Global Fits of $b \to s l^+ l^-$ Transitions

Luca Silvestrini
INFN, Rome

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- Current status
  - LUV fits
  - LU fits
- Conclusions

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and a big hug to Quim!

HEPfit

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Why $b \to s l^+ l^-$ Decays

- $b \to s l^+ l^-$ transitions are FCNC
  - cannot occur at tree-level in the SM
  - they arise at one loop through penguins and boxes
  - they are particularly sensitive to possible NP contributions
  - since $V_{ub}V_{us}^* \ll V_{cb}V_{cs}^* \approx V_{tb}V_{ts}^*$, top and charm quarks dominate loop contributions

Short- and long-distance in $b \to s l^+ l^-$ decays

- $b \to s l^+ l^-$ transitions in the SM mediated by Z-penguins, electroweak boxes and photonic penguins
- Z-penguins and electroweak boxes hard-GIM suppressed, i.e. $\propto \frac{m_{u_i}^2}{M_W^2}$; charm and up negligible
- photonic penguins soft-GIM suppressed, i.e. $\propto \log \frac{m_{u_i}^2}{M_W^2}$; logarithmic IR sensitivity: $\log \frac{\mu^2}{M_W^2}$ from mixing with charm current-current operators

Short- and long-distance in $b \to s l^+ l^-$ decays

- Two semileptonic operators at the hadronic scale, C_{9V} and C_{10A}:
  - C_{10A} purely generated at the electroweak scale, $C_{10A}(m_b) \sim C_{10A}(M_W)$
    - C_{10A} is a short distance quantity
  - C_{9V} dominated by RG log, with sizable scale dependence:

$$\frac{C_{9V}(m_b/2) - C_{9V}(2m_b)}{C_{9V}(m_b)} \sim 15\%$$

    - EW scale contribution not dominant, sensitive to scales $\leq m_b$, C_{9V} not a short distance quantity
    - scale dependence canceled by matrix element of four-quark operators
    - it is already evident in perturbation theory that the hadronic matrix element provides a contribution to C_{9V} that cannot be separated from the short-distance one, unless the matrix element can be explicitly calculated

Short- and long-distance in $b \to s l^+ l^-$ decays

The nonlocal contributions manifest as resonances in the $q^2 \equiv m^2_{\mu\mu}$ spectrum.

LHCb, 2405.17347

- Unfortunately, life is not that easy…

$$\langle 0 | J_{\rm em}(y) \phi_{K^{(*)}}(x) O^c_{1,2}(0) | B \rangle = \sum_n \langle 0 | J_{\rm em}(y) \phi_{K^{(*)}}(x) | n \rangle \langle n | O^c_{1,2}(0) | B \rangle$$

for $x_0, y_0 > 0$

- On-shell intermediate states n are not resonances in the $q^2 = m^2_{\mu\mu}$ spectrum, neither are they represented by a cut in q^2
- One should not identify long-distance contributions with charmonium and $D\bar{D}$ states. This is incomplete and leads to incorrect parameterizations, which in turn might lead to incorrect conclusions…

Short- and long-distance in $b \to sl^+l^-$ decays

The approach followed in this analysis begins with expressing the nonlocal term $Y_{q\bar{q},\lambda}(q^2)$ as a subtracted hadronic dispersion relation [41]

$$Y_{q\bar{q},\lambda}(q^2) = Y_{q\bar{q},\lambda}(q_0^2) + \frac{(q^2 - q_0^2)}{\pi} \int_{4m_\mu^2}^{\infty} \frac{\rho_{q\bar{q},\lambda}(s)}{(s - q_0^2)(s - q^2 - i\epsilon)} \mathrm{d}s, \tag{19}$$

where q_0^2 is the subtraction point discussed below and the spectral density function $\rho_{q\bar{q},\lambda}$ contains the complete information on hadronic real intermediate states that contribute to the $B^0 \to K^{*0}\mu^+\mu^-$ decay.

The expression for the nonlocal contributions given by Eq. 19 exploits the fact that $Y_{q\bar{q},\lambda}(q^2)$ is perturbatively calculable via an operator product expansion in the (unphysical) region of $q^2 \lesssim 0$ [30]. If one performs such a calculation, thereby fixing the subtraction

LHCb, 2405.17347; [41] is Khodjamirian, Mannel & Wang '12; [30] is Khodjamirian et al '10.

