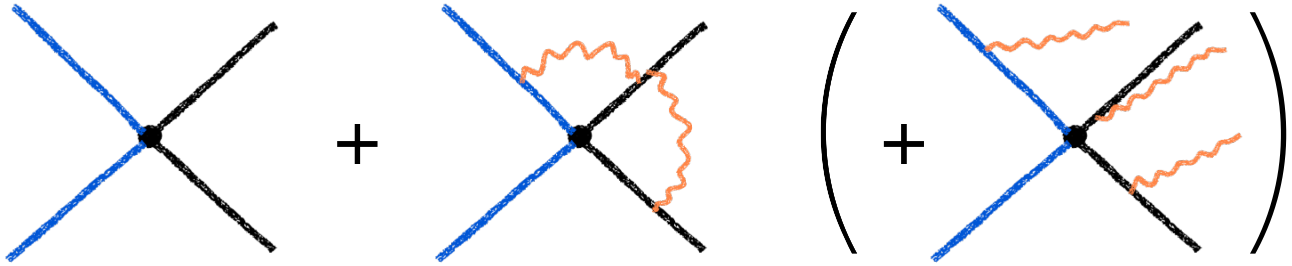
Especially relevant for muon collider, but also FCC-hh…

- $m_{W,Z} \ll E$: γ, W, Z are all similar!
- Multiple gauge boson emission is not suppressed

Sudakov factor $\frac{\alpha}{4\pi}\log^2\left(\frac{E^2}{m_W^2}\right) \times$ Casimir ≈ 1 for E ~ 10 TeV

☛ Which cross-section? Exclusive, (semi-)inclusive, depending on amount of radiation included

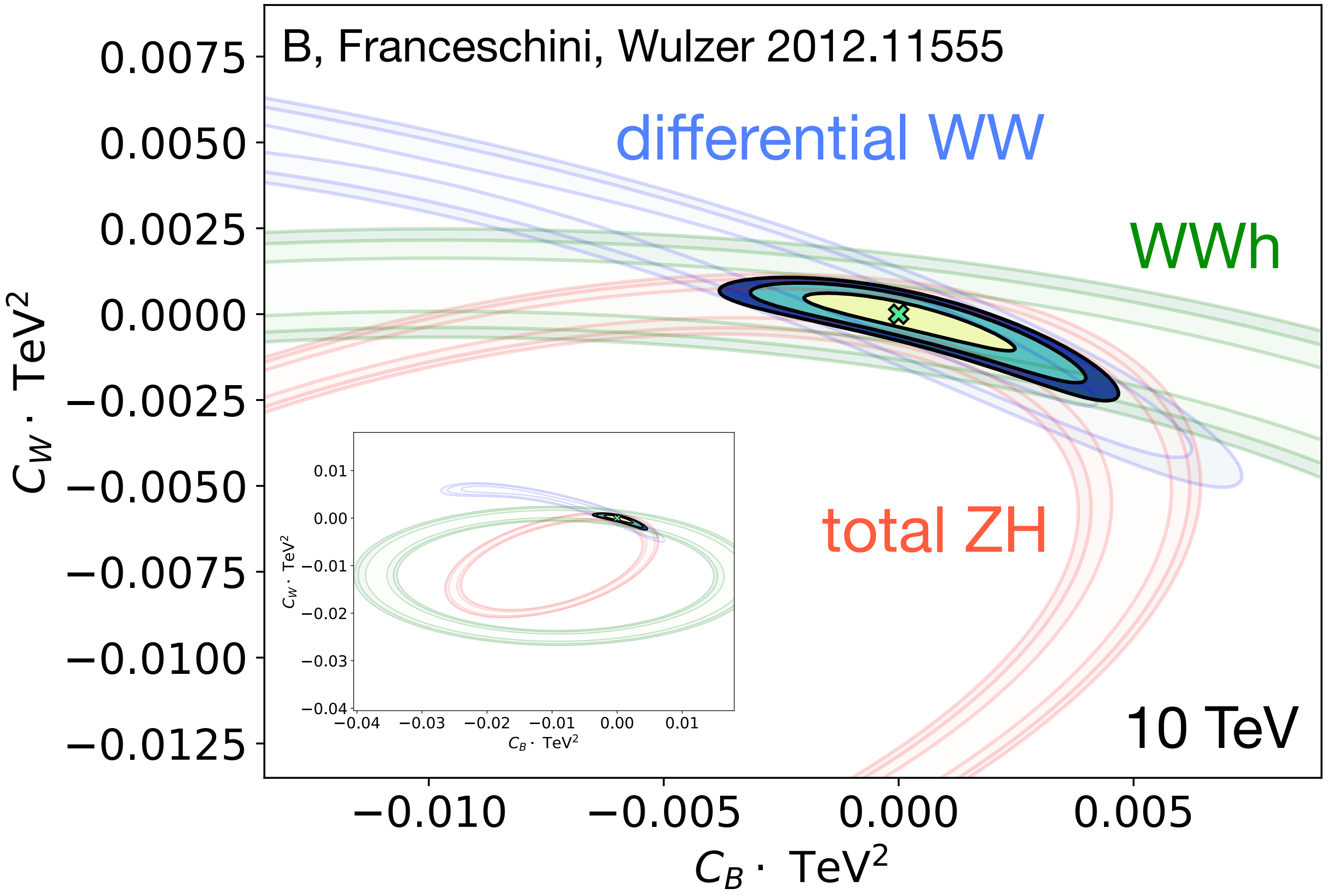
see Chen, Glioti, Rattazzi, Ricci, Wulzer 2202.10509



☛ Initial state is EW-charged:
(Precise) resummation of double logs needed. Goal: % or ‰ precision

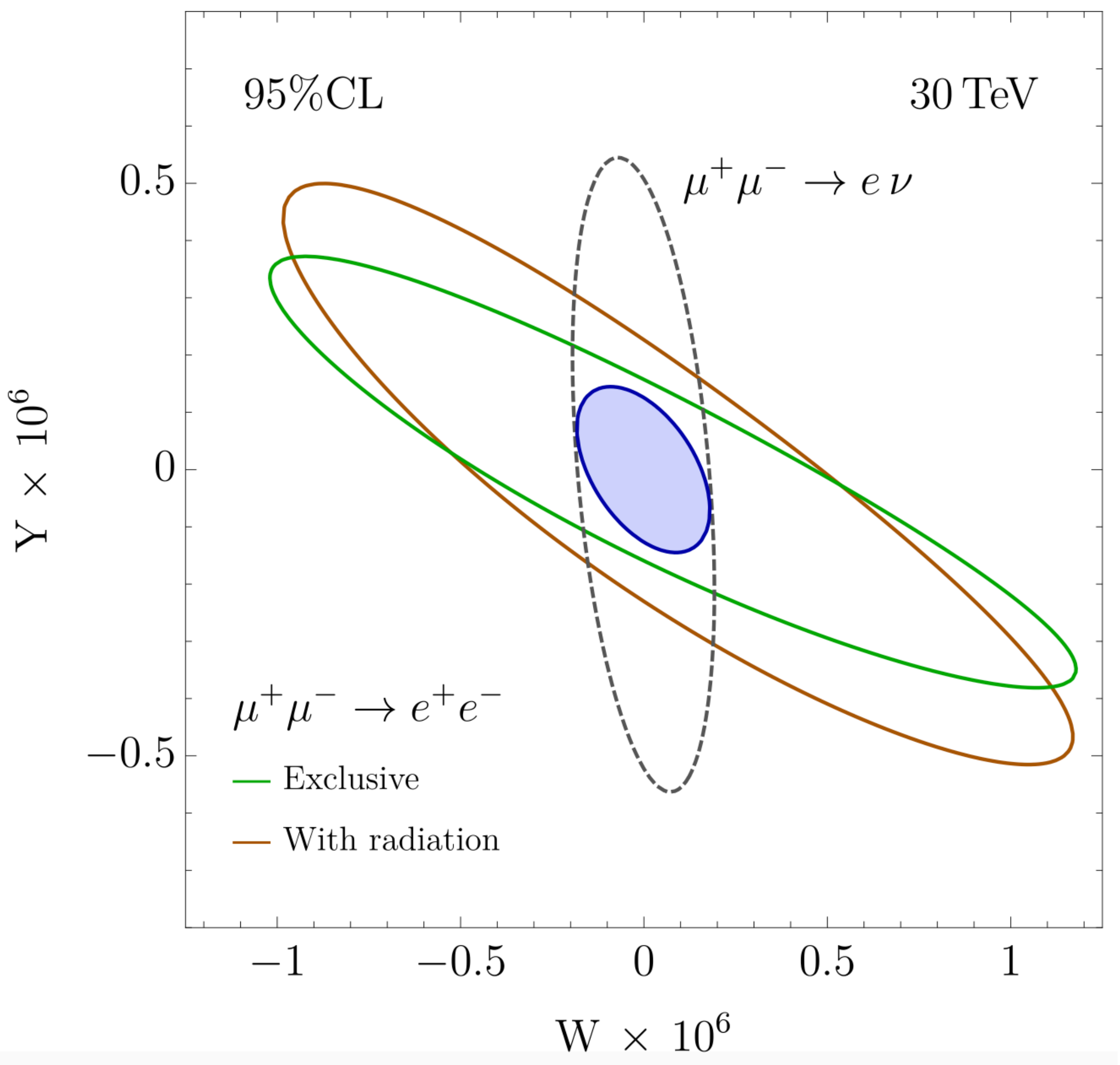
☛ Could one define EW jets? Neutrino “jet tagging”?

High-energy probes: radiation



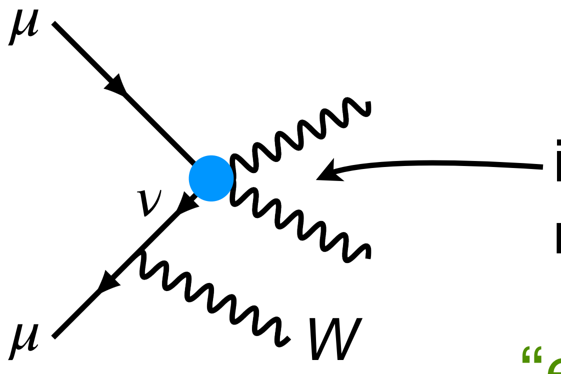
B, Franceschini, Wulzer 2012.11555

Chen, Glioti, Rattazzi, Ricci, Wulzer 2202.10509



Gauge boson radiation important:

soft W emission allows to access charged processes $\ell\nu \to W^\pm Z, W^\pm H$



independent measurement of C_W

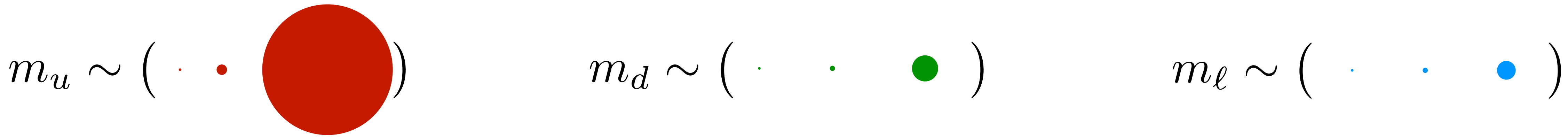
- ✦ contains new physical information!
- ✦ need to properly define inclusive observables, resummation of logs, …

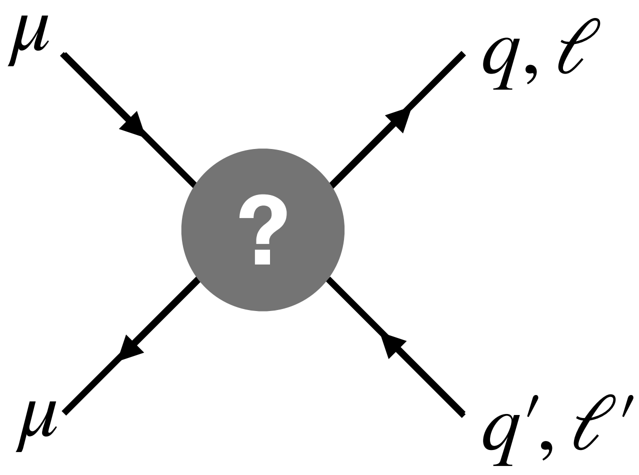
“effective neutrino approximation”

Flavour: muons vs. electrons

- ✦ New Physics (especially if related to the Higgs sector) could distinguish the different families of fermions.

- ✦ EW interactions are flavour-universal: an accidental property of the gauge lagrangian, *not* a fundamental symmetry of nature!
  - ‣ Example: Yukawa couplings, the only non-gauge interactions in the SM, violate flavour universality maximally!





A muon collider collides 2nd generation particles: could test flavour structure

➡ High-energy probes can be even more powerful in this case: enhancement wrt. low energy observables can be as large as $(E/m_\mu)^2$

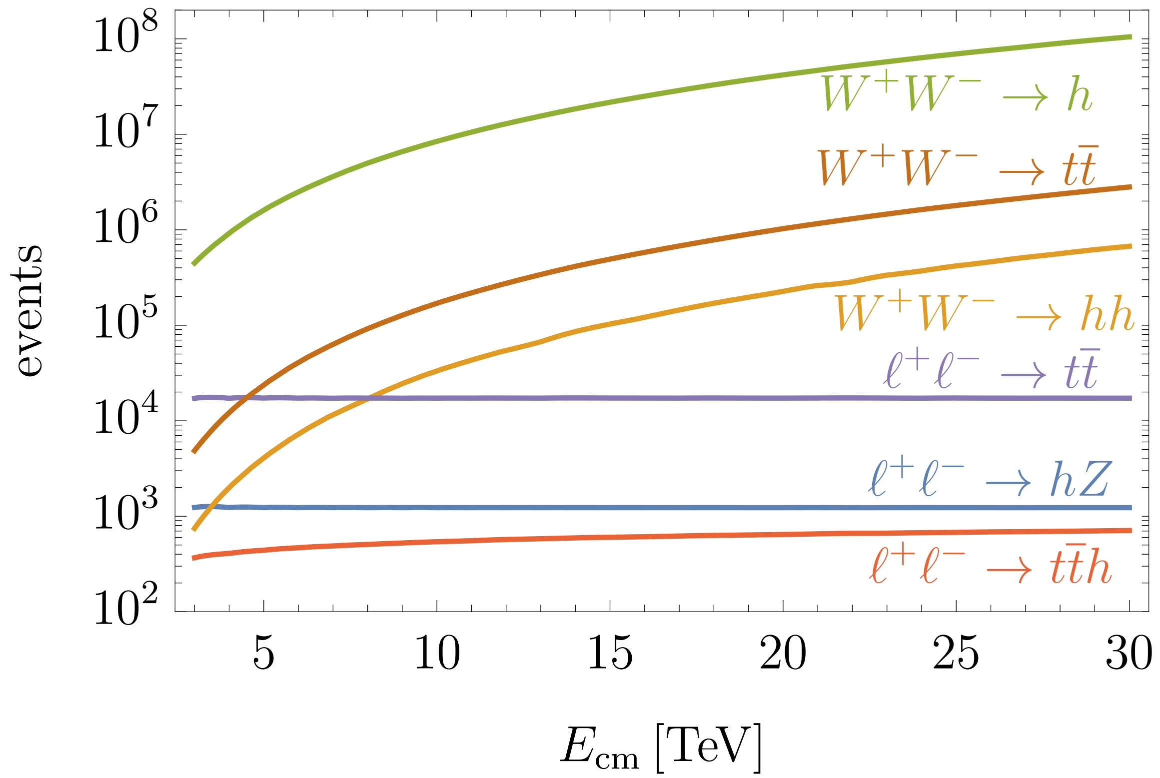
Flavor and precision

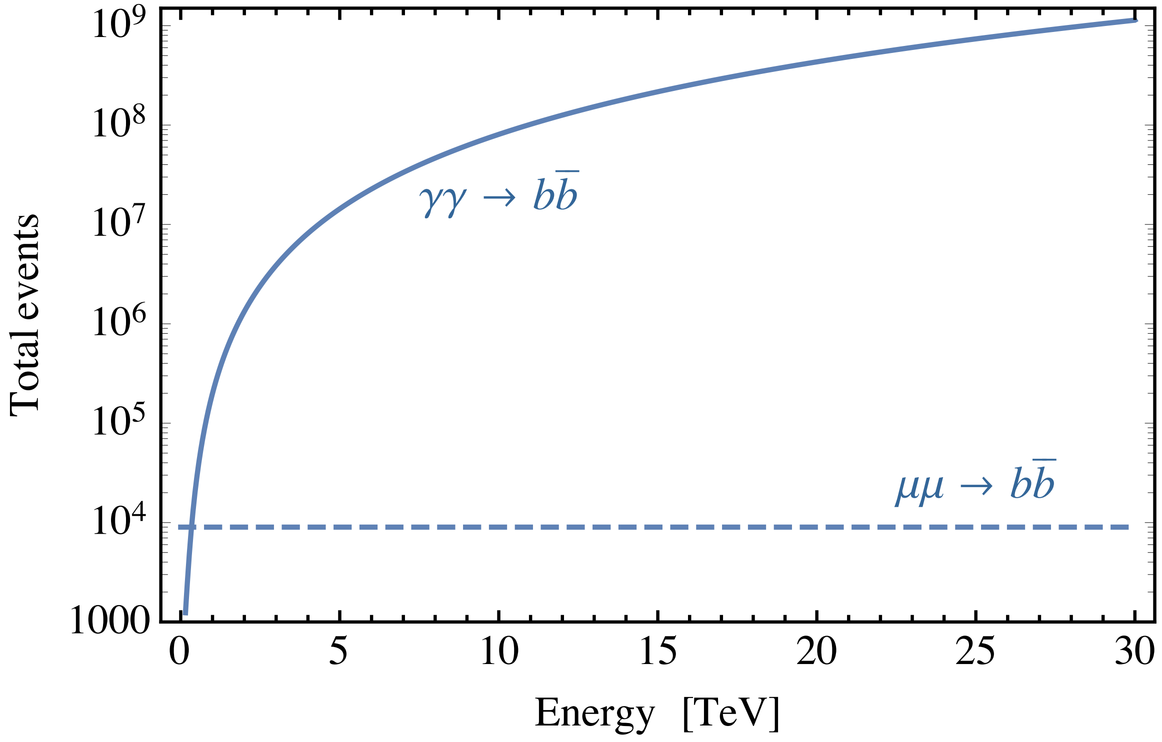
- Flavor processes: rare decays & tiny effects

$$\mathrm{BR}(B_s \to \mu\mu) \sim 10^{-9}, \qquad \mathrm{BR}(\tau \to 3\mu) \lesssim 10^{-8}, \qquad \Delta a_\mu \approx 10^{-9}$$

➡ need billions of events, usually probed by means of high-intensity experiments

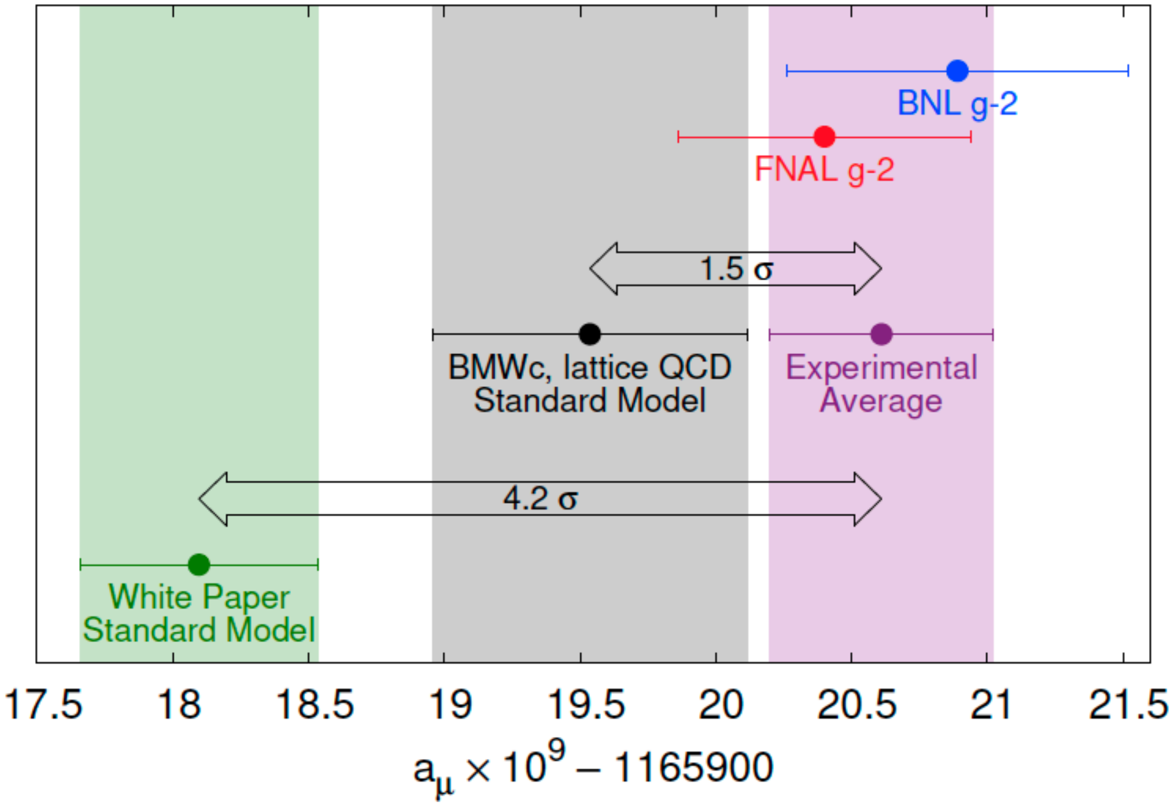
- Muon-collider: very large number of (clean) EW particles, but overall event rate not comparable to flavor factories





Flavour @ muon collider: the muon g-2

- Example: muon g-2. Can it be tested at high energies at a muon collider?



$$\Delta a_\mu = ???$$

Flavour @ muon collider: the muon g-2

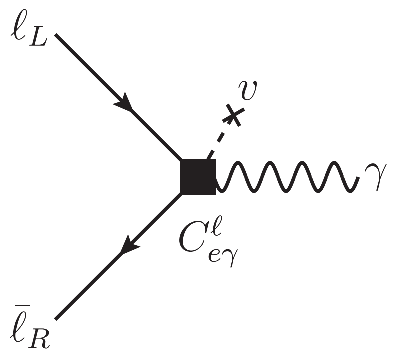
- Example: muon g-2. Can it be tested at high energies at a muon collider?

