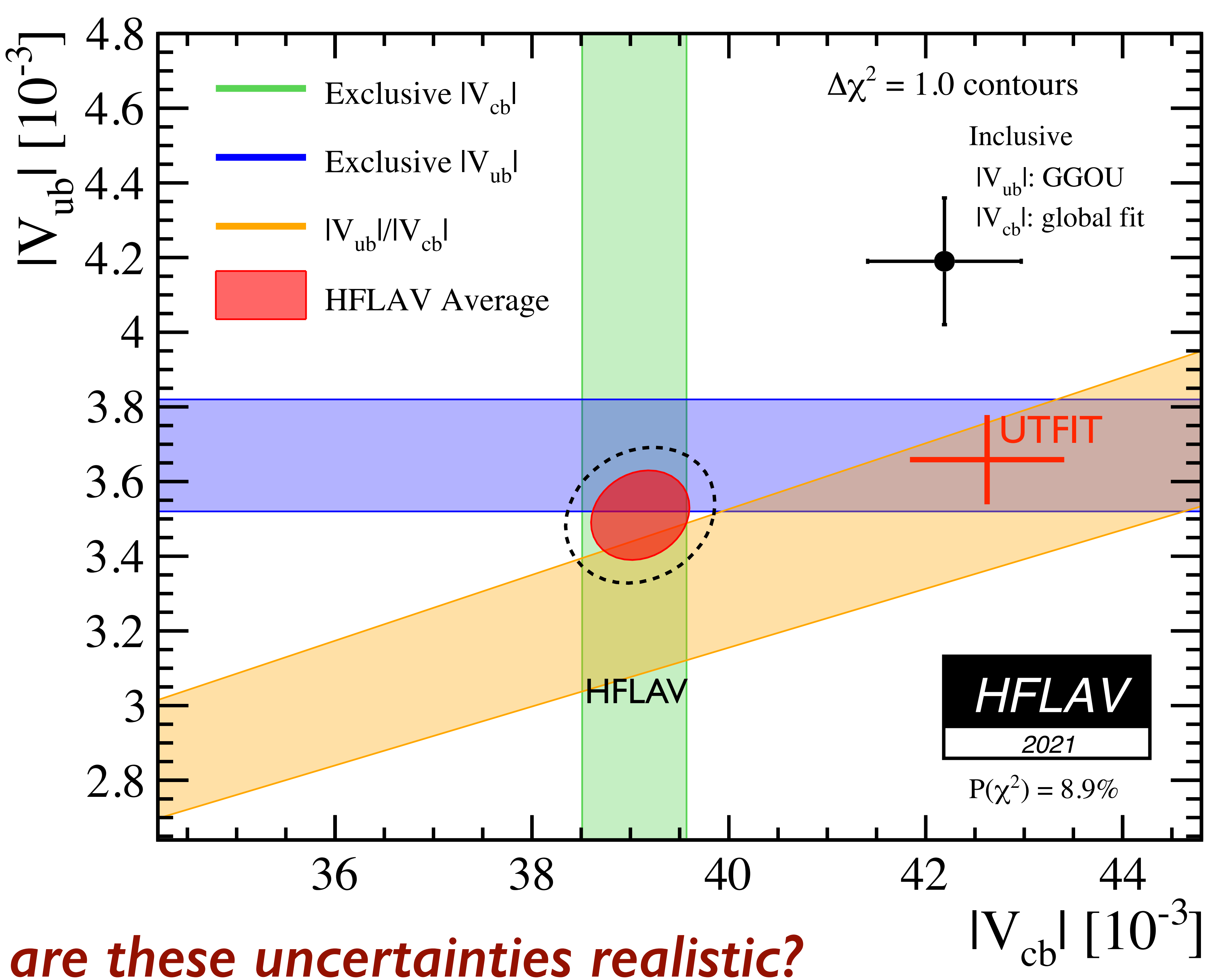
PRECISE SM PREDICTIONS FOR SEMILEPTONIC B DECAYS

Paolo Gambino
Università di Torino & INFN, Torino

The flavour path to new physics
Zurich 5-7 June 2024

THE V_{cb} (and V_{ub}) PUZZLE

Since many years the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ diverge



are these uncertainties realistic?

RECENT PROGRESS

- The last 6-7 years have seen a ***burst of activity*** in semileptonic B decays
- Many new experimental analyses by Belle, Belle II, BaBar, LHCb incl and excl
- New pert calculations at $O(\alpha_s^3)$ by Fael et al. crucial progress for inclusive V_{cb}
- 3 new lattice calculations of $B \to D^*$ form factors beyond $w = 1$, inclusive on the lattice, new $B \to \pi$, …
- Many phenomenological studies with interesting ideas (RPI methods for incl, HQET studies of form factors, …)
- There is now a clear appreciation that ~1% uncertainties require a new approach
- Not glorious work but ***work that needs to be done*** *(Bob Kowalewski)*

The importance of $|V_{cb}|$

An important CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1+\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}+\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}=0$$

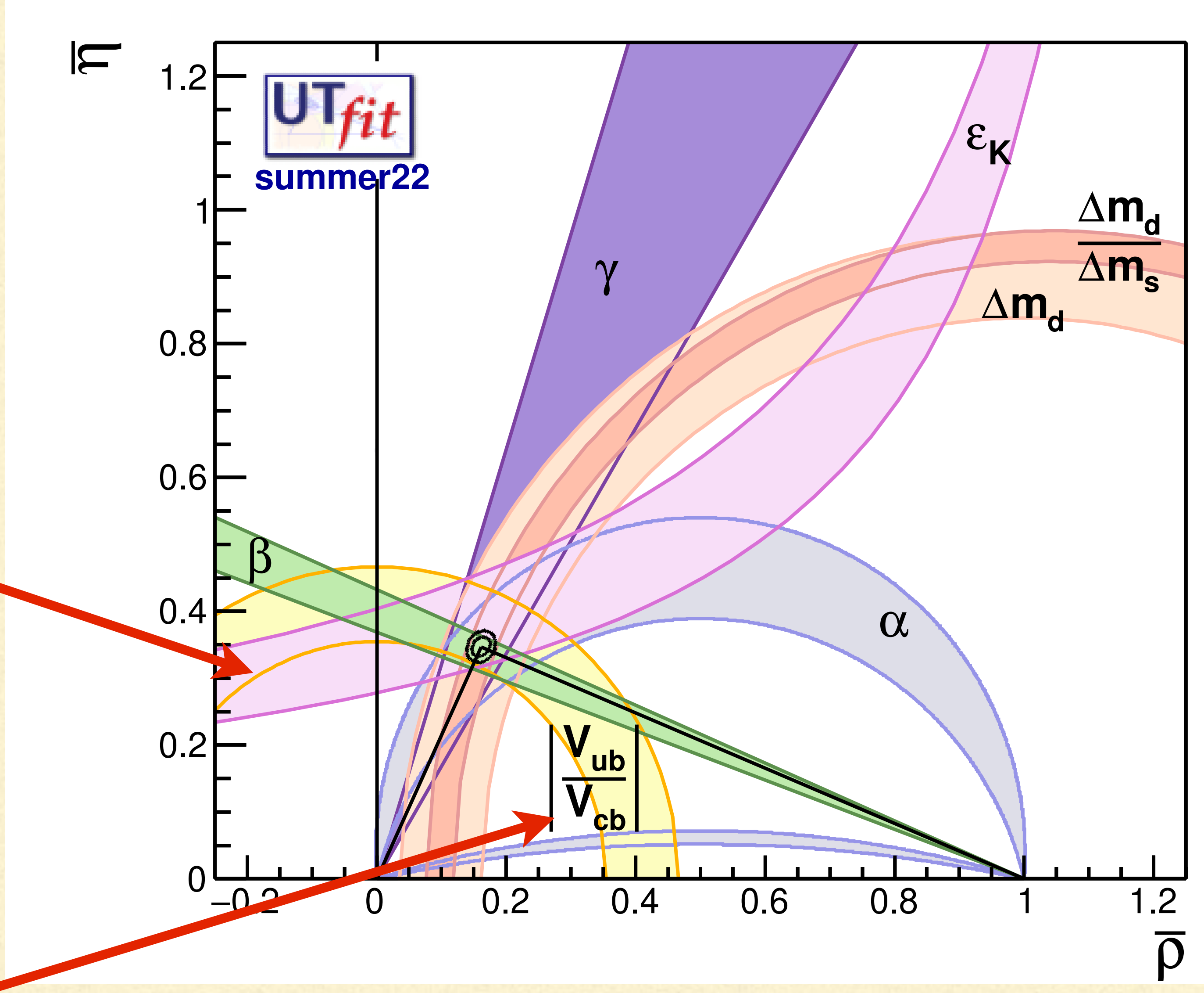
V_{cb} plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4+\ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1+O(\lambda^2)\right]$$

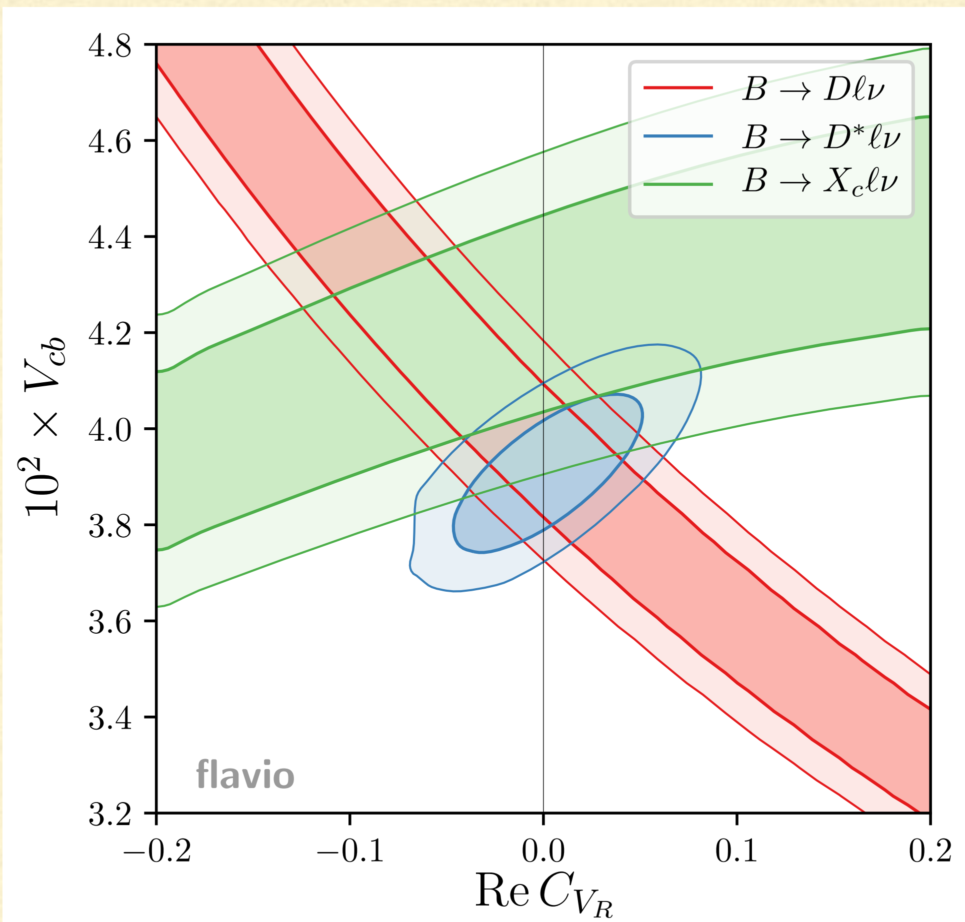
where *it often dominates the theoretical uncertainty.*
V_{ub}/V_{cb} constrains directly the UT

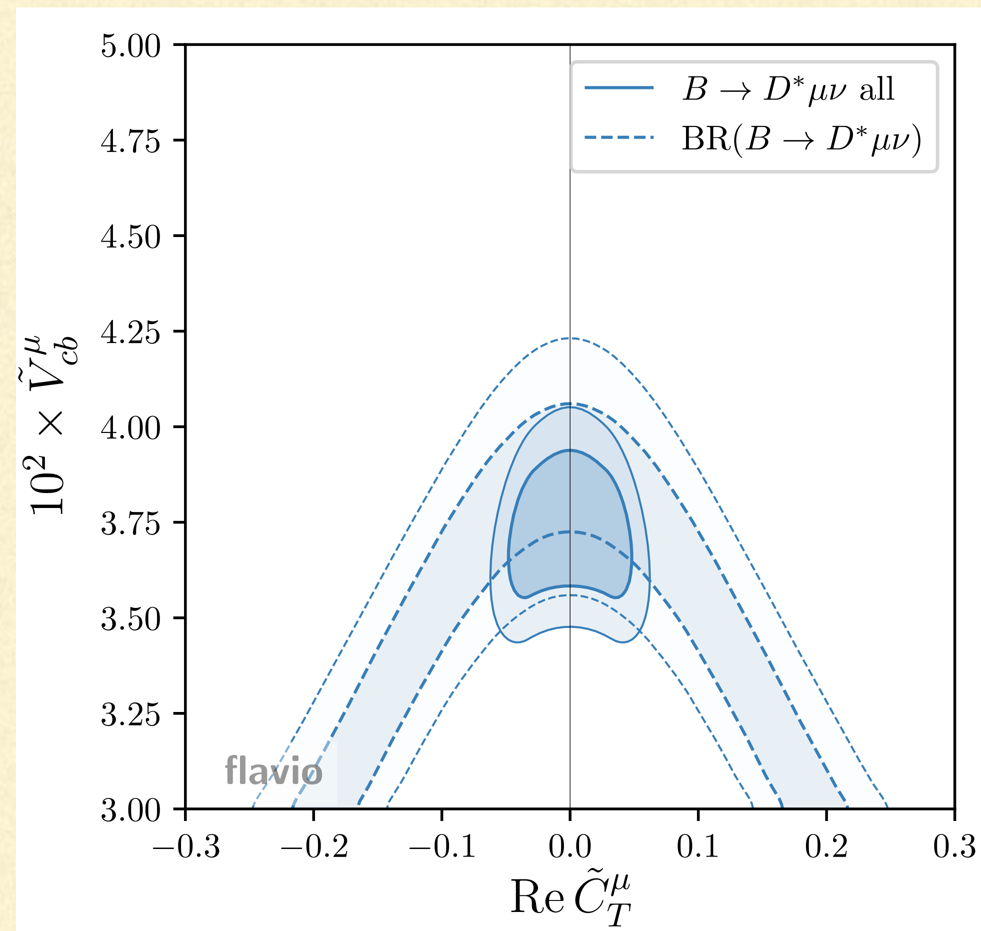


Our ability to determine precisely V_{cb} is crucial for indirect NP searches

NEW PHYSICS FOR THE V_{cb} PUZZLE?

Jung & Straub, 1801.01112



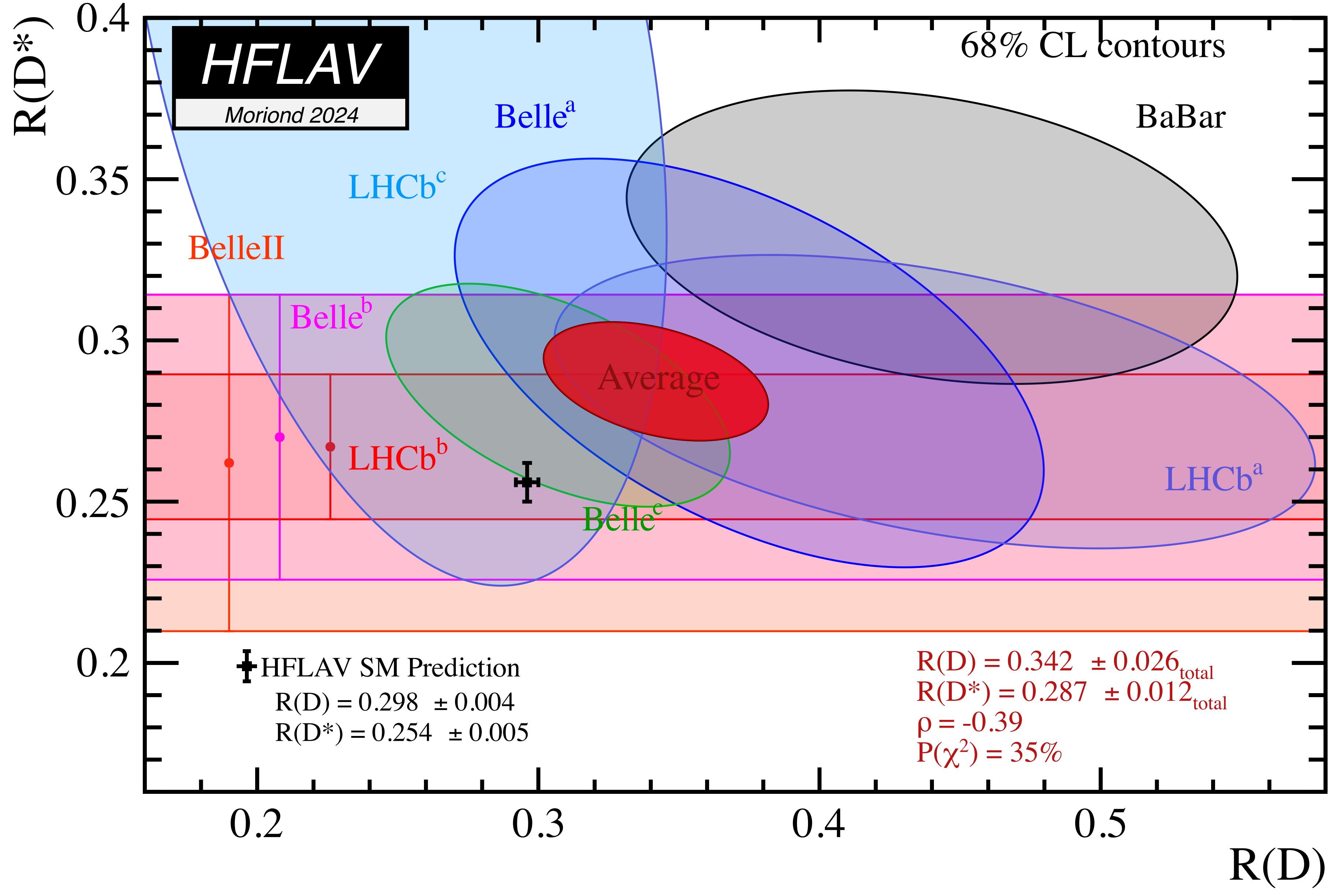


Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data: Crivellin, Pokorski, Jung, Straub…

VIOLATION of LFU with TAUS

SM predictions based on same theory as V_{cb} extraction

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

Reliability of the method depends on our control of higher order effects.
Quark-hadron duality violation would manifest itself as inconsistency in the fit.

