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Theory Predictions for Exclusive $b \to s\ell\ell$

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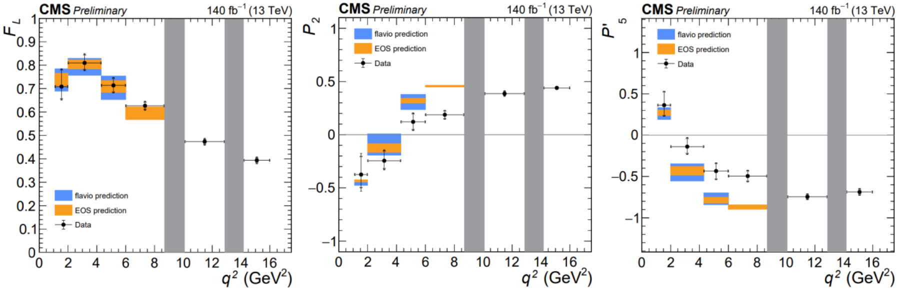
The Flavor Path to NP, Zurich, June 2024

During the last decade there has been a lot of progress in the **experimental measurements** of Exclusive $b \to s\ell\ell$ decays, a lot of efforts devoted to **their interpretation**, and a lot of advances in the **theoretical description**.



This is a talk about the **theory calculations**.

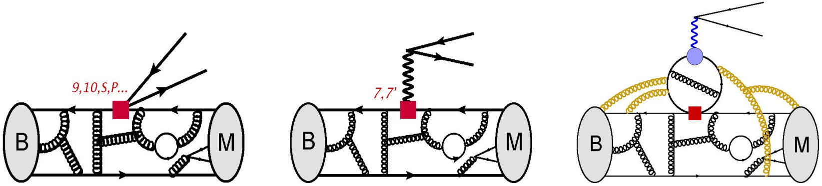
Theory Calculations:

$$\mathcal{L}_{EFT} = \mathcal{L}_{QCD+QED} + \sum_i C_i\ \mathcal{O}_i$$

C_i = Calculated through a perturbative matching calculation

$$\mathcal{A}(B \to f) = \sum_i \underbrace{C_i}_{BSM}\ \underbrace{\langle f|T\{\cdots \mathcal{O}_i \cdots\}|B\rangle}_{QCD}$$

$\langle \mathcal{O}_i \rangle$ = Non-perturbative and difficult to calculate



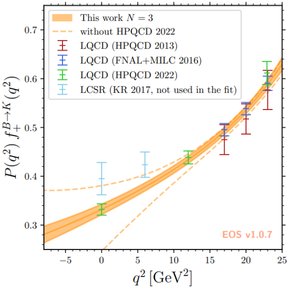
$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

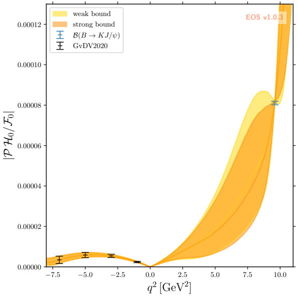
- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s} \, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \, \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \bar{M}_\lambda(k) | T\{ j^\mu_{\rm em}(x), \mathcal{C}_i \, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

Summary

Local

- Theory (LQCD / LCSRs)
- z-expansion (analyticity)
- Dispersive bounds (unitarity) [BGL/BCL]

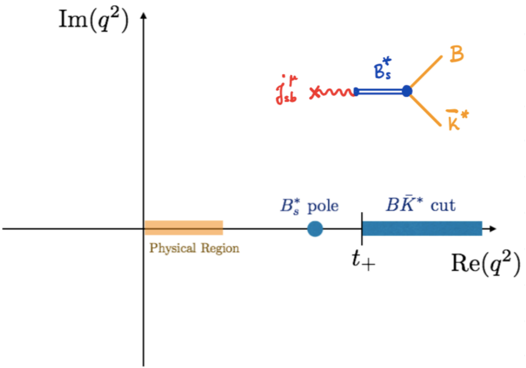




Non-Local

- Theory (QCDF, (LC)OPE, models, ...)
- q^2-dependence -z-expansion
  -dispersion relations
  -phenomenological
- Dispersive bounds
- Fits $\longrightarrow$ "data-driven" methods??

