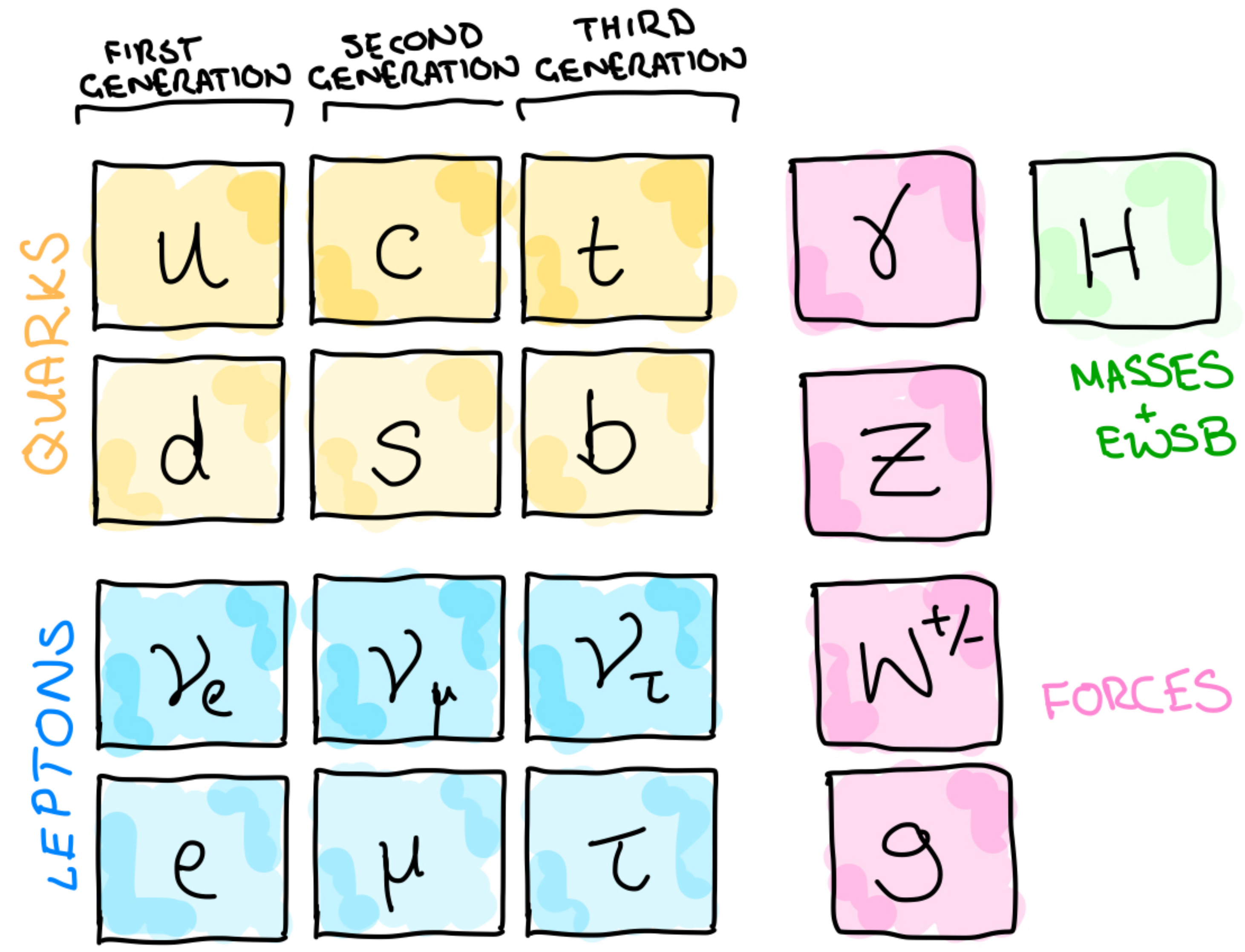
Charged-lepton flavour within and beyond the Standard Model

Innes Bigaran



To gain a full picture of the lepton flavour sector we need to combine charged-lepton (including direct leptonic decays *and* via EFT and colliders) and neutrino probes and work together.

Charged-lepton flavour within and beyond the Standard Model

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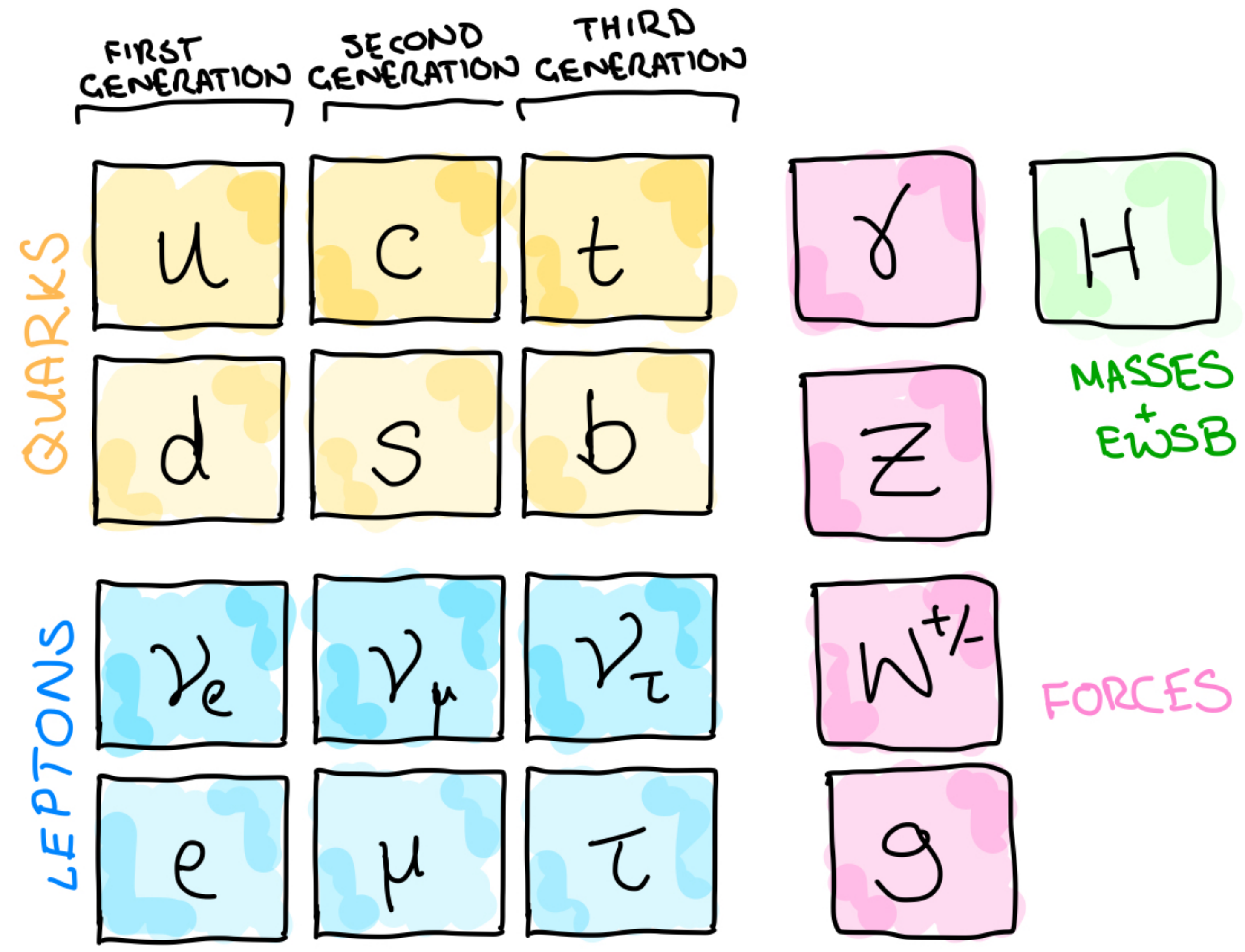
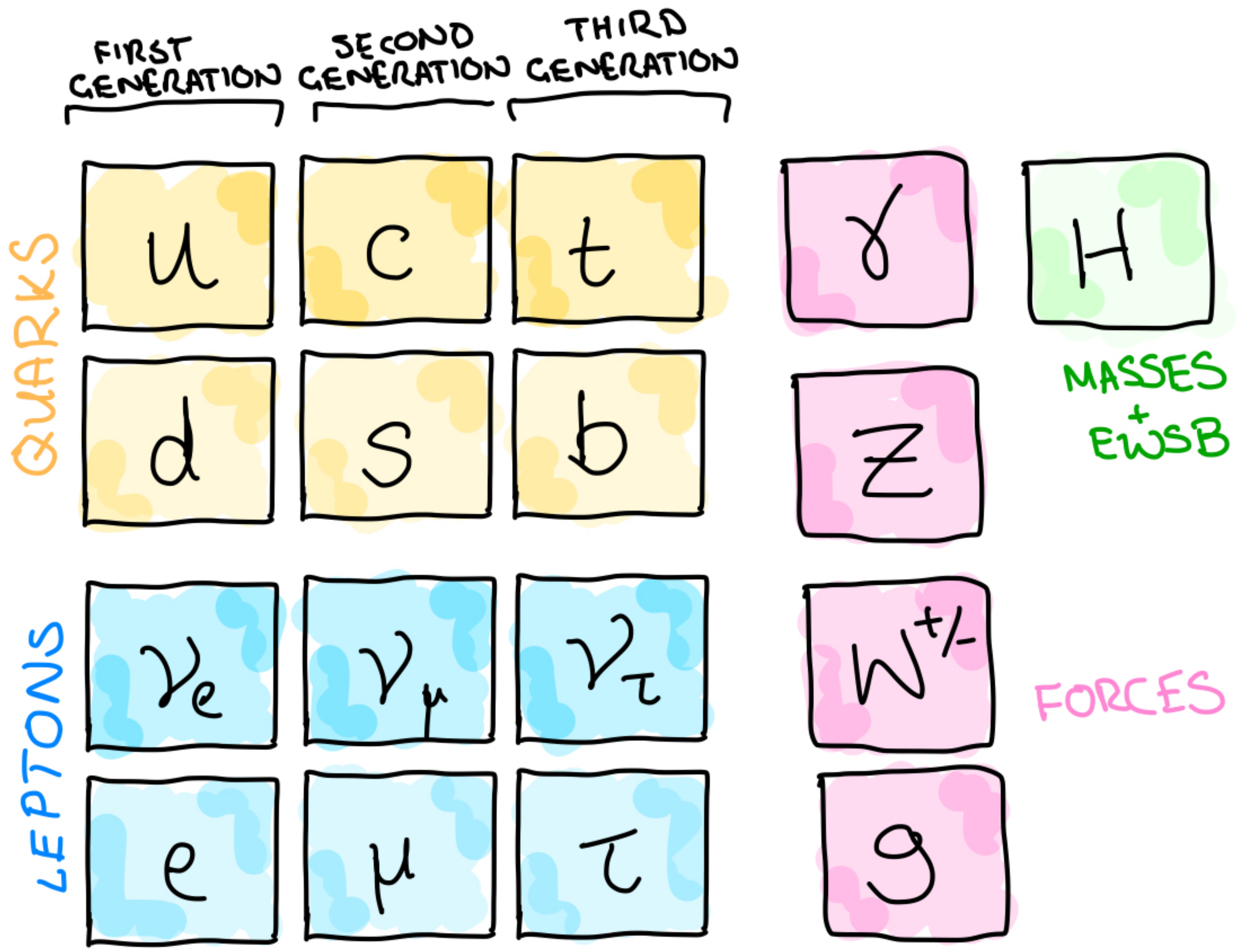
… with the tau flavour :)



Flavour in the Standard Model

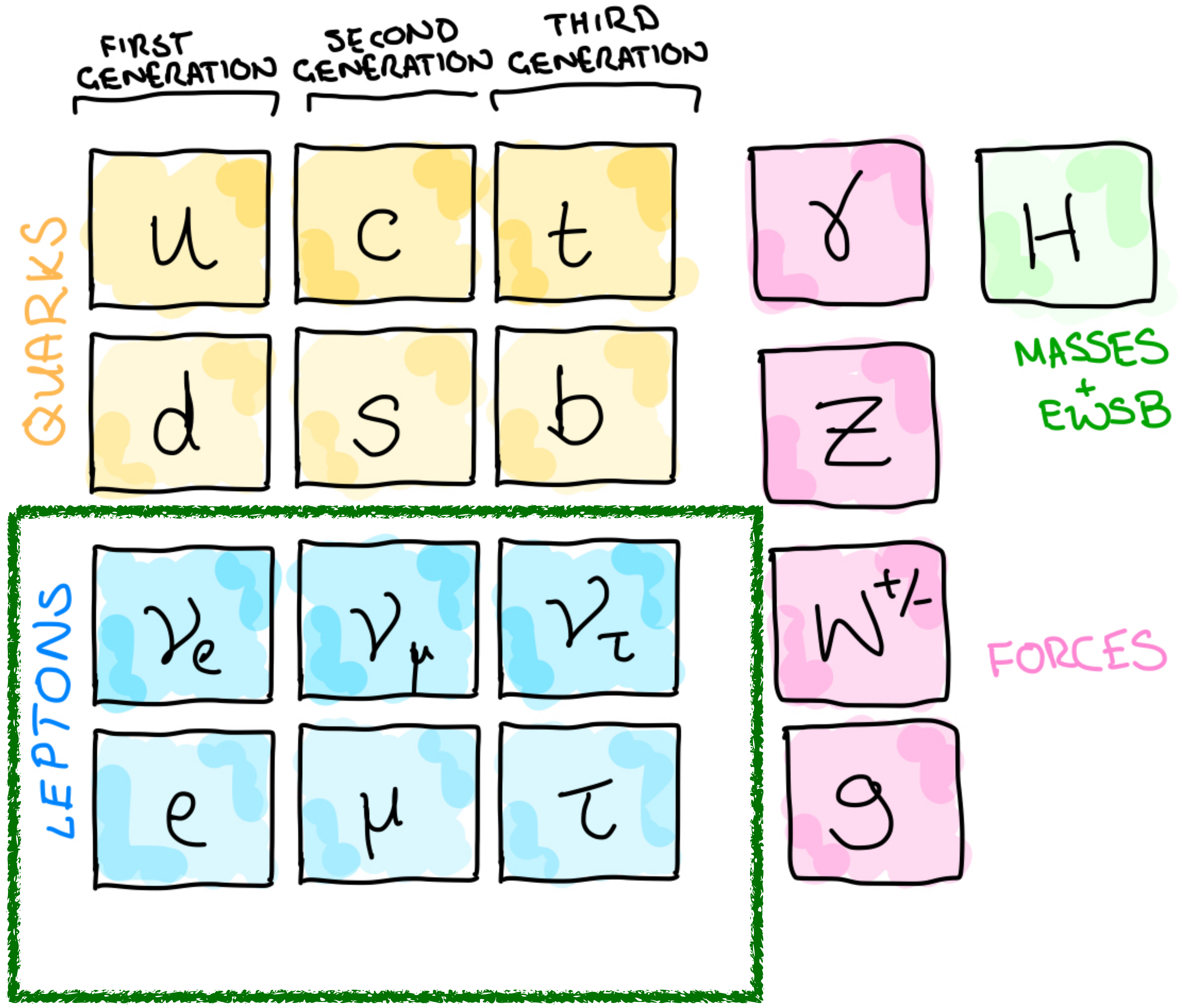


- The SM is a semi-empirical theory. Requires experimental input to fix ~ 27 free parameters to fully prescribe it

Gauge	Force interactions	3 gauge couplings
Higgs	EWSB and W/Z masses	2 Higgs-potential couplings
Flavour	Quark and lepton masses and	~ 22 free parameters

"Standard Model Flavour Puzzle"

Lepton flavour in the Standard Model



- The SM has an accidental $U(3)^5$ flavour symmetry, broken explicit by the Yukawa interactions of fermions with the Higgs
- In the Lepton sector, the remaining symmetry is

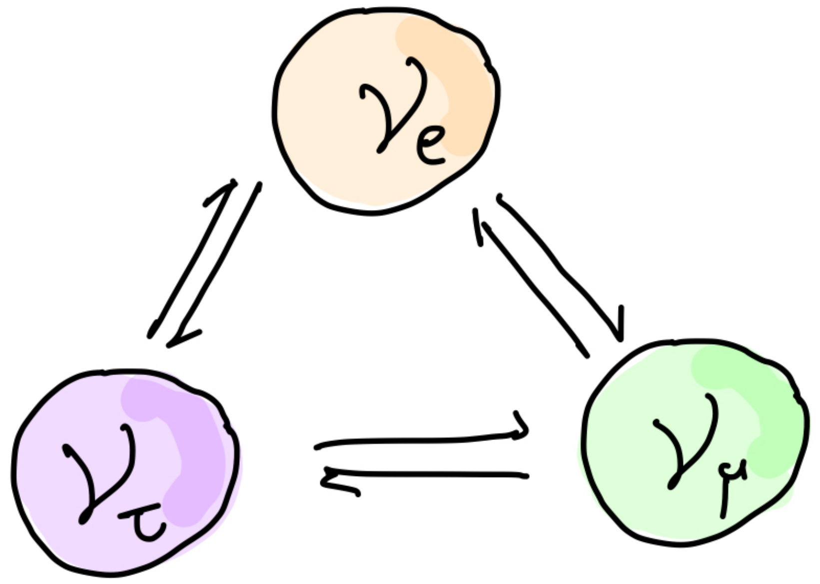
$$\mathcal{G}_{\rm L} = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

- Flavoured lepton number is conserved in [perturbative] SM interactions, thus also total (sum of flavours) lepton number

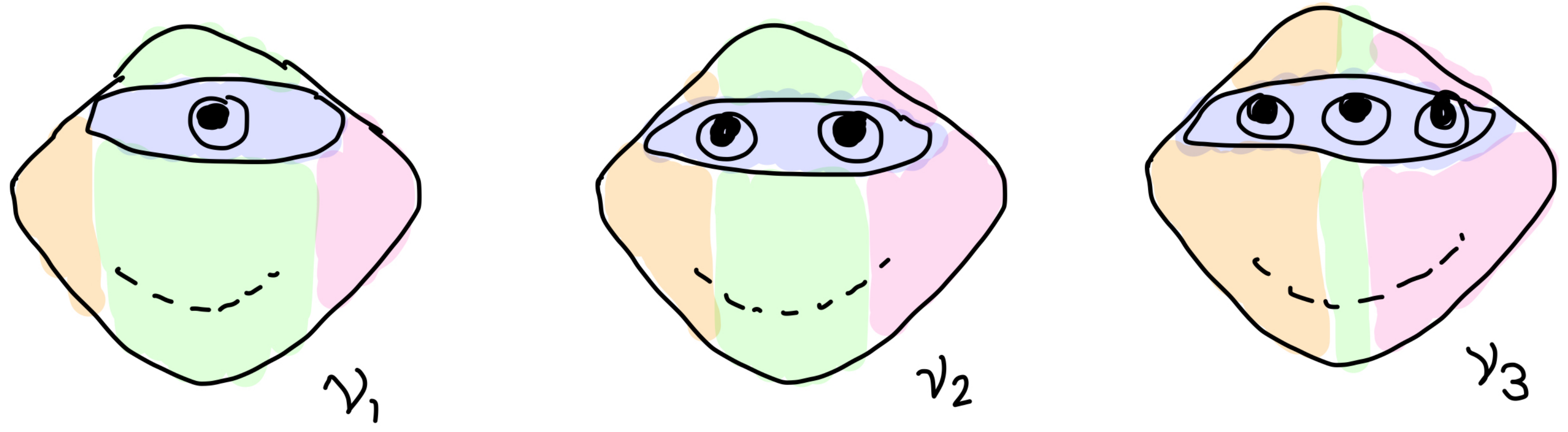
$$L = L_\mu + L_e + L_\tau$$

- Lepton flavour violation (LFV) is forbidden by accidental global symmetry

Violation of Lepton Flavour beyond the SM



Mass eigenstates are linear combinations of flavour eigenstates



- In the *vanilla* SM, neutrinos are massless.
- Neutrino and neutrino physics provide a probe of lepton flavour symmetries
- Neutrino flavour oscillations tell us that beyond the SM lepton flavour *is* violated.