3LOOP CALCULATIONS

Fael, Schoenwald, Steinhauser, 2011.11655, 2011.13654, 2205.03410

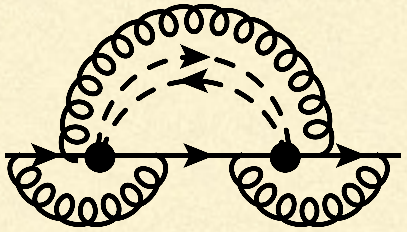
3loop and 2loop charm mass effects in relation between kinetic and $\overline{\rm MS}$ b mass

$$m_b^{kin}(1\text{GeV}) = \left[4163 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3}\right]\text{MeV} = (4526 \pm 15)\,\text{MeV}$$

Using FLAG $\overline{m}_b(\overline{m}_b) = 4.198(12)\text{GeV}$ one gets $m_b^{kin}(1\text{GeV}) = 4.565(19)\,\text{GeV}$

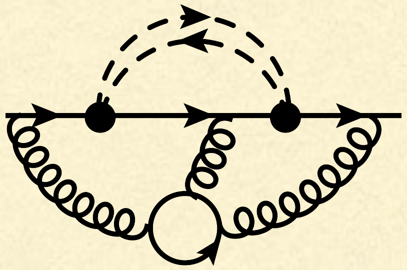
3loop correction to ***total semileptonic width***

$$\Gamma_{sl} = \Gamma_0 f(\rho)\left[0.9255 - 0.1162_{\alpha_s} - 0.0350_{\alpha_s^2} - 0.0097_{\alpha_s^3}\right]$$



(d)

in the kin scheme with $\mu = 1\text{GeV}$ and $\overline{m}_c(3\text{GeV}) = 0.987\,\text{GeV}$, $\mu_{\alpha_s} = m_b^{kin}$



(f)

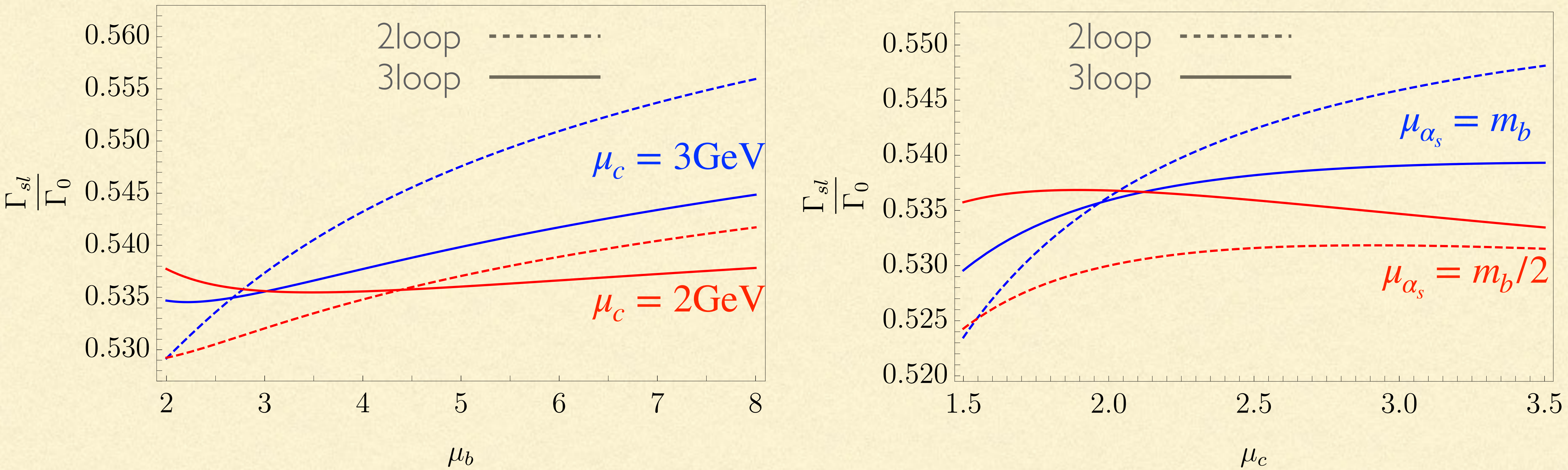
$$\Gamma_{sl} = \Gamma_0 f(\rho)\left[0.9255 - 0.1140_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0103_{\alpha_s^3}\right]$$

in the kin scheme with $\mu = 1\text{GeV}$ and $\overline{m}_c(2\text{GeV}) = 1.091\,\text{GeV}$, $\mu_{\alpha_s} = m_b^{kin}/2$

3loop correction tends to lower Γ_{sl} and therefore pushes $|V_{cb}|$ slightly up (~0.5%)

RESIDUAL UNCERTAINTY on Γ_{sl}

Bordone, Capdevila, PG, 2107.00604



Similar reduction in μ_{kin} dependence. Purely perturbative uncertainty $\pm 0.7\,\%$ (max spread), central values at $\mu_c = 2\text{GeV}, \mu_{\alpha_s} = m_b/2$.

$O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3 m_c)$, duality violation.

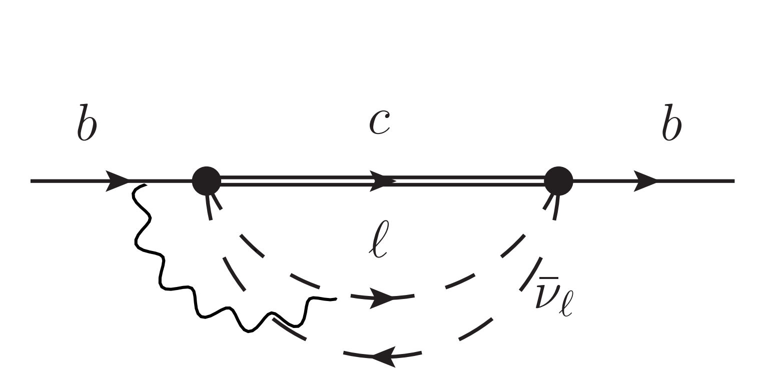
Conservatively: 1.2% overall theory uncertainty in Γ_{sl} (a ~50% reduction)

Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s \Lambda^2/m_b^2)$

QED CORRECTIONS

Bigi, Bordone, Haisch, Piccione PG
2309.02849

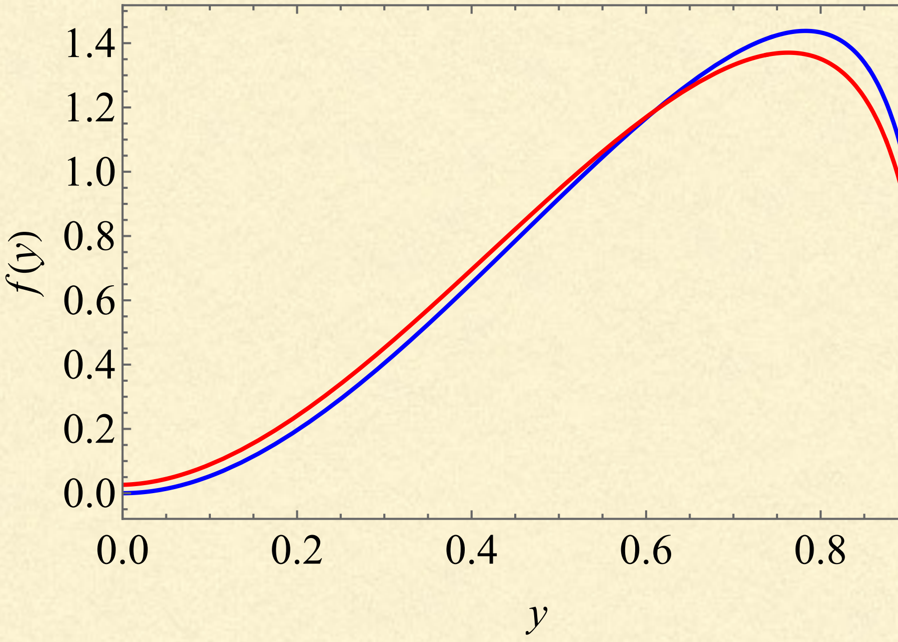
In the presence of photons, ***OPE valid only for total width*** and moments that do not resolve lepton properties (E_ℓ, q^2). Expect mass singularities and $O(\alpha\Lambda/m_b)$ corrections.



Leading logs $\alpha \ln m_e/m_b$ can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)}\left(\frac{y}{x}\right) \left(\frac{d\Gamma}{dx}\right)^{(0)}$$

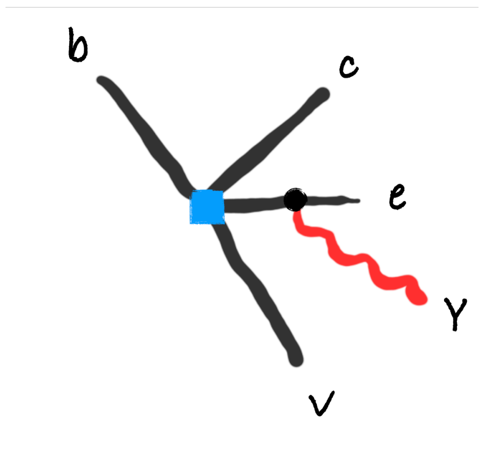
$$P_{\ell\ell}^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+$$



Electron energy spectrum

QED Leading contributions

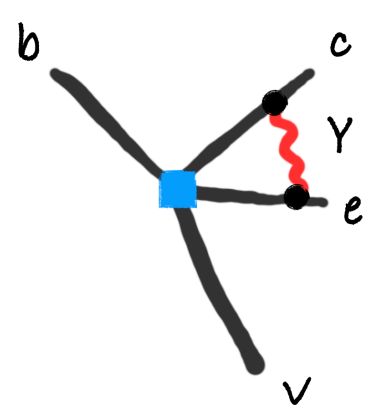
1. **Collinear logs**: captured by splitting functions



also at subleading power!

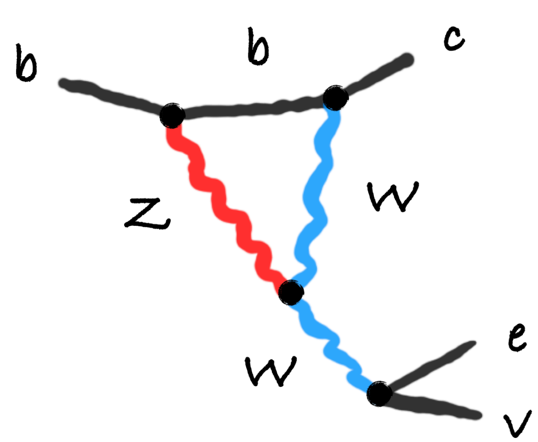
$$\sim \frac{\alpha_e}{\pi} \log \frac{m_b^2}{m_e^2}$$

2. **Threshold effects** or Coulomb terms



$$\sim \frac{4\pi\alpha_e}{9}$$

3. **Wilson Coefficient**



$$\sim \frac{\alpha_e}{\pi}\left[\log\left(\frac{M_Z^2}{\mu^2} - \frac{11}{6}\right)\right]$$

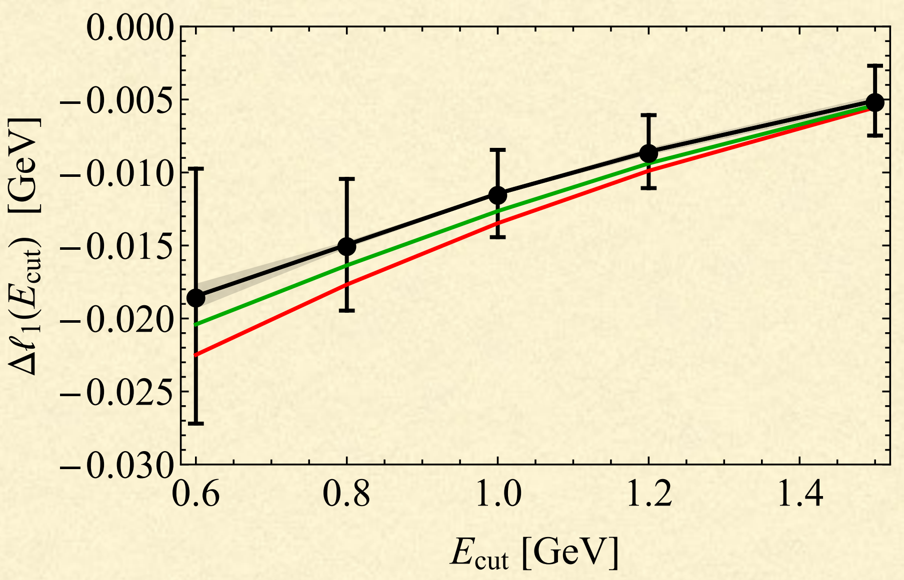
M. Bordone

COMPLETE $O(\alpha)$ EFFECTS IN LEPTONIC SPECTRUM

Typical measurements are completely inclusive, $B \to X_c \ell \nu(\gamma)$, but QED radiation is ***subtracted*** by experiments using ***PHOTOS*** (soft-collinear photon radiation to MC final states).

Small but non-negligible differences with *PHOTOS* in BaBar leptonic moments hep-ex/0403030

$E_{\rm cut}$	$\delta{\rm BR}^{\rm BaBar}_{\rm incl}$	$\delta{\rm BR}^{\rm LL}_{\rm incl}$	$\delta{\rm BR}^{\rm NLL}_{\rm incl}$	$\delta{\rm BR}^{\alpha}_{\rm incl}$	$\delta{\rm BR}^{1/m_b^2}_{\rm incl}$	$\delta{\rm BR}_{\rm incl}$	σ
0.6	-1.26%	-1.92%	-1.95%	-0.54%	-0.50%	-0.45%	$+0.34$
0.8	-1.87%	-2.88%	-2.91%	-1.36%	-1.29%	-1.22%	$+0.30$
1.0	-2.66%	-4.03%	-4.04%	-2.38%	-2.26%	-2.15%	$+0.25$
1.2	-3.56%	-5.43%	-5.41%	-3.65%	-3.43%	-3.27%	$+0.14$
1.5	-5.22%	-8.41%	-8.26%	-6.37%	-5.73%	-5.39%	-0.09



The black curve corresponds to the correction obtained by BaBar using PHOTOS, while the red (green) curve corresponds to our QED prediction including the LL terms (all QED corrections). The grey band represents the systematic uncertainty on the PHOTOS bremsstrahlungs corrections that BaBar quotes, while the black error bars correspond to the total uncertainties of the QED corrected BaBar results.

~0.2% reduction in V_{cb}

A GLOBAL FIT

Finauri, PG 2310.20324

$m_b^{\rm kin}$	$\overline{m}_c(2\,\text{GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.454	0.288	0.176	−0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48

Includes all leptonic, hadronic, and q^2 moments measured by BaBar, Belle, Belle II, Cleo, CDF, Delphi

Up to $O(\alpha_s^2)$, $O(\alpha_s/m_b^2)$, $O(1/m_b^3)$ for M_X, E_ℓ moments, up to $O(\alpha_s^2\beta_0)$, $O(\alpha_s/m_b^3)$ for q^2moments (complete $O(\alpha_s^2)$ calculation by Fael and Herren 2403.03976 to be implemented)

Subtracts QED effects beyond those computed by PHOTOS (only BaBar BR and lept moments) $\delta V_{cb} \sim -0.2\,\%$

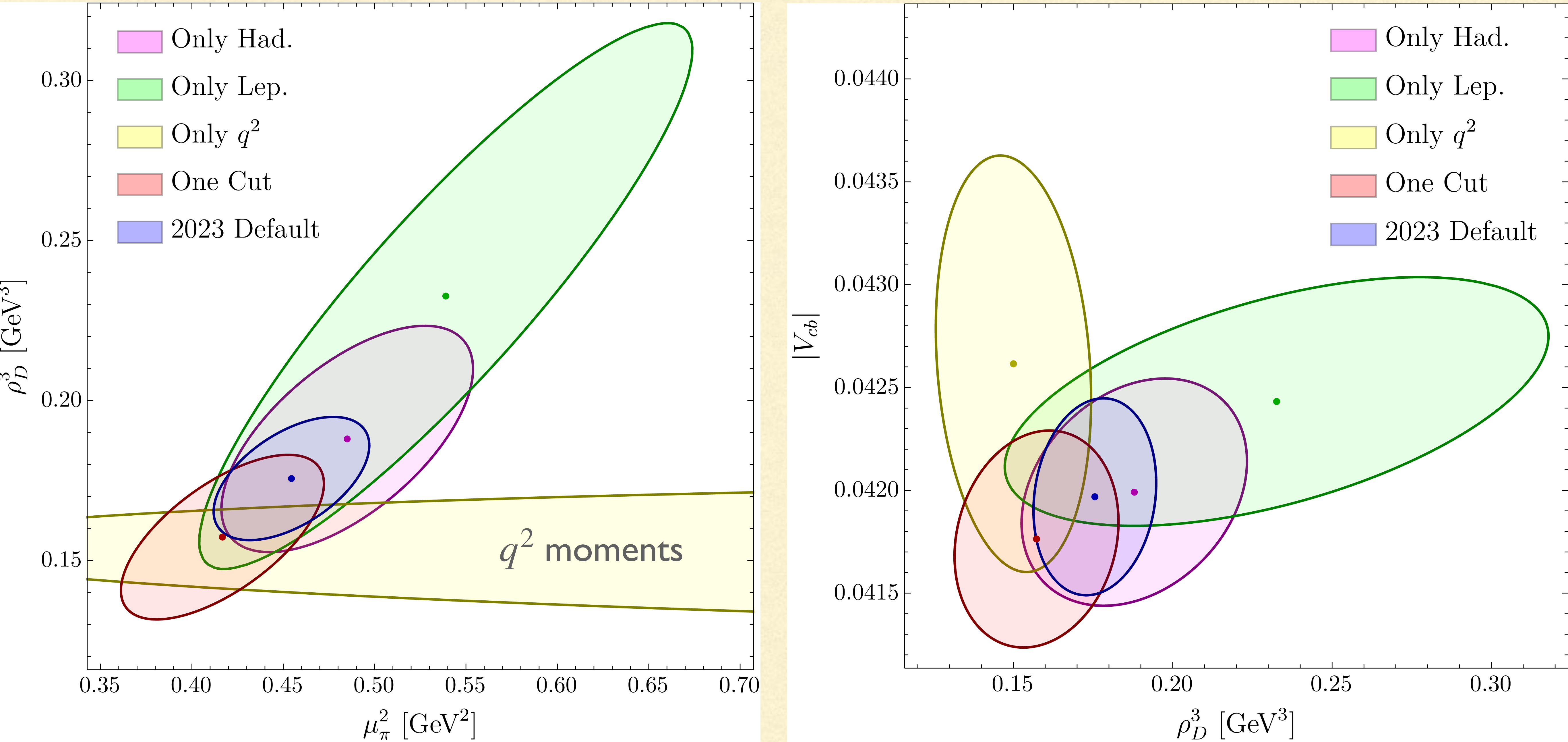
Employs $\overline{m}_b(\overline{m}_b) = 4.203(11)\text{GeV}$ and $\overline{m}_c(3\text{GeV}) = 0.989(10)\text{GeV}$ (FLAG)
$\chi^2_{min}/dof = 0.55$

$$|V_{cb}| = (41.97 \pm 0.27_{exp} \pm 0.31_{th} \pm 0.25_\Gamma) \times 10^{-3} = (41.97 \pm 0.48) \times 10^{-3}$$

consistent with analysis of q^2 moments by Bernlochner et al, 2205.10274

comparison of different datasets

Finauri, PG 2310.20324



Theory correlations are no longer an issue

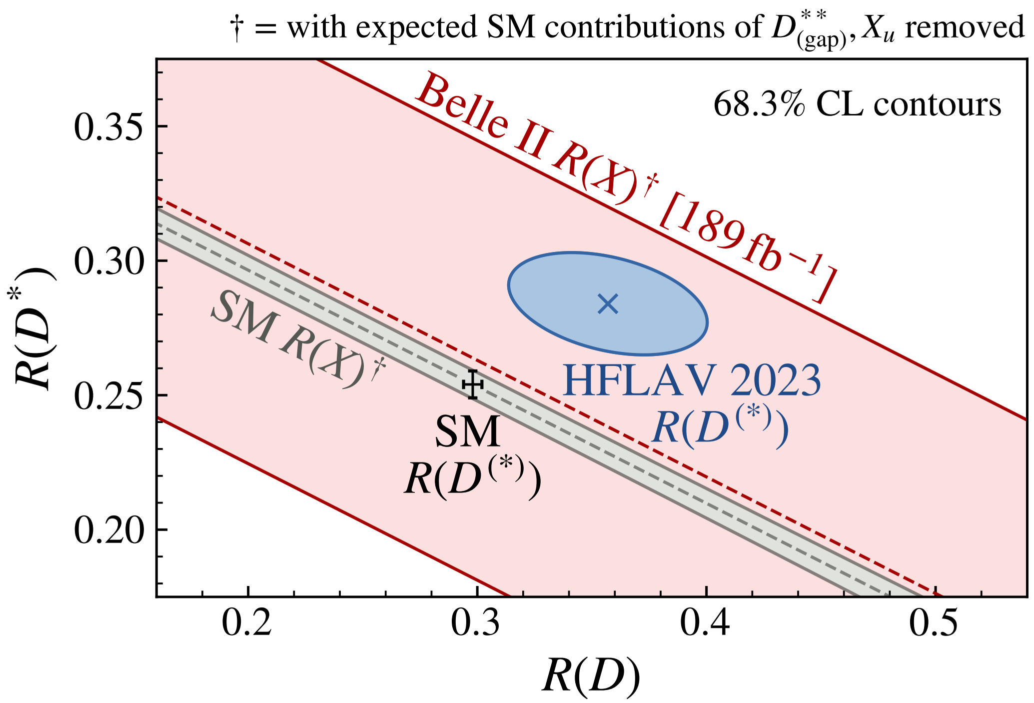
Tests of Lepton Flavor Universality

KKV, Rahimi [2207.03432]; Ligeti, Tackmann [1406.7013];Bernlocner, Sevilla, Robinson, Wormser [2101.08326]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R_{e/\mu}(X) = 1.033 \pm 0.022$ PRL131 [2023] [2301.08266]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ
- New! Belle II result: $R_{\tau/\ell}(X) = 0.228 \pm 0.016 \pm 0.036$ @EPS 2311.07248
- In agreement with SM prediction:

$$R_{\tau/\ell}(X) = 0.221 \pm 0.004$$



Belle 2311.07248

What's next for moments?