- If new physics is heavy: EFT!

One dim. 6 operator contributes at tree-level: $\mathscr{L}_{g-2} = \frac{C_{e\gamma}}{\Lambda^2} H (\bar{\ell}_L \sigma_{\mu\nu} e_R)\, e F^{\mu\nu}$ + h.c.

At low energy

$$\Delta a_\mu = \frac{4 m_\mu v}{\Lambda^2} C_{e\gamma} \approx 3 \times 10^{-9} \times \left(\frac{140\,\mathrm{TeV}}{\Lambda}\right)^2 C_{e\gamma}$$

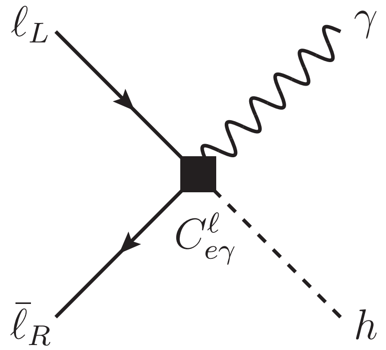


Dipole operator generates both Δa$_\mu$ and μμ → hγ

B, Paradisi 2012.02769

At high energy

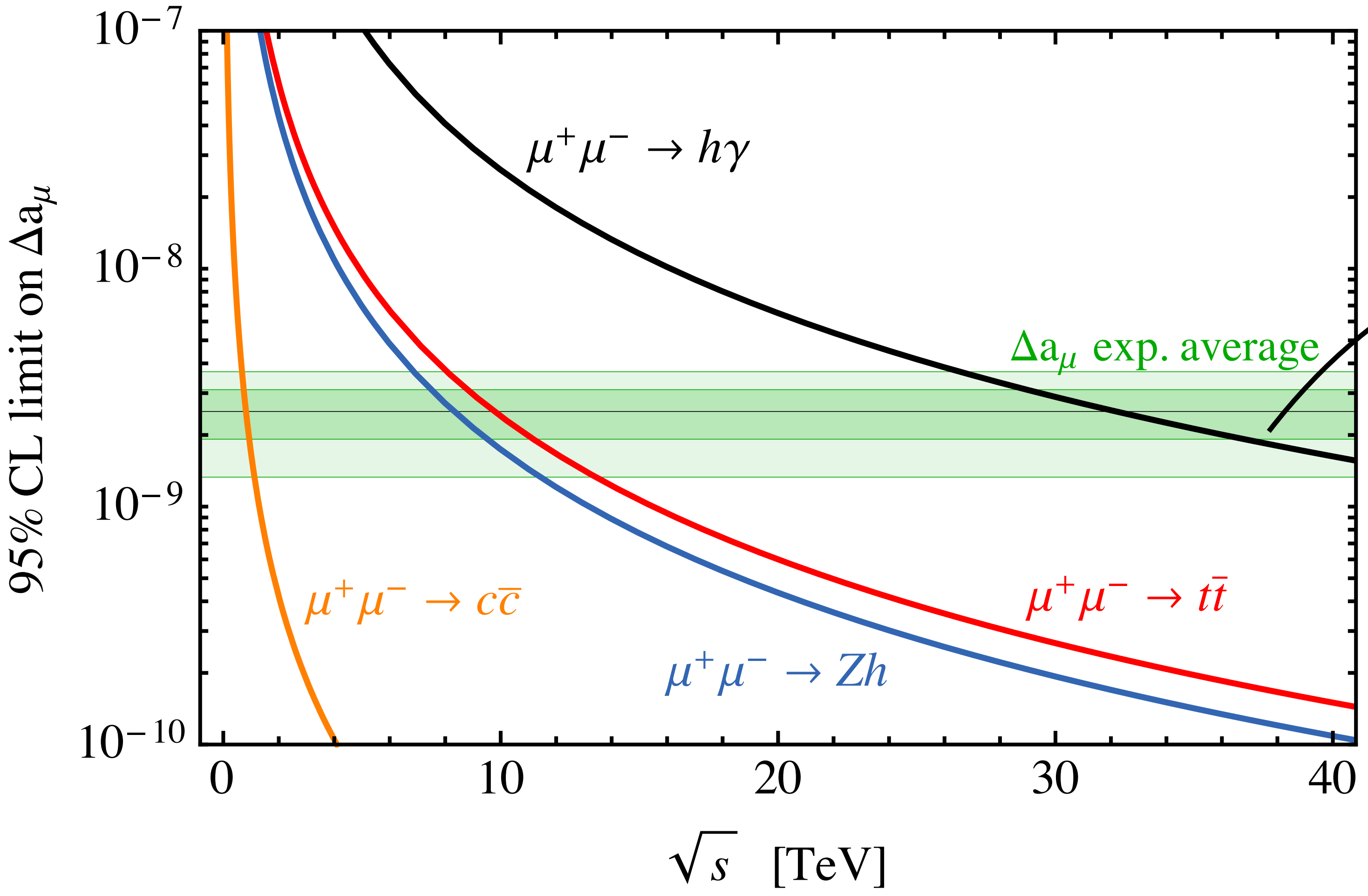
$$\sigma_{\mu^+\mu^- \to h\gamma} = \frac{s}{48\pi} \frac{|C_{e\gamma}|^2}{\Lambda^4} \approx 0.7\,\mathrm{ab} \left(\frac{\sqrt{s}}{30\,\mathrm{TeV}}\right)^2 \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2$$



$$N_{h\gamma} = \sigma \cdot \mathscr{L} \approx \left(\frac{\sqrt{s}}{10\,\mathrm{TeV}}\right)^4 \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2$$

need E > 10 TeV

Muon g-2 @ muon collider



Exp. value of Δa_μ can be tested at 95% CL at a 30 TeV collider!

(with reasonable assumptions on detector performance)

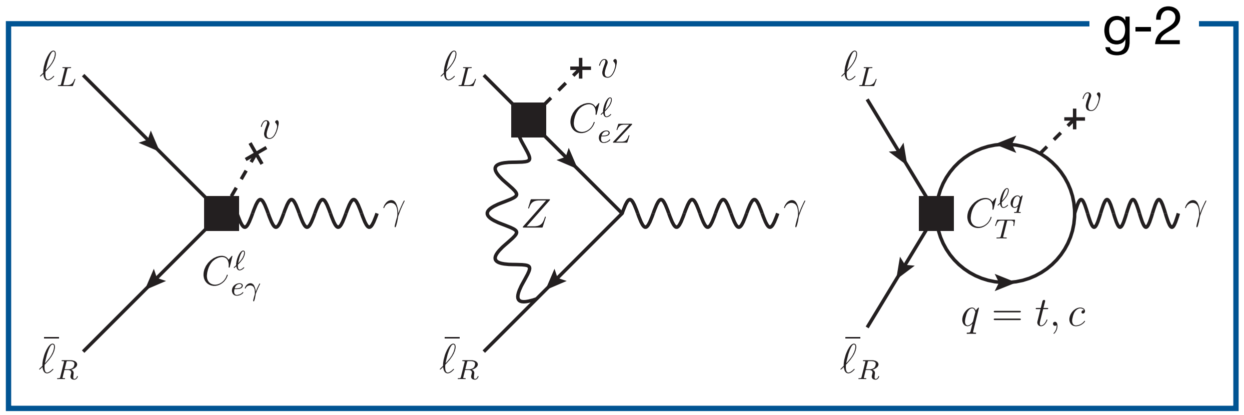
This result is completely model-independent!

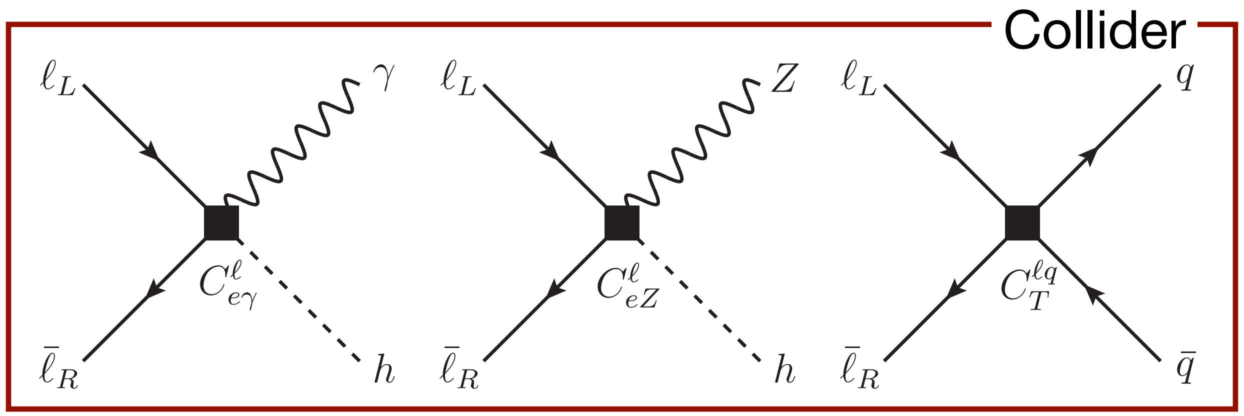
B, Paradisi 2012.02769

- Other operators enter g-2 at 1 loop:

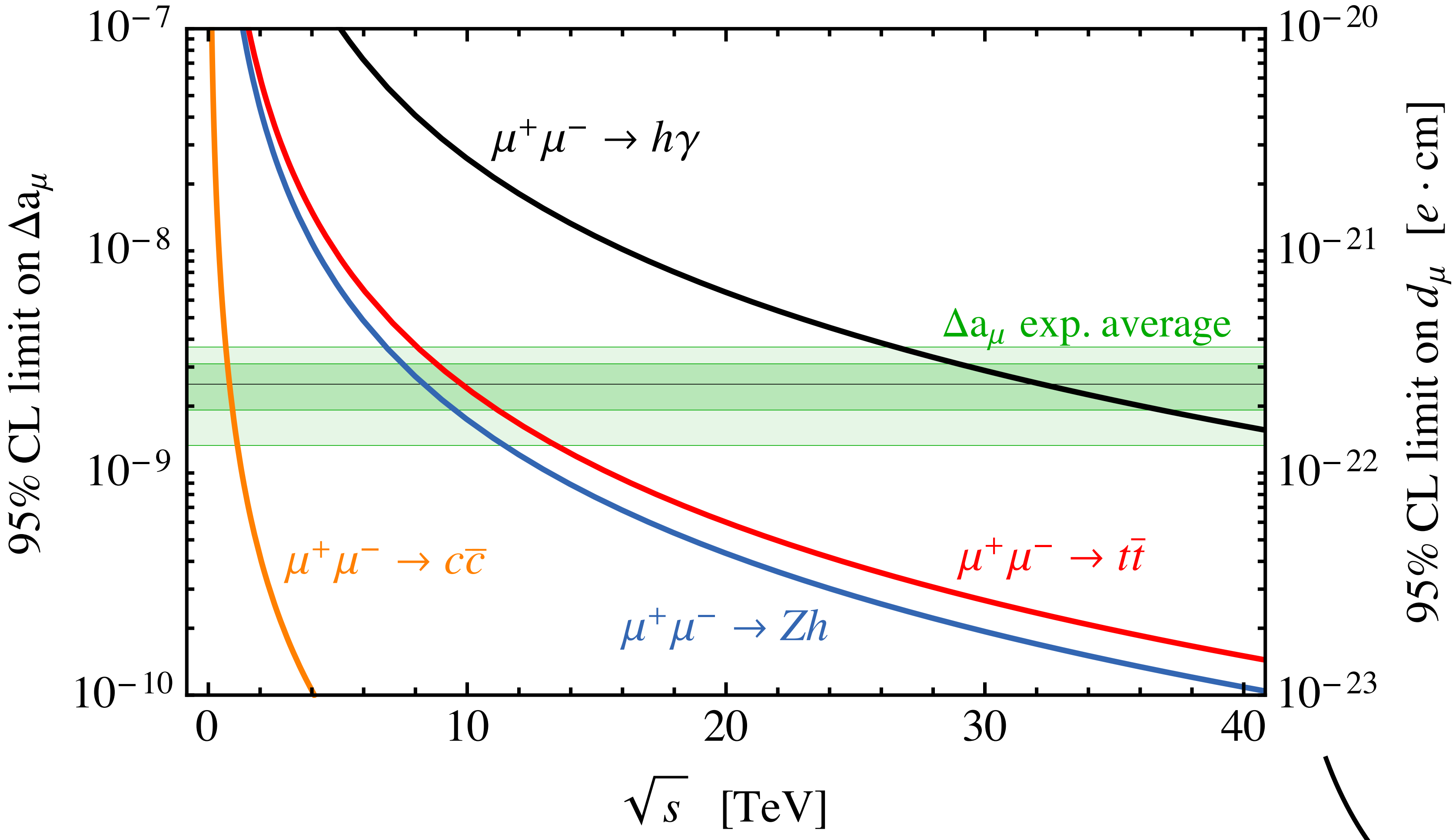
$$\Delta a_\mu \approx \left(\frac{250\,\text{TeV}}{\Lambda^2}\right)^2 \left(C_{e\gamma} - \frac{C_{Tt}}{5} - \frac{C_{Tc}}{1000} - \frac{C_{eZ}}{20}\right)$$

- Full set of operators with $\Lambda \gtrsim 100$ TeV can be probed at a high-energy muon collider





Muon g-2 @ muon collider



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This result is completely model-independent!

B, Paradisi 2012.02769

Muon EDM for free!

$$\Delta a_\mu = \frac{4 v m_\mu \mathrm{Re}(C_{e\gamma})}{\Lambda^2}$$

$$d_\mu = \frac{2v\,\mathrm{Im}(C_{e\gamma})}{\Lambda^2} = \frac{\Delta a_\mu}{2m_\mu} \tan\phi_\mu\, e$$

Collider constrains $|C_{e\gamma}|^2$

$$\Rightarrow \quad d_\mu \lesssim 10^{-22}\, e \cdot \mathrm{cm}$$

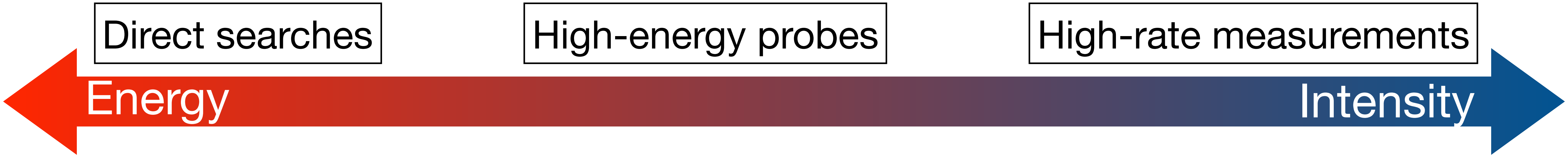
3 o.o.m. stronger than present bound!

Summary

Two colliders at once

in a high-energy muon collider

Energy AND Precision



Can probe 10 TeV EW particles directly and
100 TeV scales indirectly

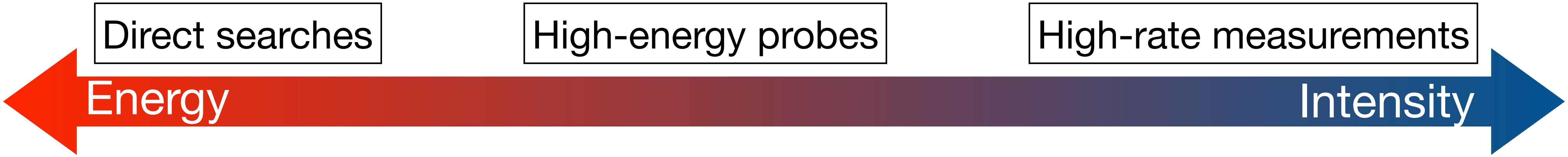
➡ the machine to discover physics of EWSB & Dark Matter

Summary

Two colliders at once

in a high-energy muon collider

Energy AND Precision

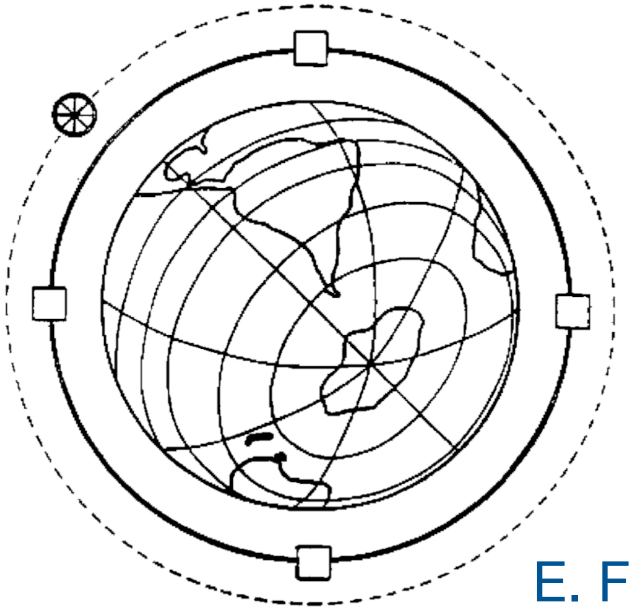

Direct searches

High-energy probes

High-rate measurements

Energy

Intensity


E. Fermi, 1954

… could become reality
if we manage to overcome
the technological challenges!