Neutrino masses/oscillation imply

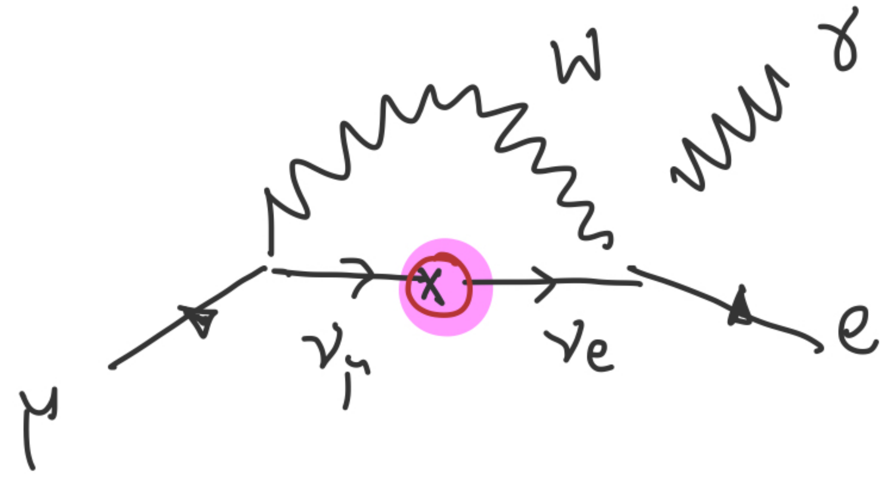
1. We need BSM physics

(At least “an extended theory beyond dim 4 SM”)

2. This new physics violates lepton flavour symmetry

Where else could we see LFV?

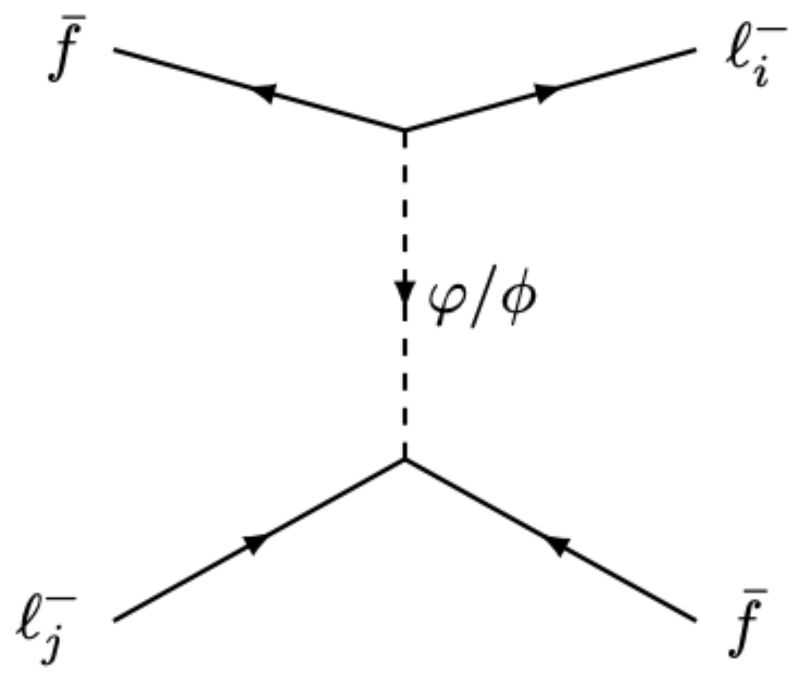
SM+ minimal neutrino mixing



< $\mathcal{O}(10^{-54})$

Petcov, 1977

SM+ model of neutrino masses?

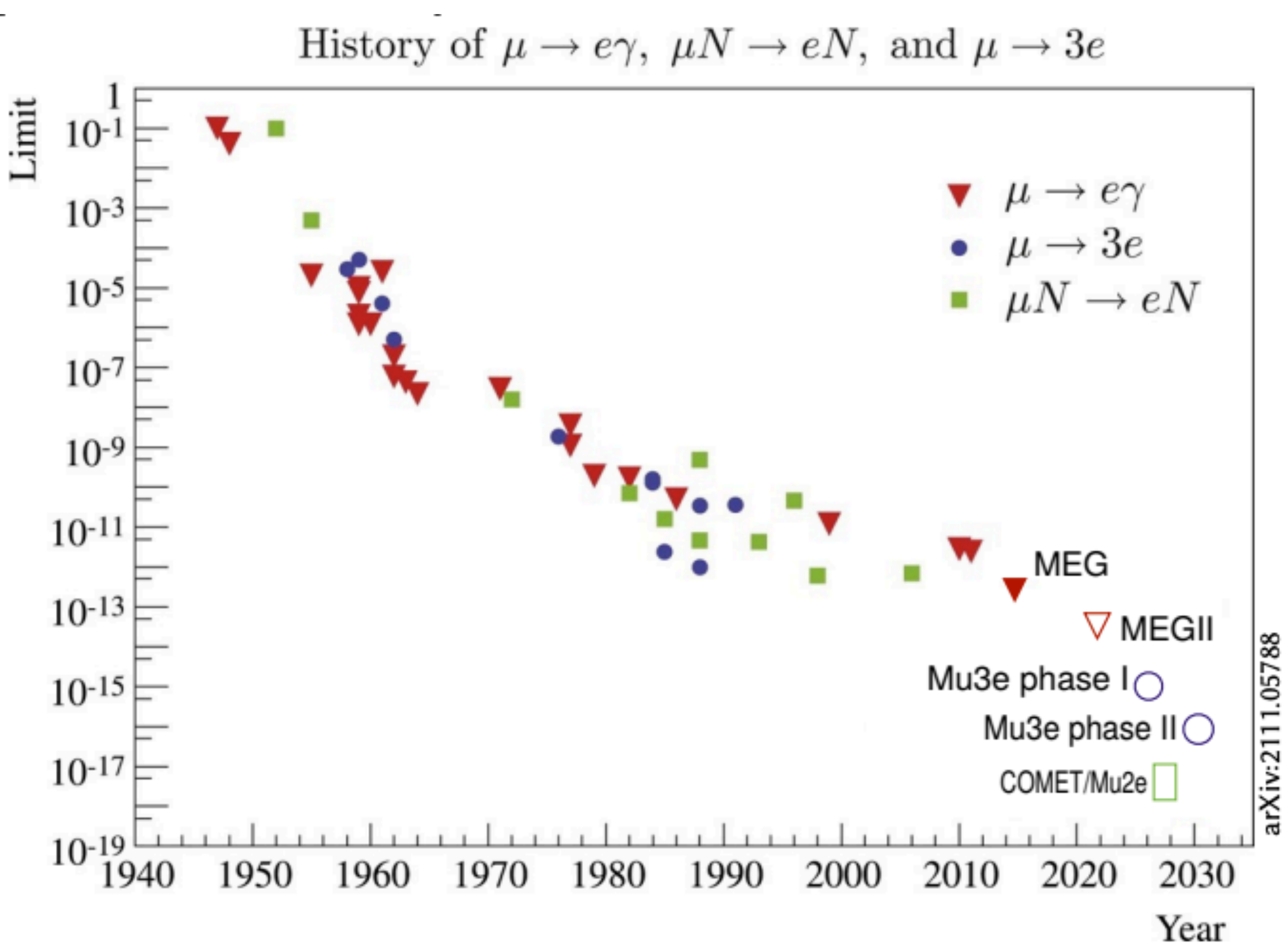


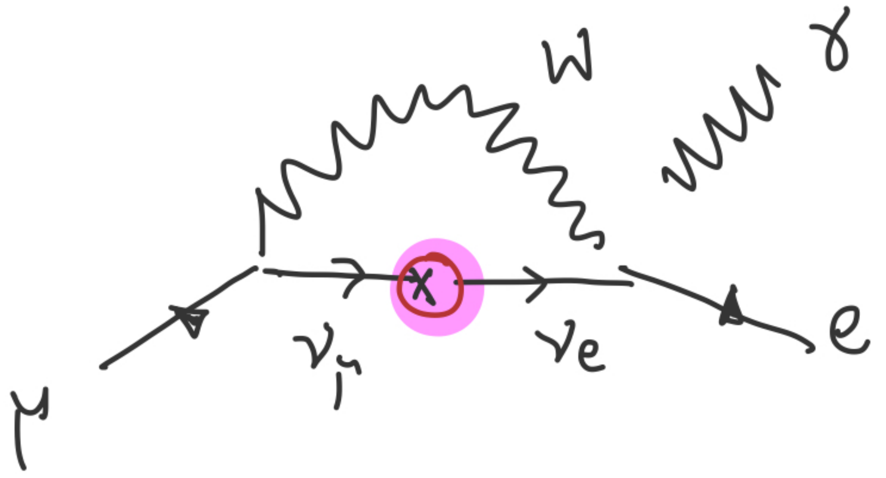
e.g. IB, Gargalionis, Volkas 1906.01870 model for neutrino masses and $b \to c\tau\nu$

- Introducing explicit flavour-mixing between neutrinos, how large will the observable effect be in charged-lepton flavour violation (cLFV)?
- SU(2) symmetry of the SM links neutrinos and (left handed) charged leptons: can we see LFV in the charged-lepton sector? Model dependent.
- By the symmetry argument presented, we expect these effects will be small *in SM+neutrino mixing*
- We don't yet know though *how* this flavour mixing is generated in a UV-complete model, incl. e.g. are neutrinos Dirac or Majorana? Depending on the model, these effects COULD be much larger…

An observable detection of cLFV is genuine sign of new physics.

Searching for charged LFV





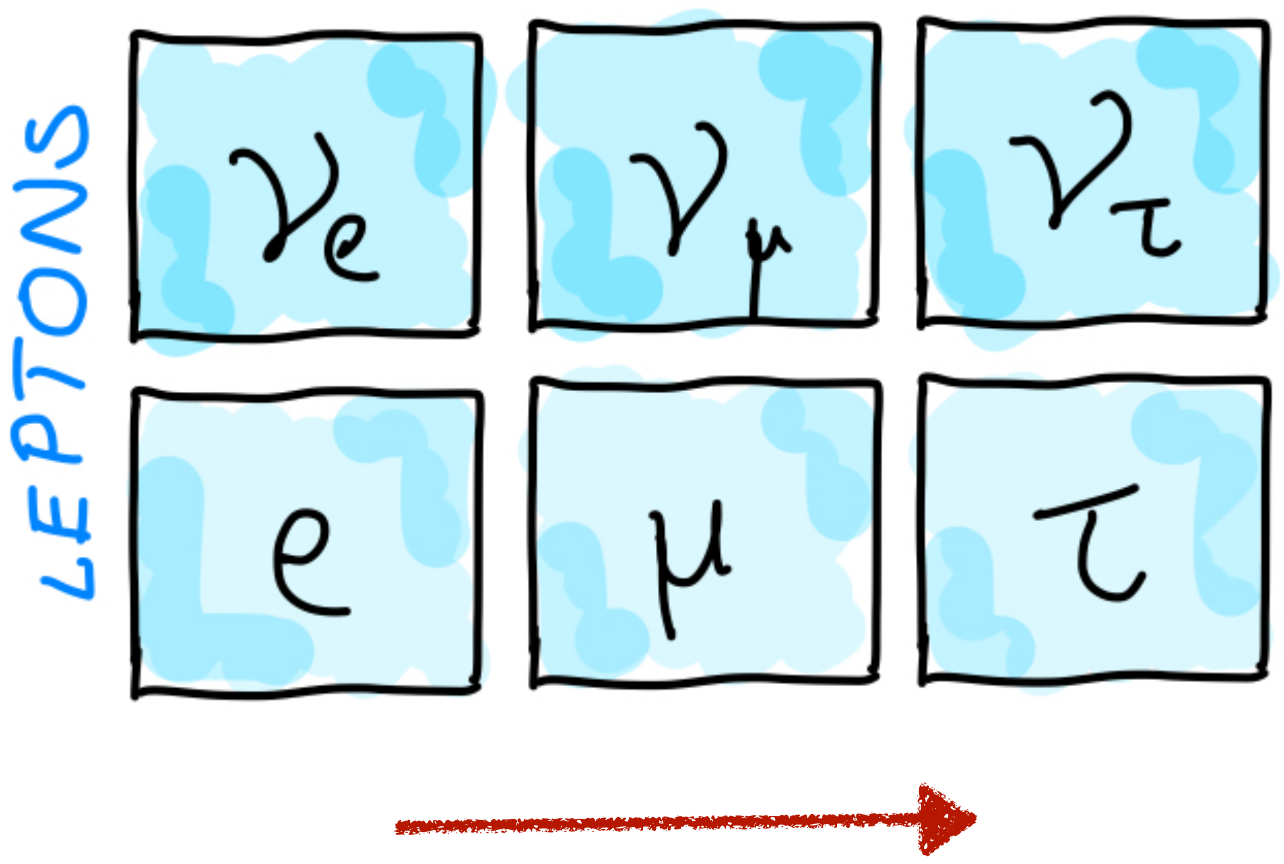
$< \mathcal{O}(10^{-54})$

Petcov, 1977

- Neutrino experiments and searches for charged LFV (cLFV) are complementary probes of the breaking of lepton flavour symmetries
- Many searches for cLFV have focussed on muon decays:
  - Muon mass ~ 105 MeV and lifetime ~ 2.2 μs

    ‘Goldilocks mass’: no hadronic decays, but does decay and we can make muons quite readily
- However, we are yet to observe any cLFV in muon experiments, and we are able to set very strong limits with present and upcoming experiments.

Searching for cLFV with the tau



Larger mass, smaller lifetime (charged leptons)

$\tau \to 3e$

$\tau \to e\mu\mu$

$\tau \to 3\mu$

$\tau \to \mu ee$

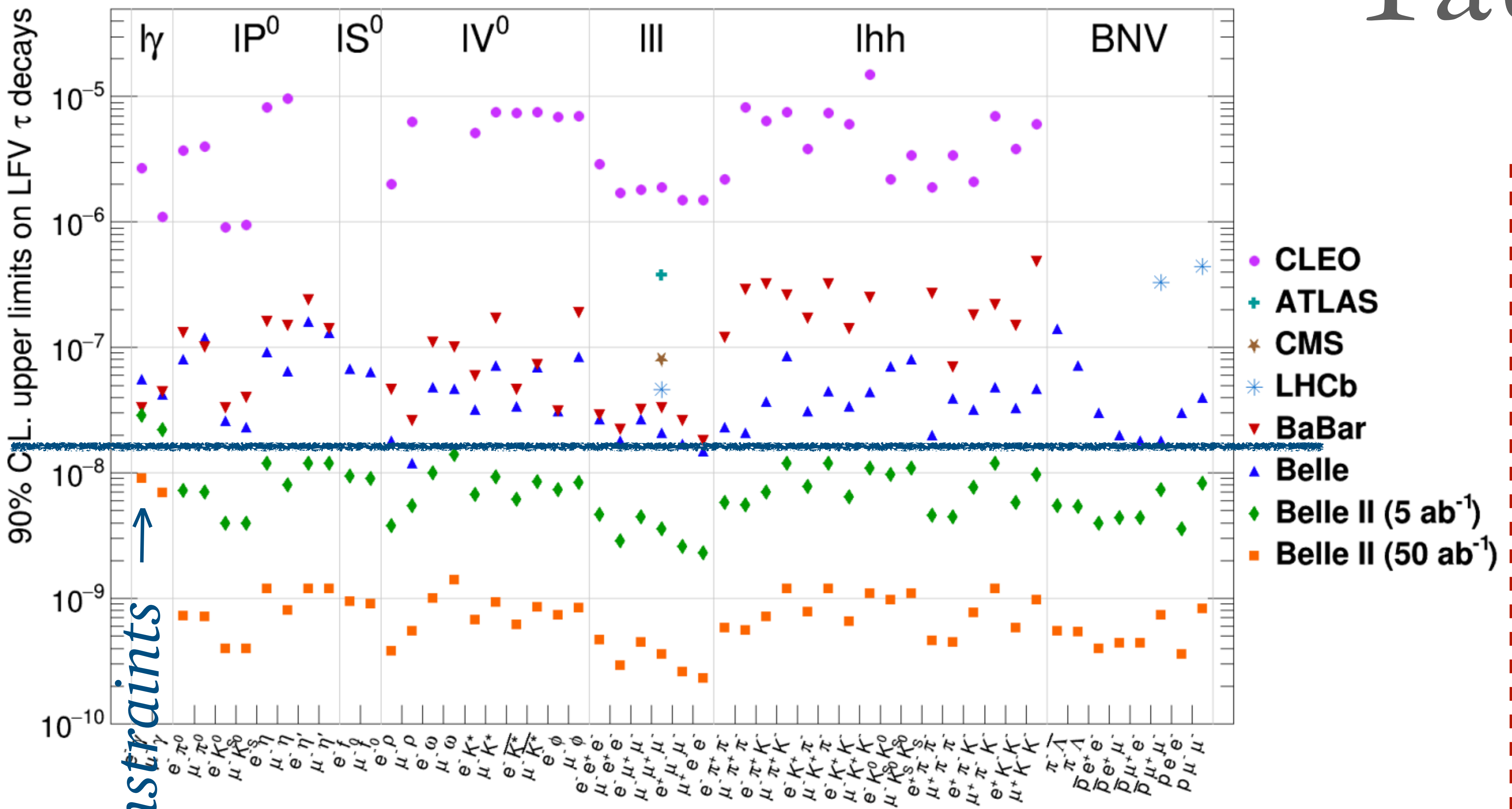
$\tau \to e\gamma$

$\tau \to \mu\gamma$.

+ more.

- The physics of the tau is much more challenging to experimentally probe.
- Tau leptons have a much shorter lifetime, and are also massive enough to decay into mesons…
- However, they are also heavy enough to decay into various combinations of other charged and neutral leptons: ideal “factory” for probing lepton symmetries beyond neutrino physics…

Tau physics at colliders



~Present constraints →

Banerjee et al. 2203.14919

$\tau \to 3e$

$\tau \to e\mu\mu$

$\tau \to 3\mu$

$\tau \to \mu ee$

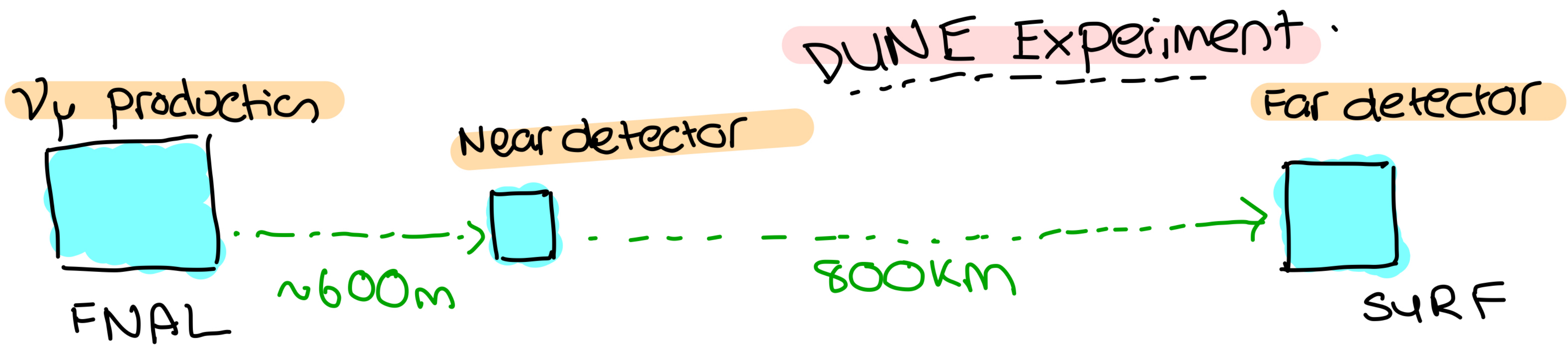
$\tau \to e\gamma$

$\tau \to \mu\gamma$.

+ more.

- Tau decays probe all types of lepton flavour transitions
- Belle II plans to improve many of the tau decay sensitivities by *up to two orders of magnitude* in the branching ratios.
- Already, current bounds from tau decays are very strong constraints on many BSM models coupling to third generation leptons. e.g. models for $b \to c\tau\nu$?