Anomalous thresholds: where do they come from?

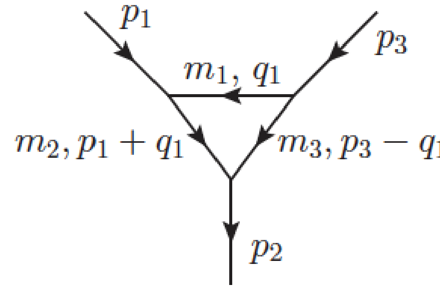
- **Landau equations**: singularities of general loop integral

$$\int \prod_{\ell=1}^{L} \frac{d^4 q_\ell}{(2\pi)^4} \prod_{i=1}^{n} \frac{i}{k_i^2 - m_i^2 + i\epsilon} \quad \text{singular when} \quad \begin{cases} \lambda_i (k_i^2 - m_i^2) = 0 & \text{for all } i = 1, \ldots, n \\ \sum_{i=1}^{n} \lambda_i k_i \cdot \frac{\partial k_i}{\partial q_\ell} = 0 & \ell = 1, \ldots, L \end{cases}$$

$\hookrightarrow$ "leading singularity" $\Leftrightarrow$ all $\lambda_i \neq 0$

- **Triangle diagram**: $L = 1$, $n = 3$, Landau equations become

$$\lambda_i (k_i^2 - m_i^2) = 0 \qquad \sum_{i=1}^{3} \lambda_i k_i = 0$$



  - **Normal thresholds**: e.g., $\lambda_3 = 0 \Rightarrow p_1^2 = (m_1 \pm m_2)^2$

    [zeros of $\lambda(p_1^2, m_1^2, m_2^2)$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$]

  - **Anomalous threshold**: all $\lambda_i \neq 0$

$$\hookrightarrow p_2^2 = s_\pm \equiv p_1^2 \frac{m_1^2 + m_3^2}{2m_1^2} + p_3^2 \frac{m_1^2 + m_2^2}{2m_1^2} - \frac{p_1^2 p_3^2}{2m_1^2} - \frac{(m_1^2 - m_2^2)(m_1^2 - m_3^2)}{2m_1^2} \pm \frac{1}{2m_1^2} \sqrt{\lambda(p_1^2, m_1^2, m_2^2)\lambda(p_3^2, m_1^2, m_3^2)}$$

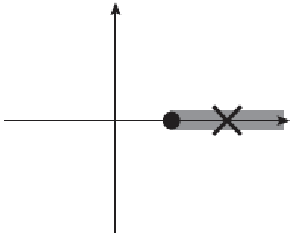
Anomalous thresholds: deformation of the integration contour

- Anomalous branch point on first sheet (can be either s_+ or s_-) requires **deformation of the integration contour**

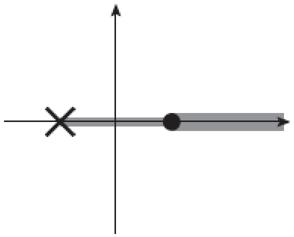
$$s_x = x(m_2 + m_3)^2 + (1 - x)s_\pm$$

- Three cases:

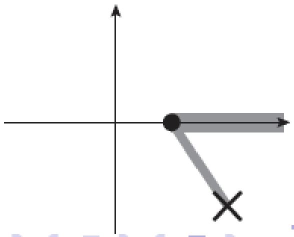
1. $s_\pm$ on normal cut
   $\hookrightarrow$ analytic continuation of normal discontinuity



2. $s_\pm$ on negative real axis
   $\hookrightarrow$ integration deformed along real axis