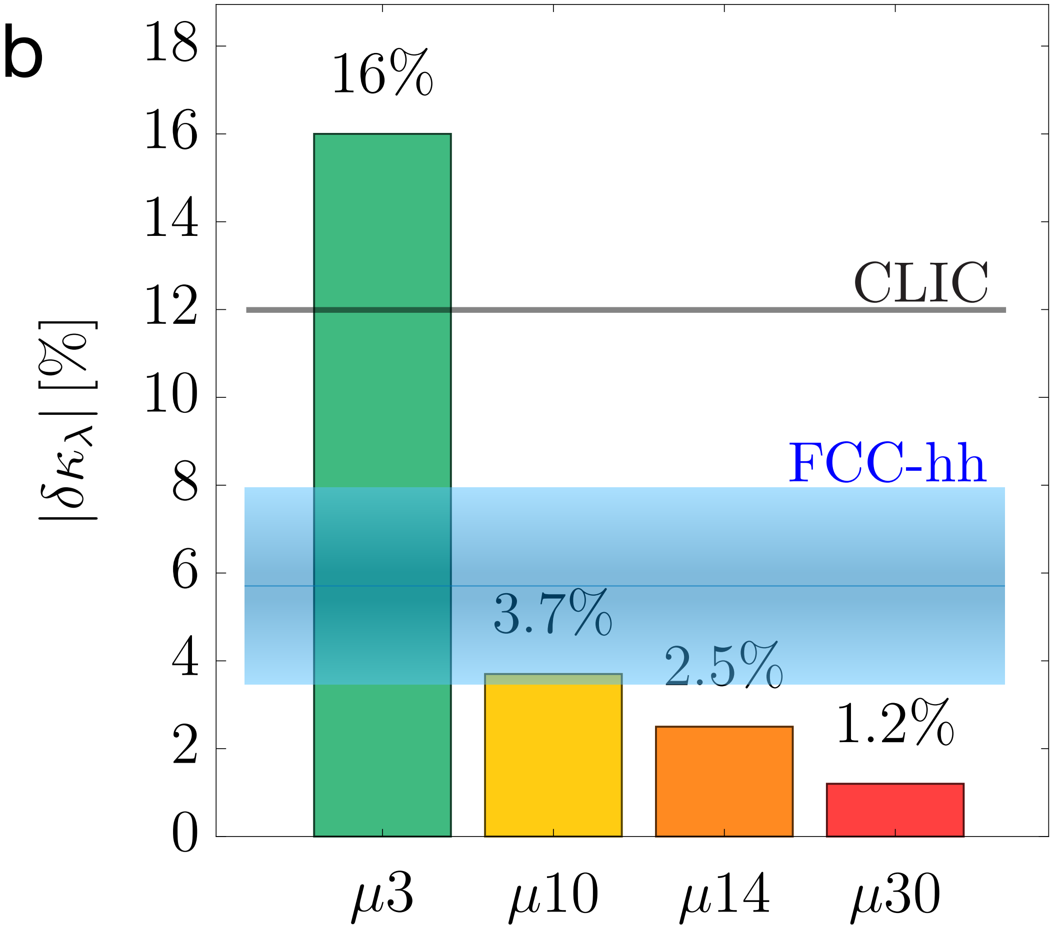
Backup

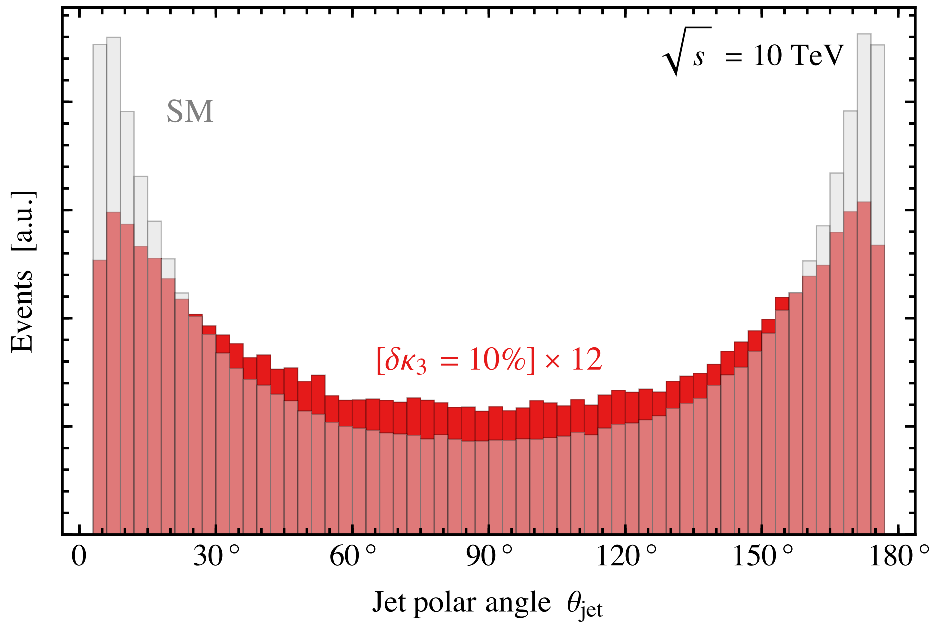
Double Higgs production

- Reach on Higgs trilinear coupling: hh → 4b

E [TeV]	$\mathscr{L}$ [ab^{-1}]	N_{rec}	$\delta\kappa_3$
3	5	170	~ 10%
10	10	620	~ 4%
14	20	1340	~ 2.5%
30	90	6'300	~ 1.2%

B, Franceschini, Wulzer 2012.11555,
Han et al. 2008.12204, Costantini et al. 2005.10289





- Weak dependence on angular acceptance (signal is in the central region)
- Some dependence on detector resolution (to remove backgrounds)

B, Franceschini, Wulzer 2012.11555
see also CLIC study 1901.05897

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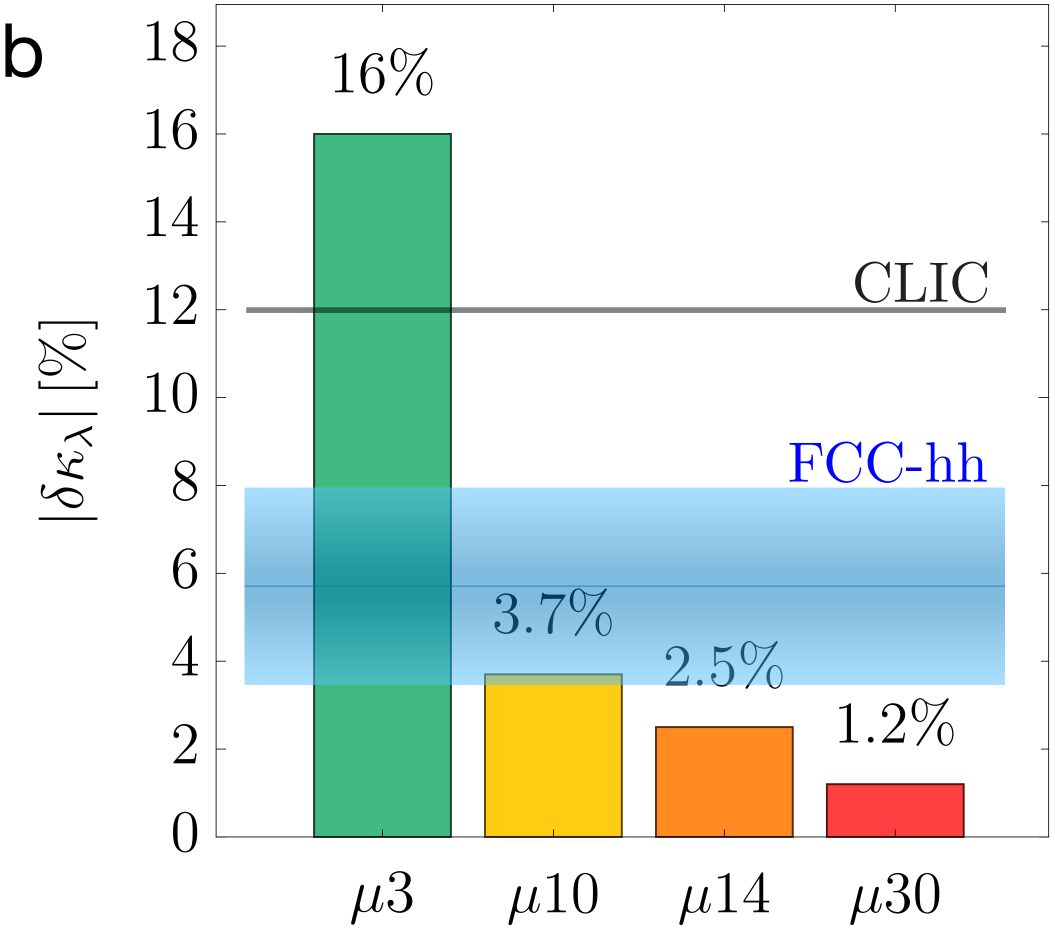

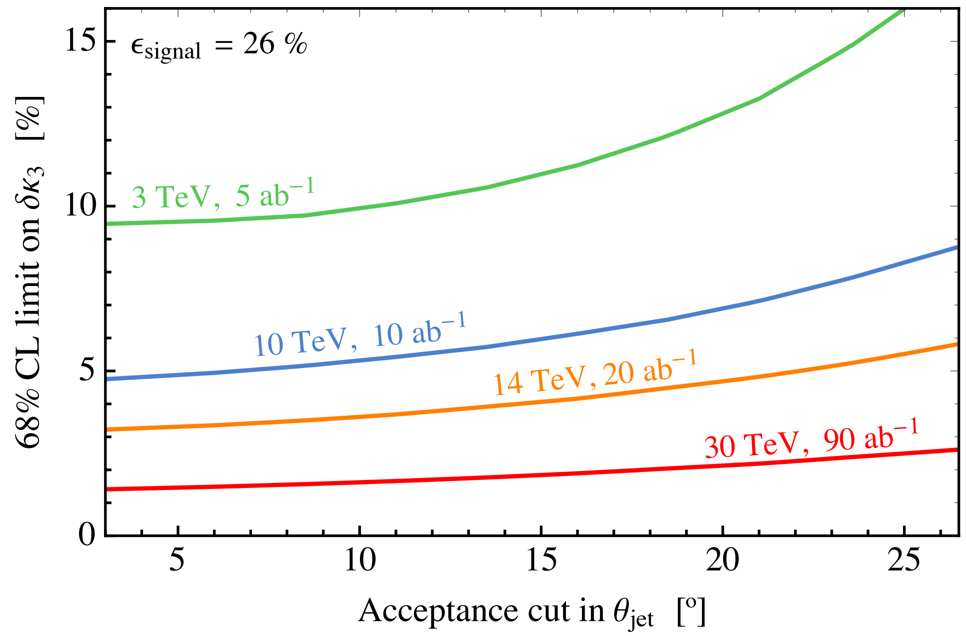

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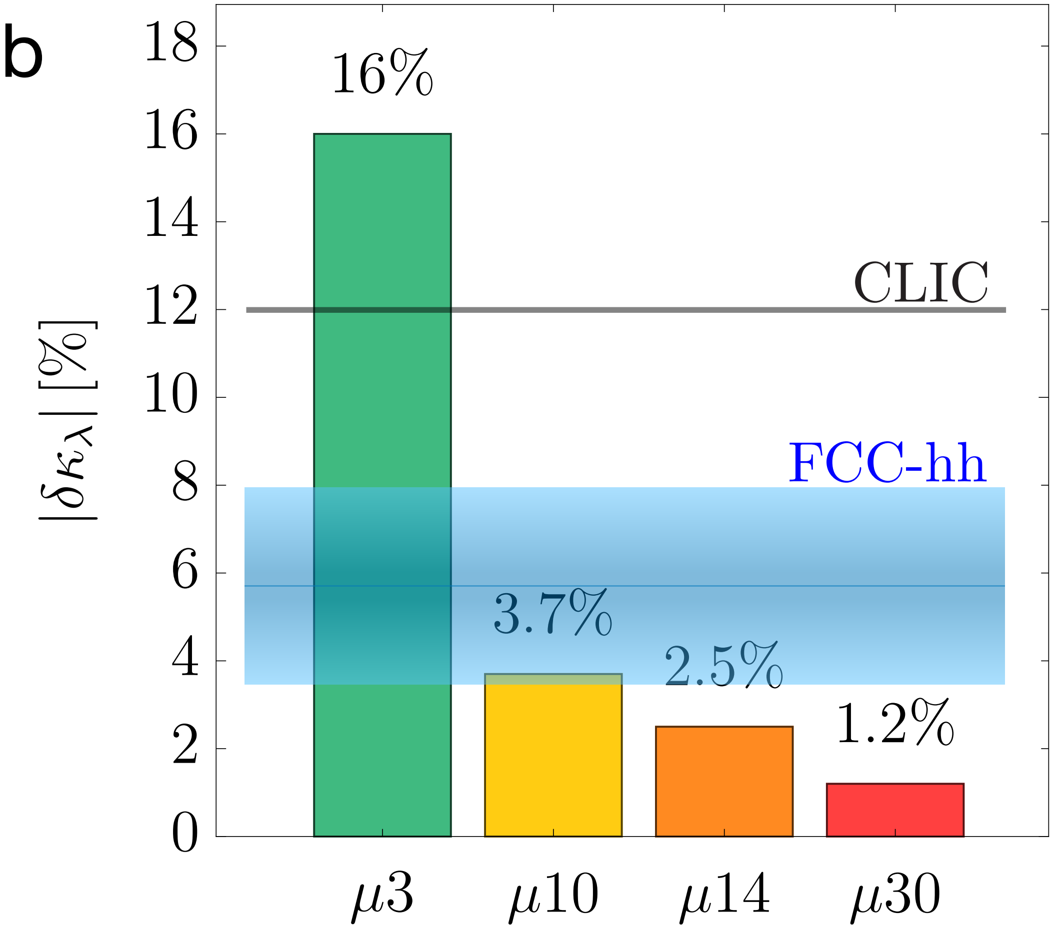

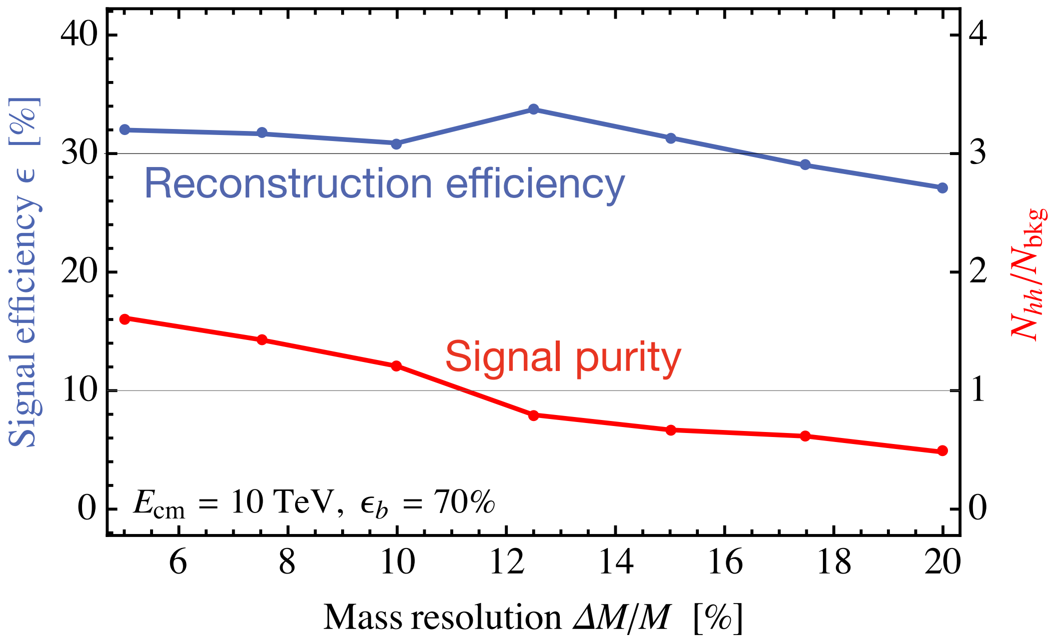

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B, Franceschini, Wulzer 2012.11555
see also CLIC study 1901.05897

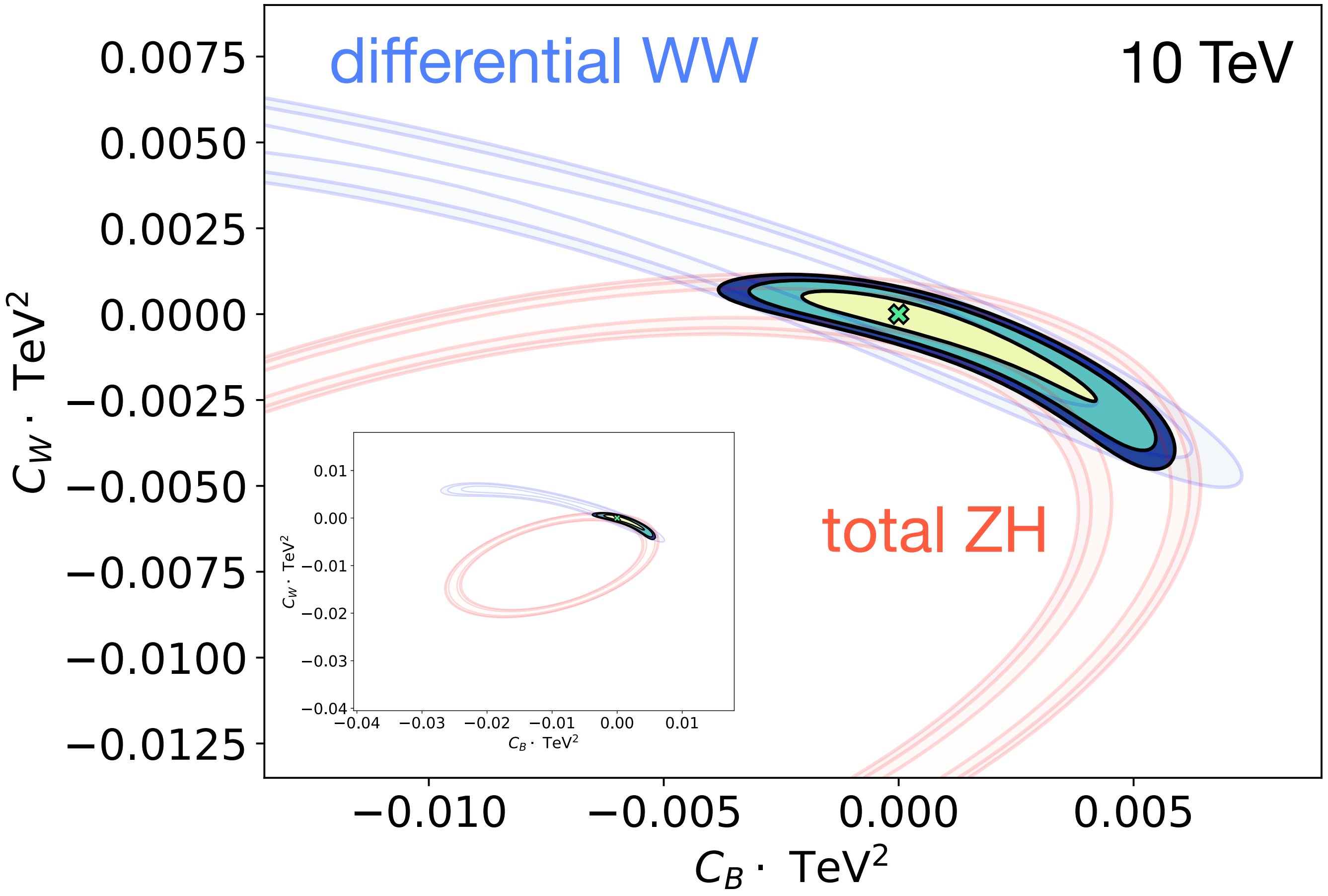
High-energy di-bosons

- C_W and C_B determined from high-energy $\mu^+\mu^- \to ZH, W^+W^-$ cross-sections

$$\sigma_{\mu\mu\to ZH} \approx 122\,\mathrm{ab}\left(\frac{10\,\mathrm{TeV}}{E_{\mathrm{cm}}}\right)^2\left[1+\#E_{\mathrm{cm}}^2 C_W+\#E_{\mathrm{cm}}^4 C_W^2\right]$$

☛ Limits on $C_{W,B}$ scale as E^2

B, Franceschini, Wulzer 2012.11555



- Fully differential WW cross-section in scattering and decay angles: can exploit the interference with transverse polarization amplitude

High-energy di-bosons

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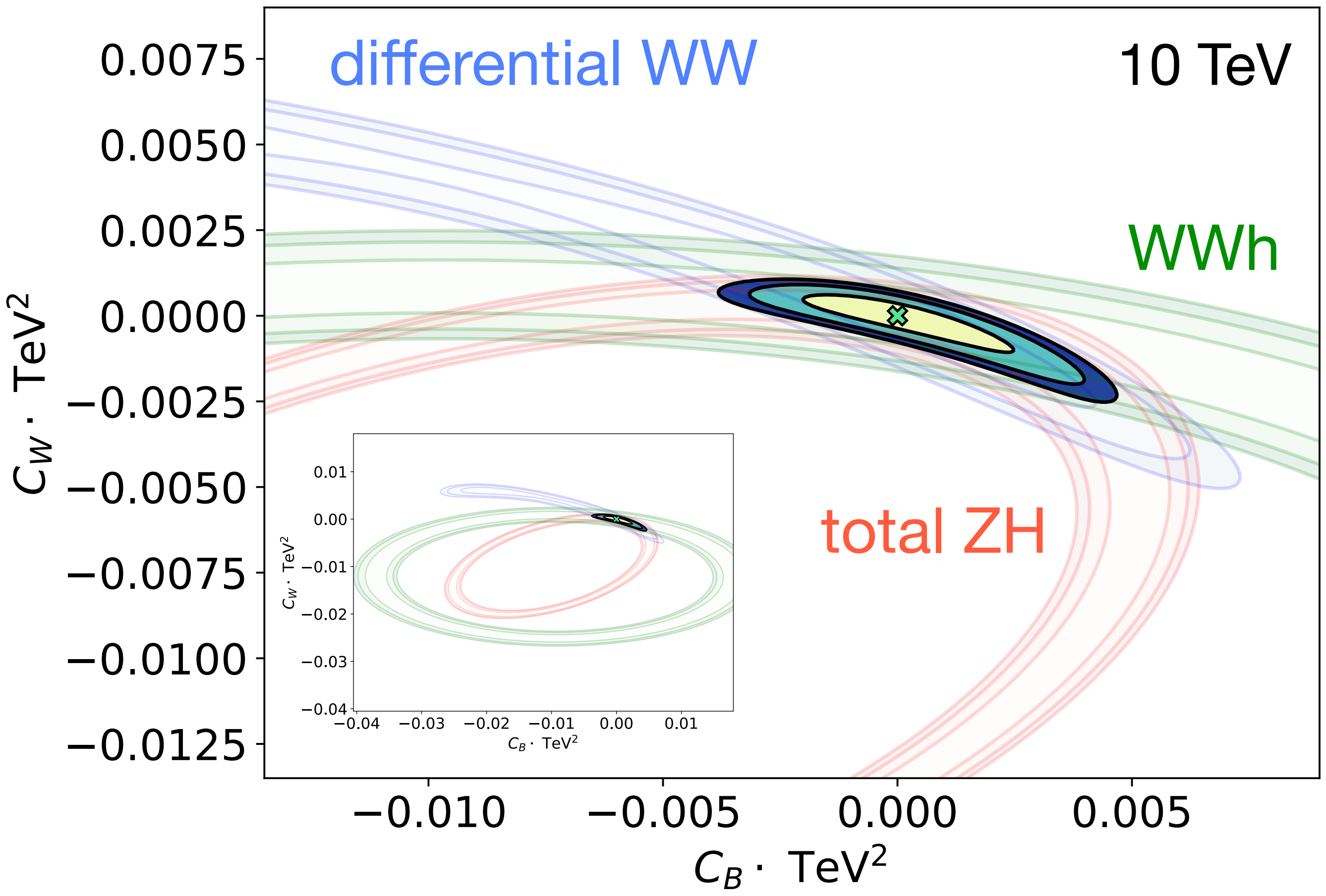
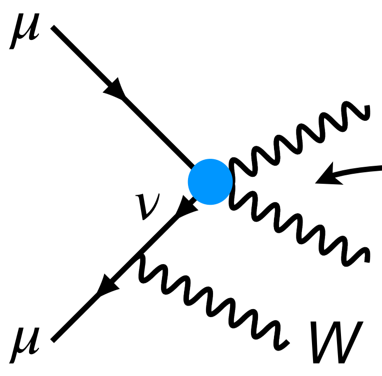


