Heavy quark diffusion from lattice QCD

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HEFTY COLLABORATION

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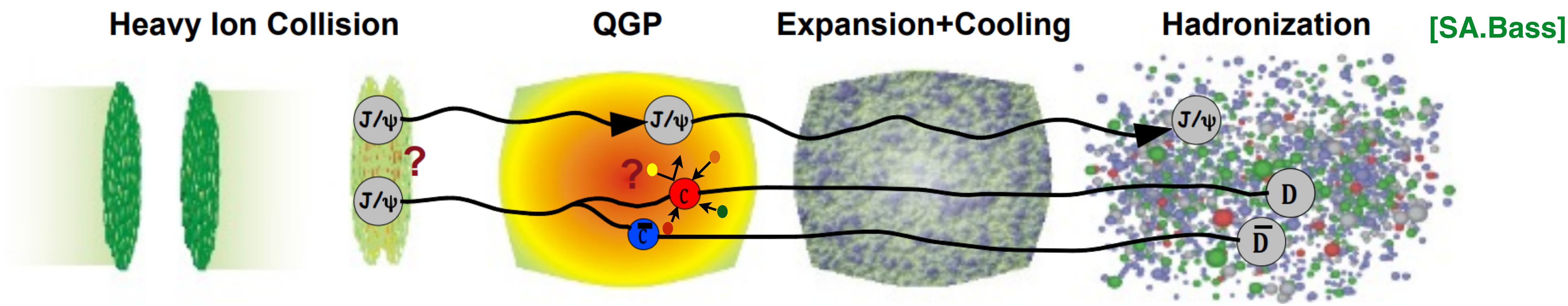
Heavy quarks in heavy ion collisions

Heavy quarkonia are produced only in the early stage of collisions



Bound state / Free quarks

Free quarks **thermalize via diffusion** within time: $\tau_{\rm kin} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3}\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ \rm GeV}\right)\frac{3\ \rm GeV}{T_c^2}$

- Perturbative estimates: [G. Moore and D. Teaney, PRC.71.064904]

  $\tau_{\rm kin,charm} \sim 6\ {\rm fm/c} \gg \tau_{\rm kin,light} \sim 1\ {\rm fm/c}$
- Experimental estimates (RHIC): [STAR Collaboration, PRL,106 (2011) 159902]

  $\tau_{\rm kin,charm} \approx \tau_{\rm kin,light}$

Need non-perturbative ab-initio determination for equilibration time!

Identify the heavy quark diffusion

Phenomenological diffusion picture of classical particle

Equilibrium -> Relaxation -> Equilibrium

$$\langle A(\mathbf{x})\rangle_{\rm eq} = 0 \qquad \partial_t \langle A(\mathbf{x},t)\rangle = D\nabla^2 \langle A(\mathbf{x},t)\rangle$$

Solution:

$$\langle A(\mathbf{k},\omega)\rangle = \frac{i}{\omega + iD\mathbf{k}^2}\langle A(\mathbf{k}, t=0)\rangle$$

Linear response theory

Perturbation to Hamiltonian:

$$H(t) = H_0 - \int \mathrm{d}\mathbf{x}\ A(\mathbf{x})h(x)e^{\epsilon t}\Theta(-t)$$

Solution:

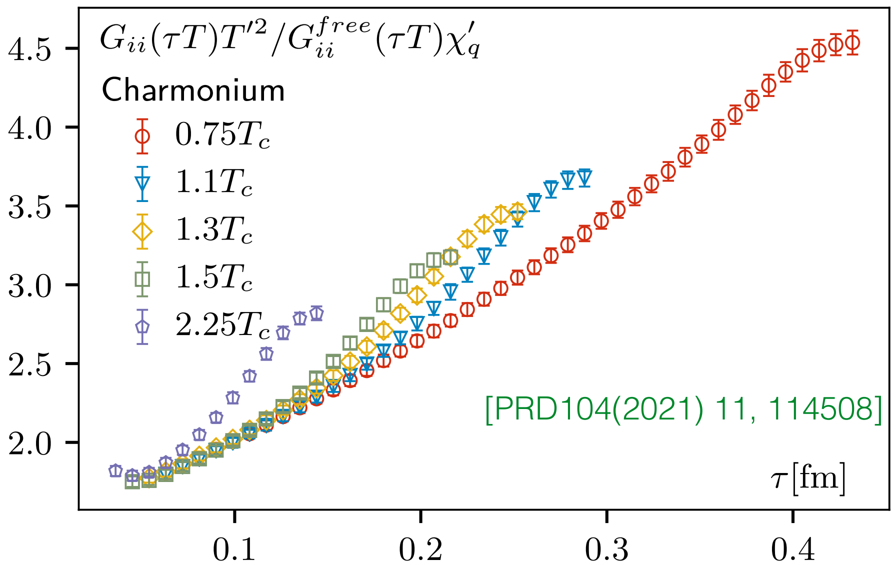
$$\frac{\partial}{\partial t}\Big(\delta\langle A(\mathbf{k}, t=0)\rangle\Big) = -\frac{G_R(\mathbf{k},t)}{\chi_q(\mathbf{k})}\delta\langle A(\mathbf{k},0)\rangle$$