- Measure all kin. moments simultaneously as a function of q^2 (E_l^B) thresholds in

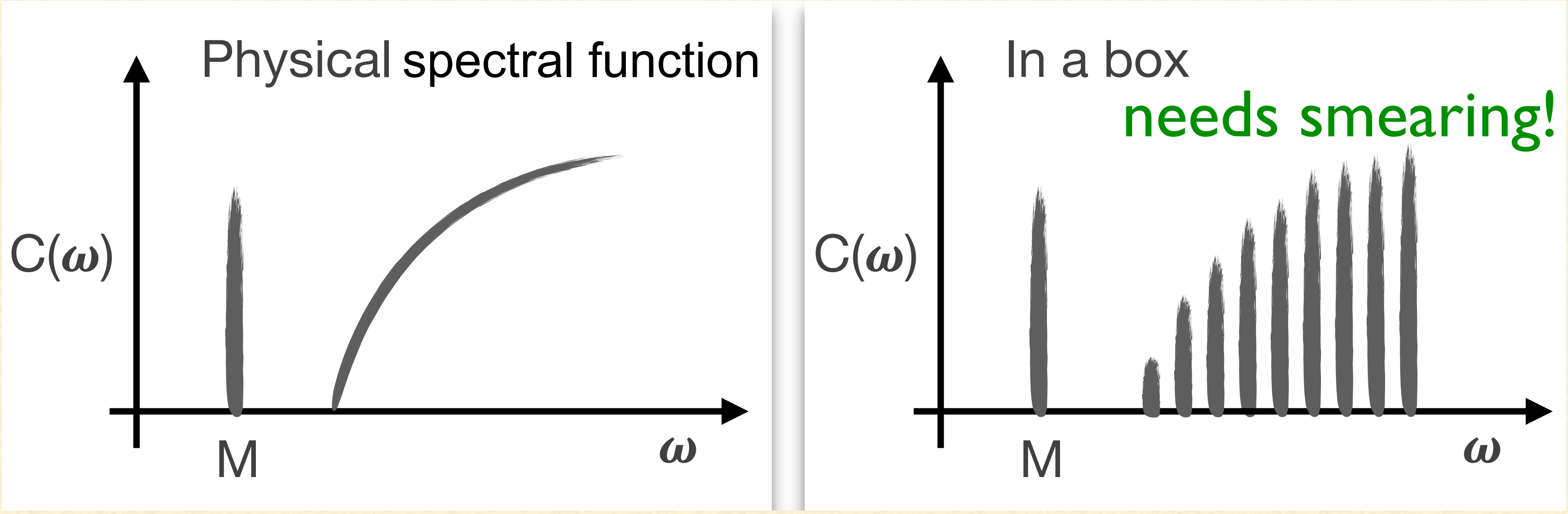
  $B \to X\ell\nu$: q^2, E_l^B, M_X, $\cos\theta_\ell$, combined variables $n_X^2\,(M_X^2, E_X)$, $P_X^\pm\,(M_X, E_X)$

- Full experimental correlations will be derived => important for global analysis
- Only shape observation (drop tagging eff. calibration, separate from $\mathscr{B}$ measurement)

INCLUSIVE DECAYS ON THE LATTICE

- Inclusive processes *impractical* to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \to$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.

- While the lattice calculation of the spectral density of hadronic correlators is an ***ill-posed problem***, the spectral density is accessible *after smearing,* as provided by phase-space integration Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa



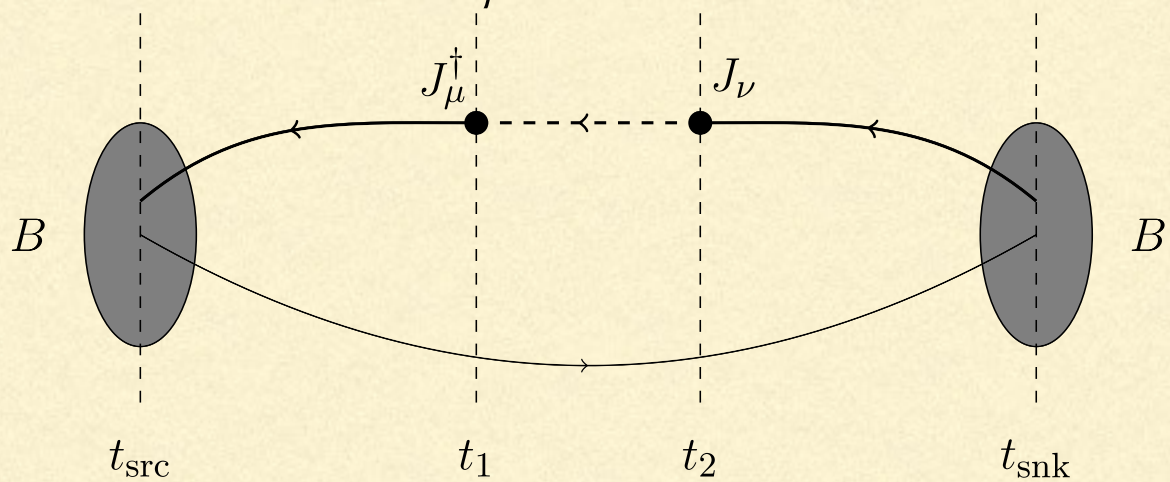
W. Jay @Snowmass workshop

A PRACTICAL APPROACH

Hashimoto, PG 2005.13730

4-point functions on the lattice are related to the hadronic tensor in euclidean

$$\sim \langle B | J_\mu^\dagger(\mathbf{x},t) J_\nu(\mathbf{0},0) | B \rangle$$

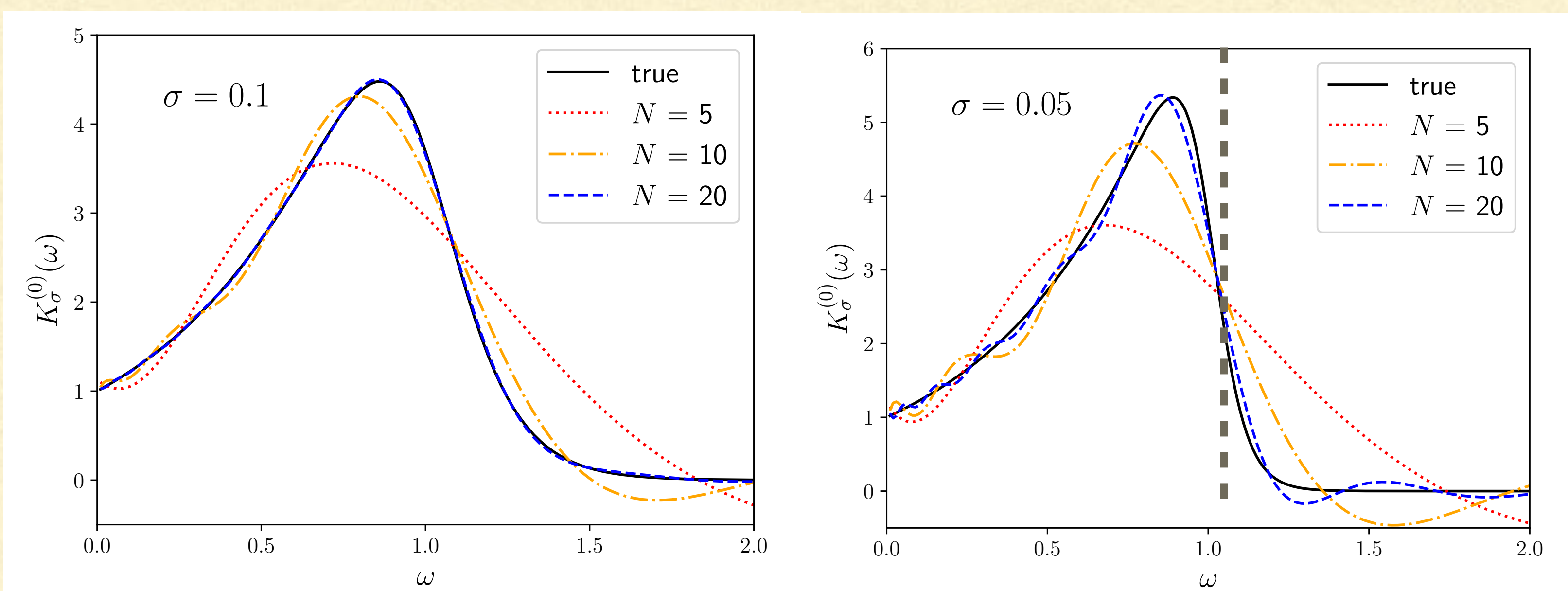


$$d\Gamma \sim L^{\mu\nu} W_{\mu\nu}, \quad W_{\mu\nu} \sim \sum_X \langle B | J_\mu^\dagger | X \rangle \langle X | J_\nu | B \rangle$$

$$\int d^3x \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{2M_B} \langle B | J_\mu^\dagger(\mathbf{x},t) J_\nu(\mathbf{0},0) | B \rangle \sim \int_0^\infty d\omega W_{\mu\nu} e^{-t\omega}$$

smearing kernel $f(\omega) = \sum_n a_n e^{-na\omega}$

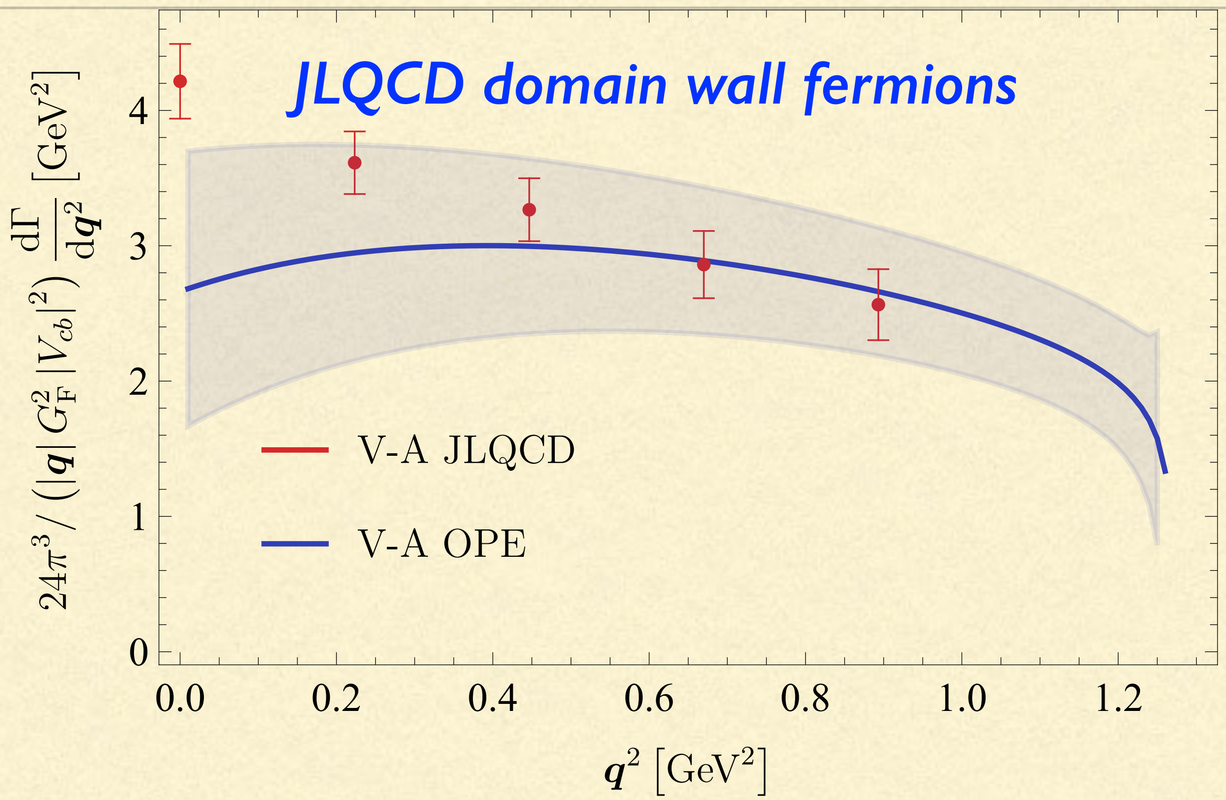
The necessary smearing is provided by phase space integration over the hadronic energy, which is cut by a θ with a sharp hedge: sigmoid $1/(1+e^{x/\sigma})$ can be used to replace kinematic $\theta(x)$ for $\sigma \to 0$. Larger number of polynomials needed for small σ

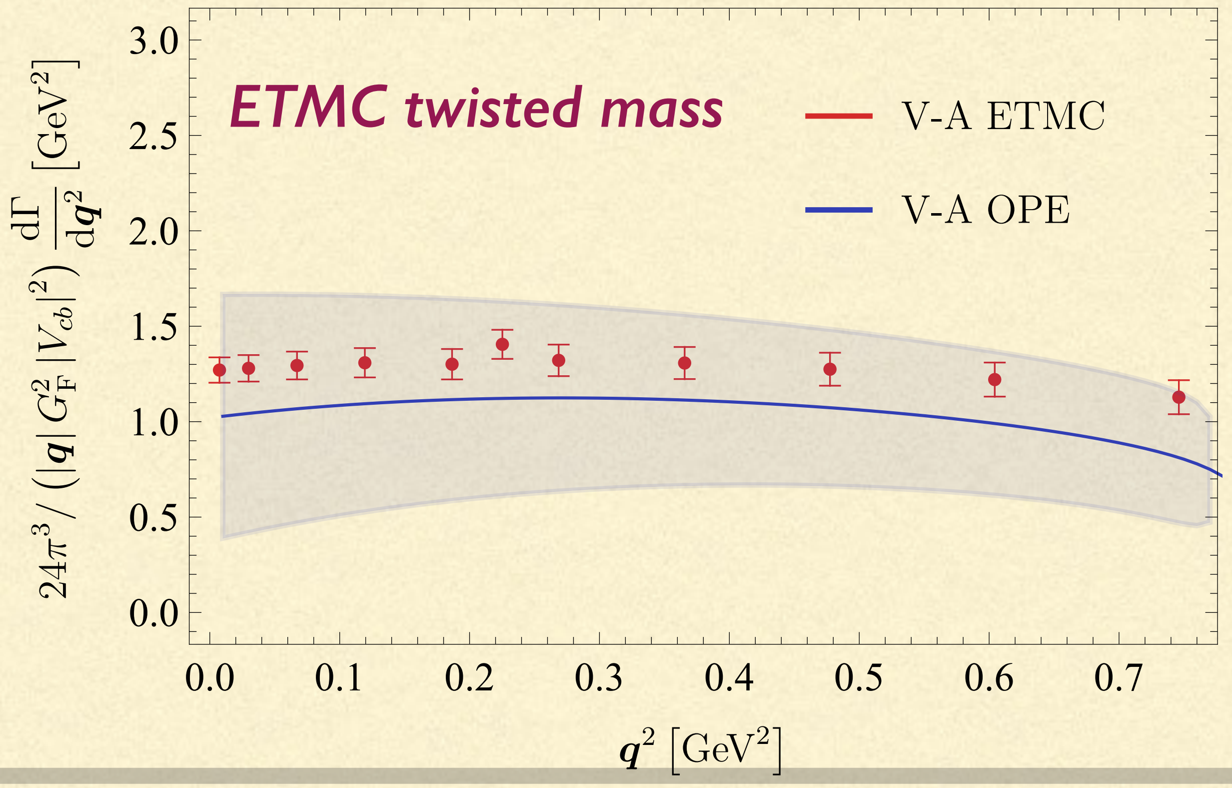


Two methods based on Chebyshev polynomials and Backus-Gilbert. Important:

$$\lim_{\sigma \to 0} \lim_{V \to \infty} X_\sigma$$

LATTICE VS OPE





m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2\ \text{GeV})$ (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2\ \text{GeV})$ (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}(2\ \text{GeV})$	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2 + \mathbf{q}^2} \sim 1-1.5\,\text{GeV}$

We do not expect OPE to work at high $|\mathbf{q}|$

Twisted boundary conditions allow for any value of $\vec{q}^2$
Smaller statistical uncertainties

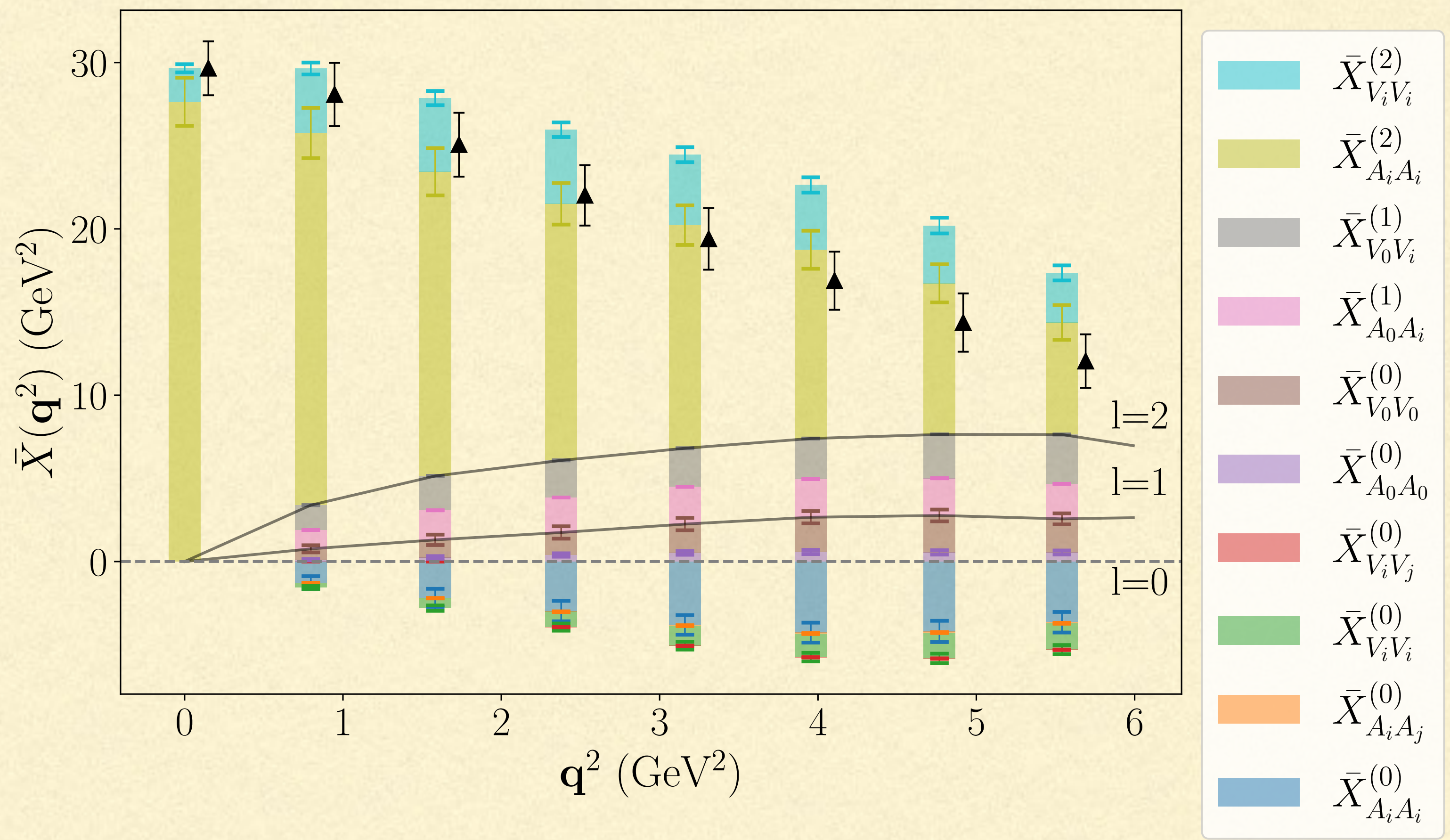
First results at the physical *b* mass

Relativistic heavy quark effective action for b

B_s decays, domain wall fermions, improved implementation of Chebychev polynomials and Backus-Gilbert

qualitative study ~5% statistical uncertainty on total width

possibly better to compare with partial width at low $\vec{q}^2$



Barone, Hashimoto, Juttner, Kaneko, Kellermann, 2305.14092

Ongoing work on semileptonic D, D_s decays by two collaborations

INCLUSIVE $|V_{ub}|$

Important Belle measurement 2102.00020

In my opinion, the cleanest measurement is the most inclusive one with $M_X < 1.7\text{GeV}, E_\ell > 1\text{GeV}$:

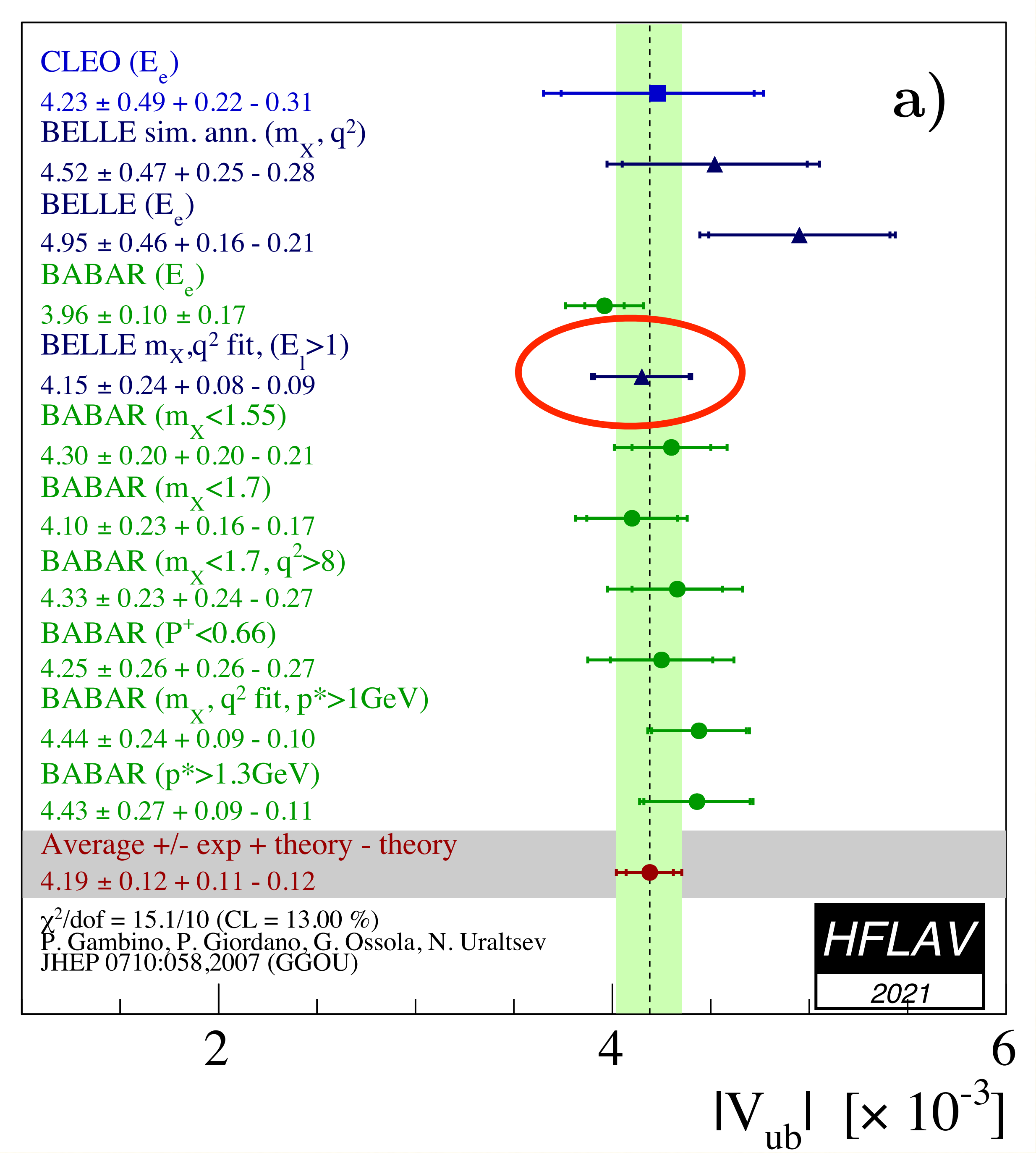
$$|V_{ub}| = (3.97 \pm 0.08 \pm 0.16 \pm 0.16)\, 10^{-3}$$

Framework	$\|V_{ub}\|[10^{-3}]$
BLNP	$4.28 \pm 0.13^{+0.20}_{-0.21}$
DGE	$3.93 \pm 0.10^{+0.09}_{-0.10}$
GGOU	$4.19 \pm 0.12^{+0.11}_{-0.12}$
ADFR	$3.92 \pm 0.1^{+0.18}_{-0.12}$
BLL (m_X/q^2 only)	$4.62 \pm 0.20 \pm 0.29$

Not all approaches at the same level
Some discrepancy hidden in the average

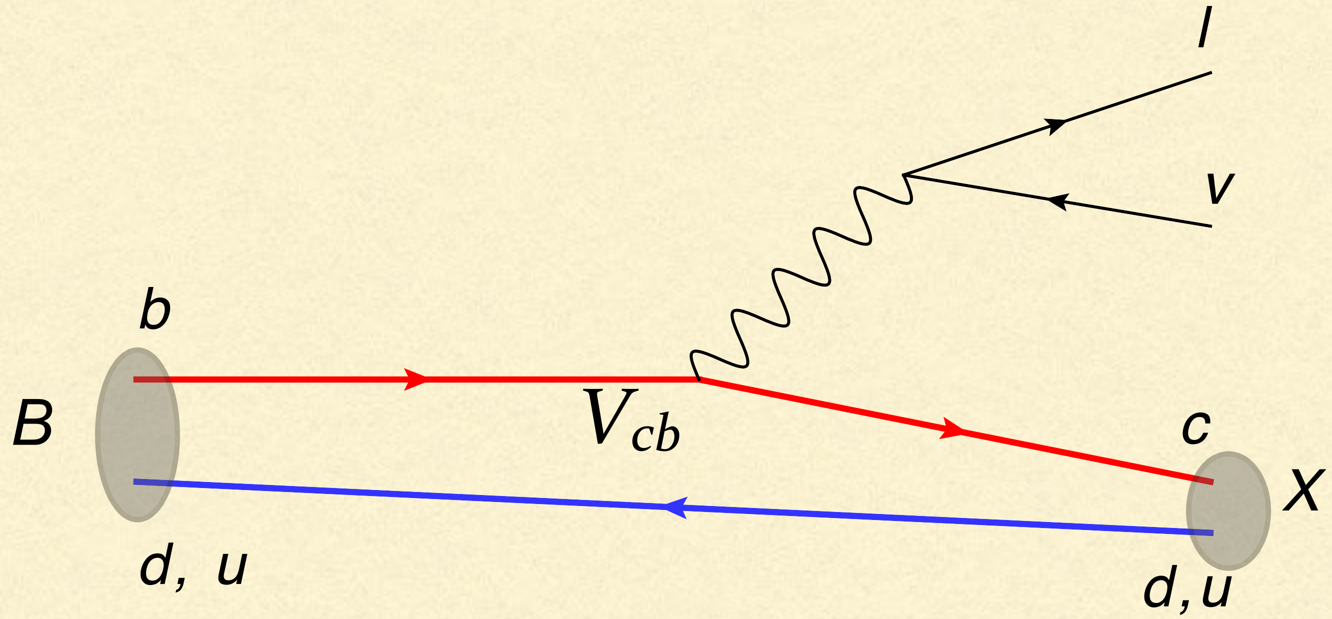
Recent calculation of the $O(\alpha_s/m_b^2)$ effects in $B \to X_u \ell \nu$, Capdevila, Nandi, PG

recent $\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 0.97 \pm 0.12$ 2303.17309



Look forward to validating approaches on Belle II data (SIMBA, NNVUB)!

EXCLUSIVE DECAYS



$= D, D^*, \ldots$

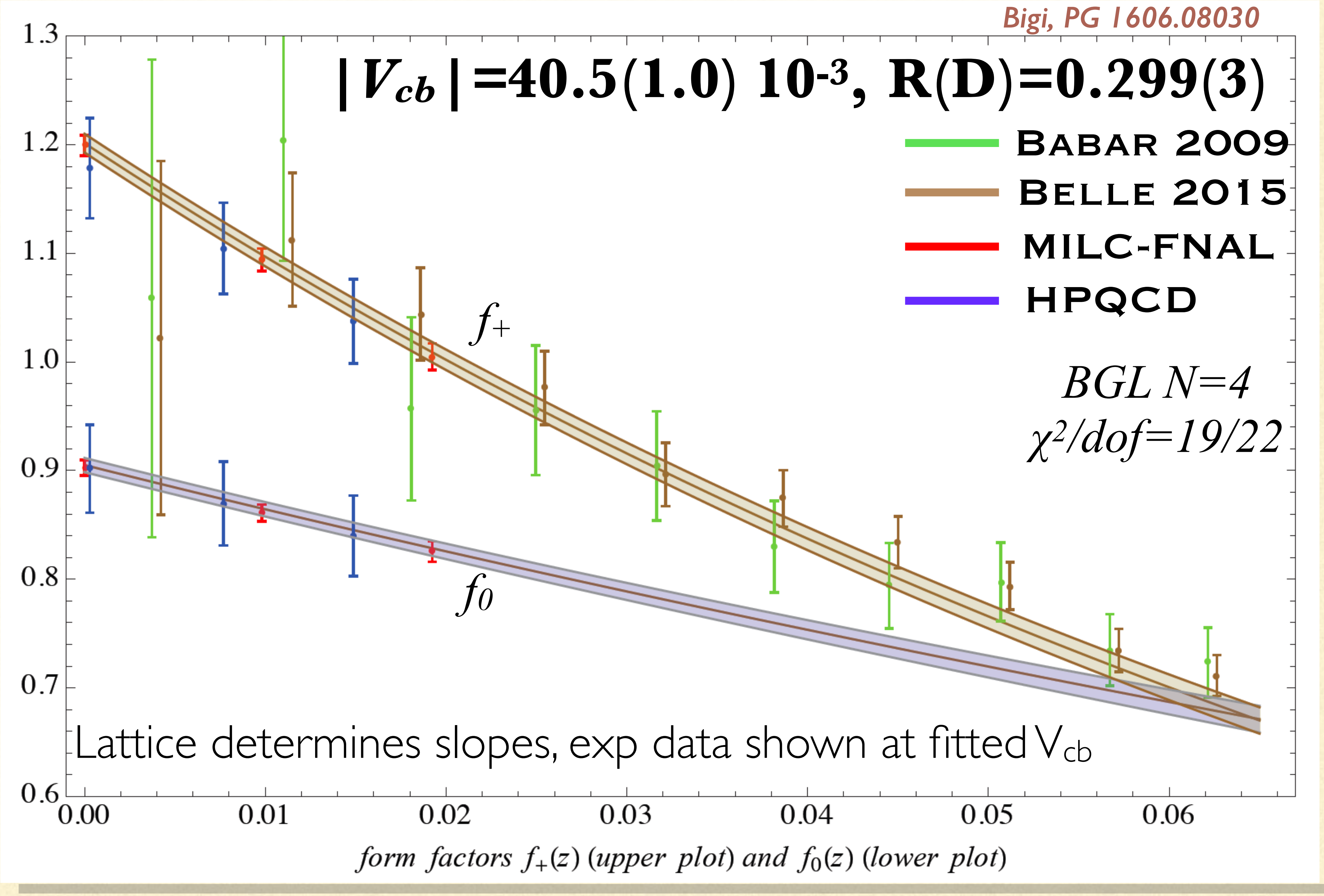
There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

$$\langle D(k)|\,\bar{c}\gamma^\mu b\,|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu\right] f_+^{B\to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0^{B\to D}(q^2)$$

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp, extrapolation, unitarity constraints, …

A ***model independent parametrization*** is very useful. In particular BGL (Boyd, Grinstein, Lebed)

LATTICE + EXP BGL FIT for $B \to D\ell\nu$



R(D)=0.299(3)

1.3σ from exp

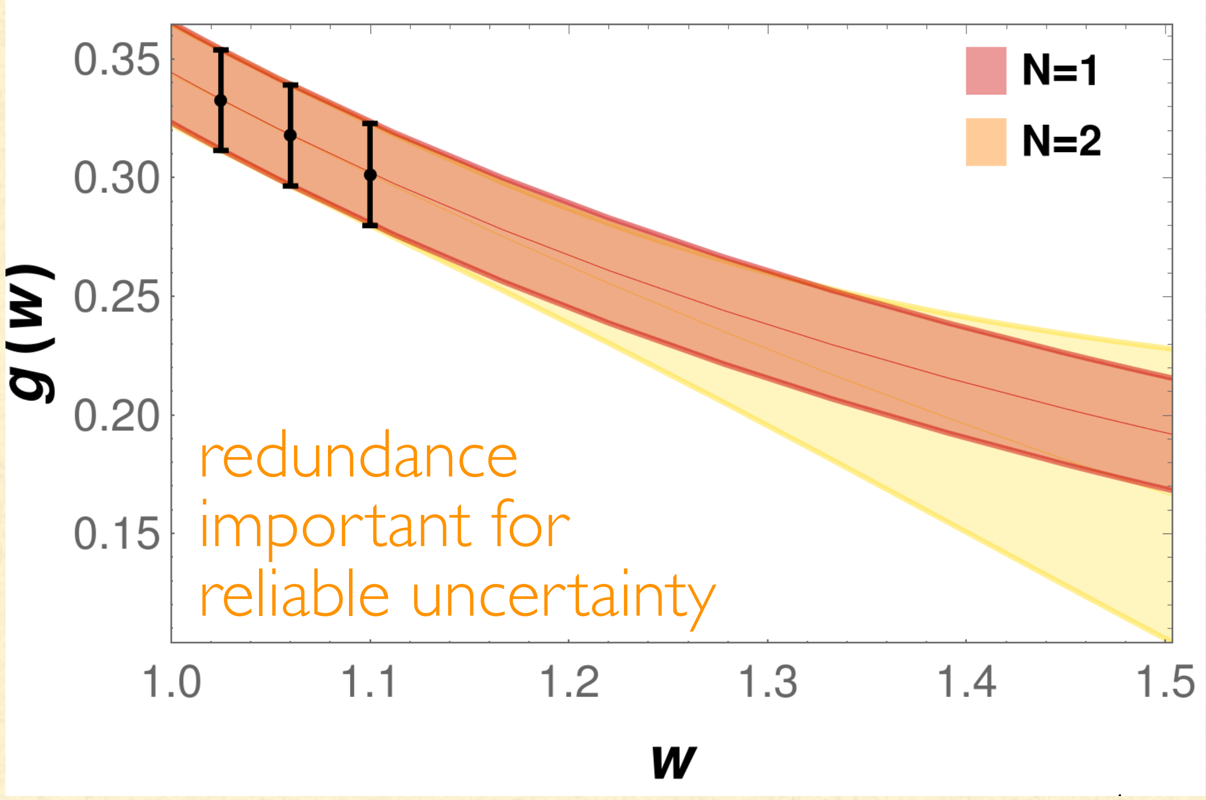
FLAG has very similar results

CLN cannot fit both ff

Model independence vs overfitting

$$f(q^2) = A(q^2) \sum_i^\infty a_i\, z(q^2)^i, \qquad \sum_i^\infty a_i^2 < 1$$

with $|z| < 0.06$. Where do we truncate the series? How can we include unitarity constraints? These questions are related.



until you have lots of precise data...

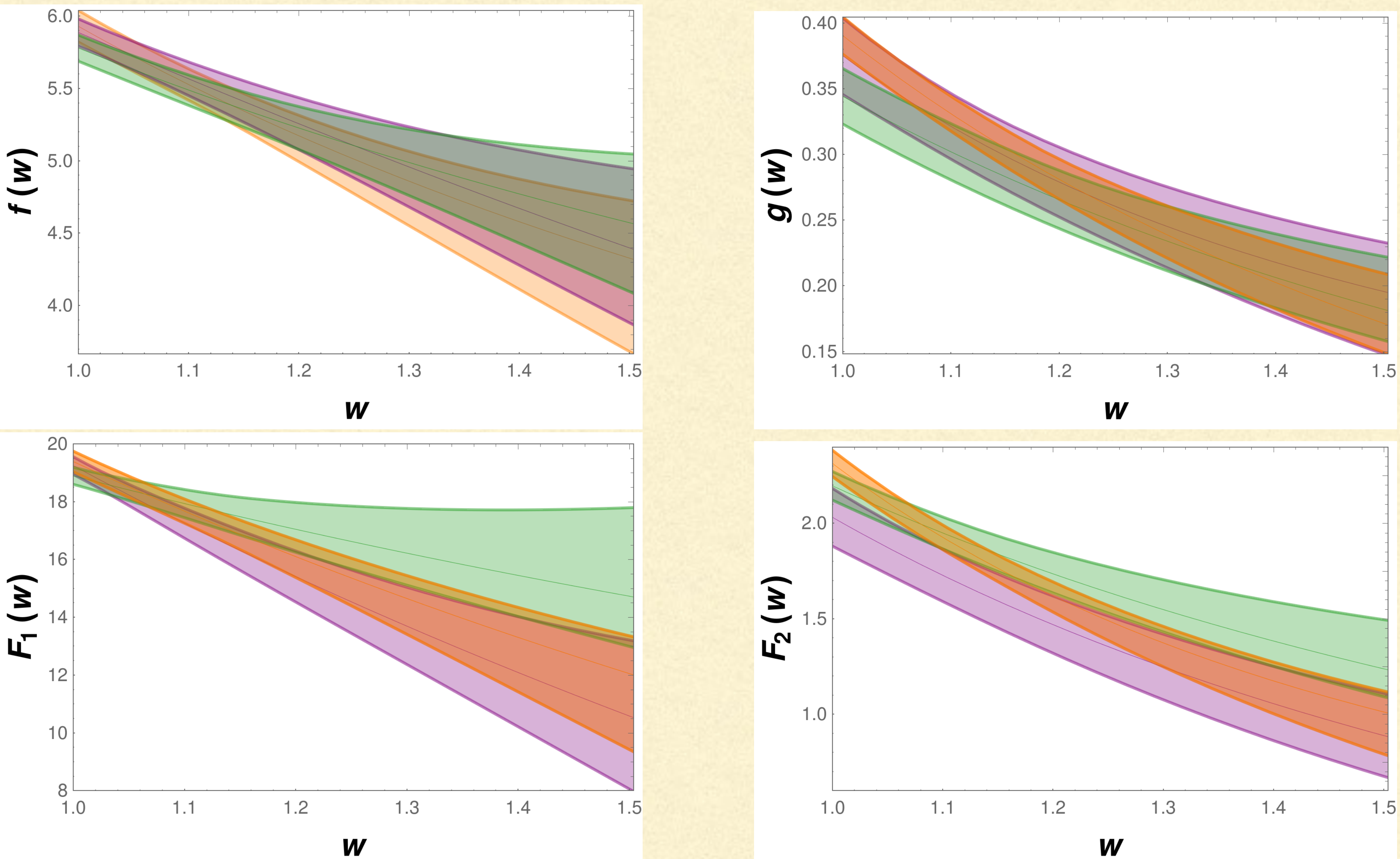
Different options with various pro/cons:

1. Frequentist fits with strong χ^2 ***penalty*** outside unitarity; increase BGL order till χ^2_{min} is stable. Can compute CL intervals Bigi, PG, 1606.08030, Jung,Schacht,PG 1905.08209 ***New: Feldman-Cousins consistent frequentist approach with well-defined CL***

2. Frequentist fit with ***Nested Hypothesis Test or AIC*** to determine optimal truncation order: go to order $N+1$ if $\Delta\chi^2 = \chi^2_{min,N} - \chi^2_{min,N+1} \geq 1,2$ Check unitarity a posteriori Bernlochner et al, 1902.09553

3. ***Bayesian inference*** using unitarity constraints as prior with BGL Flynn, Jüttner, Tsang 2303.11285 or in the ***Dispersive Matrix approach (which avoids truncation)***, Martinelli, Simula, Vittorio et al. 2105.02497

