Progress and future prospects from Lattice QCD

The Flavour Path to New Physics

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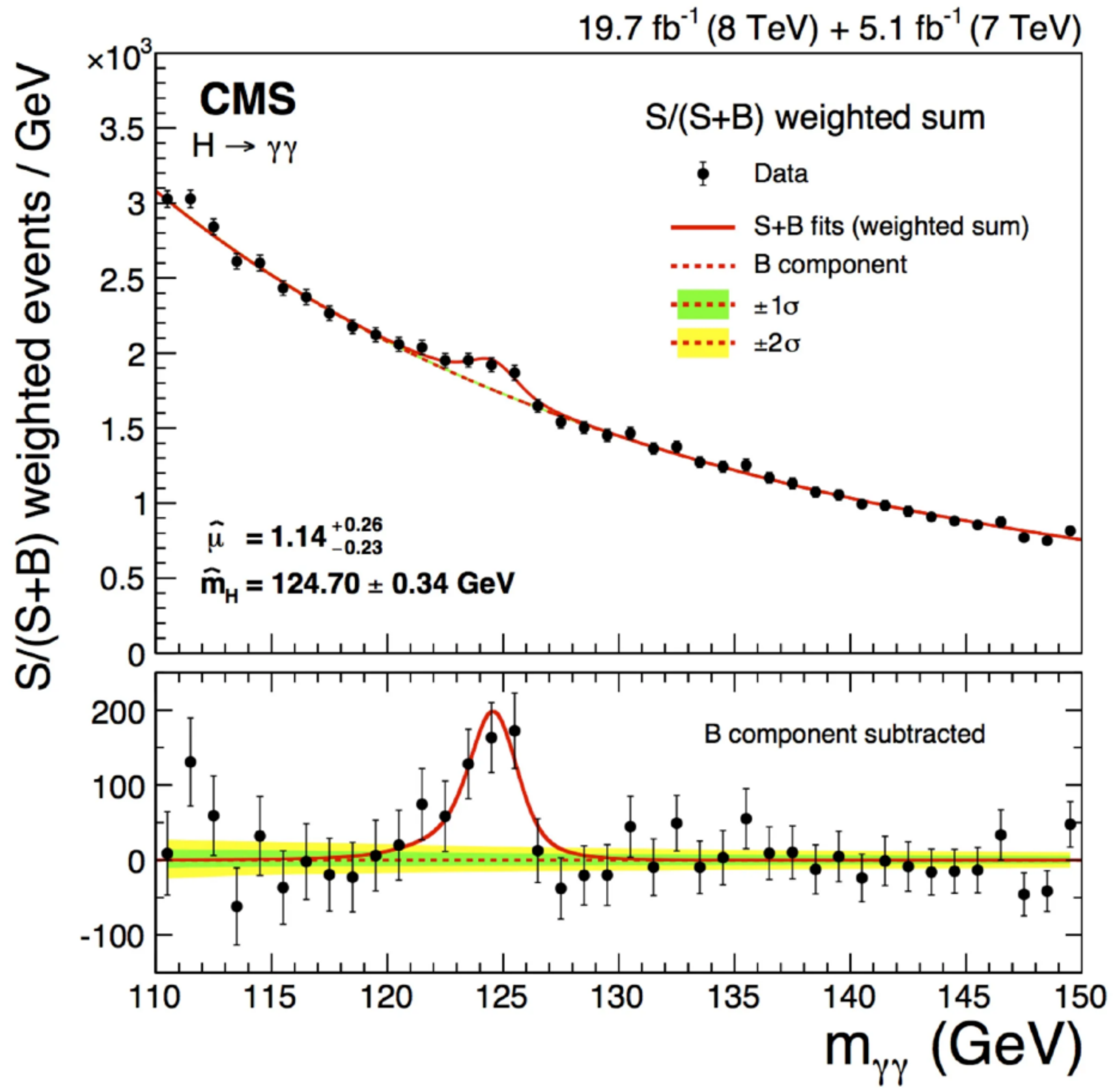
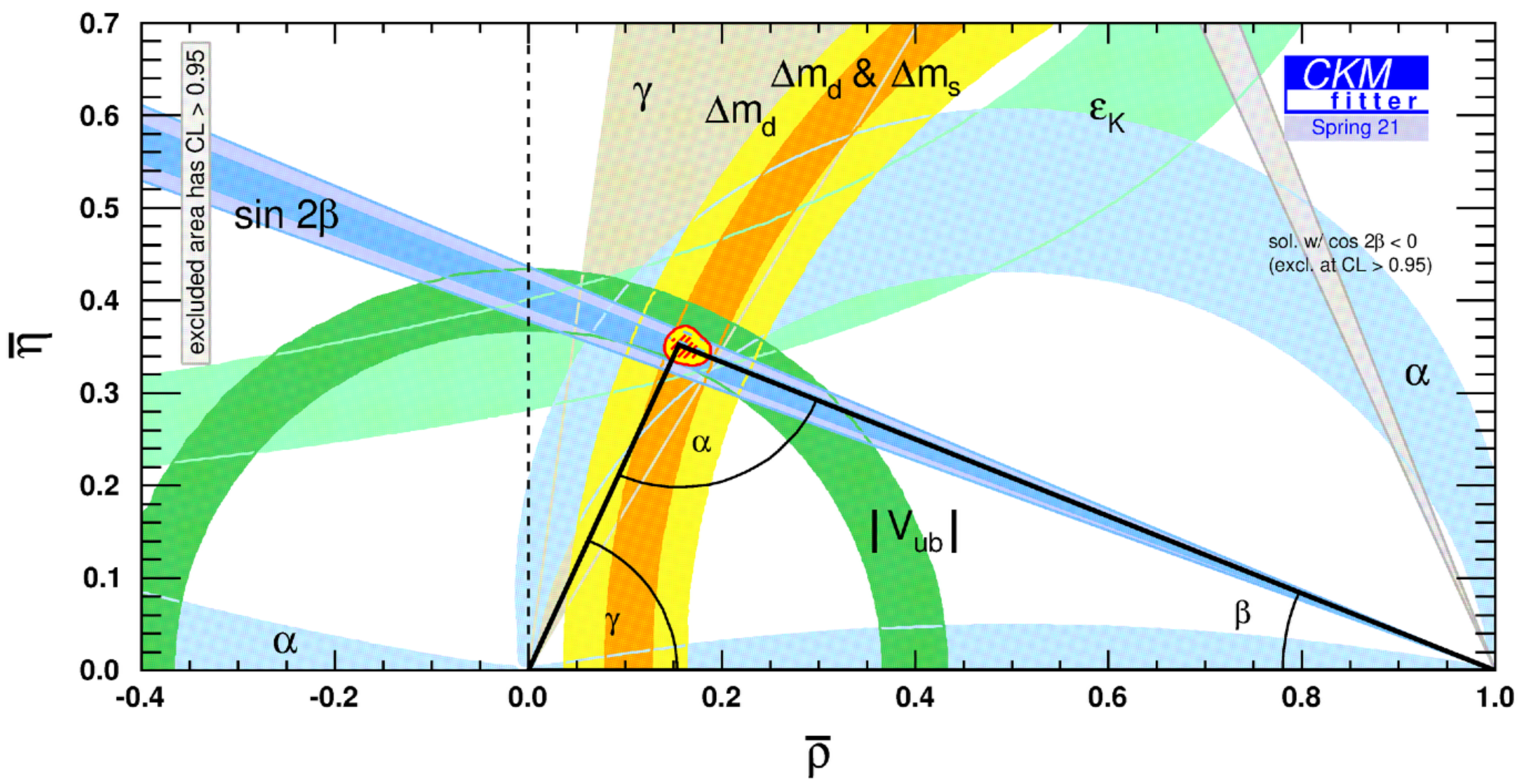
Testing the Standard Model

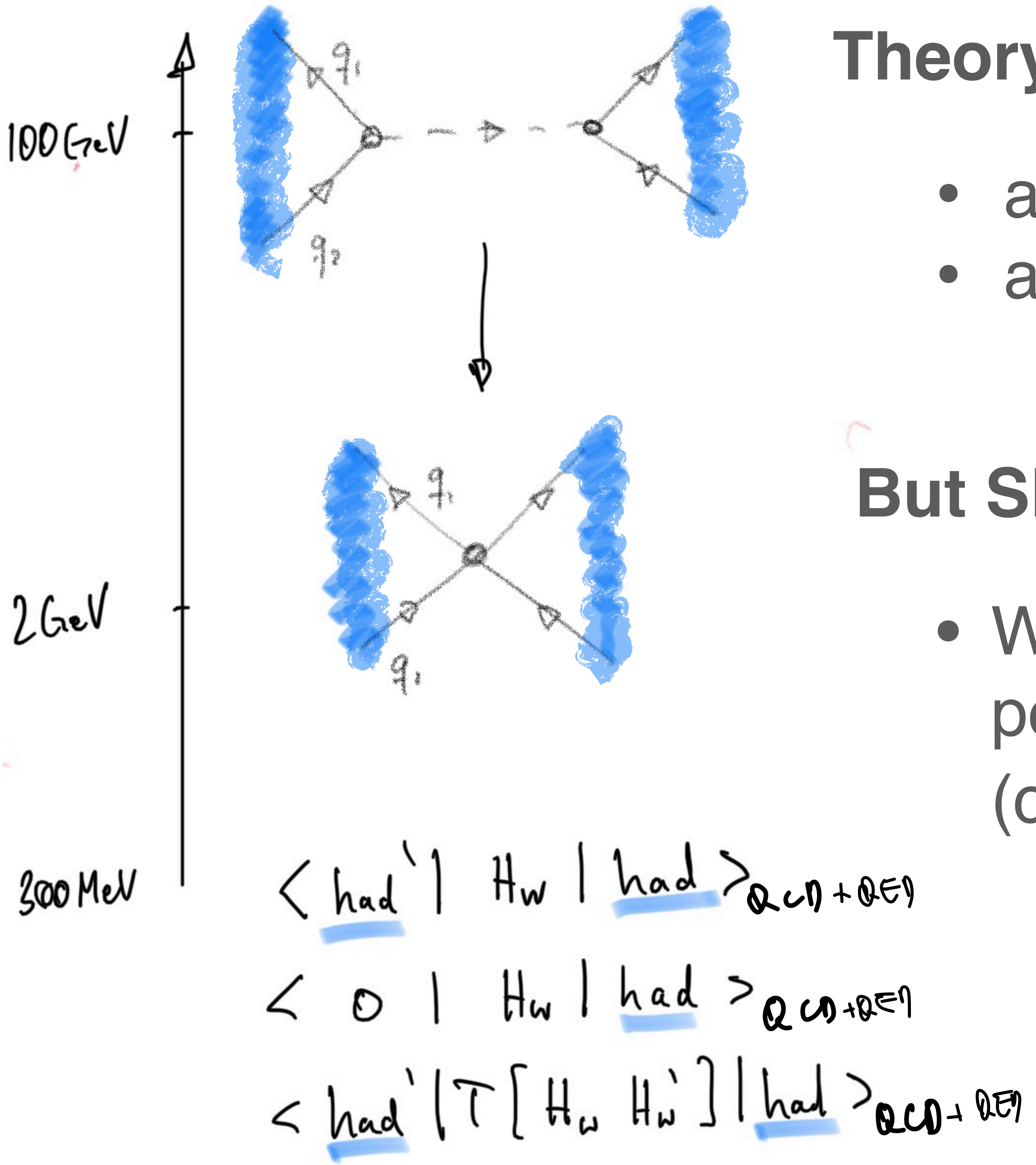
- searches for new physics
  - *direct searches* — 'bump in the spectrum'
  - *indirect searches* — SM provides relations between processes; we can therefore use experiment + theory to over-constrain SM





We will now look at Lattice QCD's role in *indirect searches*

SM theory



Theory can be hard:

- all three sectors of SM contribute
- accuracy important

But SM helps us a bit:

- Weak gauge bosons so heavy that we can replace them by point-interaction described by an Effective Hamiltonian H_W (conveniently we thereby *get rid* of a very high energy scale)

Theory predictions require computations in weak eff. theory, QCD and QED

Status with “good control” — FLAG

FLAG 21 2111.09849

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
m_{ud}[MeV]	3.1.4	3.410(43)	[6, 7]	3.381(40)	[8–12]		
m_s[MeV]	3.1.4	93.43(70)	[6, 7, 13, 14]	92.2(1.1)	[8–11, 15]		
m_s/m_{ud}	3.1.5	27.250(64)	[illegible]	[illegible]	[illegible]		
m_u[MeV]	3.1.6	2.14(8)	[illegible]	[illegible]	[illegible]		
m_d[MeV]	3.1.6	4.70(5)	[illegible]	[illegible]	[illegible]		
m_u/m_d	3.1.6	0.465(24)	[19, 21]	0.485(19)	[20]		
$\overline{m}_c$(3 GeV)[GeV]	3.2.2	0.988(11)	[6, 7, 14, 22, 23]	0.992(6)	[11, 24, 25]		
m_c/m_s	3.2.3	11.768(34)	[6, 7, 14]	11.82(16)	[24, 26]		
$\overline{m}_b(\overline{m}_b)$[GeV]	3.3	4.203(11)	[6, 27–30]	4.164(23)	[11]		
$f_+(0)$	4.3	0.9698(17)	[illegible]	[illegible]	[illegible]	0.9560(57)(62)	[35]
$f_{K^\pm}/f_{\pi^\pm}$	4.3	1.1932(21)	[illegible]	[illegible]	[illegible]	1.205(18)	[44]
$f_{\pi^\pm}$[MeV]	4.6		[illegible]	[illegible]	[illegible]		
$f_{K^\pm}$[MeV]	4.6	155.7(3)	[illegible]	[illegible]	[illegible]	157.5(2.4)	[44]
$\Sigma^{1/3}$[MeV]	5.2.4	286(23)	[45, 46]	272(5)	[12, 47–51]	266(10)	[45, 52–54]
F_π/F	5.2.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[52–54, 57]
$\bar{\ell}_3$	5.2.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[52, 53, 57]
$\bar{\ell}_4$	5.2.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[52, 53, 57, 58]
$\bar{\ell}_6$	5.2.4					15.1(1.2)	[53, 57]
$\hat{B}_K$	6.3	0.717(18)(16)	[59]	0.7625(97)	[8, 60–62]	0.727(22)(12)	[63]
B_2	6.4	0.46(1)(3)	[illegible]	[illegible]	[illegible]	0.47(2)(1)	[63]
B_3	6.4	0.79(2)(5)	[illegible]	[illegible]	[illegible]	0.78(4)(2)	[63]
B_4	6.4	0.78(2)(4)	[illegible]	[illegible]	[illegible]	0.76(2)(2)	[63]
B_5	6.4	0.49(3)(3)	[59]	0.720(38)	[62, 64]	0.58(2)(2)	[63]

quark masses

kaon decay

low-energy constants

kaon mixing

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
f_D[MeV]	7.1	212.0(7)	[17, 37]	209.0(2.4)	[65–67]	208(7)	[68]
f_{D_s}[MeV]	7.1	249.9(5)	[illegible]	[illegible]	[illegible]	246(4)	[68, 70]
$\frac{f_{D_s}}{f_D}$	7.1	[illegible]	[illegible]	[illegible]	[illegible]	1.20(2)	[68]
$f_+^{D\pi}(0)$	7.2	0.612(35)	[illegible]	[illegible]	[illegible]		
$f_+^{DK}(0)$	7.2	0.7385(44)	[71]	0.747(19)	[73]		
f_B[MeV]	8.1	[illegible]	[illegible]	[illegible]	[illegible]	188(7)	[68, 80]
f_{B_s}[MeV]	8.1	[illegible]	[illegible]	[illegible]	[illegible]	225.3(6.6)	[68, 70, 80]
$\frac{f_{B_s}}{f_B}$	8.1	1.209(5)	[illegible]	[illegible]	[illegible]	1.206(23)	[68, 80]
$f_{B_d}\sqrt{\hat{B}_{B_d}}$[MeV]	8.2			225(9)	[78, 82, 83]	216(10)	[68]
$f_{B_s}\sqrt{\hat{B}_{B_s}}$[MeV]	8.2			[illegible]	[illegible]	262(10)	[68]
$\hat{B}_{B_d}$	8.2			[illegible]	[illegible]	1.30(6)	[68]
$\hat{B}_{B_s}$	8.2			[illegible]	[illegible]	1.32(5)	[68]
ξ	8.2			1.206(17)	[78, 83]	1.225(31)	[68]
B_{B_s}/B_{B_d}	8.2			1.032(38)	[78, 83]	1.007(21)	[68]
Quantity	Sec.	$N_f = 2+1$ and $N_f = 2+1+1$			Refs.		
$\alpha^{(5)}_{\overline{\rm MS}}(M_Z)$	9.11	[illegible]			[illegible]		
$\Lambda^{(5)}_{\overline{\rm MS}}$[MeV]	9.11	[illegible]			[illegible]		
$\Lambda^{(4)}_{\overline{\rm MS}}$[MeV]	9.11	[illegible]			[illegible]		
$\Lambda^{(3)}_{\overline{\rm MS}}$[MeV]	9.11	339(12)			[11, 14, 25, 84–87]		

