University of Glasgow

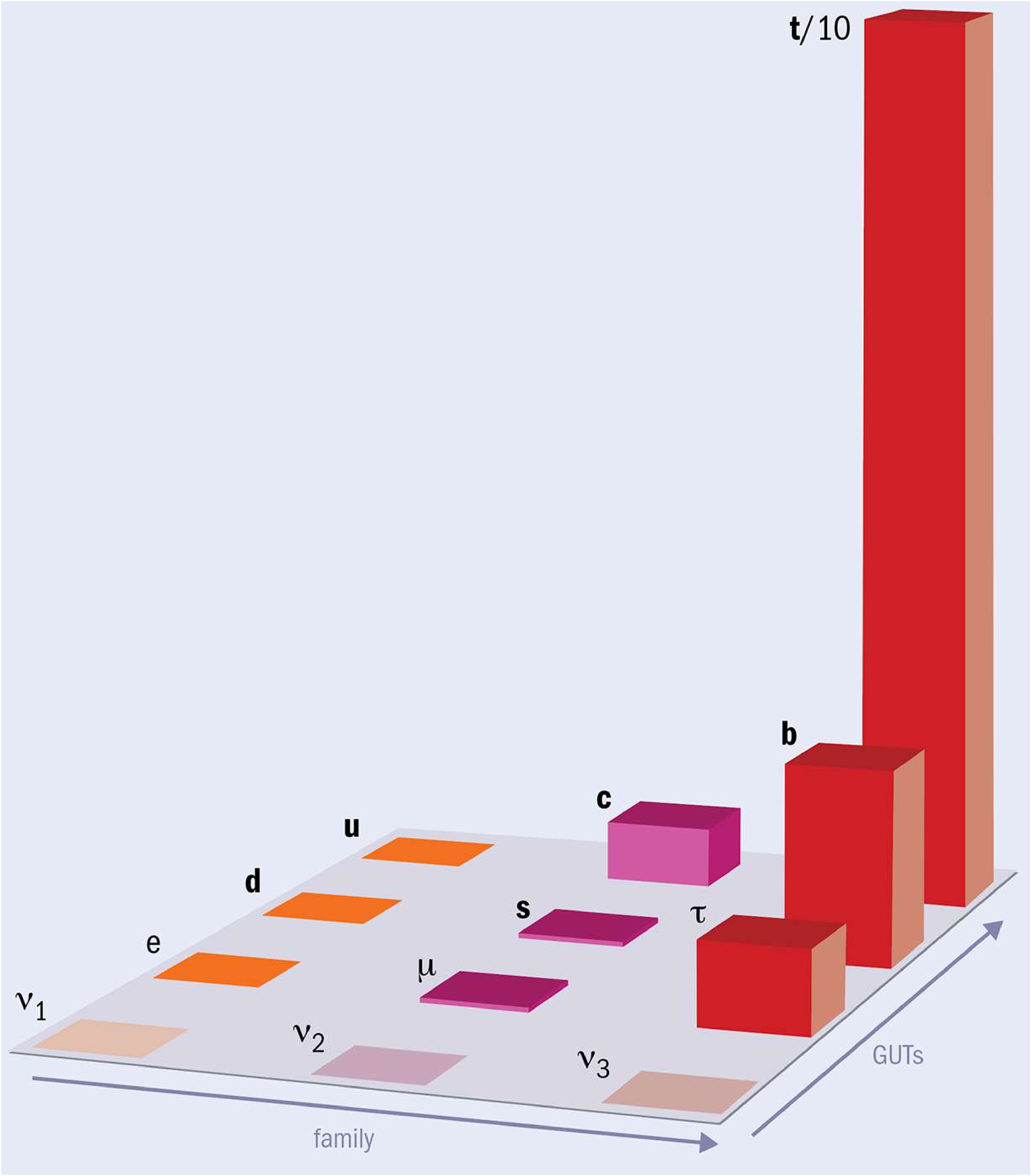
Flavour deconstruction and gauge unification

The Flavour Path to New Physics,
5th June 2024,
Zurich, Switzerland

Mario Fernández Navarro

Based on:
MFN, Stephen F. King, [2305.07690] hep-ph, JHEP 08 (2023) 020
MFN, Stephen F. King and Avelino Vicente, [2311.05683] hep-ph, JHEP 05 (2024) 130
MFN, Stephen F. King and Avelino Vicente, [2404.12442] hep-ph

The flavour puzzle



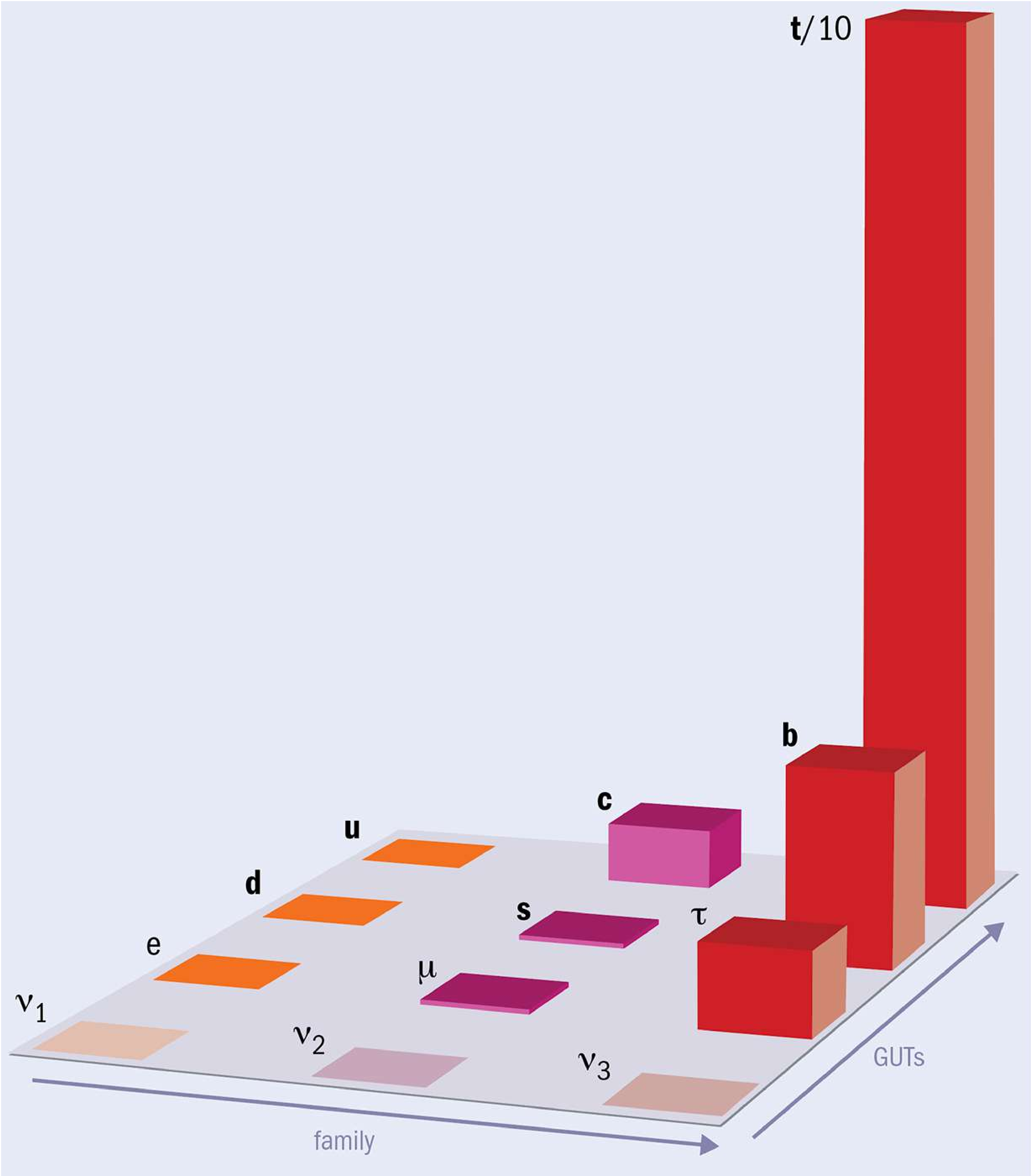
$$m_t \sim \frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_c \sim \lambda^{3.3}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_u \sim \lambda^{7.5}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$
$$m_b \sim \lambda^{2.5}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_s \sim \lambda^{5.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_d \sim \lambda^{7.0}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$
$$m_\tau \sim \lambda^{3.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_\mu \sim \lambda^{4.9}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_e \sim \lambda^{8.4}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

$$\tan\theta_{23}^\nu \sim 1\,, \; \tan\theta_{12}^\nu \sim \frac{1}{\sqrt{2}}\,, \; \theta_{13}^\nu \sim \frac{\lambda}{\sqrt{2}}\,, \qquad V_{us} \sim \lambda\,, \; V_{cb} \sim \lambda^2\,, \; V_{ub} \sim \lambda^3$$

$$\text{where } v_{\rm SM} \simeq 246\,\text{GeV} \;\text{ and }\; \boxed{\lambda \simeq \sin\theta_C \simeq 0.224}$$

- Three fermion families are identical (replicated) objects under the SM symmetry, yet they interact so differently with the Higgs

The flavour puzzle



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$$\tan\theta^\nu_{23} \sim 1\,, \; \tan\theta^\nu_{12} \sim \frac{1}{\sqrt{2}}\,, \; \theta^\nu_{13} \sim \frac{\lambda}{\sqrt{2}}\,, \qquad V_{us} \sim \lambda\,, \; V_{cb} \sim \lambda^2\,, \; V_{ub} \sim \lambda^3$$

where $v_{\rm SM} \simeq 246\,{\rm GeV}$ and $\lambda \simeq \sin\theta_C \simeq 0.224$

- Three fermion families are identical (replicated) objects under the SM symmetry, yet they interact so differently with the Higgs

→ Family (gauge) structure hidden at high energies?

Tri-hypercharge: an example of flavour deconstruction

- Flavour deconstruction: SM is embedded in a gauge symmetry that contains a separate factor for each fermion family:

$$G_{\text{universal}} \times G_1 \times G_2 \times G_3$$

[Salam 79', Rajpoot 81', Li and Ma 81', Georgi 82' … Bordone *et al* 17', Greljo and Stefanek 18', Fuentes-Martín *et al*, 20', Davighi and Isidori 23' …]

See more in Joe Davighi's talk tomorrow!

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A simple example:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3} \longrightarrow Y_{\text{SM}} \equiv Y = Y_1 + Y_2 + Y_3$$

Field	$U(1)_{Y_i}$
q_i	$1/6$
u_i^c	$-2/3$
d_i^c	$1/3$
ℓ_i	$-1/2$
e_i^c	1

q_1	q_2	q_3
u_1^c	u_2^c	u_3^c
d_1^c	d_2^c	d_3^c
ℓ_1	ℓ_2	ℓ_3
e_1^c	e_2^c	e_3^c
		H_3

‣ Light families are massless in first approximation:

$$\mathcal{L} = y_t q_3 H_3 u_3^c + y_b q_3 H_3 d_3^c + y_\tau \ell_3 H_3 e_3^c + \text{h.c.}$$

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q_1	q_2	q_3
u_1^c	u_2^c	u_3^c
d_1^c	d_2^c	d_3^c
ℓ_1	ℓ_2	ℓ_3
e_1^c	e_2^c	e_3^c
		$H_3^{u,d}$

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$$+\text{h.c.}$$

- Type II 2HDM can take care of $m_{b,\tau}/m_t$ hierarchies via

$$\tan\beta = v_u/v_d \approx \lambda^{-2} \simeq 20$$

Tri-hypercharge: EFT

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \approx 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

- E.g. $q_2 H_3^d d_2^c \sim (0, \frac{1}{2}, -\frac{1}{2})$ and $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ are forbidden by tri-hypercharge (gauge) symmetry

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- Add "23 hyperons" $\phi_{\ell 23} \sim (0, -\frac{1}{2}, \frac{1}{2})$ and $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

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$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\tilde{\phi}_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{q12}^2}{\Lambda_1^2}\frac{\phi_{q23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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- Add "12 hyperons" $\phi_{\ell 12} \sim (-\frac{1}{2}, \frac{1}{2}, 0)$ and $\phi_{q12} \sim (-\frac{1}{6}, \frac{1}{6}, 0)$

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- Add "12 hyperons" $\phi_{\ell 12} \sim (-\frac{1}{2}, \frac{1}{2}, 0)$ and $\phi_{q 12} \sim (-\frac{1}{6}, \frac{1}{6}, 0)$
- When these four hyperons (scalars) get VEVs, SM flavour structure is dynamically generated (also in up and charged lepton sectors)

Neutrino masses and mixings also included → Please ask!

Tri-hypercharge: EFT

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\tilde{\phi}_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{q12}^2}{\Lambda_1^2}\frac{\phi_{q23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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Heavy messengers needed for Λs!

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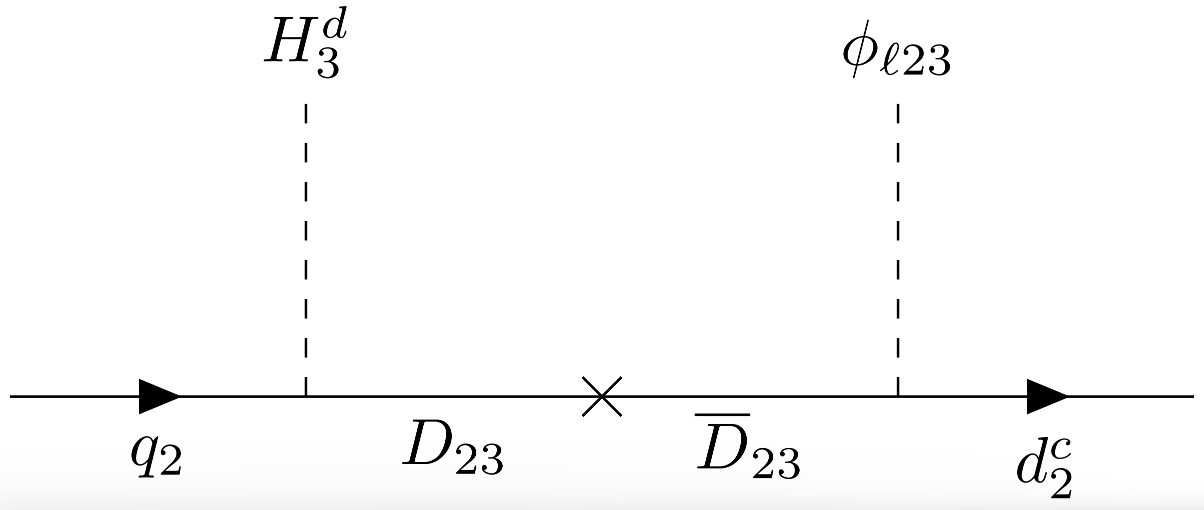
Tri-hypercharge: UV-complete models

Model 1

Field	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1},\mathbf{2})$
$\phi_{q_{12}}$	-1/6	1/6	0	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell_{12}}$	-1/2	1/2	0	$(\mathbf{1},\mathbf{1})$
$\phi_{q_{23}}$	0	-1/6	1/6	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell_{23}}$	0	-1/2	-1/2	$(\mathbf{1},\mathbf{1})$
U_{12}	-1/6	-1/2	0	$(\overline{\mathbf{3}},\mathbf{1})$
U_{13}	-1/6	0	-1/2	$(\overline{\mathbf{3}},\mathbf{1})$
U_{23}	0	-1/6	-1/2	$(\overline{\mathbf{3}},\mathbf{1})$
D_{12}	-1/6	1/2	0	$(\overline{\mathbf{3}},\mathbf{1})$
D_{13}	-1/6	0	1/2	$(\overline{\mathbf{3}},\mathbf{1})$
D_{23}	0	-1/6	1/2	$(\overline{\mathbf{3}},\mathbf{1})$
E_{12}	1/2	1/2	0	$(\mathbf{1},\mathbf{1})$
E_{13}	1/2	0	1/2	$(\mathbf{1},\mathbf{1})$
E_{23}	0	1/2	1/2	$(\mathbf{1},\mathbf{1})$

