

Gegenbauer naturalness

One idea to address the Higgs hierarchy problem

Gauthier Durieux
(CP3 – UCLouvain)

Gegenbauer Goldstones, JHEP 01 (2022) 076, [2110.06941]

Gegenbauer's Twin, JHEP 05 (2022) 140, [2202.01228]

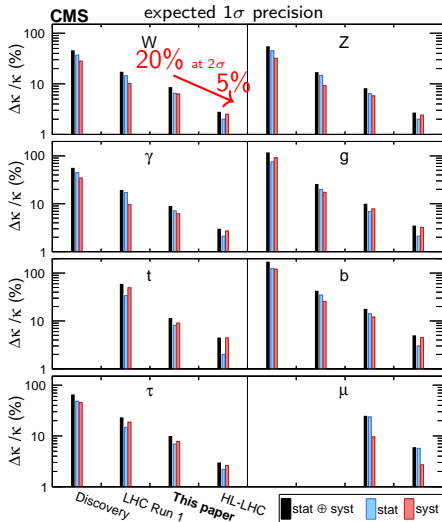
Charting the Higgs self-coupling boundaries, JHEP 12 (2022) 148, [2209.00666]

with Matthew McCullough and Ennio Salvioni

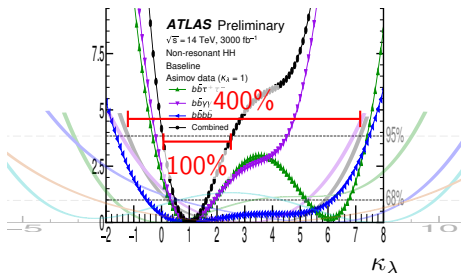
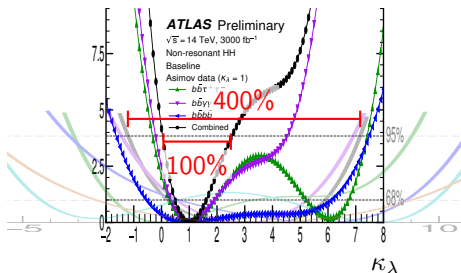
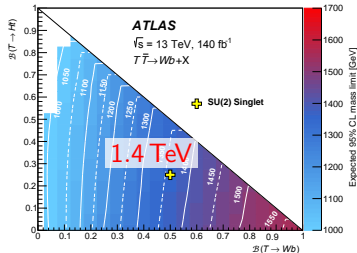


LHC present and future

[EXOT-2019-06]



[CMS-HIG-22-001]



[ATLAS-CONF-2024-006]

[ATL-PHYS-PUB-2022-053]

Naturalness exacerbated

$$\frac{m_h^2}{M_X^2} \sim \%$$

$$\delta\kappa_V \sim \%$$

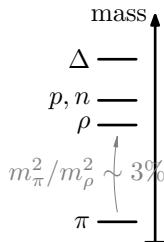
Composite Higgs

the Higgs as the
pseudo-Nambu-Goldstone boson (pNGB)
of a new strong sector

e.g. global $SO(5) \rightarrow SO(4)$ spontaneous breaking
at scale f

small mass from the
explicit $SO(5)$ breaking
by e.g. the SM

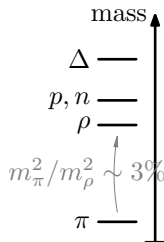
small $\delta\kappa_V$ implies $v^2/f^2 \ll 1$
and requires fine-tuning in minimal models



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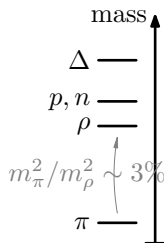
small-ish $m_h^2/M_X^2!$

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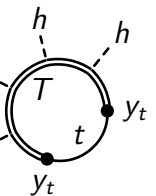
e.g. global $SO(5) \rightarrow SO(4)$ spontaneous breaking
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small-ish $m_h^2/M_X^2!$

and req **no** small $\delta\kappa_V$ or $v^2/f^2!$ models

Minimal composite Higgs

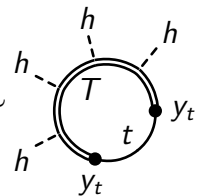


$$V(h) \sim \dots \sim \kappa \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2 \frac{h}{f} + \delta \sin^4 \frac{h}{f} \right)$$