$\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s}\, \Gamma_\lambda^{(T)}\, b \, | \bar{B}(k+q) \rangle$: Analytic structure in q^2 :

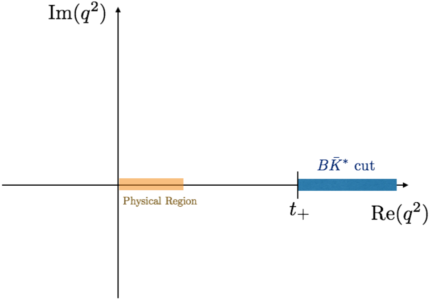


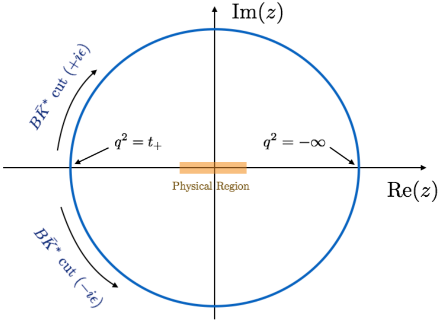
$\widehat{\mathcal{F}}_\lambda^{(T)}(q^2) \equiv (q^2 - m_{B_s^*}^2)\, \mathcal{F}_\lambda^{(T)}(q^2)$ has no pole, only cut.

Local Form Factors : q^2-dependence from analyticity

Bourrely, Caprini, Lellouch; Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert; ...

- Conformal map : $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$





- ”z-parametrization” : $\widehat{\mathcal{F}}_\lambda^{(T)}(q^2(z))$ is **analytic in** $|z| < 1$ $\quad (|z_{\text{phys}}| < 0.15)$

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{(q^2 - m_{B_s^*}^2)} \sum_k \alpha_k \, z(q^2)^k$$

Local Form Factors : Dispersive Bounds (BGL)

Boyd, Grinstein, Lebed 1997; Bharucha, Feldmann, Wick 2014

1. One starts with the two-point function

$$\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|T\{J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\}|0\rangle = \Pi_\Gamma^{(J=0)}(q^2)\left[\frac{q^\mu q^\nu}{q^2}\right] + \Pi_\Gamma^{(J=1)}(q^2)\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right]$$

2. The invariant functions fulfil a once-subtracted dispersion relation:

$$\chi_\Gamma^{(\lambda)}(Q^2) = \left[\frac{\partial}{\partial q^2}\right] \Pi_\Gamma^{(\lambda)}(q^2)\Big|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\Pi_\Gamma^{(\lambda)}(s)}{(s-Q^2)^2}\,.$$

3. The function $\chi_\Gamma^{(\lambda)}(Q^2)$ can be calculated in an OPE at a suitable substraction point Q^2
Bharucha, Feldmann, Wick 2014

4. The discontinuity of $\Pi_\Gamma^{(\lambda)}(q^2)$ is the spectral function:

$$\mathrm{Im}\Pi_\Gamma^{(\lambda)}(s) \sim \sum_H \langle 0|J^\mu|H\rangle\langle H|J^{\nu\dagger}|0\rangle \sim f_{B_s^*}^2 + |F^{BK}|^2 + |F^{BK^*}|^2 + |F^{B_s\phi}|^2 + \cdots$$

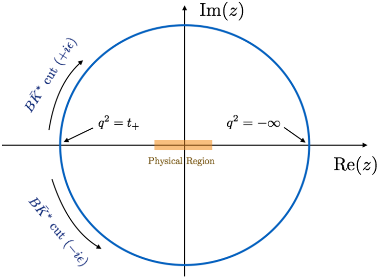
(up to phase-space functions...)