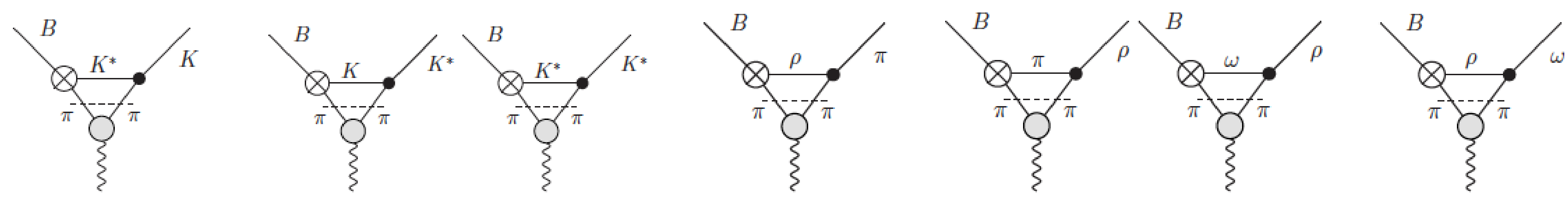


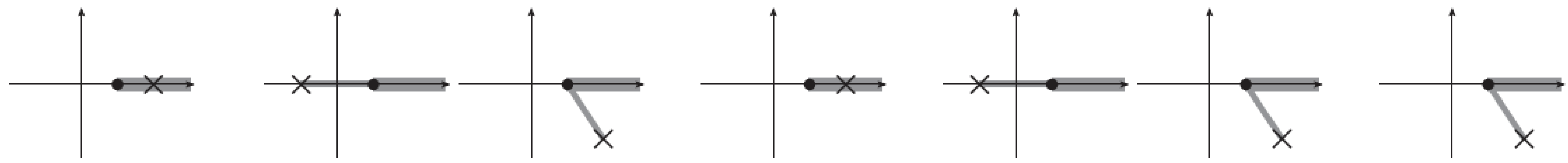
3. $s_\pm$ in complex plane
   $\hookrightarrow$ integration deformed into complex plane



Anomalous thresholds for $B \to (P, V)\gamma^*$: list of processes

$B \to K\gamma^*$	$B \to K^*\gamma^*$	$B \to \pi\gamma^*$	$B \to \rho\gamma^*$	$B \to \omega\gamma^*$



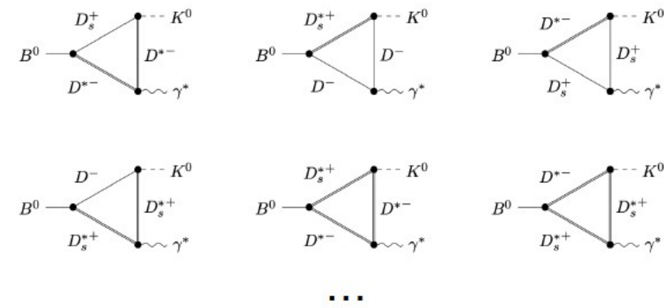


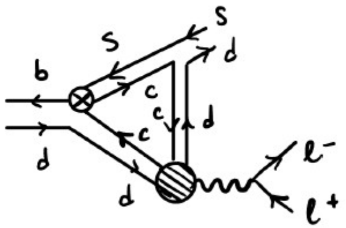
$s_+ = 18.6\,\text{GeV}^2$	-57.8 $0.5 - 4.2i$	26.4	-859.3 $0.7 - 4.8i$	$0.2 - 4.9i$
$\text{Br}[B \to K^*\pi]$	$\text{Br}[B \to K^{(*)}\pi]$	$\text{Br}[B \to \rho\pi]$	$\text{Br}[B \to \pi\pi, \pi\omega]$	$\text{Br}[B \to \rho\pi]$
$\text{Br}[B \to K\pi\pi]$	$\text{Br}[B \to K^*\pi\pi]$	$\text{Br}[B \to 3\pi]$	$\text{Br}[B \to \rho\pi\pi]$	$\text{Br}[B \to \omega\pi\pi]$

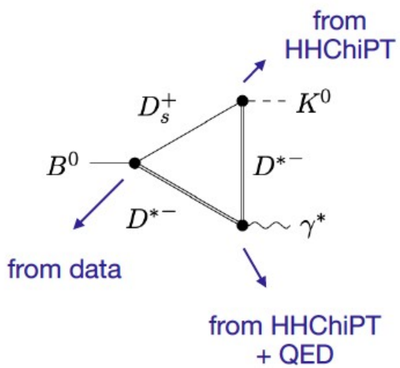
- $s_{\text{thr}} = 4M_\pi^2 = 0.08\,\text{GeV}^2$
- The branching fractions in the last line assume $\pi\pi$ in a P-wave.
- Consider K^*, ρ, ω narrow for now (could integrate over spectral functions).
- To disentangle helicity amplitudes, not only branching ratios, but polarization fractions are required.