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \quad \text{vs.} \quad |\delta \kappa_V| \simeq \frac{v^2}{2f^2} \lesssim 5\%$$

$$\rightarrow \frac{m_h^2}{M_T^2} = \kappa \underbrace{4 \frac{y_t^2 N_c}{16\pi^2} \left(1 - \frac{1}{2\delta} \right)}_{\sim 7\%} \quad \text{vs.} \quad M_T \gtrsim 2 \text{ TeV}$$

Minimal composite Higgs



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Gegenbauer fix

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \quad \boxed{1/\delta \lesssim 0.2} \quad |\delta\kappa_V| \simeq \frac{v^2}{2f^2} \lesssim 5\%$$

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Twin Higgs fix

Percent fine-tuning wrt $1/\delta \gtrsim 1$, $\kappa \simeq 1$ expectation

Naturally small vev

radiatively stable deepest minimum
close to the origin



in the low-energy pNGB potential

Radiatively stable $SO(N + 1) \rightarrow SO(N)$ potentials

$$\vec{\phi} \equiv \left(\frac{\vec{h}}{h} \sin \frac{h}{f}, \cos \frac{h}{f} \right), \quad h \equiv |\vec{h}|$$

Linear one-loop correction to $V(\frac{h}{f})$:

$$\frac{\Lambda^2}{32\pi^2 f^2} \left(\underline{V'' + (N - 1) \cot \frac{h}{f} V'} \right)$$

Radiative stability at one-loop and linear order order if $\propto V$

Differential equation of Gegenbauer polynomials

$$V\left(\frac{h}{f}\right) \propto G_n^{(N-1)/2}\left(\cos \frac{h}{f}\right)$$

Radiatively stable $SO(N + 1) \rightarrow SO(N)$ potentials

Explicit $SO(N + 1) \rightarrow SO(N)$ breaking by an irrep spurion K :

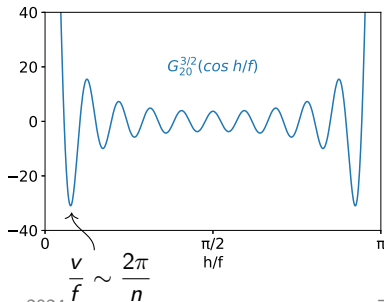
$$K^{i_1 \dots i_n} \phi_{i_1} \cdots \phi_{i_n} \quad (\text{symmetric traceless})$$

$$\vec{\phi} \equiv \left(\frac{\hbar}{h} \sin \frac{h}{f}, \cos \frac{h}{f} \right), \quad h \equiv |\vec{h}|$$

No other invariant, linear in K , can be constructed,
so all-loop linear renormalisation can only be multiplicative.

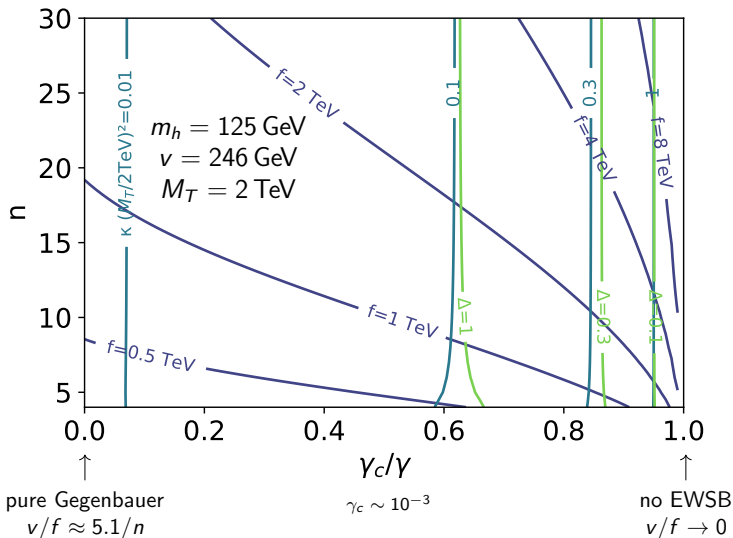
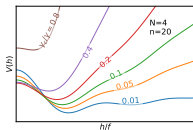
Obtain Gegenbauer polynomials:

$$K^{i_1 \dots i_n} \phi_{i_1} \cdots \phi_{i_n} \propto G_n^{(N-1)/2} \left(\cos \frac{h}{f} \right)$$