The two-body contributions are, e.g.

$$\chi_A^{(J=1)}\Big|_{BK^*} = \frac{\eta^{B\to K^*}}{24\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3}\left[s\,(M_B+M_{K^*})^2 \boxed{|A_1^{B\to K^*}|^2} + 32\, M_B^2 M_{K*}^2 \boxed{|A_{12}^{B\to K^*}|^2}\right]$$

In order to simplify the bound, it is thus convenient to reparametrize:

$$\hat{\mathcal{F}}_\lambda^{B\to M}(q^2) = \mathcal{B}_{\mathcal{F}}(z)\phi_{\mathcal{F}}(z)\mathcal{F}_\lambda^{B\to M}(q^2) = \sum_k \alpha_k^{\mathcal{F}}\, z(q^2)^k$$



$$\sum_{B\to M}\int_{-\pi}^{+\pi} d\theta \left|\hat{\mathcal{F}}_\lambda^{B\to M}(e^{i\theta})\right|^2 < 1$$

$$\boxed{\sum_{\mathcal{F},k} |\alpha_k^{\mathcal{F}}|^2 < 1}$$

Local Form Factors: 2 variations

Gubernari, Reboud, van Dyk, Virto 2023

1. "Polarized" 2-point function decomposition

$$\Pi_\Gamma^{\mu\nu}(q) = \sum_{\lambda=t,\perp,\|,0} \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*} \, \Pi_\Gamma^{(\lambda)}(q^2)$$

- This is the bound used in the literature:

$$\chi_A^{(J=1)}\Big|_{BK^*} = \frac{\eta^{B\to K^*}}{24\pi^2} \int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\text{kin}}^{1/2}}{s^2(s-Q^2)^3} \left[s\,(M_B+M_{K^*})^2 |A_1^{B\to K^*}|^2 + 32\, M_B^2 M_{K^*}^2 |A_{12}^{B\to K^*}|^2 \right]$$

- And this is what we propose:

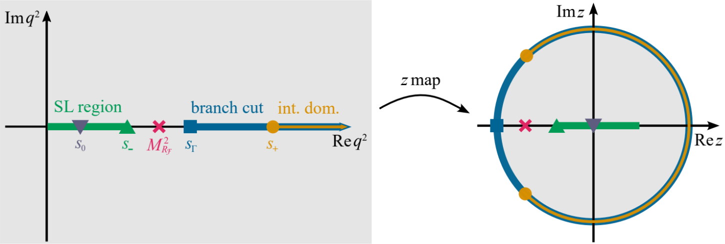
$$\chi_A^{(0)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{\pi^2} \int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\text{kin}}^{1/2}}{s^2(s-Q^2)^3} 4\, M_B^2 M_{K^*}^2 |A_{12}^{B\to K^*}|^2 ,$$

$$\chi_A^{(\|)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{8\pi^2} \int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\text{kin}}^{1/2}}{s^2(s-Q^2)^3} s\,(M_B+M_{K^*})^2 |A_1^{B\to K^*}|^2 ,$$

Local Form Factors: 2 variations

Flynn, Jüttner, Tsang 2023; Gubernari, Reboud, van Dyk, Virto 2022, 2023

2. Correct threshold, different from trivial one:



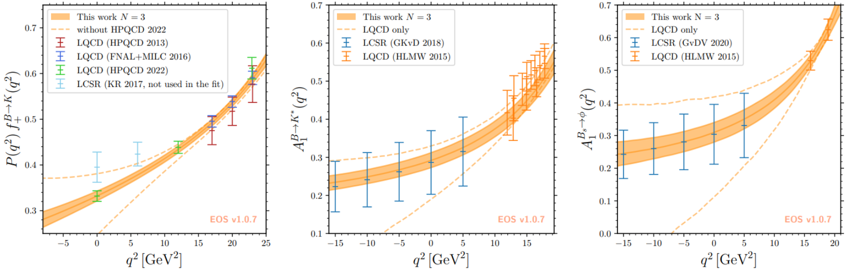
$$\int_{-\alpha_\mathcal{F}}^{+\alpha_\mathcal{F}} d\theta\, p_m^\mathcal{F}(e^{i\theta}) p_n^\mathcal{F}(e^{-i\theta}) = \delta_{mn}$$