$D_{(s)}$-meson decay

$B_{(s)}$-meson decay

$B_{(s)}$-mixing

strong-coupling constant

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
g_A^{u-d}	10.3.1	1.246(28)	[88–90]	1.248(23)	[91, 92]		
g_S^{u-d}	10.3.2	1.02(10)	[88]				
g_T^{u-d}	10.3.3	0.989(34)	[88]				
g_A^u	10.4.1	0.777(25)(30)	[93]	0.847(18)(32)	[91]		
g_A^d	10.4.1	[illegible]	[illegible]	[illegible]	[illegible]		
g_A^s	10.4.1	[illegible]	[illegible]	[illegible]	[illegible]		
$\sigma_{\pi N}$[MeV]	10.4.4	64.9(1.5)(13.2)	[22]	39.7(3.6)	[94–96]	37(8)(6)	[97]
σ_s[MeV]	10.4.4	41.0(8.8)	[98]	52.9(7.0)	[94–96, 98, 99]		
g_T^u	10.4.5	0.784(28)(10)	[100]				
g_T^d	10.4.5	−0.204(11)(10)	[100]				
g_T^s	10.4.5	−0.0027(16)	[100]				

nucleon matrix elements

$D_{(s)}$ SL decays

$D \to \pi \ell \nu \qquad D \to K \ell \nu$

$B \to \pi \ell \nu \qquad B_s \to K \ell \nu \qquad R(D_s)$

$B_{(s,c)}$ SL decays

$B_{(s)} \to D_{(s)} \ell \nu \qquad B \to D^* \ell \nu \qquad R(D)$

$B_c \to (\eta_c, J/\psi) \ell \nu$

Λ_b SL decays

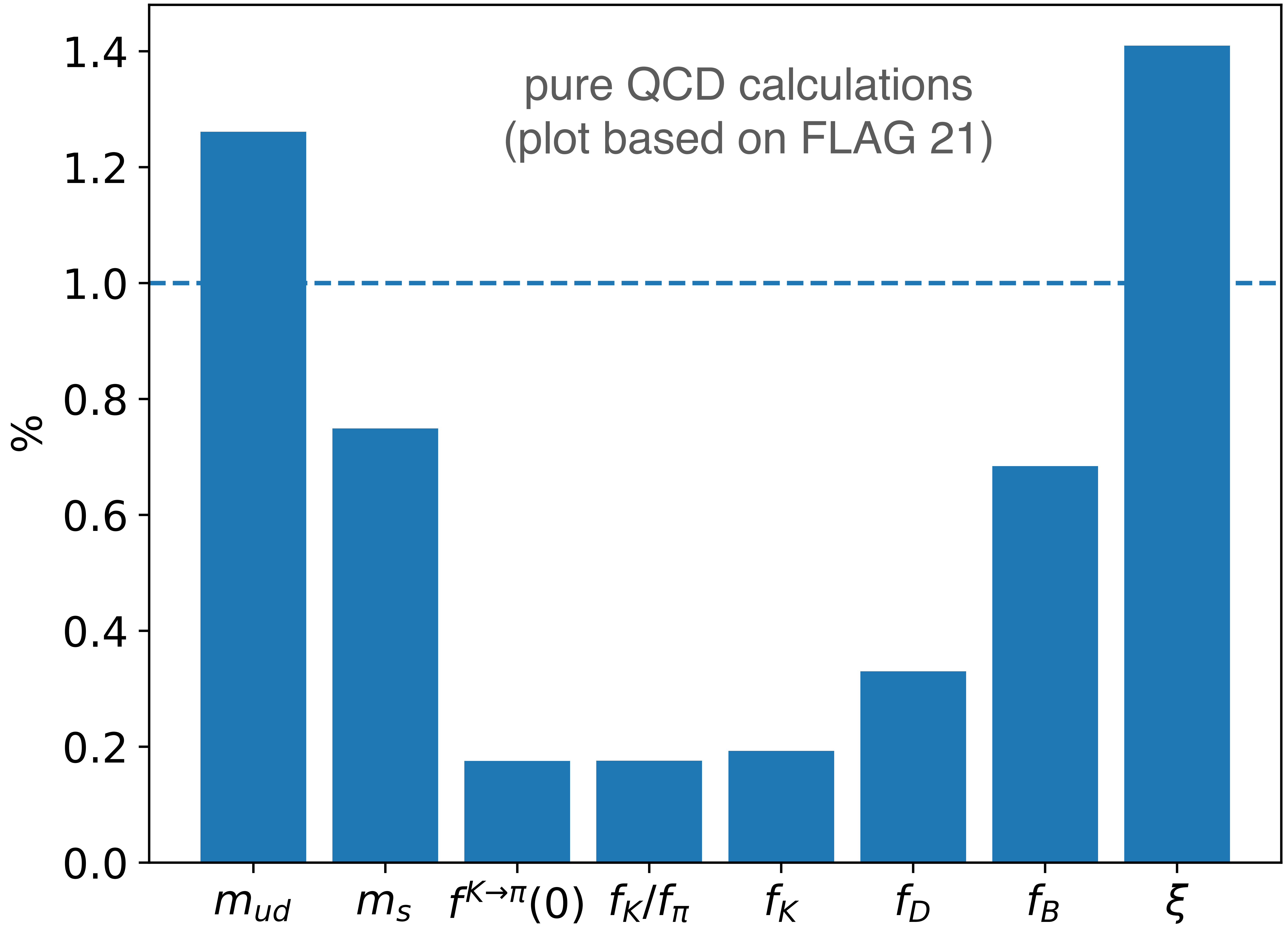
$\Lambda_b \to (p, \Lambda_c^{(*)}) \ell \bar{\nu} \qquad \Lambda_b \to \Lambda^{(*)} \ell \ell$

1st, 2nd row CKM ME

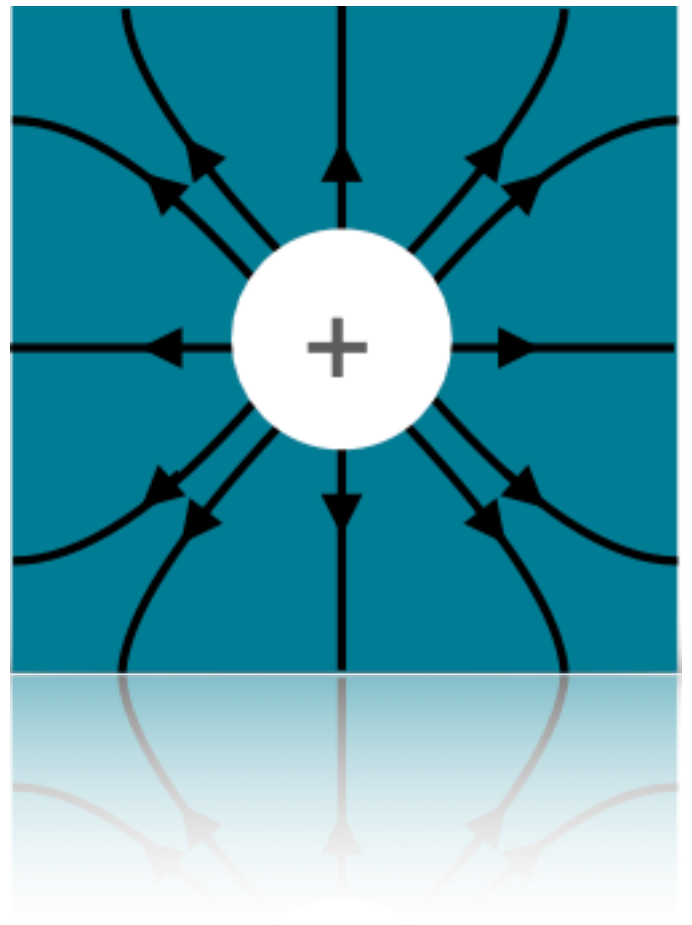
$|V_{ud}|, |V_{us}|, |V_{ub}|, |V_{cd}|, |V_{cs}|, |V_{cb}|$

What to do if precision too high?

Current accuracy on some quantities



QCD+QED+strong isospin breaking

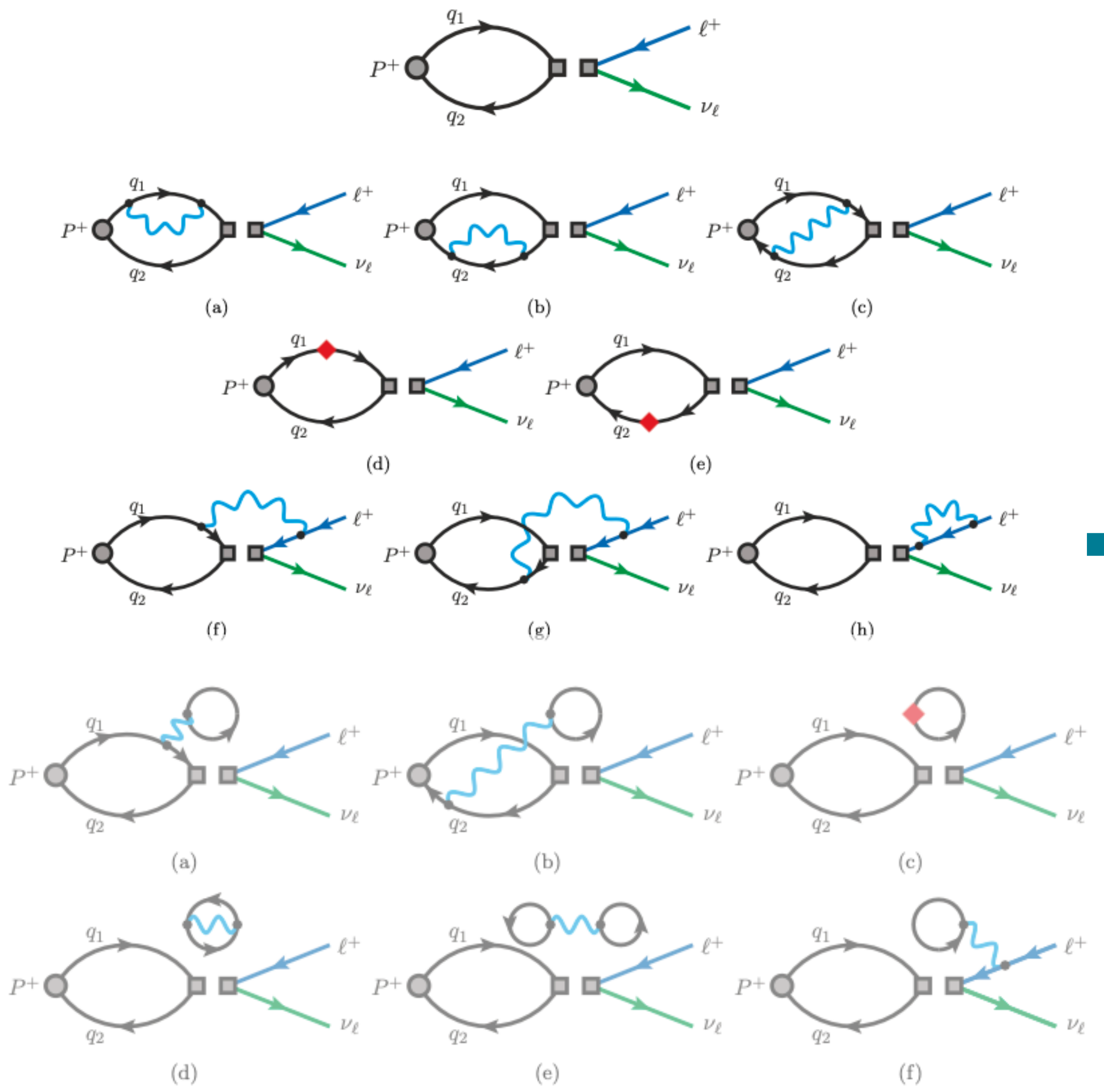


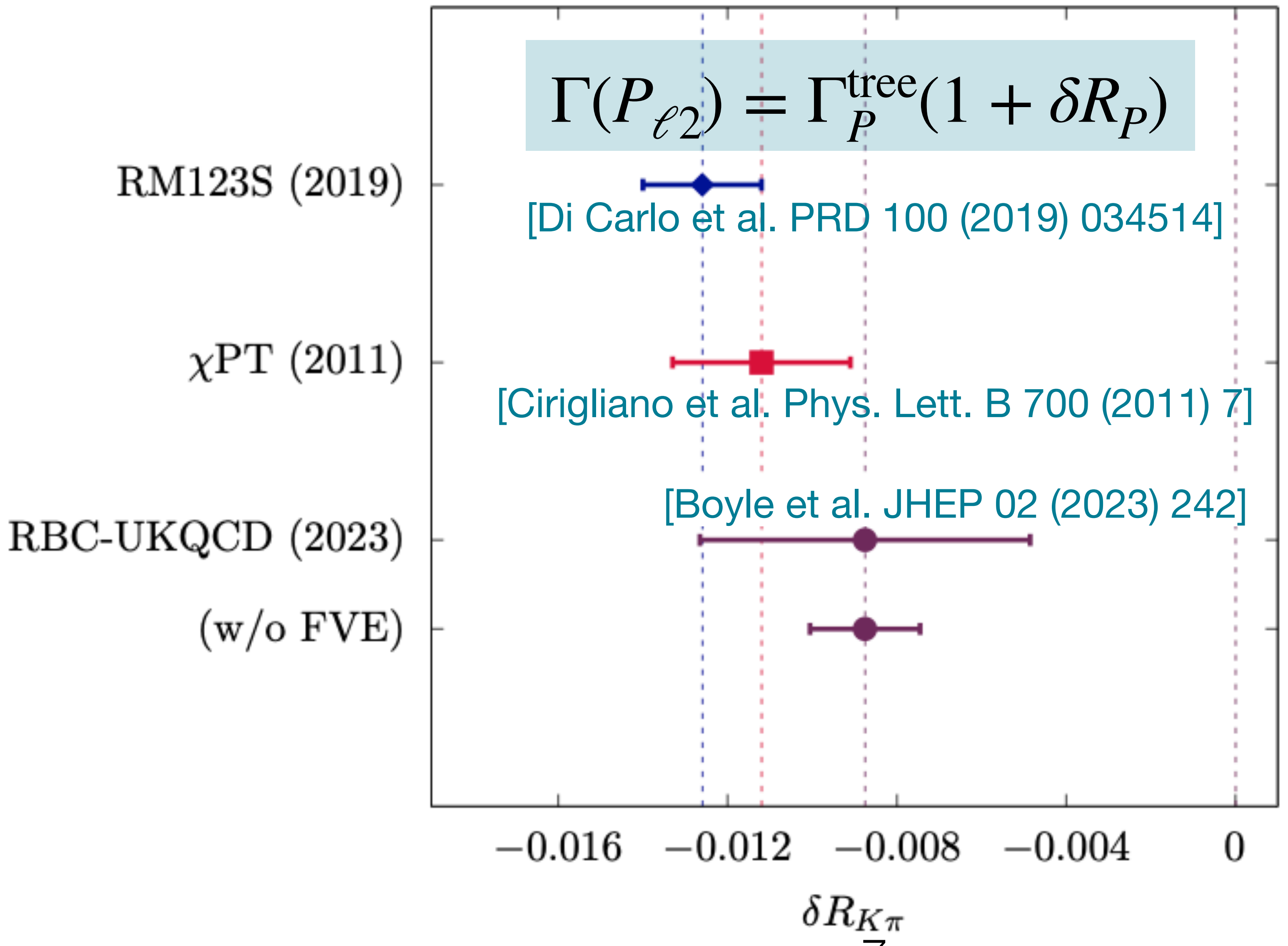
quantities: $(g-2)_\mu$, hadron spectra (e.g. $M_n - M_p$), decay constants

theoretically challenging: formulations: QED_{TL}, QED_L, QED_m, QED_{C*}, QED_∞, …
[Duncan, Eichten, Thacker, PRL 76 (1996)], [Hayakawa, Uni PTP 120 (2008)]
[Endres et al., PRL 117 (2016)], [Lucini et al. JHEP 02 (2016)]
[Feng et al. PRD 100 (2019), PRD 108 (2023)]

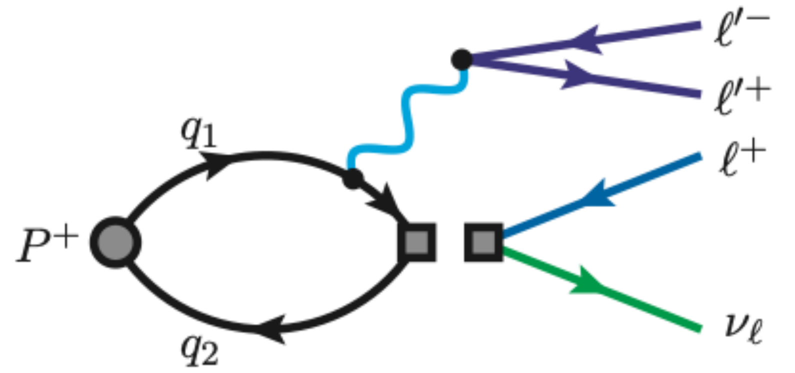
Decay constant:

see M. Di Carlo's Lattice 23 plenary [arXiv:2401.07666]

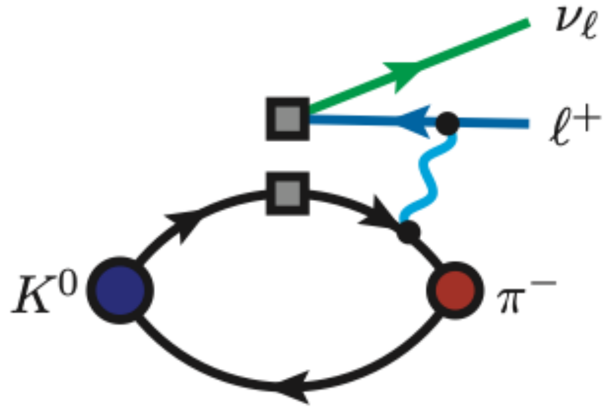




other quantities:



virtual photon emission
[Frezzotti et al. *PRD* 108 (2023) and arXiv:2403.05404]



semileptonic decay
[Christ et al., arXiv:2402.08915]

Leaving precision behind for a moment

two examples for new directions

higher-order electro-weak ME

$$\langle f \,|\, T\{\mathcal{O}_{\mathrm{EW},2}\,\mathcal{O}_{\mathrm{EW},1}\}\,|\, i\rangle$$

Exploration

- $K \to \pi\ell^+\ell^-$ [Isidori et al. PLB 633 (2006) 75-83] [RBC/UKQCD, e.g PRD 107 (2023) 1, L011503]
- $K \to \pi\nu\bar{\nu}$ [RBC/UKQCD, e.g. PRD 100 (2019) 11, 114506]
- $B \to \mu^+\mu^-\gamma$ [Frezzotti et al. arXiv:2402.03262]
- …

Minkowski-space amplitude:

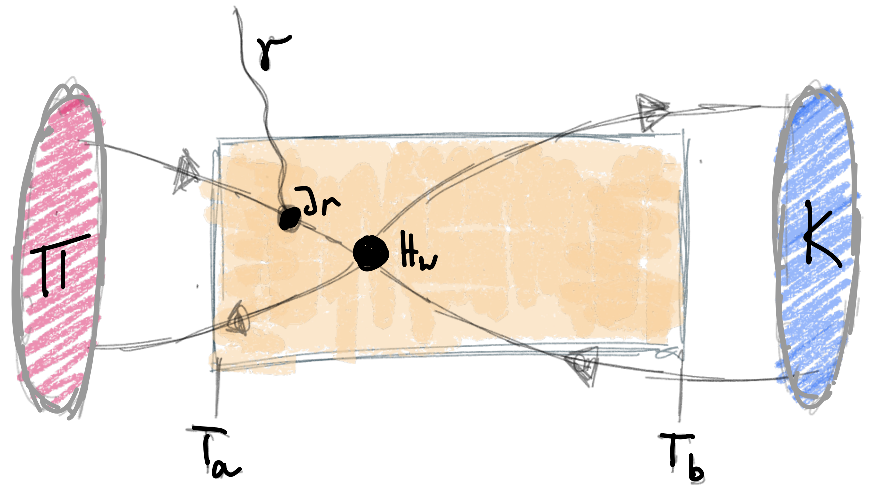
$$\mathcal{A}^c_\mu(q^2) = \int d^4x \langle \pi^c(p)|T\left[J_\mu(0)H_W(x)\right]|K^c(k)\rangle$$

complications arise when considering the amplitude in **Euclidean space …**

Expression in Euclidean space:

$$A^c_\mu(T_a, T_b, q^2) = \int_0^\infty dE \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E}\left(1 - e^{(E_K(\mathbf{k})-E)T_a}\right)$$

$$+ \int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E - E_\pi(\mathbf{p})}\left(1 - e^{-(E-E_\pi(\mathbf{k}))T_b}\right)$$



requires complex control of

- divergent terms,
- on-shell intermediate states
- renormalisation
- …

technically extremely challenging but we are learning how to do these calculations, and there is more to come!