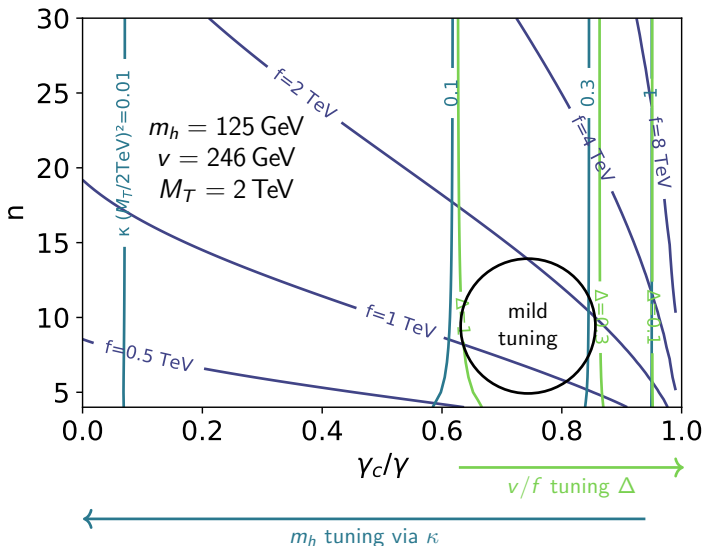
The Gegenbauer Higgs

$$V(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma G_n^{3/2} \left(\cos \frac{h}{f} \right) \right]$$



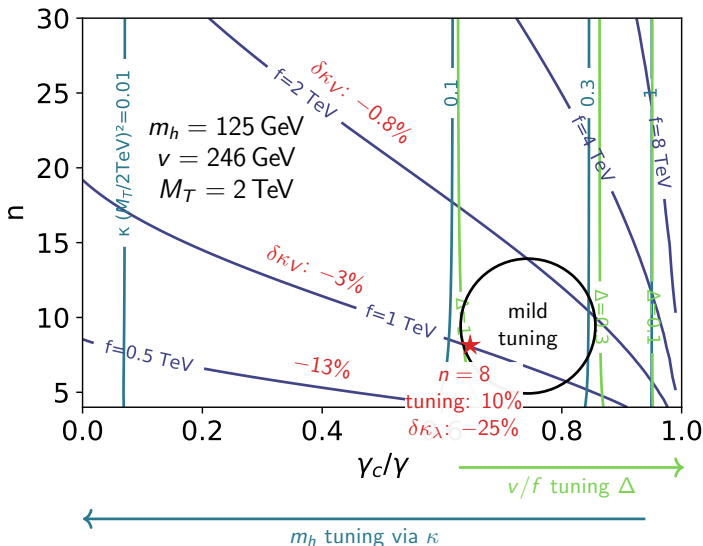
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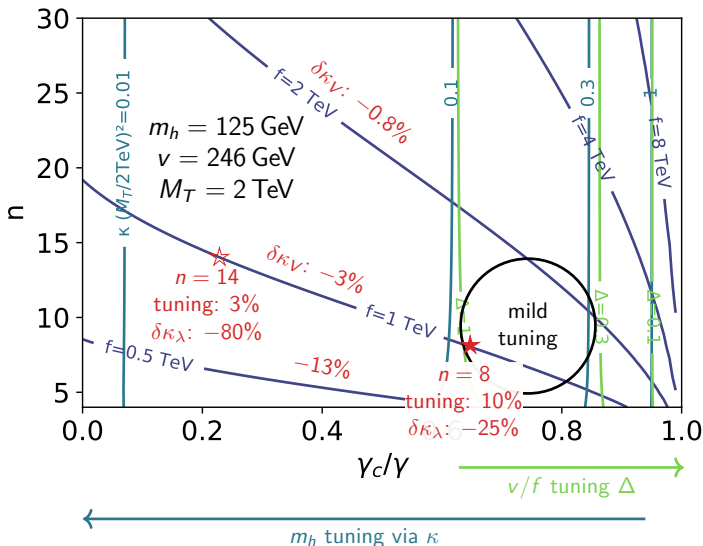
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The Gegenbauer Higgs