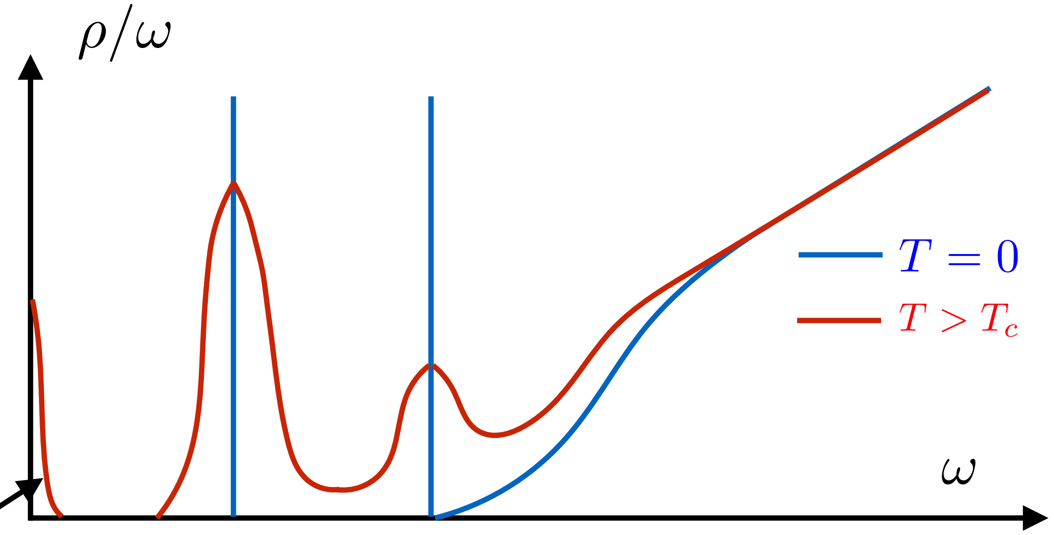
Kubo formula: $$G_R(\mathbf{k},\omega) = \frac{iD\mathbf{k}^2}{\omega + iD\mathbf{k}^2}\chi_q(\mathbf{k}) \sim \rho(\vec{k},\omega)$$

$$A \to J^\mu = \bar{\psi}\gamma^\mu\psi$$

$$D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\rho^{ii}(\omega)}{\omega}$$

Difficult from mesonic correlators



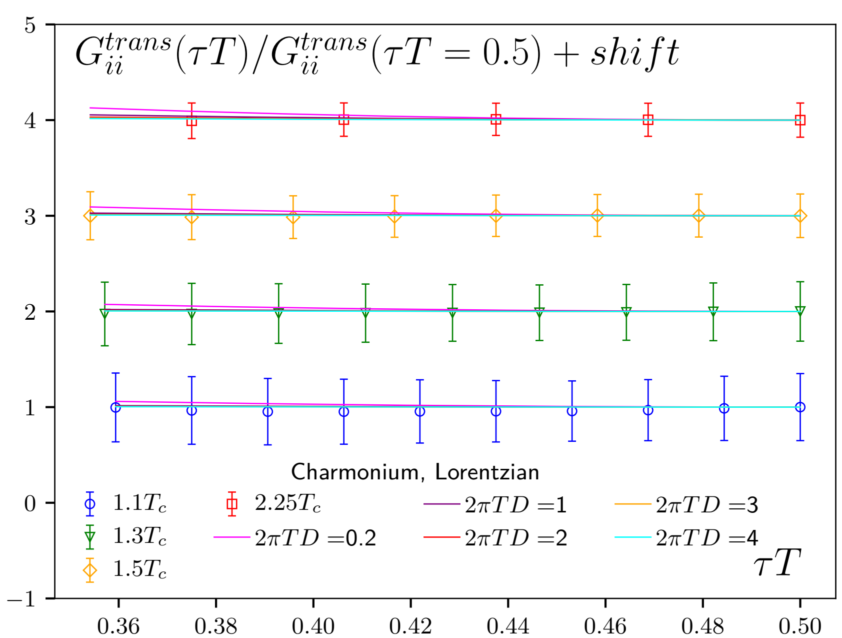


$$\rho_{ii}^{trans}(\omega) = 3\chi_q D \frac{\omega\eta^2}{\omega^2 + \eta^2}$$

$$G_H(\tau, \vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x}) = \int \frac{d\omega}{2\pi} \frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/2T)} \rho_H(\omega, \vec{p}, T)$$

ill-posed

$K(\omega, \tau, T)$



- Insensitivity of transport peak to tau makes it hard to determine D

[PRD104(2021) 11, 114508]

Simpler case: infinite heavy quark mass limit

Heavy quark *momentum* diffusion coefficient in HQET

J. Casalderrey-Solana and D. Teaney, PRD 74, 085012

S. Caron-Huot et al., JHEP 0904 (2009) 053

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

$$\partial_t p_i = -\eta_D p_i + \xi_i(t)$$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa \delta_{ij}\delta(t-t')$$

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = \mathbf{F}$$

- Mass dependent momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta\ll|\omega|\lesssim\omega_{\rm UV}}$$