unstable final sates

$$\langle f_1 f_2 \,|\, \mathcal{O}_{\mathrm{EW},1} \,|\, i \rangle$$

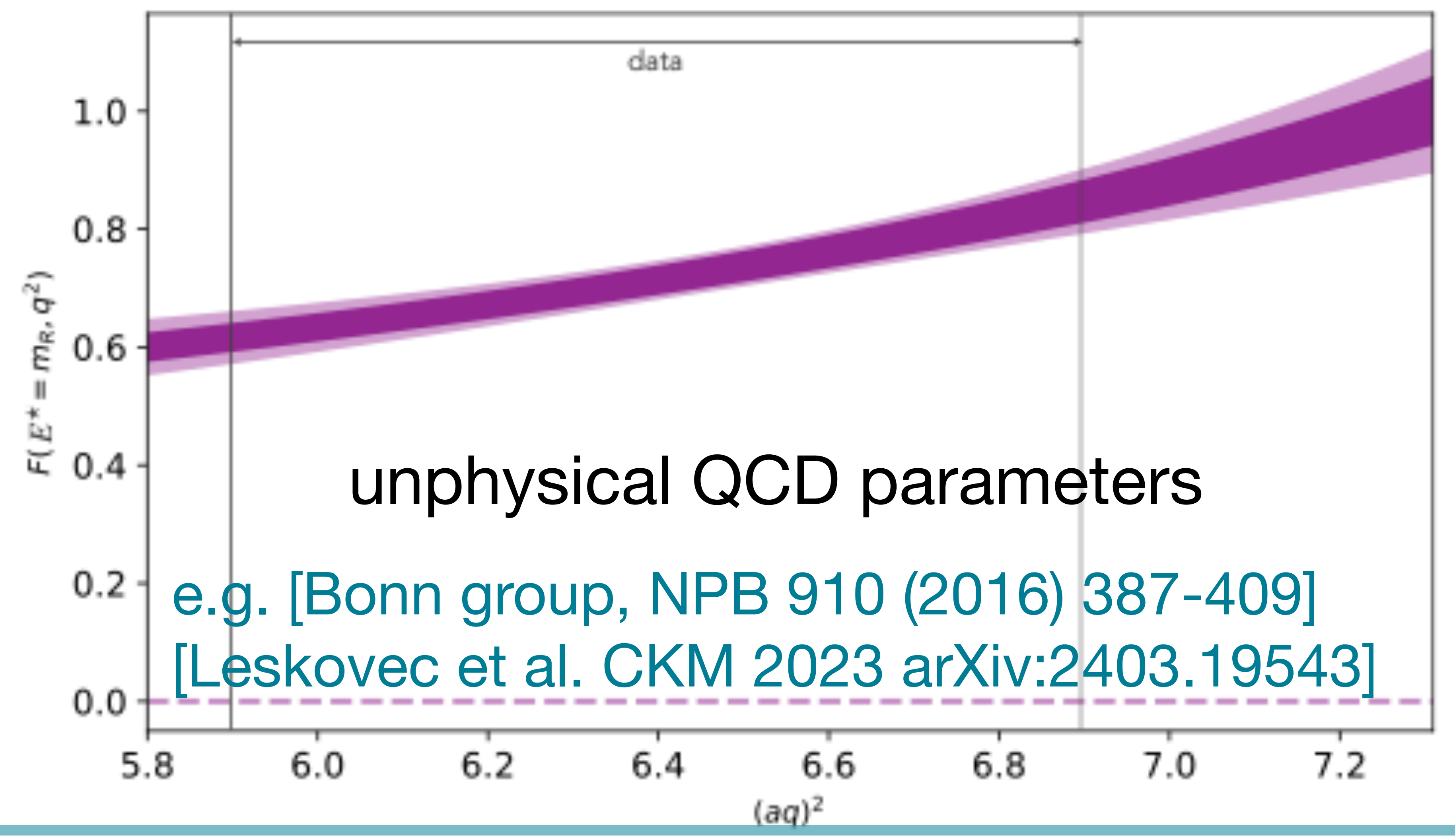
e.g.:

- $B \to \rho \ell \bar{\nu}_\ell \to \pi\pi \ell \bar{\nu}_\ell$
- $B \to K^* \ell^+ \ell^- \to K\pi \ell^+ \ell^-$

go beyond the narrow-width approximation, take rescattering of final states into account

$$\langle f_1 f_2(P,\ell) \,|\, J^\mu \,|\, B(p) \rangle = \sum_i K_i^\mu F_i(E^*, q^2) \frac{T(E^*)}{k^\ell}$$

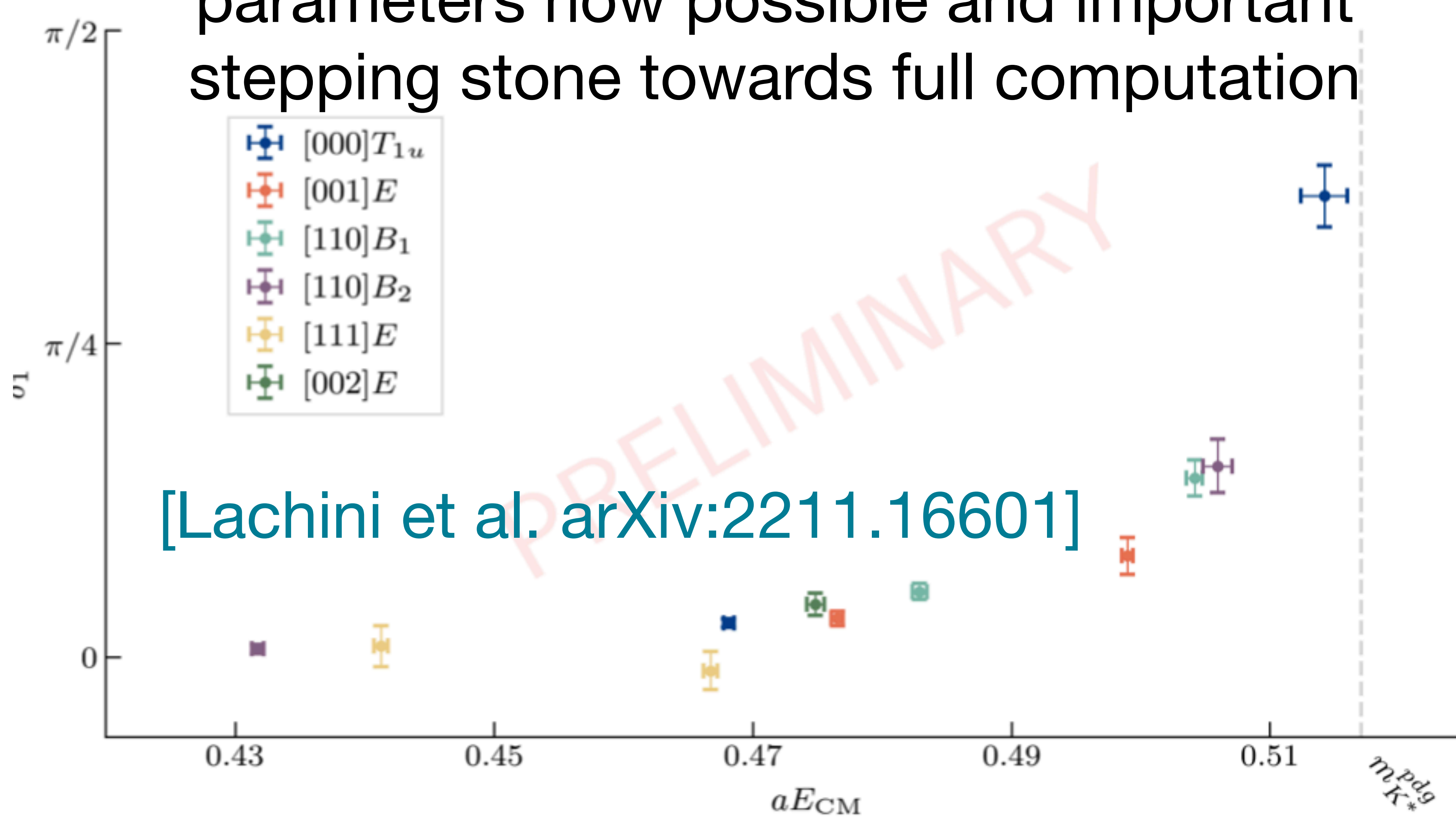
E^* two-hadron energy in CMF
K_i^μ Lorentz decomposition
T scattering amplitude



unphysical QCD parameters

e.g. [Bonn group, NPB 910 (2016) 387-409]
[Leskovec et al. CKM 2023 arXiv:2403.19543]

$K\pi$ scattering phase shift with physical QCD parameters now possible and important stepping stone towards full computation



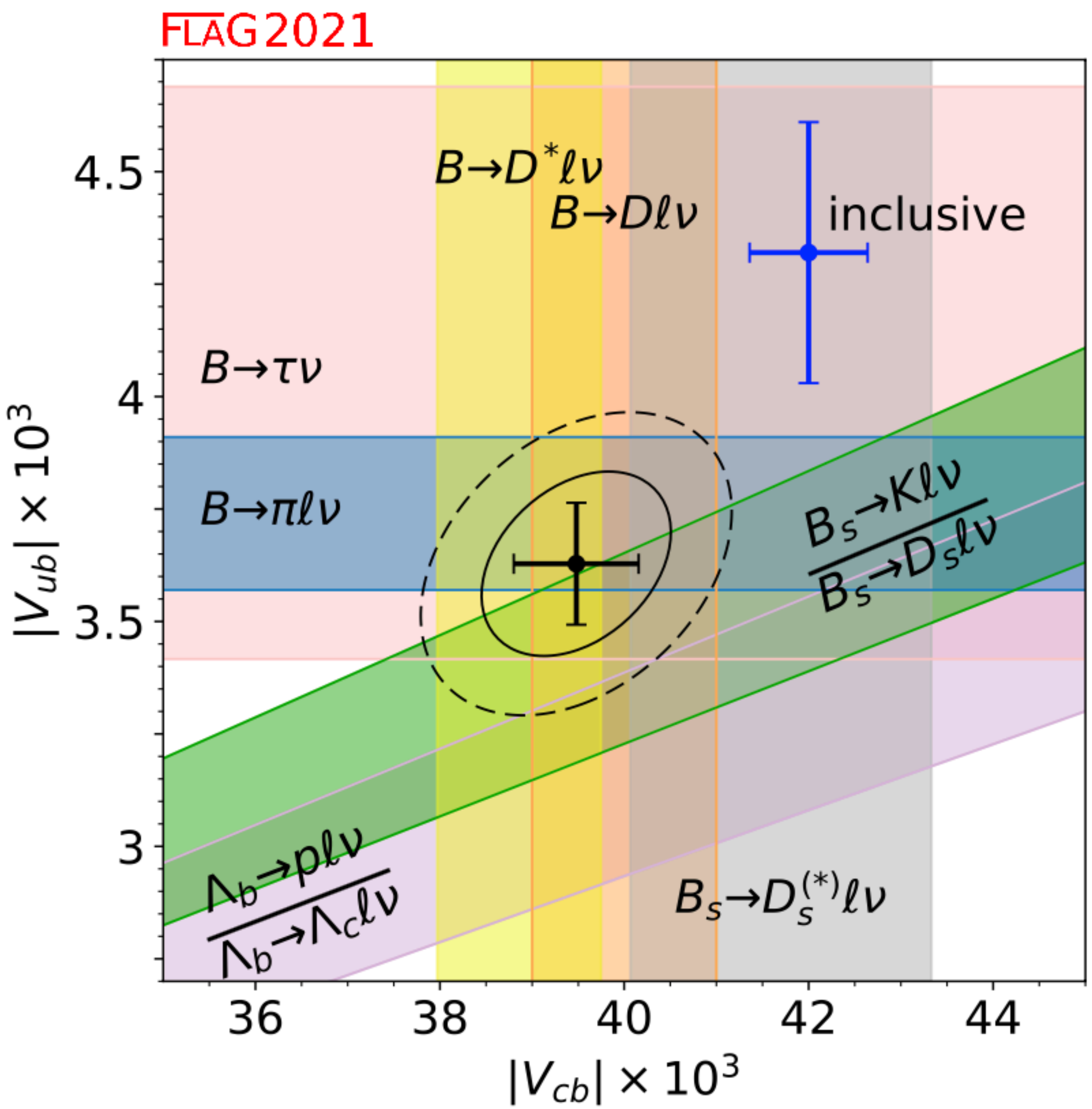
[Lachini et al. arXiv:2211.16601]

comprehensive lattice predictions for $B \to \rho \ell \nu_\ell$ and $B \to K^* \ell^+ \ell^-$ are coming within reach

Coming back to precision

new exp. data, new theory data

… a long-standing puzzle



A) Exclusive decay $B \to D^* \ell \bar{\nu}_\ell$:

- new quality of experimental data
- new quality of lattice data

→ **new and improved analysis techniques**

B) Inclusive decay $B \to X_c \ell \bar{\nu}_\ell$:

- existing determinations OPE based
- new ideas allow for lattice computations

→ **discuss new ideas and preliminary results**

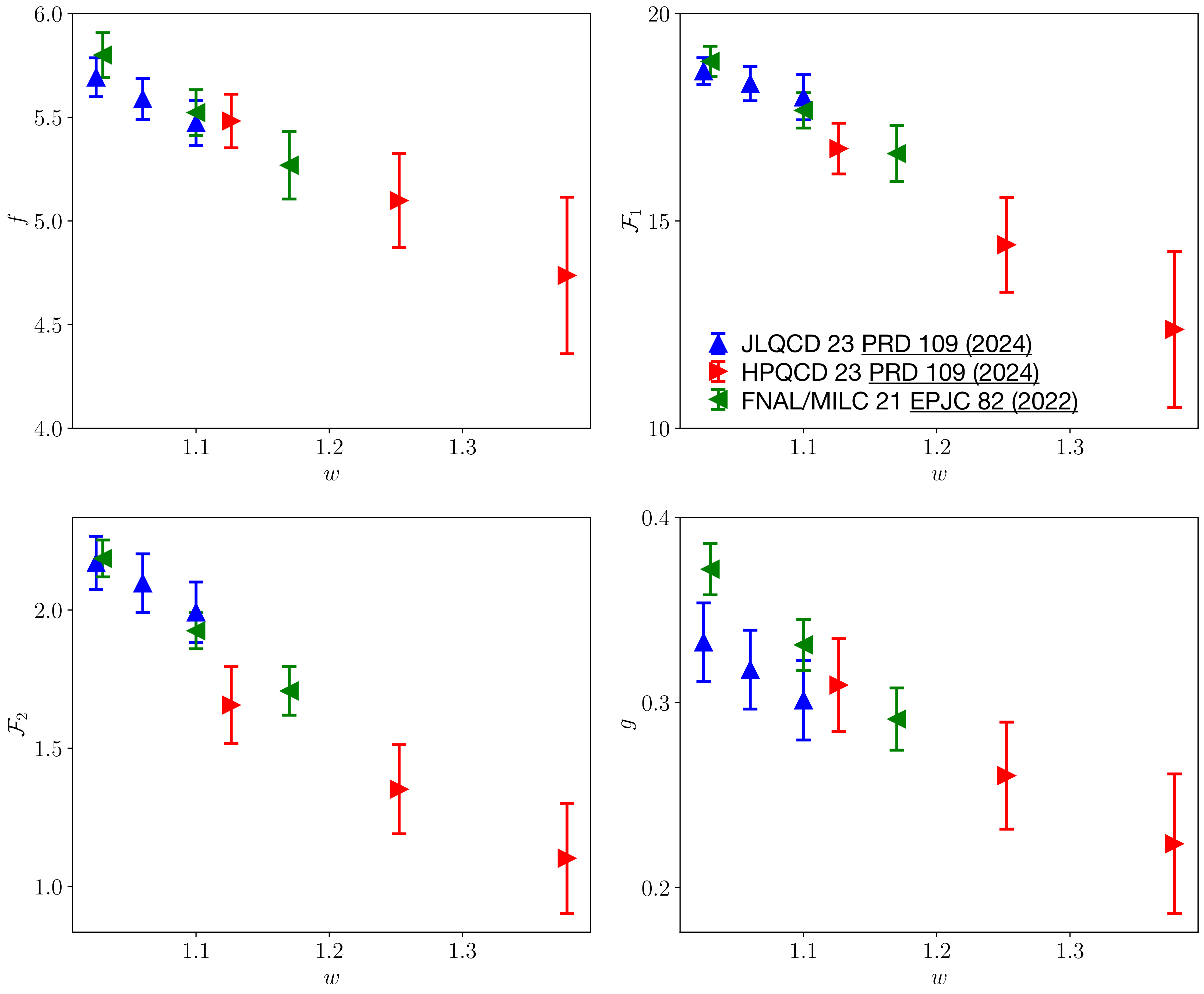
See also Paolo's talk!