$$\hat{\mathcal{F}}_\lambda^{B\to M}(q^2) = \mathcal{B}_\mathcal{F}(z)\phi_\mathcal{F}(z)\mathcal{F}_\lambda^{B\to M}(q^2) = \sum_k \alpha_k^\mathcal{F}\; p_k^\mathcal{F}(z)$$

$$\sum_{B\to M} \int_{-\alpha_\mathcal{F}}^{+\alpha_\mathcal{F}} d\theta \left|\hat{\mathcal{F}}_\lambda^{B\to M}(e^{i\theta})\right|^2 < 1 \quad\Rightarrow\quad \boxed{\sum_{\mathcal{F},k} |\alpha_k^\mathcal{F}|^2 < 1}$$

Local Form Factor Fits

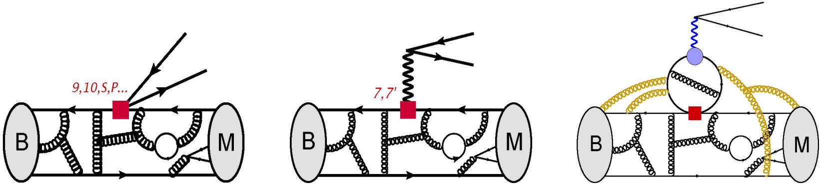
Gubernari, Reboud, van Dyk, Virto, 2305.06301



Truncate the series expansion to $N = 2, 3, 4$

Uncertainties stable for $N > 2$

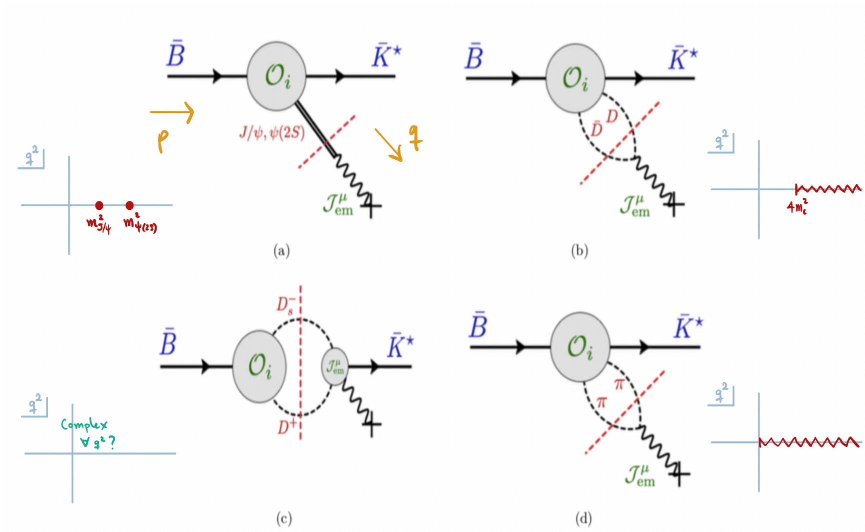
Non-Local Form Factors



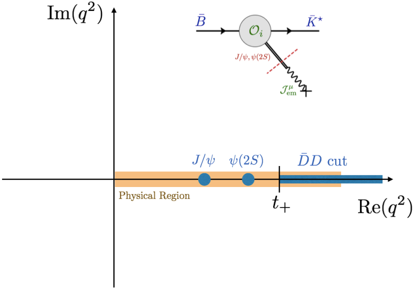
$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

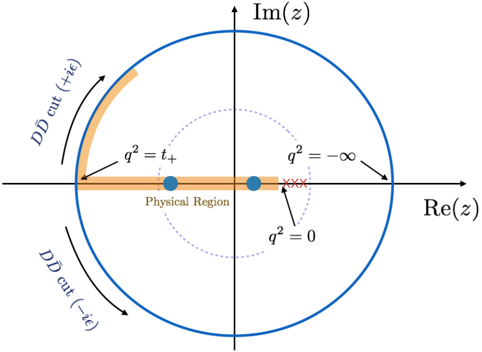
- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s} \, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \, \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \bar{M}_\lambda(k) | \mathcal{T}\{ \mathcal{J}_{\rm em}^\mu(x), \mathcal{C}_i \, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