- Completion via $SU(2)$-singlet VL fermions
- Most simple scalar sector and potential



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$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1}, \mathbf{2})$
ϕ_{q12}	-1/6	1/6	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell 12}$	-1/2	1/2	0	$(\mathbf{1}, \mathbf{1})$
ϕ_{q23}	0	-1/6	1/6	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell 23}$	0	-1/2	-1/2	$(\mathbf{1}, \mathbf{1})$
U_{12}	-1/6	-1/2	0	$(\overline{\mathbf{3}}, \mathbf{1})$
U_{13}	-1/6	0	-1/2	$(\overline{\mathbf{3}}, \mathbf{1})$
U_{23}	0	-1/6	-1/2	$(\overline{\mathbf{3}}, \mathbf{1})$
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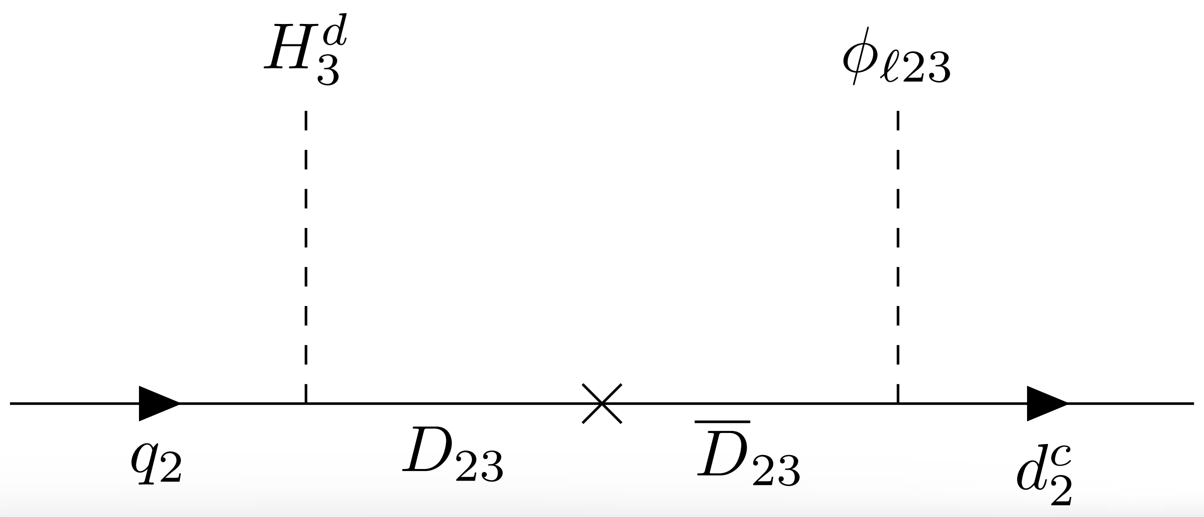

Model 2

Field	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1}, \mathbf{2})$
H_2^d	0	$-1/2$	0	$(\mathbf{1}, \mathbf{2})$
H_1^d	$-1/2$	0	0	$(\mathbf{1}, \mathbf{2})$
ϕ_{q12}	-1/6	1/6	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell 12}$	-1/2	1/2	0	$(\mathbf{1}, \mathbf{1})$
ϕ_{q23}	0	-1/6	1/6	$(\mathbf{1}, \mathbf{1})$
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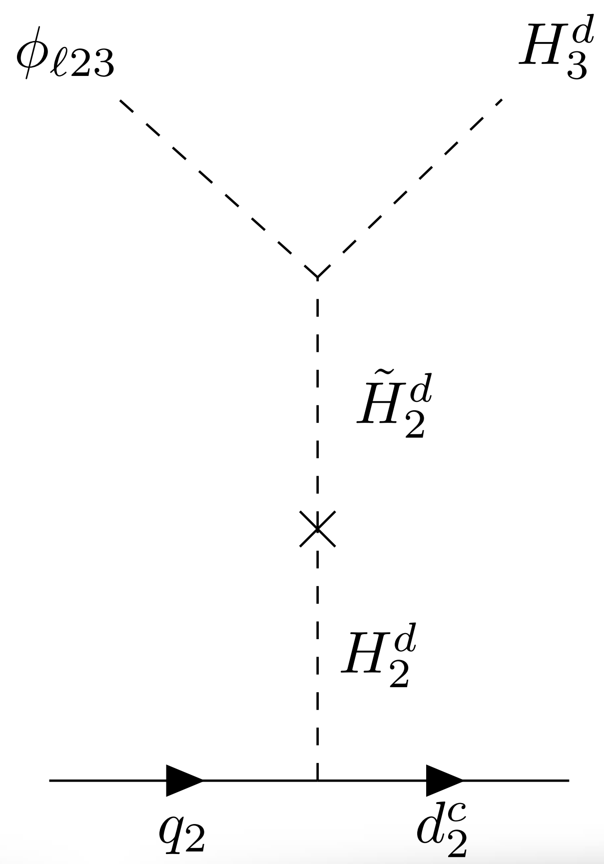
- Minimal number of degrees of freedom and representations
- Diagonal down-quark and charged lepton mass matrices



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$\phi_{q_{12}}$	-1/6	1/6	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell_{12}}$	-1/2	1/2	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{q_{23}}$	0	-1/6	1/6	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell_{23}}$	0	-1/2	-1/2	$(\mathbf{1}, \mathbf{1})$
U_{12}	-1/6	-1/2	0	$(\overline{\mathbf{3}}, \mathbf{1})$
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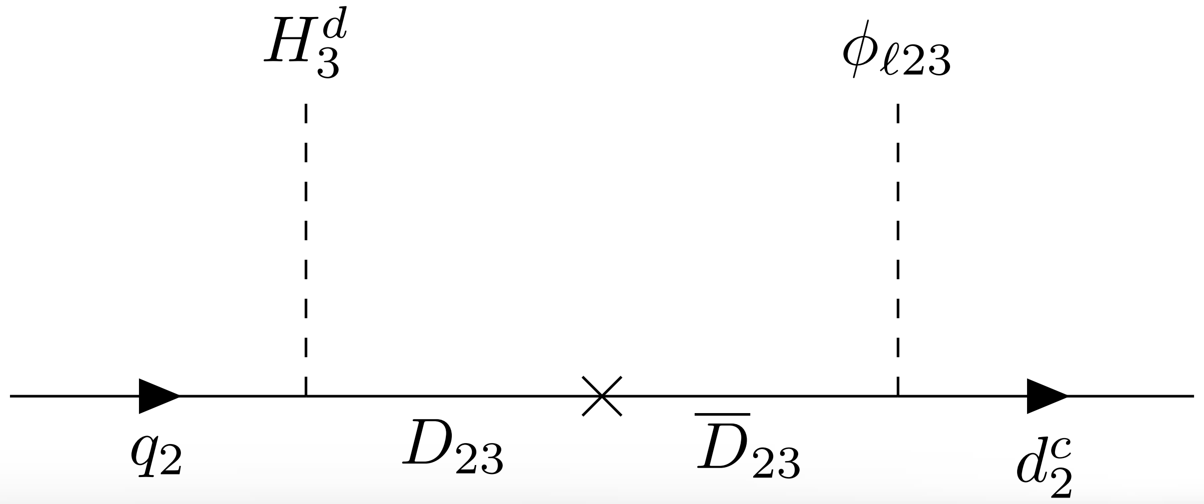

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H_2^d	0	$-1/2$	0	$(\mathbf{1}, \mathbf{2})$
H_1^d	$-1/2$	0	0	$(\mathbf{1}, \mathbf{2})$
$\phi_{q_{12}}$	-1/6	1/6	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell_{12}}$	-1/2	1/2	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{q_{23}}$	0	-1/6	1/6	$(\mathbf{1}, \mathbf{1})$
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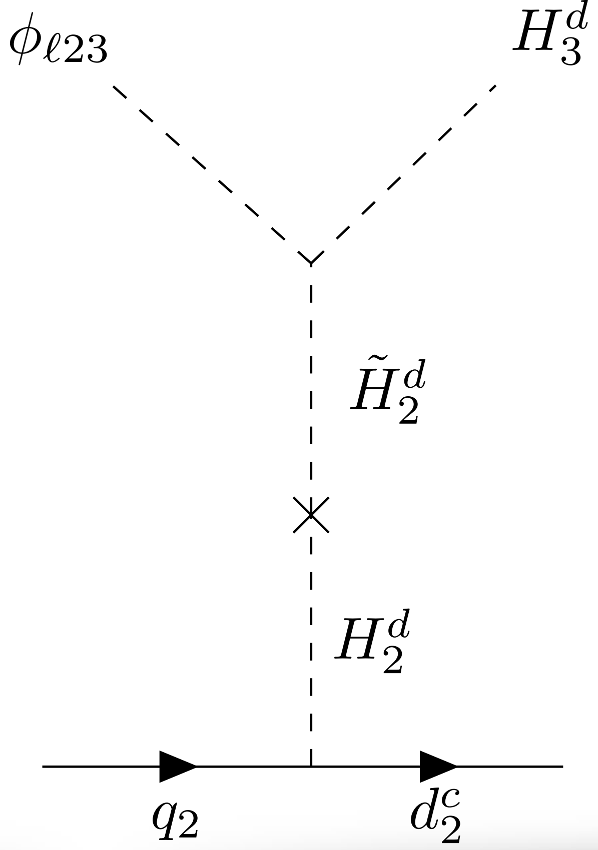
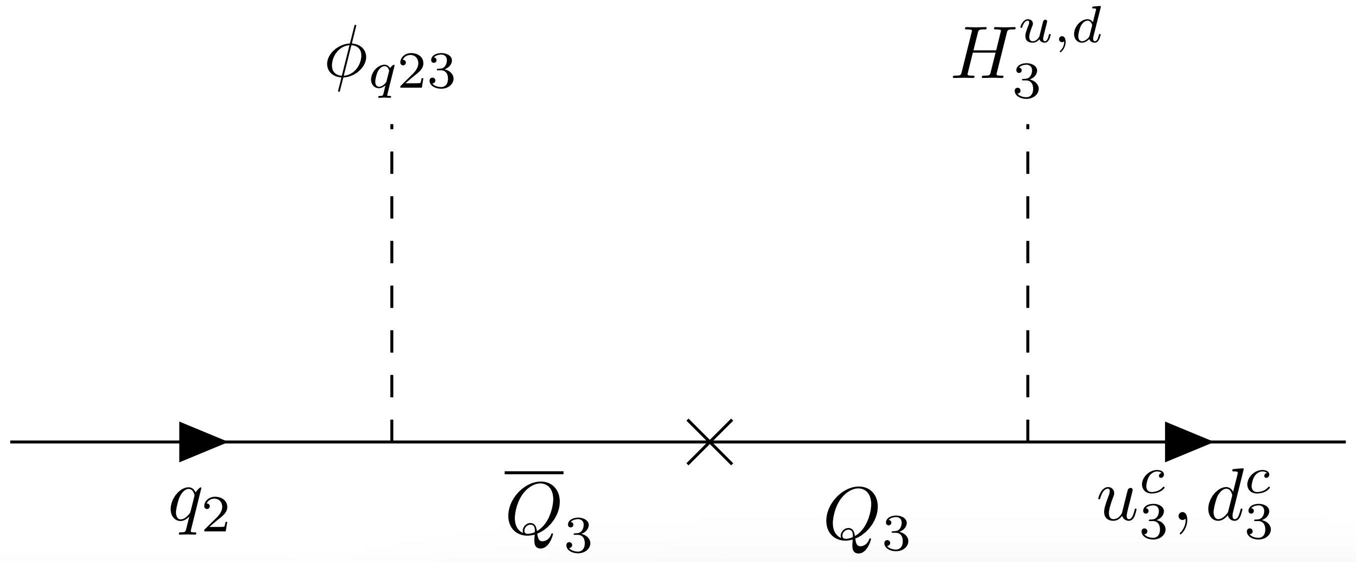


Model 3

	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1}, \mathbf{2})$
$H_2^{u,d}$	0	$\pm 1/2$	0	$(\mathbf{1}, \mathbf{2})$
$H_1^{u,d}$	$\pm 1/2$	0	0	$(\mathbf{1}, \mathbf{2})$
$\phi_{q_{12}}$	-1/6	1/6	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{q_{13}}$	-1/6	0	1/6	$(\mathbf{1}, \mathbf{1})$
$\phi_{q_{23}}$	0	-1/6	1/6	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell 12}$	-1/2	1/2	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell 13}$	-1/2	0	1/2	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell 23}$	0	-1/2	1/2	$(\mathbf{1}, \mathbf{1})$
Q_1	1/6	0	0	$(\mathbf{3}, \mathbf{2})$
Q_2	0	1/6	0	$(\mathbf{3}, \mathbf{2})$
Q_3	0	0	1/6	$(\mathbf{3}, \mathbf{2})$

- Same matter under each hypercharge
- Pheno safe (minimal breaking of $U(2)^5$)
- $SU(2)$-doublets VL quarks



Model 1 spectrum

$$Y_d = \begin{pmatrix} c^d_{11}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{12}}} & c^d_{12}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{13}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{q23}}{M_{D_{23}}} \\ c^d_{21}\frac{\phi_{\ell 12}}{M_{D_{12}}}\frac{\tilde{\phi}_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{22}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{23}\frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c^d_{33} \end{pmatrix}$$

$$Y_u = Y_d(d \to u, D \to U) \qquad Y_e = Y_d(d \to e, q \to \ell, D \to E)$$

Model 1 spectrum

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{12}}} & c_{12}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{13}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{q23}}{M_{D_{23}}} \\ c_{21}^d \frac{\phi_{\ell 12}}{M_{D_{12}}} \frac{\tilde{\phi}_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{23}^d \frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c_{33}^d \end{pmatrix}$$

$$Y_u = Y_d(d \to u, D \to U) \qquad Y_e = Y_d(d \to e, q \to \ell, D \to E)$$

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Model 1 spectrum

$$Y_d = \begin{pmatrix} c^d_{11}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{12}}} & c^d_{12}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{13}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{q23}}{M_{D_{23}}} \\ c^d_{21}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\tilde{\phi}_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{22}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{23}\frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c^d_{33} \end{pmatrix}$$

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$$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$$

→ Highly non-generic

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$Y_u = Y_d(d \to u, D \to U)$ $\quad Y_e = Y_d(d \to e, q \to \ell, D \to E)$

Spectrum up to $\mathcal{O}(1)$ variations

- Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

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E

$\mathcal{O}(10^4\,\text{TeV})$ — $M_{U_{12,13},D_{12,13},E_{12,13}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

↓

$\mathcal{O}(1000\,\text{TeV})$ — $v_{12} \sim \langle \phi_{q12} \rangle, \langle \phi_{\ell 12} \rangle, M_{U_{23},D_{23},E_{23}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_{12} \equiv Y_1 + Y_2} \times U(1)_{Y_3}$

↓

$\mathcal{O}(10\,\text{TeV})$ — $v_{23} \sim \langle \phi_{q23} \rangle, \langle \phi_{\ell 23} \rangle$

$SU(3)_c \times SU(2)_L \times U(1)_{Y \equiv Y_1 + Y_2 + Y_3}$

$174\,\text{GeV}$ — $v_{\text{SM}} \sim \langle H_{u,d} \rangle$

Phenomenology

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

$$\xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$

$$\xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

Phenomenology

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$
$$\overset{v_{12}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\overset{v_{23}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