Part I:
QFT constraints for exclusive semileptonic meson decays

based on work in collaboration with

- **Jonathan Flynn (Southampton)** and **Tobi Tsang (CERN)** [JHEP 12 (2023) 175]
- **Marzia Bordone (CERN)**, in preparation

New lattice data – $B \to D^* \ell \bar{\nu}_\ell$



New lattice data

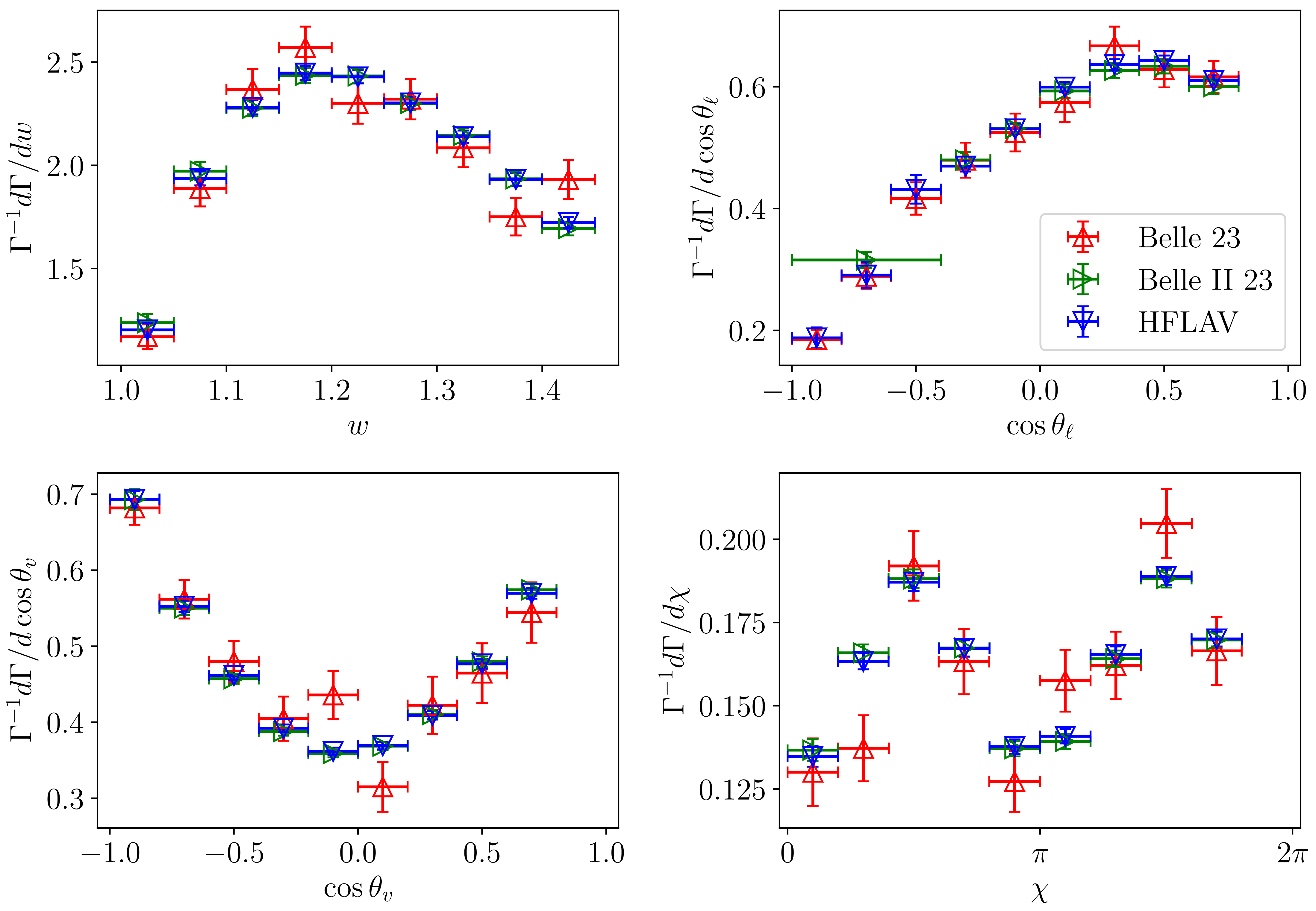
- four form factors $f, \mathscr{F}_1, \mathscr{F}_2, g$

$$w = \frac{M_B^2 + M_{D*}^2 - q^2}{2M_B M_{D*}} \qquad q_\mu = (p_B - p_{D*})_\mu$$

- first time that lattice data covers kinematical range
- three different and independent collaborations
- just in time for new experimental data ...

New experimental data – $B \to D^* \ell \bar{\nu}_\ell$

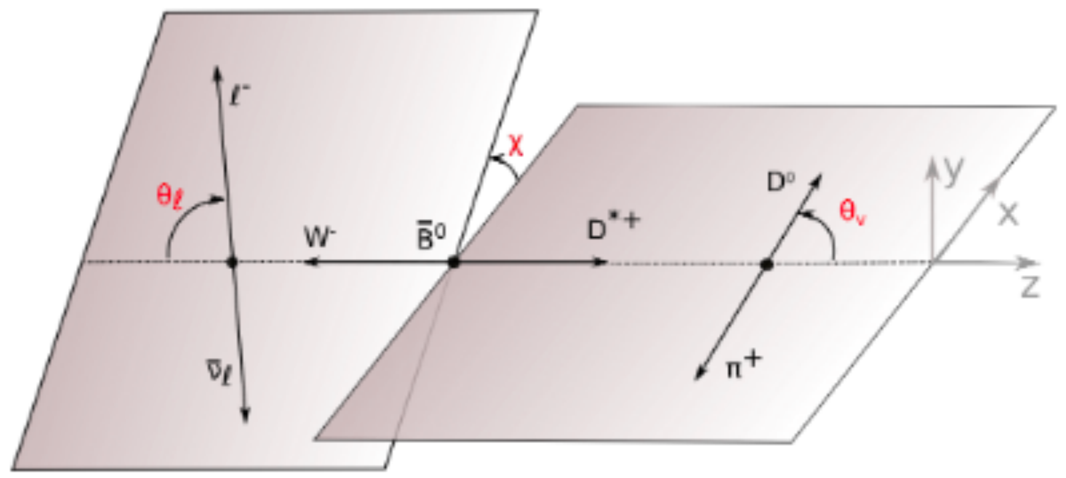
Belle Phys.Rev.D 108 (2023) 1, 012002
Belle II Phys.Rev.D 108 (2023) 9, 9



New experimental data

- four (normalised) differential decay rates in channels

$$\alpha = w,\ \cos\theta_\ell,\ \cos\theta_v, \chi$$



- between 7 and 10 bins per α
- data available on HEPData
- two experimental collaborations
- just in time for new lattice data …

$$\frac{d\Gamma}{dw d\cos(\theta_\ell) d\cos(\theta_v) d\chi} = \frac{3G_F^2}{1024\pi^4}|V_{cb}|^2\eta_{EW}^2 M_B r^2\sqrt{w^2-1}q^2$$
$$\times \{(1-\cos(\theta_\ell))^2\sin^2(\theta_v)H_+^2(w) + (1+\cos(\theta_\ell))^2\sin^2(\theta_v)H_-^2(w)$$
$$+4\sin^2(\theta_\ell)\cos^2(\theta_v)H_0^2(w) - 2\sin^2(\theta_\ell)\sin^2(\theta_v)\cos(2\chi)H_+(w)H_-(w)$$
$$-4\sin(\theta_\ell)(1-\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi)H_+(w)H_0(w)$$
$$+4\sin(\theta_\ell)(1+\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi)H_-(w)H_0(w)\}$$

How to best analyse this new quality of data as part of a precision test of the SM?

Form-factor parameterisation

$$f_X(q_i^2) = \frac{1}{B_X(q_i^2)\phi_X(q_i^2, t_0)} \sum_{n=0}^{K_X-1} a_{X,n} z(q_i^2)^n$$

unitarity constraint: $|\mathbf{a}_X|^2 \leq 1$

[Boyd, Grinstein, Lebed, PRL 74 (1995)]

[Okubo, PRD 3, 2807 (1971), PRD 4, 725 (1971)]
[Okubo, Shih, PRD 4, 2020 (1971)]
[Boyd, Grinstein, Lebed, PLB 353, 306 (1995), NPB461, 493 (1996). PRD 56, 6895 (1997)]

Determine all $a_{X,n}$ from finite set of theory data

Frequentist fit:
- $N_{\text{dof}} = N_{\text{data}} - K_X \geq 1$
  $\rightarrow$ in practice truncation K at low order
- induced systematic difficult to estimate
- meaning of Frequentist with unitarity constraint?

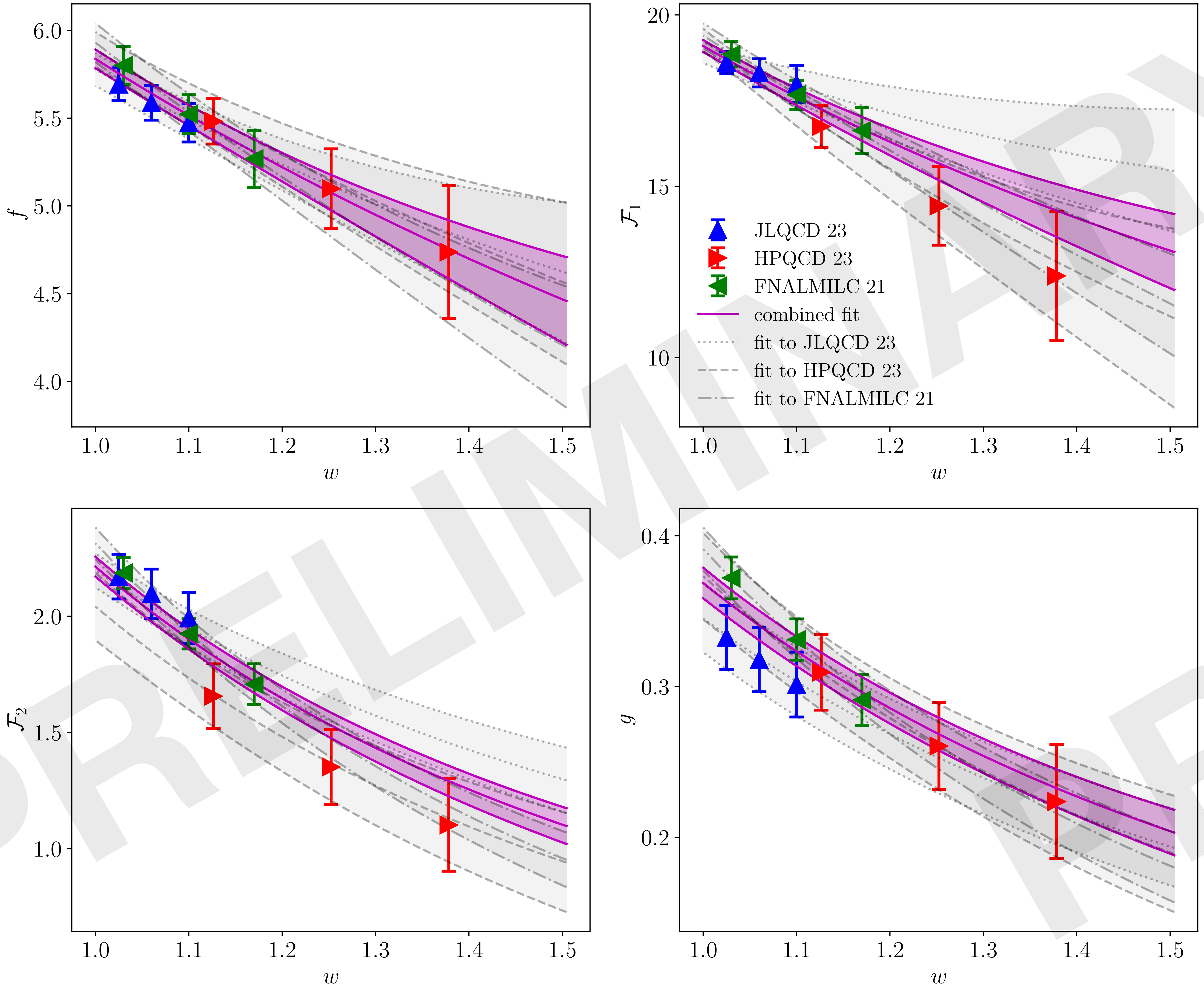
Bayesian fit:
- fit including higher order z expansion meaningful
- unitarity regulates and controls higher-order coefficients [Flynn, AJ, Tsang JHEP 12 (2023) 175]
- well-defined meaning of unitarity constraint

Recommendation: Combined Frequentist + Bayesian perspective

Also have a look: Dispersive-matrix method, Di Carlo et al. PRD 2021

Strategy A: Fit to lattice data

[Bordone, AJ in preparation]



Frequentist fit

K_f	K_{F_1}	K_{F_2}	K_g	$a_{g,0}$	$a_{g,1}$	$a_{g,2}$	$a_{g,3}$	p	$\chi^2/N_{\rm dof}$	$N_{\rm dof}$
2	2	2	2	0.03138(87)	-0.059(24)	-	-	0.95	0.62	30
3	3	3	3	0.03131(87)	-0.046(36)	-1.2(1.8)	-	0.90	0.67	26
4	4	4	4	0.03126(87)	-0.017(48)	-3.7(3.3)	49.9(53.6)	0.79	0.75	22

- good fit quality
- lattice data *superficially* compatible (covariance contains systematic effects…)
- no unitarity constraint

Bayesian inference

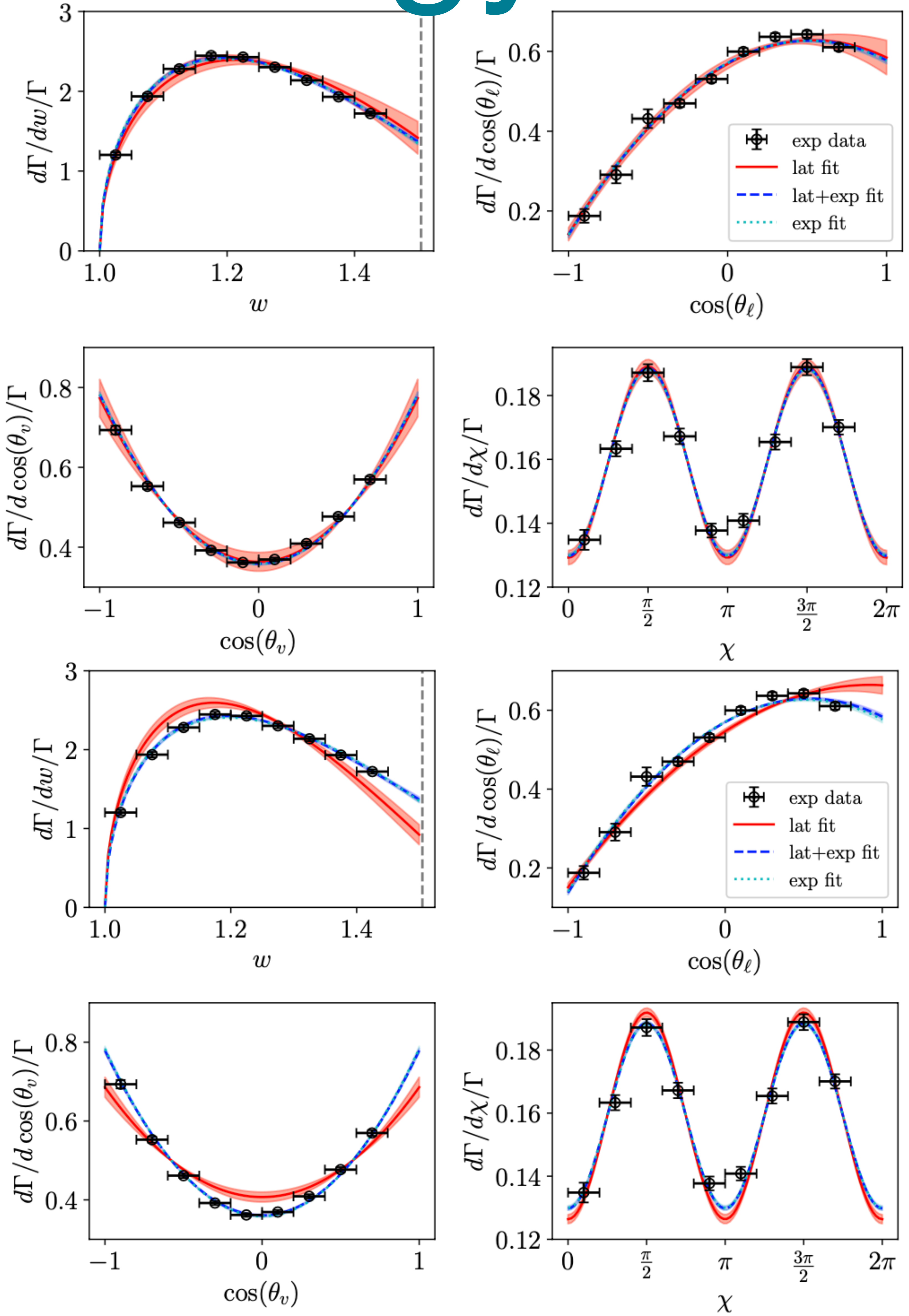
K_f	K_{F_1}	K_{F_2}	K_g	$a_{g,0}$	$a_{g,1}$	$a_{g,2}$	$a_{g,3}$
2	2	2	2	0.03133(80)	-0.058(25)	-	-
3	3	3	3	0.03129(81)	-0.062(27)	-0.10(55)	-
4	4	4	4	0.03134(86)	-0.061(25)	-0.10(50)	-0.04(49)

- unitarity constraint regulates higher-order coefficients
- truncation independent

Strategy B: Fit to lattice + exp.data

JLQCD 23 + Belle II 23

FNAL/MILC 21 + HPQCD 23 + Belle II 23



[Bordone, AJ in preparation]

- BGL fit to only lattice data (strategy A) misses experimental points for two of the lattice data sets
- BGL fit to experimental and lattice data of good quality

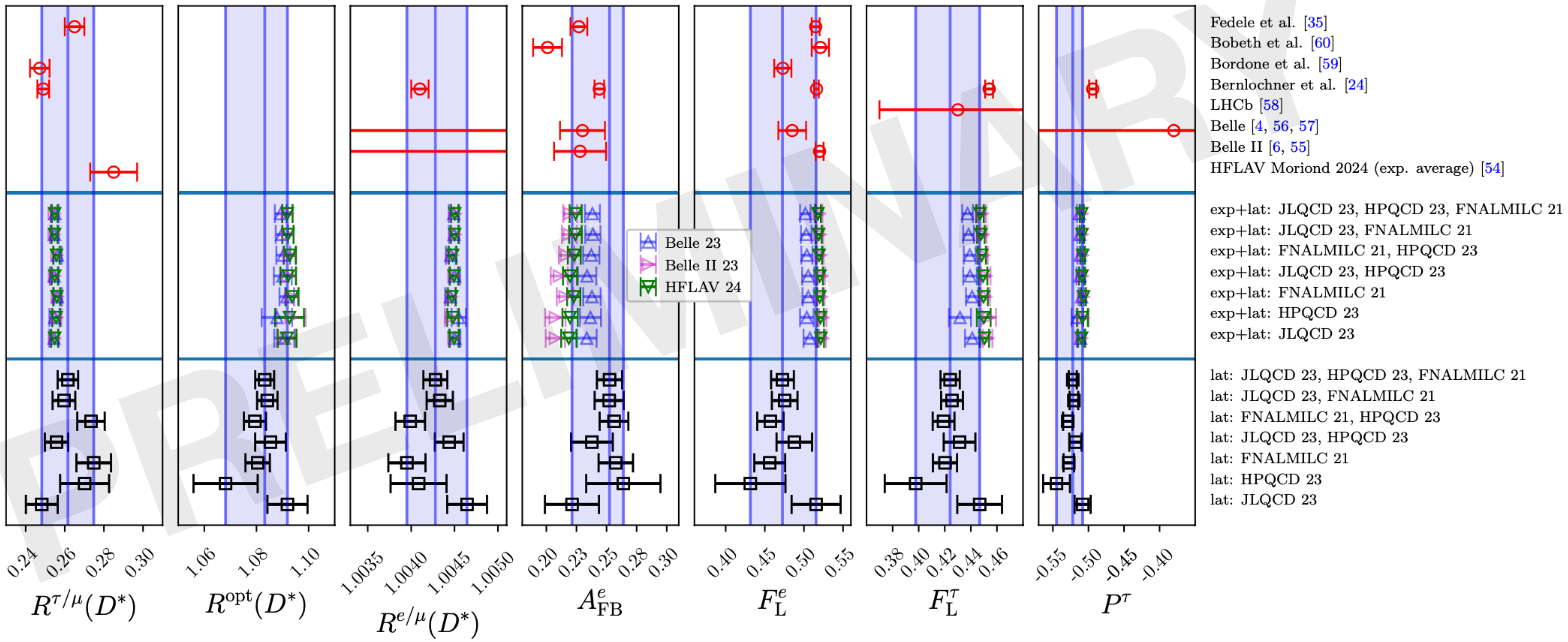
  Frequentist fit quality good

  lat $(p, \chi^2/N_{\rm dof}, N_{\rm dof}) = (0.79, 0.75, 22)$

  lat+exp $(p, \chi^2/N_{\rm dof}, N_{\rm dof}) = (0.18, 1.15, 56)$
- some BGL coefficients shift between strategy A) and B) by up to a few σ → but precision of lattice data allows for enough wiggle room