Non-Local Form Factors: Analytic structure



z-parametrisation for $\mathcal{H}_\lambda(q^2)$





- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$ is analytic in $|z| < 1$
- Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$: $\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{H}_\lambda(z)$
- Expansion needed for $|z| < 0.52$ ($-7\,\mathrm{GeV}^2 \leq q^2 \leq 14\mathrm{GeV}^2$)

Dispersive bound

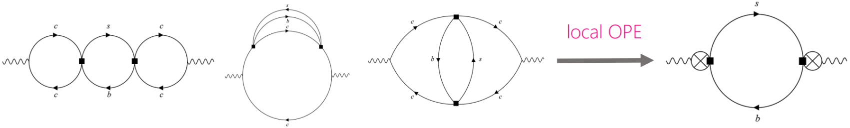
1. Consider the correlation function

$$\Pi(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0 | T\left\{ O^{\mu}(q;x), O^{\mu,\dagger}(q;0) \right\} |0\rangle$$

where

$$O^{\mu}(q;x) = -i \int d^4y\, e^{+iq\cdot y}\; T\{ j^{\mu}_{\text{em}}(x+y), (C_1\mathcal{O}_1 + C_2\mathcal{O}_2)(x) \}$$

2. Calculate in OPE region



$$\chi^{\text{OPE}}(-m_b^2) = (1.81 \pm 0.02) \times 10^4 \text{GeV}^{-2}$$

3. Twice-subtracted dispersion relation:

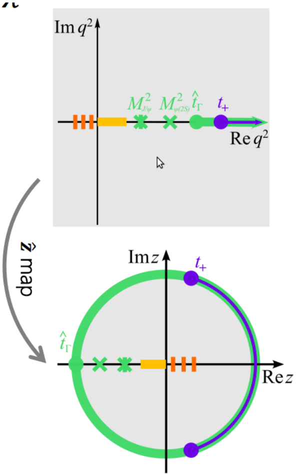
$$\chi^{\text{OPE}}(Q^2) \equiv \frac{1}{2i\pi} \int_0^\infty ds \, \frac{\text{Disc}_{b\bar{s}}\Pi^{\text{had}}(s)}{(s-Q^2)^3}$$

$$\frac{3}{32i\pi^3} \text{Disc}_{b\bar{s}}\Pi^{\text{had}}(s) = \frac{2M_B^4\,\lambda^{3/2}(M_B^2, M_K^2, s)}{s^4} \left|\mathcal{H}_0^{B\to K}(s)\right|^2 \theta(s - s_{BK})$$
$$+ \frac{2M_B^6\,\sqrt{\lambda(M_B^2, M_{K^*}^2, s)}}{s^3} \left(\left|\mathcal{H}_\perp^{B\to K^*}(s)\right|^2 + \left|\mathcal{H}_\parallel^{B\to K^*}(s)\right|^2 + \frac{M_B^2}{s}\left|\mathcal{H}_0^{B\to K^*}(s)\right|^2 \right) \theta(s - s_{BK^*})$$
$$+ \frac{M_B^6\,\sqrt{\lambda(M_{B_s}^2, M_\phi^2, s)}}{s^3} \left(\left|\mathcal{H}_\perp^{B_s\to \phi}(s)\right|^2 + \left|\mathcal{H}_\parallel^{B_s\to \phi}(s)\right|^2 + \frac{M_{B_s}^2}{s}\left|\mathcal{H}_0^{B_s\to \phi}(s)\right|^2 \right) \theta(s - s_{B_s\phi})$$

$+$ further positive terms

Dispersive bound

Gubernari, van Dyk, Virto 2020



Redefine $\mathcal{H}_i$ as before:

$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) \equiv \phi_\lambda^{B\to M}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to M}(z)\,,$$

Expand in ortogonal polynomials in **arc**:

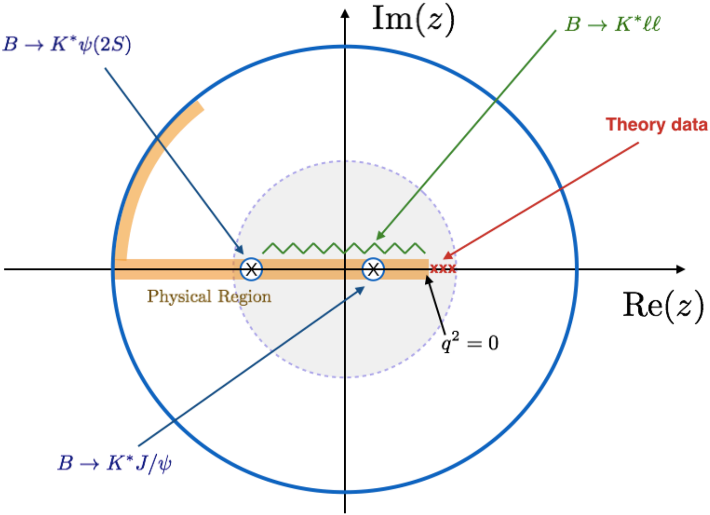
$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, p_n^{B\to M}(z)$$

The dispersive bound then takes the simple form

$$\sum_{n=0}^{\infty}\left\{2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\|,0}\left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right]\right\} < 1\,.$$

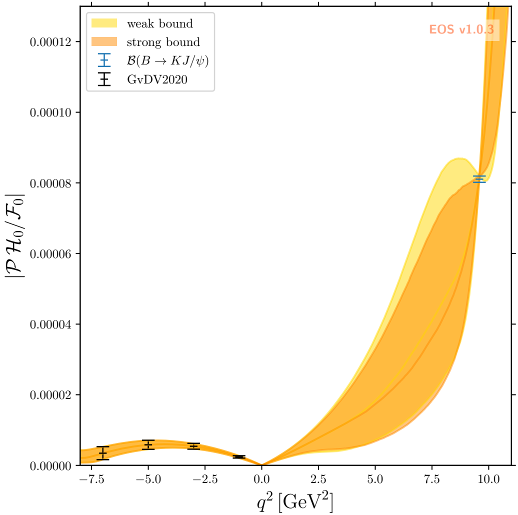
Fit to z-parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017

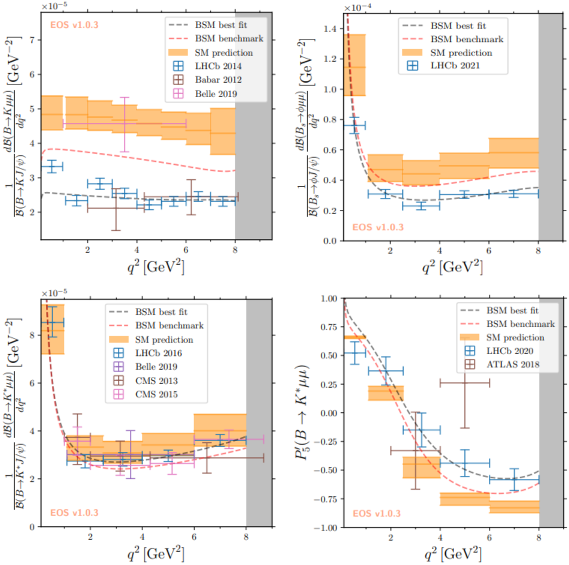


Results

Gubernari, Reboud, van Dyk, Virto 2022



Gubernari, Reboud, van Dyk, Virto 2022



Issues:

1. Is the **theory data** reliable?
2. Validity of z-expansion: Do we understand the **analytic structure**?
3. Truncation of z-expansion $\rightarrow$ **dispersive bound**
4. Technical aspects of **fits** (convergence, interpretation, ...)

Any **concern** should be linked to one of these points **clearly**.