Charm rescattering in $B \to K\bar{\ell}\ell$

- We cannot exclude a sizable long-distance contribution with a reduced q^2- or λ- dependence which would mimic a short-distance effect.
- For this reason, we tried to estimate the rescattering contribution from the leading two-body intermediate state $D_s D^*$ and $D_s^* D$.







- We estimate this diagram using data on $B \to DD^*$ and Heavy Hadron Chiral Perturbation Theory (valid for soft kaons).
- Our result is most reliable close to the q^2 end-point (small kaon momentum), and satisfies constraints from gauge invariance.
- The absorptive part is finite and "exact" (no approximations) at the end-point.

Our analysis partially confirms the findings of Ref. [21] that these rescattering contributions, usually neglected in theory-driven estimates of $B \to K^{(*)}\mu^+\mu^-$ amplitudes, are relatively flat in q^2 (far from the narrow charmonium states) and can mimic a short-distance effect. On

Isidori, Polonski & Tinari 2405.17551

My Pedestrian Attitude

- Waiting for a calculation of charming penguins in QCD, let us take a conservative attitude and use data

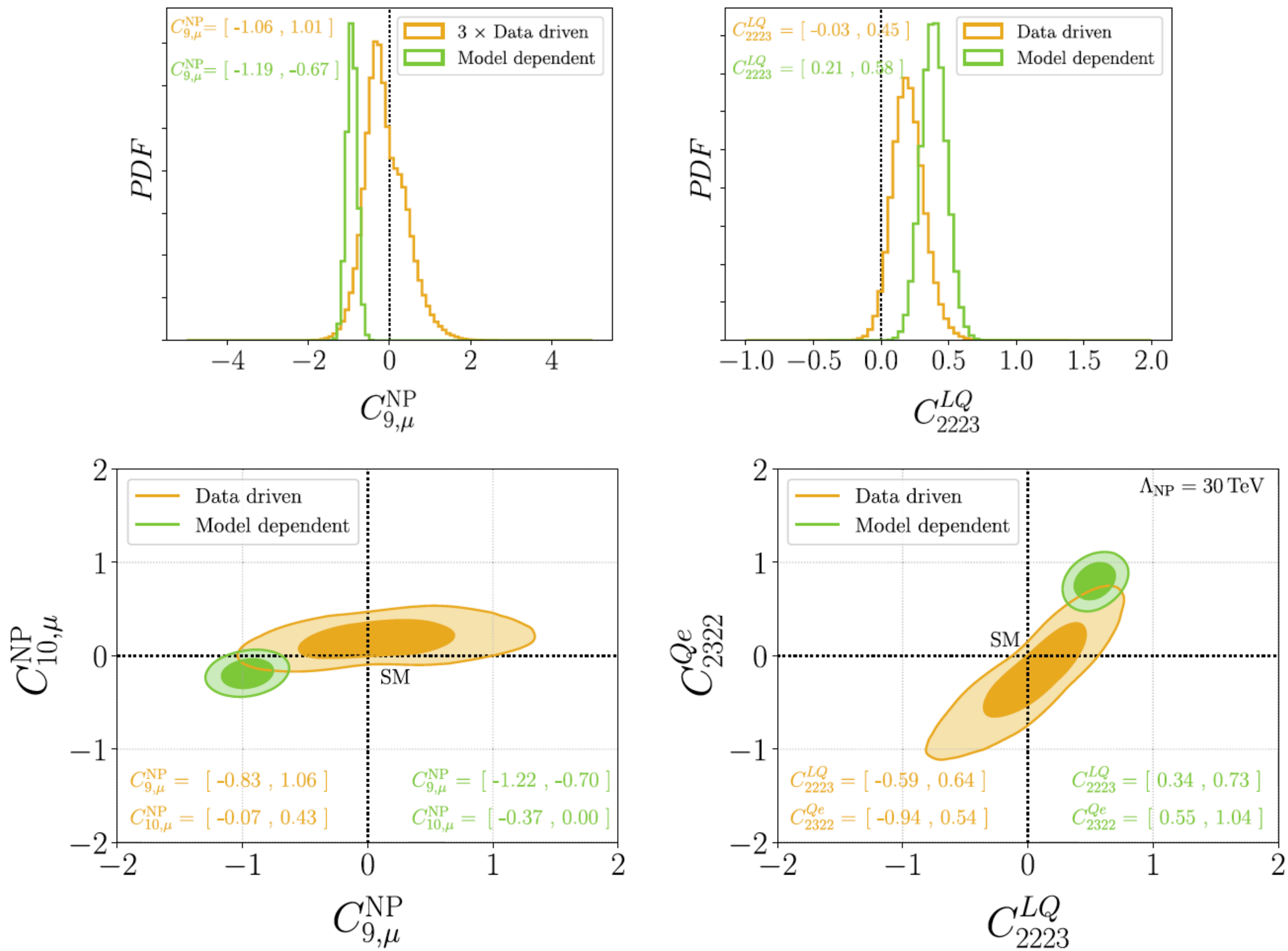
- Parametrize charming penguins as smooth functions of q^2, Taylor-expanded for small q^2:

$$
\begin{aligned}
H_V^- &\propto \left\{ \left(C_9^{\rm eff} + h_-^{(1)} \right) \tilde{V}_{L-} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\rm eff} + h_-^{(0)} \right) \tilde{T}_{L-} - 16\pi^2 h_-^{(2)} q^4 \right] \right\} , \\
H_V^+ &\propto \left\{ \left(C_9^{\rm eff} + h_-^{(1)} \right) \tilde{V}_{L+} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\rm eff} + h_-^{(0)} \right) \tilde{T}_{L+} - 16\pi^2 \left(h_+^{(0)} + h_+^{(1)} q^2 + h_+^{(2)} q^4 \right) \right] \right\} , \\
H_V^0 &\propto \left\{ \left(C_9^{\rm eff} + h_-^{(1)} \right) \tilde{V}_{L0} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\rm eff} + h_-^{(0)} \right) \tilde{T}_{L0} - 16\pi^2 \sqrt{q^2} \left(h_0^{(0)} + h_0^{(1)} q^2 \right) \right] \right\} . \qquad (3.4)
\end{aligned}
$$

- Evidently with this pedestrian but efficient choice $h^{(0)}_-$ is equivalent to a shift of C_7 and $h^{(1)}_-$ is equivalent to a shift of C_9. All other parameters represent genuine hadronic contributions.

Data-Driven vs Model-Dependent: LUV

MARCO CIUCHINI *et al.* PHYS. REV. D **107,** 055036 (2023)



Data-Driven vs Model-Dependent: LUV

	95% HPDI	ΔIC
$C_{9,\mu}^{\rm NP}$	[−1.06, 1.01] **[−1.19, −0.67]**	−2.4 **43**
$\{C_{9,\mu}^{\rm NP}, C_{10,\mu}^{\rm NP}\}$	{[−0.83, 1.06], [−0.07, 0.43]} **{[−1.22, −0.70],[−0.37,0.00]}**	−3.4 **41**
$\{C_{9,\mu}^{\rm NP}, C_{9,\mu}^{\prime,\rm NP}\}$	{[−1.06, 1.40], [−2.20, 1.31], **{[−1.33, −0.79],[0.08,0.88],**	−4.1 **45**
$\{C_{9,\mu}^{\rm NP}, C_{10,\mu}^{\prime,\rm NP}\}$	{[−1.07, 1.20], [−0.28, 0.20], **{[−1.34, −0.77],[−0.39,0.02],**	−5.1 **41**
$\{C_{9,\mu}^{\rm NP}, C_{10,\mu}^{\rm NP}, C_{9,\mu}^{\prime,\rm NP}, C_{10,\mu}^{\prime,\rm NP}\}$	{[−0.90, 1.49], [−0.15, 0.62], [−2.27, 1.18], [−0.33, 0.47]} **{[−1.38, −0.82],[−0.39,0.02],** **[−0.49,0.79],[−0.46,0.17]}**	−8.1 **57**