- Z'_{23} is lighter and protected by accidental $U(2)^5$ symmetry

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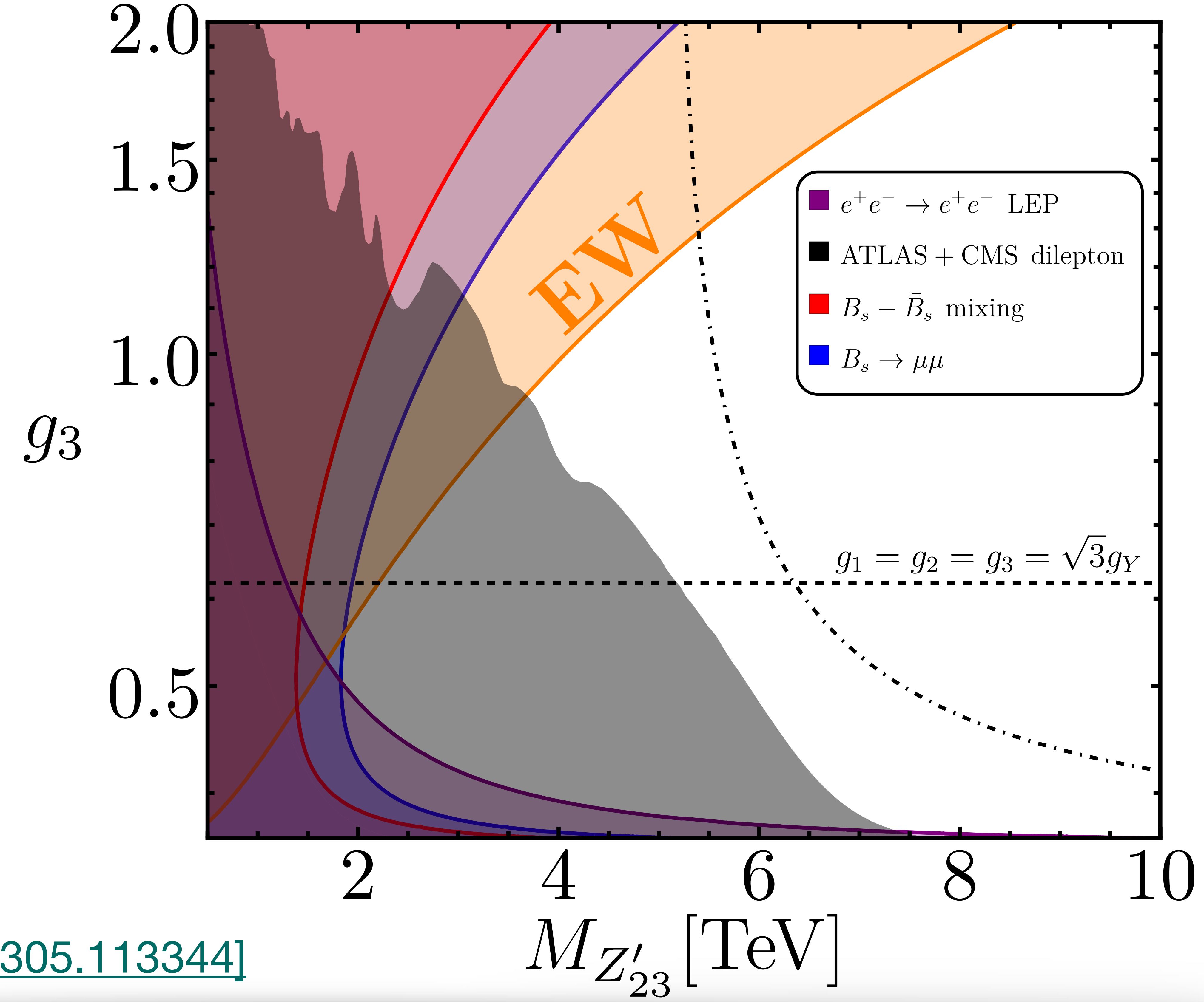
- Z'_{23} is lighter and protected by accidental $U(2)^5$ symmetry
  - Tested by dilepton tails at hadron colliders or EWPOs (independent of UV-completion) - bounds of order TeV

$$g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \qquad g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36\,(M_Z)$$



[see EW global fit and FCC-ee projections in Davighi and Stefanek, 2305.113344]

Phenomenology

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- Z'_{12} is heavier and tested via FCNCs sensitive to the different complete models - bounds typically beyond 100 TeV

Model	Observable	Mediator	Bound (TeV)
1	$K-\bar{K}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 340 \times \left\lvert \mathrm{Re}\begin{bmatrix} c^d_{12} & c^d_{21} \\ c^d_{22} & c^d_{22} \end{bmatrix} \right\rvert$
	$K-\bar{K}$ (Im)	Z'_{12}	$M_{Z'_{12}}/g_1 > 3 \cdot 10^3 \times \left\lvert \mathrm{Im}\begin{bmatrix} c^d_{12} & c^d_{21} \\ c^d_{22} & c^d_{22} \end{bmatrix} \right\rvert$
	$\mu \to e\gamma$	Z'_{12}	$M_{Z'_{12}}/g_1 > 30 \times \lvert c^e_{12}/c^e_{22} \rvert$
		Z'_{23}	$M_{Z'_{23}}/g_3 > 8 \times \lvert y^e_{62}(y^e_{65}y^e_{15})^* \rvert$
	$\mu \to 3\,e$	Z'_{12}	$M_{Z'_{12}}/g_1 > 30 \times \lvert c^e_{12}/c^e_{22} \rvert$
2	$D-\bar{D}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 150 \times \left\lvert \mathrm{Re}\begin{bmatrix} c^u_{12} & c^u_{21} \\ c^u_{22} & c^u_{22} \end{bmatrix} \right\rvert$
	$D-\bar{D}$ (Im)	Z'_{12}	$M_{Z'_{12}}/g_1 > 500 \times \left\lvert \mathrm{Im}\begin{bmatrix} c^u_{12} & c^u_{21} \\ c^u_{22} & c^u_{22} \end{bmatrix} \right\rvert$

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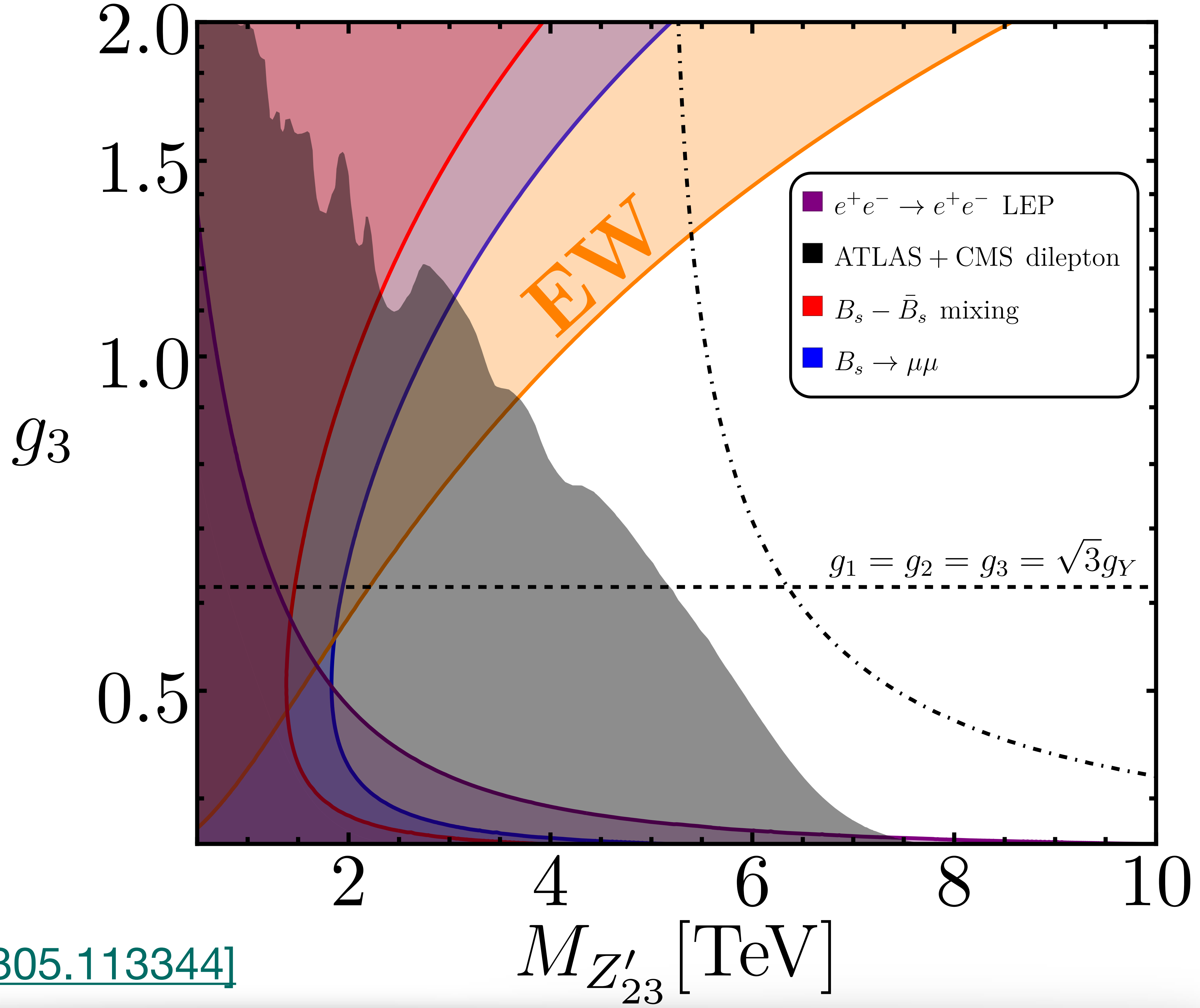

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"Deconstructed" GUT?

- Gauge sector of flavour deconstructed models may contain up to 9 gauge couplings:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

[This talk]

$$SU(3)_c \times SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \times U(1)_Y$$

[Li and Ma, PRL 81'; Muller and Nandi, hep-ph/9602390 … Chiang *et al*, 0911.1480; Allwicher *et al*, 2011.01946; Davighi *et al* 2312.13346; Capdevila *et al*, 2401.00848 …]

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- "Deconstructed" theories seem to preserve an approximate $\mathbb{Z}_3$ (cyclic permutation symmetry) relating the three sites (i.e. approx. same matter content under the three sites):
   - E.g. $\{\phi_{\ell 12}^{(\frac{1}{2},-\frac{1}{2},0)}, \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}, \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\}$, $\{H_1^{(\frac{1}{2},0,0)}, H_2^{(0,\frac{1}{2},0)}, H_3^{(0,0,\frac{1}{2})}\}$, $\{D_{12}^{(-\frac{1}{6},\frac{1}{2},0)}, D_{13}^{(-\frac{1}{6},0,\frac{1}{2})}, D_{23}^{(0,-\frac{1}{6},\frac{1}{2})}\}$

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  - If $\mathbb{Z}_3$ is exact at very high energies, then:

$$\boxed{SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3}$$

[Salam 79', Rajpoot 81', Georgi 82',
de Rújula, Georgi, Glashow 84', $SU(3)_c \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$ …]

with $\mathbb{Z}_3$ permuting the three $SU(5)$, contains a single gauge coupling in the UV.

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✓ Deconstructed GUTs may be the origin of low energy flavour deconstructed models.

$SU(5)^3$: an explicit example

- Note that the model must have the same matter under each site.

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

+ hyperons and VL fermions of tri-hypercharge model 3

$F_i \to d_i^c \oplus \ell_i \quad T_i \to q_i \oplus u_i^c \oplus e_i^c$

$SU(5)^3$: an explicit example

- Note that the model must have the same matter under each site.
- An example: embedding tri-hypercharge at the GUT scale into $SU(5)^3$.

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	**1**	**1**
F_2	**1**	$\bar{\mathbf{5}}$	**1**
F_3	**1**	**1**	$\bar{\mathbf{5}}$
T_1	**10**	**1**	**1**
T_2	**1**	**10**	**1**
T_3	**1**	**1**	**10**
Ω_1	**24**	**1**	**1**
Ω_2	**1**	**24**	**1**
Ω_3	**1**	**1**	**24**
H_1	**5**	**1**	**1**
H_2	**1**	**5**	**1**
H_3	**1**	**1**	**5**

$$SU(5)^3 \xrightarrow{v_{24}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$$
$$\xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$$
$$\xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3} \,.$$

Can do both diagonal and vertical symmetry breaking thanks to cyclic symmetry

+ hyperons and VL fermions of tri-hypercharge model 3

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Example of gauge coupling unification

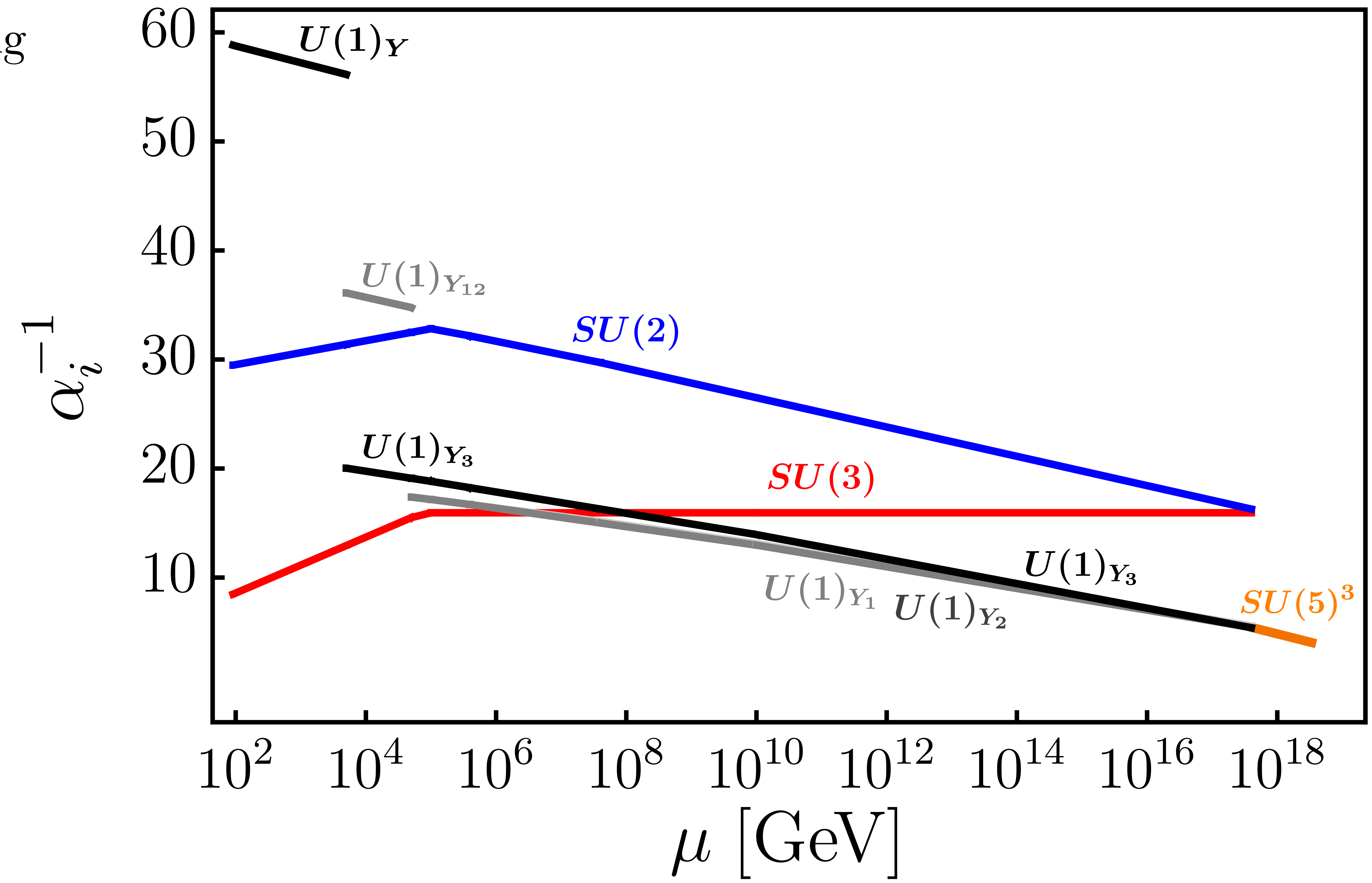
- Discontinuities due to gauge coupling matching conditions:

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}(v_{23})$$

$$\alpha_i = \frac{g_i^2}{4\pi}$$

$$\alpha_{Y_1}^{-1} + \alpha_{Y_2}^{-1} = \alpha_{Y_{12}}^{-1}(v_{12})$$

$$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(v_{\mathrm{SM}^3})$$



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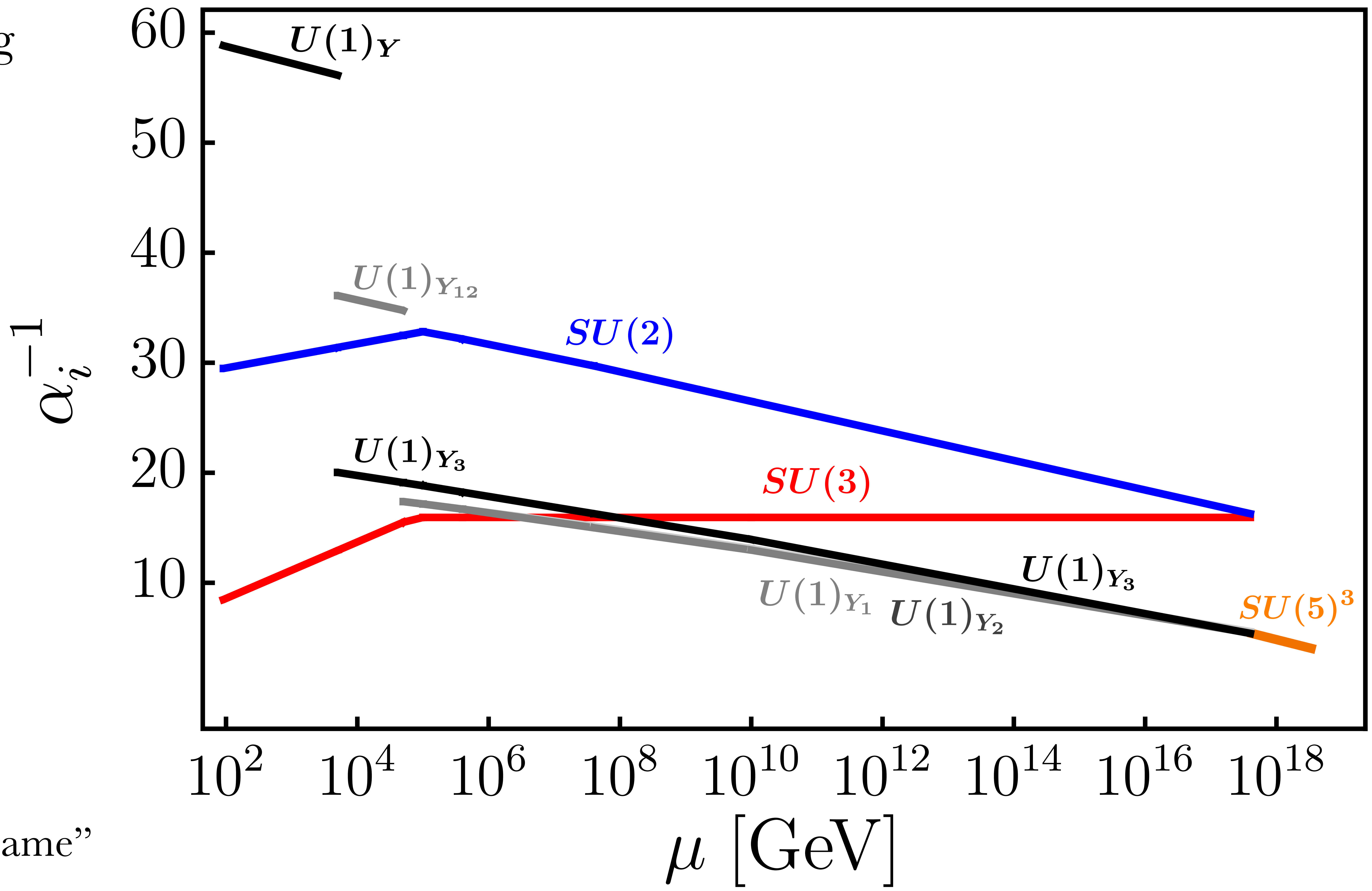
$$\alpha^{-1}_{s,L,1} + \alpha^{-1}_{s,L,2} + \alpha^{-1}_{s,L,3} = \alpha^{-1}_{s,L}(v_{\mathrm{SM}^3})$$

- VL quarks Q_i help bend SU(2).
- Colour octet $\Theta_i \sim (\mathbf{8},\mathbf{1},0)_i$ from cyclic **24** at v_{12} scale to bend SU(3) (non-SUSY).
- Gauge couplings approximately "run the same" thanks to approximate $\mathbb{Z}_3$ at low energies, which becomes exact at high energies.



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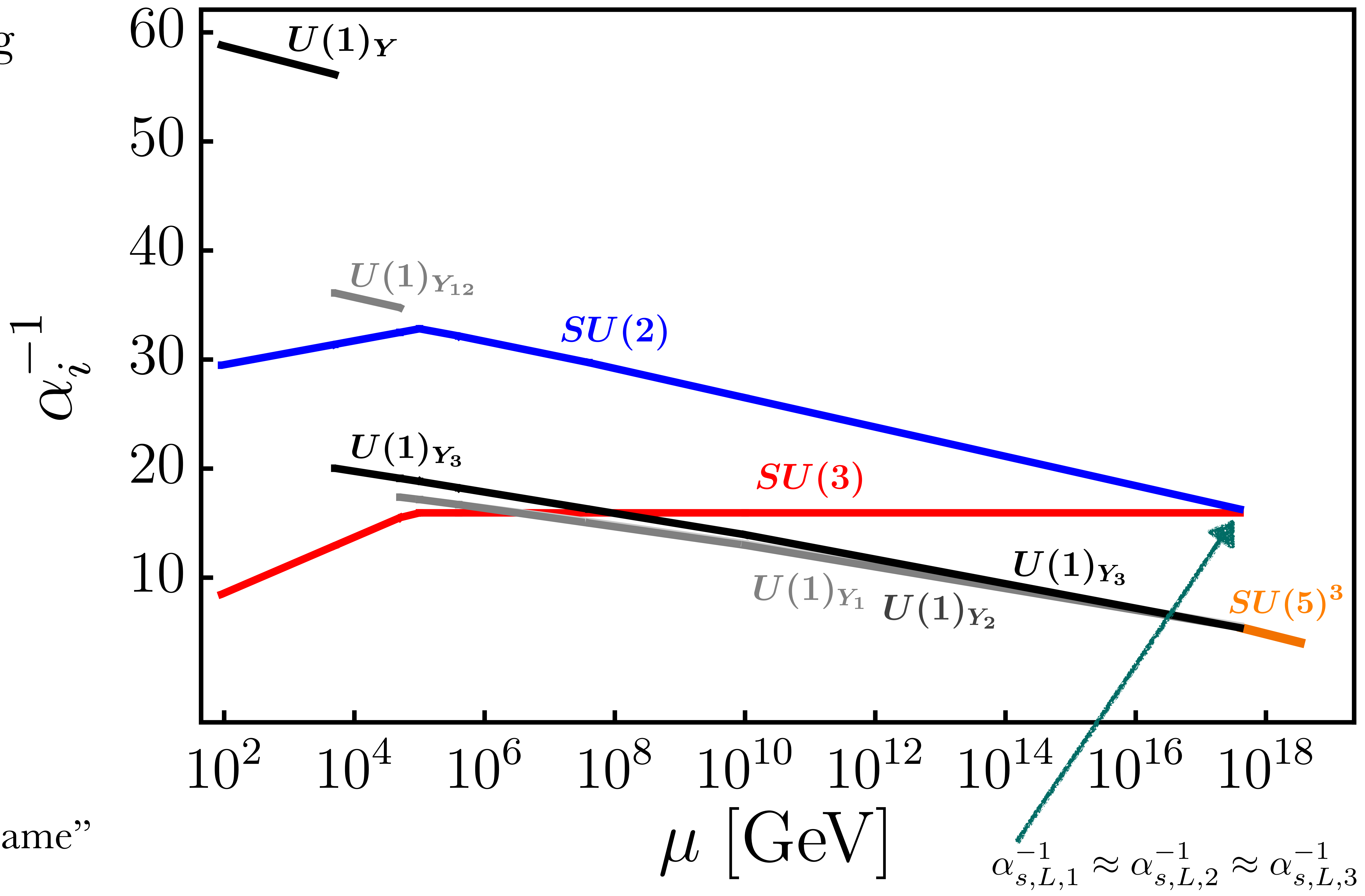

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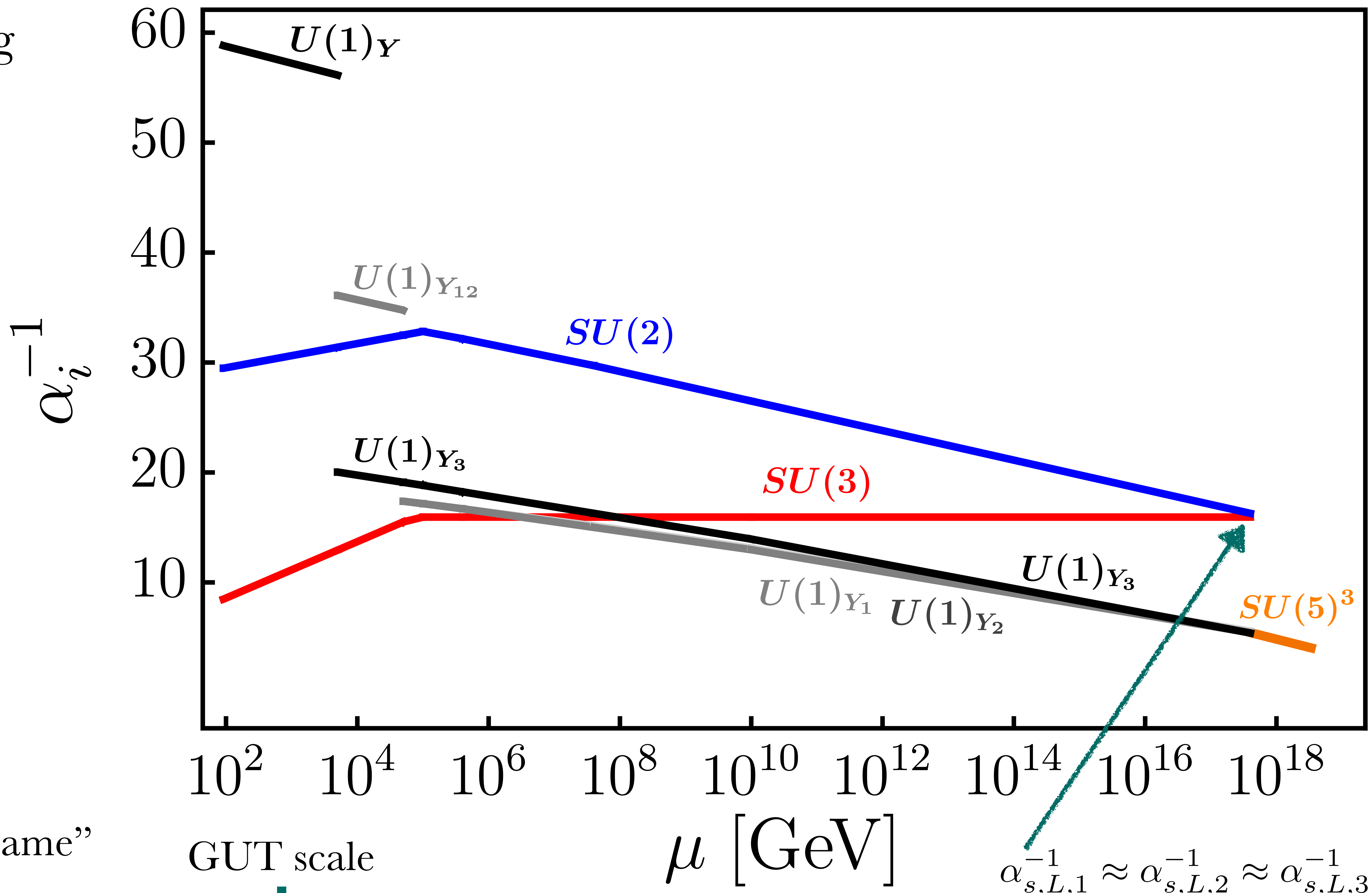

GUT scale

$M_{\mathrm{GUT}} \simeq 5 \times 10^{17}$ GeV → Lower if we consider intermediate scale (ask!)

Take home messages

✓ A simple option for flavour deconstruction is the tri-hypercharge group:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

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Thank you!

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✓ Flavour deconstructed theories consistent all the way from the EW scale to the GUT scale!

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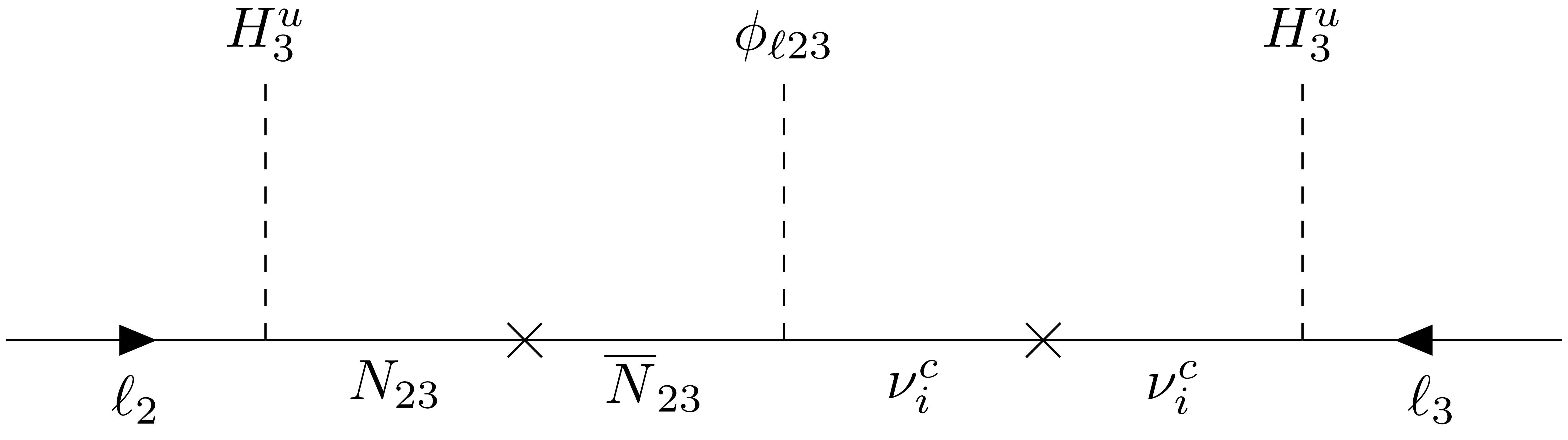

$i = 1, 2$

$$\mathcal{L} = c_{3i}\ell_3 H_3^u \nu_i^c + c_{2i}\frac{\phi_{\ell 23}}{M_{N_{23}}}\ell_2 H_3^u \nu_i^c + \ldots + M_{ij}\nu_i^c \nu_j^c + \text{h.c.}$$