LATTICE FORM FACTORS FOR $B \to D^*$



FERMILAB/MILC

JLQCD

HPQCD

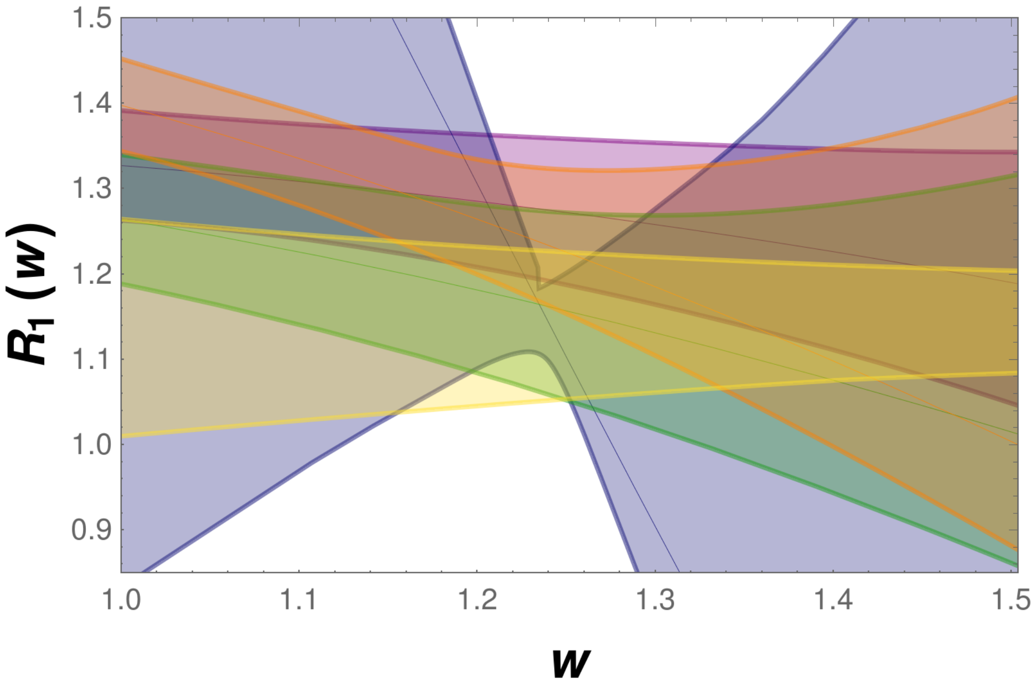
2105.14019, 2112.13775, 2304.03137

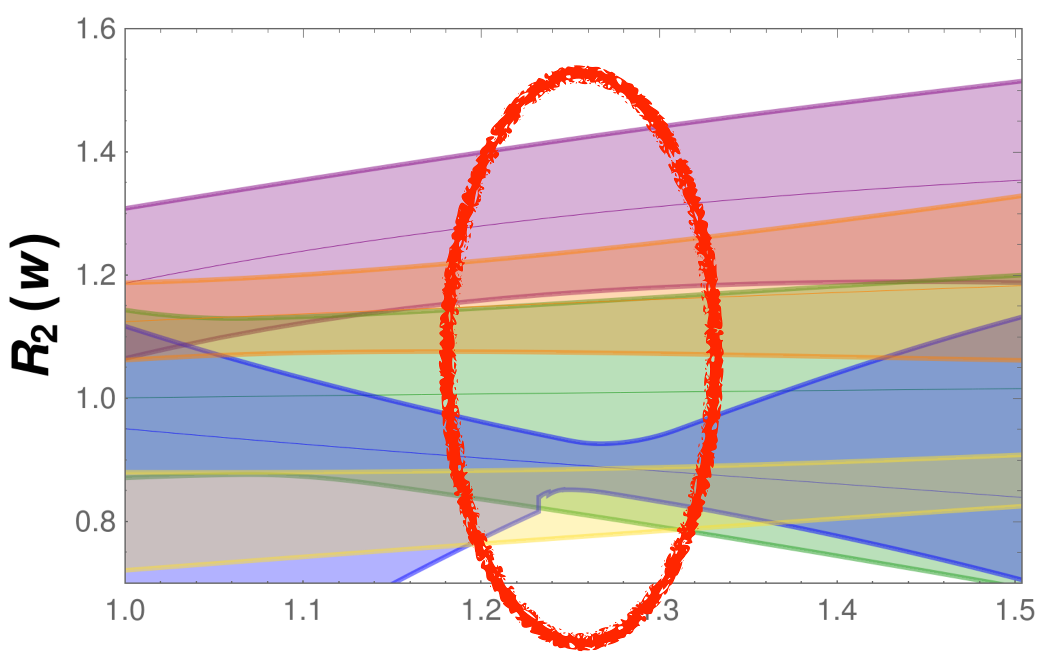
No major discrepancy

but differences may get amplified in certain combinations of ffs

see Andreas Juttner talk

RATIOS OF FORM FACTORS





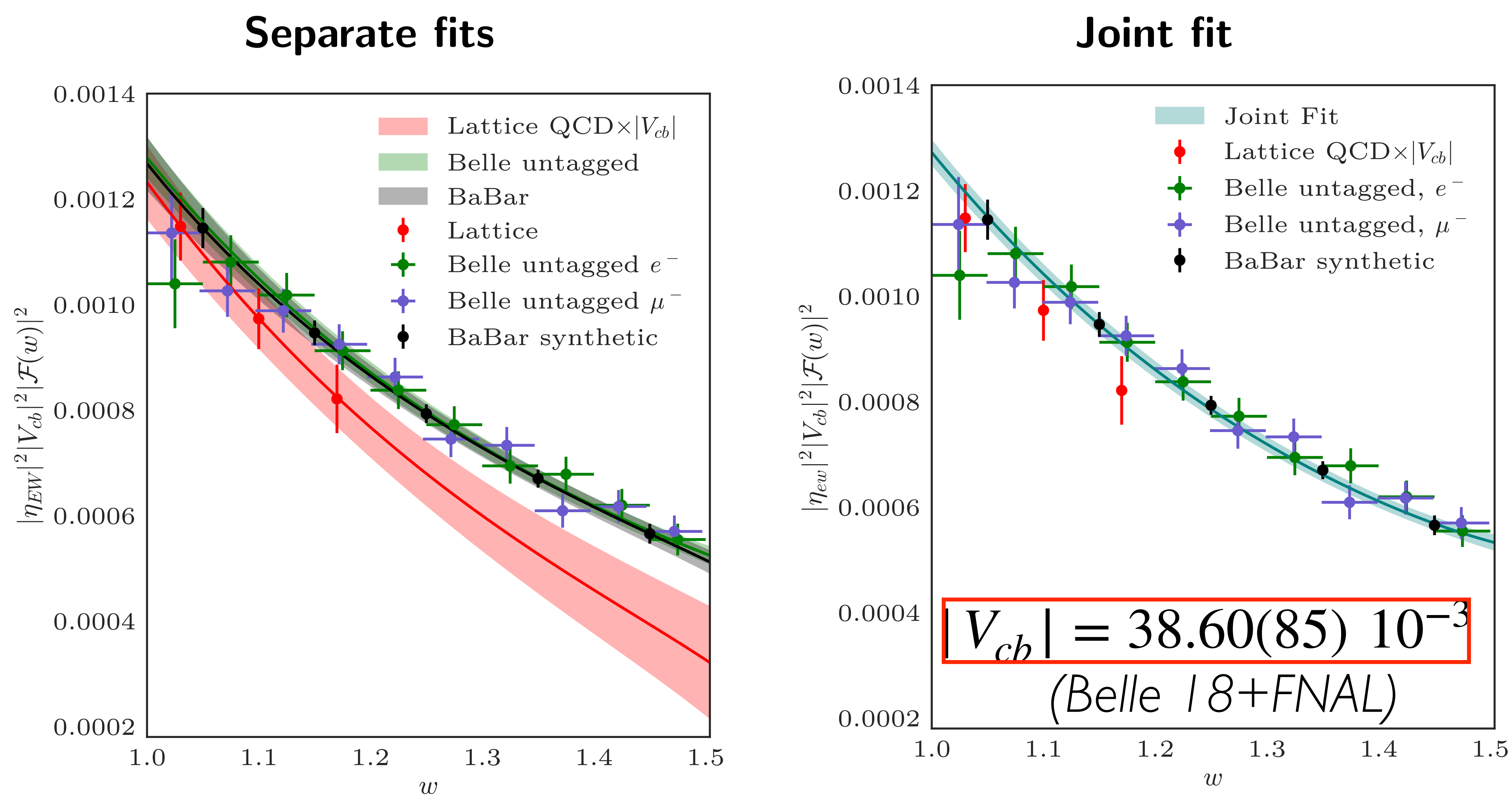
M.Jung

FERMILAB/MILC
JLQCD
HPQCD
HQE (LCSR+SR+lat<2019)
EXP (Belle 2018)

Form factor ratios more sensitive to differences. Stark tension between F/M & HPQCD and HQE & EXP in R_2

FERMILAB/MILC

2105.14019



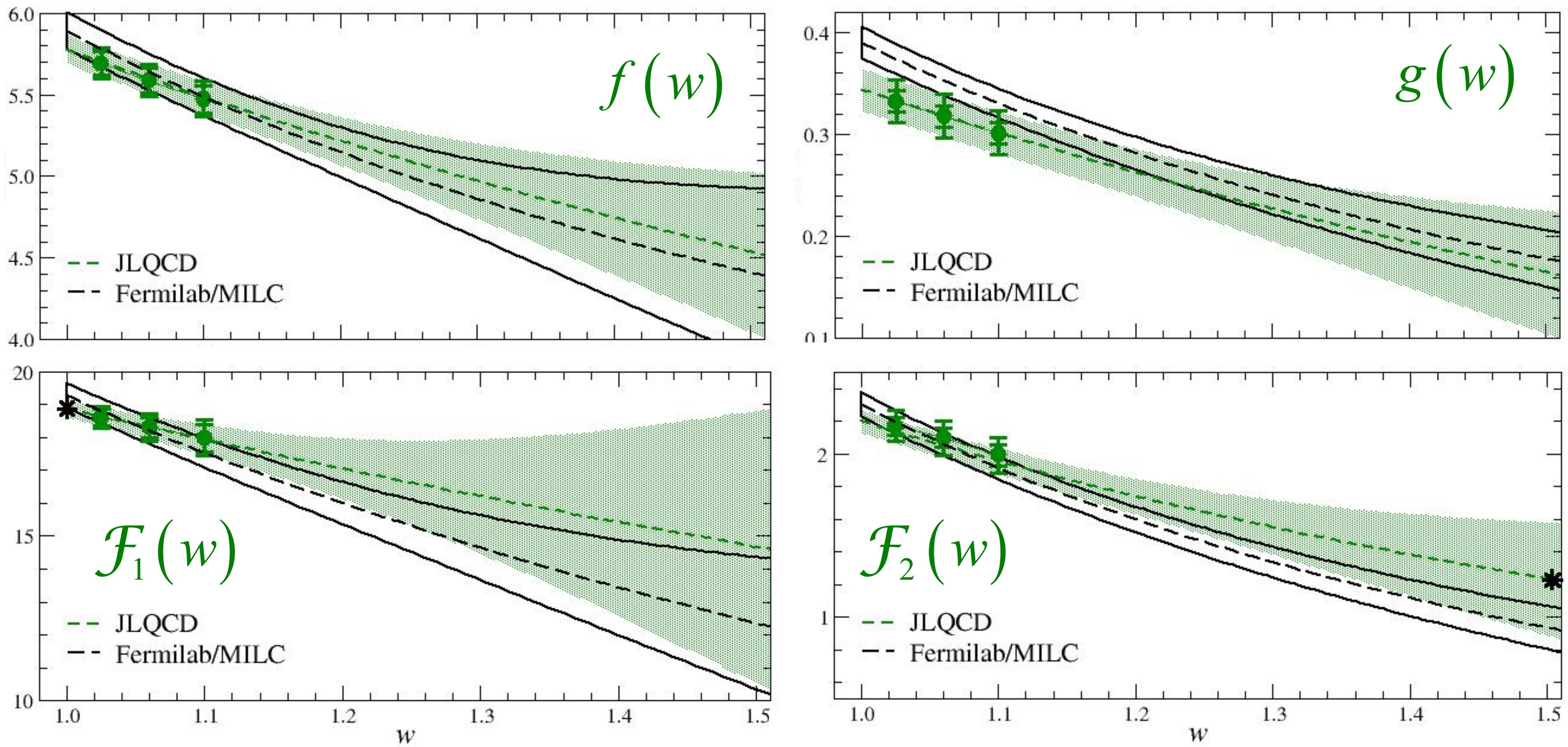
Fit	Lattice	Exp	Lat + Belle	Lat + BaBar	Lat + Exp
p-Value	0.88	0.037	0.015	0.088	0.002

First lattice calculation beyond zero recoil for this mode

Our analysis of Belle 18+ FNAL data (Jung, PG):
$|V_{cb}|$=39.4(9) 10^{-3} ($\chi^2_{min} = 50$) *using only total rate* $|V_{cb}|$=$42.2^{+2.8}_{-1.7}\ 10^{-3}$

JLQCD RESULTS

JLQCD vs Fermilab/MILC



- reasonably consistent

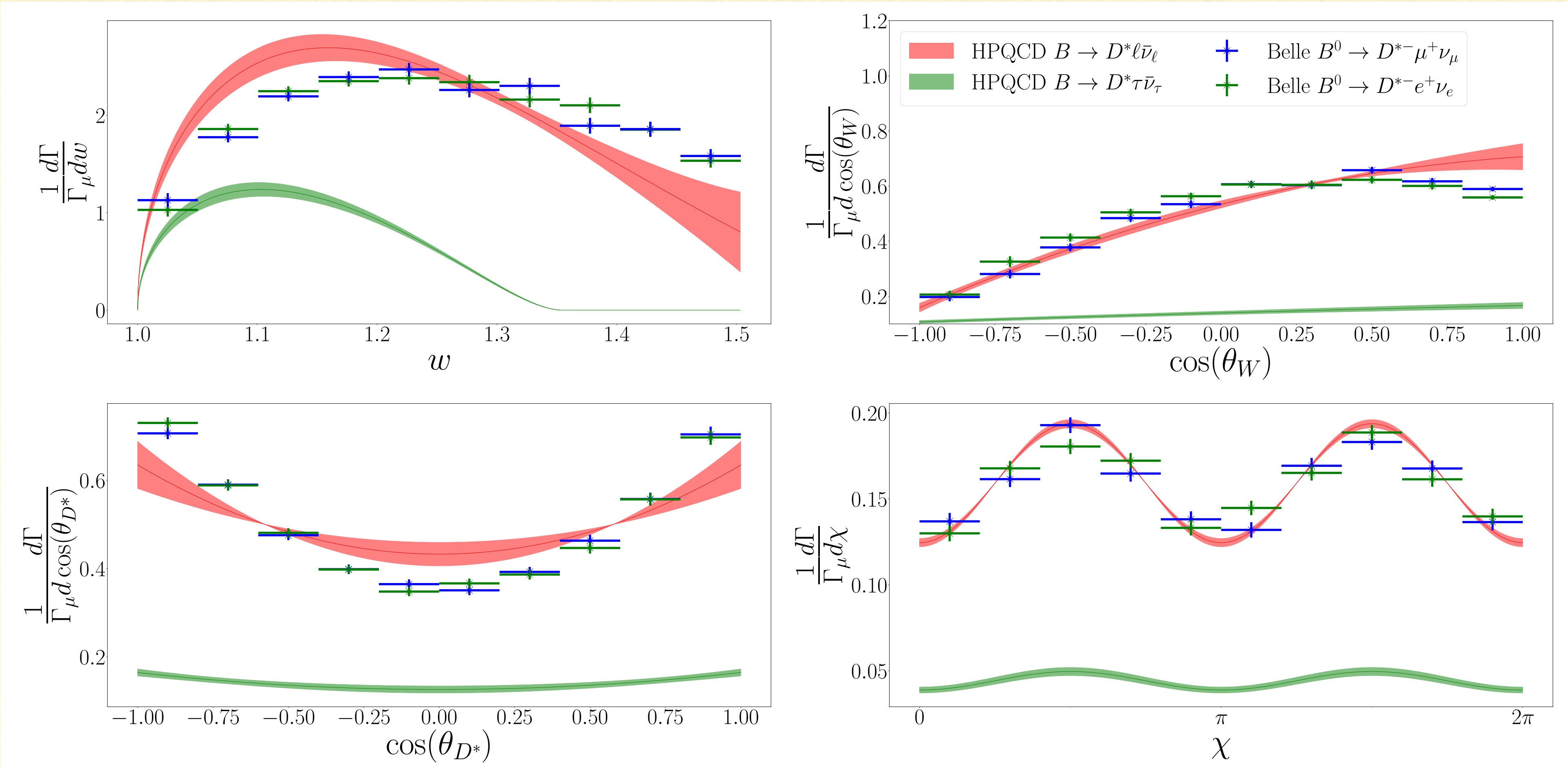
$\Leftrightarrow$ g @ $w \sim 1$

T. Kaneko @ Barolo workshop 4/2021
Kaneko et al 2112.13775

Our analysis of Belle18+ JLQCD data (Jung, PG):
$|V_{cb}|=40.7(9)\ 10^{-3}$ *($\chi^2_{min} = 33$) using only total rate* $|V_{cb}|=40.8^{+1.8}_{-2.3}\ 10^{-3}$

HPQCD

2304.03137v2



Tension with Belle 2018 data similar to FNAL

Belle18+HPQCD		
BGL exp	χ^2	$\lvert V_{cb}\rvert$
0001	59	41.0(9)
0101	55	41.1(9)
0111	44	40.7(9)
1111	43	40.5(9)
1121	42	40.4(9)
1222	42	40.4(9)
2222	40	40.2(9)
2232	40	40.2(9)
3333	40	40.2(9)

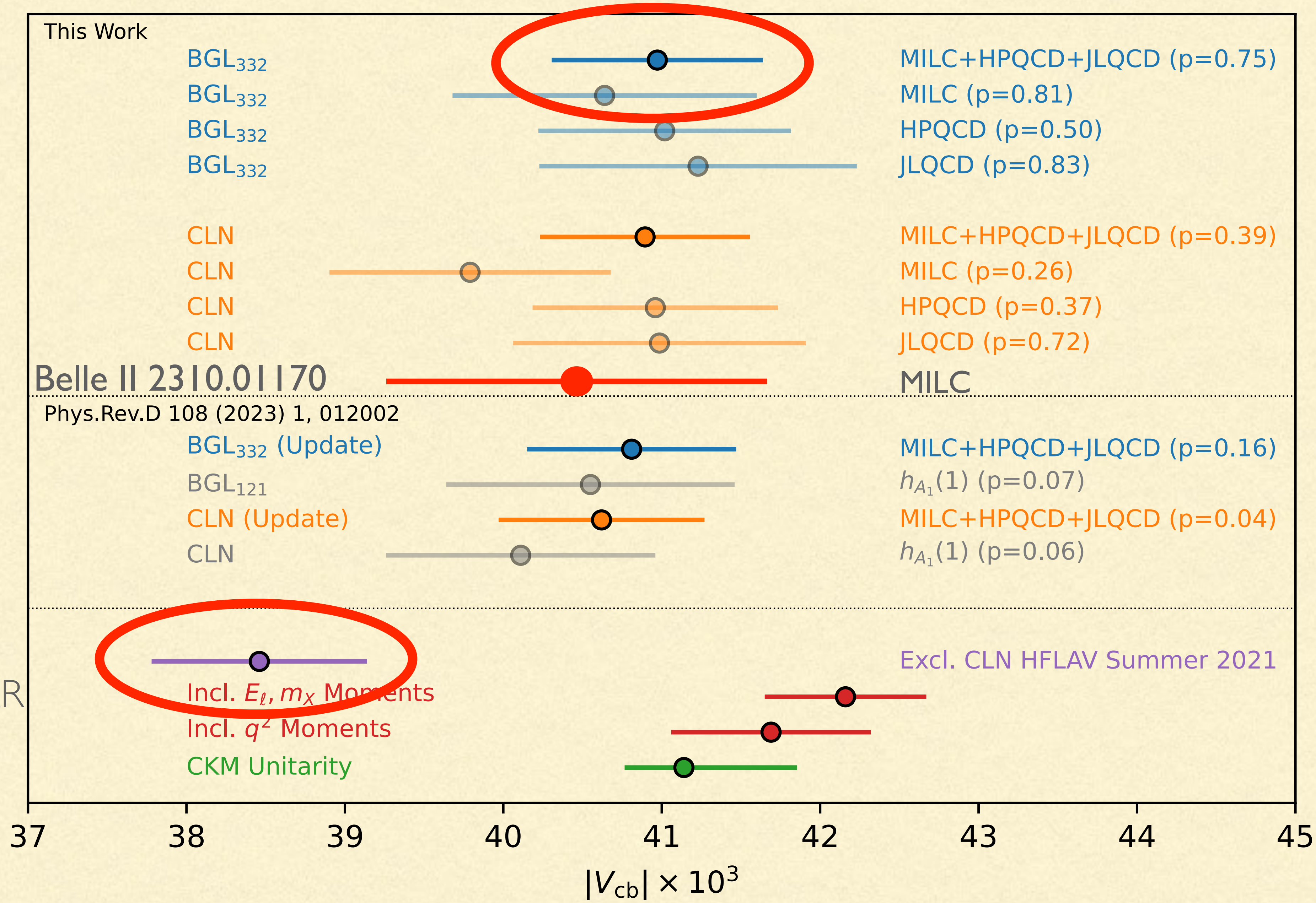
Extrapolation in m_h, data cover the whole w region

Our analysis of Belle18+ HPQCD data (Jung, PG):
$|V_{cb}|=40.2(9)\ 10^{-3}$ *using only total rate* $|V_{cb}|=43.2 \pm 2.2\ 10^{-3}$

Global BGL fit to Belle18+FNAL+JLQCD+HPQCD data:
$|V_{cb}|=40.2(7)\ 10^{-3} (\chi^2_{min} = 71.4)$ *using only total rate* $|V_{cb}|=41.6(1.3)\ 10^{-3}$

New from Belle! [2310.20286]

$$|V_{cb}| = 41.0(7) \times 10^{-3}$$



normalisation from HFLAV total rate!