- Large quark mass limit in effective field theory

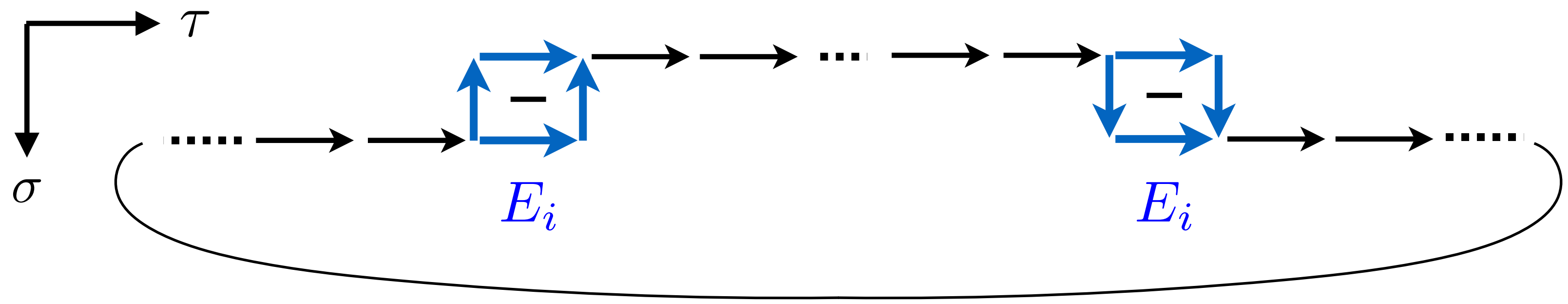
$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty}{\rm d}t\, e^{i\omega(t-t')}\int {\rm d}^3\vec{x}\,\left\langle\frac{1}{2}\{\mathcal{F}^i(t,\vec{x}),\mathcal{F}^i(0,\vec{0})\}\right\rangle\right]$$

Heavy quark momentum diffusion on the lattice

$$\frac{\kappa_{E,B}}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega\to 0} \frac{\rho_{E,B}(\omega)}{\omega}$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}}$$

$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2\rangle}{3MT}$$



Color-electric field correlation function

$$G_{E,B}(\tau,T) = \int \frac{\mathrm{d}\omega}{\pi} K(\omega,\tau,T)\rho_{E,B}(\omega,T)$$

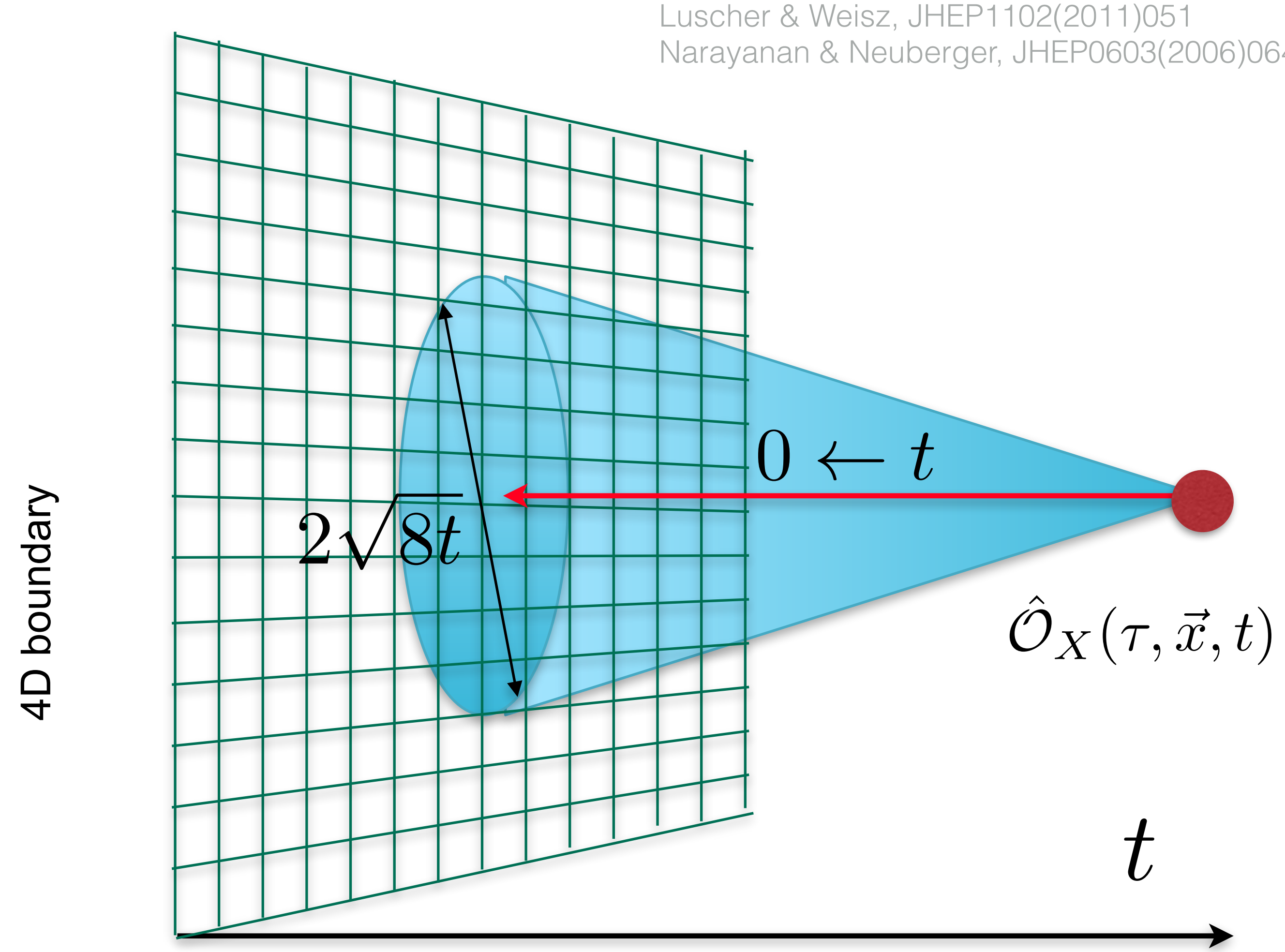
- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak

Gradient flow

Smear fields according to diffusion equation:

$$\frac{\mathrm{d}B_\mu(x,t)}{\mathrm{d}t} \sim -\frac{\delta S_G[B_\mu(x,t)]}{\delta B_\mu(x,t)} \sim D_\nu G_{\nu\mu}(x,t) \qquad B_\nu(x,t)|_{t=0} = A_\nu(x)$$

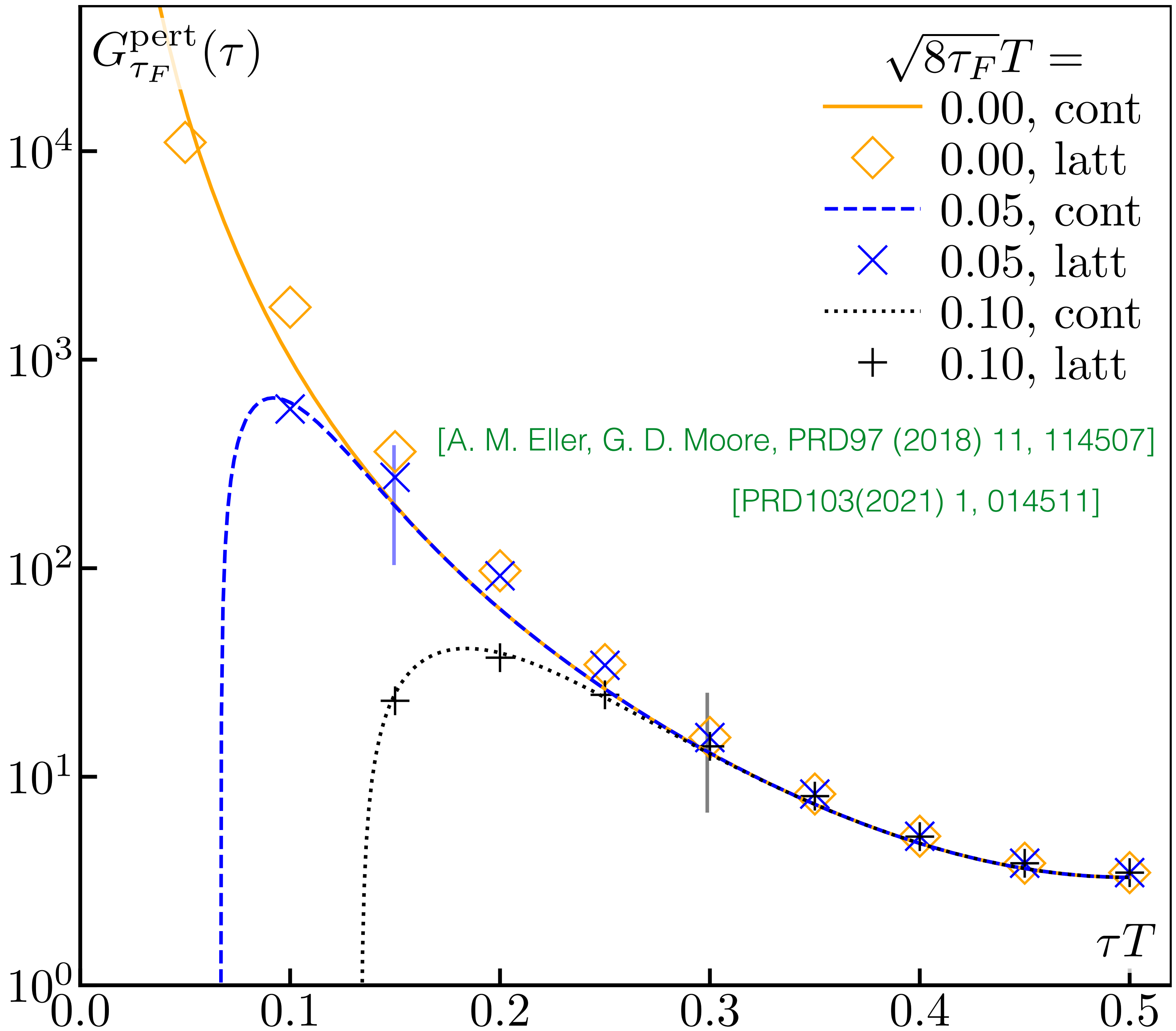
Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064



- Need good enough signal at finite flow time
- Need to know how much one can flow
- Need to know how to go back to zero flow