Other observables

[Bordone, AJ in preparation]



[Isidori, Sumensari, EPJC 80 (2020) 11, 1078]

- lat: scatter from different lattice collaborations concerning (2-3σ) (see also [Fedele et al. PRD 108, 055037 (2023)])
- lat+exp: lattice consistent, experiments inconsistent
- parameterisation-based observables show high degree of sensitivity

analysis reveals tensions amongst lattice as well as amongst experimental data sets

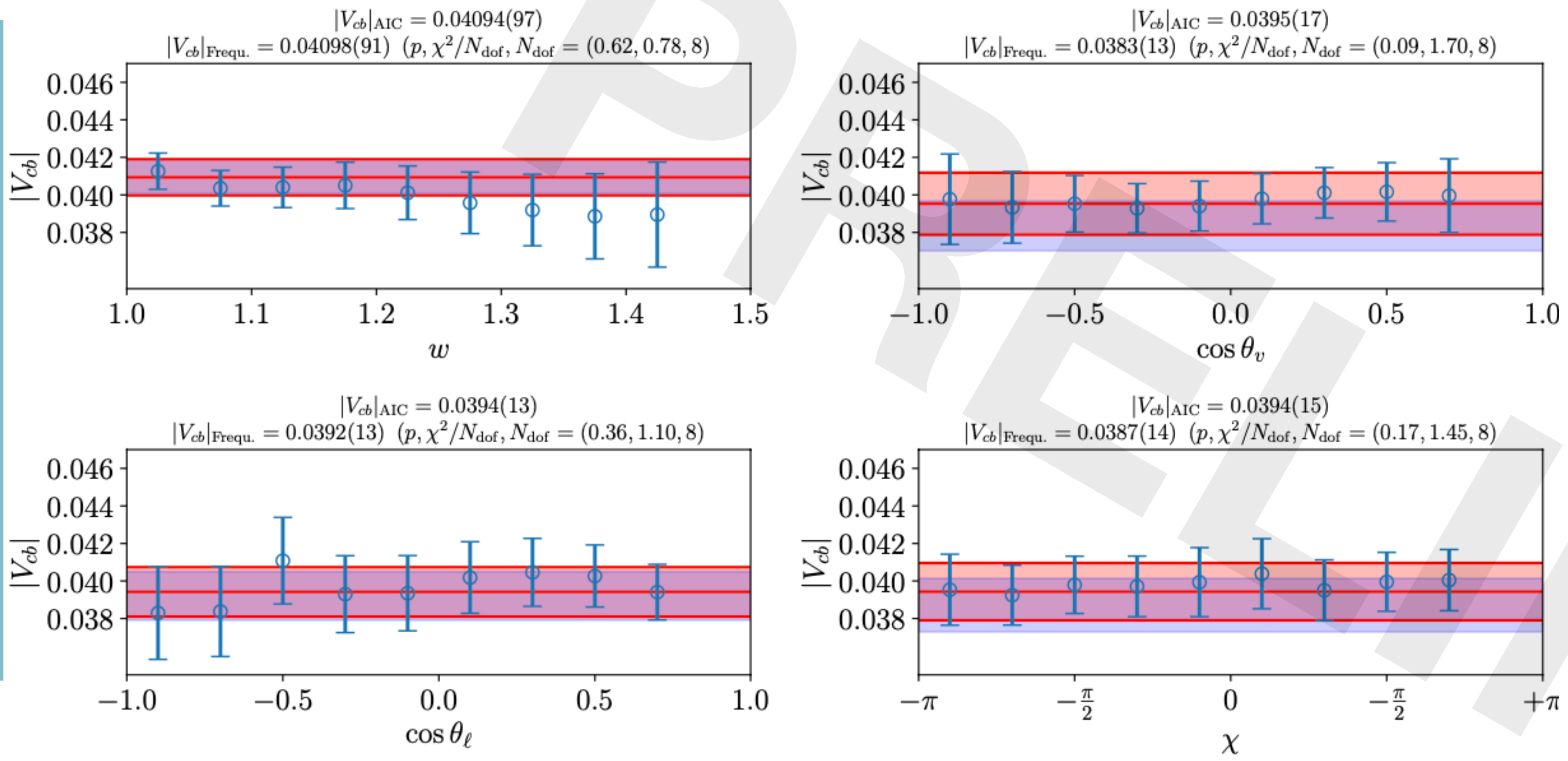
$|V_{cb}|$ — Strategy A: different lattice input — different

[Bordone, AJ in preparation]

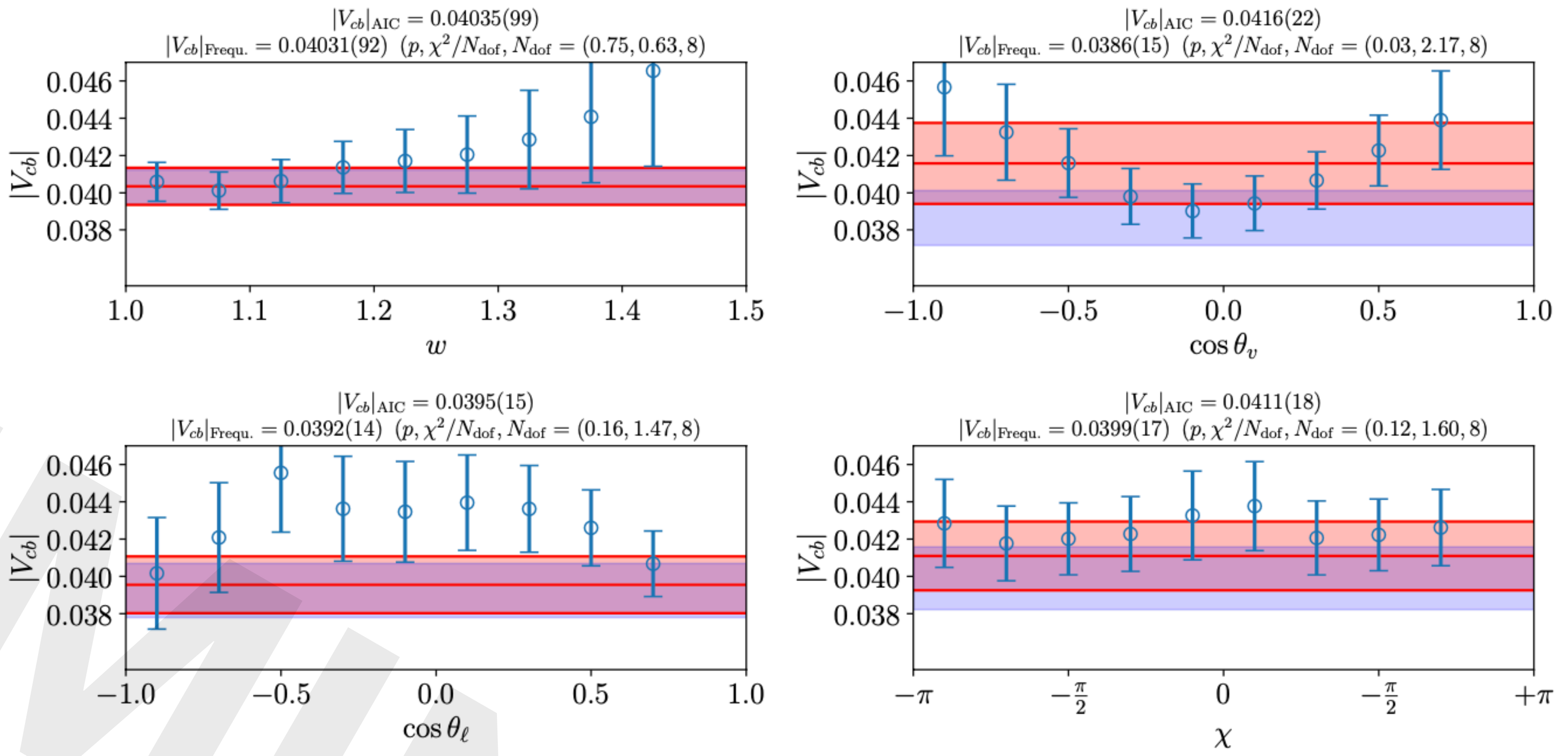
$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\rm exp}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\alpha}\right]^{(i)}_{\rm exp} \Big/ \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]^{(i)}_{\rm lat}\right)^{1/2}$$

$$\Gamma_{\rm exp} = \frac{\mathscr{B}(B^0 \to D^{*,-}\ell^+\nu_\ell)}{\tau(B^0)},$$

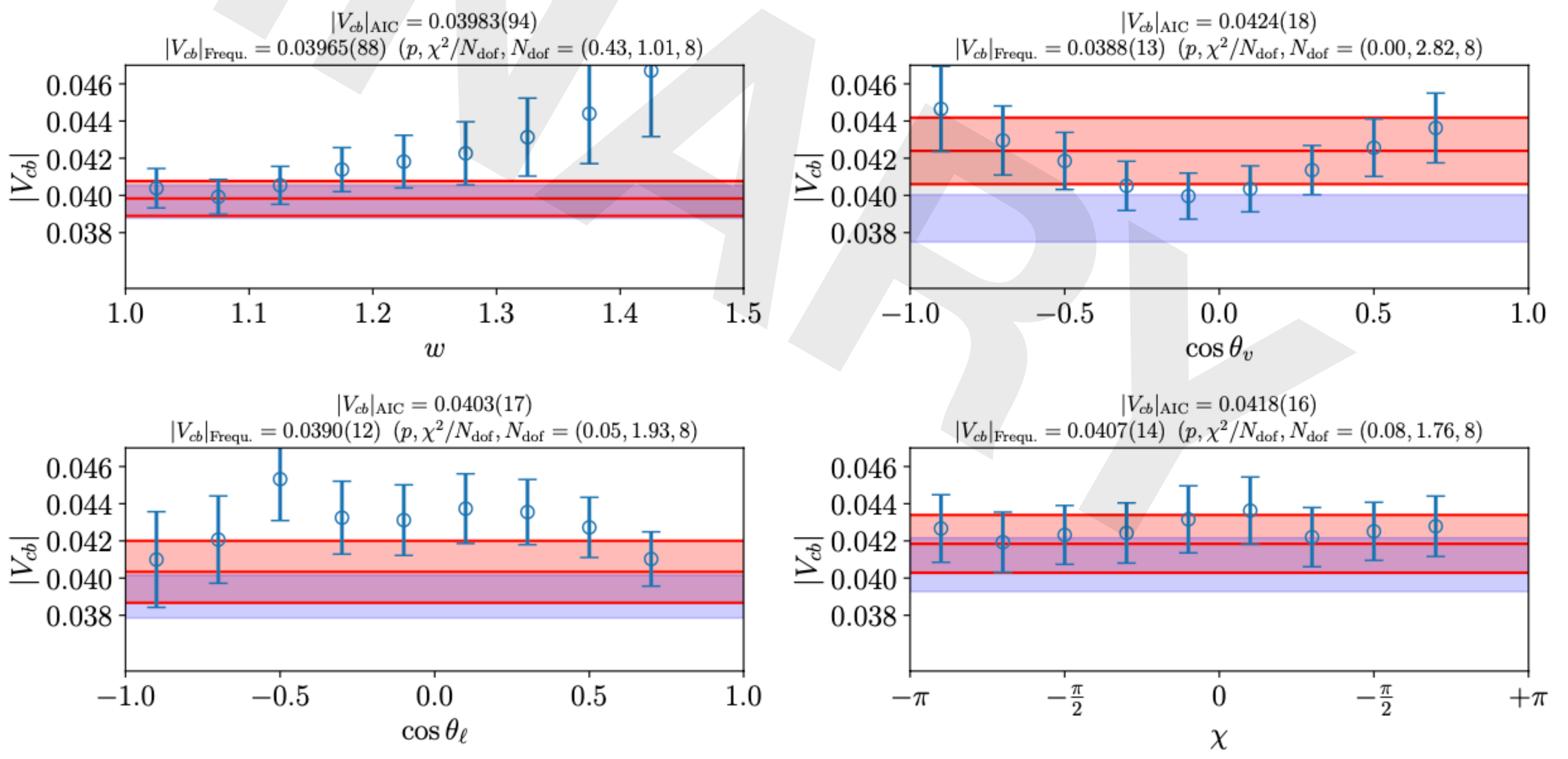
JLQCD 23



HPQCD 23



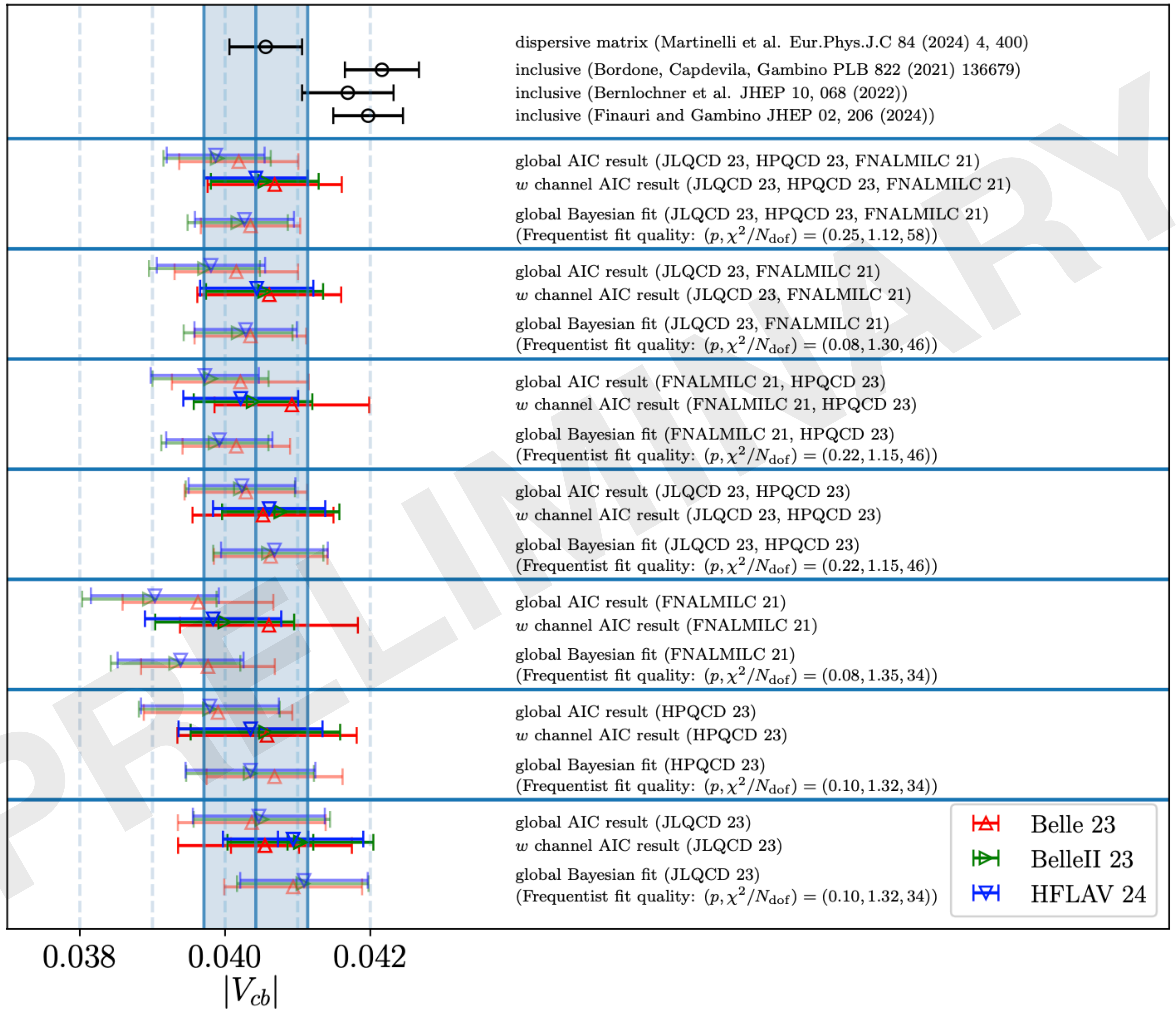
FNAL/MILC 21



- New AIC-based approach works nicely and reduces bias likely due to d'Agostini
- some lattice data however problematic and at odds with expectation
- in particular analysis of angular distributions problematic?
- discard analysis $X = \cos\theta_v, \cos\theta_\ell, \chi$?