- ✦ Gauge boson radiation important at high energies: soft W emission allows to access the charged processes $\ell^\pm\nu \to W^\pm Z, W^\pm H$



independent measurement of G_{3L}

“effective neutrino approximation”

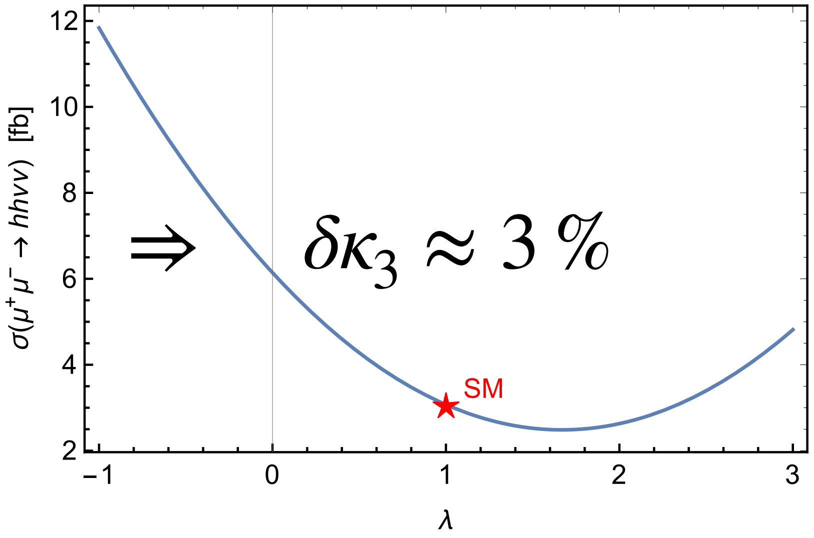
need to properly include higher-order effects
inclusive observables, resummation, …

Double Higgs production

Number of events $\sim s\log(s/m_h^2) \approx 10^5$ at 14 TeV

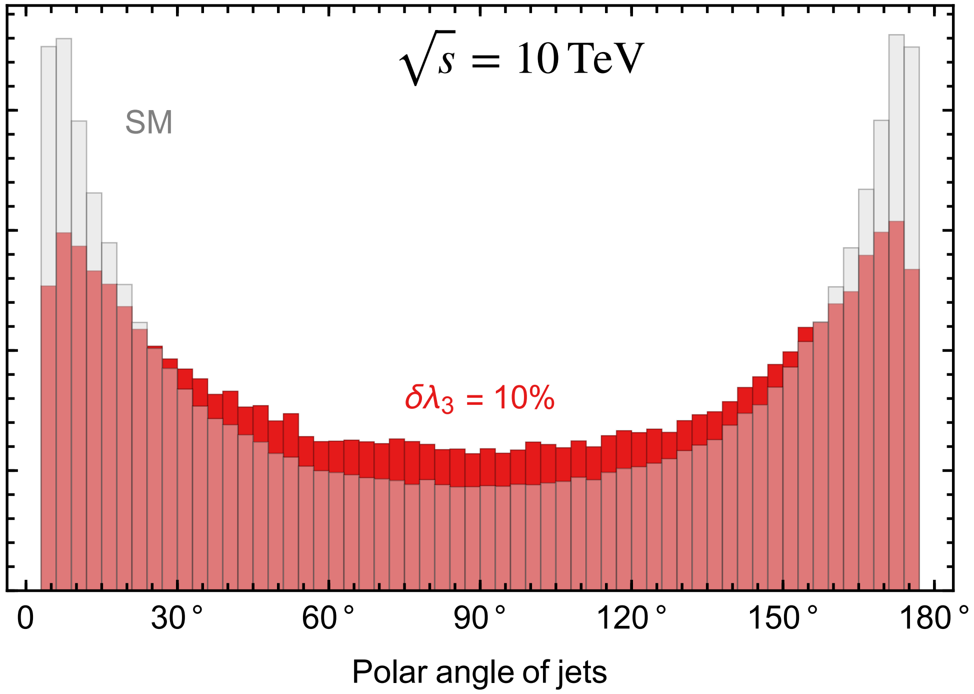
Naïve estimate of the reach: $\delta\sigma \sim (N \times \epsilon)^{-1/2} \approx 1\,\%$

$$\left.\begin{array}{r}\text{reconstruction eff.} \sim 30\,\% \\ \mathrm{BR}(hh \to 4b) = 34\,\%\end{array}\right\}\ \epsilon \sim 10\,\%$$

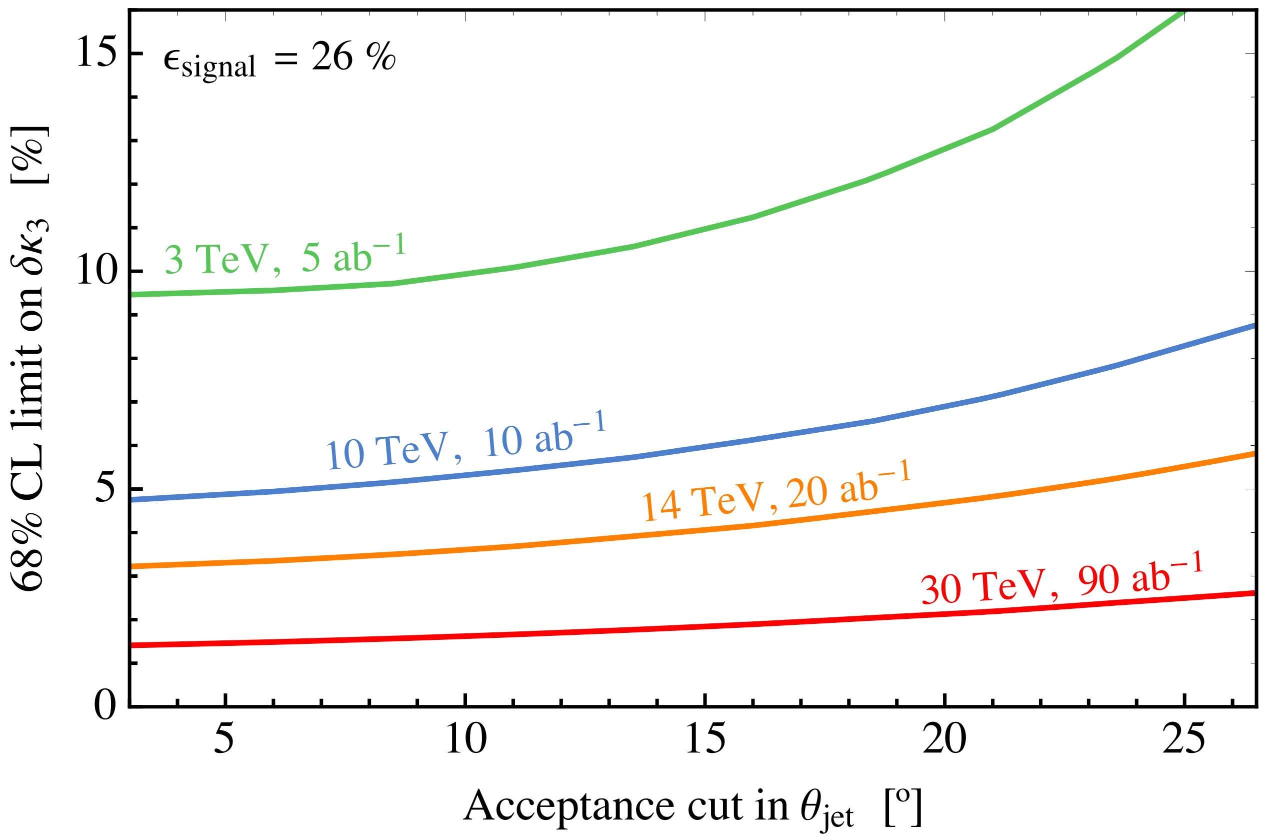


$\Rightarrow \delta\kappa_3 \approx 3\,\%$

- ✦ **Acceptance cuts** in polar angle θ and p_T of jets:
  - ‣ *hh* signal is strongly peaked in forward region



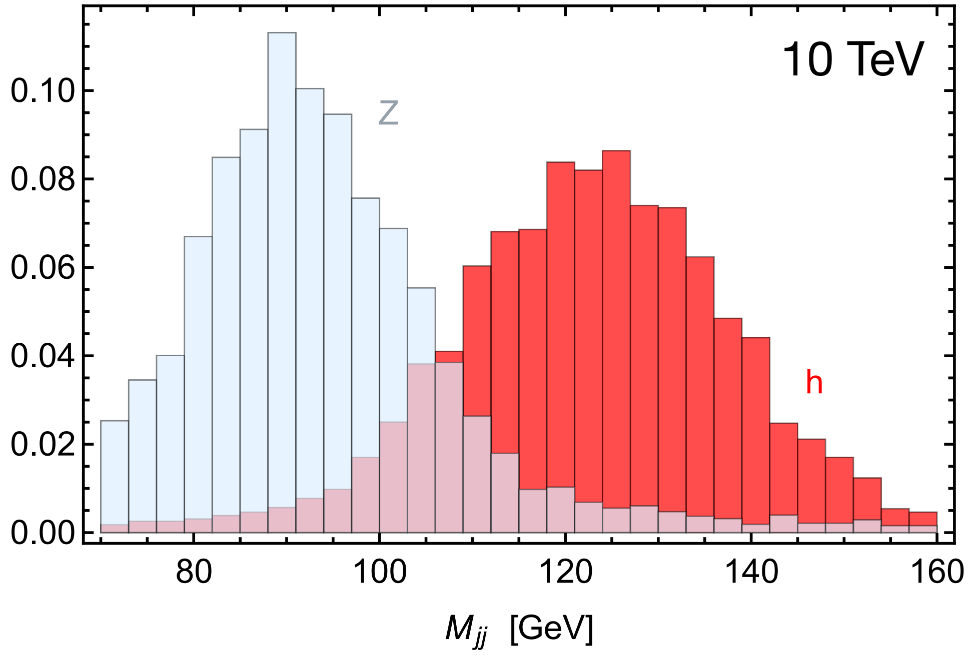
B, Franceschini, Wulzer 2012.11555

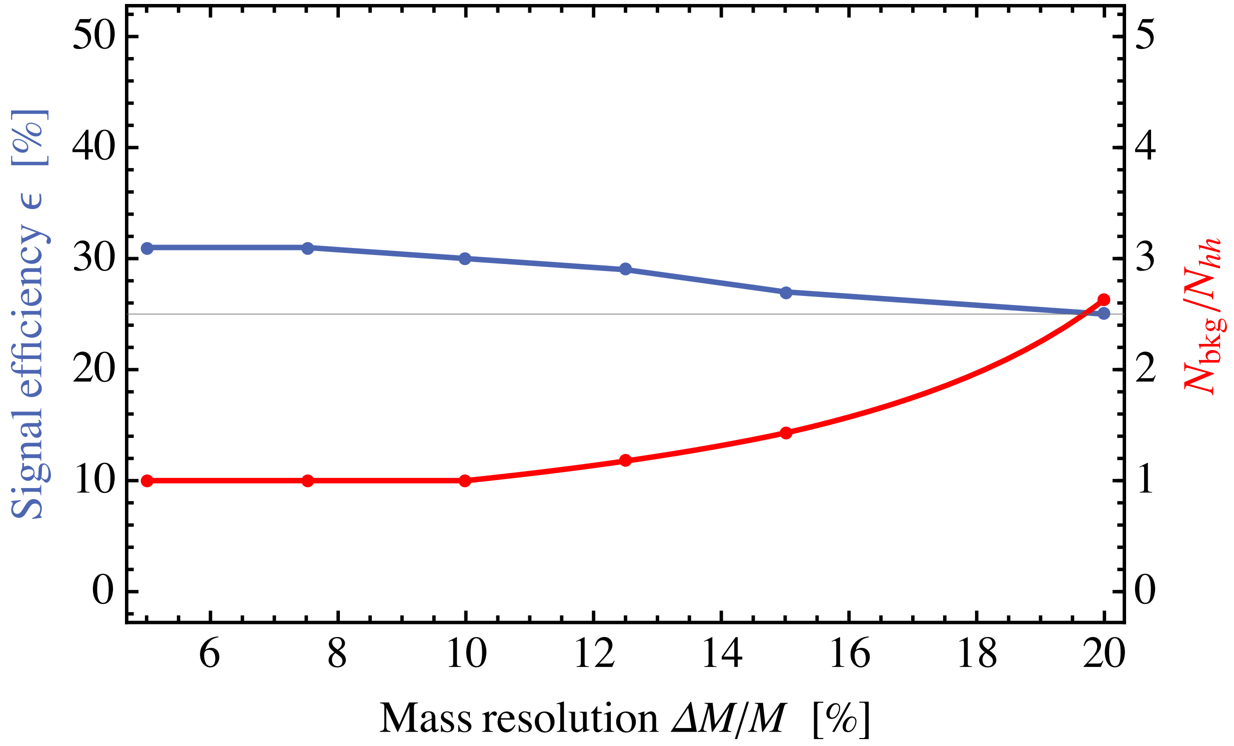


- ‣ Contribution from trilinear coupling is more central: loss due to angular cut is less important

Double Higgs production

- **Backgrounds are important** and cannot be neglected (see also CLIC study 1901.05897)
  - Mainly VBF di-boson production: Zh & ZZ, but also WW, Wh, WZ…
  - Precise invariant mass reconstruction is crucial to isolate signal





NB: (Very!) simplified background analysis *(at parton level!)*

All this should be done properly with a detector simulation

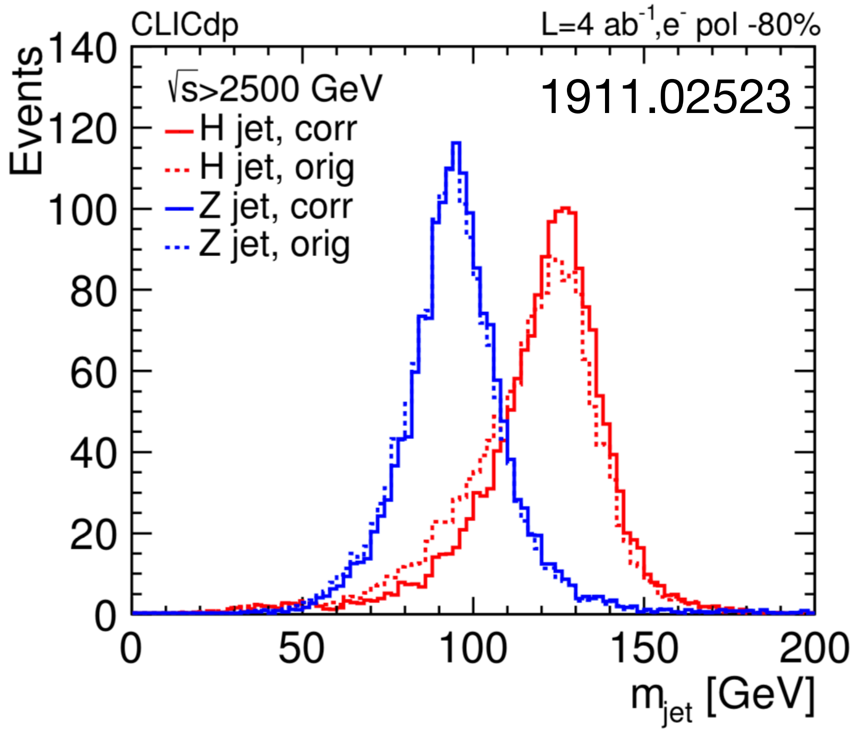
However, perfect agreement with 1901.05897! (3 TeV CLIC)

B, Franceschini, Wulzer 2012.11555

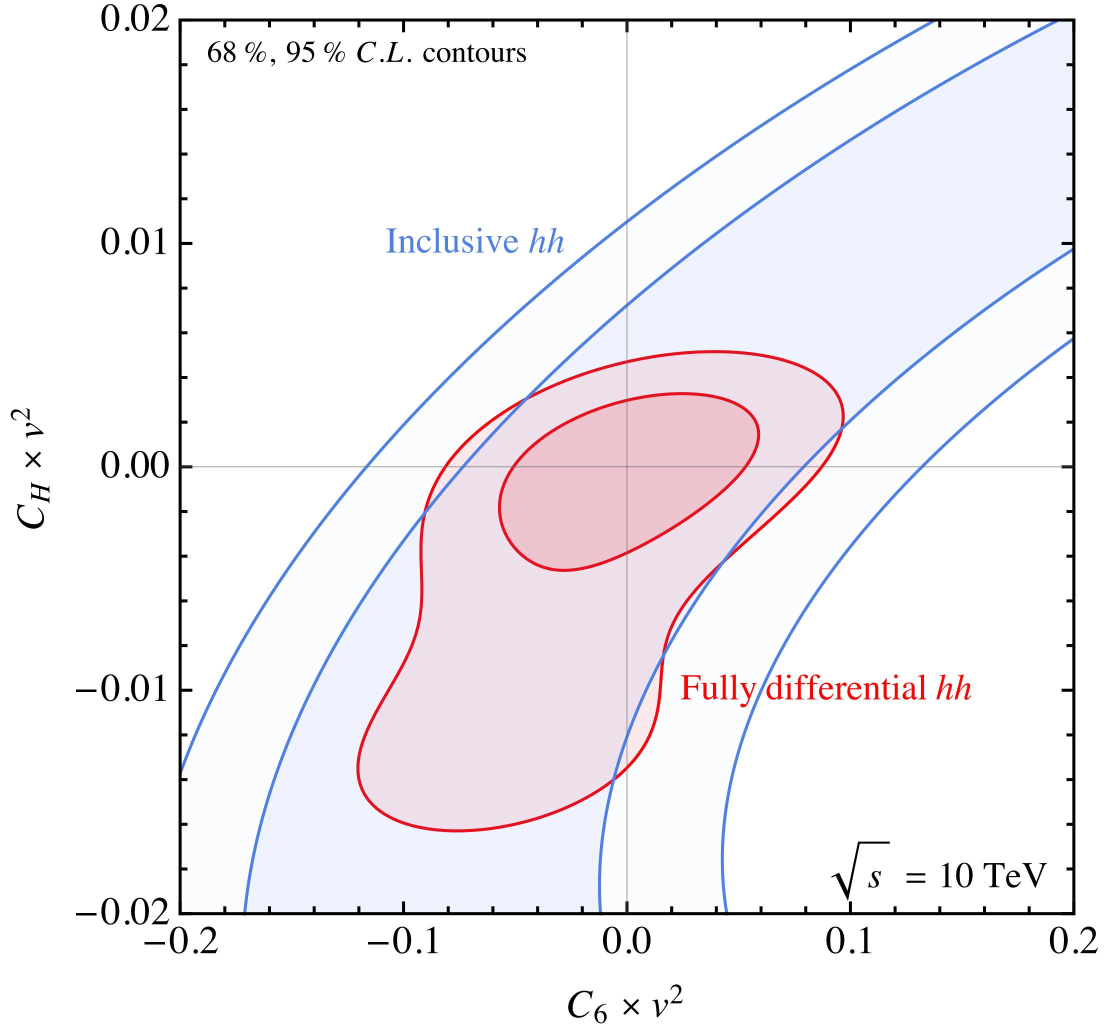
Double Higgs at high mass

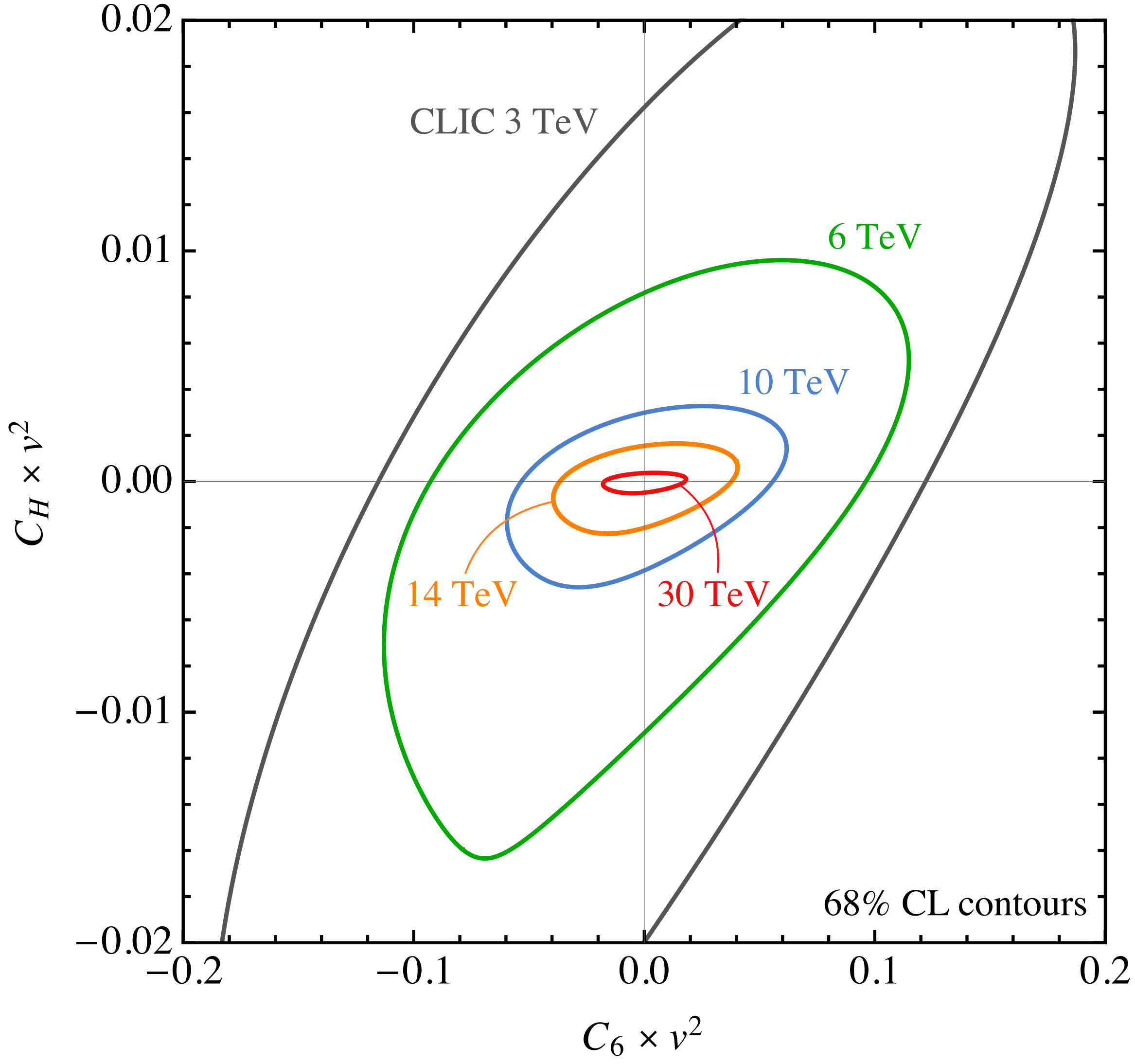
- Fully differential analysis in p_T and M_{hh} to optimize combined sensitivity to C_H and C_6
- Very boosted Higgs bosons: treat them as a single h-jet, without reconstructing the 4 b's.