Maximum flow time limit

color-electric correlators



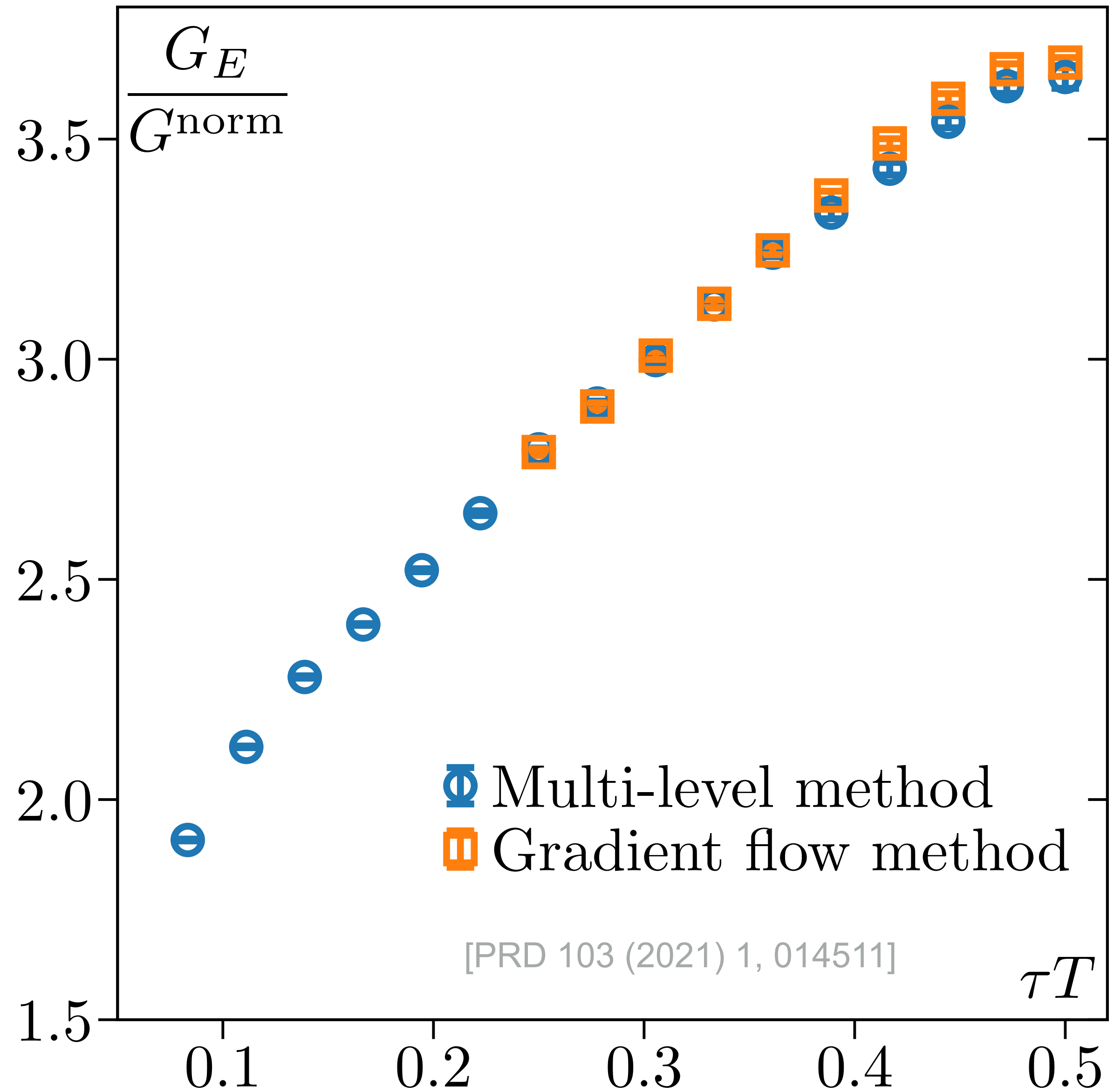
- Flow destroys the signal at small distances
- Large distance parts are not affected
- More points are destroyed at larger flow times
- At most 1% deviation of flowed correlators from unflowed ones determines maximum flow time:

$$a \lesssim \sqrt{8t} \lesssim \frac{\tau - a}{3}$$

a : to suppress lattice effects

a : lattice version of the perturbative flow time limit

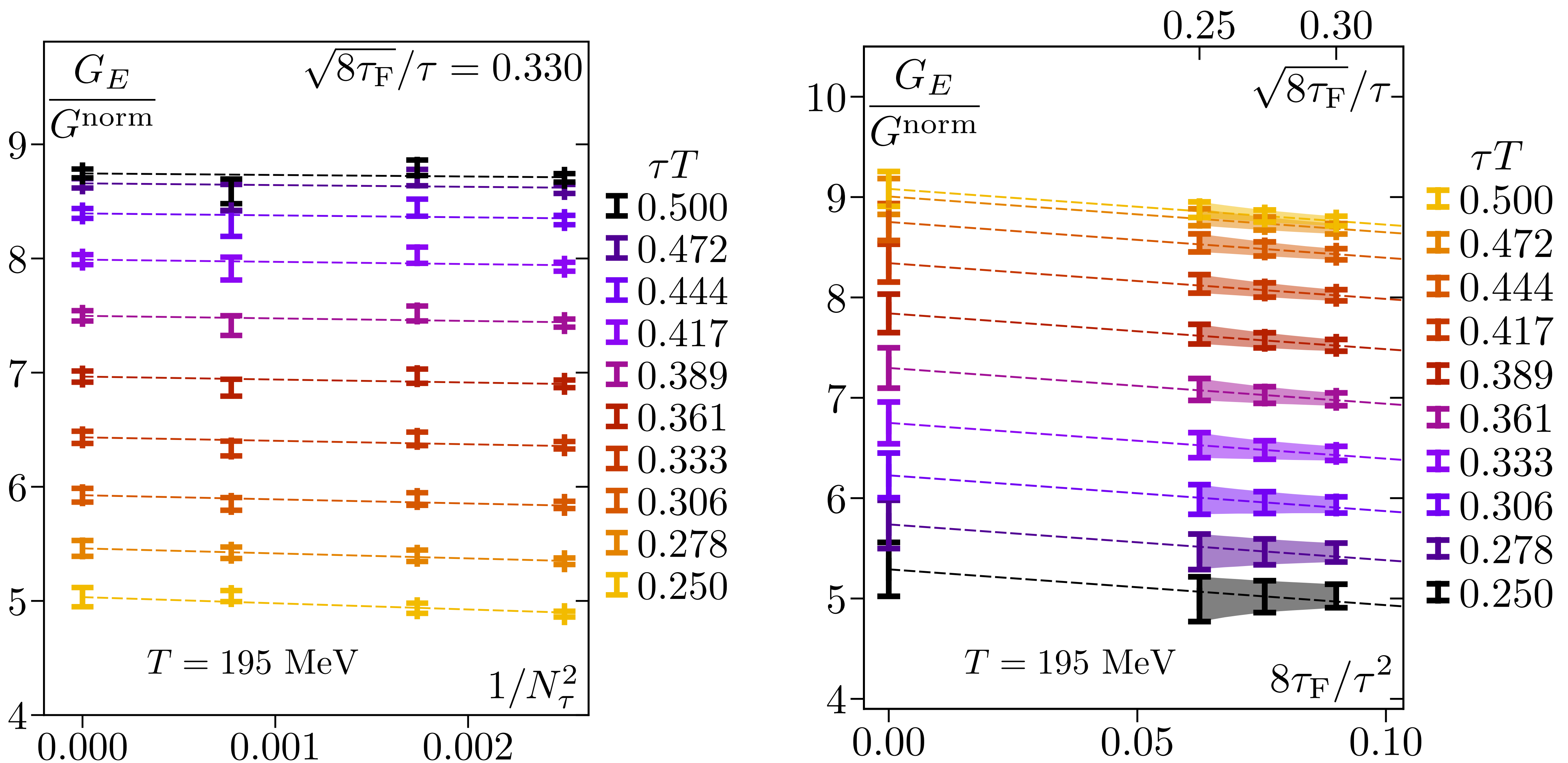
Develop methodology in quenched approximation