SEE ALSO

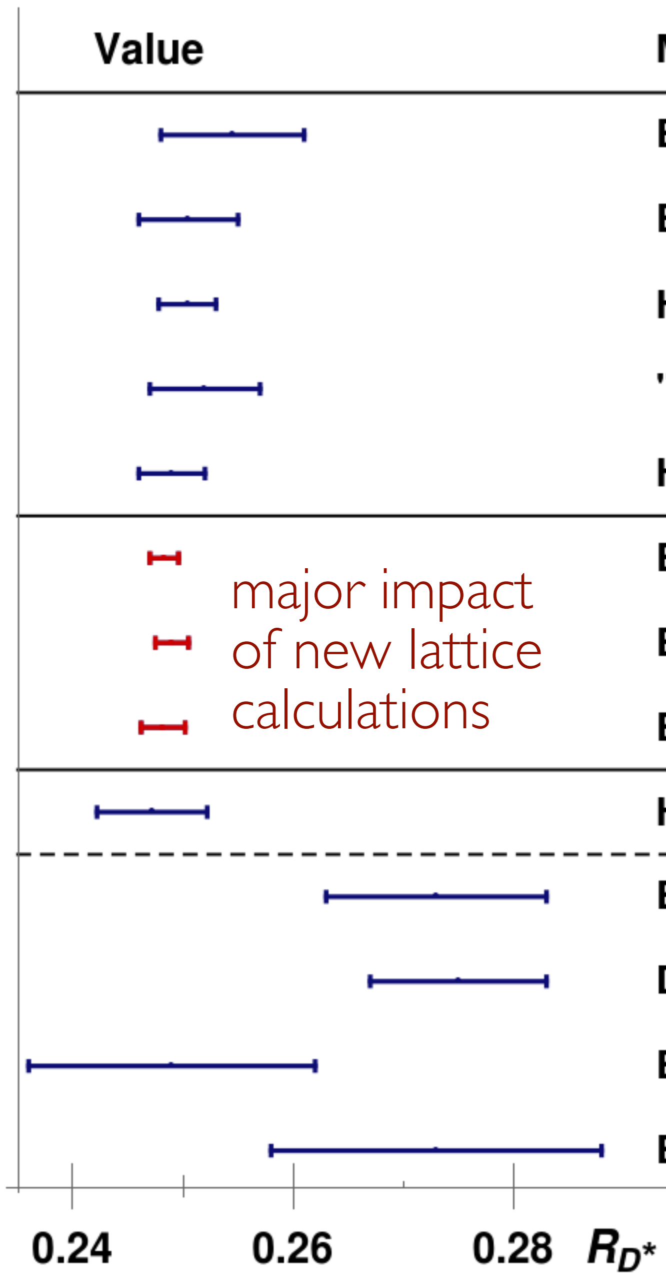
1903.10002, LHCB

2001.03225 BABAR

Do we understand why they differ so much?

Should we average them?

$R(D^*)$ PREDICTIONS



Value	Method	Input Theo	Input Exp	Reference
	BGL	Lattice, HQET	Belle'17'18	Gambino et al.'19
	BGL	Lattice, HQET	Belle'18	Jaiswal et al.'20
	HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	Belle'17'18	Bordone et al.'20
	"Average"			HFLAV'21
	HQET$_{RC}$@$1/m^2,\alpha_s^{(2)}$	Belle'17'18	Lattice	Bernlochner et al.'22
	BGL	Lattice	Belle'18, Babar'19	Vaquero et al.'21v2
	BGL	Lattice	Belle'18	JLQCD
	BGL	Lattice	Belle'18	Davies, Harrison'23
	HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	---	Bordone et al.'20
	BGL	Lattice	---	Vaquero et al.'21v2 FNAL/MILC
	DM	Lattice	---	Martinelli et al.
	BGL	Lattice	---	JLQCD (+unitarity)
	BGL	Lattice	---	Davies, Harrison'23

M.Jung

Predictions based only on Fermilab & HPQCD lead to larger R(D*), in better agreement with exp, mostly because of the suppression at high w of the denominator. ***No reason not to use experimental data for a SM test***, especially in presence of tensions in lattice data.

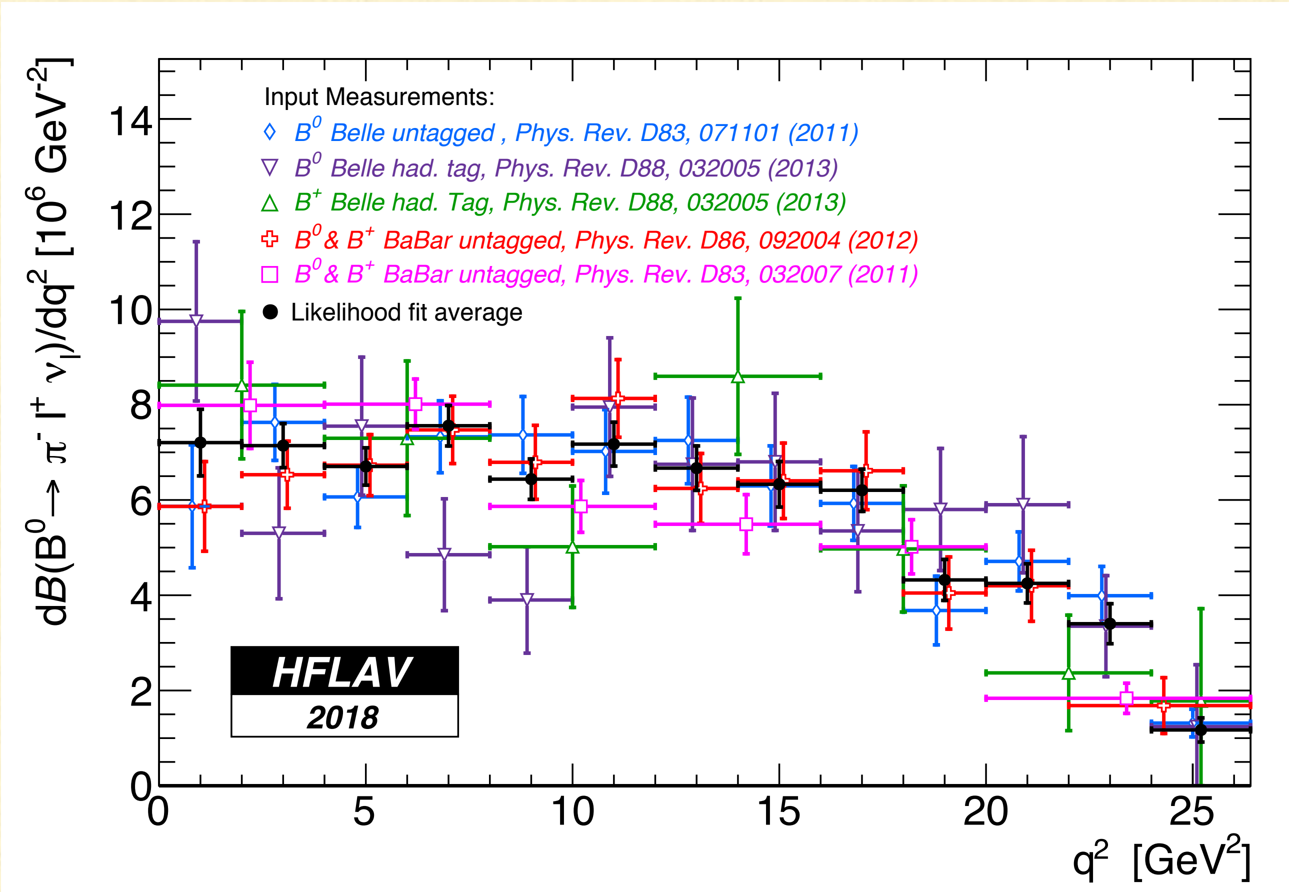
What about the DM results applied to other FFs?

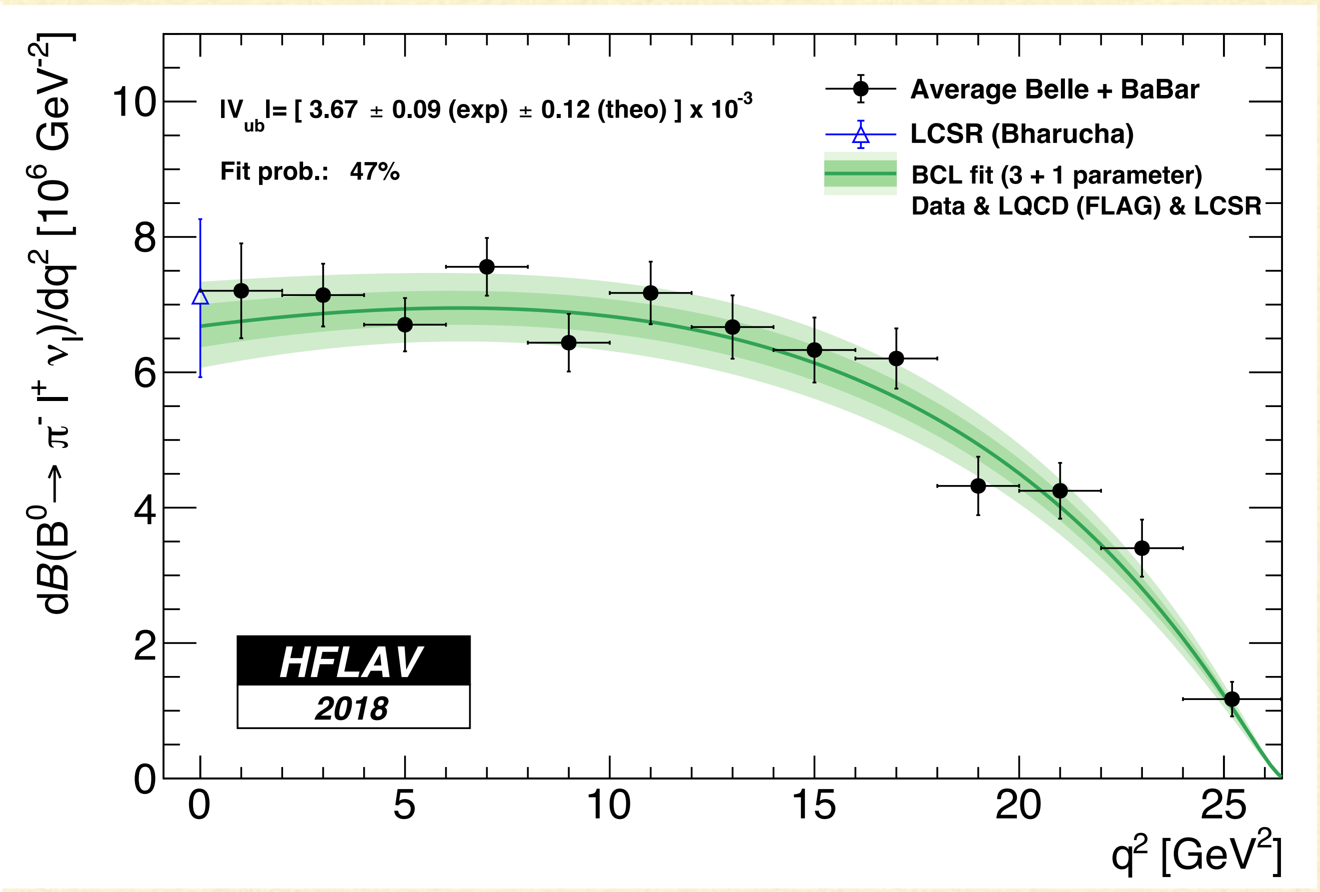
Lattice FFs	$R(D^*)$	$P_\tau(D^*)$	$F_{L,\tau}$	$F_{L,\ell}$	$A_{FB,\ell}$
FNAL/MILC [14]	0.275(8)	-0.529(7)	0.418(9)	0.450(19)	0.261(14)
HPQCD [15]	0.276(8)	-0.558(13)	0.448(16)	0.426(30)	0.272(21)
JLQCD [16]	0.248(8)	-0.508(11)	0.398(16)	0.561(29)	0.220(21)
Average [14]-[16] (PDG scale factor)	0.266(9) (2.0)	-0.529(11) (2.1)	0.420(11) (1.6)	0.471(36) (2.6)	0.254(14) (1.3)
Combined [14]-[16]	0.262(5)	-0.525(5)	0.436(8)	0.468(14)	0.253(10)
Experimental value	0.284(12) [32]	$-0.38 \pm 0.51^{+0.21}_{-0.16}$ [37]	0.49(8) [34, 35]	0.523(8) [13, 36]	0.231(17) [13, 36]

We have an analogous pattern: either we reproduce $R(D^*)$ but observe a tension with new F_L^ℓ and $A_{\rm FB}^\ell$ data (HPQCD) or viceversa (JLQCD)!

2310.03680
Martinelli, Simula, Vittorio
Fedele et al. 2305.15457

EXCLUSIVE V_{ub} $B \to \pi \ell \nu$





HFLAV

$$|V_{ub}| = (3.70 \pm 0.10\,(\mathrm{exp}) \pm 0.12\,(\mathrm{theo})) \times 10^{-3} \quad (\mathrm{data} + \mathrm{LQCD}),$$
$$|V_{ub}| = (3.67 \pm 0.09\,(\mathrm{exp}) \pm 0.12\,(\mathrm{theo})) \times 10^{-3} \quad (\mathrm{data} + \mathrm{LQCD} + \mathrm{LCSR}),$$

- ***New LCSR results*** (1811.00983) have been included for the first time in global fits to lattice and experimental data on $B \to \pi \ell \nu$ in 2103.01809 and 2102.07233, leading to $|V_{ub}| = 3.77(15)10^{-3}$ and $|V_{ub}| = 3.88(13)10^{-3}$. The latter removes outliers and is within 1σ from most recent inclusive results.
- HFLAV adopts a 2stage procedure, first making averages at different q^2 (low p) and fitting to extract V_{ub}

$B \to \pi$ form factors

recent JLQCD FF calculation 2203.04938
$|V_{ub}| = 3.93(41)\, 10^{-3}$

Small impact on $|V_{ub}|$ after including experimental data (information at small q^2 / large z)

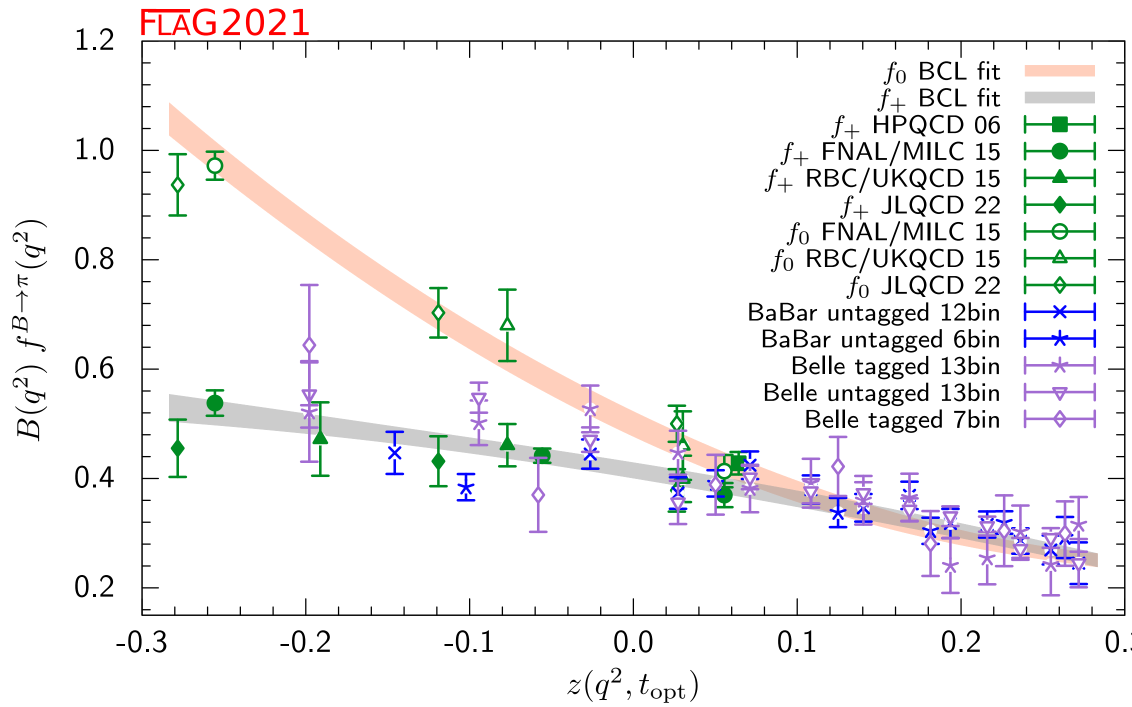
FLAG5 Web update

http://flag.unibe.ch/2021/Media?action=AttachFile&do=get&target=FLAG_2023_webupdate.pdf

$B \to \pi \ell \nu$ ($N_f = 2+1$)

	Central Values	Correlation Matrix					
$\lvert V_{ub}\rvert \times 10^3$	3.64 (16)	1	-0.812	-0.107	0.127	-0.325	-0.151
a_0^+	0.425 (15)	-0.812	1	-0.189	-0.308	0.409	0.00937
a_1^+	-0.443 (39)	-0.107	-0.189	1	-0.499	-0.0345	0.150
a_2^+	-0.51 (13)	0.127	-0.308	-0.499	1	-0.189	0.128
a_0^0	0.560 (17)	-0.325	0.409	-0.0345	-0.189	1	-0.772
a_1^0	-1.346 (53)	-0.151	0.00937	0.150	0.128	-0.772	1

χ^2/dof $= 116.6/62$: error rescaled by $\sqrt{\chi^2/\text{dof}} = 1.37$

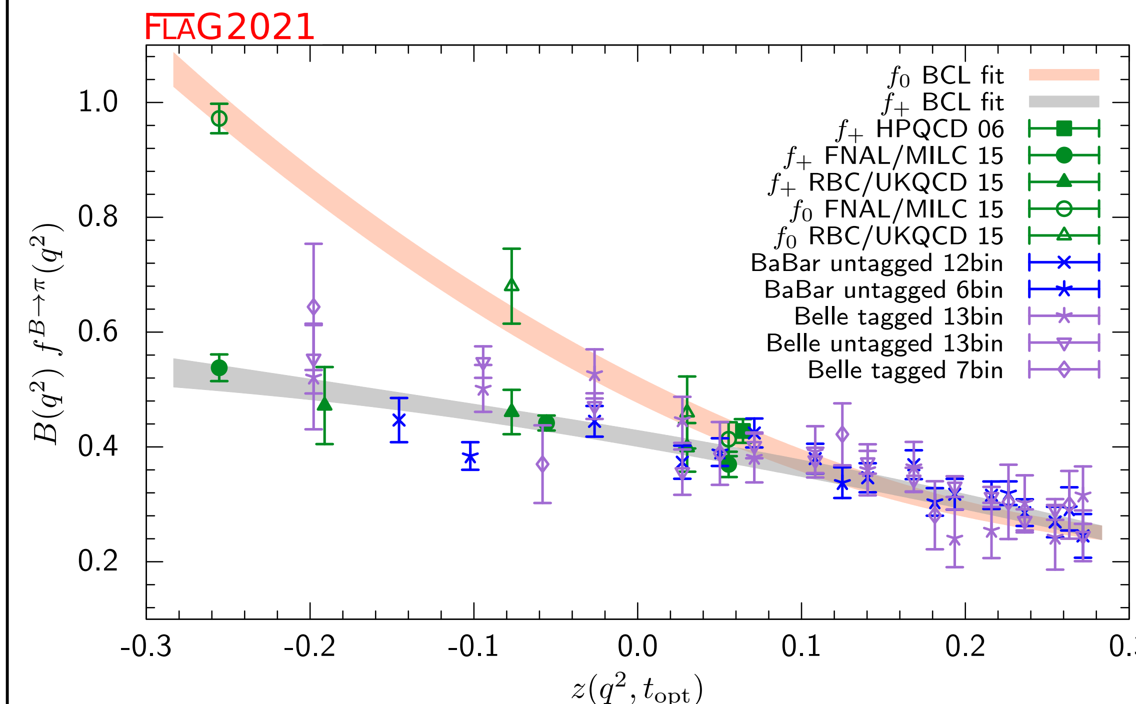


FLAG5 (2111.09849)

$B \to \pi \ell \nu$ ($N_f = 2+1$)

	Central Values	Correlation Matrix					
$\lvert V_{ub}\rvert \times 10^3$	3.74 (17)	1	−0.851	−0.349	0.375	−0.211	−0.246
a_0^+	0.415 (14)	−0.851	1	0.155	−0.454	0.260	0.144
a_1^+	−0.488 (53)	−0.349	0.155	1	−0.802	−0.0962	0.220
a_2^+	−0.31 (18)	0.375	−0.454	−0.802	1	0.0131	−0.100
a_0^0	0.500 (23)	−0.211	0.260	−0.0962	0.0131	1	−0.453
a_1^0	−1.424 (54)	−0.246	0.144	0.220	−0.100	−0.453	1