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ν_1^c	0	0	0	$(\mathbf{1}, \mathbf{1})$
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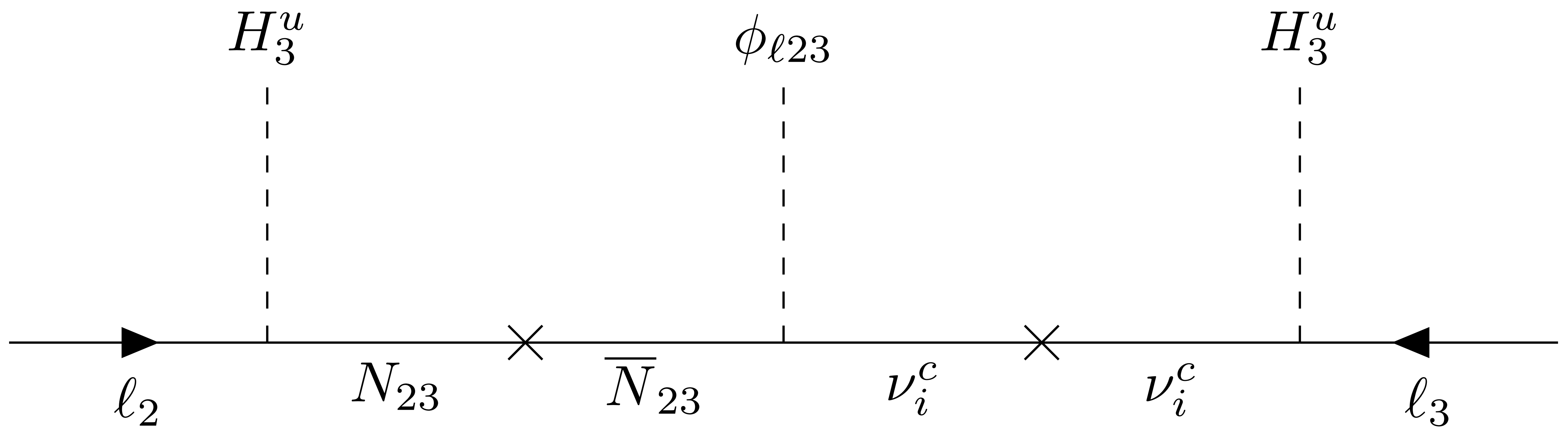

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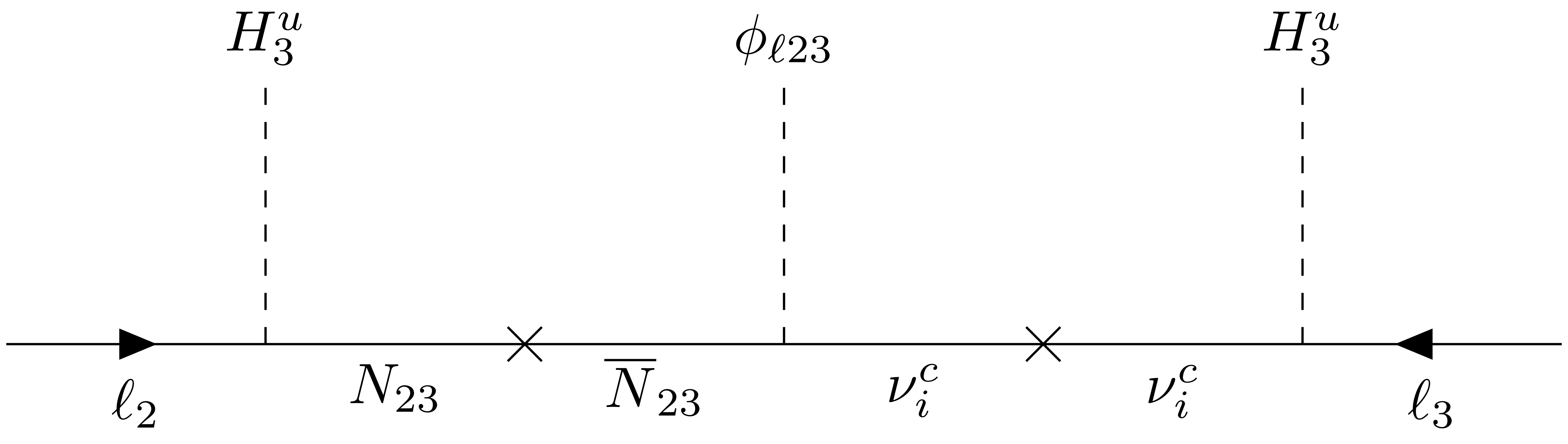

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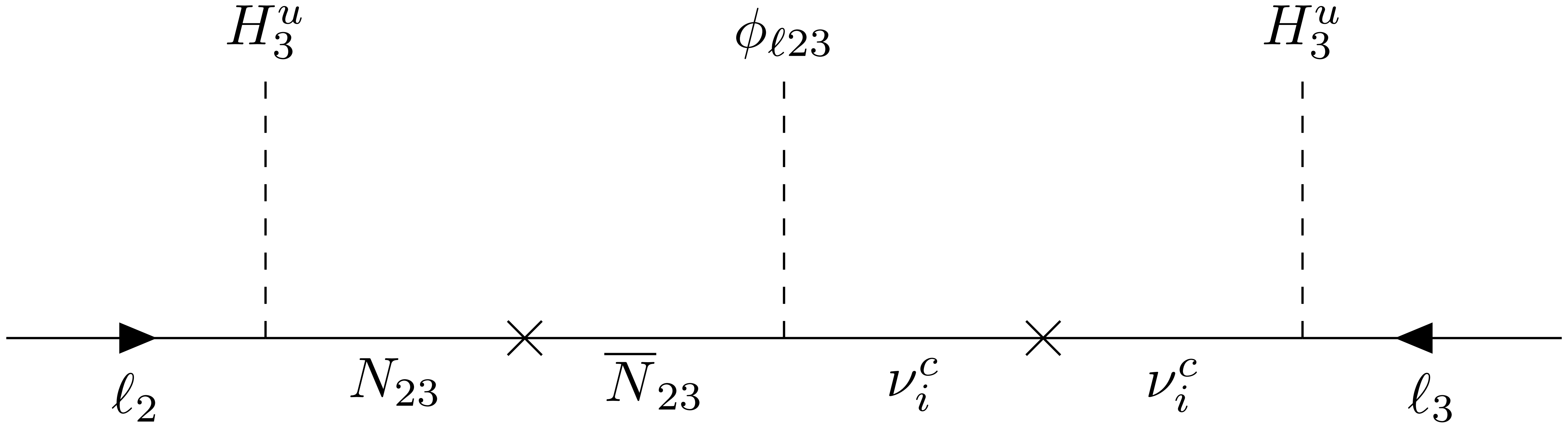

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Seesaw mechanism!

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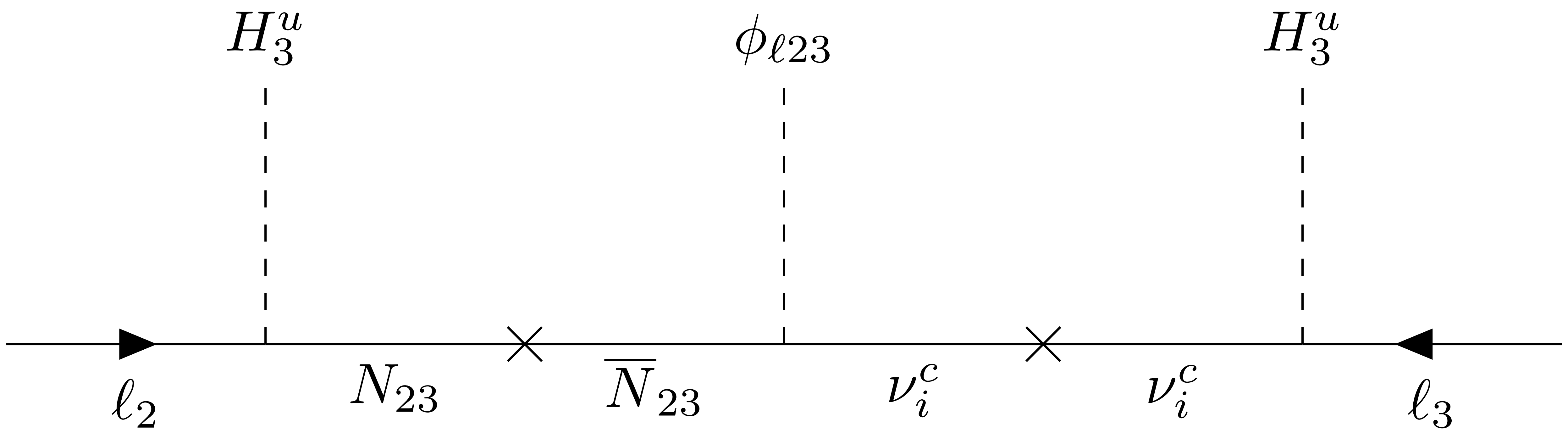

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Seesaw mechanism!

- ✓ $M \approx 10^{15}$ GeV
- ✓ No need of small couplings nor v_{12}, v_{23} being very heavy
- ✓ No need of adding extra scalars
- ✓ $M_{N_{23}} \approx v_{23} \gtrsim \mathcal{O}(10\ \text{TeV})$

Backup: Gauge coupling unification

- Discontinuities due to gauge coupling matching conditions:

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}(v_{23})$$

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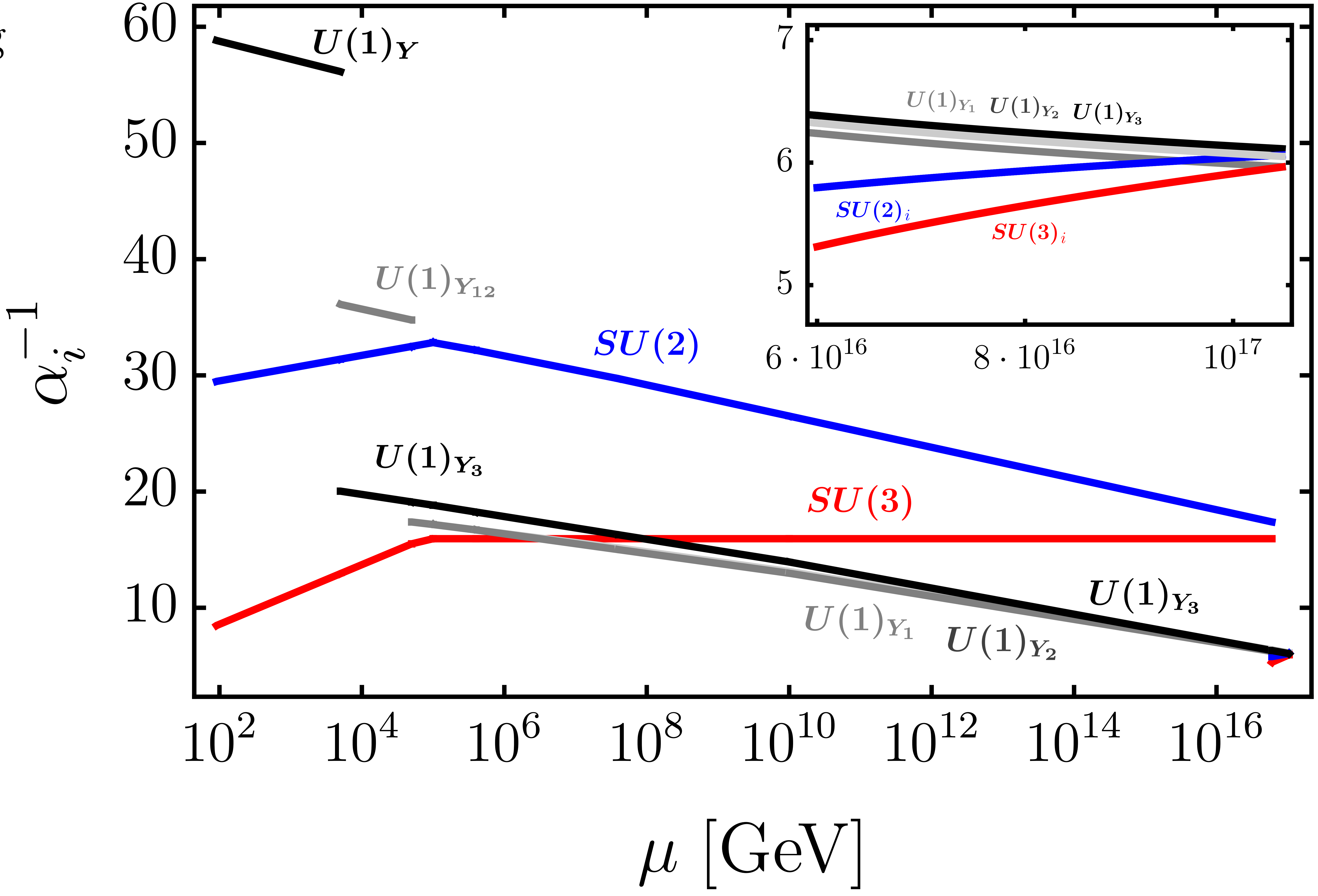

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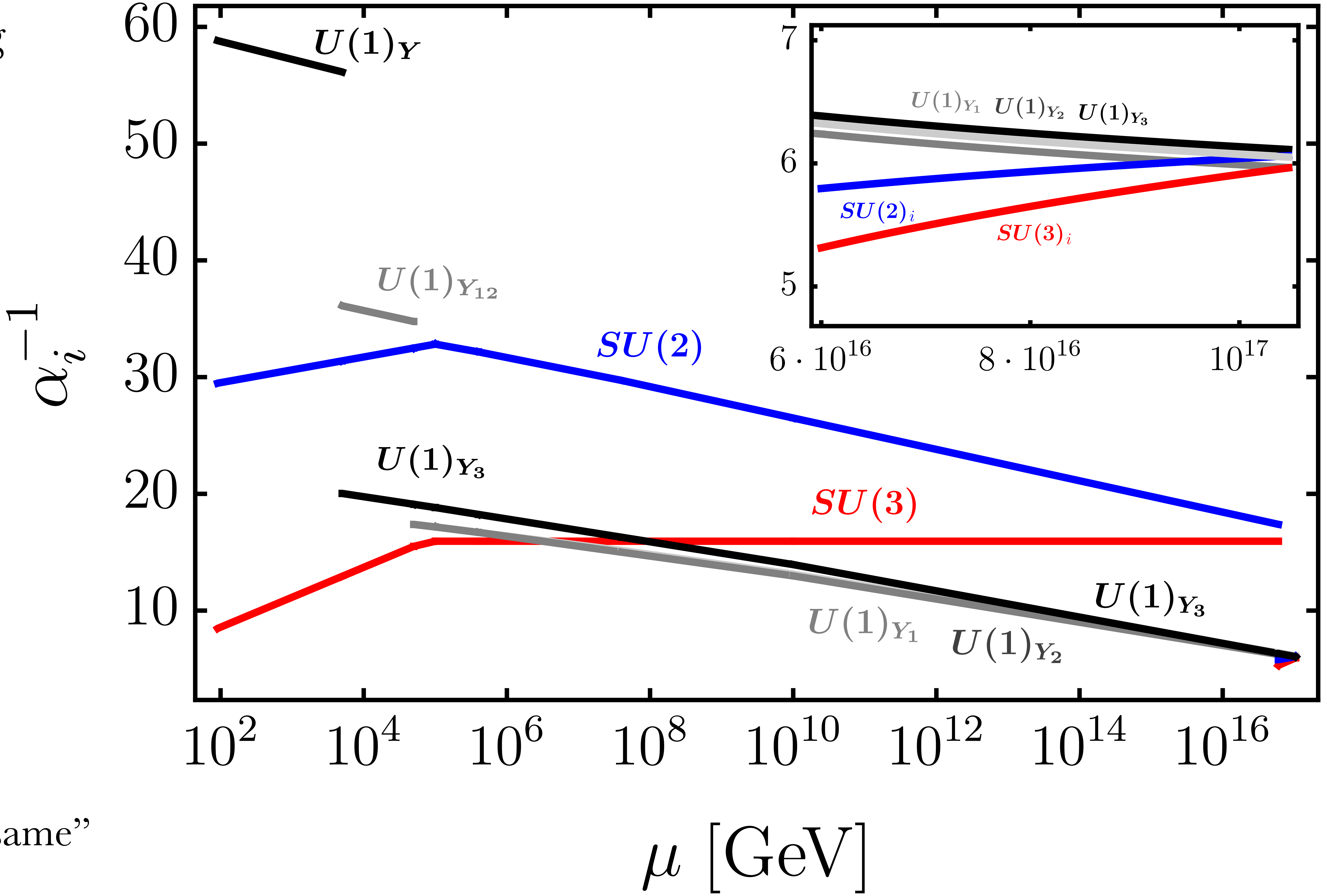