- Consistent results from ML & GF
- Gradient flow paves the way to full QCD

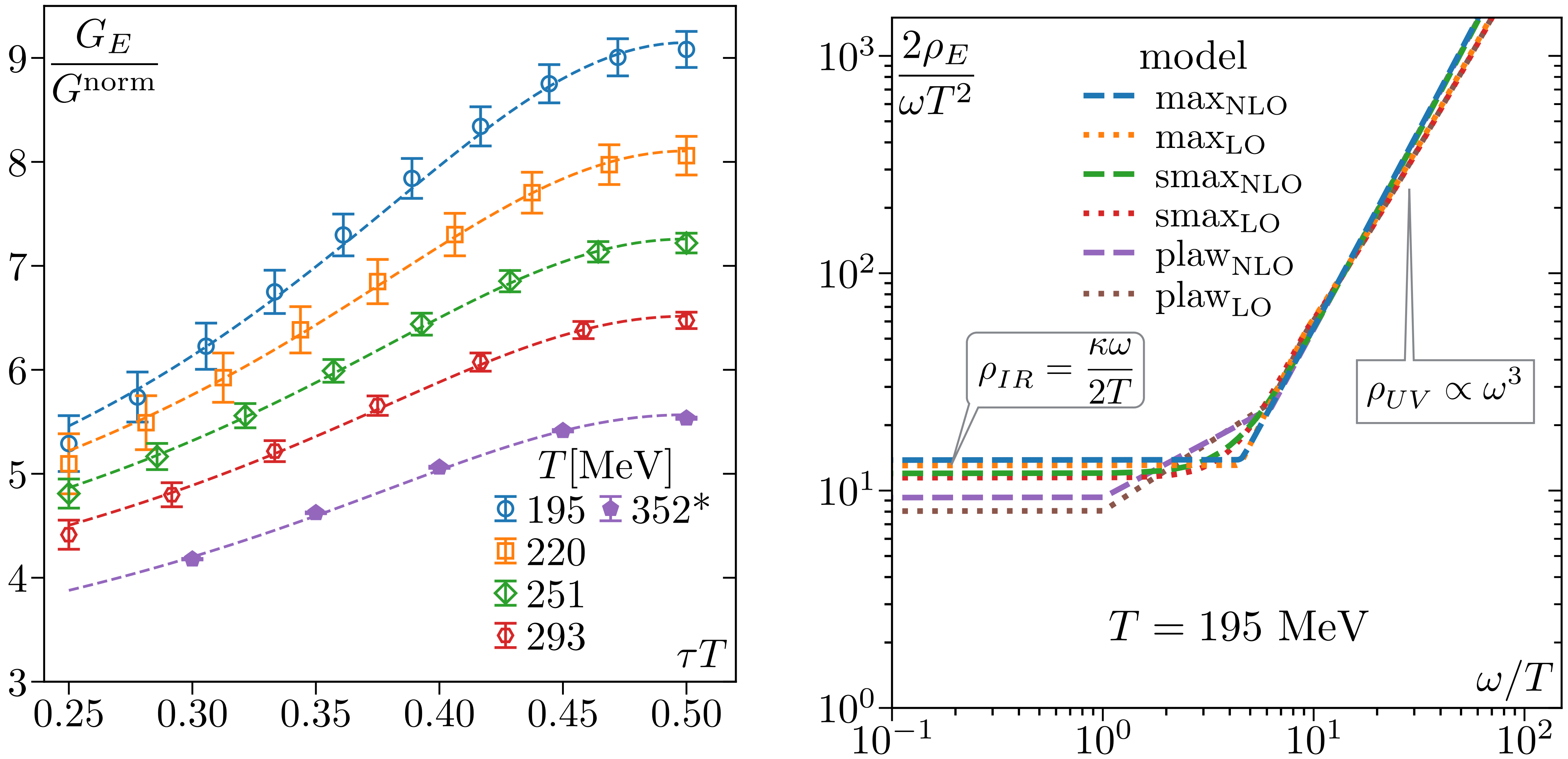
Kappa_E in QCD: double extrapolation

- First full QCD calculation of kappa!
- Wide temperature range 195 MeV - 352 MeV
- Mpion=320 MeV
- Extrapolation Ansatz describes lattice data well