Tau physics at neutrino experiments



$$P \propto \sin^2\Big(\underbrace{\frac{\Delta m^2 L}{4E_\nu}}_{1.27\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E_\nu}\right)}\Big)\sin^2\theta$$

$\mu \to \tau$?

$\Delta m_{23}^2 \sim 10^{-3}\ eV^2$
(atmospheric)

$E_\nu \sim GeV$

$\Rightarrow L \sim 1000\ Km$
to observe oscillations
into the τ.

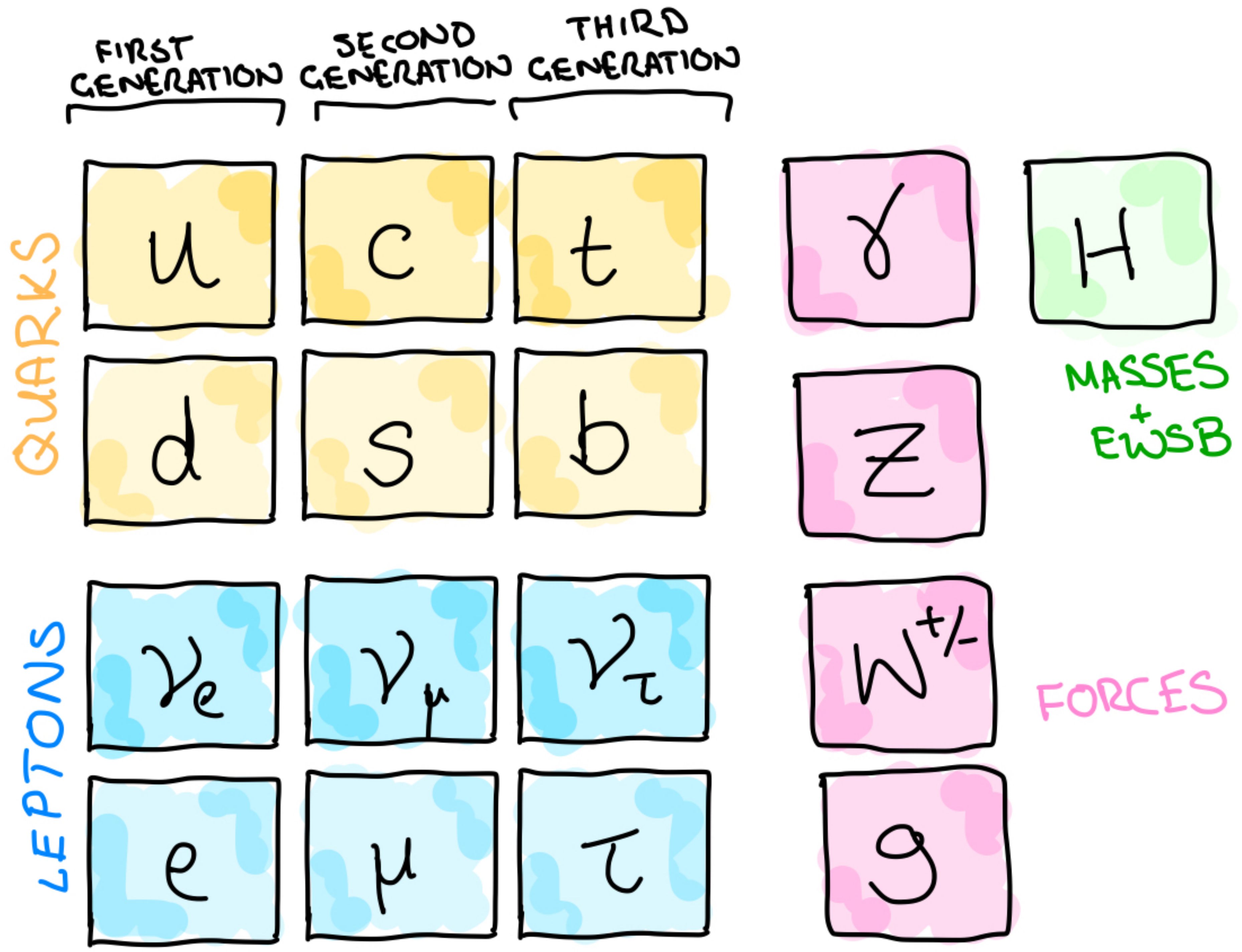
Production of tau neutrinos at the near detector kinematically suppressed in scattering, and the baseline is too short for production by oscillation.

- Many neutrino experiments aim to measure oscillation parameters: narrowing down the structure of the PMNS matrix. Tau appearance requires long baseline.
- But also precision measurements of neutrino scattering off targets. Production of the tau via scattering is limited by its large mass.
- If SM processes are so small, BSM models could enhance tau/tau neutrino production and “anomalous tau event” appearance at the near detector could be a complementary probe for cLFV new physics?

Part I

Why may BSM physics preference the tau?

Based on IB, XG He, M.A. Schmidt, G. Valencia, R.Volkas
Phys.Rev.D 107 (2023) 5, 055001
arXiv: 2212.09760



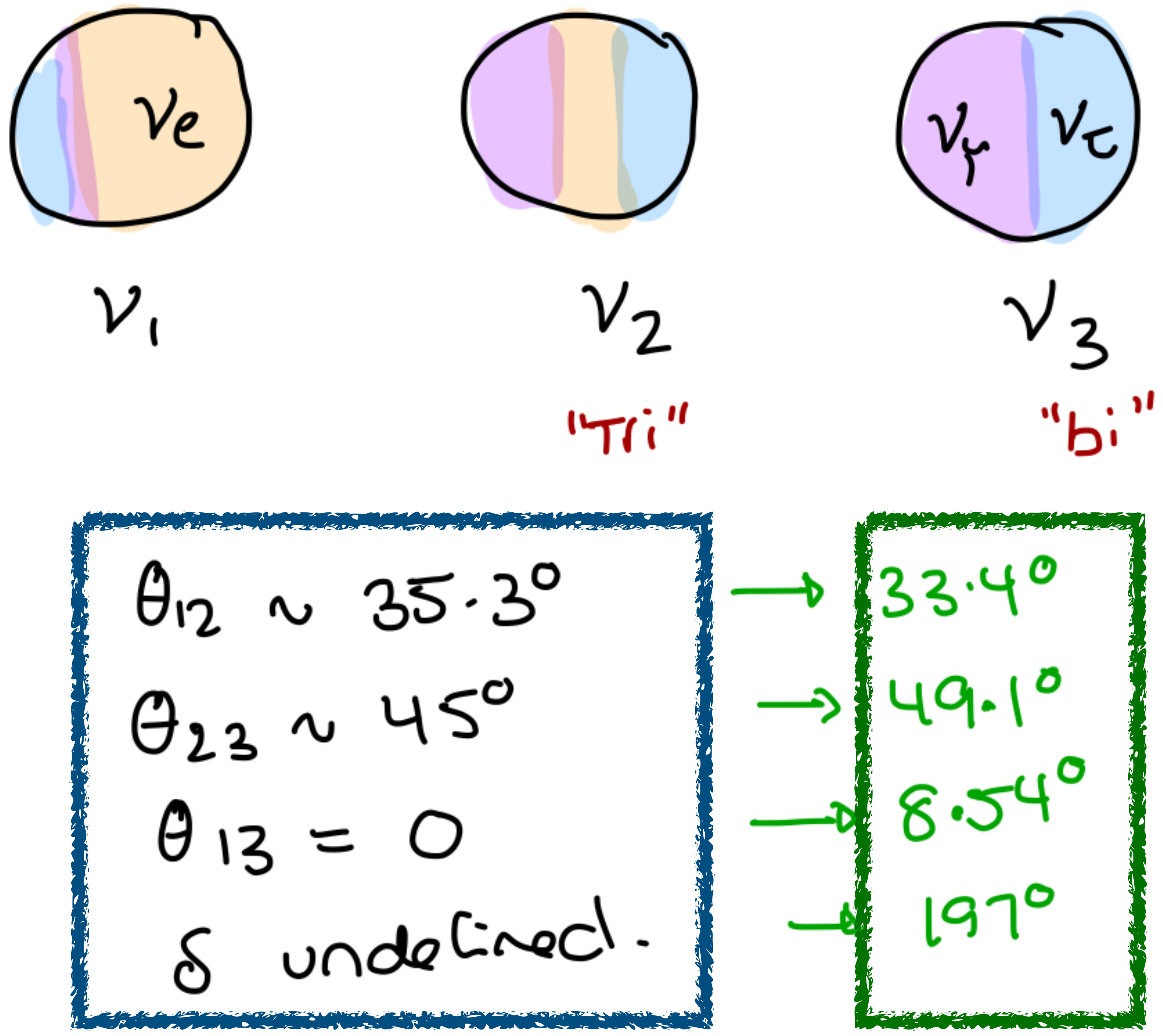
Lepton flavour triality

Ernest Ma, 2010 “Quark and lepton flavour triality” 1006.3524

- Many models seek to explain the flavour symmetries of the SM and beyond. In particular, they often begin with a large symmetry and step-wise break this symmetry into preserved *residual* subgroups
- In the SM+ neutrino masses, experimentally we know that cLFV is small and so are neutrino masses hinting at the usefulness of residual symmetries in BSM model building.
- Here we focus on a hypothetical subgroup in the lepton sector: *lepton flavour triality*

Lepton flavour triality

Ernest Ma, 2010 “Quark and lepton flavour triality” 1006.3524



Predicted by tribimaximal mixing

Present CVs (NuFit 2023)

- Neutrino flavour mixing should be explained by a flavour symmetry model of the lepton sector
- Prior to 2011, it was quite possible to explain observed neutrino mixing using a *tribimaximal* hypothesis.
- A very successful model for explaining this mixing was based on the tetrahedral group A_4, breaking $A_4 \rightarrow Z_3$ in the charged-lepton sector and $A_4 \rightarrow Z_2$ in the neutrino sector
- In 2011, Daya Bay and Reno measured a nonzero θ_{13}, inconsistent with tribimaximal mixing. But we can still learn from this model-building effort.

(Altarelli + Feriglio 0512103, He, Kuem +Volkas 0601001)

What is lepton flavour triality?

- Approximate Z_3 “lepton triality”: a discrete subgroup of lepton flavour.
- Other flavour models may also exhibit this residual subgroup structure. In fact, one may seek to build this into a more complete flavour model if it proves to be phenomenologically motivated…
- Each charged lepton is assigned a triality charge (e.g. in line with their generation)

The idea: each charged lepton is *charged* under a Z_3 (flavour triality)

$$\psi_T \to \left(e^{2\pi i/3}\right)^T \psi .$$

T is a triality charge.

electron : T=1
muon : T=2
tau : T=3

} In line with generation.

One can trace the rotations through a Lagrangian, or as I will do here we can note that *triality sums modulo three* and look at explicit processes with this in mind…

What does this say about cLFV?

Charge assignments:

L_i has triality $T=i$

$e_{R,i}$ has triality $T=i$

& Triality sums modulo 3

Implications:

$\mu \to e \gamma$

$T=2$ $T=1$

$\Delta T \neq 0$

Triality breaking!

$\tau^- \to \mu^+ e^- e^-$

$T=3$ $T=-2+1+1=0$

$0 \bmod 3 = 3$

$\Rightarrow \Delta T = 0$!

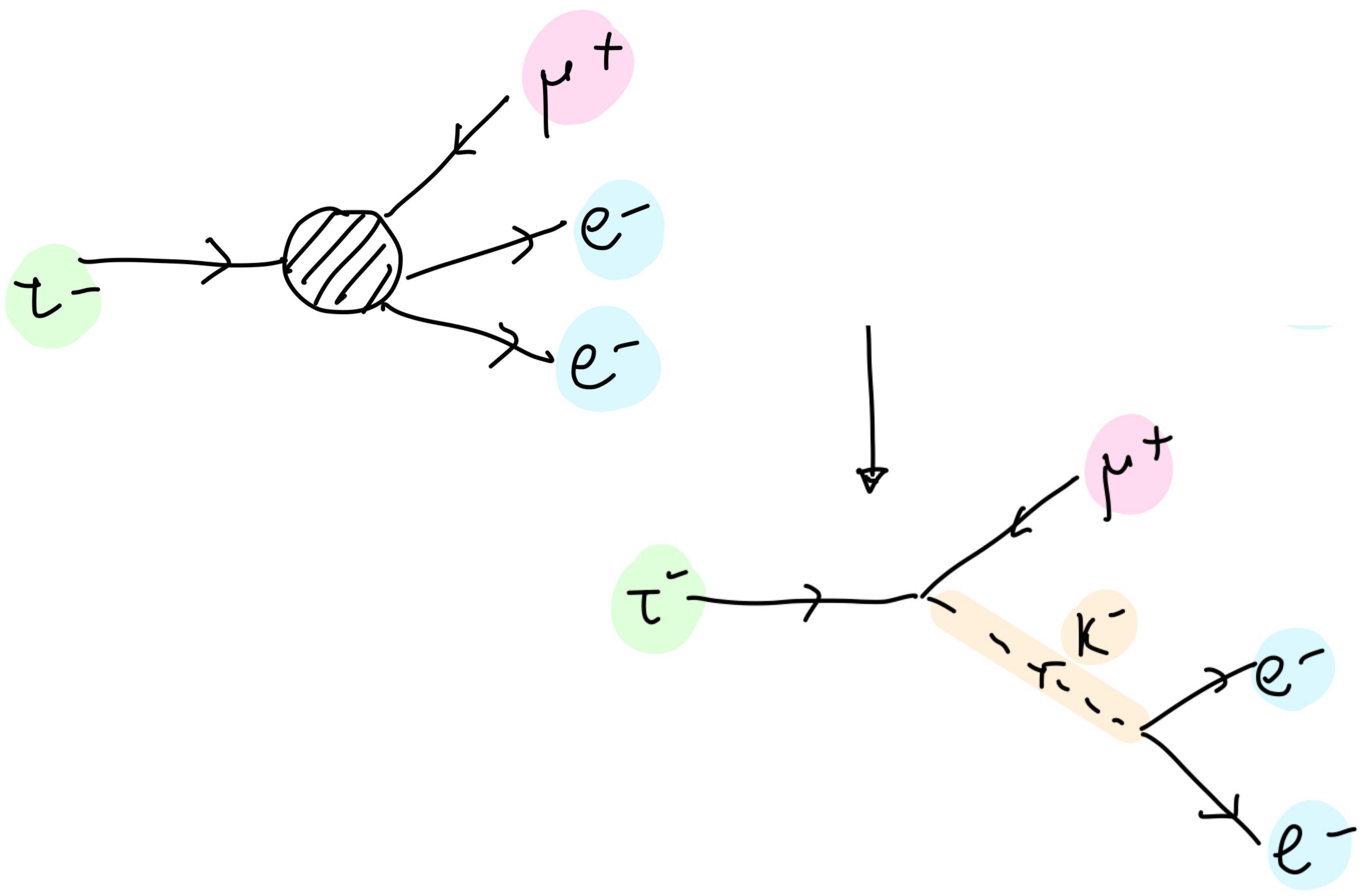
Triality preserving!

- If triality is a good symmetry, then it should be conserved.
- This permits certain cLFV processes, and “forbids” others.
- If triality is ultimately broken by some small parameter, then this approximate symmetry suppresses the size of any triality breaking processes [observable ν mass requires ultimate breaking]

If the lepton flavour symmetry has triality as a feature, then we are MUCH more likely to see cLFV in tau decays!

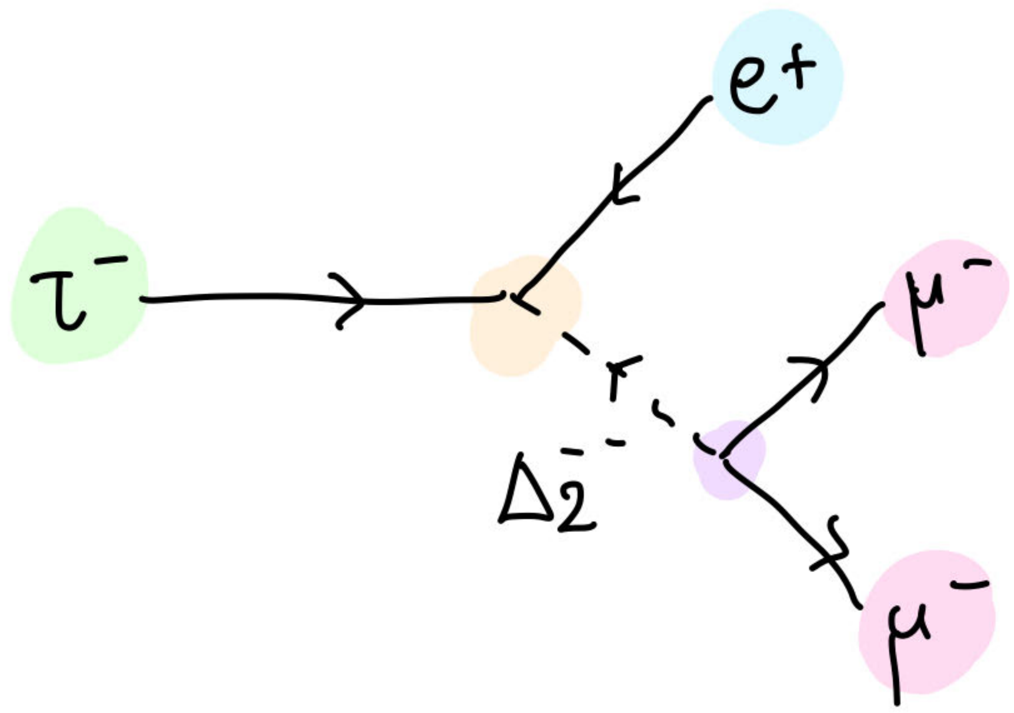
So where should we look for cLFV?