  We assumed a boosted-H tagging efficiency ~ 50%



B, Franceschini, Wulzer 2012.11555

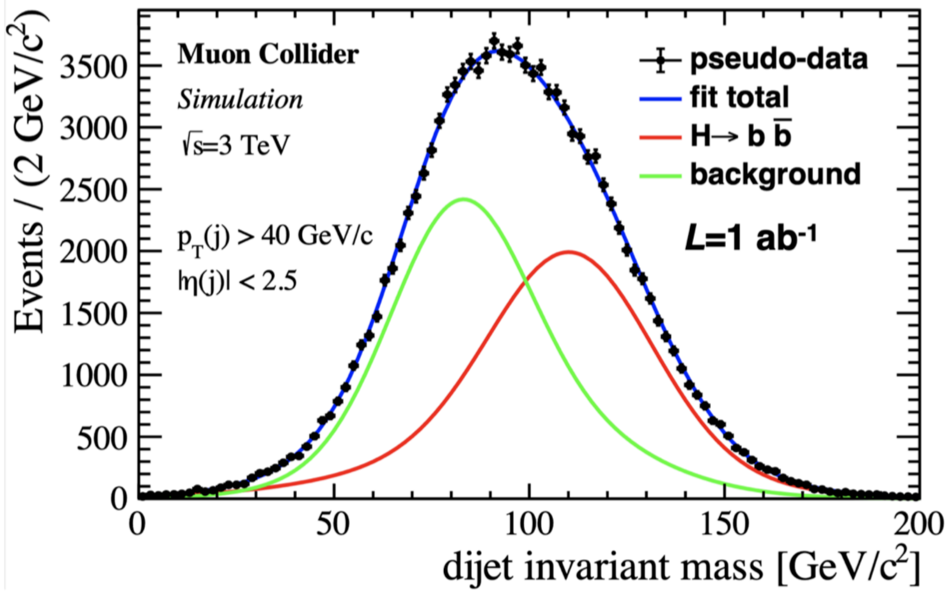




Single Higgs: backgrounds

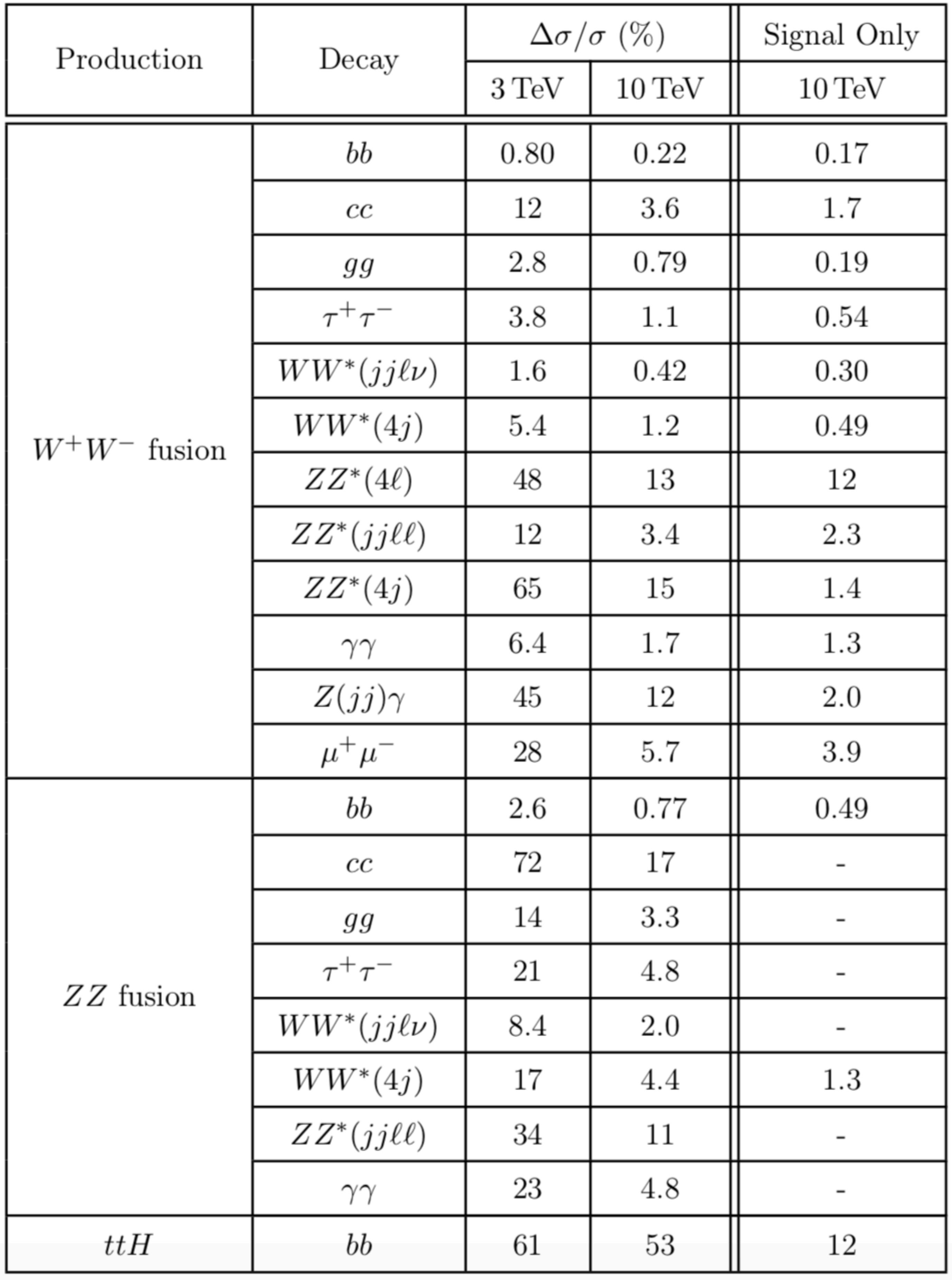
- Physics backgrounds (including the Higgs itself!)
- Beam-induced background

L. Sestini et al.



- Detector performance
  - “soft” and forward particles

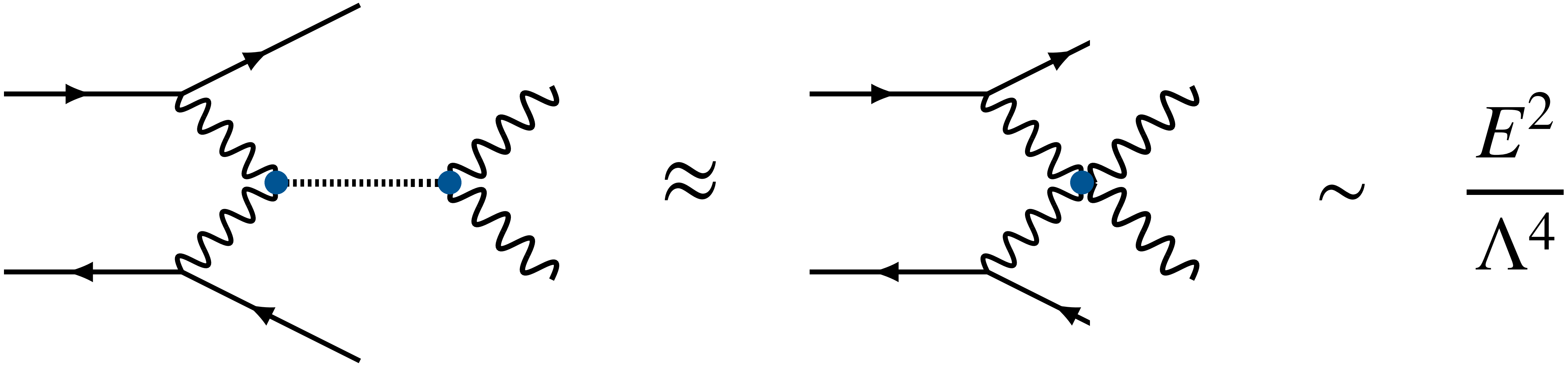
Forslund, Meade
2203.09425



Production	Decay	$\Delta\sigma/\sigma$ (%)		Signal Only
		3 TeV	10 TeV	10 TeV
W^+W^- fusion	bb	0.80	0.22	0.17
	cc	12	3.6	1.7
	gg	2.8	0.79	0.19
	$\tau^+\tau^-$	3.8	1.1	0.54
	$WW^*(jj\ell\nu)$	1.6	0.42	0.30
	$WW^*(4j)$	5.4	1.2	0.49
	$ZZ^*(4\ell)$	48	13	12
	$ZZ^*(jj\ell\ell)$	12	3.4	2.3
	$ZZ^*(4j)$	65	15	1.4
	$\gamma\gamma$	6.4	1.7	1.3
	$Z(jj)\gamma$	45	12	2.0
	$\mu^+\mu^-$	28	5.7	3.9
ZZ fusion	bb	2.6	0.77	0.49
	cc	72	17	-
	gg	14	3.3	-
	$\tau^+\tau^-$	21	4.8	-
	$WW^*(jj\ell\nu)$	8.4	2.0	-
	$WW^*(4j)$	17	4.4	1.3
	$ZZ^*(jj\ell\ell)$	34	11	-
	$\gamma\gamma$	23	4.8	-
ttH	bb	61	53	12

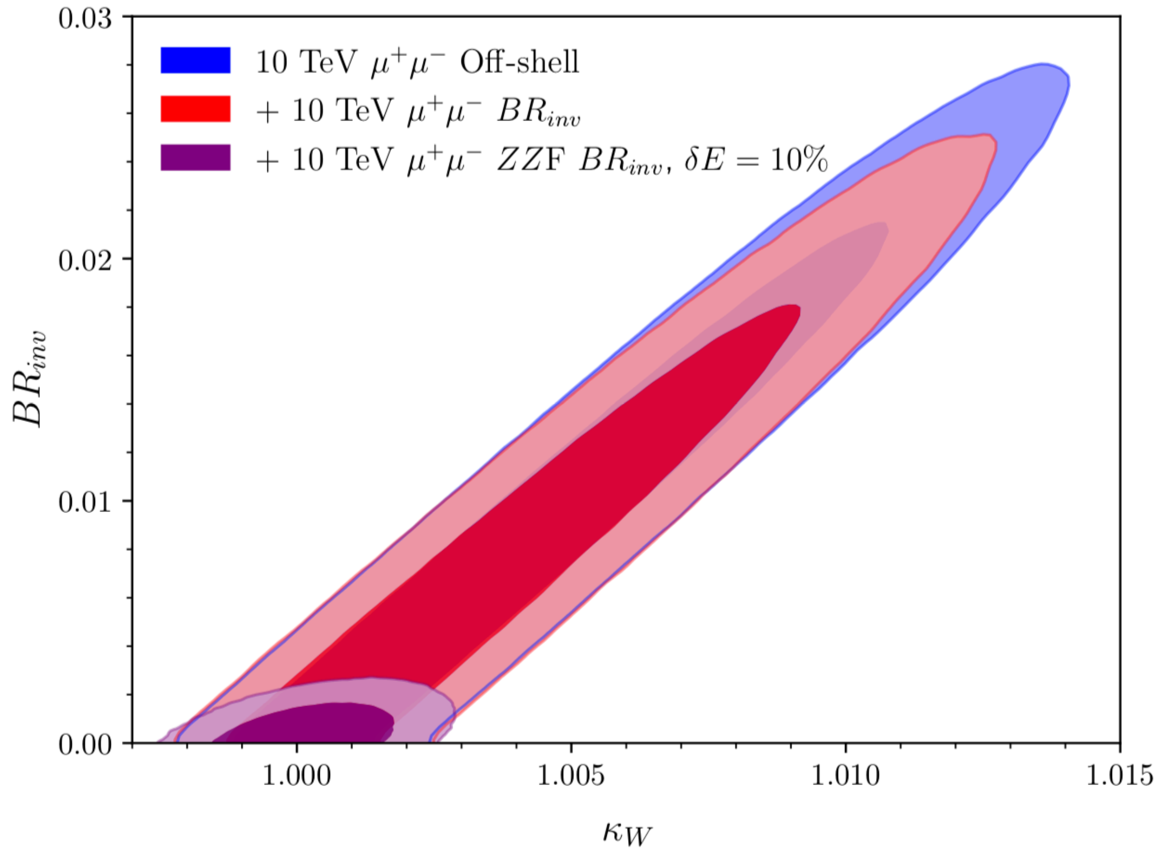
Single Higgs at high mass (off-shell)

- Off-shell single Higgs production: independent of width



$$\approx \qquad \sim \quad \frac{E^2}{\Lambda^4}$$

Forslund, Meade 2308.02633



precision limited (~ 3%) due to backgrounds: not possible to determine κ_W precisely through WW scattering

➔ correlation width vs. coupling

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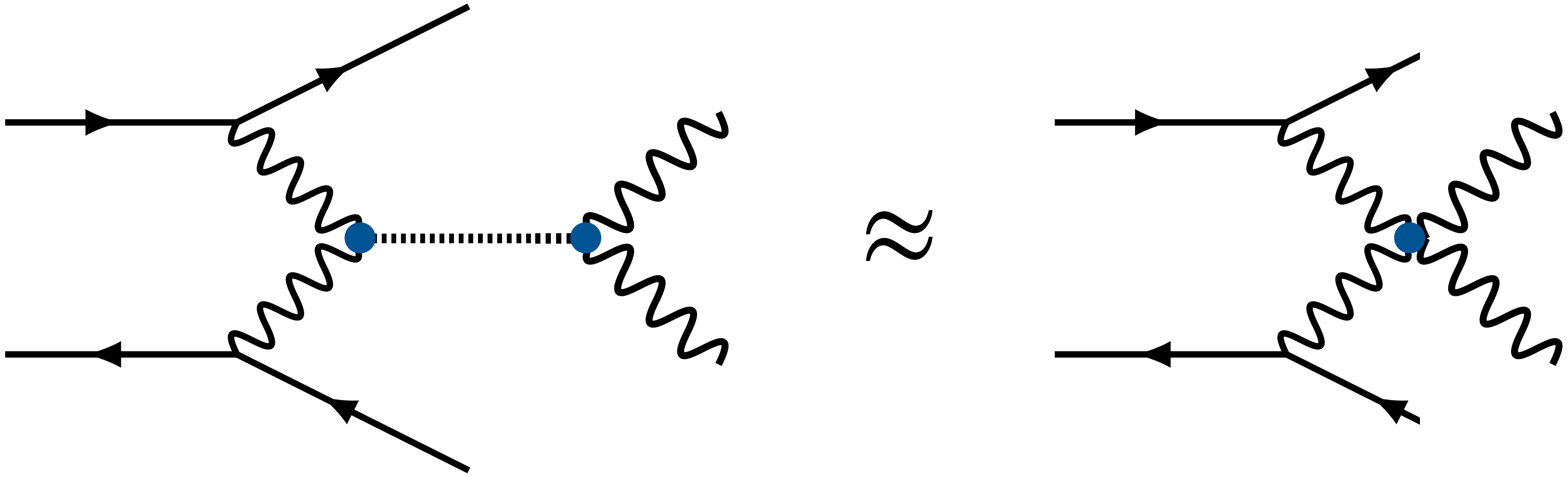
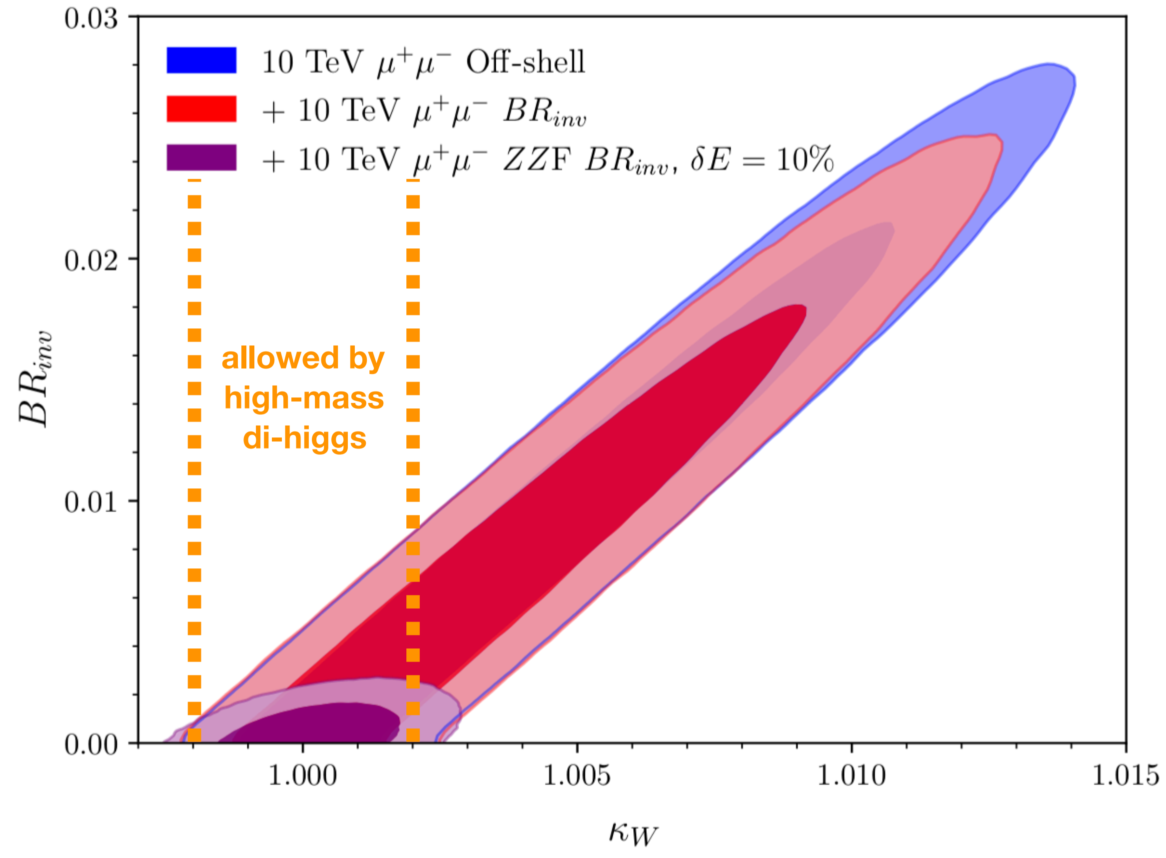