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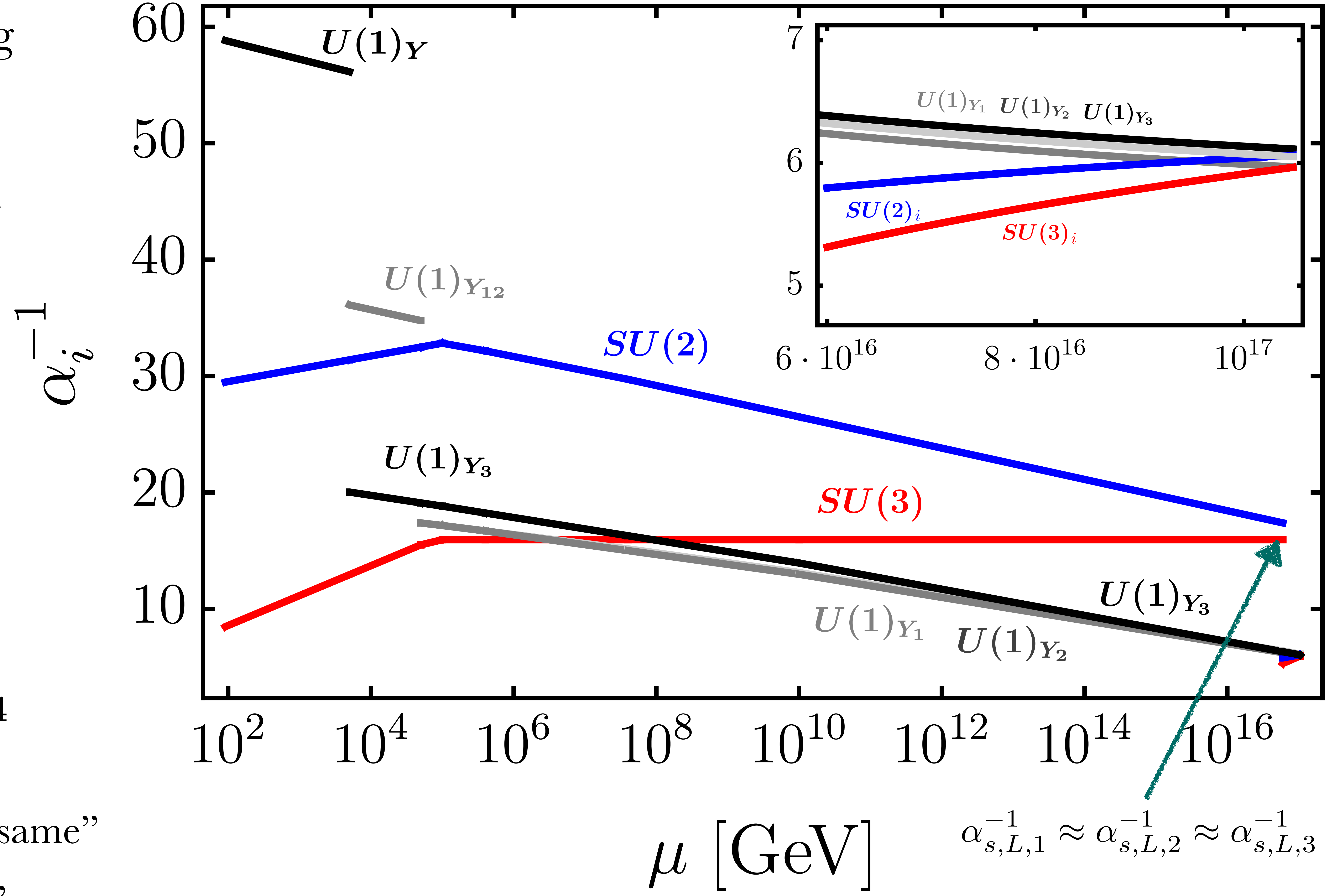
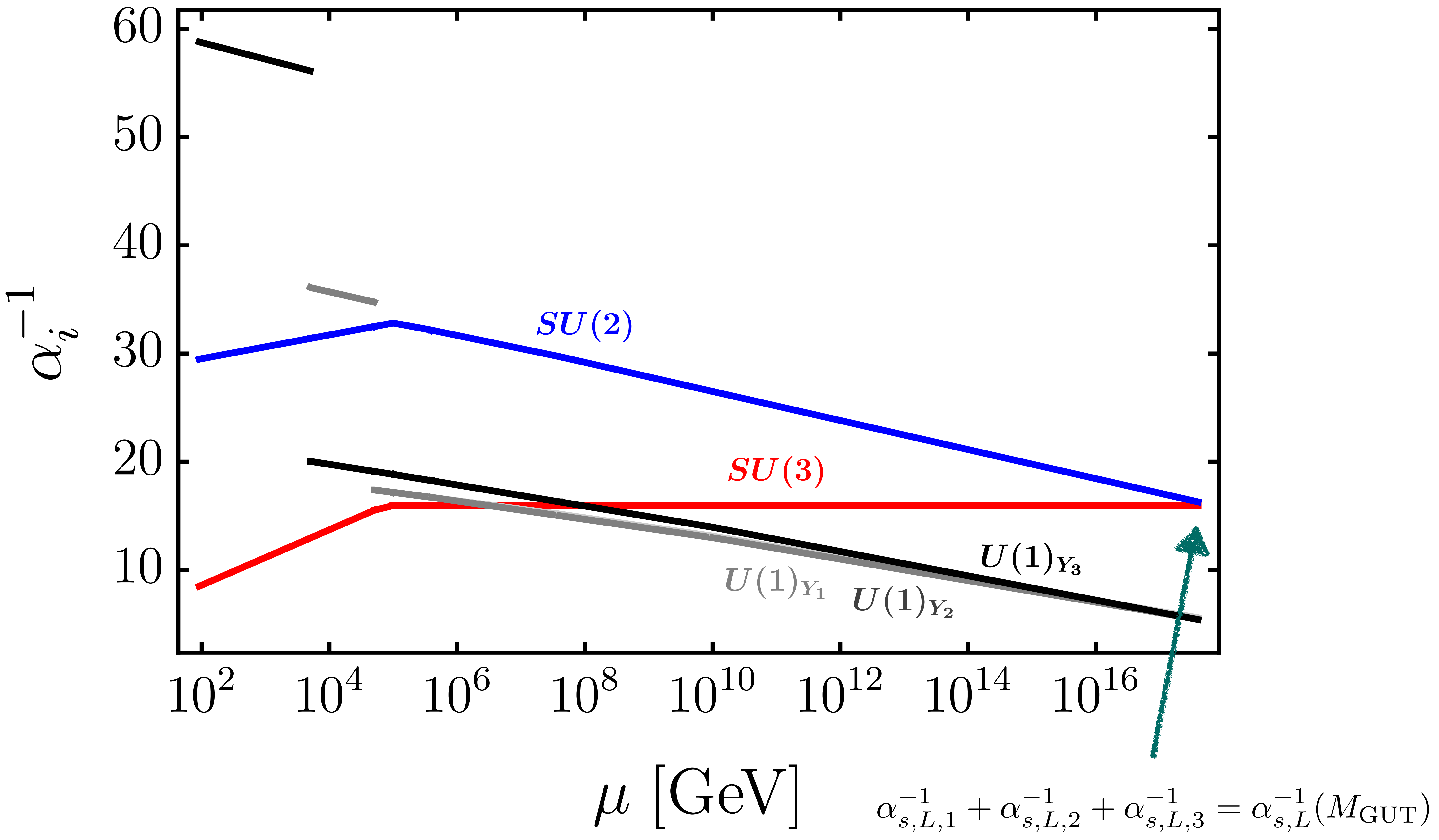


$v_{\mathrm{SM}^3} = 6 \times 10^{16}$ GeV → $M_{\mathrm{GUT}} = 10^{17}$ GeV

Backup: Gauge coupling unification

No intermediate SM[3] scale: $SU(5)^3 \xrightarrow{M_{\rm GUT}} SU(3)_c \times SU(2)_L \times U(1)_Y^3$



Robust prediction for GUT scale: $M_{\rm GUT} \simeq 5 \times 10^{17}$ GeV

Backup: Gauge coupling unification

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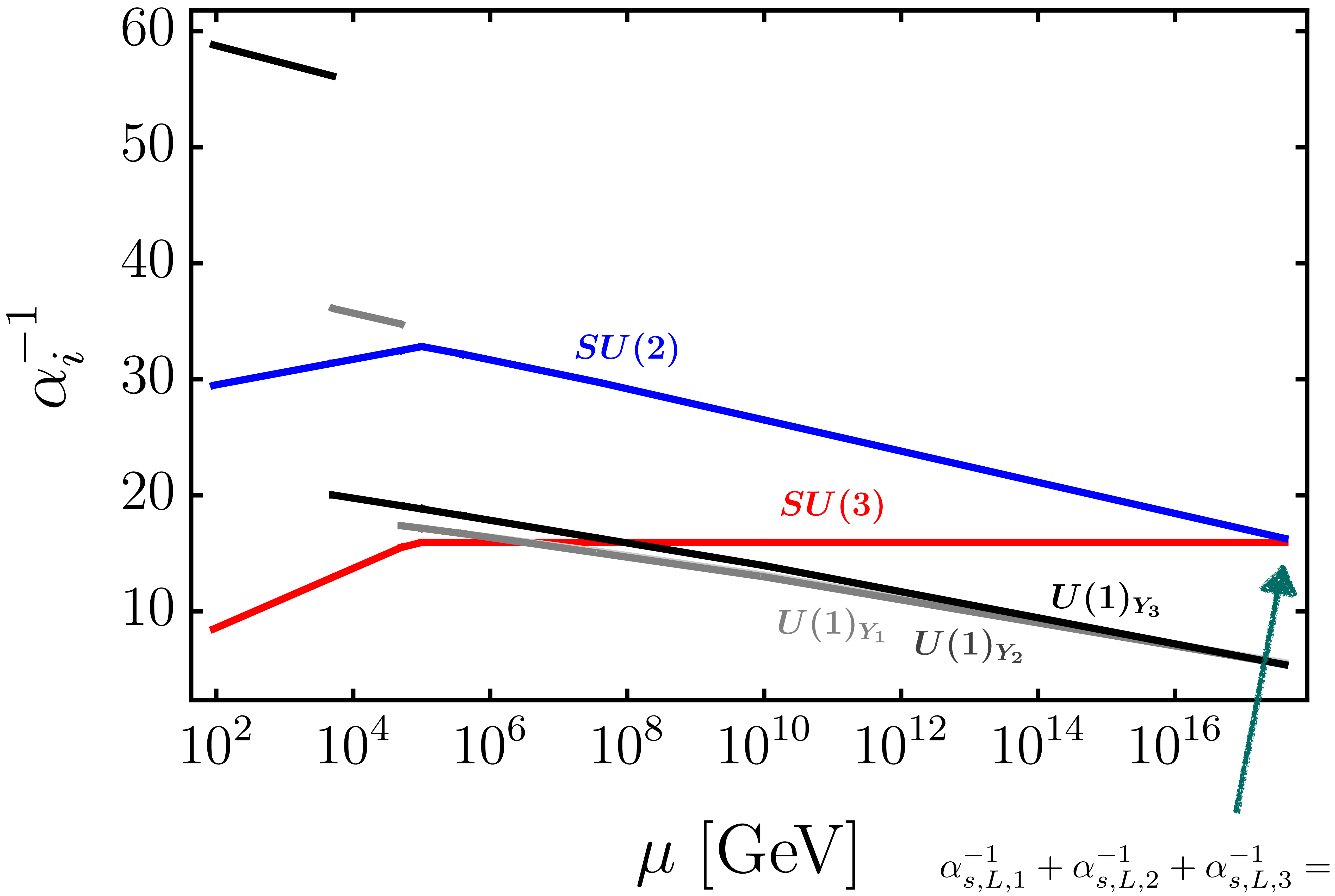
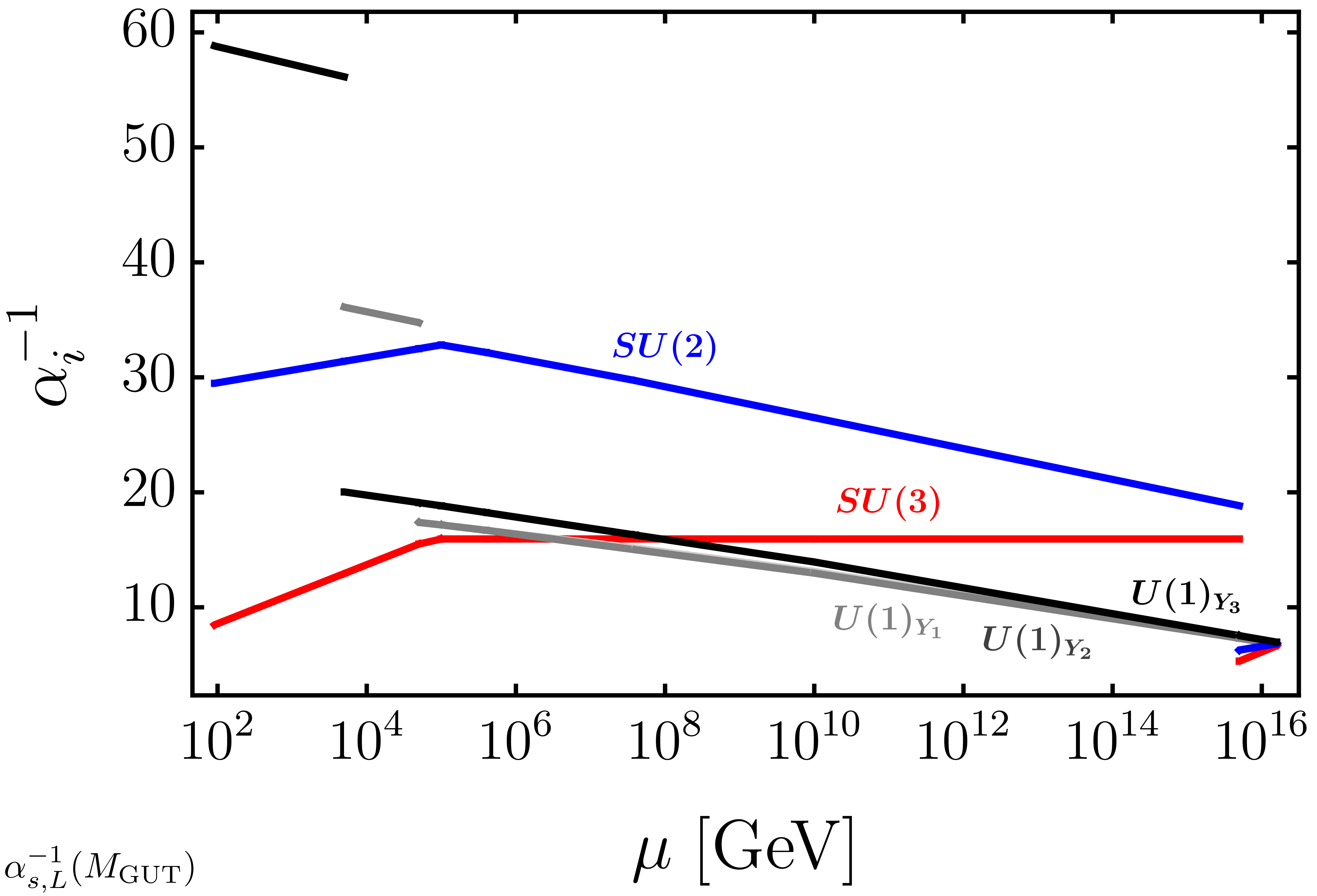
How low can can we deconstruct $SU(3)_c$ and $SU(2)_L$?