Observable	Present constraint	Projected sensitivity
$\mathrm{BR}(\tau^- \to \mu^-\mu^-e^+)$	$< 1.7 \times 10^{-8}$ [1]	2.6×10^{-10} [2]
$\mathrm{BR}(\tau^- \to \mu^+e^-e^-)$	$< 1.5 \times 10^{-8}$ [1]	2.3×10^{-10} [2]



- There are two triality-preserving cLFV decays
- The sensitivity to these decays at experiment is expected to be improved significantly at Belle II
- If triality is a ‘good’ lepton flavour symmetry, these are our best bet for finding signs of the underlying new physics violating lepton flavour beyond the SM
- Simplest models which generate these effective interactions involved *scalar bileptons:* particles which couple directly to two leptons See review e.g. Davidson, Cupyers hep-ph/9609487
- We study other constraints on these models under the triality assumption in

Neutrino masses triality-based models



$$\Delta_T \sim \begin{pmatrix} \Delta_T{}^+ & \Delta_T{}^{++} \\ \Delta_T{}^0 & -\Delta_T{}^+ \end{pmatrix}$$

doubly charged scalar.

isospin partners

$$\mathcal{L}_{\Delta_1} = \frac{1}{2}\left(2f_1 \overline{L_3^c}\, \Delta_1 L_2 + f_2 \overline{L_1^c}\, \Delta_1 L_1\right) + h.c.$$

$$\overline{L_3^c}\, i\sigma_2 \mathbf{\Delta}_1 L_2 = -\overline{(\tau_L)^c}\,\mu_L\,\Delta_1^{++} - \frac{1}{\sqrt{2}}\left[\overline{(\tau_L)^c}\,\nu_{\mu L} + \overline{(\nu_{\tau L})^c}\,\mu_L\right]\Delta_1^{+} + \boxed{\overline{(\nu_{\tau L})^c}\,\nu_{\mu L}\,\Delta_1^{0}}$$

$$\overline{L_1^c}\, i\sigma_2 \mathbf{\Delta}_1 L_1 = -\overline{(e_L)^c}\,e_L\,\Delta_1^{++} - \sqrt{2}\,\overline{(e_L)^c}\,\nu_{eL}\,\Delta_1^{+} + \boxed{\overline{(\nu_{eL})^c}\,\nu_{eL}\,\Delta_1^{0}}$$

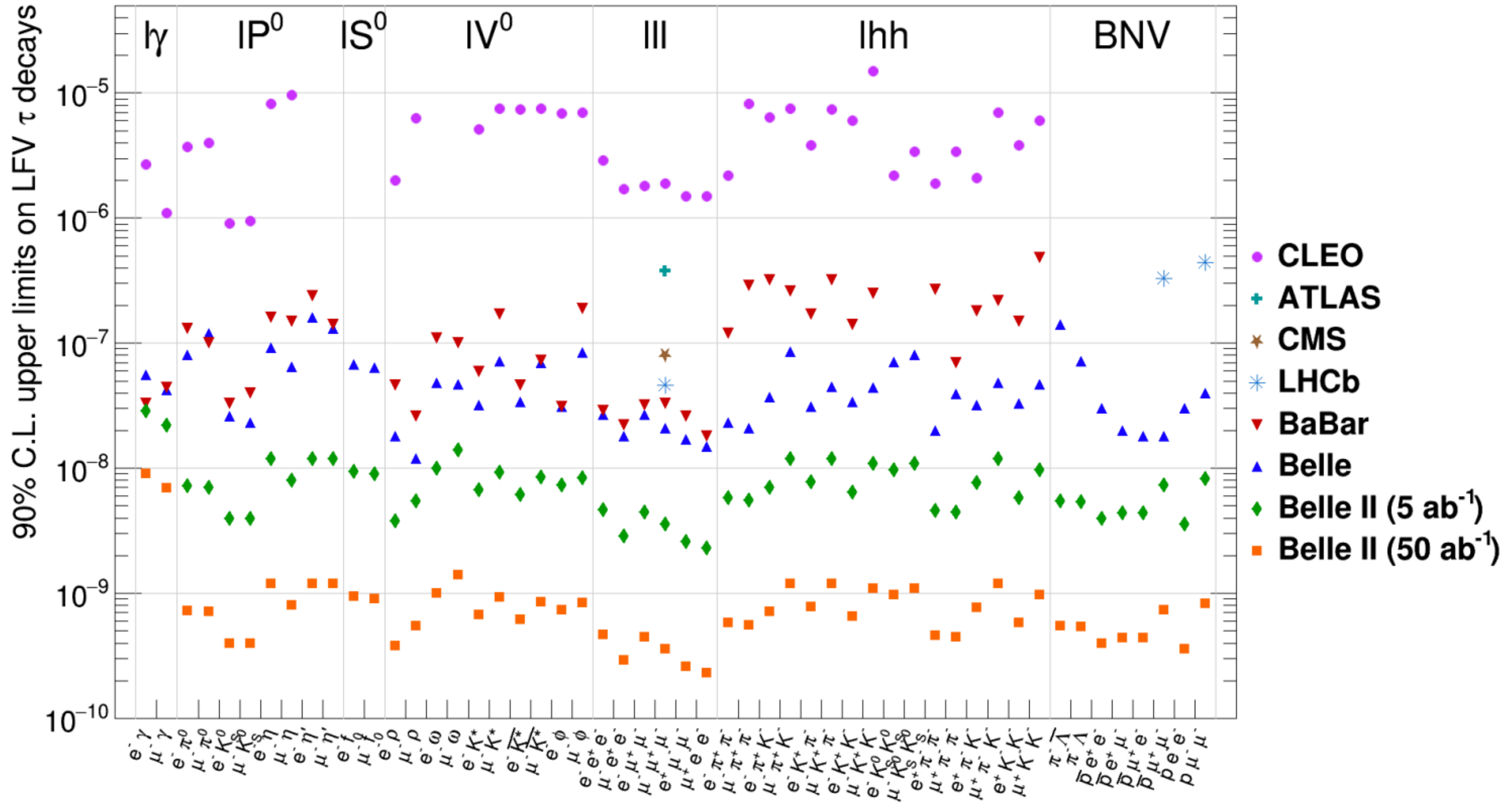
- Lepton triality needs to be broken to achieve observed neutrino mass texture.

Breaking triality to generate neutrino masses isn't bad, neutrino masses are small so triality protection helps to keep triality breaking effect small.

  - e.g. Triplet "bilepton": if neutral component gets a vev then we obtain Majorana neutrino mass, though with a restricted flavour structure (Type II seesaw). Naturally small *vev* due to soft triality breaking by a cubic Higgs coupling.

- Triality-breaking processes (e.g. muonic cLFV) are generated by this breaking, but their influence is suppressed and tau transitions remain dominant signals

Summary of Part I



$\tau \to 3e$

$\tau \to 3\mu$

$\tau \to e\gamma$

$\tau \to e\mu\mu$

$\tau \to \mu e e$

$\tau \to \mu\gamma$.

+ more.

- Consider lepton flavour-specific symmetries to motivate why muon-electron cLFV experiments may not be the "smoking-gun" location for lepton flavour signals
- Lepton flavour triality is an *example* of this
- In such a model, it becomes clear that there is interesting complementarity between neutrino experimental probes and "high-energy" LFV probes, esp. via tau decays
- With the prospects of upcoming tau experiments, could we look for LFV probes at neutrino experiments which may *not* be suppressed by the neutrino mass scale?

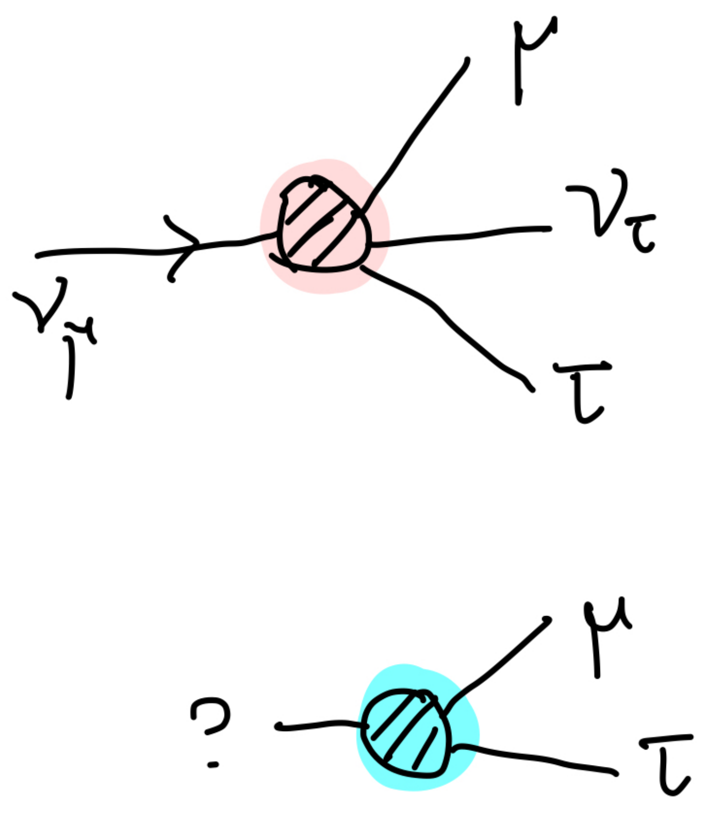
Part II

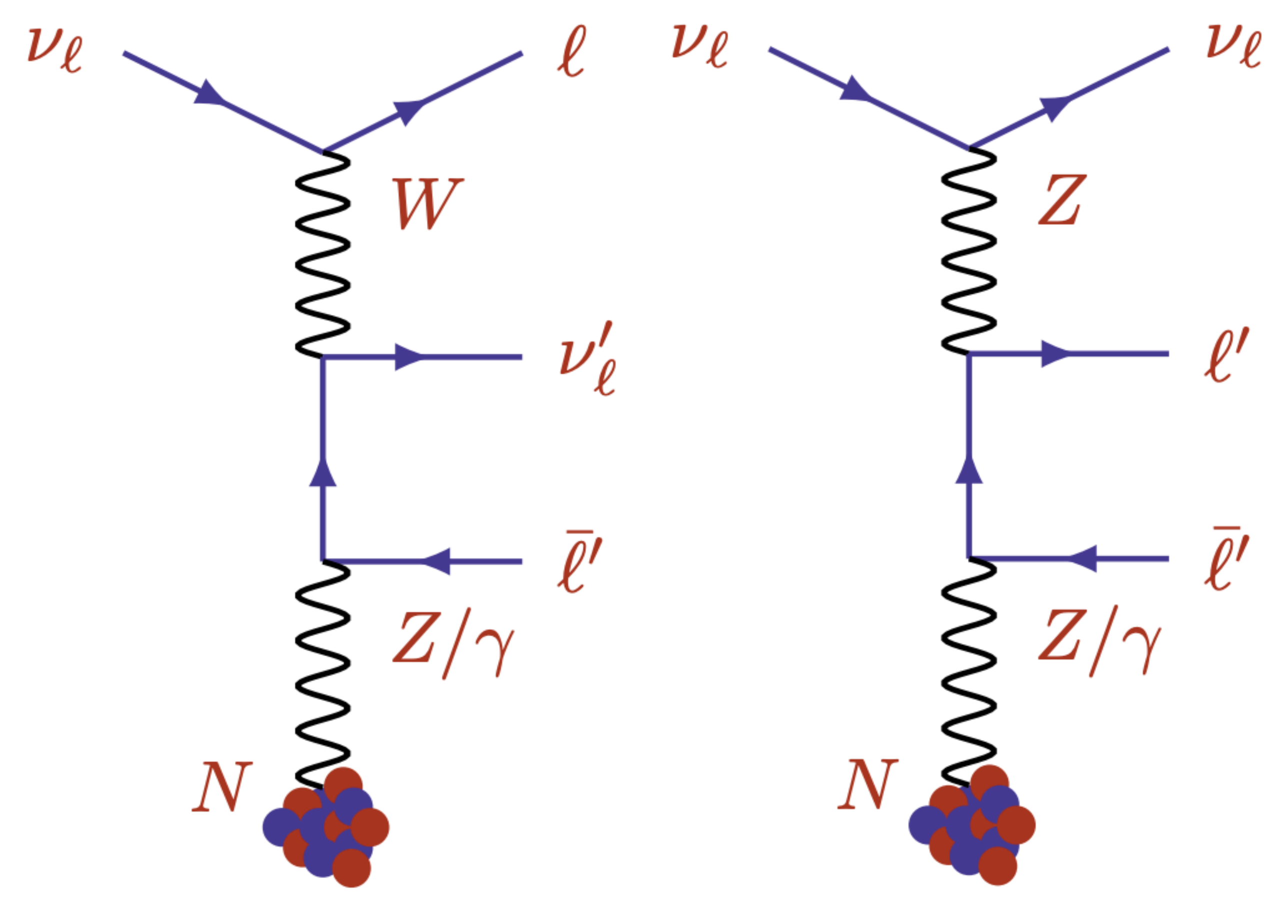
Are “anomalous” taus at high-intensity neutrino BSM physics?

Based on IB, B. Dev, D. Lopez Gutierrez, P. A. Machado
arXiv: 2406.xxxx (Coming ~ next fortnight!)

* unrelated to Triality*

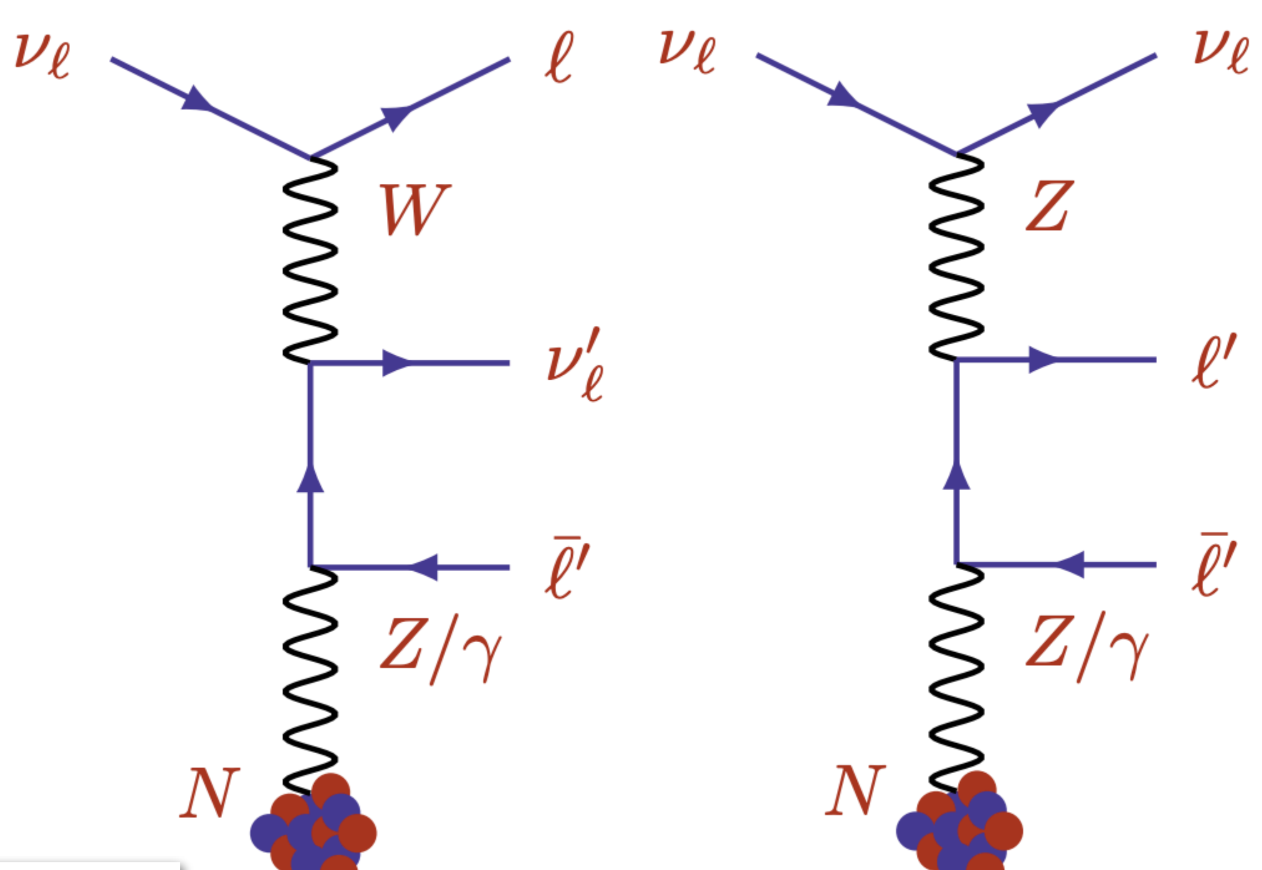
Producing and detecting tau neutrinos

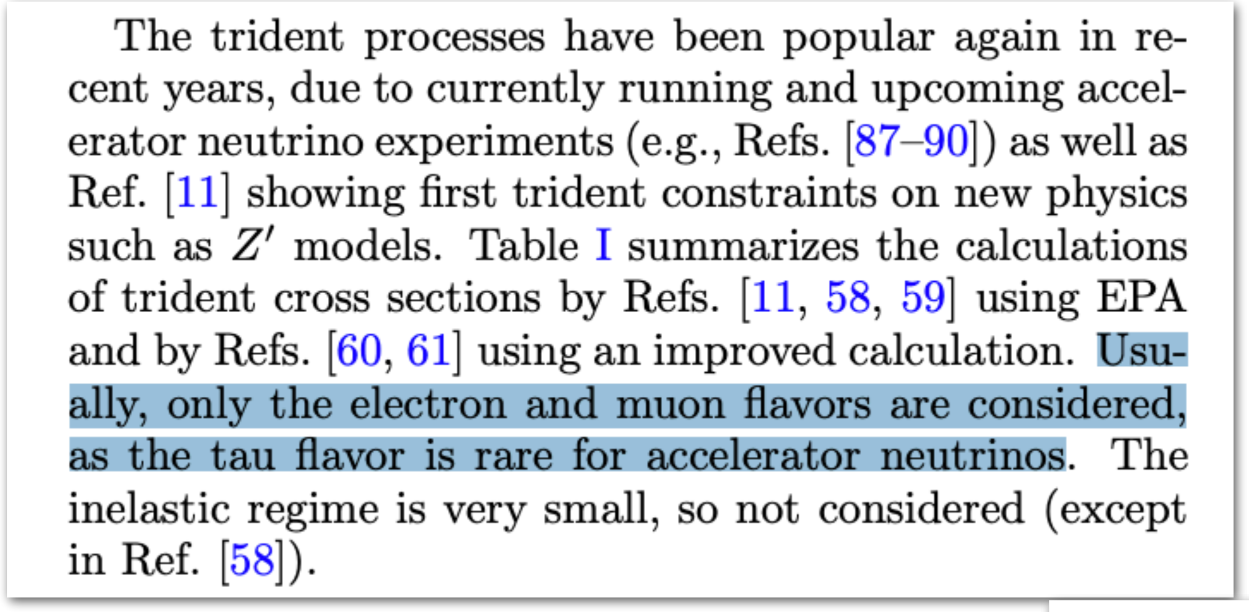




- The least well-studied standard model particle ~ O(10) events ever observed
- A tau neutrino detection is associated with observable tau production, requiring sufficient energy to create a tau at rest
- Neutrinos solely interacting via electroweak interactions imply very small cross sections, require large incoming neutrino fluxes to probe scattering
- Idea: Tau neutrino *trident production* at the near detector could "mimic" cLFV BSM effects. What do we know about these processes with the tau?

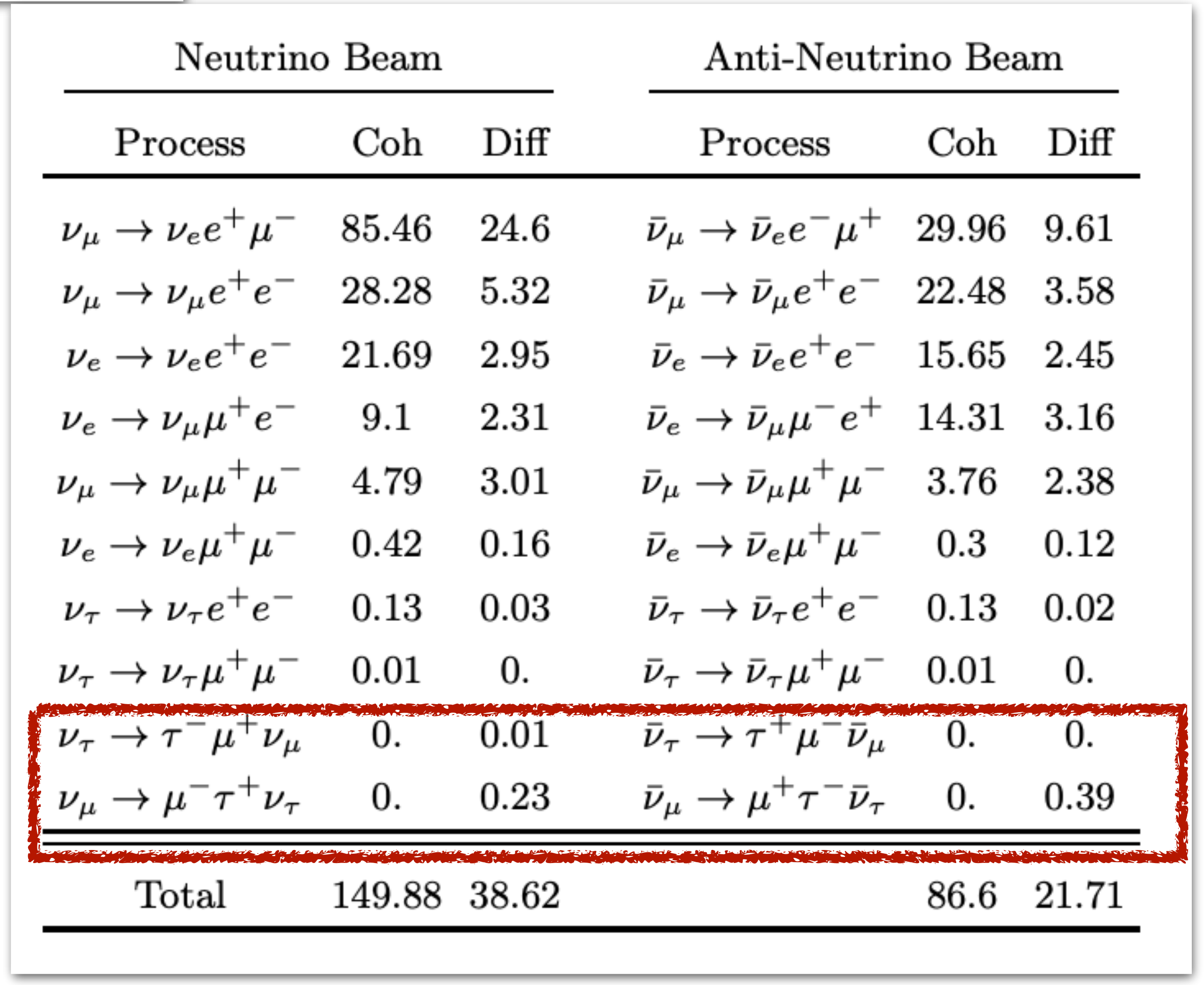
Neutrino Trident Production (NTP)





The trident processes have been popular again in recent years, due to currently running and upcoming accelerator neutrino experiments (e.g., Refs. [87–90]) as well as Ref. [11] showing first trident constraints on new physics such as Z' models. Table I summarizes the calculations of trident cross sections by Refs. [11, 58, 59] using EPA and by Refs. [60, 61] using an improved calculation. Usually, only the electron and muon flavors are considered, as the tau flavor is rare for accelerator neutrinos. The inelastic regime is very small, so not considered (except in Ref. [58]).