	95% HPDI	ΔIC
C_{2223}^{LQ}	[−0.03, 0.45] **[0.21, 0.58]**	−1.6 **3**
$\{C_{2223}^{LQ}, C_{2322}^{Qe}\}$	{[−0.59, 0.64], [−0.94, 0.54]} **{[0.34,0.73],[0.55,1.04]}**	−3.4 **41**
$\{C_{2223}^{LQ}, C_{2223}^{ed}\}$	{[−0.03, 0.48], [−0.39, 0.32]} **{[0.24,0.63],[−0.95, −0.09]}**	−4.0 **6**
$\{C_{2223}^{LQ}, C_{2223}^{Ld}\}$	{[−0.06, 0.65], [−0.24, 0.49]} **{[0.18,0.57],[−0.14,0.23]}**	−5.1 **−2**
$\{C_{2223}^{LQ}, C_{2322}^{Qe}, C_{2223}^{Ld}, C_{2223}^{ed}\}$	{[−0.88, 0.78], [−1.26, 0.57], [−0.76, 1.58], [−0.98, 1.64]} **{[0.43,0.84],[0.62,1.16],** **[−0.50,0.10],[−0.64,0.63]}**	−8.1 **57**

Bold: model-dependent. The second and last scenarios are fully equivalent in the SMEFT and LEFT parameterizations, so ΔICs are identical.

The SM is always slightly preferred in the Data-Driven approach, while NP is strongly preferred in the Model-Dependent case.

LU ΔC_9 or Charming Penguins?

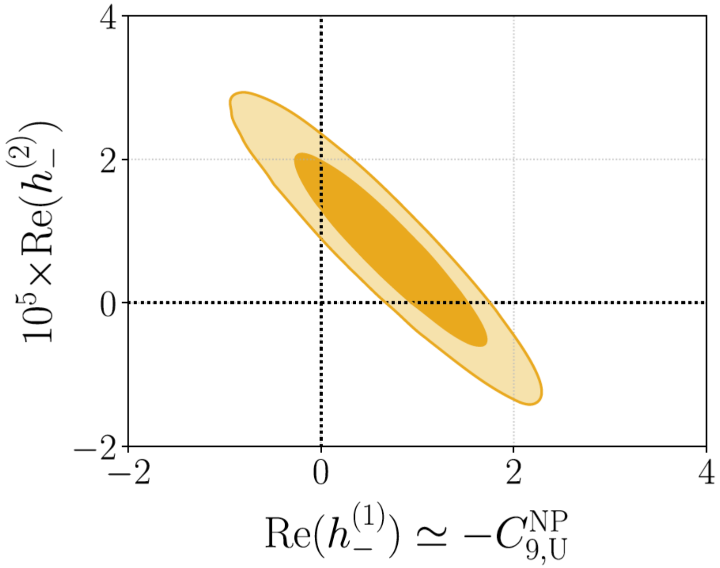


FIG. 7. Joint posterior PDF for $\mathrm{Re}(h_-^{(1)})$ and $\mathrm{Re}(h_-^{(2)})$ in a SM fit in the data-driven scenario. Darker (lighter) regions correspond to 68% (95%) probability. Notice that according to our hadronic parametrization given in Eq. (5), $\mathrm{Re}(h_-^{(1)})$ can be reinterpreted as a lepton universal NP contribution, $C_{9,\mathrm{U}}^{\mathrm{NP}}$.

- ΔC_9 is not only degenerate with the corresponding term in the hadronic amplitude, but also with other terms in the expansion
- More precise data could help resolving this kind of degeneracy; however, the fundamental one between ΔC_9 and $h^{(1)}_-$ requires a theoretical calculation.