$$\sim \frac{E^2}{\Lambda^4}$$

Forslund, Meade 2308.02633



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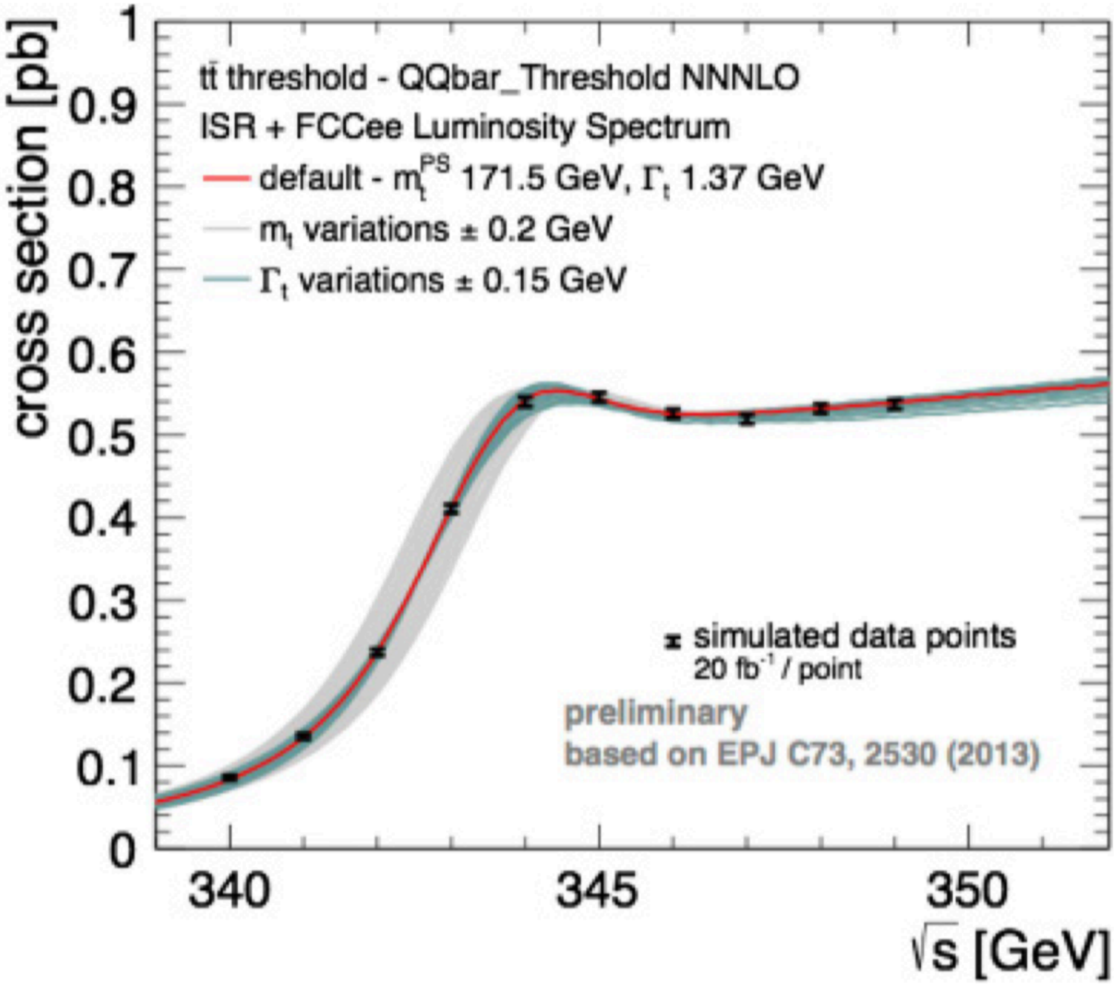
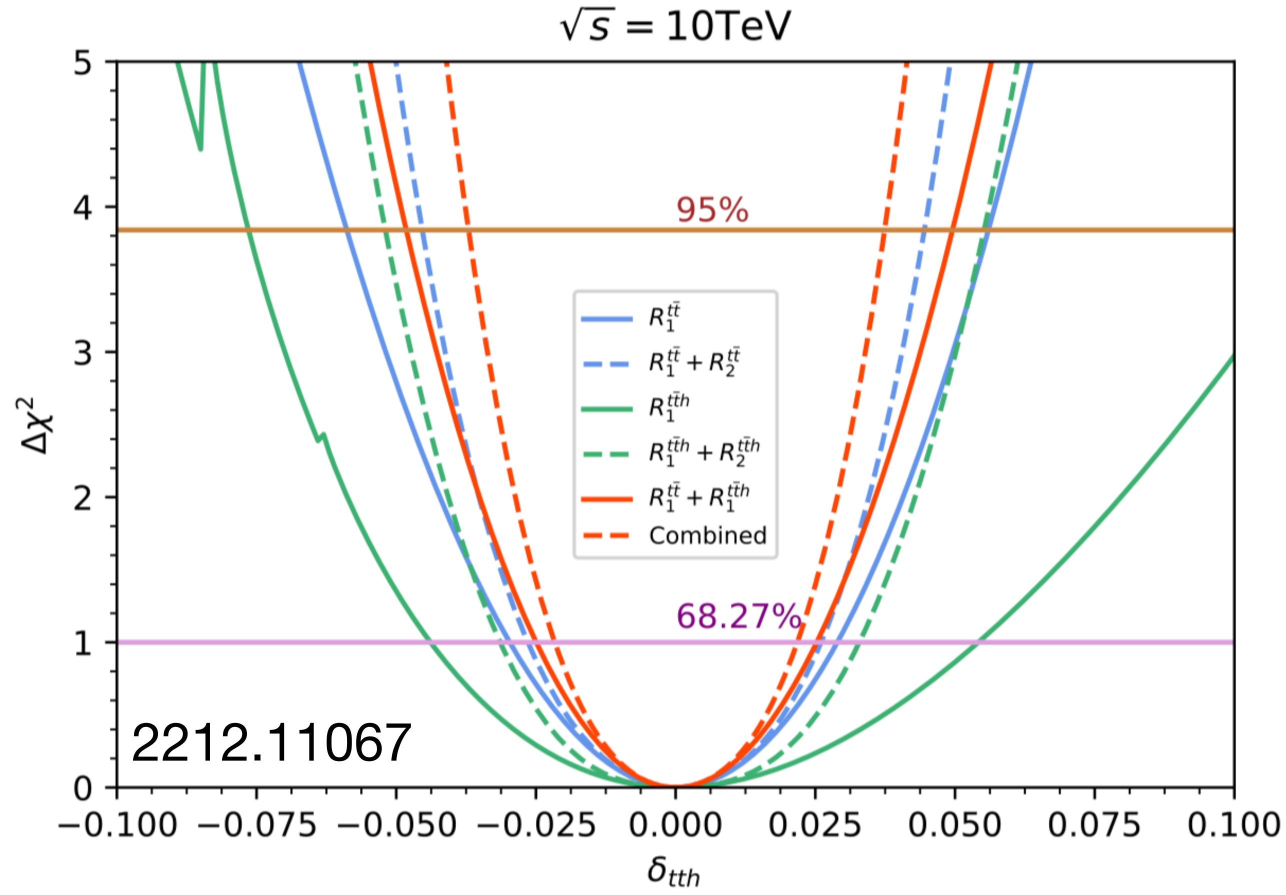
➔ correlation width vs. coupling

Top quark Yukawa



threshold scan @ FCC

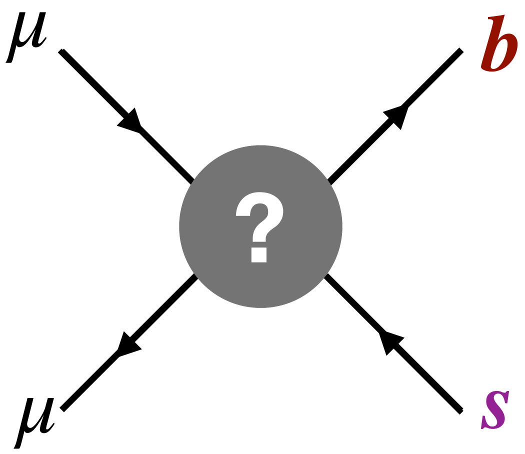
tth @ muon collider



2212.11067

(a) $\mu^+\mu^- \to t\bar{t}\nu\bar{\nu}$ with $\sqrt{s} = 10$ TeV and $L = 10\,\mathrm{ab}^{-1}$.

Quark flavor violation



Four-fermion interactions: muon current coupled to flavor-violating bilinear

$$\frac{c_{bs}}{\Lambda^2}(\bar{b}_{L,R}\gamma^\rho s_{L,R})(\bar{\mu}_{L,R}\gamma_\rho \mu_{L,R})$$

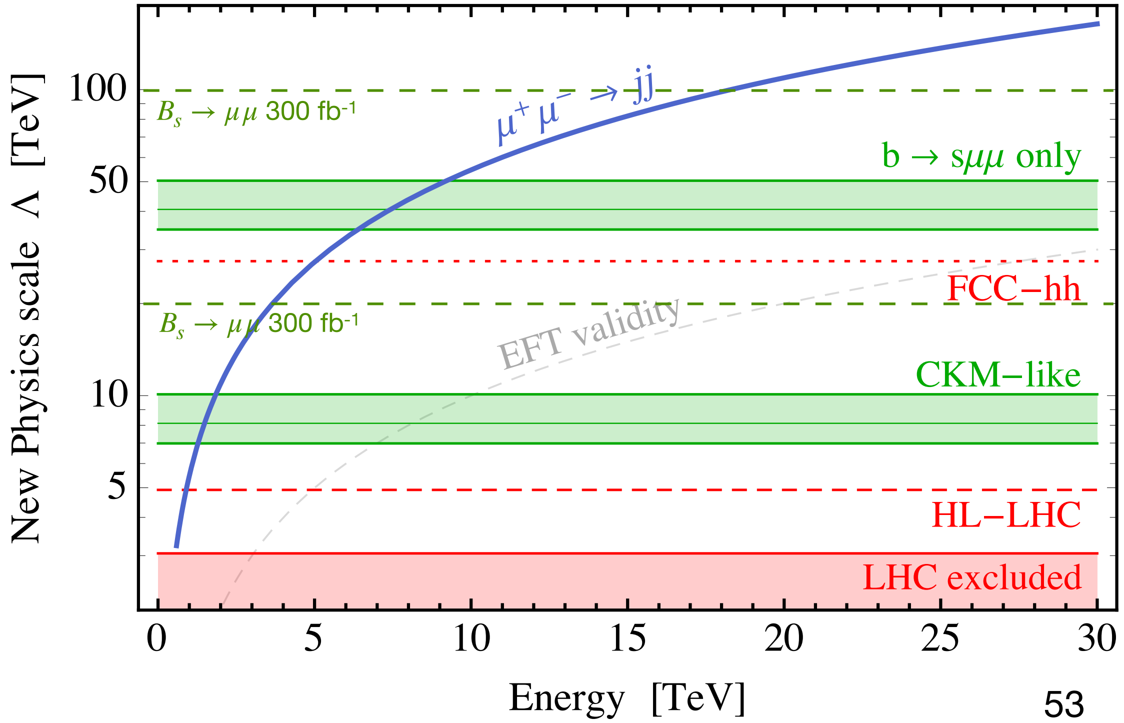
- Contributes to (semi-)leptonic rare B decays $b \to s\mu\mu$: branching ratios & angular observables of various hadronic processes

$$B_s \to \mu\mu, \qquad B \to K^{(*)}\mu\mu, \qquad B_s \to \phi\mu\mu, \qquad \Lambda_b \to \Lambda\mu\mu$$

- Theory uncertainties: cannot improve indefinitely with rare decays

$$\mathrm{BR}(B \to K\mu\mu) \sim \frac{m_W^4}{\Lambda^4}, \quad \sigma(\mu\bar{\mu} \to jj) \sim \frac{E^2}{\Lambda^4}$$

Azatov, Garosi, Greljo, Marzocca, Salko, Trifinopoulos 2205.13552



Lepton g-2 from rare Higgs decays

- Tau magnetic dipole moment: enhanced due to the larger mass

$$\Delta a_\tau = \frac{4v\, m_\tau}{\Lambda^2} C^\tau_{e\gamma} \approx \Delta a_\mu \frac{m_\tau^2}{m_\mu^2} \approx 10^{-6}$$

if $C^\ell_{e\gamma}$ scales as y_ℓ

Present bound: $\Delta a_\tau \lesssim 10^{-2}$

from LEP $e^+e^- \to e^+e^-\tau^+\tau^-$

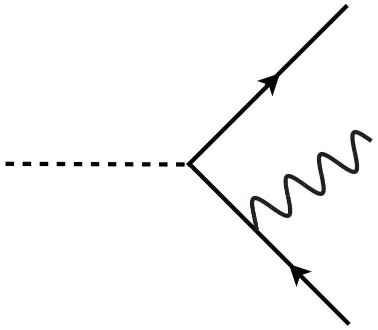
hep-ex/0406010

Can be improved to few 10-3
at HL-LHC 1908.05180

- Contribution to h → $\tau\tau\gamma$ decays:

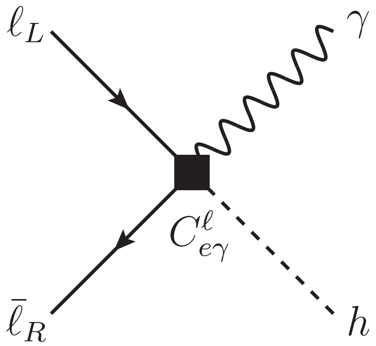
$$\mathrm{BR}^{(\mathrm{SM})}_{h\to\tau^+\tau^-\gamma} \approx 5 \times 10^{-4}$$

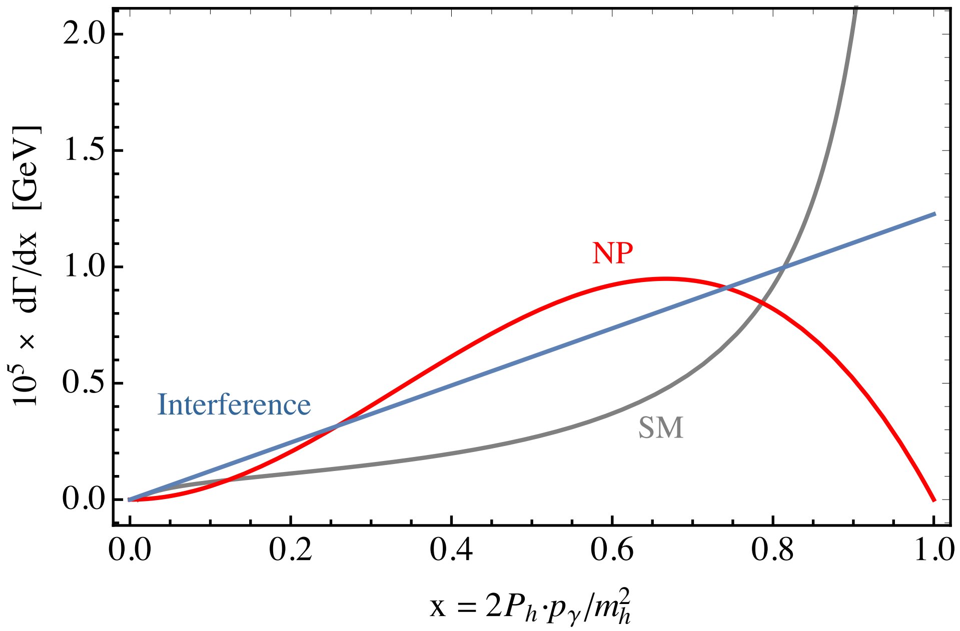
(with cut on soft collinear photon)



could be measured at few % level by Higgs factory

$$\mathrm{BR}^{(\mathrm{NP})}_{h\to\tau^+\tau^-\gamma} \approx 0.2 \times \Delta a_\tau$$



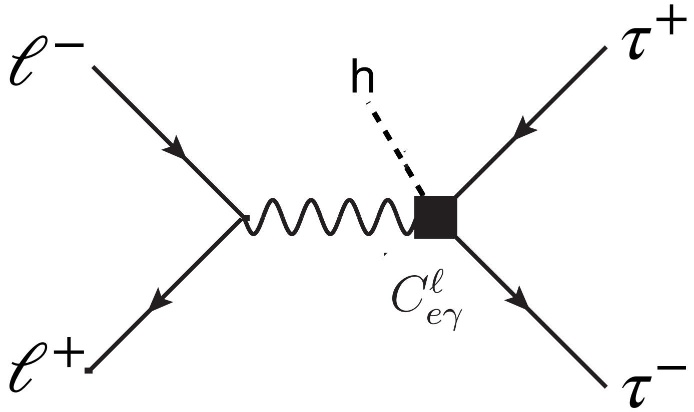


Tau g-2 from high-energy probes

Further possibilities to measure Δa_τ precisely from high-energy probes

- $H\tau\tau$ associated production

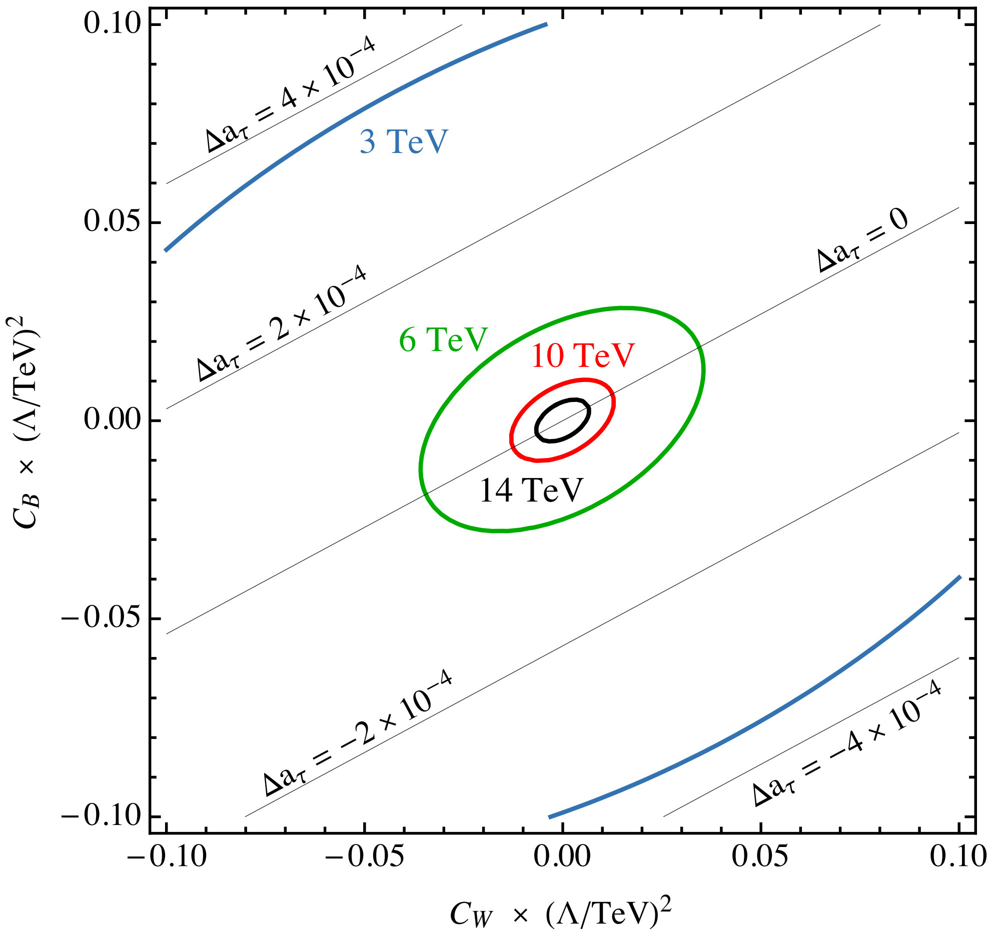
work in progress with Levati, Paradisi, Maltoni, Wang

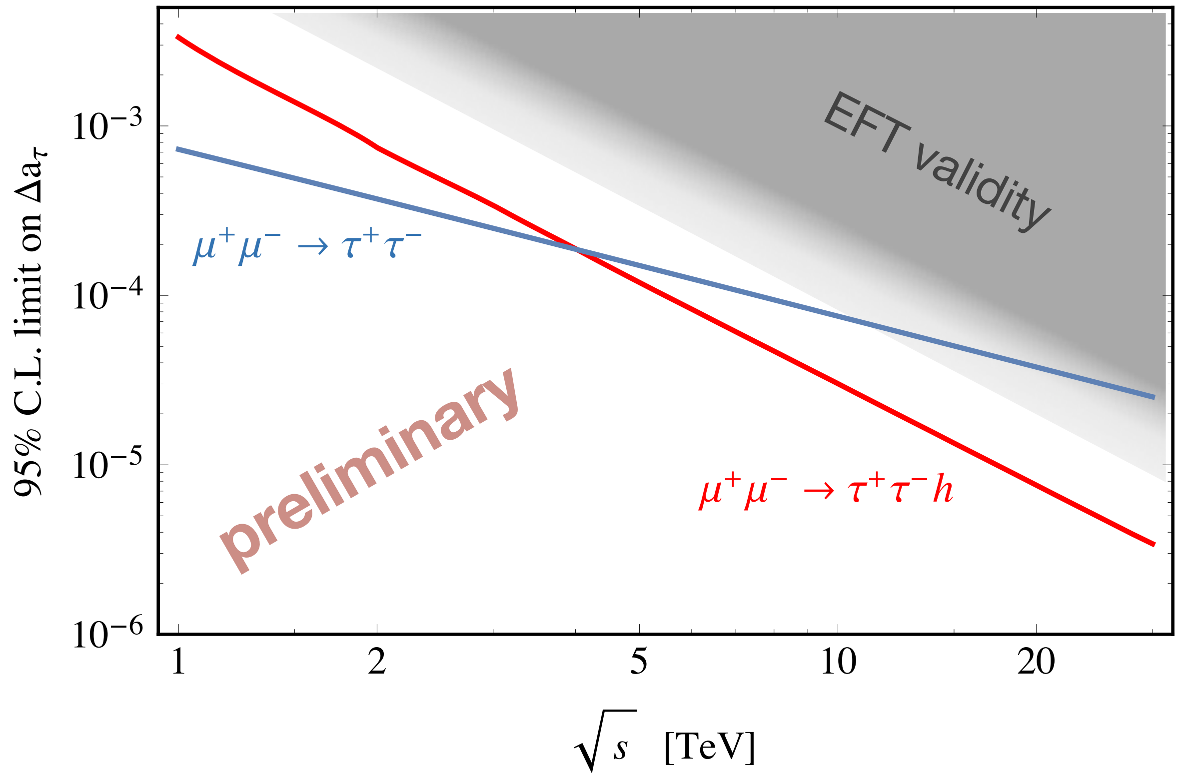


- Main background from μμ → $Z\gamma$

(where Z is mistaken for H)

Could probe $\Delta a_\tau \sim 10^{-5}$ @ 10 TeV





also a bound on tau EDM!