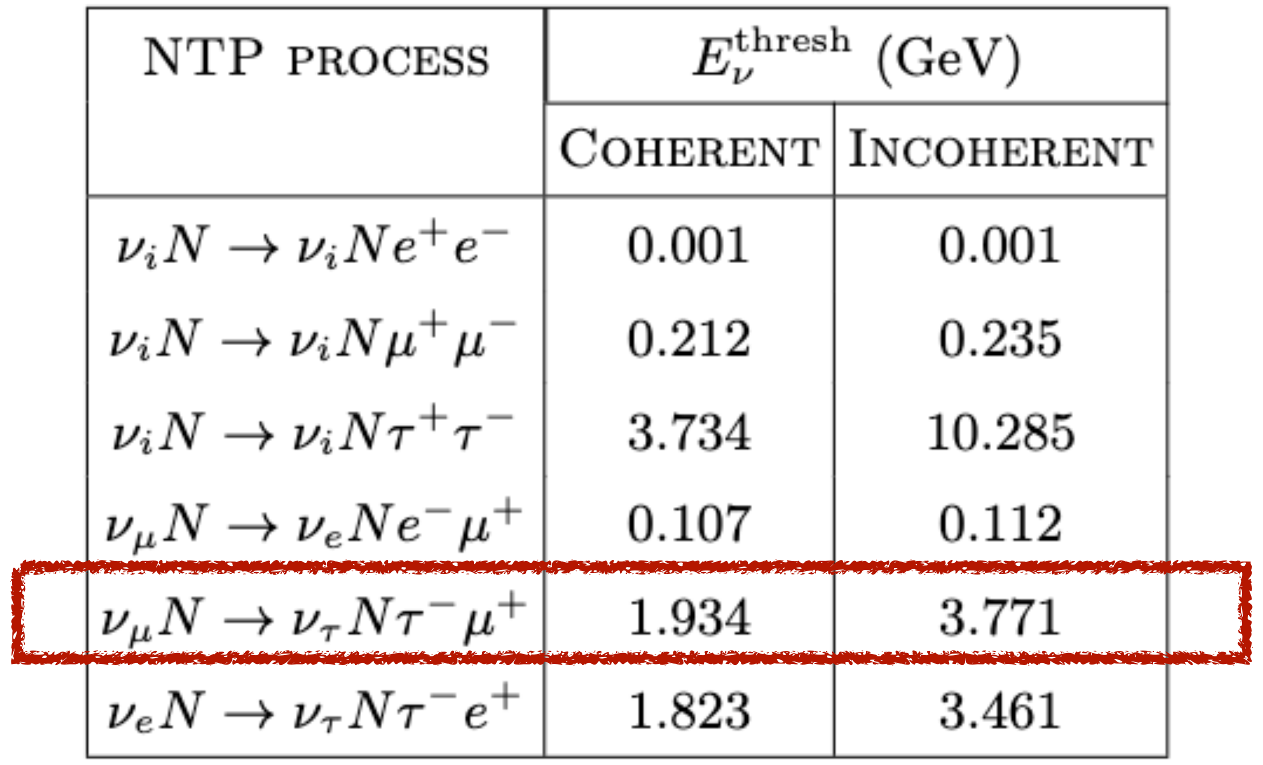
Zhou, Beacom 1910.08090



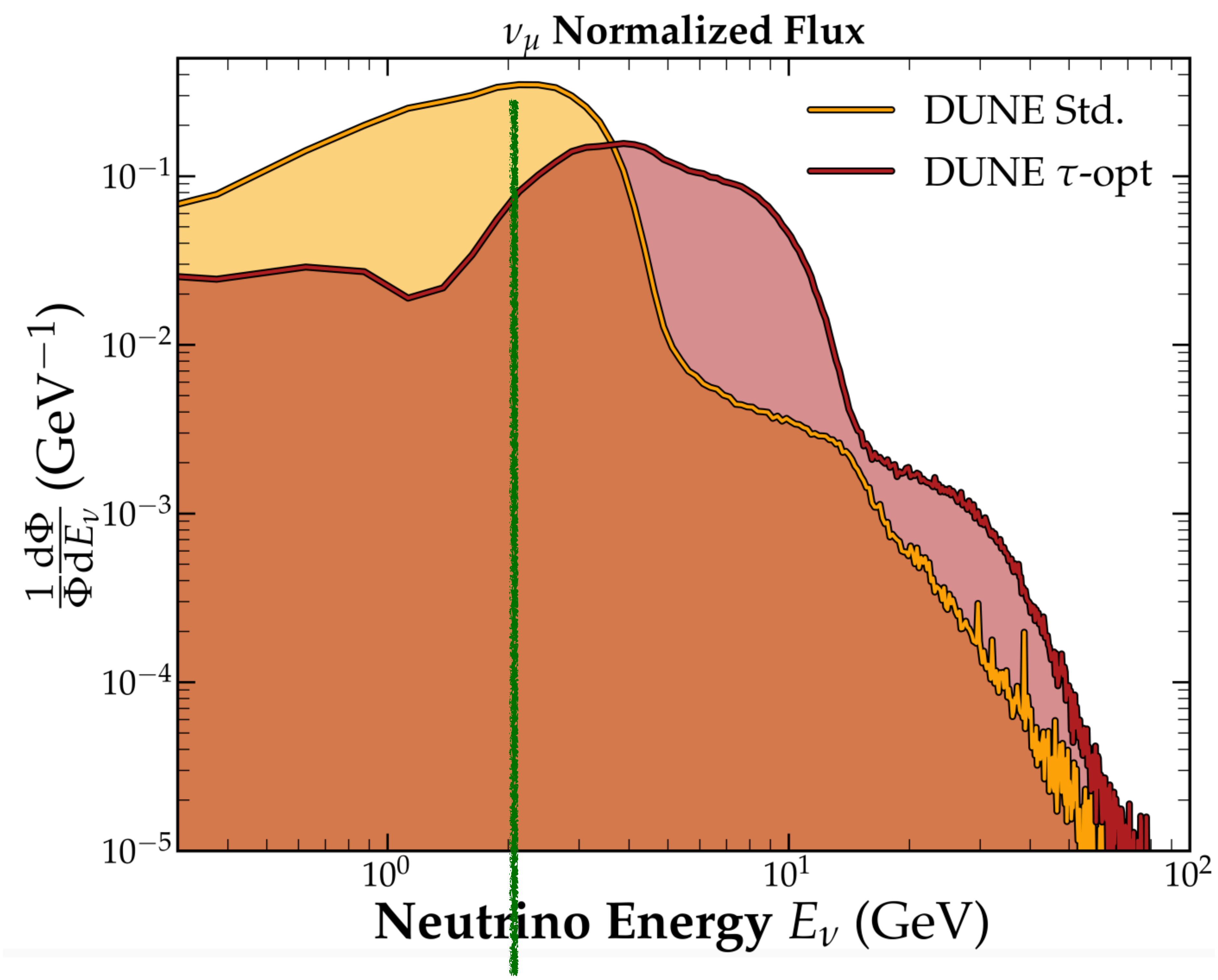
Neutrino Beam			Anti-Neutrino Beam		
Process	Coh	Diff	Process	Coh	Diff
$\nu_\mu \to \nu_e e^+ \mu^-$	85.46	24.6	$\bar{\nu}_\mu \to \bar{\nu}_e e^- \mu^+$	29.96	9.61
$\nu_\mu \to \nu_\mu e^+ e^-$	28.28	5.32	$\bar{\nu}_\mu \to \bar{\nu}_\mu e^+ e^-$	22.48	3.58
$\nu_e \to \nu_e e^+ e^-$	21.69	2.95	$\bar{\nu}_e \to \bar{\nu}_e e^+ e^-$	15.65	2.45
$\nu_e \to \nu_\mu \mu^+ e^-$	9.1	2.31	$\bar{\nu}_e \to \bar{\nu}_\mu \mu^- e^+$	14.31	3.16
$\nu_\mu \to \nu_\mu \mu^+ \mu^-$	4.79	3.01	$\bar{\nu}_\mu \to \bar{\nu}_\mu \mu^+ \mu^-$	3.76	2.38
$\nu_e \to \nu_e \mu^+ \mu^-$	0.42	0.16	$\bar{\nu}_e \to \bar{\nu}_e \mu^+ \mu^-$	0.3	0.12
$\nu_\tau \to \nu_\tau e^+ e^-$	0.13	0.03	$\bar{\nu}_\tau \to \bar{\nu}_\tau e^+ e^-$	0.13	0.02
$\nu_\tau \to \nu_\tau \mu^+ \mu^-$	0.01	0.	$\bar{\nu}_\tau \to \bar{\nu}_\tau \mu^+ \mu^-$	0.01	0.
$\nu_\tau \to \tau^- \mu^+ \nu_\mu$	0.	0.01	$\bar{\nu}_\tau \to \tau^+ \mu^- \bar{\nu}_\mu$	0.	0.
$\nu_\mu \to \mu^- \tau^+ \nu_\tau$	0.	0.23	$\bar{\nu}_\mu \to \mu^+ \tau^- \bar{\nu}_\tau$	0.	0.39
Total	149.88	38.62		86.6	21.71

Magill, Plastid 1612.05642 - utilising Effective Photon Approximation.

- Exceptionally rare process in the SM. Neutrino scatters off a nuclear target and produces a lepton pair.
  - Only $\nu_\mu \mu^+ \mu^-$ observed by Charm-II (~55) and CCFR (~37) experiments. NuTeV sets upper bound.
  - Results are consistent with the SM
- Focus on muon and electron tridents, lower production and detection thresholds and larger cross-sections
- Tau trident production negligible at e.g ND of DUNE?
- Is this really true?



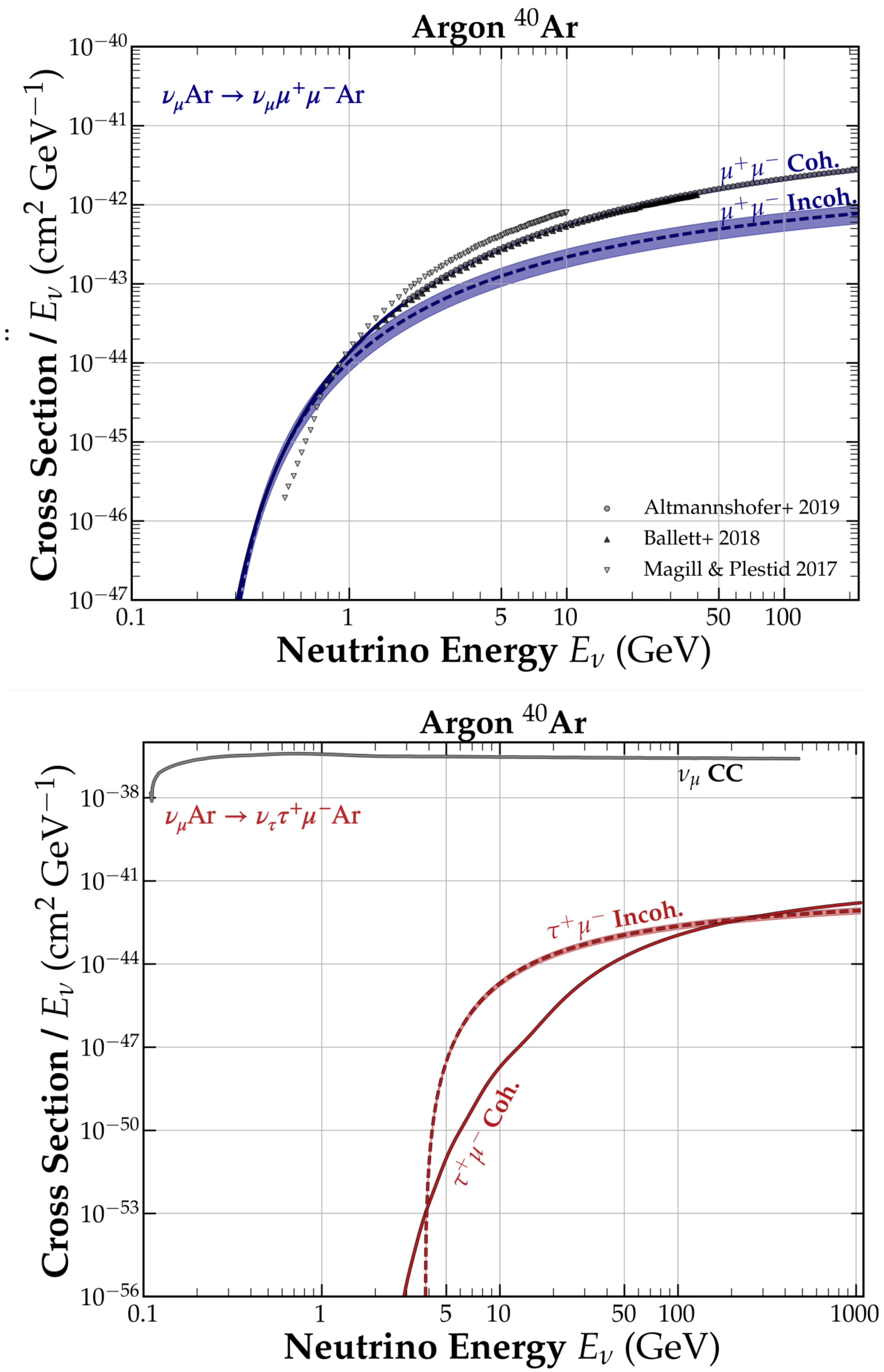
NTP PROCESS	E_ν^{thresh} (GeV)	
	COHERENT	INCOHERENT
$\nu_i N \to \nu_i N e^+ e^-$	0.001	0.001
$\nu_i N \to \nu_i N \mu^+ \mu^-$	0.212	0.235
$\nu_i N \to \nu_i N \tau^+ \tau^-$	3.734	10.285
$\nu_\mu N \to \nu_e N e^- \mu^+$	0.107	0.112
$\nu_\mu N \to \nu_\tau N \tau^- \mu^+$	1.934	3.771
$\nu_e N \to \nu_\tau N \tau^- e^+$	1.823	3.461