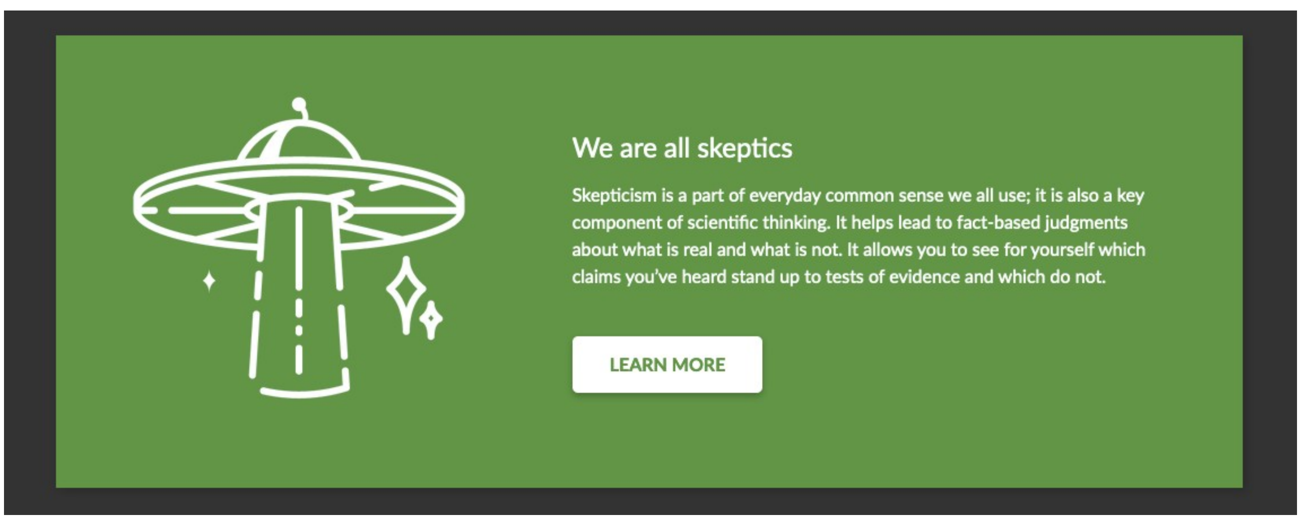
Miracles…

function of q^2 with a polynomial expansion. Wilson coefficients and nonlocal hadronic parameters are determined under two alternative hypotheses: the first relies on experimental information alone, while the

TABLE V. Best fit value, confidence intervals and deviation from the SM predictions [29,30] for the four Wilson coefficients and the two fit configurations. For each Wilson coefficient, the likelihood has been profiled over the other coefficients. The SM predictions at the b-quark energy scale [29,30] are also reported for reference.

	$q^2 > 0$ only				
	Best fit value	68% C.I.	95% C.I.	SM value	Deviation from SM (σ)
$\mathcal{C}_9$	3.34	[2.77, 3.87]	[2.30, 4.33]	4.27	1.9
$\mathcal{C}_{10}$	−3.69	[−4.00, −3.40]	[−4.33, −3.12]	−4.17	1.5
$\mathcal{C}'_9$	0.48	[−0.07, 0.97]	[−0.62, 1.45]	0	0.9
$\mathcal{C}'_{10}$	0.38	[0.13, 0.66]	[−0.14, 0.92]	0	1.5

LHCb, 2312.09102



Let's check this…

skepticalinquirer.org

ΔC_9, our pedestrian q^2 expansion and the z-expansion

- LHCb writes the amplitudes as

$$\mathcal{A}_\perp^{L,R} = \mathcal{N}\left\{[(\mathcal{C}_9 + \mathcal{C}_9') \mp (\mathcal{C}_{10} + \mathcal{C}_{10}')]\mathcal{F}_\perp(q^2,k^2) + \frac{2m_b M_B}{q^2}\left[(\mathcal{C}_7 + \mathcal{C}_7')\mathcal{F}_\perp^T(q^2,k^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_\perp(q^2)\right]\right\},$$

$$\mathcal{A}_\parallel^{L,R} = -\mathcal{N}\left\{[(\mathcal{C}_9 - \mathcal{C}_9') \mp (\mathcal{C}_{10} - \mathcal{C}_{10}')]\mathcal{F}_\parallel(q^2,k^2) + \frac{2m_b M_B}{q^2}\left[(\mathcal{C}_7 - \mathcal{C}_7')\mathcal{F}_\parallel^T(q^2,k^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_\parallel(q^2)\right]\right\},$$