$|V_{cb}|$ – Summary

[Bordone, AJ in preparation]



- comparison of different lattice and experimental input
- by and large good agreement (especially if angular bins discarded from AIC)

strategy A) BGL fit to lattice data, then combination with experiment
strategy B) BGL fit to both lattice and experiment

- **We find no noteworthy tensions between the results from both strategies**
- **This analysis confirms a slight tension with inclusive analyses**

See also analysis within dispersive-matrix method, Martinelli et al. EPJC (2024)

Part II:
QFT constraints for inclusive meson decays (on the lattice)

ongoing work in collaboration with

Alessandro Barone (Soton → Mainz)
Ahmed Elgazhari (Soton)
Shoji Hashimoto (KEK)
Takashi Kaneko (KEK)
Ryan Kellermann (KEK)
Hu Zhi (KEK)

JHEP 07 (2023) 145

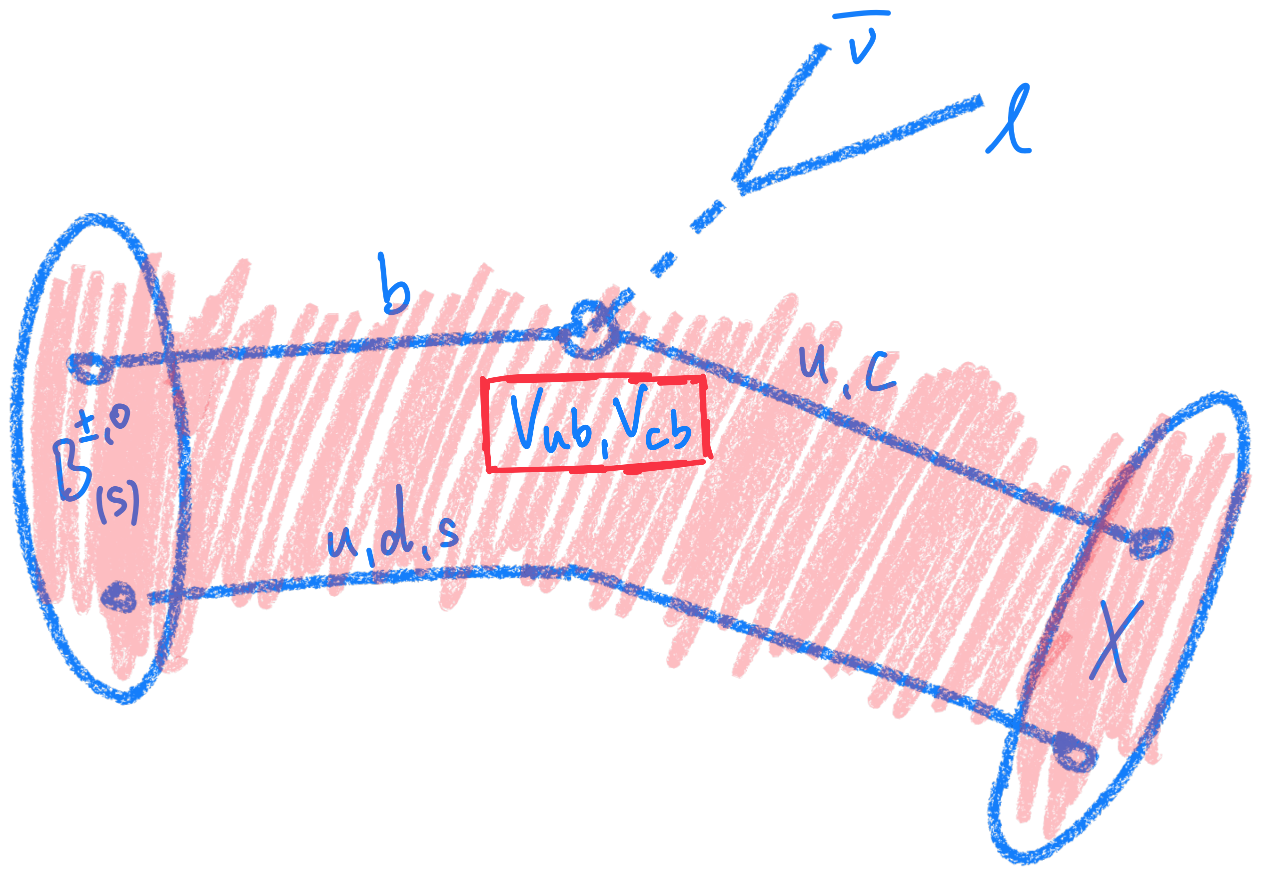
Hansen et al. (2017) PRD 96 094513 (2017)
Hashimoto PTEP 53-56 (2017)
Bailas et al. PTEP 43-50 (2020)
Gambino and Hashimoto PRL 125 32001 (2020)
Barone et al. JHEP 07 (2023) 145

Inclusive SL decay in the SM

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{qQ}|^2}{8\pi} L^{\mu\nu} W_{\mu\nu}$$

hadronic tensor

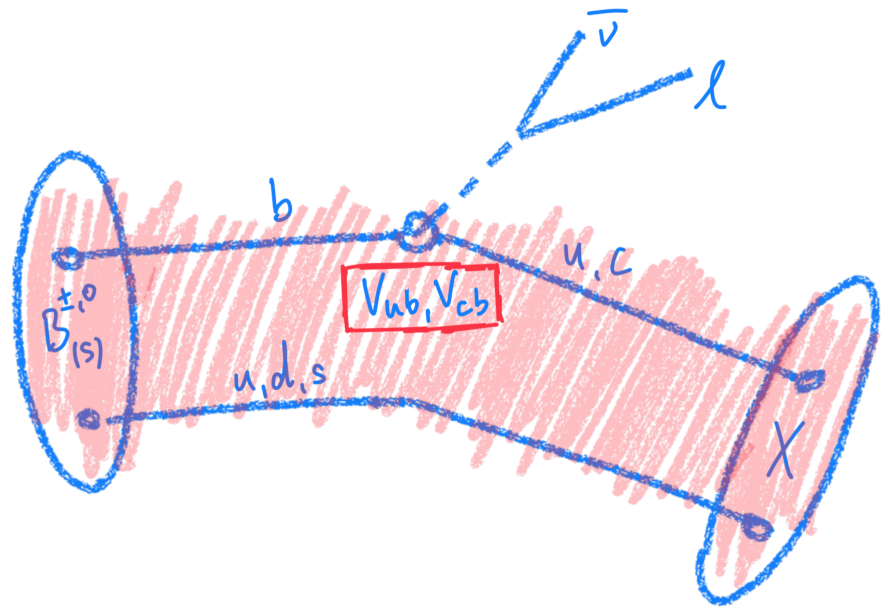
leptonic tensor



We consider the case $B_s \to X_c \ell \nu$:

$$W^{\mu\nu}(p_{B_s}, q) = \frac{1}{2E_{B_s}} \sum_{X_c} (2\pi)^3 \delta^{(4)}(p_{B_s} - q - p_{X_c}) \langle B_s(\mathbf{p}_{B_s}) | (\tilde{J}^\mu(q^2))^\dagger | X_c(p_{X_c}) \rangle \langle X_c(p_{X_c}) | \tilde{J}^\nu(q^2) | B_s(\mathbf{p}_{B_s}) \rangle$$

from now on B_s at rest ($\mathbf{p}_{B_s} = \mathbf{0}$)



Inclusive SL decay

kinematics:

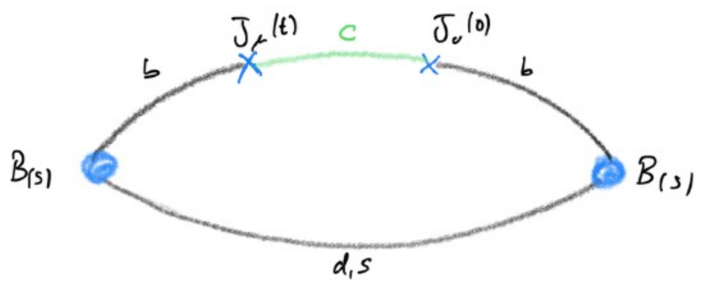
$$\omega_{\min} = \sqrt{M_{D_s}^2 + \mathbf{q}^2} \qquad \mathbf{q}^2_{\max} = \frac{(M_{B_s}^2 - M_{D_s}^2)^2}{4M_{B_s}^2}$$

$$\omega_{\max} = M_{B_s} - \sqrt{\mathbf{q}^2}$$

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi} L^{\mu\nu} W_{\mu\nu} \rightarrow \Gamma(B_s \to X_c l\nu) = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\mathbf{q}^2_{\max}} d\mathbf{q}^2 \sqrt{\mathbf{q}^2} \int_{\omega_{\min}}^{\omega_{\max}} d\omega\, W_{\mu\nu}(\omega, \mathbf{q}) k^{\mu\nu}(\omega, \mathbf{q})$$

$\bar{X}(\mathbf{q}^2)$

- integration over lepton energy $E_l \rightarrow k^{\mu\nu}$
- ω is energy of intermediate state X_c
- $\mathbf{q}$ is three-momentum transfer



lattice computation of 4pt function

$$C_{\mu\nu}(t, \mathbf{q}) = \int_{\omega_{\min}}^{\infty} d\omega\, W_{\mu\nu}(\omega, \mathbf{q})\, e^{-\omega t}$$

expansion of leptonic kernel:

$$k^{\mu\nu}(\omega, \mathbf{q}) = \sum_n c_{\mu\nu}(\mathbf{q})(e^{-a\omega})^n$$

$$\bar{X}(\mathbf{q}) = \sum_k c_{\mu\nu,k}(\mathbf{q}) \int d\omega W_{\mu\nu}(\omega, \mathbf{q})\, e^{-\omega k}$$

$$= \sum_k c_{\mu\nu,k}(\mathbf{q})\, C_{\mu\nu}(ak, \mathbf{q})$$

Useless in practice due to deteriorating signal-to-noise in 4pt function → need to improve

Chebyshev reconstruction

Barata, Fredenhagen, Commun.Math.Phys. 138 (1991) 507-520, Bailas et al. PTEP 43-50 (2020), Gambino and Hashimoto PRL 125 32001 (2020)

Improvement: expand in shifted Chebyshev polynomials $\tilde{T}(e^{na\omega})$

$$\bar{X}(\mathbf{q}) = \sum_k \tilde{c}_{\mu\nu,k}(\mathbf{q}) \int d\omega W_{\mu\nu}(\omega, \mathbf{q})\, \tilde{T}_k(\omega)$$
$$= \sum_{k,j} \tilde{c}_{\mu\nu,k}(\mathbf{q})\, \langle \tilde{T}_k \rangle_{\mu\nu}$$

$$\langle \tilde{T}_k \rangle_{\mu\nu} = \frac{\sum_{j=0}^{k} \tilde{t}_j^{(k)} C_{\mu\nu}(j + 2t_0)}{C_{\mu\nu}(2t_0)}$$

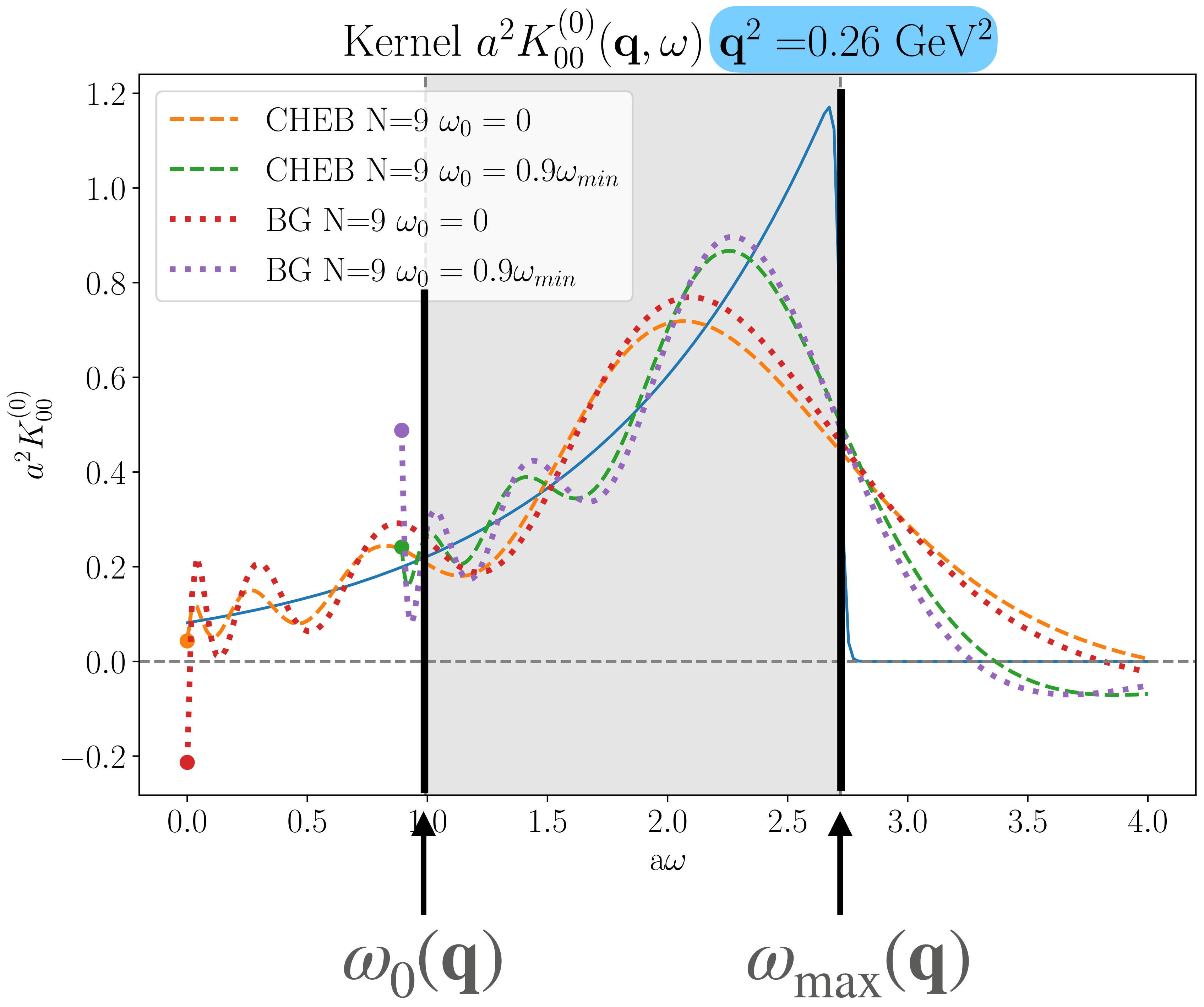
- $C_{\mu\nu}(t)$ monotonously decreasing with t
- **we use $|\langle \tilde{T}_k \rangle_{\mu\nu}| \leq 1$ as uniform Bayesian prior (i.e. regulator)**
- $\tilde{c}_{\mu\nu,k}$ turn out to be nicely behaved — suppression of higher-order terms

Kernel approximation

$$K_{\mu\nu}(\omega, \mathbf{q}; t_0) = e^{2\omega t_0} k_{\mu\nu}(\omega, \mathbf{q},)\theta_\sigma(\omega_{\max} - \omega)$$

$$\tilde{c}_k = \langle K, \tilde{T}_k \rangle = \int_{\omega_0}^{\infty} d\omega\, K_{\mu\nu}(\omega, \mathbf{q}) \tilde{T}_k(\omega) \tilde{\Omega}(\omega)$$

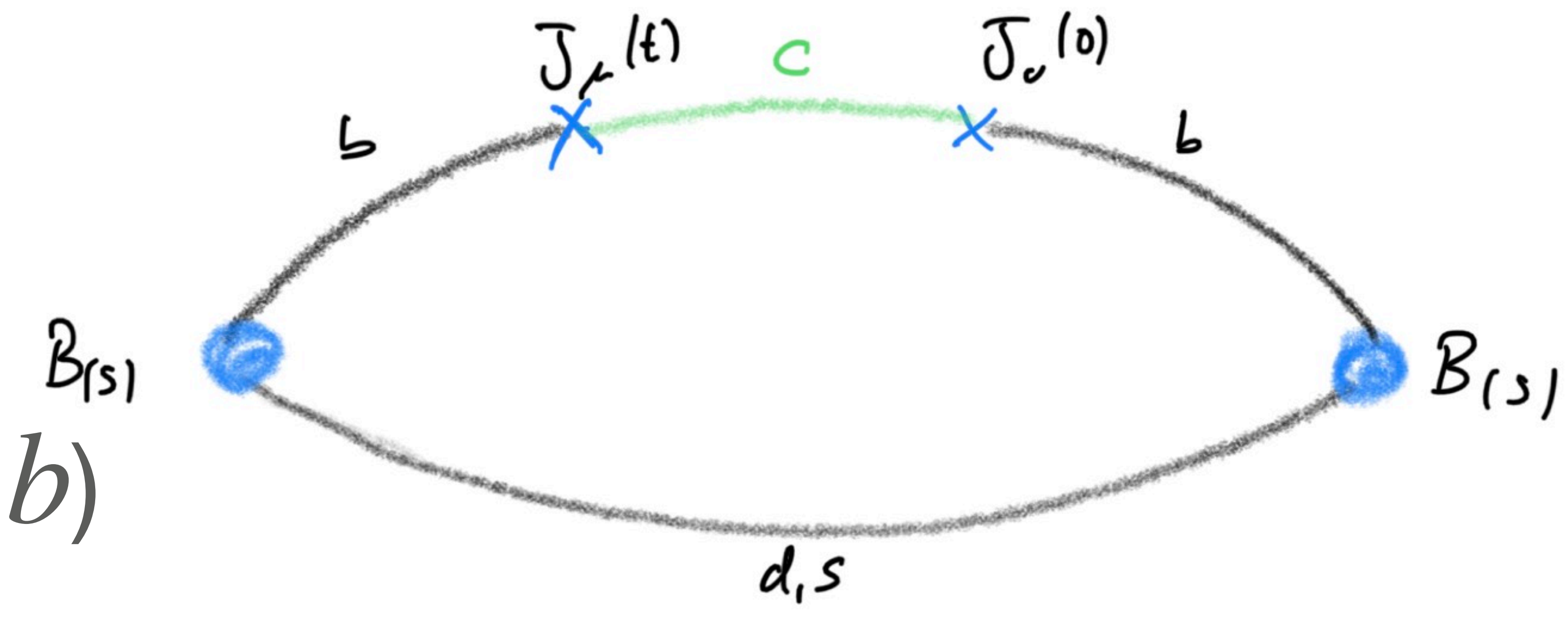
e.g. $K^{(0)}_{\sigma,00}(\mathbf{q}, \omega; t_0) = e^{2\omega t_0}\, \mathbf{q}^2\, \theta_\sigma(\omega_{\max} - \omega)$