$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(M_{\rm GUT})$



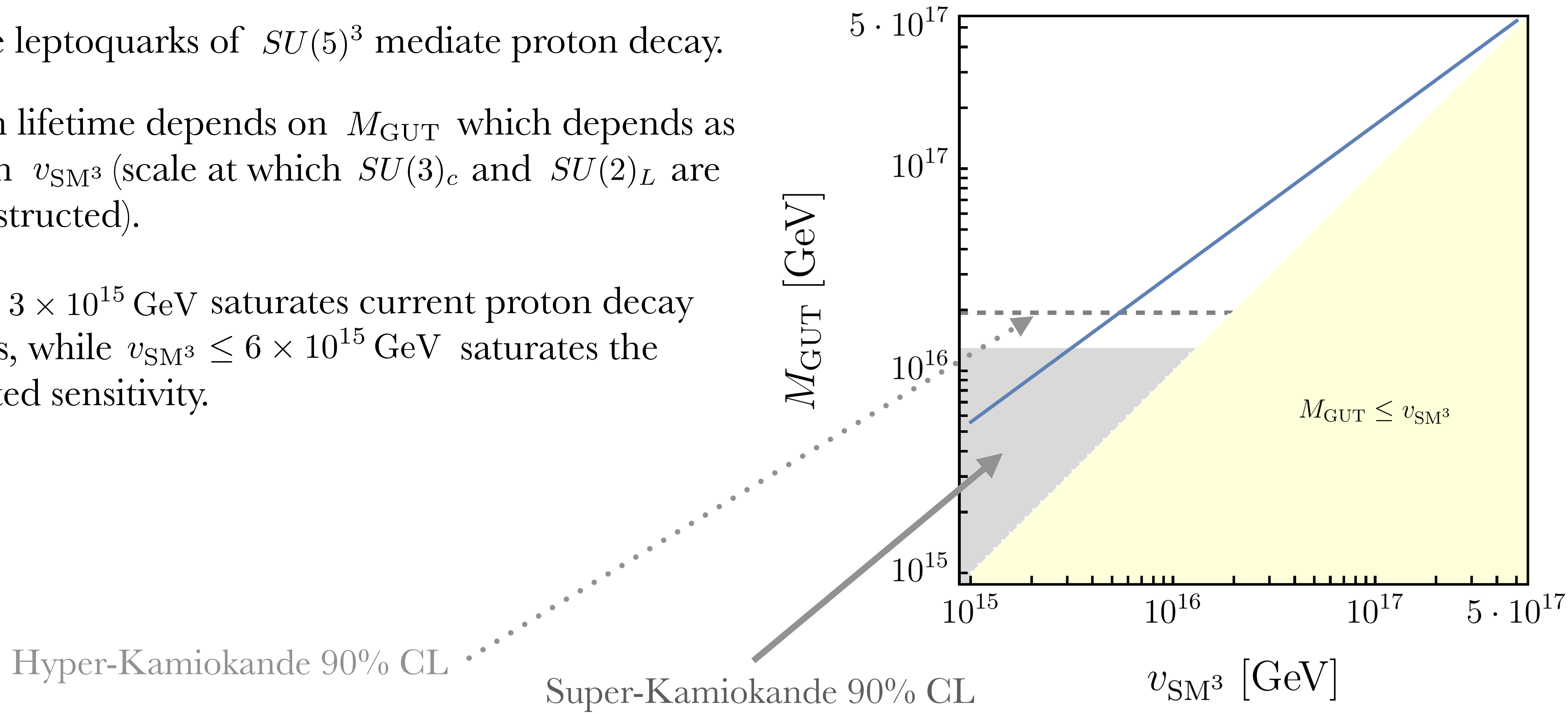
Robust prediction for GUT scale: $M_{\rm GUT} \simeq 5 \times 10^{17}$ GeV

$v_{\rm SM^3} = 5 \times 10^{15}$ GeV $\longrightarrow$ $M_{\rm GUT} \simeq 1.8 \times 10^{16}$ GeV

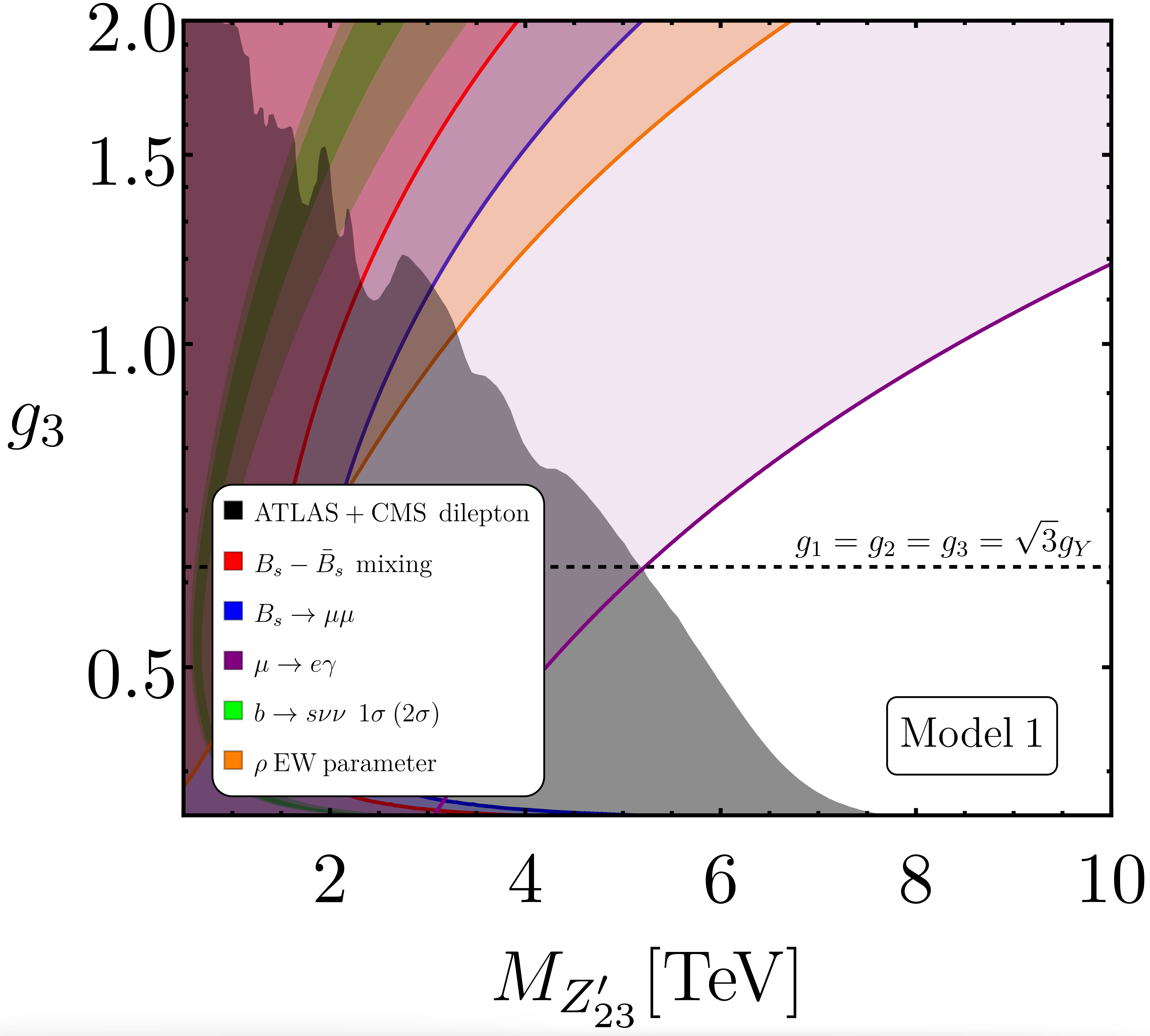
proton decay!

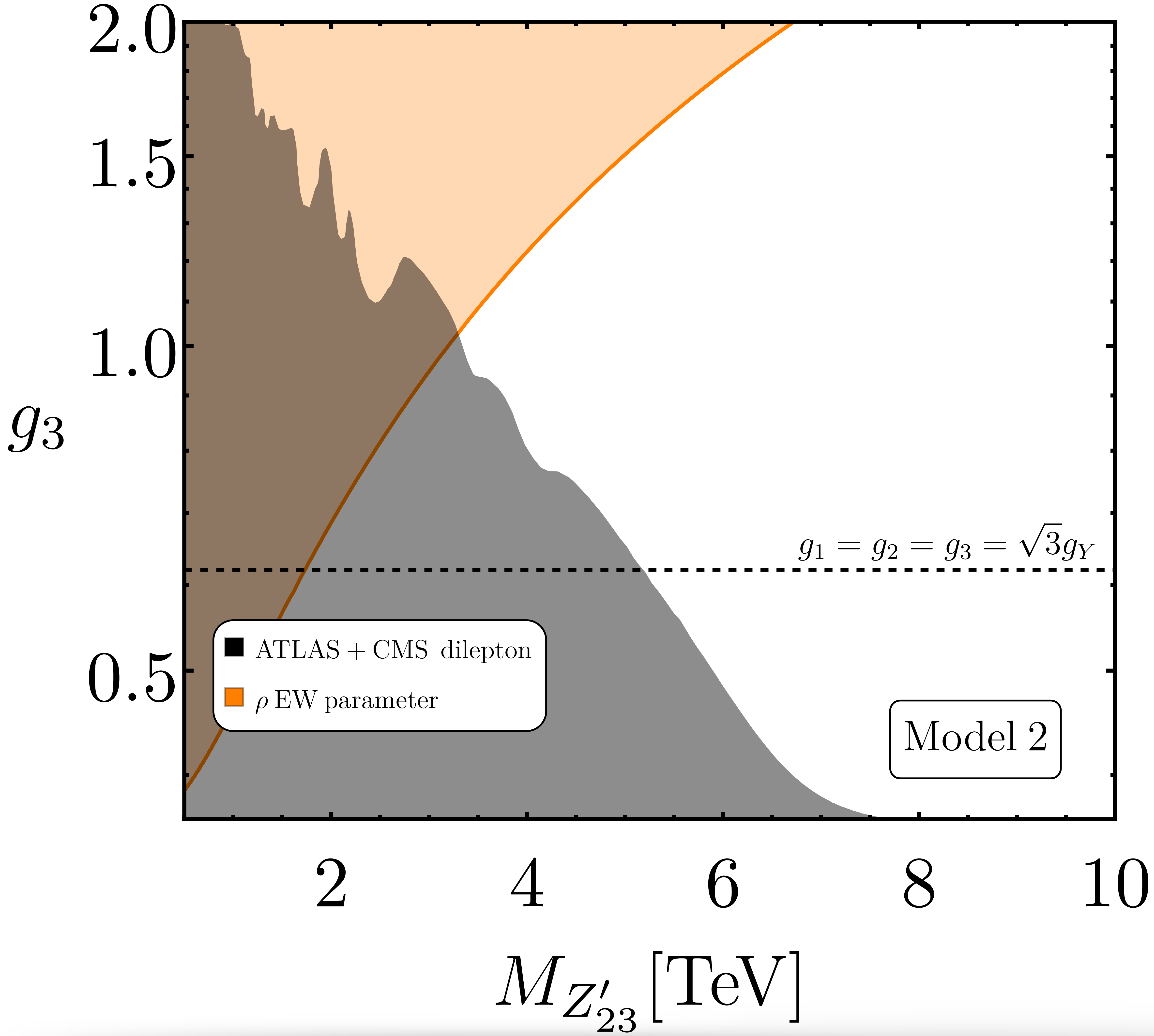
Backup: Proton decay

- Gauge leptoquarks of $SU(5)^3$ mediate proton decay.
- Proton lifetime depends on $M_{\rm GUT}$ which depends as well on $v_{\rm SM^3}$ (scale at which $SU(3)_c$ and $SU(2)_L$ are deconstructed).
- $v_{\rm SM^3} \leq 3\times 10^{15}\,{\rm GeV}$ saturates current proton decay bounds, while $v_{\rm SM^3} \leq 6\times 10^{15}\,{\rm GeV}$ saturates the projected sensitivity.