~ Threshold for any tau production

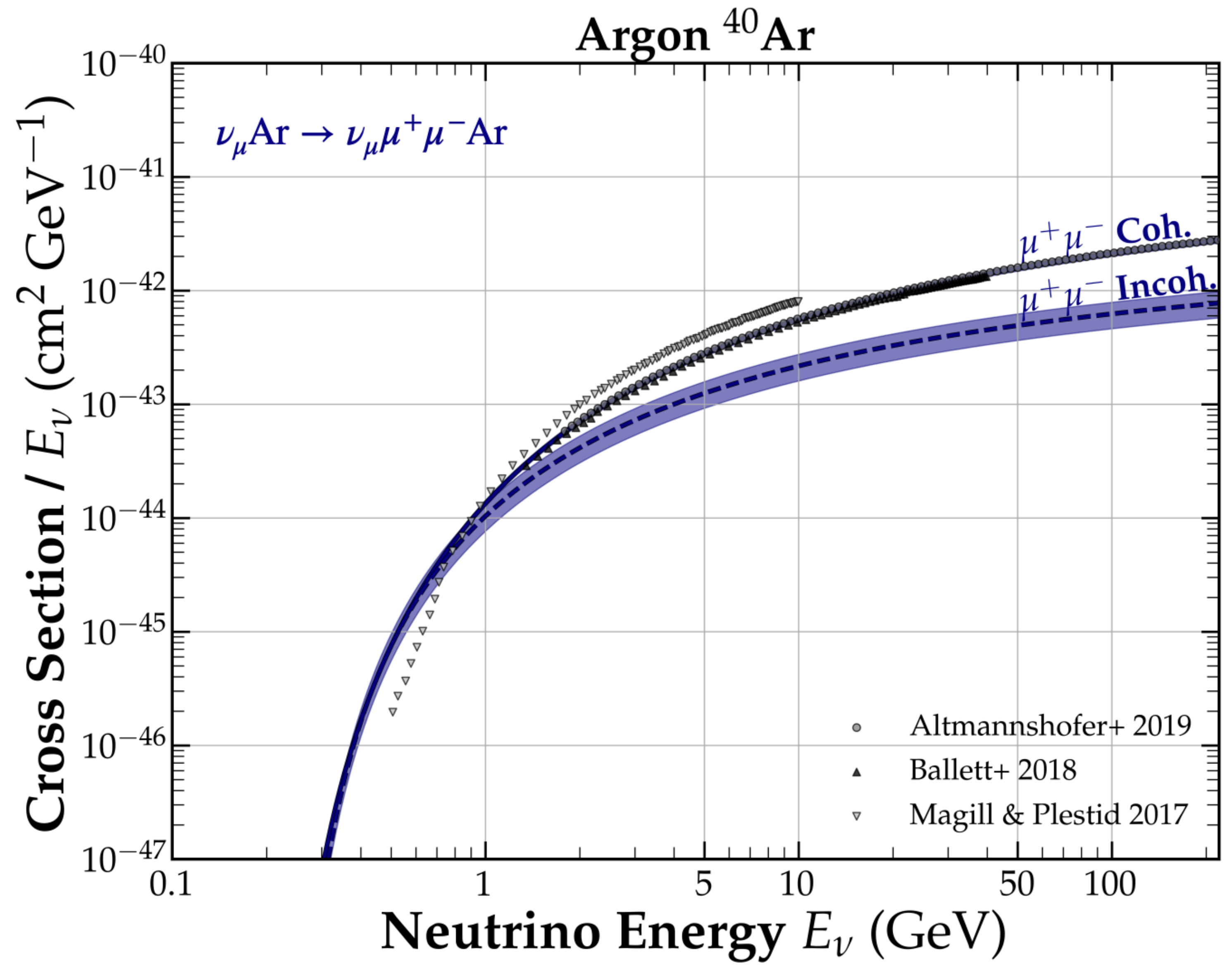
Neutrino Trident Production

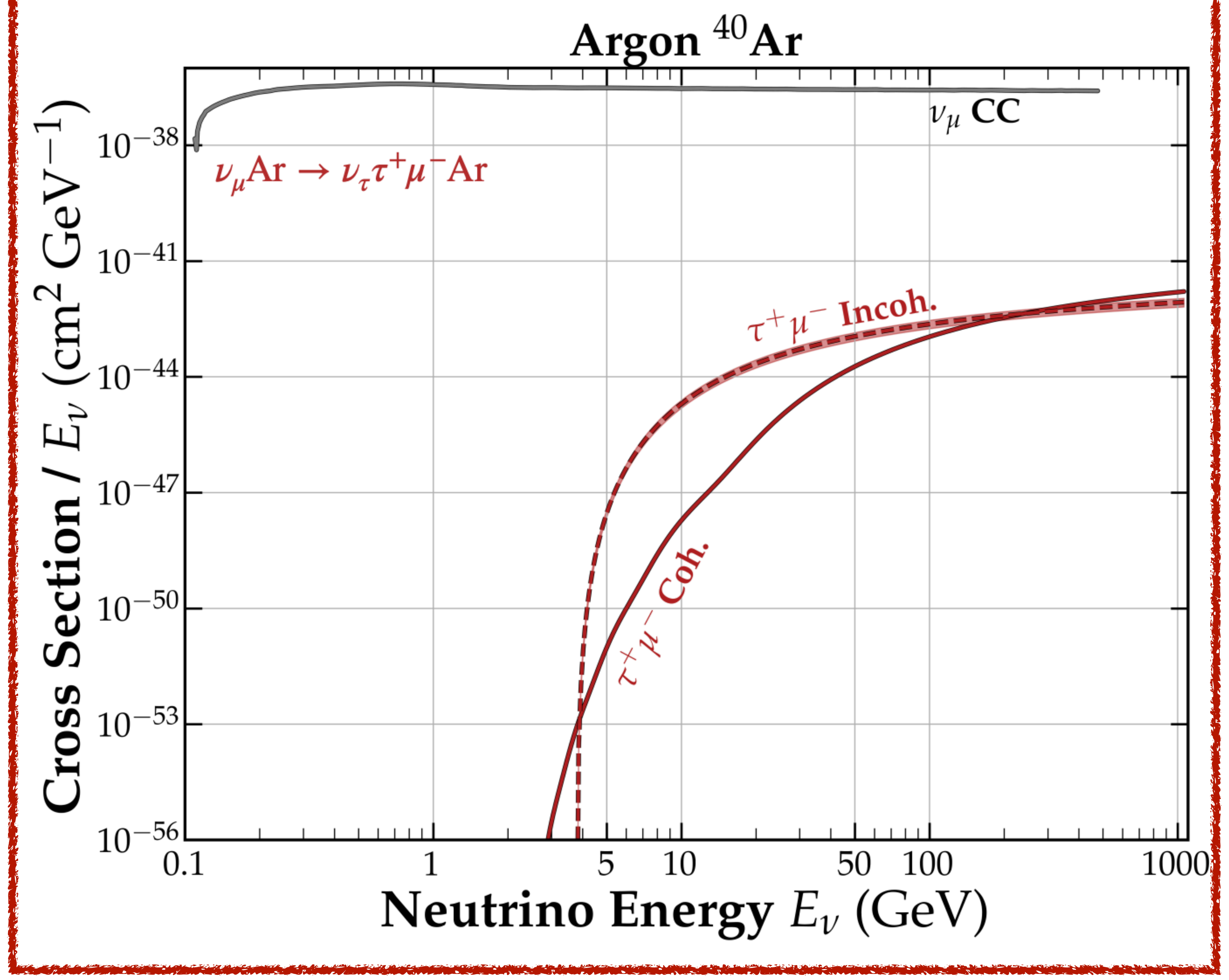
- DUNE will produce a high intensity muon-neutrino beam, with the two modes with oscillation measurement targets
  - **Standard** (CP-optimised) mode
  - **Tau** (neutrino) optimised mode
- The threshold for tau production is high, and in regions of lower flux for the standard mode but the neutrino flux for tau-optimised will peak after the threshold for the tau-optimised mode.
- Focus on the muon-tau trident, lower threshold initiated by a muon-neutrino beam.



Neutrino Trident Production at DUNE

- Significant work done by Altmannshofer et al. 1902.06765 on the muon and electron tridents
- At DUNE neutrino energies, two production mechanisms for the neutrino trident dominate for the hadronic interaction: **incoherent** (*diffractive*) and **coherent** elastic scattering off the nucleus.
- Coherent scattering: the neutrino "sees" the nucleus as a whole, scales as $\sim Z^2$ and with nuclear form factors.
- Incoherent scattering: the neutrino "sees" individual nucleons, $\sim 2Z$ nucleons, depends on nucleon form factors



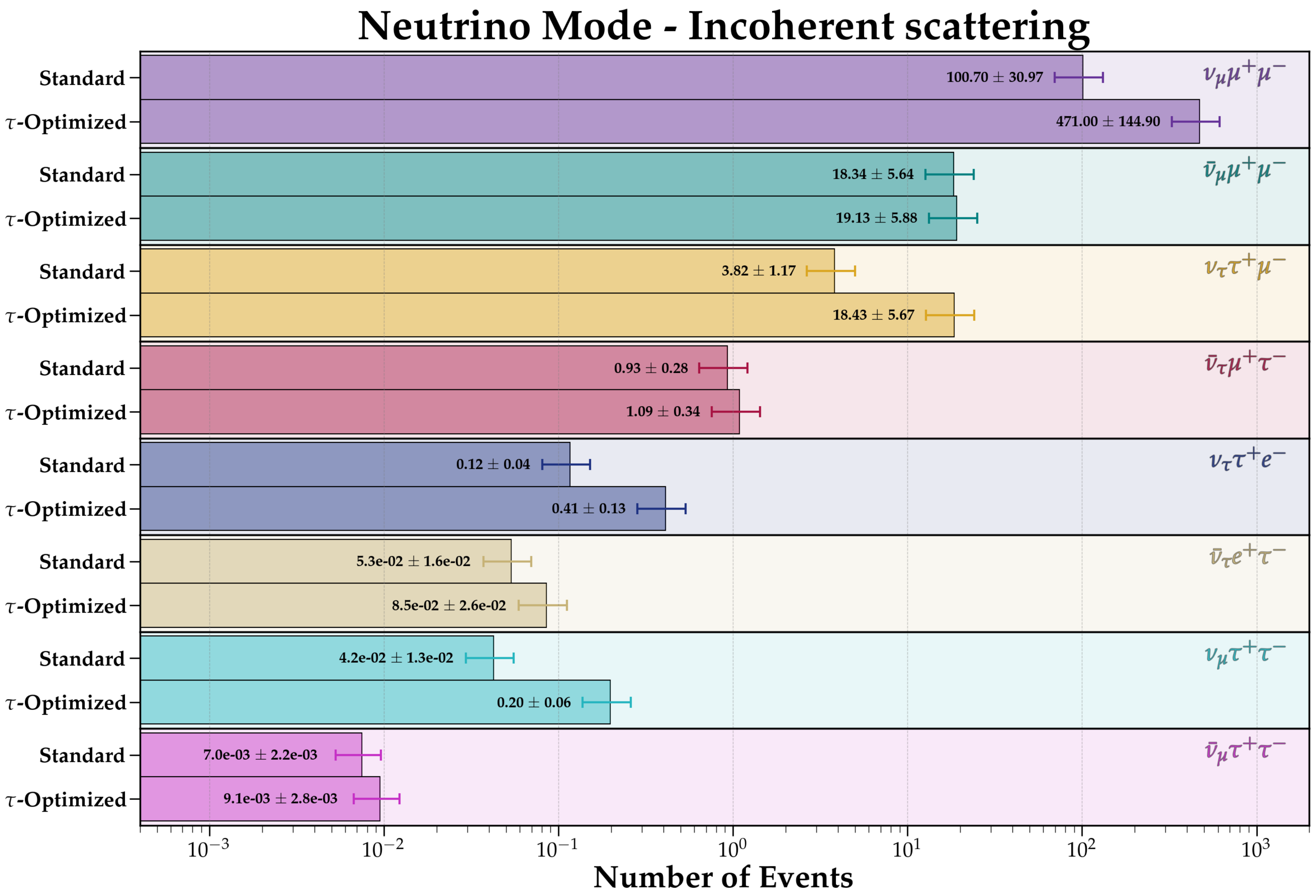


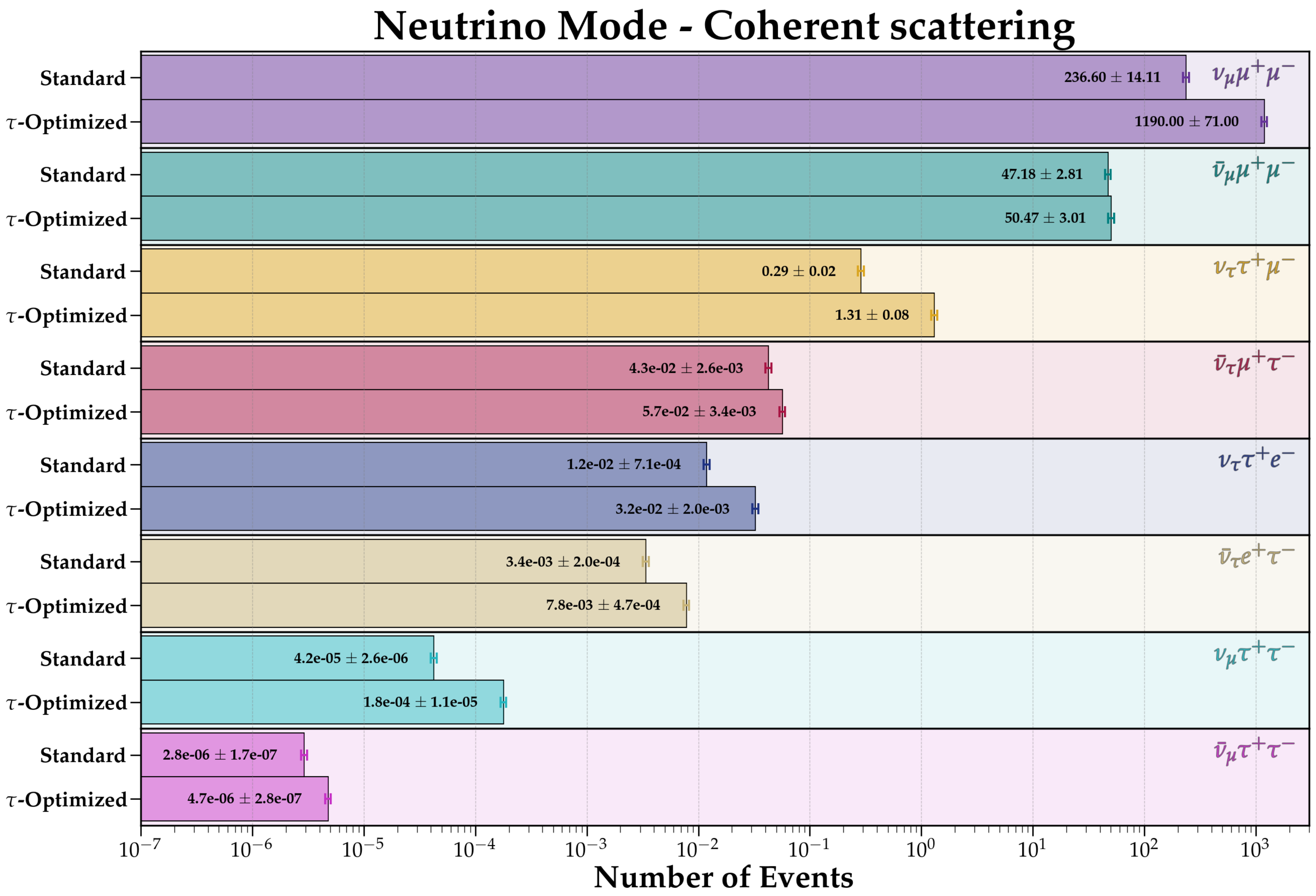
Neutrino Trident Production at DUNE

1. The threshold for coherent NTP is lower, lower energy neutrinos only access the full nuclear structure
2. Incoherent NTP quickly (briefly) overtakes, as the neutrino has sufficient energy to access nucleons and their form-factors dominate
3. Due to the ultimate $\sim Z^2$ vs $\sim Z$ enhancement of the coherent process leads to a second crossing point. This happens over a larger energy range because the tau is *heavy*

We cannot assume coherent scattering dominates for the tau-containing NTP, so also cannot just utilise EPA which is only ~ valid for coherent muon NTP. Can't completely trust previous tau NTP predictions for DUNE.

Neutrino Trident Production at DUNE





Validated against Altmannshofer et al. 1902.06765 for muon and electron tridents. Extended to tau NTP events, focus on tau-muon trident. Expect ~ 4 events (standard) and ~20 events (tau optimised)

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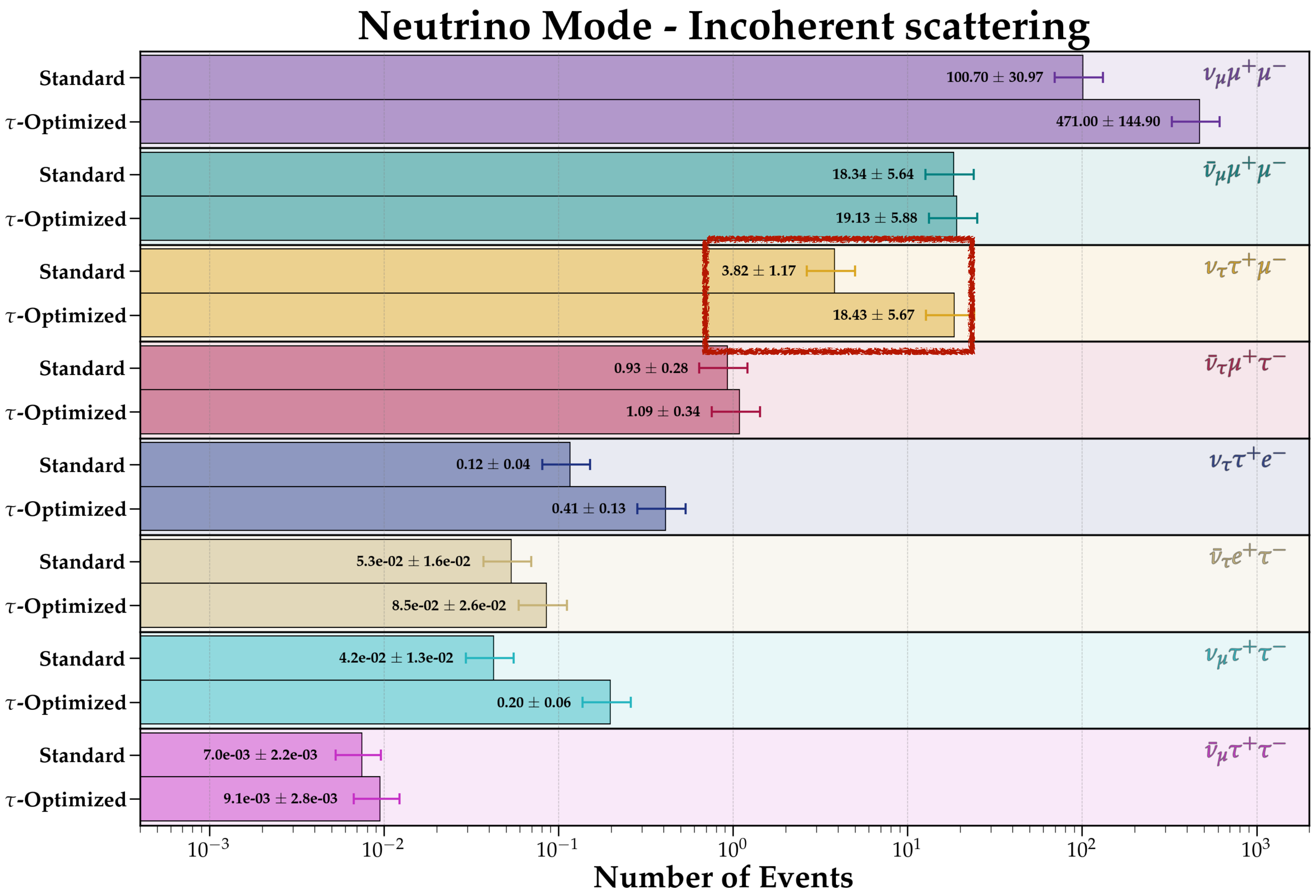
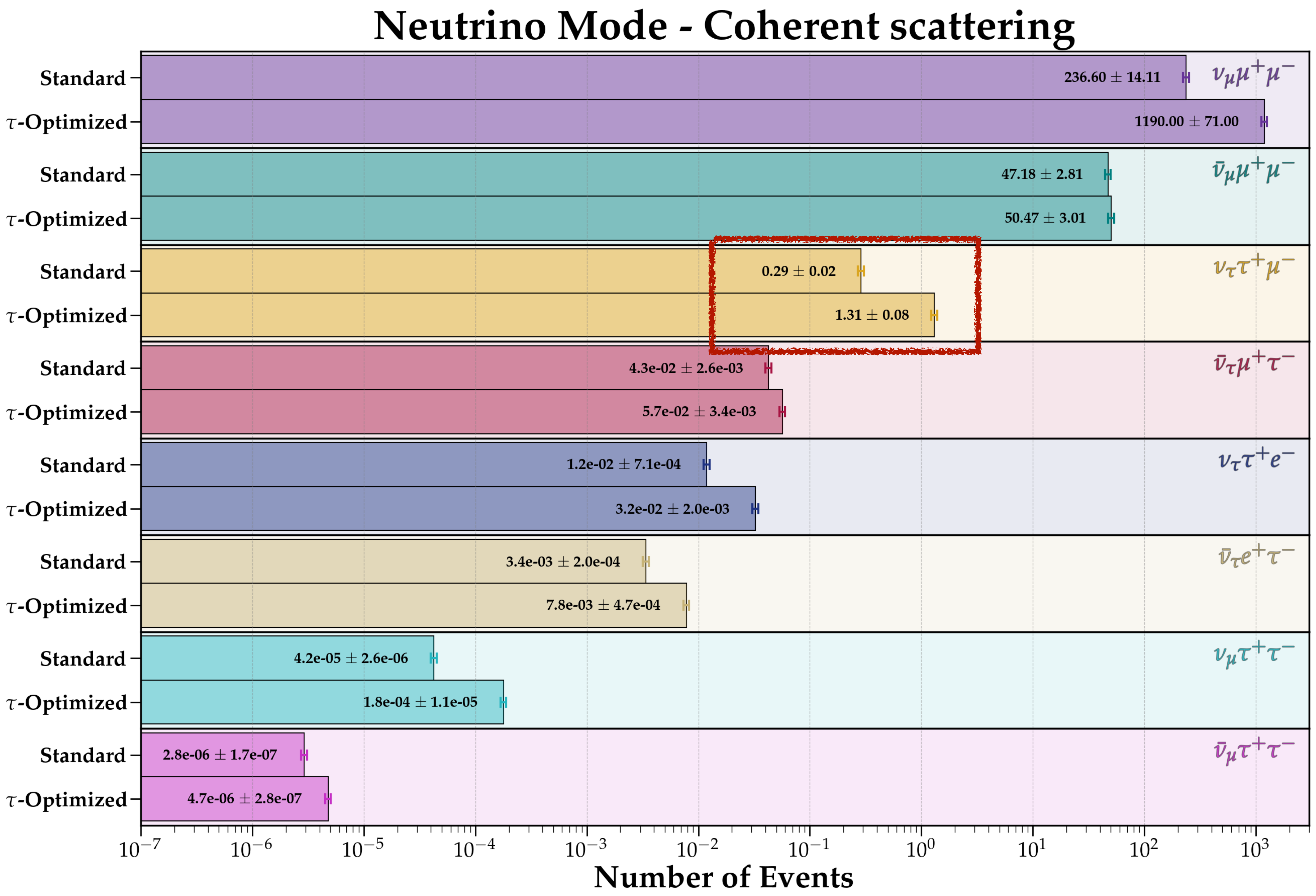

Neutrino Mode - Incoherent scattering


Neutrino Mode - Coherent scattering

Validated against Altmannshofer et al. 1902.06765 for muon and electron tridents. Extended to tau NTP events, focus on tau-muon trident. Expect ~ 4 events (standard) and ~20 events (tau optimised) , *these are not "anomalous taus" but rather SM taus!*

Conclusions

Tau lepton physics already provides strong and useful constraints on BSM models for flavour.
There is plenty of (new) physics to explore in tau flavour beyond just considering it as a constraint, especially as a complement to neutrino physics experiments.