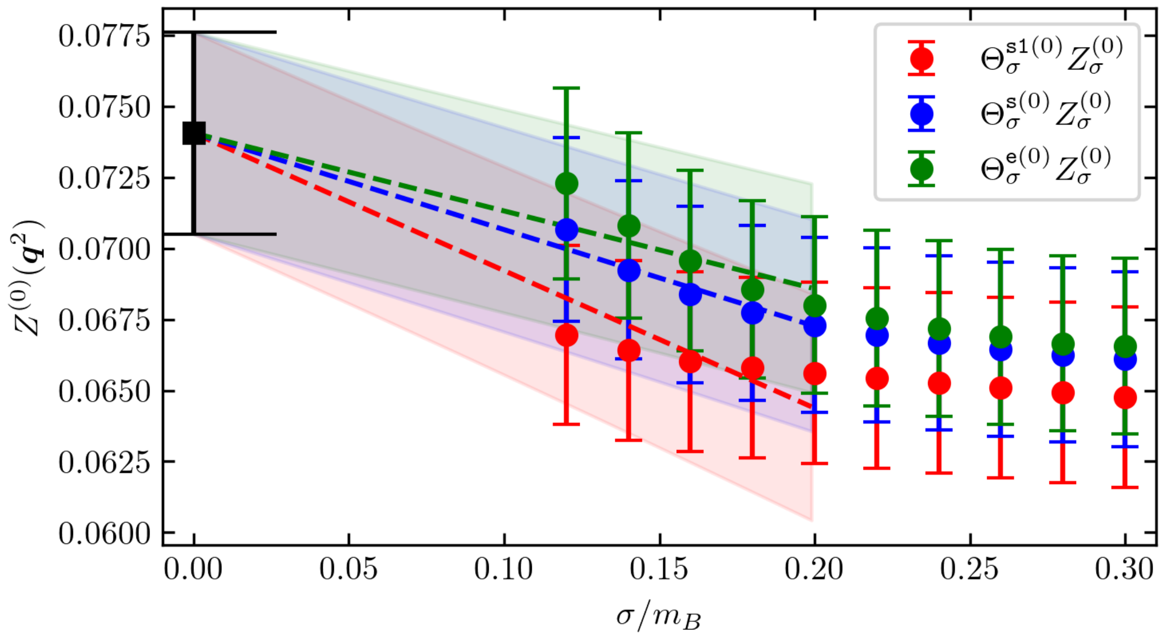
χ^2/dof $= 78.7/56 = 1.41$: error rescale by $\sqrt{\chi^2/\text{dof}} = 1.19$

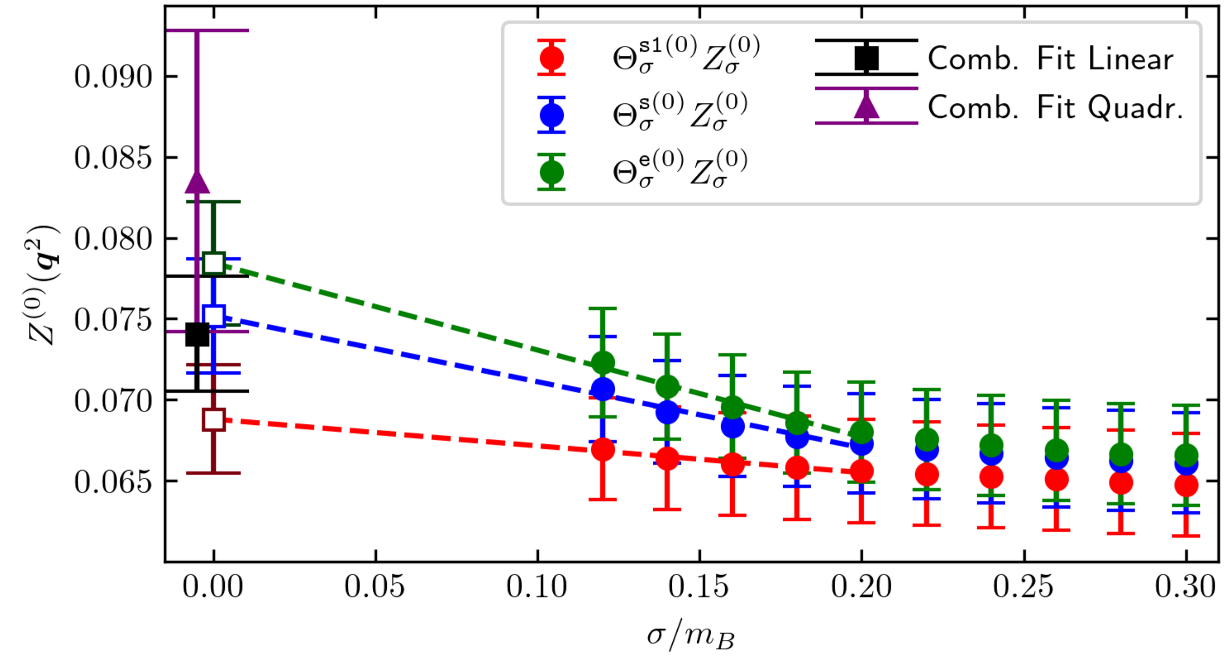


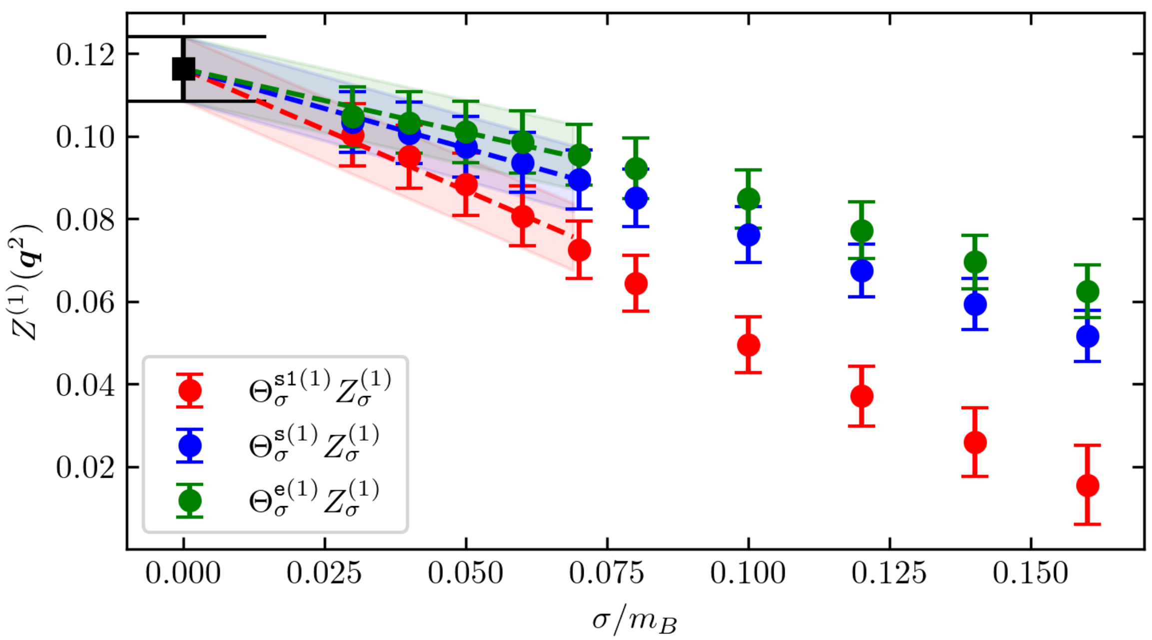
SUMMARY AND OUTLOOK

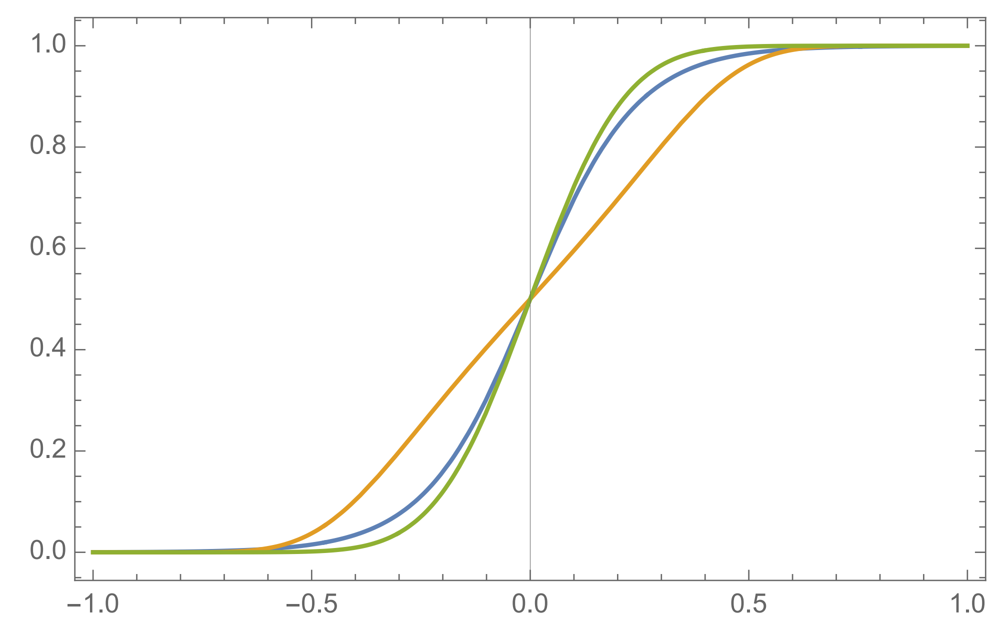
- *Inclusive $b \to c$ seems OK: q^2 moments consistent with leptonic and hadronic ones, perturbation theory generally OK; higher powers appear small. But don't dream of going below 1%…*
- *Calculations of inclusive semileptonic meson decays on the lattice have started. Precision to be seen, but you can count they will, at some point, contribute.*
- *Inclusive V_{ub} converging towards exclusive V_{ub} and waiting for more data*
- *Exclusive V_{cb}: parametrisations and related uncertainties require great care. Uncertainties were underestimated. Consensus that BGL is the most appropriate framework for fits. Ongoing discussions on how exactly use it.*
- *Lattice $B \to D^*$ form factors: situation still unclear, 2 calculations in tension with exp and HQE. Don't underestimate their difficulty.*
- *Many new ideas on how to improve the exp analyses and reduce/control errors: I bet some cloud will soon disappear.*

in detail









ETMC at $|\mathbf{q}| = 0.5\mathrm{GeV}$

Using different approx to the kernel
improves the $\boldsymbol{\sigma} \to 0$ extrapolation
Interplay with continuum and infinite volume limits

D'AGOSTINI BIAS

Standard χ^2 fits
sometimes lead
to paradoxical results

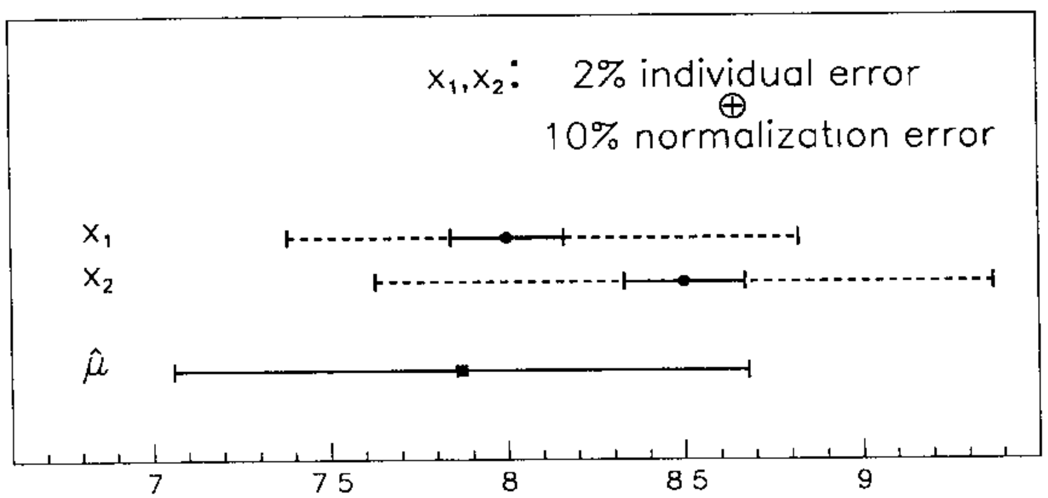


Fig. 1. Best estimate of the true value from two correlated data points, using in the χ^2 the empirical covariance matrix of the meaurements. The error bars show individual and total errors.

$$\hat{k} = \frac{x_1\sigma_2^2 + x_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2 + (x_1 - x_2)^2\sigma_f^2},$$

Many exp systematics are highly correlated. Bias is stronger with more bins

On the use of the covariance matrix to fit correlated data

G. D'Agostini

Dipartimento di Fisica, Università "La Sapienza" and INFN, Roma, Italy

(Received 10 December 1993; revised form received 18 February 1994)

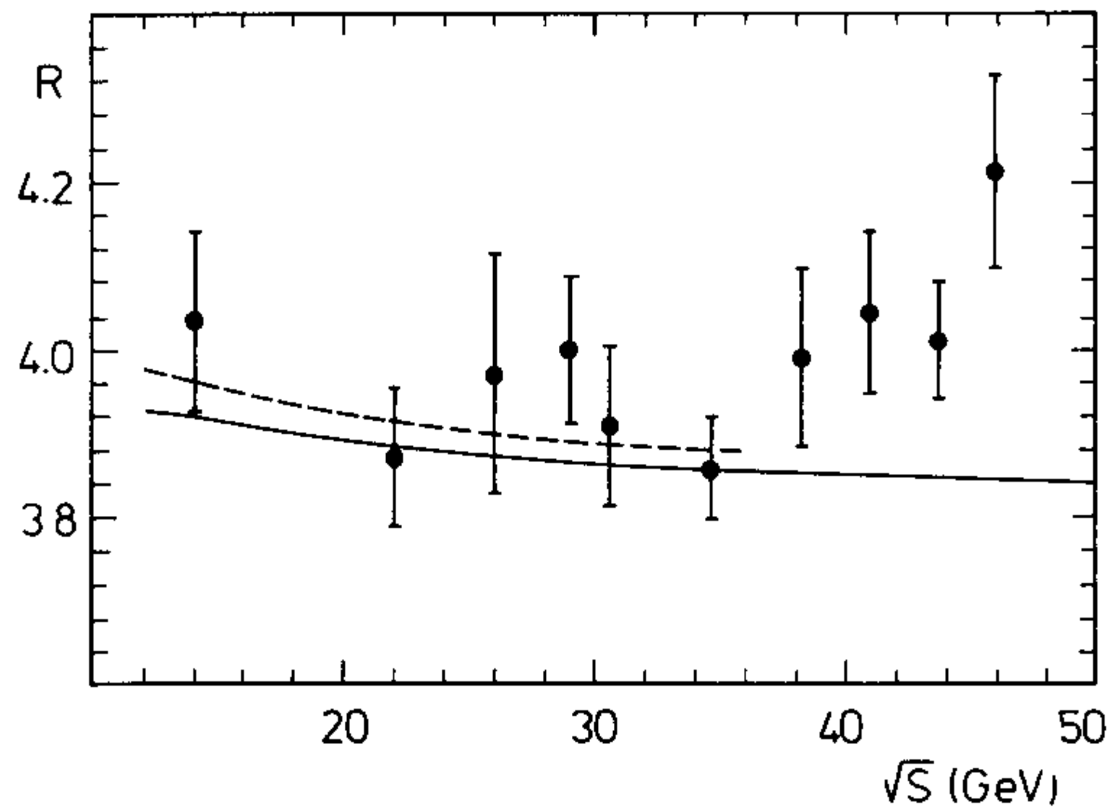
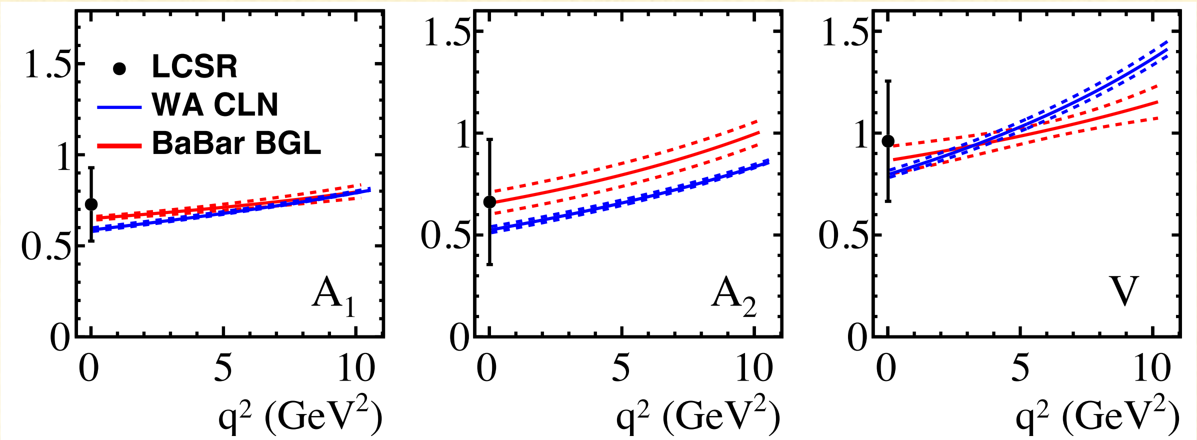


Fig. 2. R measurements from PETRA and PEP experiments with the best fits of QED + QCD to all the data (full line) and only below 36 GeV (dashed line). All data points are correlated (see text).

RESULTS BY BABAR AND LHCb

1903.10002, 2001.03225

Reanalysis of tagged B^0 and B^+ data, unbinned 4 dimensional fit with simplified BGL and CLN
About 6000 events
No data provided yet



No clear BGL$^{(111)}$/CLN difference but disagreement with HFLAV CLN ffs

V_{cb}=0.0384(9)

Measurement of $|V_{cb}|$ with $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ decays

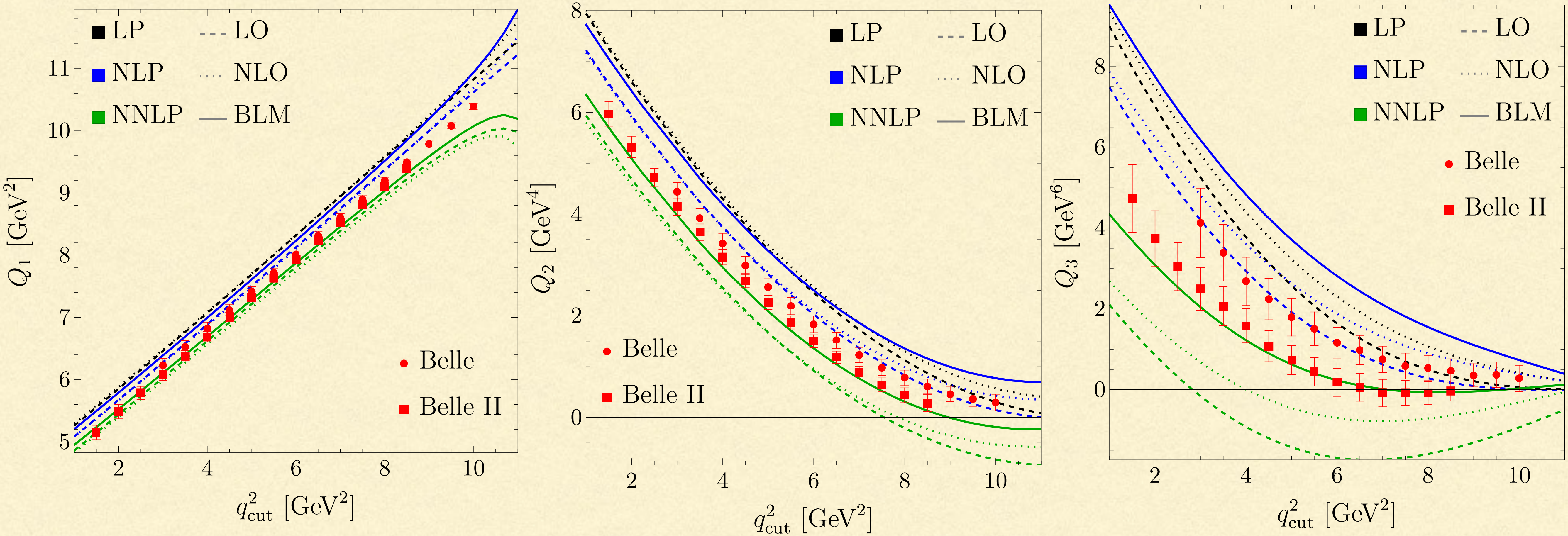
$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)}{\mathcal{B}(B^0 \to D^-\mu^+\nu_\mu)},$$
$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \to D_s^{*-}\mu^+\nu_\mu)}{\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)}$$

V_{cb}=0.0414(16) CLN
V_{cb}=0.0423(17) BGL$^{(222)}$

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL$^{(222)}$

$O(\alpha_s^2\beta_0)$ CORRECTIONS TO q^2 MOMENTS

Finauri, PG 2310.20324

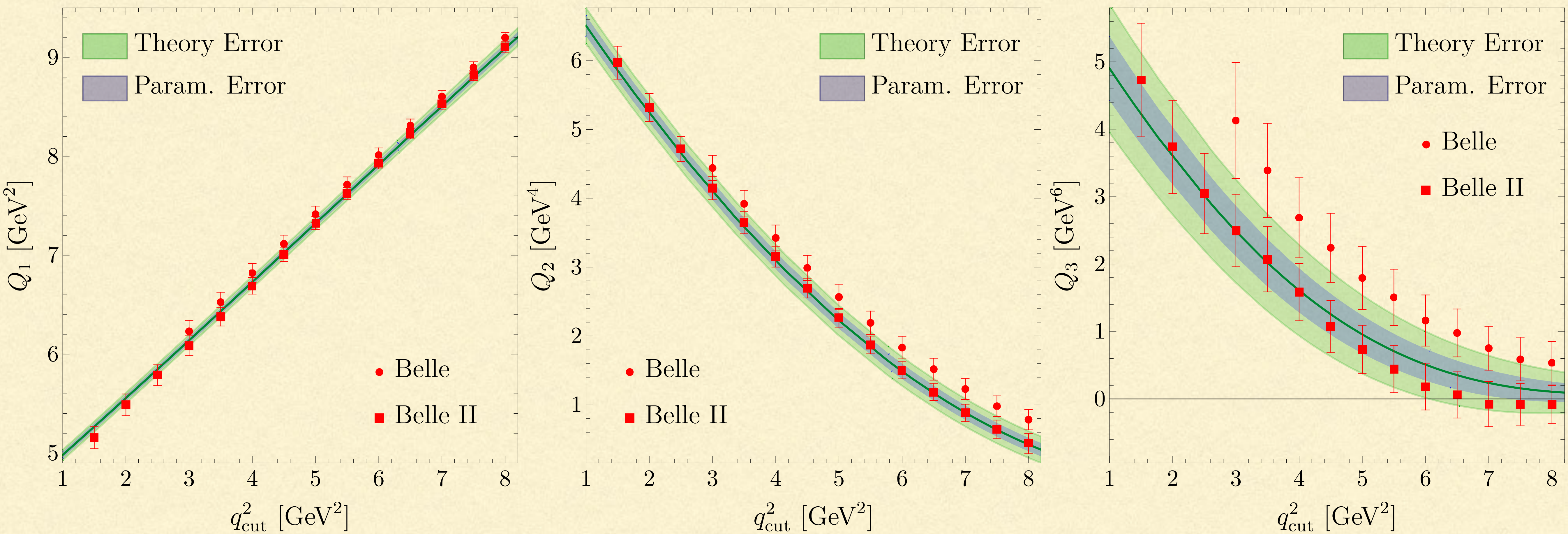


sizeable for 2nd and 3rd moments

Belle and Belle II moments differ by $\sim 2\sigma$

New $O(\alpha_s^2)$ calculation Fael and Herren 2403.03976

MINOR TENSIONS IN HIGHER q^2 MOMENTS



HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$ Mannel,Turczyk,Uraltsev 1009.4622

Lowest Lying State Saturation Approx (LLSA) truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen,Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\,\mu_G^2 \qquad \epsilon \sim 0.4\text{GeV}$$

$\boldsymbol{\epsilon}$ excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

still without q^2 moments!