Theory data

Operator Product Expansion in selected regions:

$$\mathcal{H}^\mu(q,k) = i\int d^4x\, e^{iq\cdot x}\,\langle \bar{M}_\lambda(k)|\mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\,\mathcal{O}_i(0)\}|\bar{B}(q+k)\rangle$$

- Large-q^2: Dominated by $x\sim 0$ (short-distance dominance - OPE)

  Grinstein, Pirjol; Beylich, Buchalla, Feldmann

- Low-q^2: Dominated by $x^2\sim 0$ (light-cone dominance - LCOPE)

  Khodjamirian, Mannel, Pivovarov, Wang

In both cases,

$$\mathcal{K}^\mu(q) = i\int d^4x\, e^{iq\cdot x}\,\mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\,\mathcal{O}_i(0)\}$$
$$= \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\,\bar{s}\gamma_\nu P_L b + \Delta C_7(q^2)\,2im_b\,\bar{s}\sigma^{\mu\nu}q_\nu P_R b + \cdots$$

Thus,

$$\mathcal{H}^\mu_{\rm OPE}(q,k) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\mathcal{F}_\nu + 2im_b\,\Delta C_7(q^2)\mathcal{F}^{T\mu} + \cdots$$

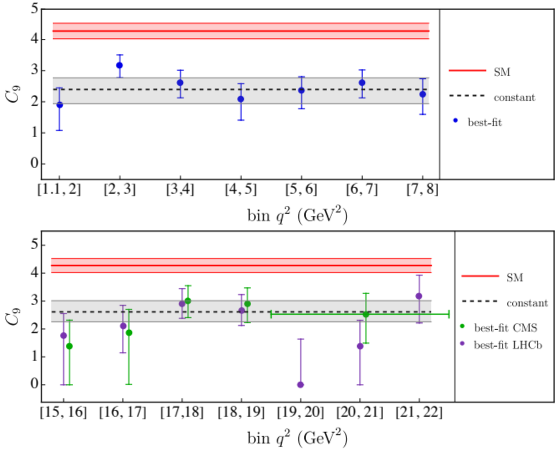
Theory data

- Unfortunately, no LQCD calculations here (so far).
- Theory is always based on some form of **factorization** (e.g. QCD Factorization in HQL, OPE in smart place, etc).
- Thus any problem you have in e.g. $B \to \pi\pi$, likely here too.
- But you can choose q^2 (e.g. $q^2 \ll 0$). This improves things.
- The leading term here is exactly factorizable ($C_9\mathcal{F}$) thus the **error** is "subleading".
- One may also try fitting the z-expansion / dispersion relation / model **without theory data**

  Mauri, Blake, Owen, Petridis, LHCb... 1709.03921, 2312.09102, 2405.17347

Tests

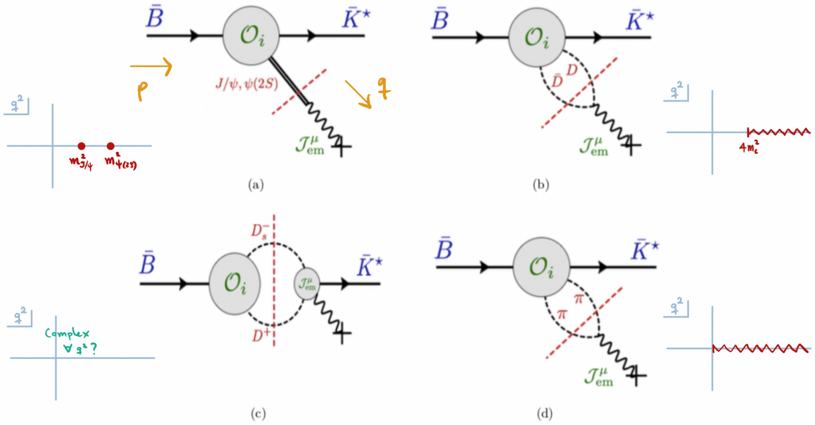
There is one important property that distinguishes C_9 from $\Delta C_9^\lambda(q^2)$:



Bordone, Isidori, Mächler, Tinari 2024

(see also Altmannshofer, Straub 2014; Descotes-Genon, Hofer, Matias, Virto 2015.)