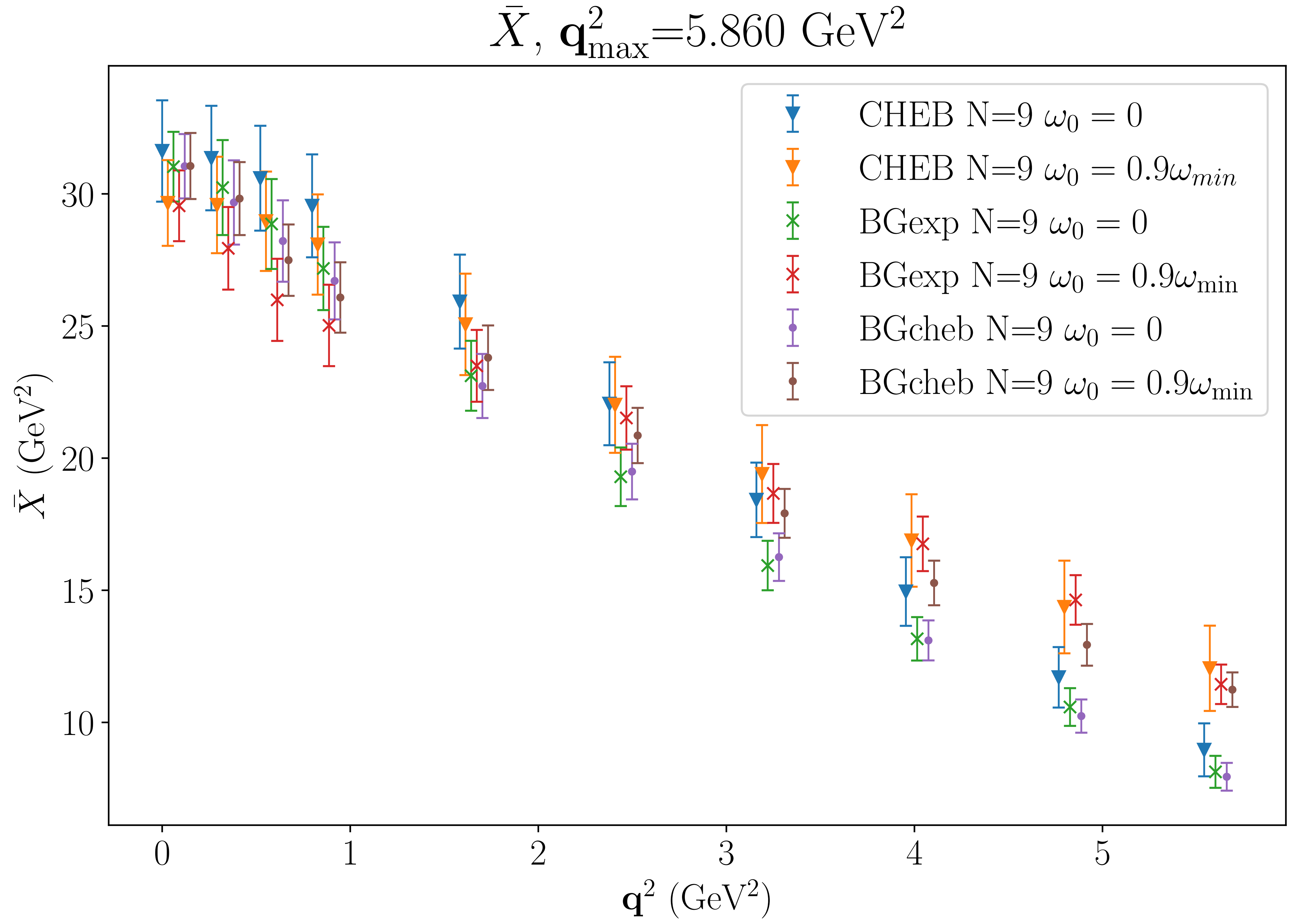
- this analysis stage independent of data
- smearing σ
- order of approximation $N \leftrightarrow C_{\mu\nu}(t)$
- we can play with ω_0

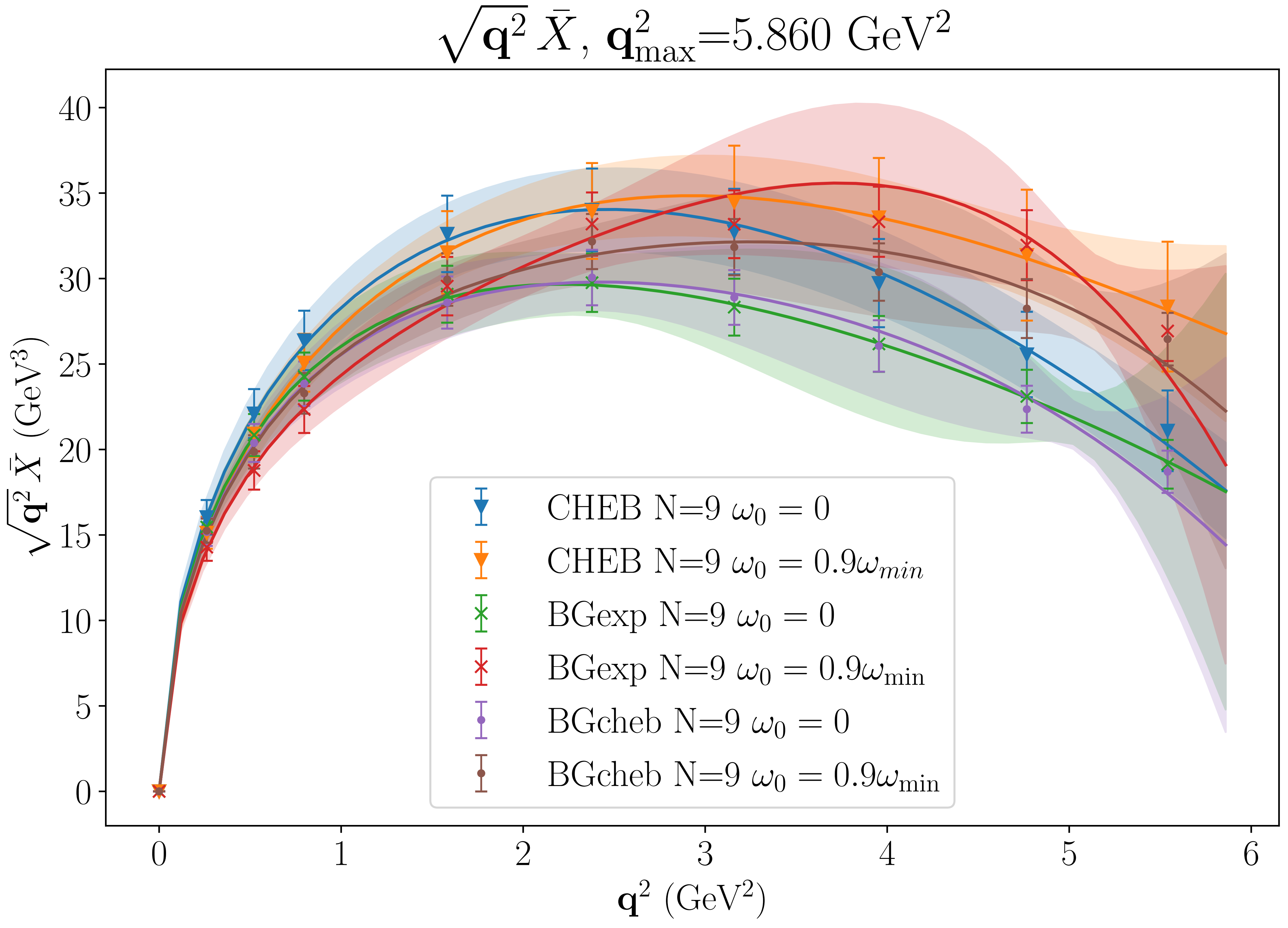
Exploratory study

- $B_s \to X_c \ell \nu$
- lattice study on $24^3 \times 64$ RBC/UKQCD DWF ensemble ($M_\pi^{\mathrm{sea}} \approx 330\,\mathrm{MeV}$)
- physical m_s- and m_b-quark masses (RHQ action for b) near-physical m_c (domain-wall)
- implemented in Grid/Hadrons
- run on DiRAC Extreme-scaling service Tursa (A100-40 nodes)
- 120 gauge configs, 8 Z_2 noise-source planes



Results for $\bar{X}(\mathrm{q})$



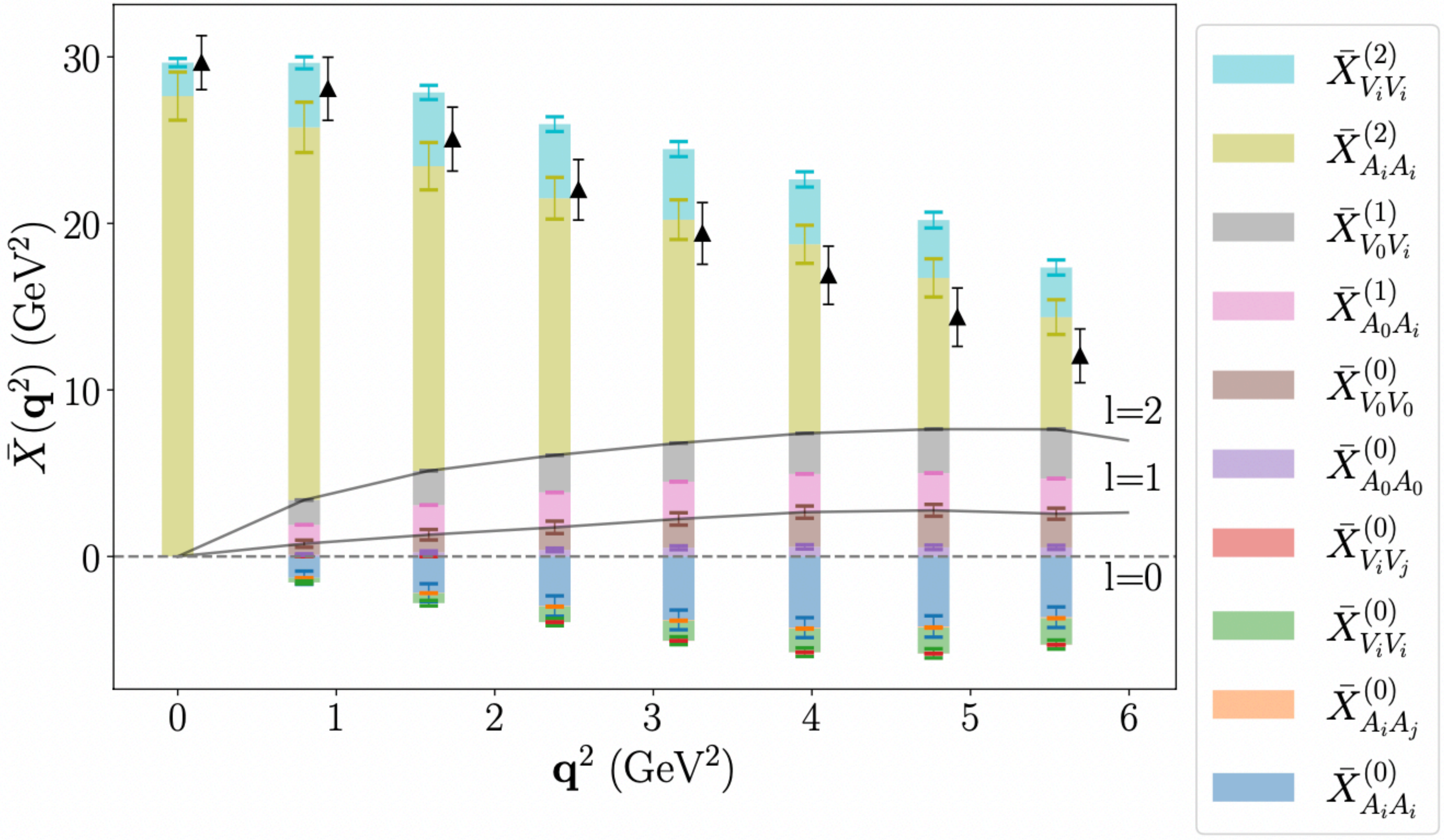


variations of analysis techniques largely consistent — tension at larger $\mathbf{q}^2$ visible

Integral of $\sqrt{\mathbf{q}^2}\,\bar{X}(\mathbf{q}^2)$ proportional to Γ;

data opens up opportunities for a number of novel studies!

Contributions from various channels



Approach provides for nice laboratory to understand and probe contributions to inclusive decay from various sources

Here: A_iA_i channel appears dominant

Ground-state contribution?

How big is the ground-state contribution to inclusive decay? (D_s for $B_s \to X_c \ell \bar{\nu}_\ell$)

Contribution to hadronic tensor:

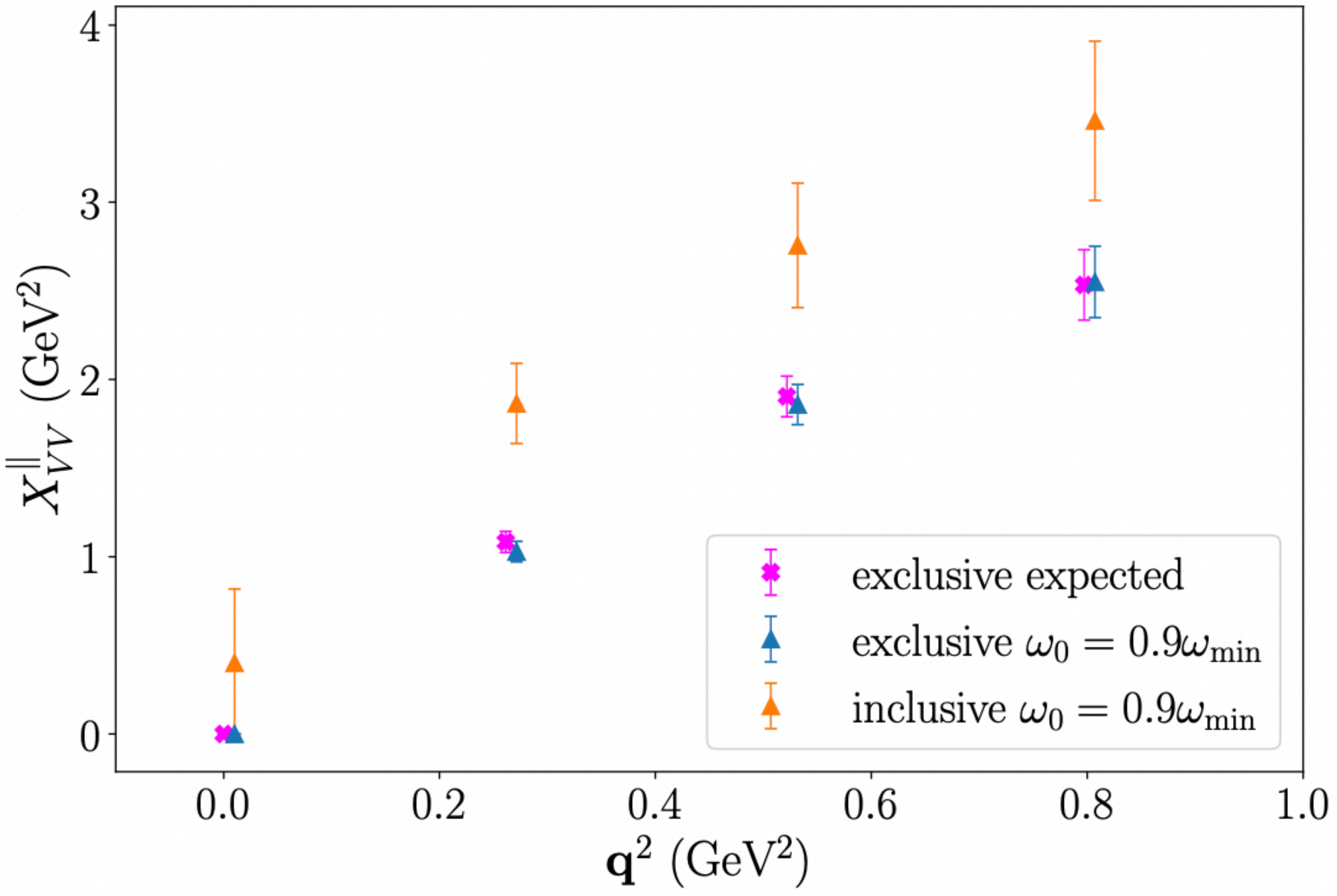
$$W_{\mu\nu} \to \delta(\omega - E_{D_s}) \frac{1}{4 M_{B_s} E_{D_s}} \langle B_s | J_\mu^\dagger | D_s \rangle \langle D_s | J_\nu | B_s \rangle$$

Compute $B_s \to D_s$ matrix elements on the lattice:

$$\langle D_s | V_\mu | B_s \rangle = f_+(q^2)(p_{B_s} + p_{D_s})_\mu + f_-(q^2)(p_{B_s} - p_{D_s})_\mu$$

$$\bar{X}^{\parallel}_{VV} \to \frac{M_{B_s}}{E_{D_s}} \mathbf{q}^2 \, | f_+(q^2) |^2$$

Ground-state contribution?