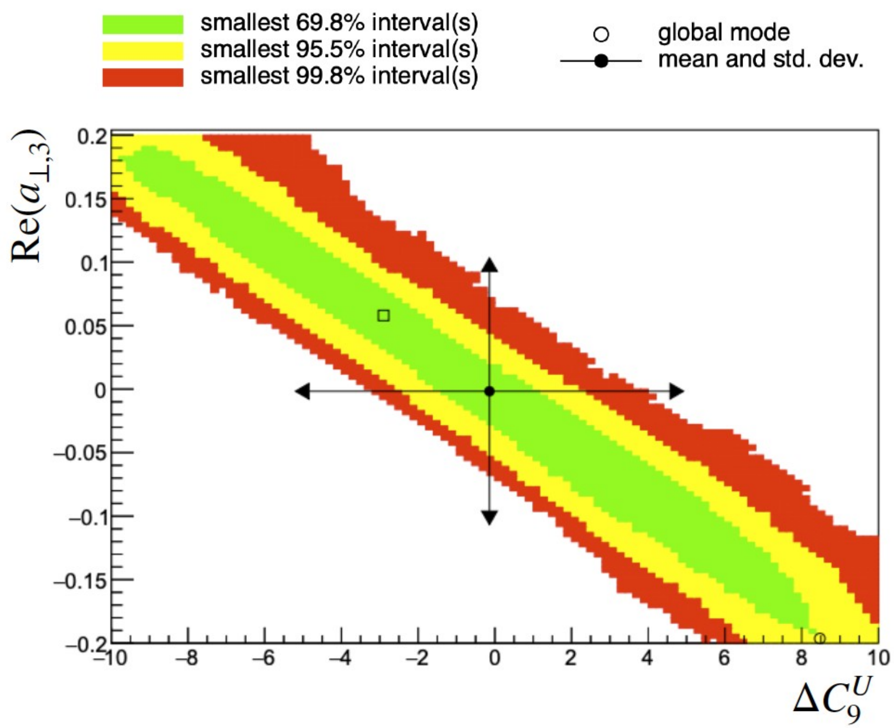
$$\mathcal{A}_0^{L,R} = -\mathcal{N}\frac{M_B}{\sqrt{q^2}}\left\{[(\mathcal{C}_9 - \mathcal{C}_9') \mp (\mathcal{C}_{10} - \mathcal{C}_{10}')]\mathcal{F}_0(q^2,k^2) + \frac{2m_b M_B}{q^2}\left[(\mathcal{C}_7 - \mathcal{C}_7')\mathcal{F}_0^T(q^2,k^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_0(q^2)\right]\right\},$$

without explicitly identifying the possible ΔC_7 and ΔC_9 components in H_λ. Then uses the analyticity-inspired conformal mapping $q^2 \mapsto z(q^2) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$, in local and non-local form factors:

$$\hat{\mathcal{H}}_\lambda(z) = \phi_\lambda^{-1}(z)\sum_k \alpha_{\lambda,k} z^k$$

ΔC_9, our pedestrian q^2 expansion and the z-expansion

- It is easy to check that each order of the polynomial expansion of $H_\lambda(z)$ contains a q^2-independent term, so that the hadronic ΔC_9 receives new contributions at each order in z;
- Truncating the expansion too early might lead to an apparent model-independent determination of ΔC_9.



NP CLAIMS

- M. Ciuchini

But ... really a reliable estimate of uncertainties is missing and theory must be improved otherwise we will continue to generate anomalies out of our ingnorance

G. Martinelli @ Planck2024

Conclusions

- $b \to s l^+ l^-$ transitions potentially affected by charming penguins
- In spite of the many (very) recent theoretical efforts, we still have to rely on models if we want to disentangle possible NP in C_9 from hadronic effects
- With current data, no firm model-independent conclusion can be drawn on the presence (or absence) of evidence for sizable hadronic effects. But please remember: absence of evidence is not evidence of absence!
- Forthcoming experimental and theoretical progress will allow us to improve our understanding of charming penguins and hopefully clarify whether NP is hiding there or not!