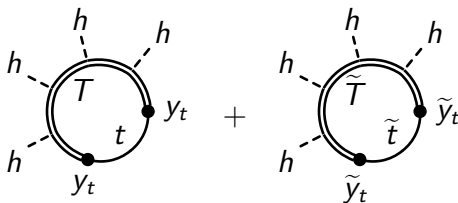
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Naturally smaller mass

[Chacko, Goh, Harnik '05]

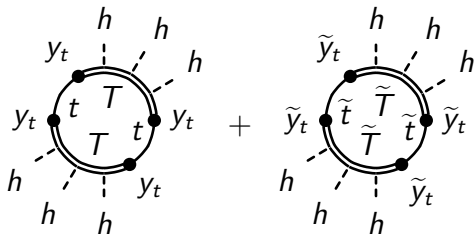
[Barbieri, Greco, Rattazzi, Wulzer '15]



$$\frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \sin^2 \frac{h}{f} + \frac{N_c \tilde{y}_t^2}{16\pi^2} f^2 M_{\tilde{T}}^2 \cos^2 \frac{h}{f}$$

if twin parity enforces $y_t = \tilde{y}_t$ and $M_T = M_{\tilde{T}}$
no M_T^2 sensitivity

Naturally smaller mass



$$\frac{N_c y_t^4}{16\pi^2} f^4 \sin^4 \frac{h}{f} \log M_T \quad + \quad \frac{N_c \tilde{y}_t^4}{16\pi^2} f^4 \cos^4 \frac{h}{f} \log M_{\tilde{T}}$$

retaining $\log M_T$ sensitivity only

Gegenbauer's Twin

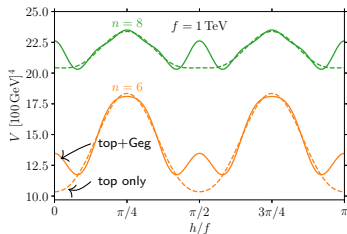
- global $SO(8) \supset SO(4) \times \widetilde{SO(4)}$
- spontaneous $SO(8) \rightarrow SO(7)$
 - 7 NGBs
 - 6 eaten by W^\pm, Z and $\widetilde{W}^\pm, \widetilde{Z}$
 - 1 Higgs: $\vec{\phi} = (\vec{0}_3, \sin \frac{h}{f}; \vec{0}_3, \cos \frac{h}{f})^T$ in unitary gauge



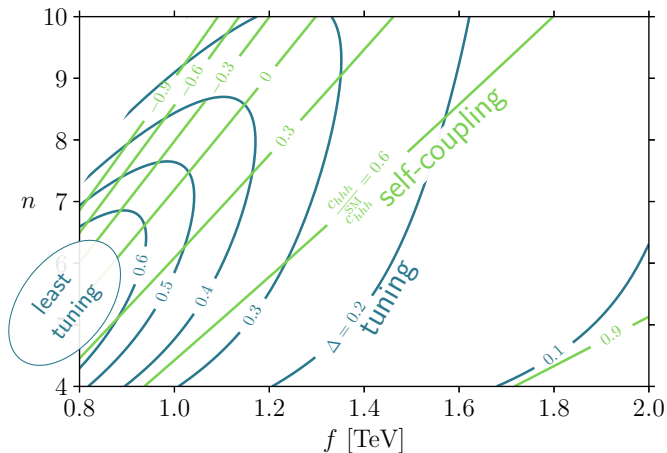
Leopold B. Gegenbauer
1849–1903

- minimal explicit breaking is insufficient

- explicit $SO(8) \rightarrow SO(4) \times \widetilde{SO(4)}$
 - radiative stability from irrep spurion
 - $G_n^{3/2}(\cos \frac{2h}{f})$ potential