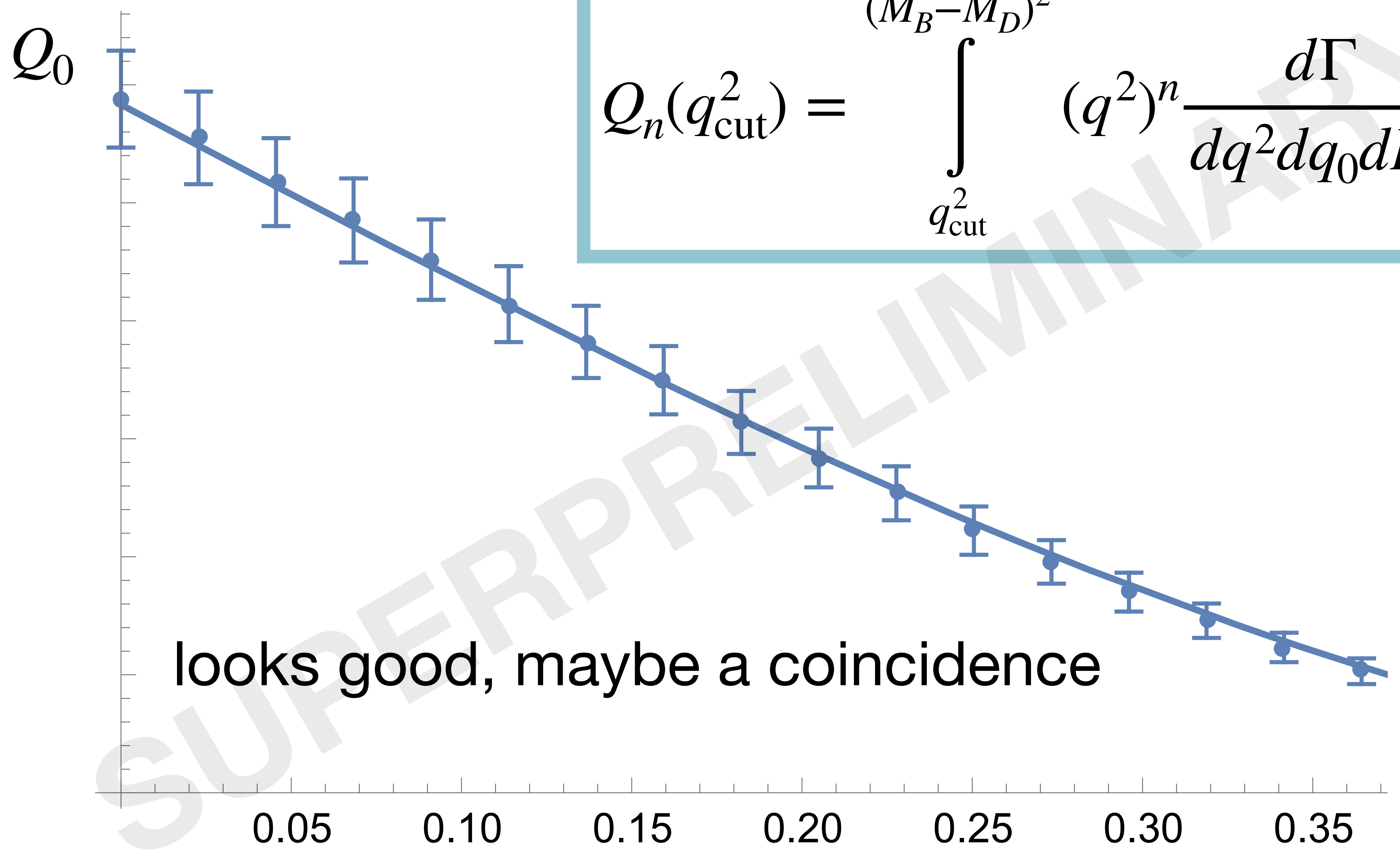


- Results for exclusive channel agree for both ways of data analysis (standard 3pt vs. Chebychev)
- clear distinction between ground-state and full inclusive determination

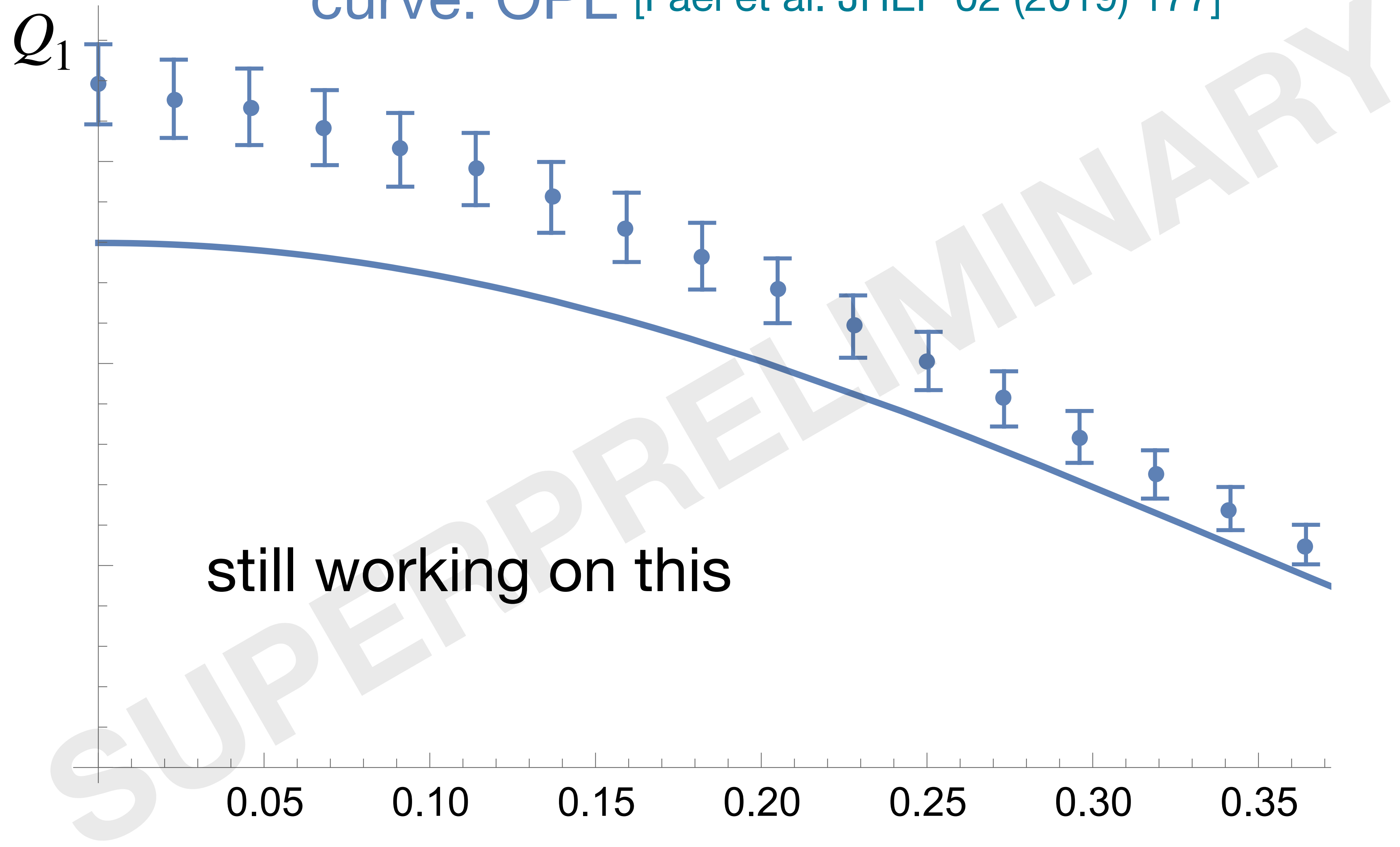
Hadronic moments

[Barone, Fael, Jüttner, ongoing]

$$Q_n(q^2_{\mathrm{cut}}) = \int_{q^2_{\mathrm{cut}}}^{(M_B-M_D)^2} (q^2)^n \frac{d\Gamma}{dq^2 dq_0 dE_\ell} dq^2 dq_0 dF$$



data points: lattice [Barone et al. ongoing]
curve: OPE [Fael et al. JHEP 02 (2019) 177]



Detailed study lattice vs. OPE:

- Lattice data without systematics, i.e. comparison super-preliminary and qualitative (one lattice spacing, one volume, unphysical charm mass, …)
- Plots only to convey idea of potential for future detailed studies
- Could allow understanding origin of puzzle (if on theory side)

Summary & Outlook

New precision

- FLAG remains the goto-place to get a good impression of where LQCD has good control of systematic effects for quantities relevant to flavour physics
- sub-percent precision requires new thinking and new developments: simulations of QCD+QED+strong IBB

New directions

- long-distance effects in EW processes require developing new ideas and pushing frontiers — but developments show that challenges can be tackled (e.g. rare decays)
- $P \to V + \mathrm{leptons}$ now being attacked by community
- inclusive decays on the lattice might shed light on incl./excl. puzzle

New data

- new quality of experimental and lattice data requires new analysis techniques — more data to come in…

CONTINUUM FOUNDATIONS OF LATTICE GAUGE THEORIES

FIRST CERN-NORDIC SCHOOL
22-26 July 2024, CERN, Geneva

Lectures

Dispersive methods	Gilberto Colangelo (Bern)
Inverse Problems	Luigi Del Debbio (Edinburgh)
Quantum Computing	Zohreh Davoudi (Maryland)
Resurgence	Gerald Dunne (Connecticut)

CERN

University of Stavanger

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QUANTUM THEORY CENTER

Organisers

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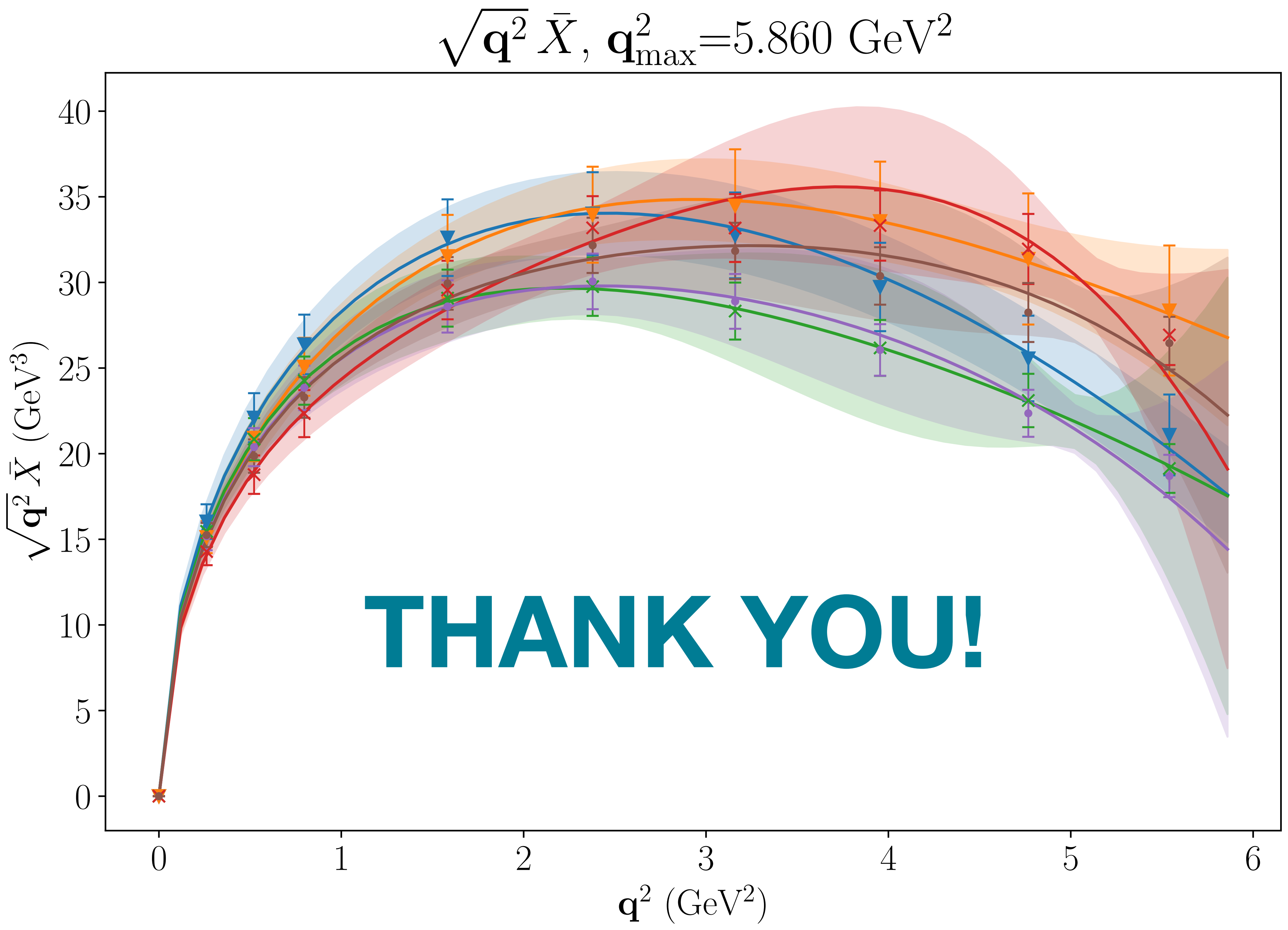
Michele Della Morte (Odense)
Matteo Di Carlo (CERN)
Felix Erben (CERN)
Andreas Jüttner (CERN/Southampton)
Simon Kuberski (CERN)
Alexander Rothkopf (Stavanger)

Registration at: https://indico.cern.ch/event/1342488


$\sqrt{\mathbf{q}^2}\,\bar{X}$, $\mathbf{q}^2_{\text{max}}$=5.860 GeV^2
$\sqrt{\mathbf{q}^2}\,\bar{X}$ (GeV^3)
$\mathbf{q}^2$ (GeV^2)
0
5
10
15
20
25
30
35
40
0
1
2
3
4
5
6

THANK YOU!

Bayesian form-factor fit

Flynn, AJ, Tsang, JHEP 12 (2023) 175

Compute BGL parameters as expectation values

$$\langle g(\mathbf{a})\rangle = \mathcal{N}\int d\mathbf{a}\, g(\mathbf{a})\,\pi(\mathbf{a}\,|\,\mathbf{f}, C_{\mathbf{f}})\,\pi_{\mathbf{a}}$$

where *probability for parameters given model and data (assume input Gaussian)*

$$\pi(\mathbf{a}\,|\,\mathbf{f}, C_{\mathbf{f}}) \propto \exp\left(-\frac{1}{2}\chi^2(\mathbf{a},\mathbf{f})\right) \quad \text{where} \quad \chi^2(\mathbf{a},\mathbf{f}) = (\mathbf{f}-\mathbf{f}_{\mathrm{BGL}})^T C_{\mathbf{f}}^{-1}(\mathbf{f}-\mathbf{f}_{\mathrm{BGL}})$$

where *prior knowledge is* ***only*** *QFT unitarity constraint (flat prior for BGL params):*

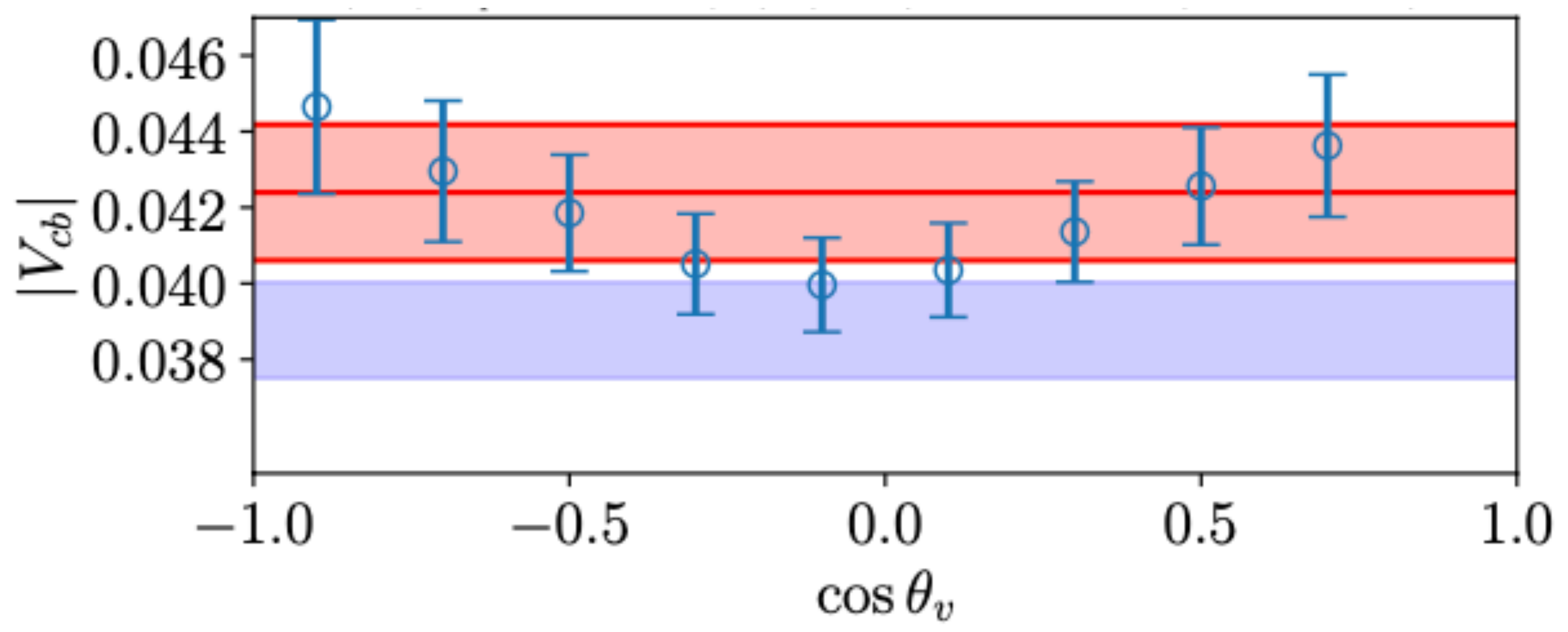
$$\pi_{\mathbf{a}} \propto \theta\left(1-|\mathbf{a}_X|^2\right)$$

In practice MC integration: draw samples for $\mathbf{a}$ from multivariate normal distribution and drop samples not compatible with unitarity

$|V_{cb}|$ — Strategy A: Fit to lattice data

[Bordone, AJ in preparation]

$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\rm exp}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\alpha}\right]^{(i)}_{\rm exp} / \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]^{(i)}_{\rm lat}\right)^{1/2}, \quad \text{where} \quad \Gamma_{\rm exp} = \frac{\mathcal{B}(B^0 \to D^{*,-}\ell^+\nu_\ell)}{\tau(B^0)},$$



blue:
- Frequentist fit $(p, \chi^2/N_{\rm dof}, N_{\rm dof}) = (0.00, 2.82, 8)$
- d'Agostini Bias? [d'Agostini, Nucl.Instrum.Meth.A 346 (1994)]

red:
- Akaike-Information-Criterion analysis [H. Akaike IEEE TAC (19,6,1974)] average over all possible fits with at least two data points and then weighted average:

$$w_{\{\alpha,i\}} = \mathcal{N}^{-1}\exp\left(-\frac{1}{2}(\chi^2_{\{\alpha,i\}} - 2N_{{\rm dof},\{\alpha,i\}})\right) \qquad \mathcal{N} = \sum_{{\rm set}\in\{\alpha,i\}} w_{\rm set}$$

$$|V_{cb}| = \langle |V_{cb}| \rangle \equiv \sum_{{\rm set}\in\{\alpha,i\}} w_{\rm set}|V_{cb}|_{\rm set}$$

- result more *sensible* and bias apparently reduced

Kernel approximation

$$\bar{X}(\mathbf{q}) \approx c_{\mu\nu,0}(\mathbf{q})\ \ C_{\mu\nu}(0,\mathbf{q}) + c_{\mu\nu,1}(\mathbf{q})\ \ C_{\mu\nu}(a,\mathbf{q}) + c_{\mu\nu,2}(\mathbf{q})\ \ C_{\mu\nu}(2a,\mathbf{q}) + \ldots$$

instead of in $(e^{-a\omega})^n$ we now expand in shifted Chebyshev polynomials

Barata, Fredenhagen, Commun.Math.Phys. 138 (1991) 507-520, Bailas et al. PTEP 43-50 (2020), Gambino and Hashimoto PRL 125 32001 (2020)

$$\tilde{T}_k(\omega)\ :\ [\omega_0, \infty] \to [-1,1]$$

$$\tilde{T}_k(\omega) = \sum_{j=0}^{k} \tilde{t}_j^{(k)}\, e^{-ja\omega}$$

$$K_{\mu\nu}(\omega,\mathbf{q}) \approx \frac{\tilde{c}_{\mu\nu,0}}{2} + \sum_{k=1}^{N} \tilde{c}_{\mu\nu,k}\tilde{T}_k(\omega)$$

$$\tilde{c}_{\mu\nu,k} = \int_{\omega_0}^{\infty} d\omega\, K_{\mu\nu}(\omega,\mathbf{q})\tilde{T}_k(\omega)\tilde{\Omega}(\omega)$$