Tau g-2 from high-energy probes

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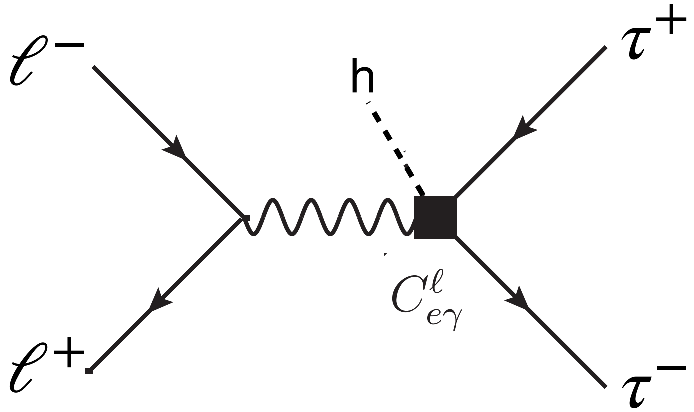

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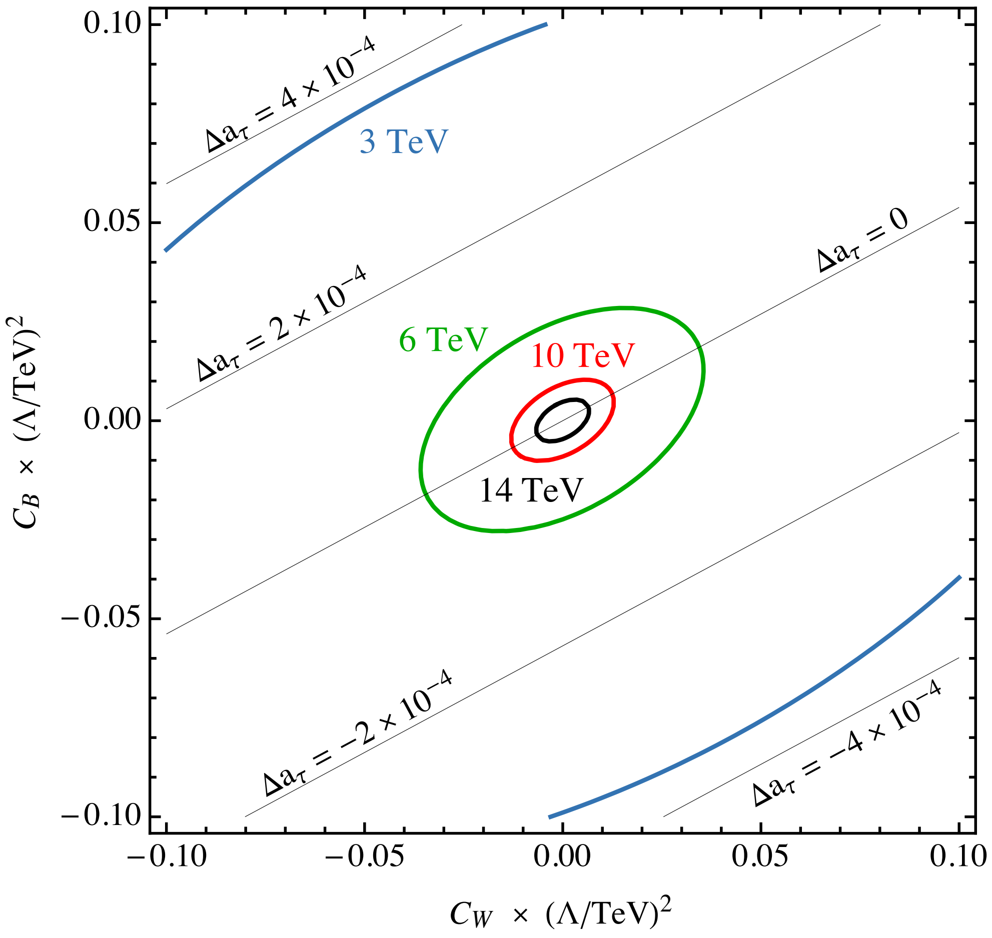
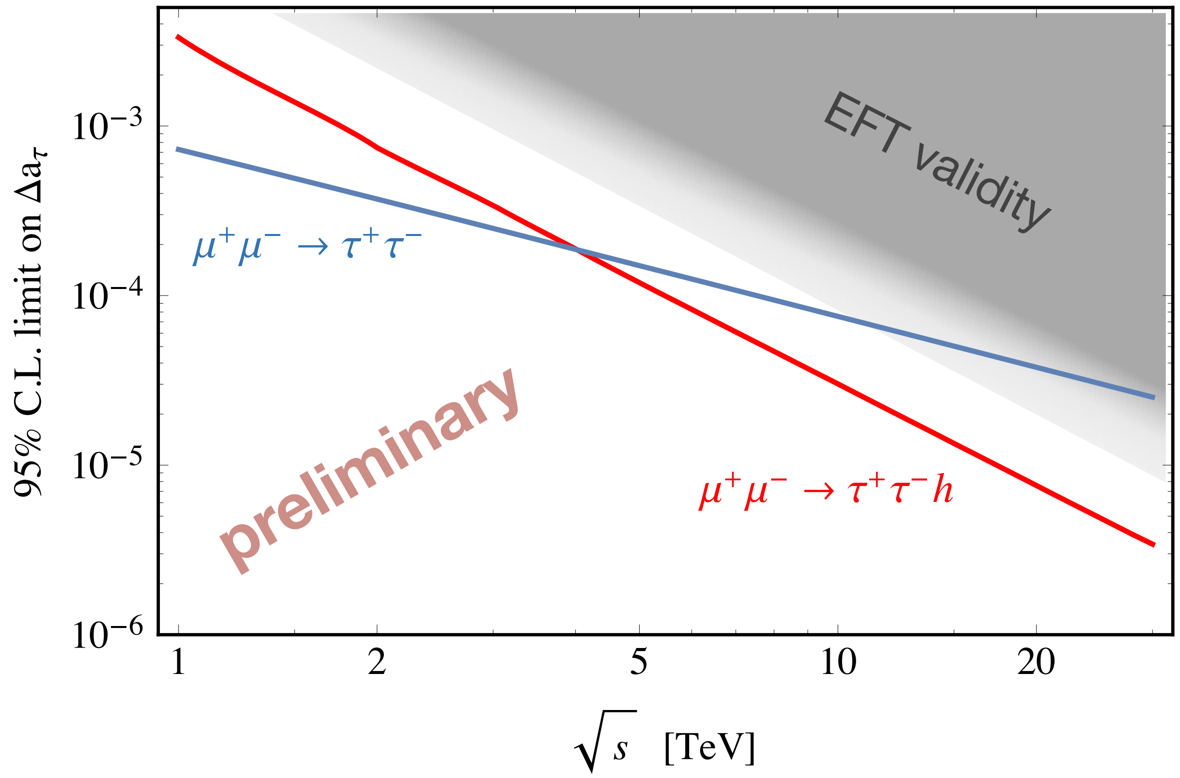


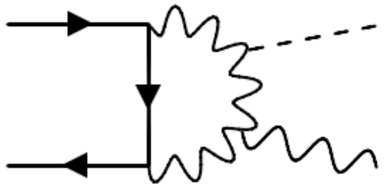


also a bound on tau EDM!

Muon g-2 @ muon collider

- SM irreducible bakground is small: $\sigma^{(\mathrm{SM})}_{\mu^+\mu^-\to h\gamma} \approx 10^{-2}\,\mathrm{ab}\left(\frac{30\,\mathrm{TeV}}{\sqrt{s}}\right)^2$

  tree-level is suppressed by muon mass; loop contribution dominant

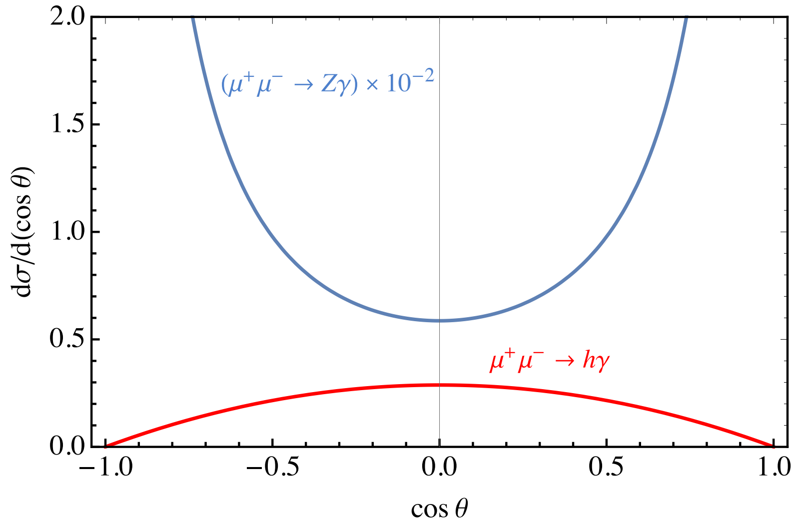


- Main background from *μμ* → Z*γ* (where Z is mistaken for H)

  (large due to transverse Z polarizations)

$$\frac{d\sigma_{\mu\mu\to h\gamma}}{d\cos\theta} = \frac{|C^{\mu}_{e\gamma}(\Lambda)|^2}{\Lambda^4}\frac{s}{64\pi}(1-\cos^2\theta)$$

$$\frac{d\sigma_{\mu\mu\to Z\gamma}}{d\cos\theta} = \frac{\pi\alpha^2}{4s}\frac{1+\cos^2\theta}{\sin^2\theta}\frac{1-4s_W^2+8s_W^4}{s_W^2c_W^2}$$



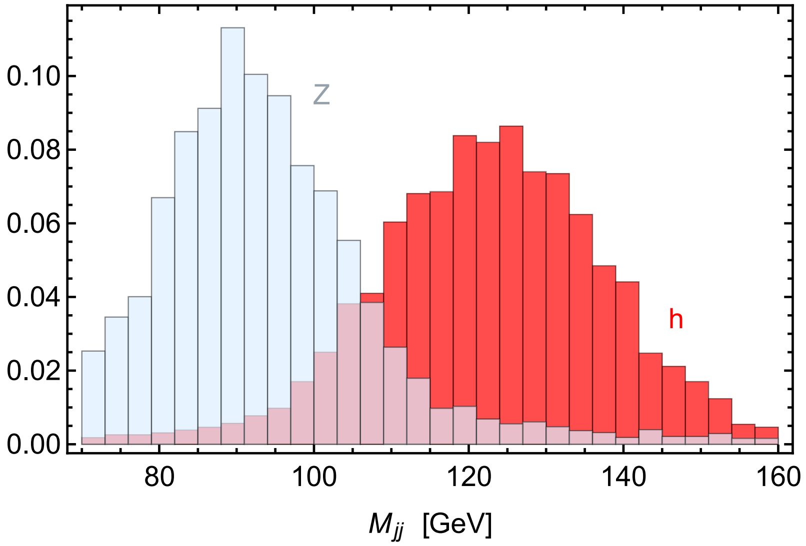
Search in h → bb channel:

$\epsilon_b \approx 80\,\%$ $\quad |\cos\theta_{\mathrm{cut}}| < 0.6$ $\quad \mathrm{BR}_{h\to b\bar{b}} = 58\,\%$

At 30 TeV, 90 ab^{-1}, for $\Delta a_\mu = 3\times 10^{-9}$:

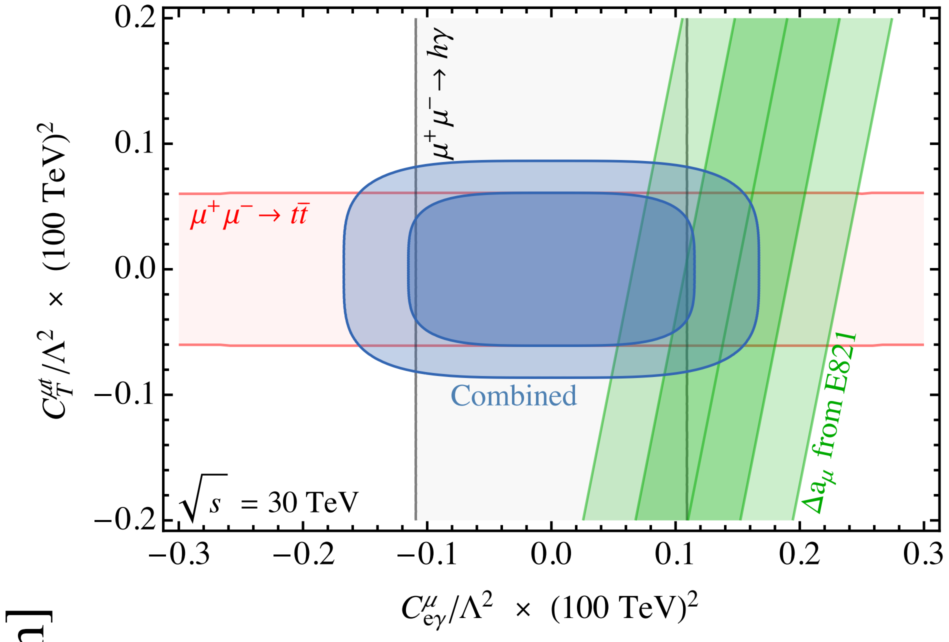
$$N_S = 22, \quad N_B = 886\times p_{Z\to h}$$

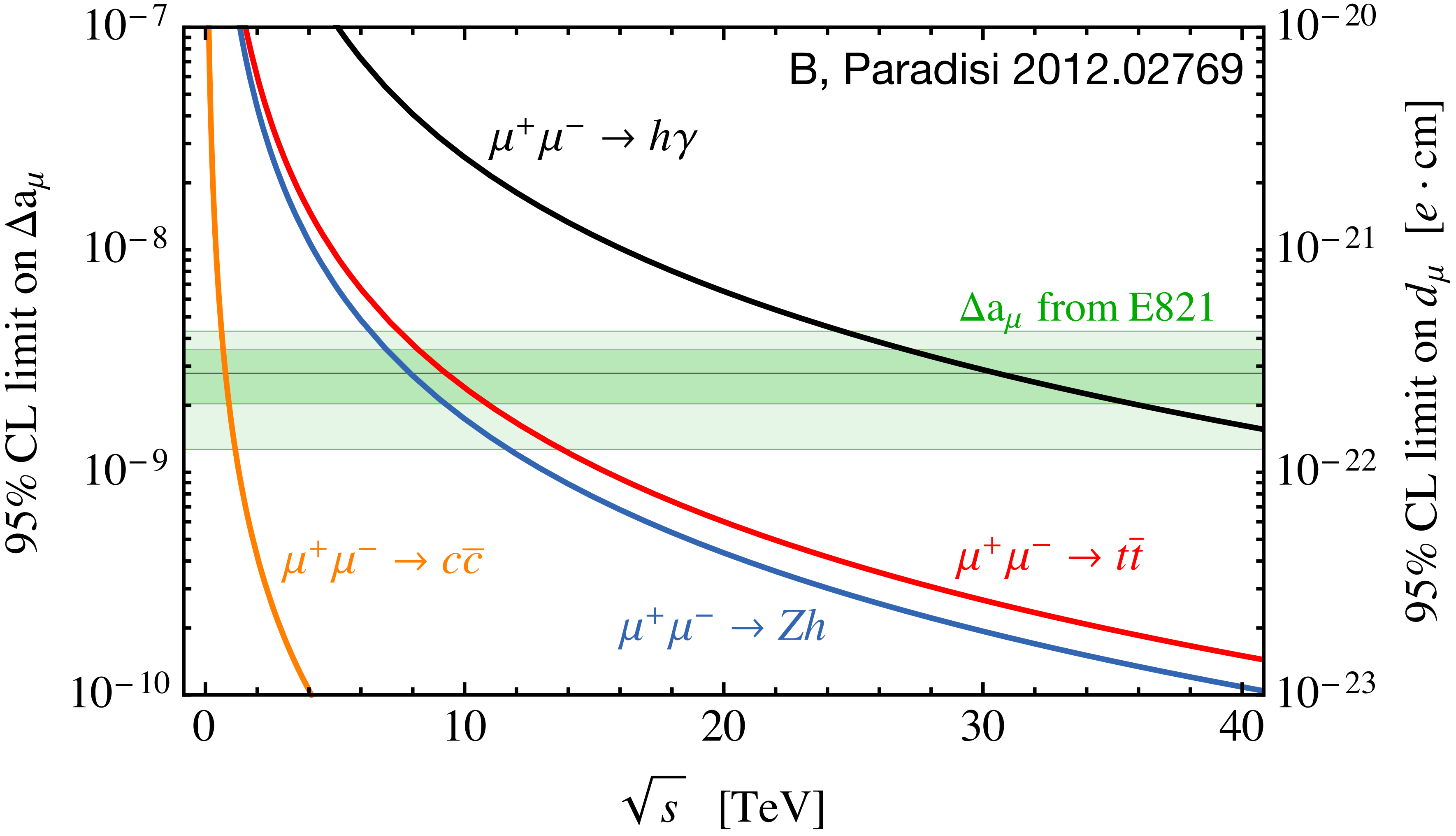
Δa$_\mu$ can be tested at 95% CL at a 30 TeV collider if Z→h mistag probability < 10-15%



Muon g-2 @ muon collider

- Full set of operators with Λ ≳ 100 TeV can be probed at a high energy muon collider





Muon EDM for free!

$$d_\mu = \frac{\Delta a_\mu \tan\phi_\mu}{2m_\mu} e = \frac{2v\,\mathrm{Im}(C_{e\gamma})}{\Lambda^2}$$

Collider constrains $|C_{e\gamma}|^2 \Rightarrow d_\mu \lesssim 10^{-22}\, e \cdot \mathrm{cm}$

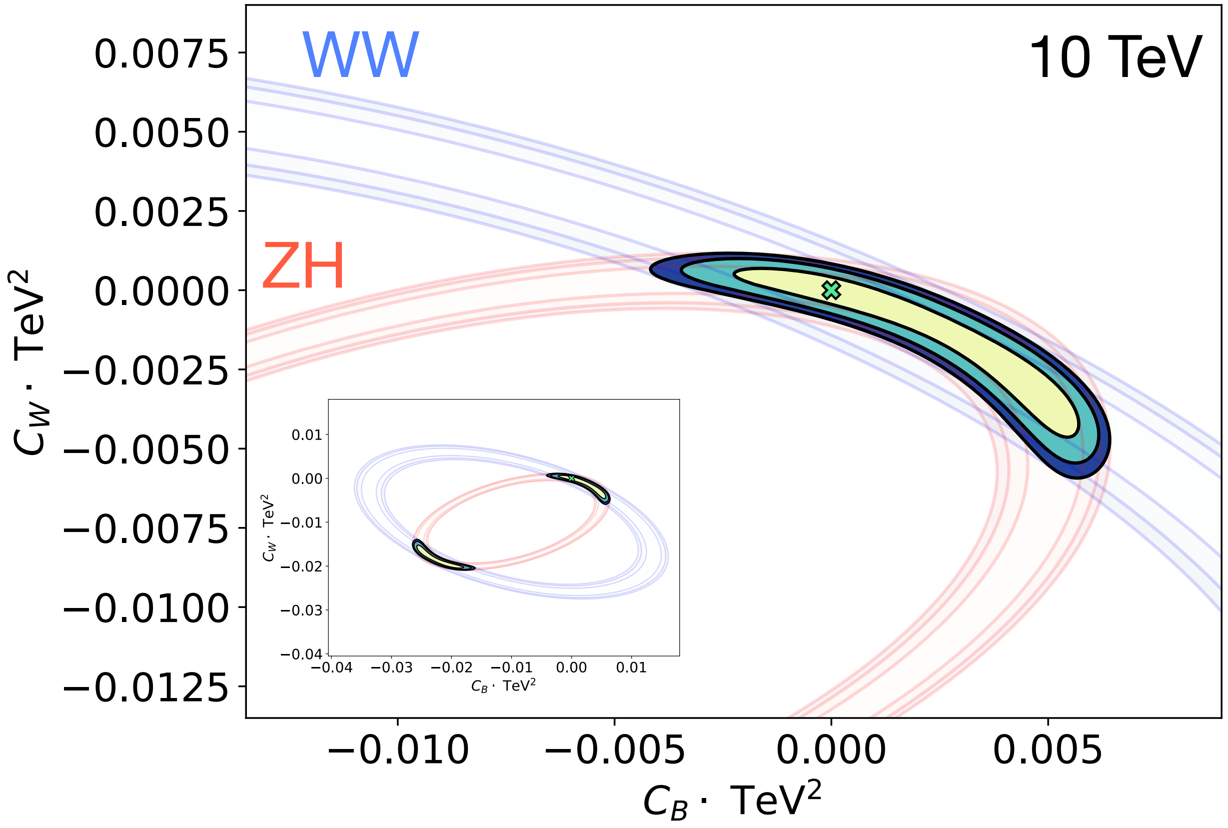
3 o.o.m. stronger than present bound!