Part I: Tau physics and triality

IB, XG He, M.A. Schmidt, G. Valencia, R.Volkas
Phys.Rev.D 107 (2023) 5, 055001 arXiv: 2212.09760

Residual, softly broken, Z_3 triality symmetry could explain why cLFV has not yet been observed. This would suggest cLFV is most likely to be observed in triality-preserving tau decays, e.g. at Belle-II

Part II: Anomalous Taus or the SM?

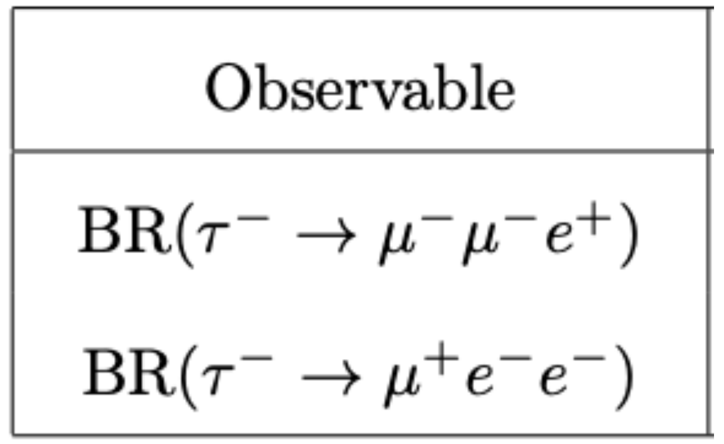
IB, B. Dev, D. Lopez Gutierrez, P. A. Machado
arXiv: 2406.xxxx (Coming ~ next fortnight!)

The SM contribution to tau production at the near detector via trident production is non-neglible for DUNE energies. If DUNE switches on and sees taus at the ND, this is not necessarily new physics.

Thank you!

To gain a full picture of the lepton flavour sector we need to combine charged-lepton (including direct leptonic decays *and* via EFT and colliders) and neutrino probes and work together.

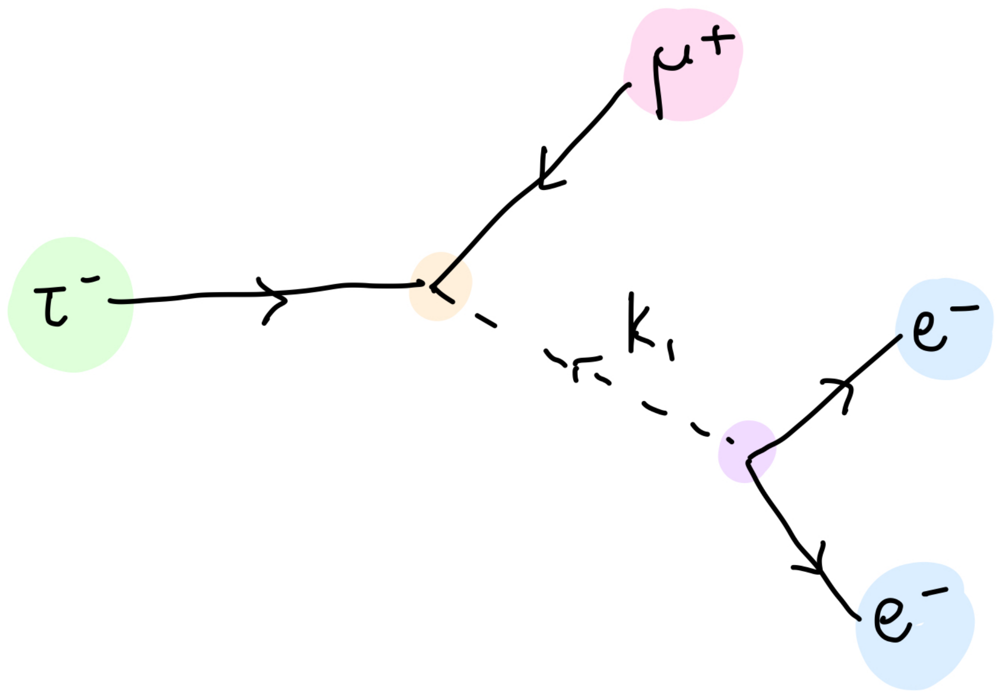
Backup



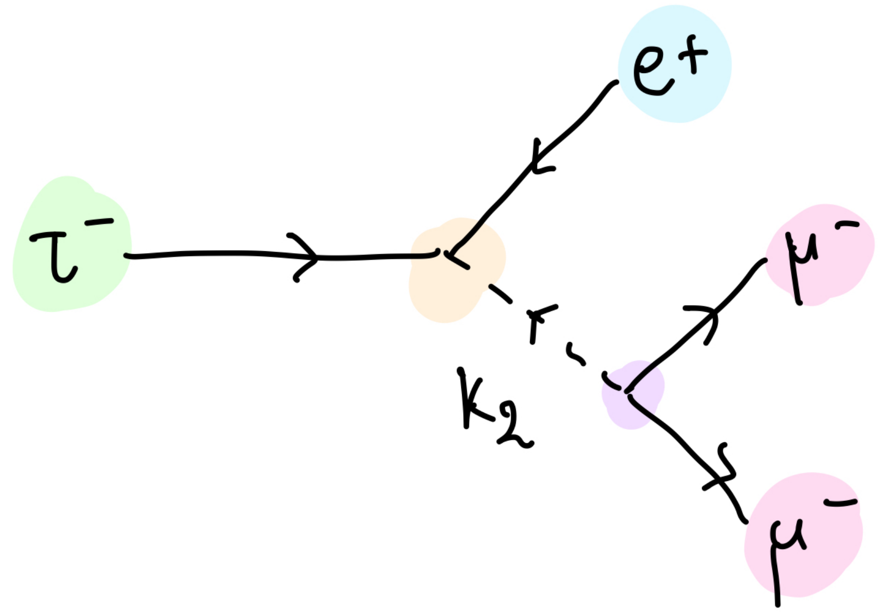
Observable
$\mathrm{BR}(\tau^- \to \mu^-\mu^-e^+)$
$\mathrm{BR}(\tau^- \to \mu^+e^-e^-)$

Electroweak Singlet Scalars

T=1 scalar



T=2 scalar



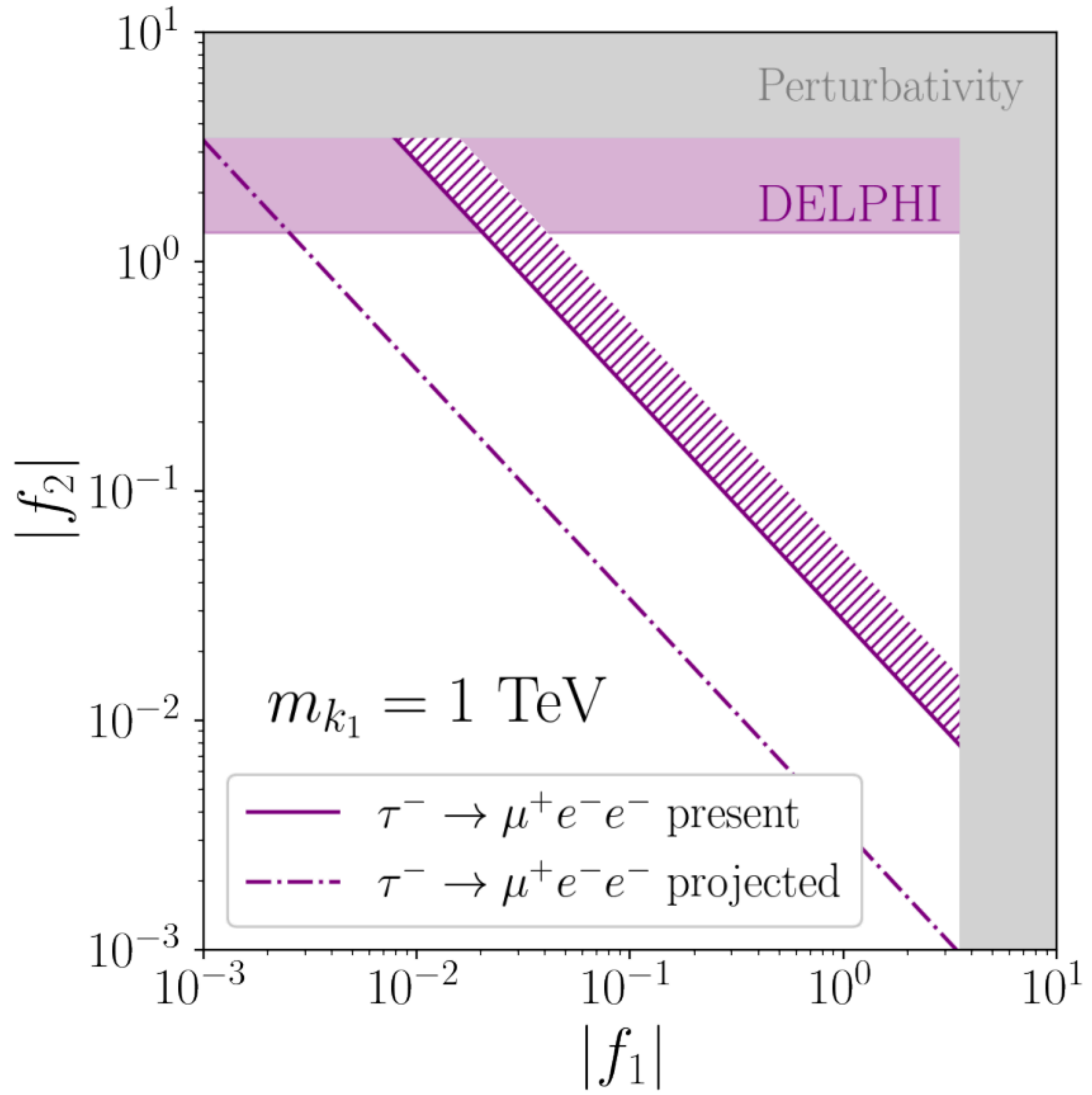
- Electroweak singlet scalar *bileptons* , doubly electrically charged and carrying a triality charge — triality forbids other SM interactions, making their phenomenology straightforward.
- Mediating this interaction at tree-level, we can either assign the scalar a triality of T=2 or T=1
- The T=1 scalar k_1 mediates $\tau^- \to \mu^+e^-e^-$

$$\mathcal{L}_{k_1} = \frac{1}{2}\left(2f_1\,\overline{\tau_R^c}\,\mu_R + f_2\,\overline{e_R^c}\,e_R\right)k_1 + h.c.$$

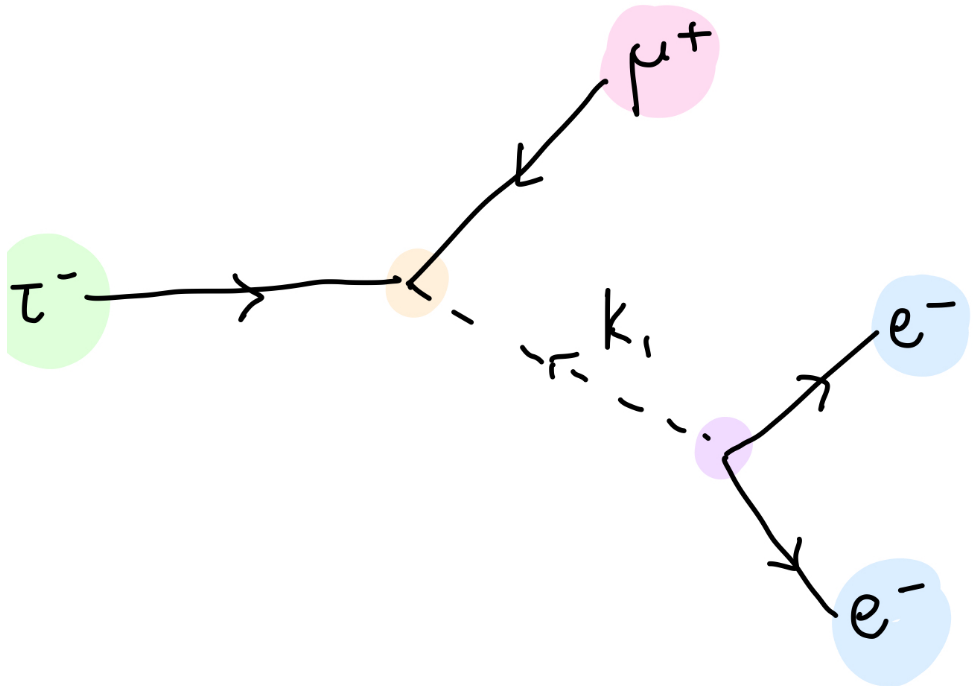
- The T=2 scalar k_2 mediates $\tau^- \to e^+\mu^-\mu^-$

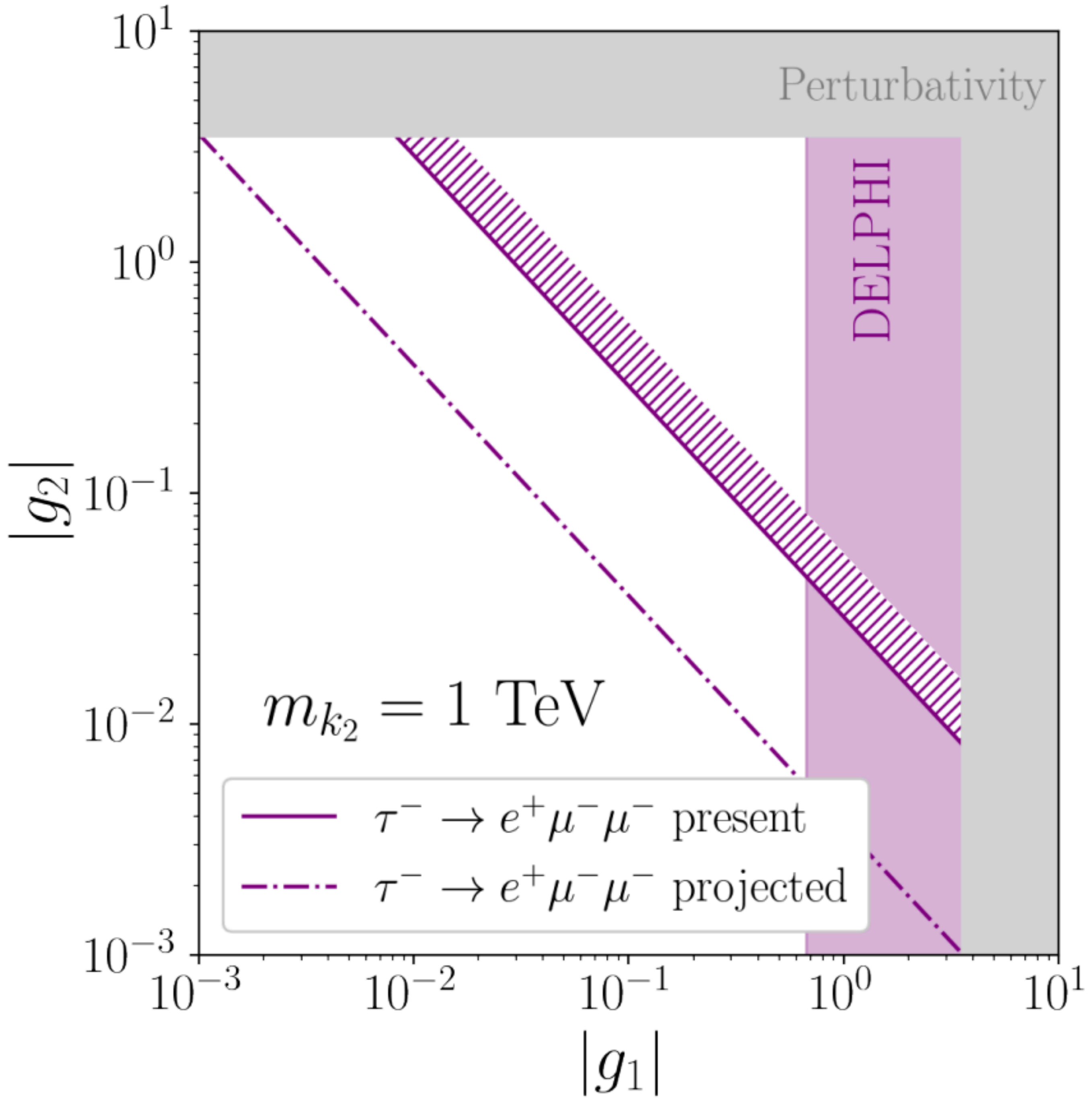
$$\mathcal{L}_{k_2} = \frac{1}{2}\left(2g_1\,\overline{\tau_R^c}\,e_R + g_2\,\overline{\mu_R^c}\,\mu_R\right)k_2 + h.c.$$

Electroweak Singlet Scalars

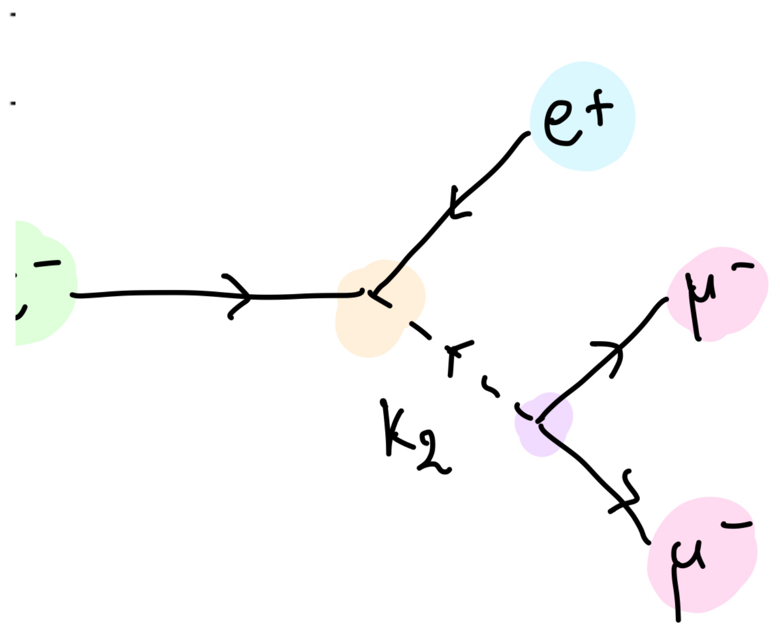


T=1 scalar





T=2 scalar

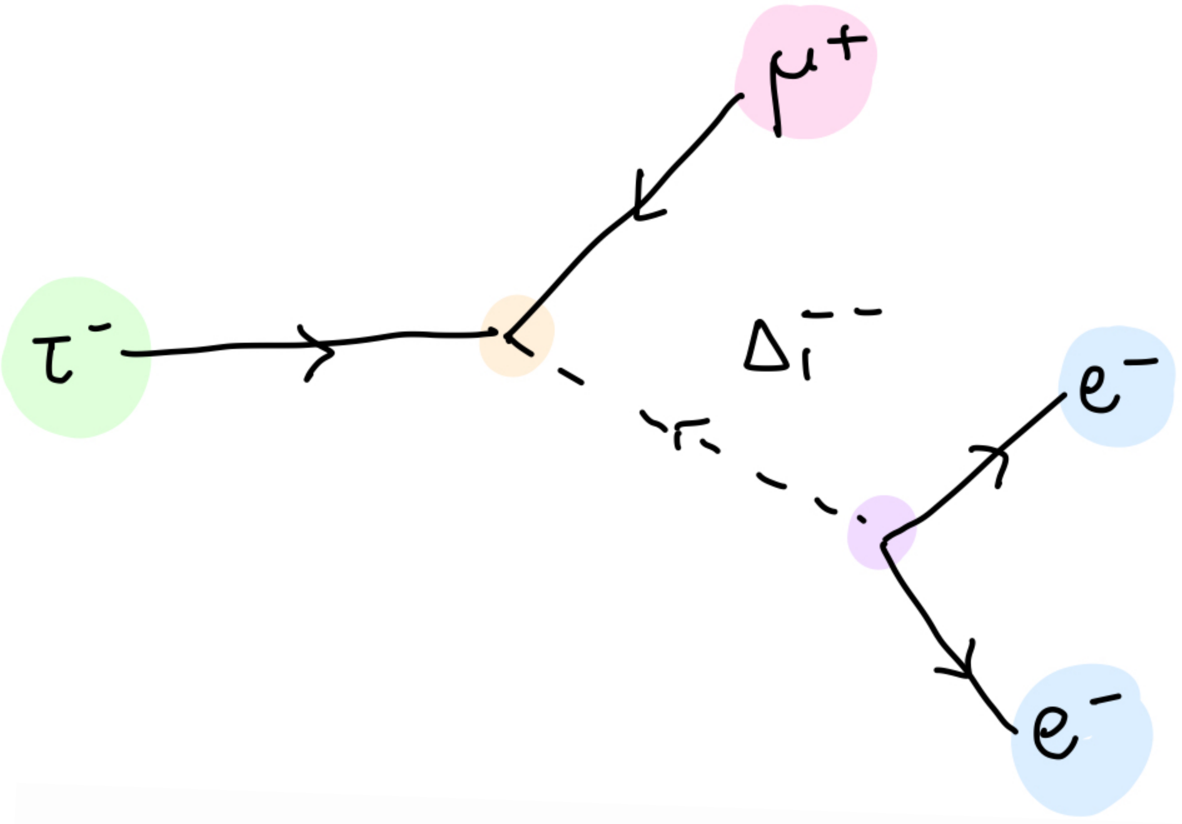


$$\mathcal{L}_{k_1} = \frac{1}{2}\left(2f_1 \overline{\tau_R^c}\mu_R + f_2 \overline{e_R^c} e_R\right) k_1 + h.c.$$

$$\mathcal{L}_{k_2} = \frac{1}{2}\left(2g_1 \overline{\tau_R^c} e_R + g_2 \overline{\mu_R^c} \mu_R\right) k_2 + h.c.$$