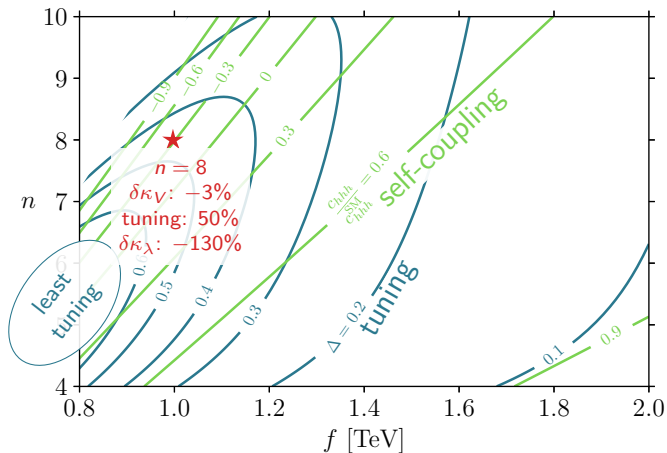


Gegenbauer's Twin



(and possibly large M_T , with unitarity violating H scattering towards 6 TeV)

Gegenbauer's Twin

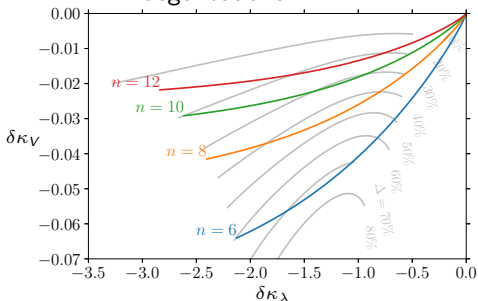


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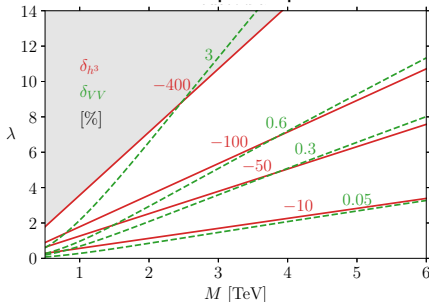
Naturally large $\delta\kappa_\lambda/\delta\kappa_V$

see also: [Di Luzio, Gröber, Spannowsky '17]
 [Gupta, Rzehak, Wells '13] [Falkowski, Rattazzi '19]
 [Logan, Rental '15] [Chala, Krause, Nardini '18] [etc.]

Gegenbauer's Twin



custodial weak-quadruplet scalar



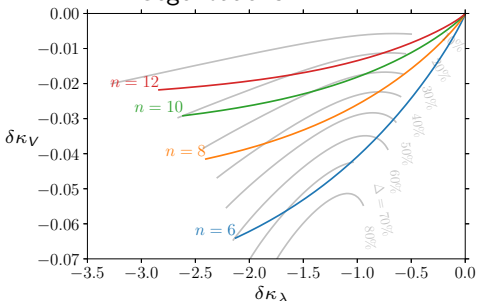
$$\lambda H^* H^* (\epsilon H) \Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \tilde{\Phi}$$

- a loop factor allowed dimensionally (or v^2/M_χ^2 if dim-6/dim-8)
- $\text{dim} \gg 6$ operators may be very relevant
- vacuum stability as limiting constraint

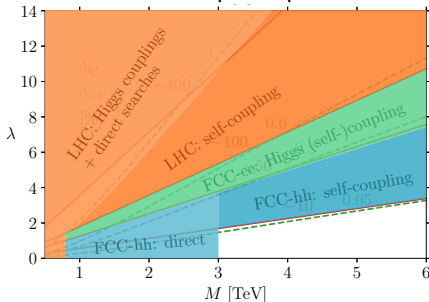
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Gegenbauer naturalness

The linear renormalisation of pNGB potentials admits (remarkable) eigenfunctions.

An example theory exists with natural $\frac{m_h^2}{M_X^2} \sim \%$

natural $\delta\kappa_V \sim \%$

natural $\frac{\delta\kappa_V}{\delta\kappa_\lambda} \sim \%$ (bonus)

Thus, LHC still probes symmetry-based naturalness, and $\delta\kappa_\lambda$ could be the first signal.

Key is a non-minimal breaking of the global pNGB-Higgs symmetry.

UV realisation?