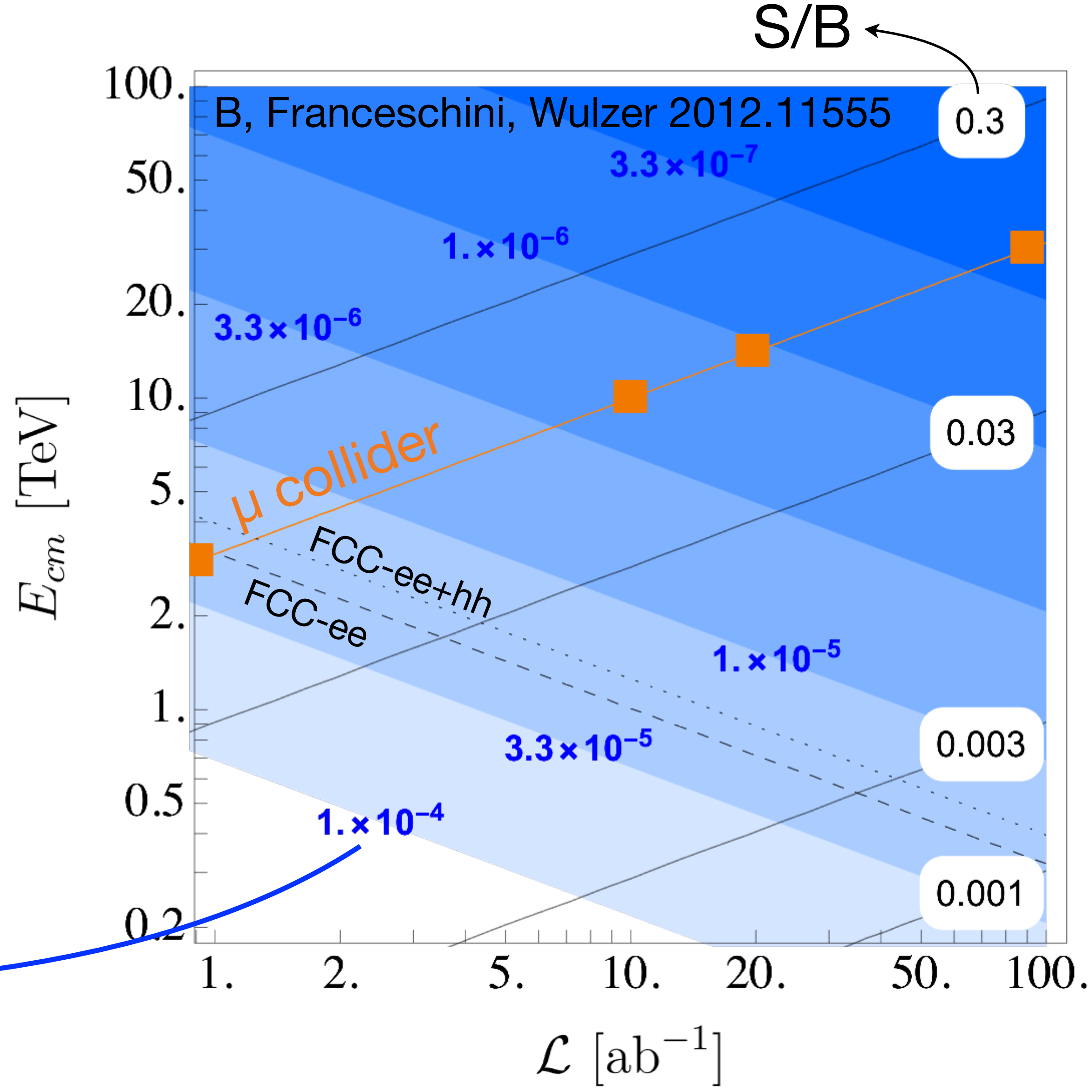
High-energy di-bosons

- C_W and C_B determined from high-energy $\mu^+\mu^- \to ZH$, W^+W^- total cross-sections

Limits on $C_{W,B}$ scale as E^2

$$\sigma_{\mu\mu\to ZH} \approx 122\,\text{ab}\left(\frac{10\,\text{TeV}}{E_\text{cm}}\right)^2\left[1+\#\,E_\text{cm}^2 C_W+\#\,E_\text{cm}^4 C_W^2\right]$$

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- In universal theories, $C_{W,B}$ related with Z-pole and other EW observables

$$\hat S = m_W^2(C_W + C_B)$$

Muon collider:

$$10\,\text{TeV}:\quad C_W \lesssim (40\,\text{TeV})^{-2},\qquad \hat S \lesssim 10^{-6}$$

$$30\,\text{TeV}:\quad C_W \lesssim (120\,\text{TeV})^{-2},\qquad \hat S \lesssim 10^{-7}$$

LEP : $\hat S \lesssim 10^{-3}$

FCC : $\hat S \lesssim 10^{-5}$ ⟵ ultimate precision at Z pole

High-energy WW: angular analysis

- $O_{W,B}$ contribute to longitudinal scattering amplitudes:

$$\mathscr{A}_{00}^{(\mathrm{NP})} = s\,(G_{1L} - G_{3L})\,\sin\theta_\star$$
$$\mathscr{A}_{-+} = -\frac{g^2}{2}\sin\theta_\star$$
$$\mathscr{A}_{+-} = g^2\cos^2\frac{\theta_\star}{2}\cot^2\frac{\theta_\star}{2}$$

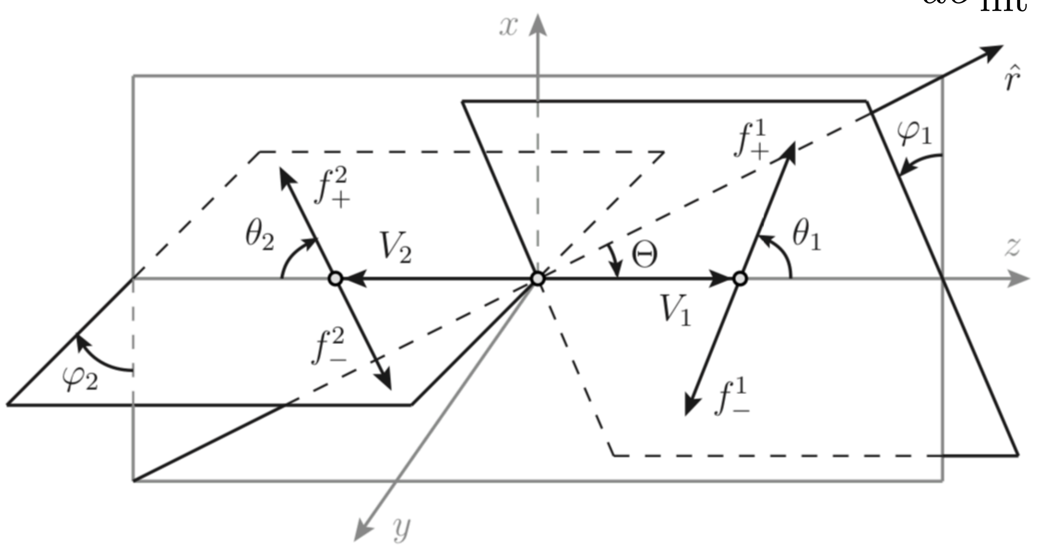
- In the SM, large contribution to $\mu^+\mu^- \to W^+W^-$ from transverse polarizations.

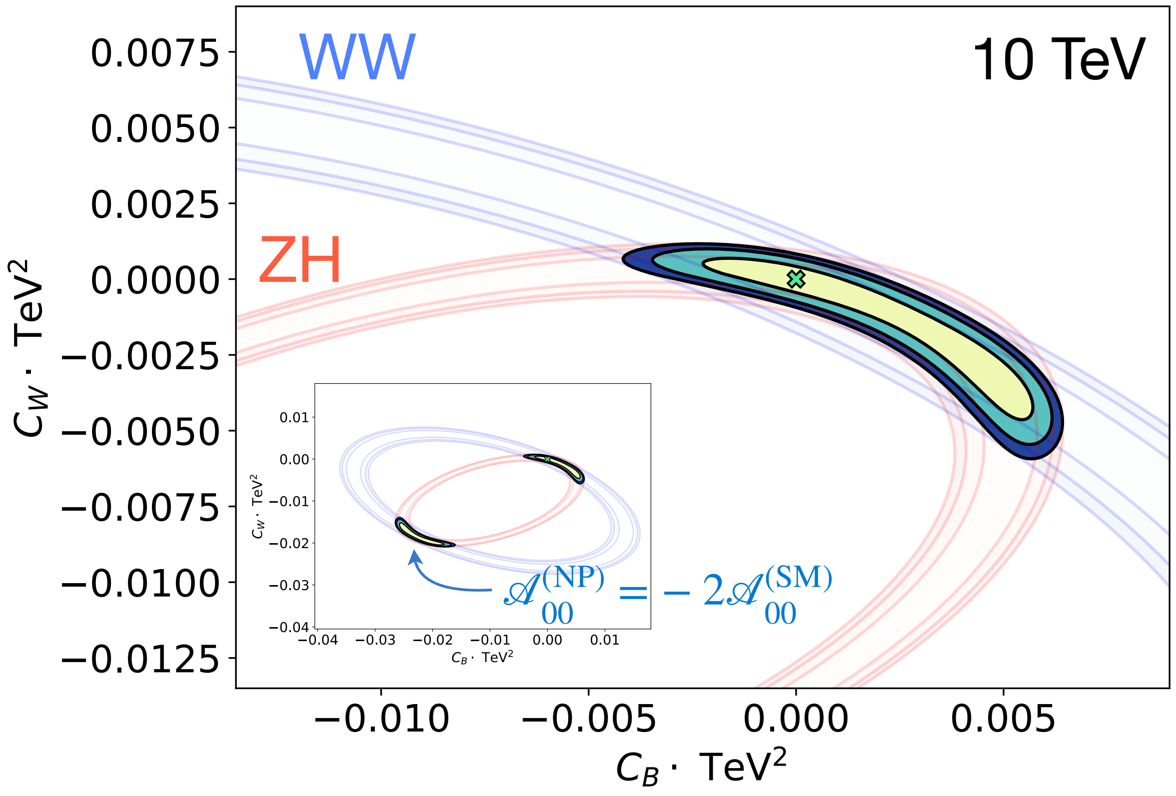
Interference between ±∓ and 00 helicity amplitudes cancels in the total cross-section ⇒ signal suppressed!

see also Panico et al. 1708.07823, 2007.10356

$$d\sigma_{\mathrm{int}} \propto \mathcal{M}_{00}\mathcal{M}_{+-}\cos(\varphi_+ - \varphi_-)\sin\theta_+(1+\cos\theta_+)\sin\theta_-(1-\cos\theta_-)$$
$$+\mathcal{M}_{00}\mathcal{M}_{-+}\cos(\varphi_+ - \varphi_-)\sin\theta_+(1-\cos\theta_+)\sin\theta_-(1+\cos\theta_-)$$

($\theta_\pm, \varphi_\pm$ polar and azimuthal angle of $W^\pm$ decay products)





- Can exploit the SM/BSM interference by looking at fully differential WW cross-section in scattering and decay angles!

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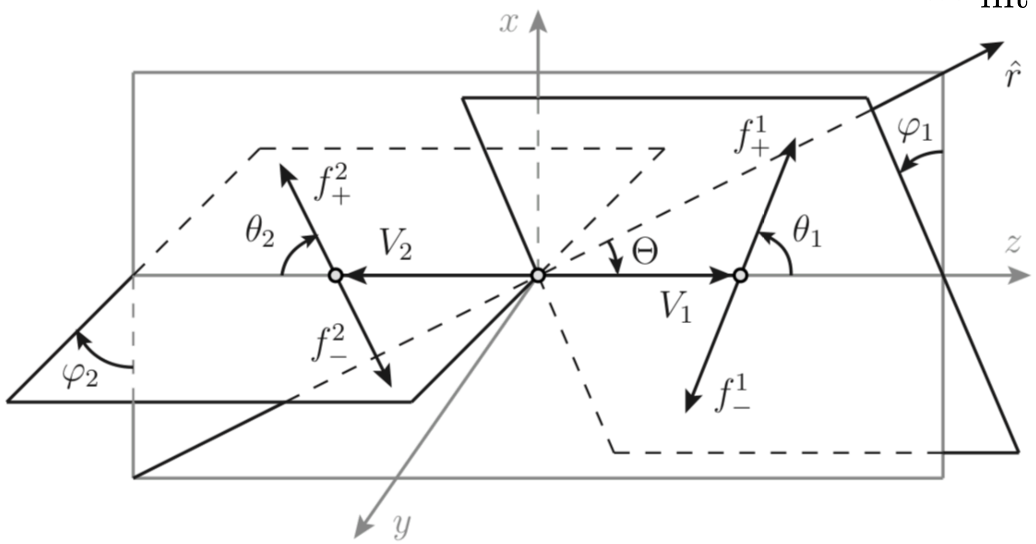

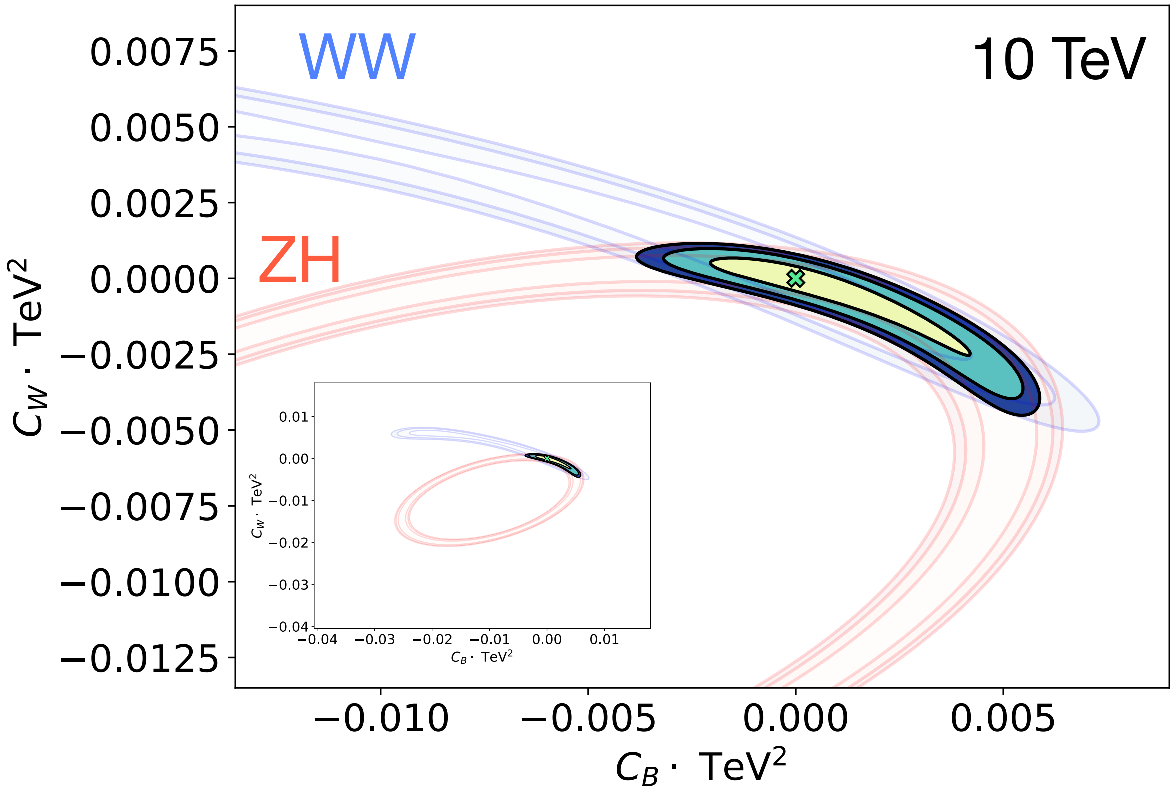

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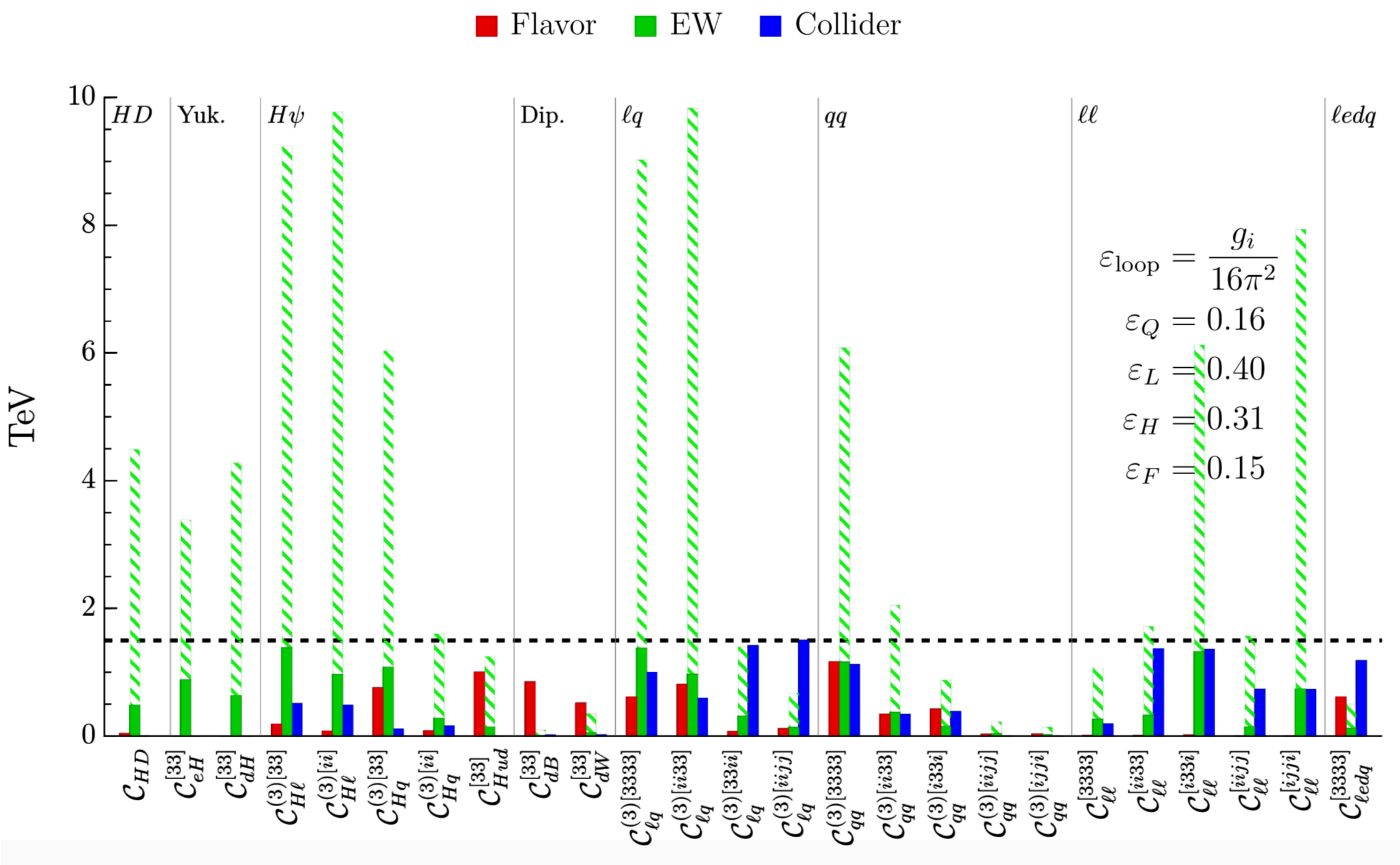
EW precision



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