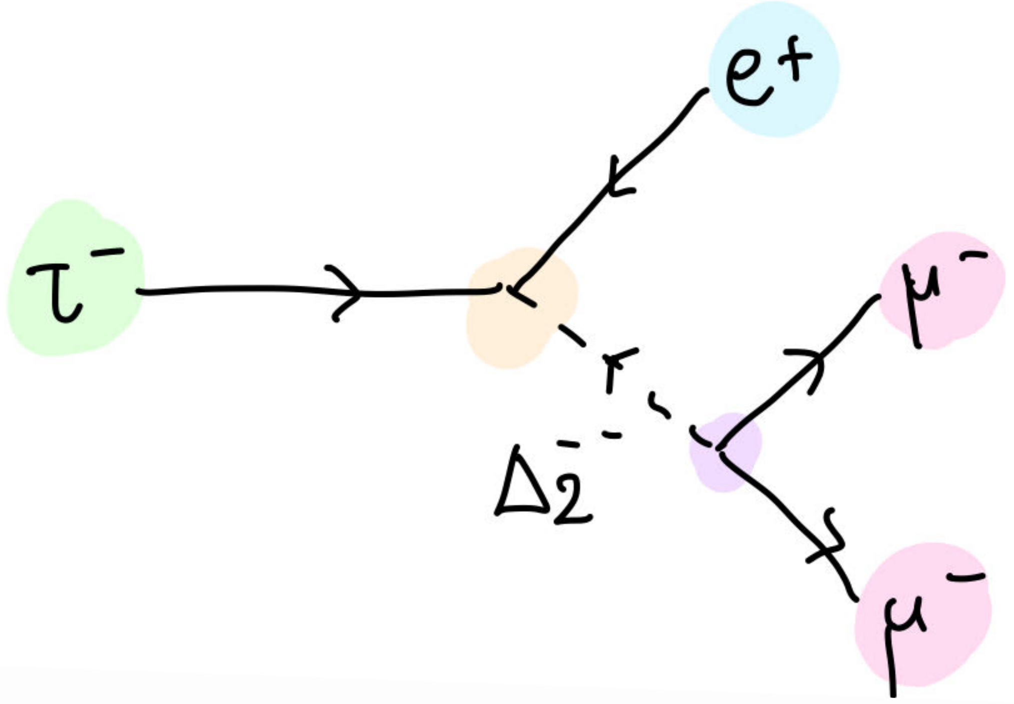
- Doubly-charged scalar bilepton is constrained to be TeV scale by direct searches
- Contribution from (T=1) [T=2] to e+ e- → (e+ e-) [τ+ τ-] constrained by DELPHI

Electroweak Triplet Scalars



T=1

$$\mathcal{L}_{\Delta_1} = \frac{1}{2}\left(2f_1\,\overline{L^c_3}\,\Delta_1 L_2 + f_2\,\overline{L^c_1}\,\Delta_1 L_1\right) + h.c.$$



T=2

$$\mathcal{L}_{\Delta_2} = \frac{1}{2}\left(2g_1\,\overline{L^c_3}\,\Delta_2 L_1 + g_2\,\overline{L^c_2}\,\Delta_2 L_2\right) + h.c.$$

- Much richer phenomenology. Can be thought of as an extension of the singlet scalar study.
- Couple to lepton doublet: constraints also from neutrino interactions
- Electroweak triplet representation:

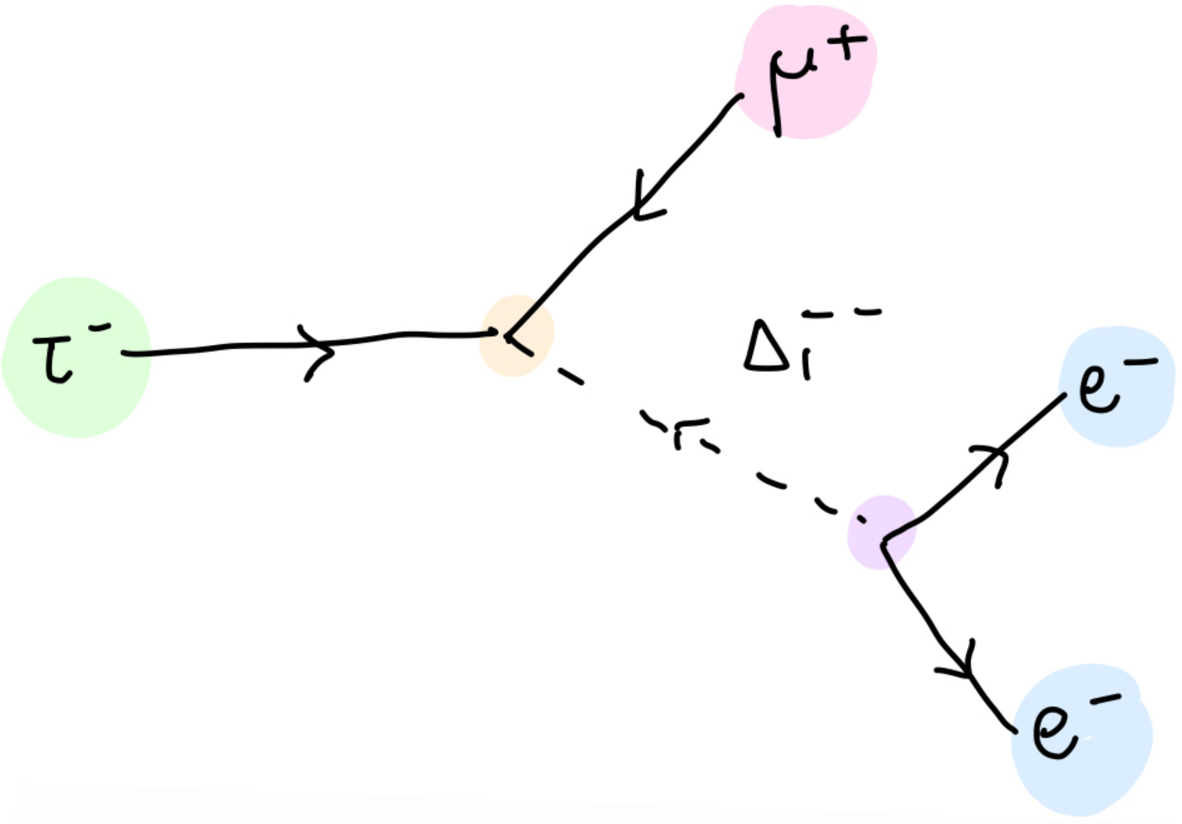
$$\Delta_T \sim \begin{pmatrix} \Delta_T^+ & \Delta_T^{++} \\ \Delta_T^0 & -\Delta_T^+ \end{pmatrix}$$

doubly charged scalar.

isospin partners

- Similarly constrained to have TeV scale masses by bilepton direct collider searches

Electroweak Triplet Scalars



T=1

$$\mathcal{L}_{\Delta_1} = \frac{1}{2}\left(2f_1\,\overline{L_3^c}\,\Delta_1 L_2 + f_2\,\overline{L_1^c}\,\Delta_1 L_1\right) + h.c.$$

Example of expanded interaction terms:

$$\overline{L_3^c}\, i\sigma_2 \mathbf{\Delta}_1\, L_2 = -\overline{(\tau_L)^c}\,\mu_L\,\Delta_1^{++} \boxed{-\frac{1}{\sqrt{2}}\left[\overline{(\tau_L)^c}\,\nu_{\mu L} + \overline{(\nu_{\tau L})^c}\,\mu_L\right]\Delta_1^{+}} + \overline{(\nu_{\tau L})^c}\,\nu_{\mu L}\,\Delta_1^0,$$

$$\overline{L_1^c}\, i\sigma_2 \mathbf{\Delta}_1\, L_1 = -\overline{(e_L)^c}\,e_L\,\Delta_1^{++} - \sqrt{2}\,\overline{(e_L)^c}\,\nu_{eL}\,\Delta_1^{+} + \overline{(\nu_{eL})^c}\,\nu_{eL}\,\Delta_1^0.$$

$$\Delta_T \sim \begin{pmatrix} \Delta_T^{+} & \Delta_T^{++} \\ \Delta_T^{0} & -\Delta_T^{+} \end{pmatrix}$$

doubly charged scalar.

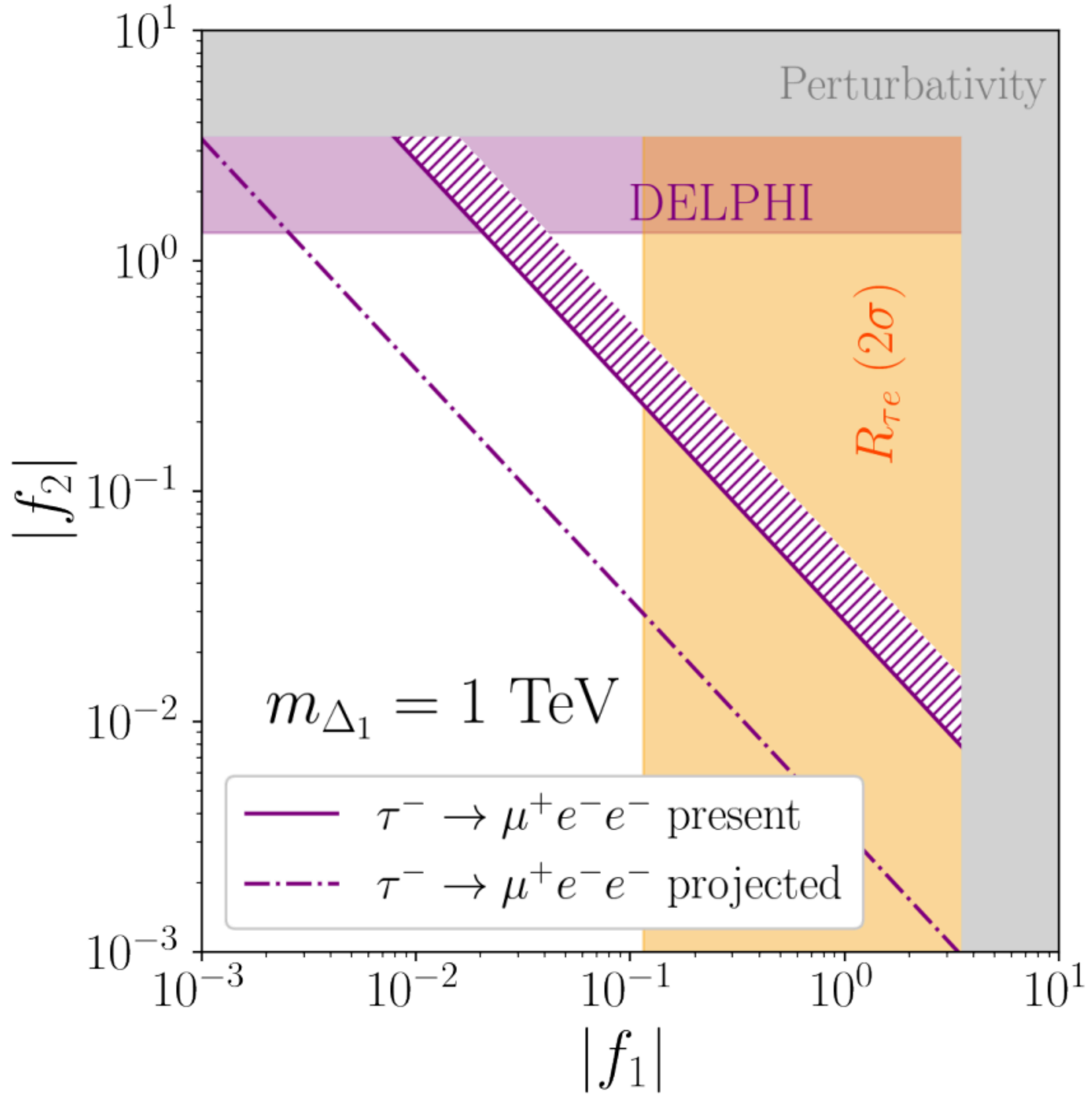
isospin partners

- Isospin partners: single and doubly charged components
- LFU ratios of tau decays also constrain parameter space, e.g.

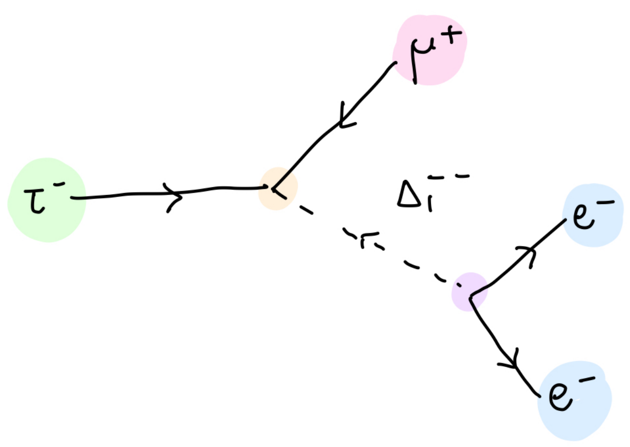
$$R_{\tau e} = \frac{\Gamma(\tau\to\mu+inv)}{\Gamma_{SM}(\tau\to\mu+inv)} \times \frac{\Gamma_{SM}(\mu\to e+inv)}{\Gamma(\mu\to e+inv)}$$

- Majorana-mass type interaction induced by neutral component. Constrained in structure by triality…[we will return to this later]

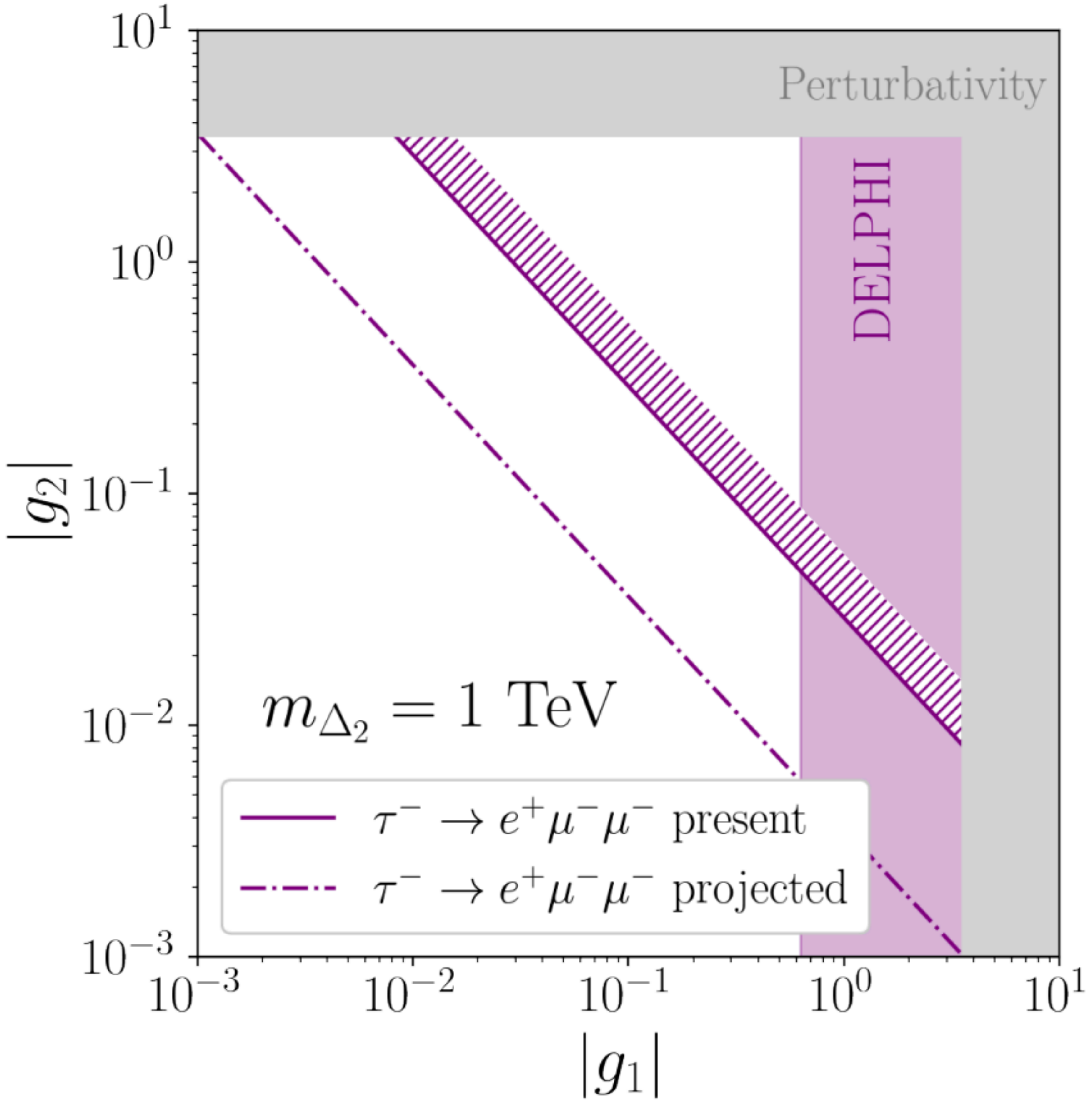
Electroweak Triplet Scalars



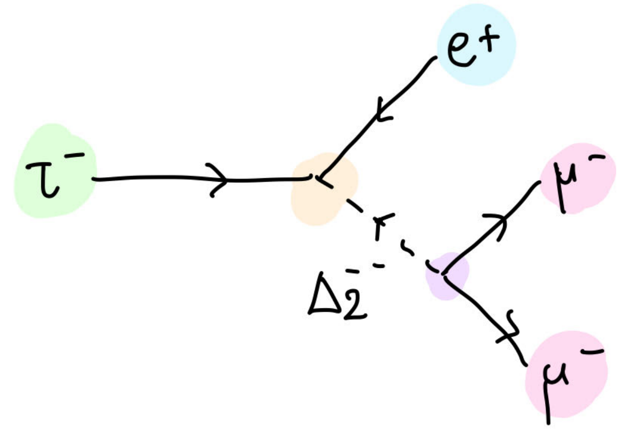
T=1 scalar



$$\mathcal{L}_{\Delta_1} = \frac{1}{2}(2f_1 \overline{L_3^c}\Delta_1 L_2 + f_2 \overline{L_1^c}\Delta_1 L_1) + h.c.$$



T=2 scalar



$$\mathcal{L}_{\Delta_2} = \frac{1}{2}(2g_1 \overline{L_3^c}\Delta_2 L_1 + g_2 \overline{L_2^c}\Delta_2 L_2) + h.c.$$