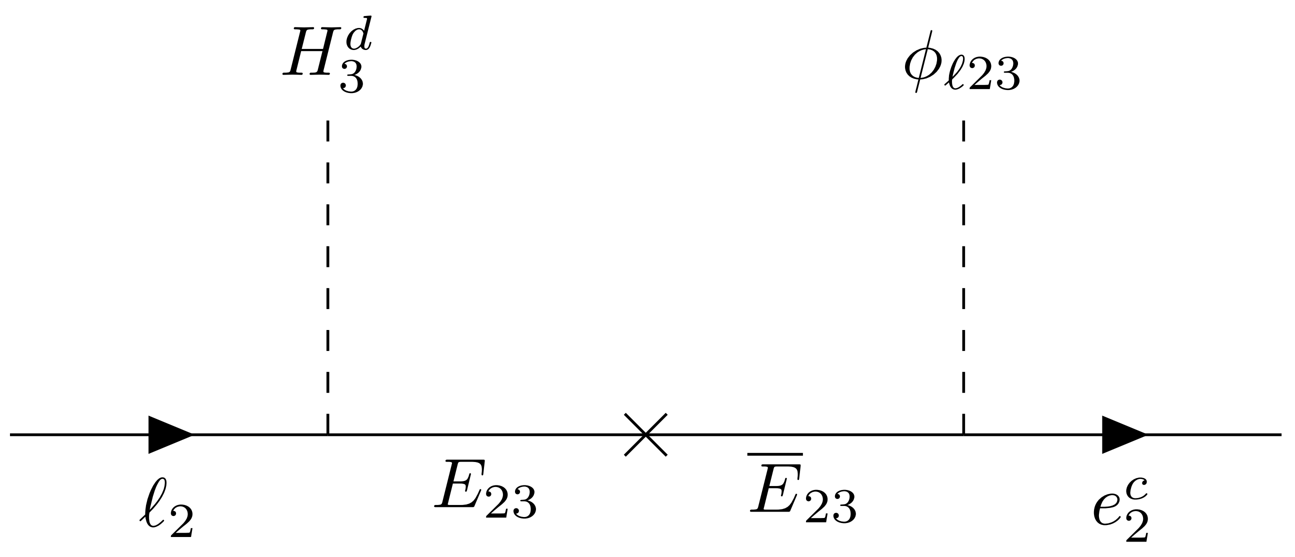
Backup: TeV scale pheno



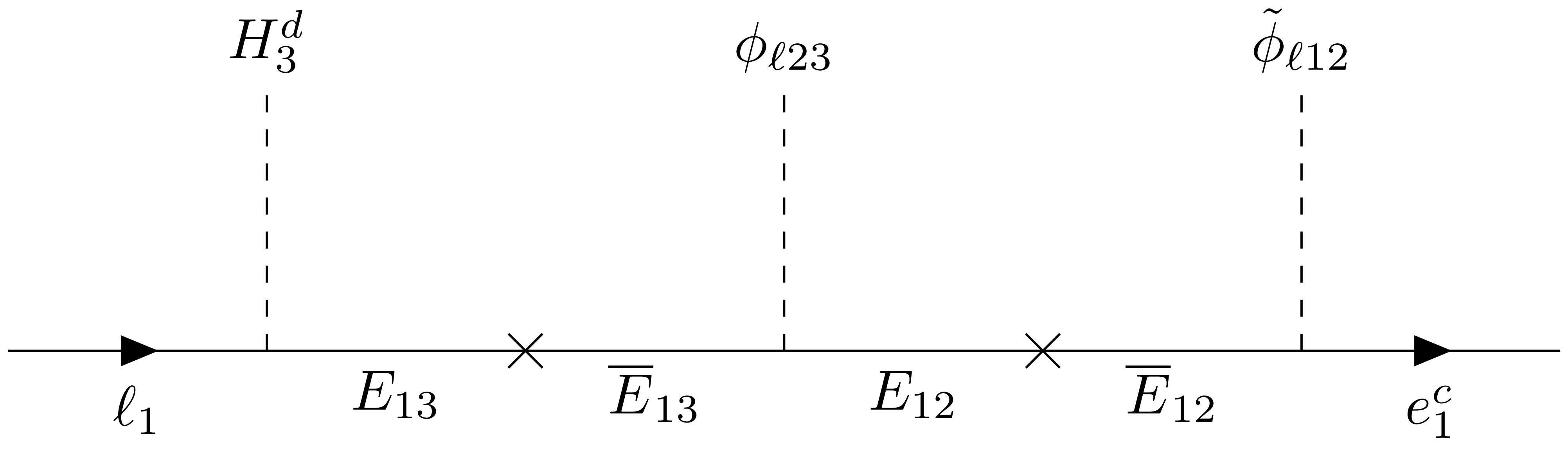


Backup: VL charged leptons

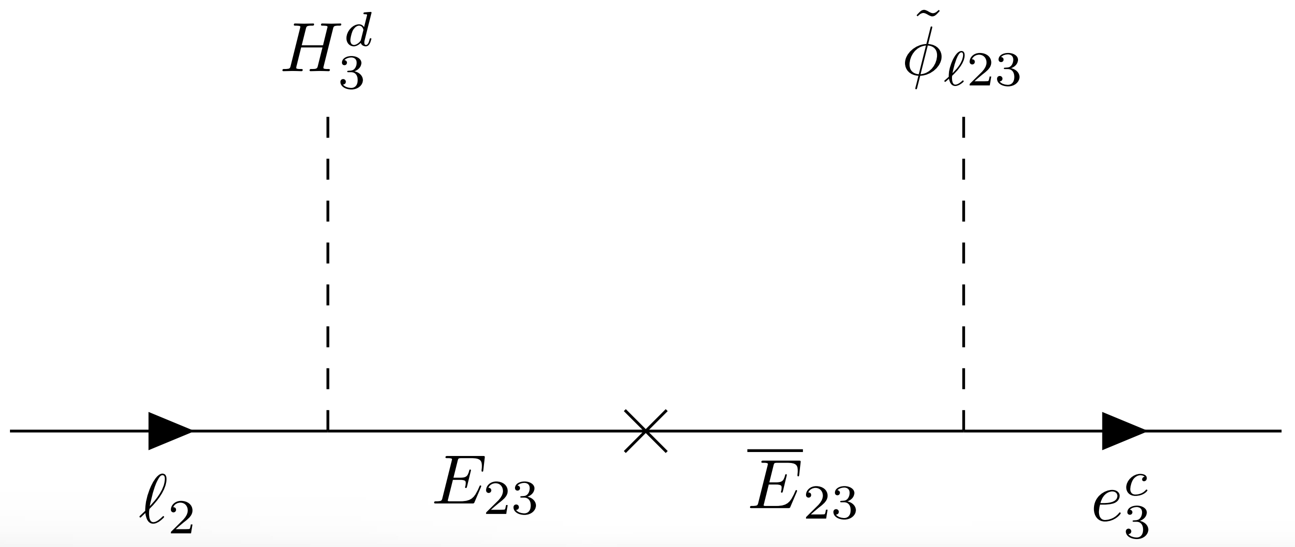
$$\mathcal{L}_e = \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \end{pmatrix} \begin{pmatrix} c^e_{11} \dfrac{\phi_{\ell 12}}{M_{E_{13}}} \dfrac{\phi_{\ell 23}}{M_{E_{12}}} & c^e_{12} \dfrac{\tilde{\phi}_{\ell 12}}{M_{E_{13}}} \dfrac{\phi_{\ell 23}}{M_{E_{23}}} & c^e_{13} \dfrac{\tilde{\phi}_{\ell 12}}{M_{E_{13}}} \dfrac{\tilde{\phi}_{\ell 23}}{M_{E_{23}}} \\ c^e_{21} \dfrac{\phi_{\ell 12}}{M_{E_{12}}} \dfrac{\phi_{\ell 12}}{M_{E_{13}}} \dfrac{\phi_{\ell 23}}{M_{E_{23}}} & c^e_{22} \dfrac{\phi_{\ell 23}}{M_{E_{23}}} & c^e_{23} \dfrac{\tilde{\phi}_{\ell 23}}{M_{E_{23}}} \\ 0 & 0 & c^e_{33} \end{pmatrix} \begin{pmatrix} e^c_1 \\ e^c_2 \\ e^c_3 \end{pmatrix} H^d_3 + \text{h.c.} \,,$$

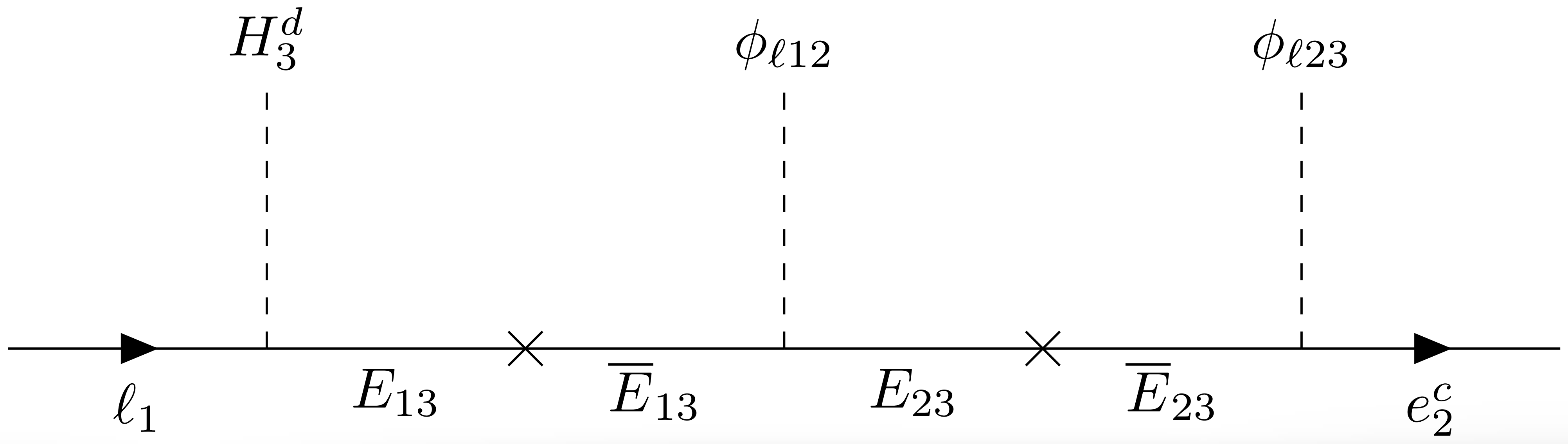


(a)



(b)





Backup: Yukawa couplings models 2 and 3

Model 2

$$\mathcal{L}_u = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} c^u_{11}\dfrac{\phi_{\ell 12}}{M_{U_{13}}}\dfrac{\phi_{\ell 23}}{M_{U_{12}}} & c^u_{12}\dfrac{\phi_{q12}}{M_{U_{13}}}\dfrac{\phi_{\ell 23}}{M_{U_{23}}} & c^u_{13}\dfrac{\phi_{q12}}{M_{U_{13}}}\dfrac{\phi_{q23}}{M_{U_{23}}} \\ c^u_{21}\dfrac{\phi_{\ell 12}}{M_{U_{12}}}\dfrac{\phi_{q12}}{M_{U_{13}}}\dfrac{\phi_{\ell 23}}{M_{U_{23}}} & c^u_{22}\dfrac{\phi_{\ell 23}}{M_{U_{23}}} & c^u_{23}\dfrac{\phi_{q23}}{M_{U_{23}}} \\ 0 & 0 & c^u_{33} \end{pmatrix} \begin{pmatrix} u^c_1 \\ u^c_2 \\ u^c_3 \end{pmatrix} H^u_3 + \text{h.c.}$$

$$\mathcal{L}_{d,e} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \operatorname{diag}\left(c^d_{11}\frac{\tilde\phi_{\ell 12}}{M_{H^d_1}}\frac{\tilde\phi_{\ell 23}}{M_{H^d_2}},\ c^d_{22}\frac{\tilde\phi_{\ell 23}}{M_{H^d_2}},\ c^d_{33} \right) \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \end{pmatrix} H^d_3$$
$$+ \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \end{pmatrix} \operatorname{diag}\left(c^e_{11}\frac{\tilde\phi_{\ell 12}}{M_{H^d_1}}\frac{\tilde\phi_{\ell 23}}{M_{H^d_2}},\ c^e_{22}\frac{\tilde\phi_{\ell 23}}{M_{H^d_2}},\ c^e_{33} \right) \begin{pmatrix} e^c_1 \\ e^c_2 \\ e^c_3 \end{pmatrix} H^d_3 + \text{h.c.}\,,$$

Model 3

$$\mathcal{L} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} c^u_{11}\dfrac{\phi_{\ell 13}}{M_{H^u_1}} & c^u_{12}\dfrac{\phi_{\ell 23}}{M_{H^u_2}}\dfrac{\phi_{q12}}{M_{Q_2}} & c^u_{13}\dfrac{\phi_{q13}}{M_{Q_3}} \\ c^u_{21}\dfrac{\phi_{\ell 13}}{M_{H^u_1}}\dfrac{\phi_{q12}}{M_{Q_1}} & c^u_{22}\dfrac{\phi_{\ell 23}}{M_{H^u_2}} & c^u_{23}\dfrac{\phi_{q23}}{M_{Q_3}} \\ c^u_{31}\dfrac{\phi_{\ell 13}}{M_{H^u_1}}\dfrac{\widetilde{\phi}_{q13}}{M_{Q_1}} & c^u_{32}\dfrac{\phi_{\ell 23}}{M_{H^u_2}}\dfrac{\widetilde{\phi}_{q23}}{M_{Q_2}} & c^u_{33} \end{pmatrix} \begin{pmatrix} u^c_1 \\ u^c_2 \\ u^c_3 \end{pmatrix} H^u_3$$
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$$+ \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \end{pmatrix} \begin{pmatrix} c^e_{11}\dfrac{\widetilde{\phi}_{\ell 13}}{M_{H^d_1}} & 0 & 0 \\ 0 & c^e_{22}\dfrac{\widetilde{\phi}_{\ell 23}}{M_{H^d_2}} & 0 \\ 0 & 0 & c^e_{33} \end{pmatrix} \begin{pmatrix} e^c_1 \\ e^c_2 \\ e^c_3 \end{pmatrix} H^d_3 + \text{h.c.}\,,$$

Backup: SU(5) cube model table

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
χ_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
χ_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
χ_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
N_{12}	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
N_{13}	$\mathbf{5}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
N_{23}	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
$H_1^{u,d}$	$\mathbf{5},\ \overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{u,d}$	$\mathbf{1}$	$\mathbf{5},\ \overline{\mathbf{5}}$	$\mathbf{1}$
$H_3^{u,d}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5},\ \overline{\mathbf{5}}$
Φ_{12}^F	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
Φ_{13}^F	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Φ_{23}^F	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$
Φ_{12}^T	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{1}$
Φ_{13}^T	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
Φ_{23}^T	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$H_1^{\mathbf{45}}$	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{45}$	$\mathbf{1}$
$H_3^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{45}$

Backup: SU(5) cube model table

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
χ_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
χ_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
χ_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
N_{12}	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
N_{13}	$\mathbf{5}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
N_{23}	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
$H_1^{u,d}$	$\mathbf{5},\ \overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{u,d}$	$\mathbf{1}$	$\mathbf{5},\ \overline{\mathbf{5}}$	$\mathbf{1}$
$H_3^{u,d}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5},\ \overline{\mathbf{5}}$
Φ_{12}^F	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
Φ_{13}^F	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Φ_{23}^F	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$
Φ_{12}^T	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{1}$
Φ_{13}^T	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
Φ_{23}^T	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$H_1^{\mathbf{45}}$	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{45}$	$\mathbf{1}$
$H_3^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{45}$

cyclic **45** to split down/charged lepton masses as in conventional $SU(5)$

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Phi(0, -\frac{1}{6}, \frac{1}{6}) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.
- Introduce a spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$, then we can write $\Phi q_2 H_3^d d_3^c$. Repeat for every entry in the matrix.

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\tfrac{1}{2},0,\tfrac{1}{2}) & \Phi(-\tfrac{1}{6},-\tfrac{1}{3},\tfrac{1}{2}) & \Phi(-\tfrac{1}{6},0,\tfrac{1}{6}) \\ \Phi(-\tfrac{1}{3},-\tfrac{1}{6},\tfrac{1}{2}) & \Phi(0,-\tfrac{1}{2},\tfrac{1}{2}) & \Phi(0,-\tfrac{1}{6},\tfrac{1}{6}) \\ \Phi(-\tfrac{1}{3},0,\tfrac{1}{3}) & \Phi(0,-\tfrac{1}{3},\tfrac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \mathrm{h.c.}$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.
- Introduce a spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$, then we can write $\Phi q_2 H_3^d d_3^c$. Repeat for every entry in the matrix.

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \text{h.c.}$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.
- Introduce a spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$, then we can write $\Phi q_2 H_3^d d_3^c$. Repeat for every entry in the matrix.
- Promote spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$ to the physical scalar ("hyperon") , $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

$$\frac{\phi_{q23}}{\Lambda_2} q_2 H_3^d d_3^c$$

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \mathrm{h.c.}$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.
- Introduce a spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$, then we can write $\Phi q_2 H_3^d d_3^c$. Repeat for every entry in the matrix.
- Promote spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$ to the physical scalar ("hyperon") , $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

$$\frac{\phi_{q23}}{\Lambda_2} q_2 H_3^d d_3^c$$

- We also promote the spurion in the (2,2) entry to hyperon $\Phi \sim (0, -\frac{1}{2}, \frac{1}{2}) \sim \phi_{\ell 23}$:

$$\frac{\phi_{\ell 23}}{\Lambda_2} q_2 H_3^d d_2^c$$

Backup: Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \text{h.c.}$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.
- Introduce a spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$, then we can write $\Phi q_2 H_3^d d_3^c$. Repeat for every entry in the matrix.
- Promote spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$ to the physical scalar (“hyperon”), $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

$$\frac{\phi_{q23}}{\Lambda_2} q_2 H_3^d d_3^c$$

- We also promote the spurion in the (2,2) entry to hyperon $\Phi \sim (0, -\frac{1}{2}, \frac{1}{2}) \sim \phi_{\ell 23}$:

$$\frac{\phi_{\ell 23}}{\Lambda_2} q_2 H_3^d d_2^c$$

Flavour structure dynamically generated via tri-hypercharge SSB

Backup: Literature review

GUT product groups

"tribal group" to motivate multiple scales vs SO(10)

- **1979** Abdus Salam; EPS conference 1979, footnote 41 ⟶ $SU(5)_1 \times SU(5)_2 \times SU(5)_3 \xrightarrow{M_i} \text{SM}_i \times SU(5)_j$
- **1981** Subhash Rajpoot; PRD 24 (1981) 1890. ⟶ numerics + study different breakings + discrete symmetries for 1 gauge coupling
- **1982** Howard Georgi; Nucl. Phys. B 202 (1982) 397 ⟶ $SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times SU(5)_{\text{TC}}$ + cyclic permutation (also $SO(10)^5$)
- **1984** de Rújula, Georgi, Glashow; Fifth worksop on GUT ⟶ $SU(3)_c \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$
- **1995** Barbieri, Dvali and Strumia, hep-ph/9407239 ⟶ SUSY $SU(5)^3$ $SO(10)^3$ + $(\mathbf{5}_i, \overline{\mathbf{5}}_j)$ scalars + discrete symmetries ⟶ d=5 proton decay!
- **1998-2007** C.L. Chou, [hep-ph/9804325]; Asaka and Takanashi, [hep-ph/0409147]; Babu, Barr and Gogoladze [0709.3491] (↑ $SO(10)^3$)
- **2023** MFN, Stephen F. King, Avelino Vicente [2311.05683]; ⟶ $SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$ Broken via **24** to "deconstructed" theory of flavour (e.g. tri-hypercharge)