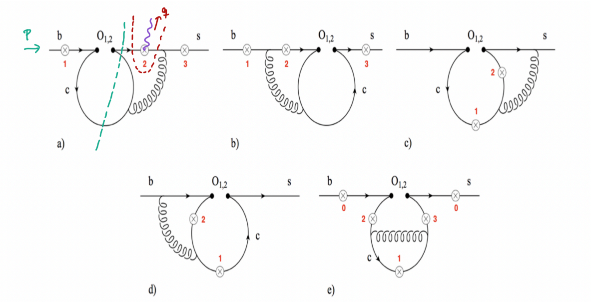
Analytic structure



Analytic structure

- There is a "light-hadron" cut for $q^2 > 0$, but it is OZI suppressed.
- p^2 cut makes $\mathcal{H}(q^2)$ complex everywhere, but does it affect q^2?
- Partonic calculation mimics all singularities (must be a Theorem)
- Two-loop partonic calculation confirms analytic structure

  Asatrian, Greub, Virto 2019



Analytic structure

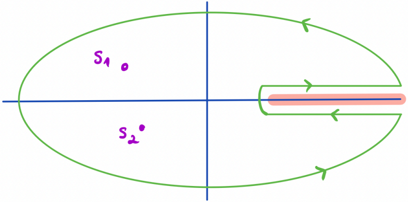
Direct check of analytic structure at two loops: Asatian, Greub, Virto 2019

$$F(s_1) - F(s_2) = \frac{s_1 - s_2}{2\pi i} \int_{s_{\rm th}}^{\infty} dt \frac{F(t+i0) - F(t-i0)}{(t-s_1)(t-s_0)}$$

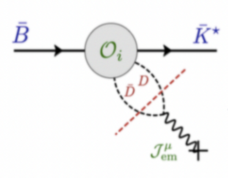
Example:

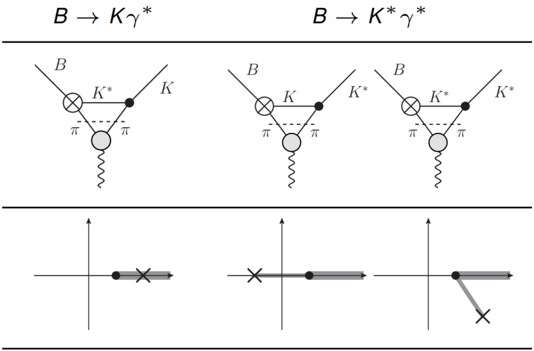
$$F^{(7)}_{2,(b)}(-3+i) - F^{(7)}_{2,(b)}(-1-2i) = 0.0894864 - 0.160827\, i\,,$$

$$\frac{-2+3i}{2\pi i} \int_4^{\infty} dt \, \frac{{\rm Disc}\, F^{(7)}_{2,(b)}(t)}{(t+3-i)(t+1+2i)} = 0.0894966 - 0.160839\, i\,.$$



Left-Hand Cuts?





$$\Pi_{P,\lambda}(s) = \underbrace{\frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\nu_{P,\lambda}(s')\, g_{P,\lambda}(s')\, F_\pi^{V*}(s')}{s'-s}}_{\equiv \Pi^{\rm norm}_{P,\lambda}(s)} + \underbrace{\frac{1}{\pi}\int_0^1 dx \, \frac{\partial s_x}{\partial x}\, \frac{\nu_{P,\lambda}(s_x)\, {\rm disc}\, g_{P,\lambda}(s_x)\, F_\pi^{V*}(s_x)}{s_x - s}}_{\equiv \Pi^{\rm anom}_{P,\lambda}(s)}$$

First mentioned by Ciuchini et al 2022 but in the context of the p^2 cut

Are they there? Are they sizable? Can we modify the z-expansion?

Conclusion

- It really looks like $C_9 \sim C_9^{\rm SM} - 1 \simeq 3$
- But this is a difficult scenario for theory
- The winning strategy is most probably a **wise use of theory in a data-driven approach**...
- ... but it must be done **model-independently**.
- So far, theory + z-expansion + dispersive bounds holds some promise in analogy to the local form factors, which are increasingly established.

Thank you