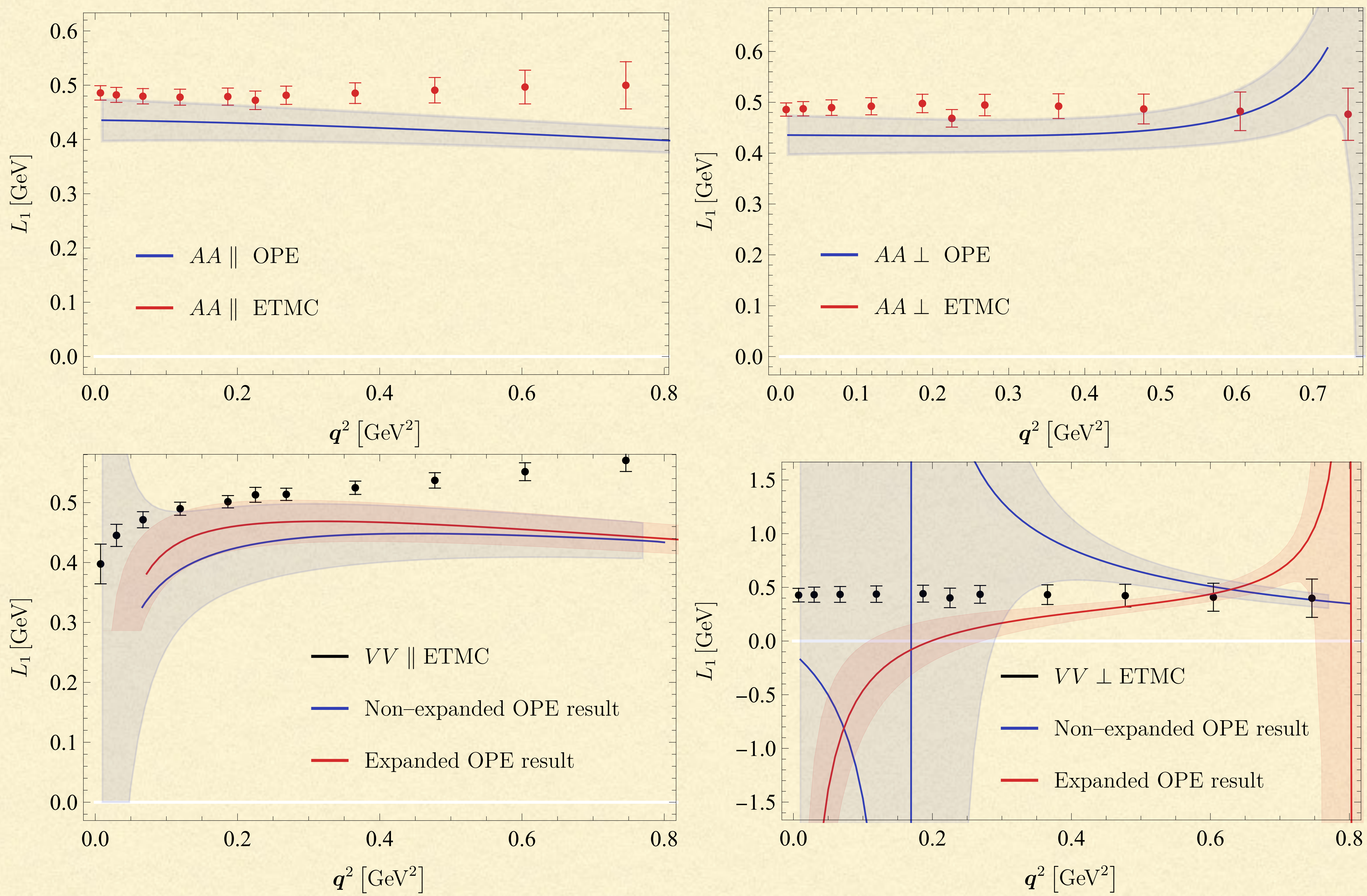
$$|V_{cb}| = 42.00(53) \times 10^{-3}$$

Bordone, Capdevila, PG, 2107.00604
Update of 1606.06174

MOMENTS

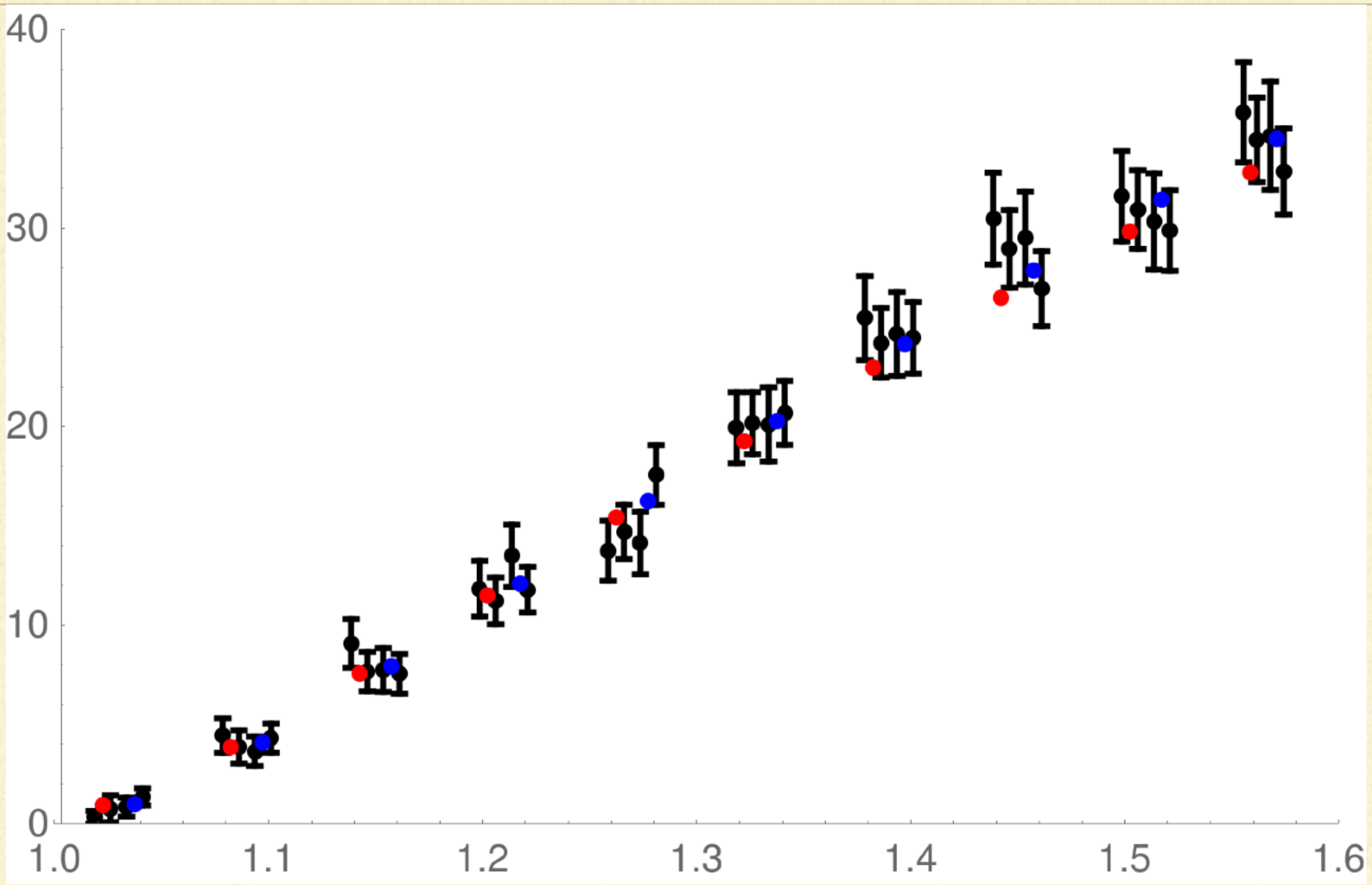
PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762

$$L_1 = \langle E_\ell(\mathbf{q}^2) \rangle$$



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting its parameters

w DISTRIBUTION for $B \to D\ell\nu$



Belle 2015 consider 4 channels ($B^{0,+}, e, \mu$) for each bin.
Average (red points) usually lower than all central values. ***D'Agostini bias?***
Blue points are average of normalised bins.

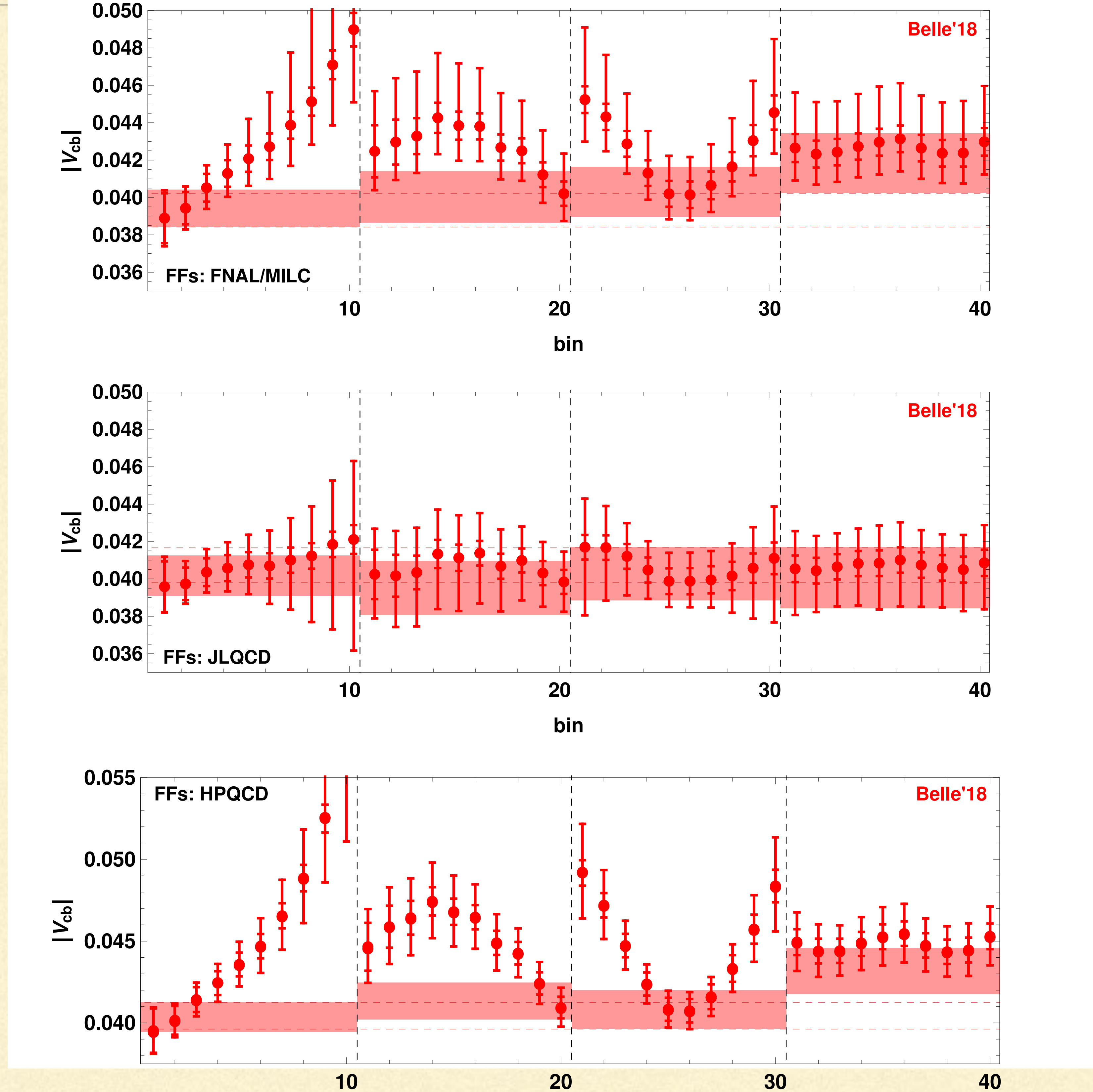
Standard fit to Belle15+FNAL+HPQCD: $|V_{cb}| = 40.9(1.2)\, 10^{-3}$ Jung, PG
Fit to normalised bins+width Belle15+FNAL+HPQCD: $|V_{cb}| = 41.9(1.2)\, 10^{-3}$

Binned V_{cb} from Belle'18 data: FNAL/MILC vs JLQCD

FNAL/MILC

JLQCD

HPQCD



Binned analysis proposed by Martinelli, Simula, Vittorio in DM approach
2105.08674
2109.15248

Extracting V_{cb} from each bin, FFs only determined by lattice QCD

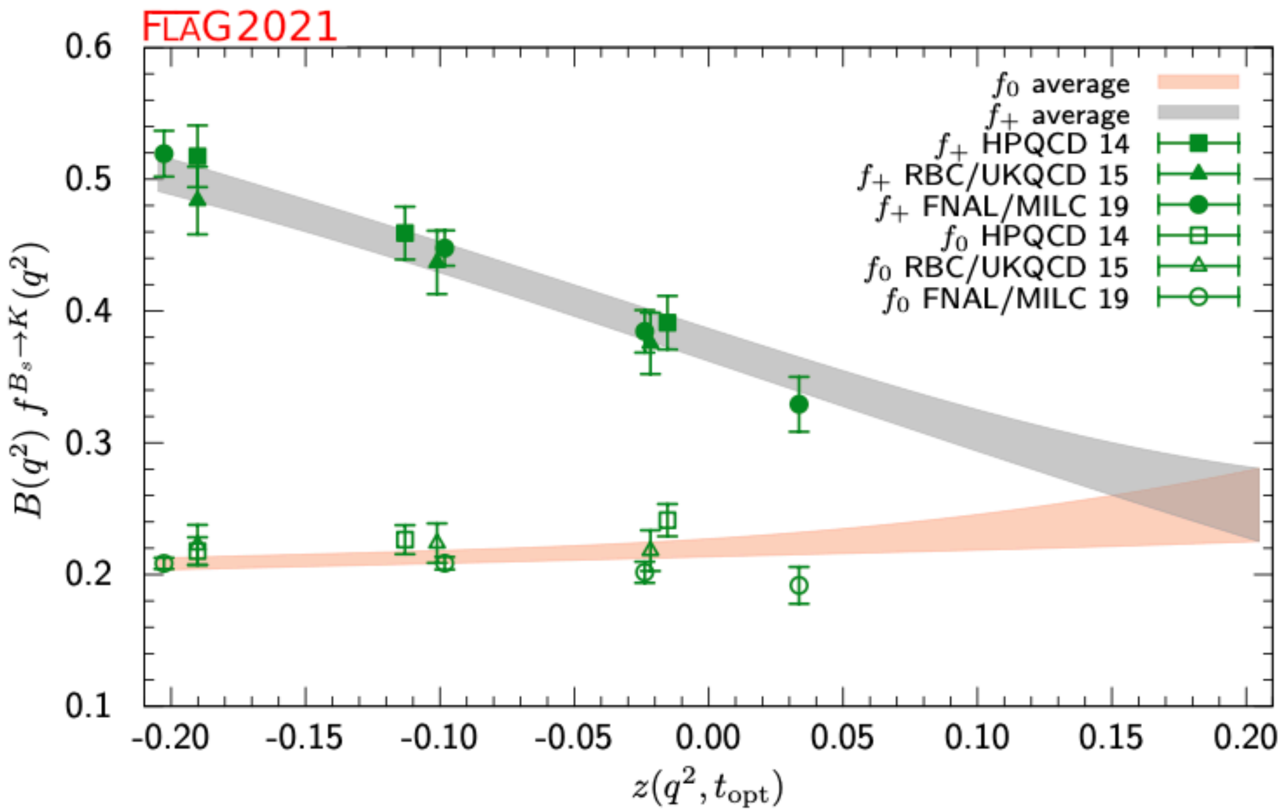
M. Jung

Global BGL fit to Belle18+FNAL+JLQCD+HPQCD data:

$|V_{cb}|=40.2(7)\ 10^{-3}$ ($\chi^2_{min} = 71.4$) *using only total rate* $|V_{cb}|=41.6(1.3)\ 10^{-3}$

$B_s \to K$ form factors and $|V_{ub}/V_{cb}|$

- FLAG5 combined form factors:



[LHCb 2012.05143]

$$\frac{1}{|V_{ub}|^2}\int_{q^2_{\rm min}=m_\mu^2}^{7\,{\rm GeV}^2} \frac{d\Gamma(B_s \to K^-\mu^+\nu_\mu)}{dq^2} = (2.26 \pm 0.38)\ {\rm ps}^{-1}$$

$$\frac{1}{|V_{ub}|^2}\int_{7\,{\rm GeV}^2}^{q^2_{\rm max}=(m_{B_s}-m_K)^2} \frac{d\Gamma(B_s \to K^-\mu^+\nu_\mu)}{dq^2} = (4.02 \pm 0.31)\ {\rm ps}^{-1}$$

$$\frac{|V_{ub}|}{|V_{cb}|}({\rm low}) = 0.0819 \pm 0.0072_{\rm lat.} \pm 0.0029_{\rm exp}$$

$$\frac{|V_{ub}|}{|V_{cb}|}({\rm high}) = 0.0860 \pm 0.0037_{\rm lat.} \pm 0.0038_{\rm exp}$$

$$\frac{|V_{ub}|}{|V_{cb}|}({\rm low}) = 0.061(4)$$ ***using LCSR***

Khodjamirian, Rusov

Note: RBC/UKQCD provides synthetic data points, HPQCD and FNAL/MILC only z-fit results

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.079(4)(4) \qquad (\Lambda_b \to p\mu\nu)$$ ***LHCb + Meinel et al***

My average of inclusive $\frac{|V_{ub}|}{|V_{cb}|} = 0.094(6)$ ***and exclusive*** $\frac{|V_{ub}|}{|V_{cb}|} = 0.094(4)$