hotQCD, PRL 130 (2023) 23, 231902

Kappa_E in QCD: spectra modelling



hotQCD, PRL 130 (2023) 23, 231902

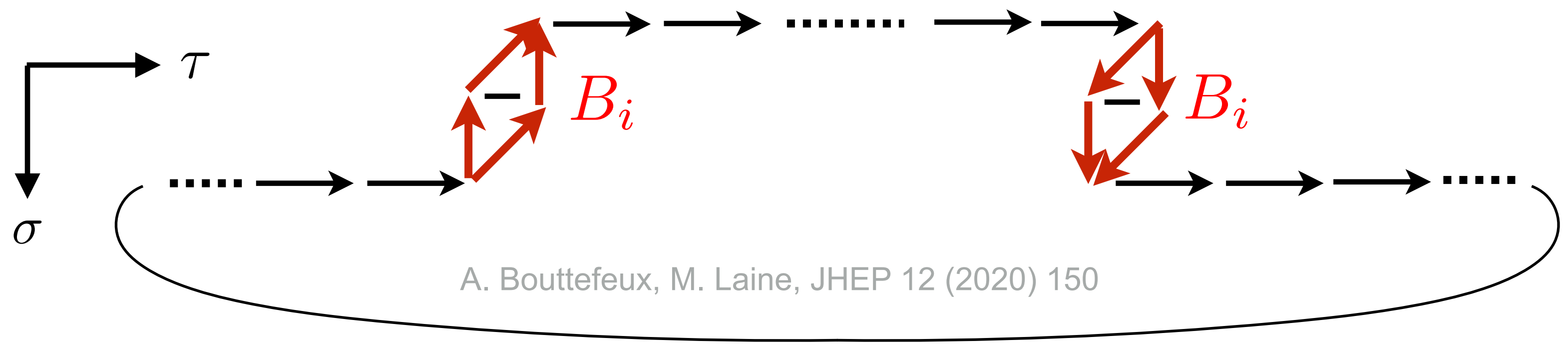
- Good description of lattice data using different models

$$\rho_{\text{max}} \equiv \max(\phi_{\text{IR}}, \phi_{\text{UV}})$$

$$\rho_{\text{smax}} \equiv \sqrt{\phi_{\text{IR}}^2 + \phi_{\text{UV}}^2}$$

$$\rho_{\text{plaw}} \equiv \begin{cases} \phi_{\text{IR}} & \omega \leq \omega_{\text{IR}}, \\ a\omega^b \quad \text{for} & \omega_{\text{IR}} < \omega < \omega_{\text{UV}}, \\ \phi_{\text{UV}} & \omega \geq \omega_{\text{UV}}, \end{cases}$$

Finite mass correction



Color-magnetic field correlation function

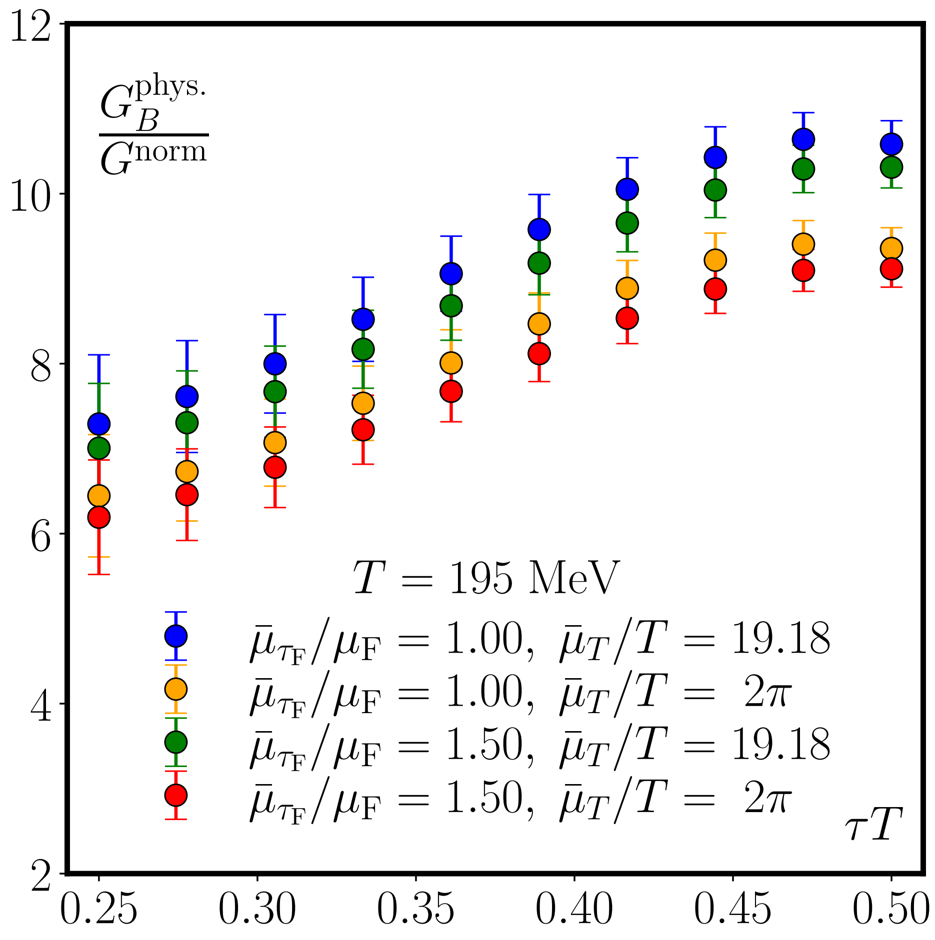
D. Guazzini, et al., JHEP 10 (2007) 081

Charm & bottom quark not heavy enough

Banerjee '22: D. Banerjee, et al., JHEP 08 (2022) 128

TUMQCD '22: N. Brambilla, et al., PRD107, 054508

M. Laine, JHEP 06 (2021) 139



Anomalous dimension at 1-loop order in MSbar:

$$\rho_B(\omega) = \frac{g^2 C_F \omega^3}{6\pi}\left[1 - \frac{g^2 C_A}{(4\pi)^2}\frac{2}{\epsilon} + (\text{finite})\right] + \mathcal{O}(g^6)$$

Logarithmic divergence in flow time:

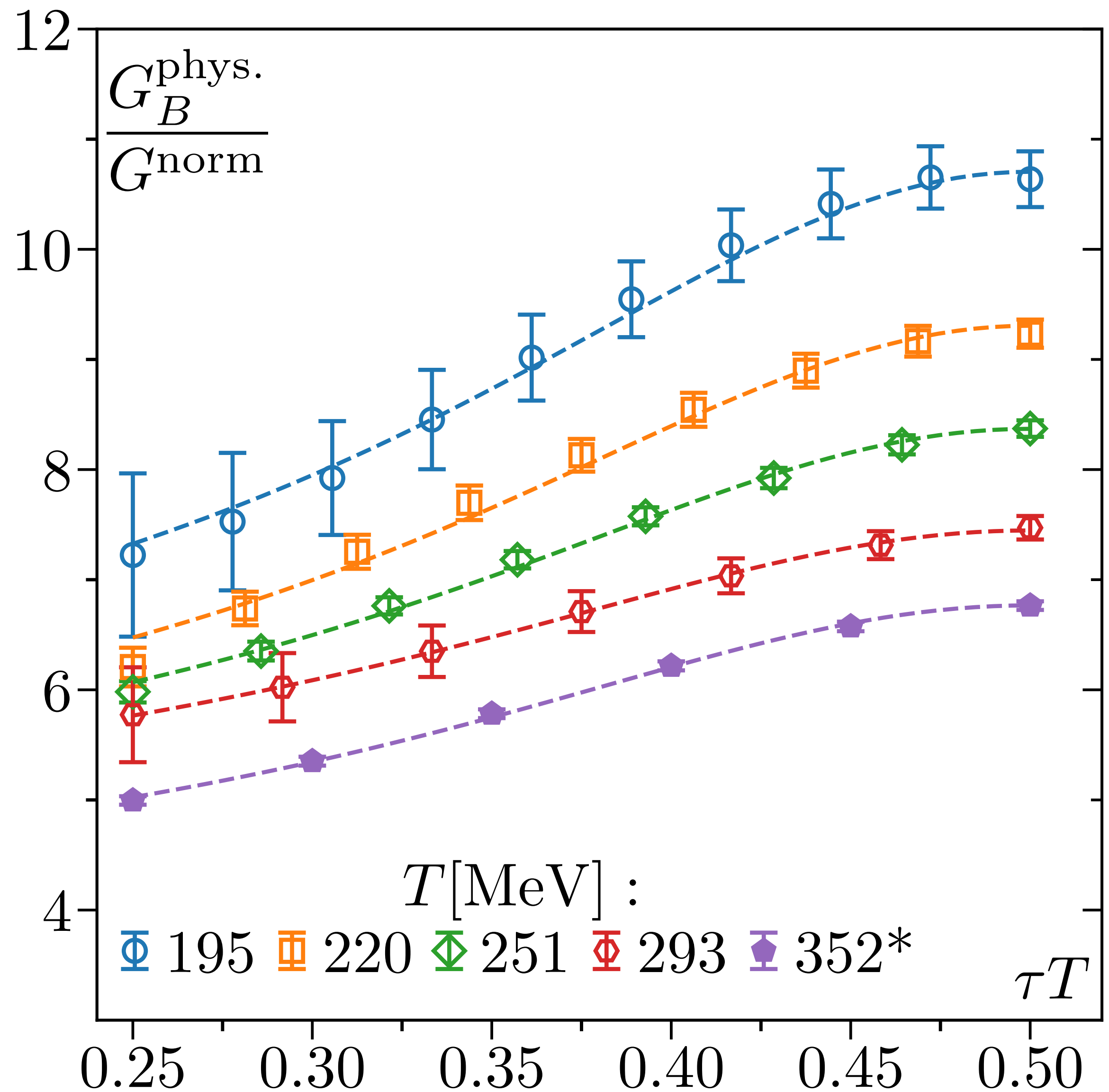
$$G_B^{\text{flow}}(\tau, \tau_{\text{F}}) \sim 1 + 2\gamma_0 g^2 \ln(\mu/\mu_{\text{F}})$$

Regulated by a matching procedure:

$$G_B^{\text{phys}}(\tau, T) = \lim_{\tau_{\text{F}} \to 0} Z_{\text{match}}(\bar{\mu}_T, \bar{\mu}_{\tau_{\text{F}}}, \mu_{\text{F}})\, G_B^{\text{flow}}(\tau, T, \tau_{\text{F}})$$

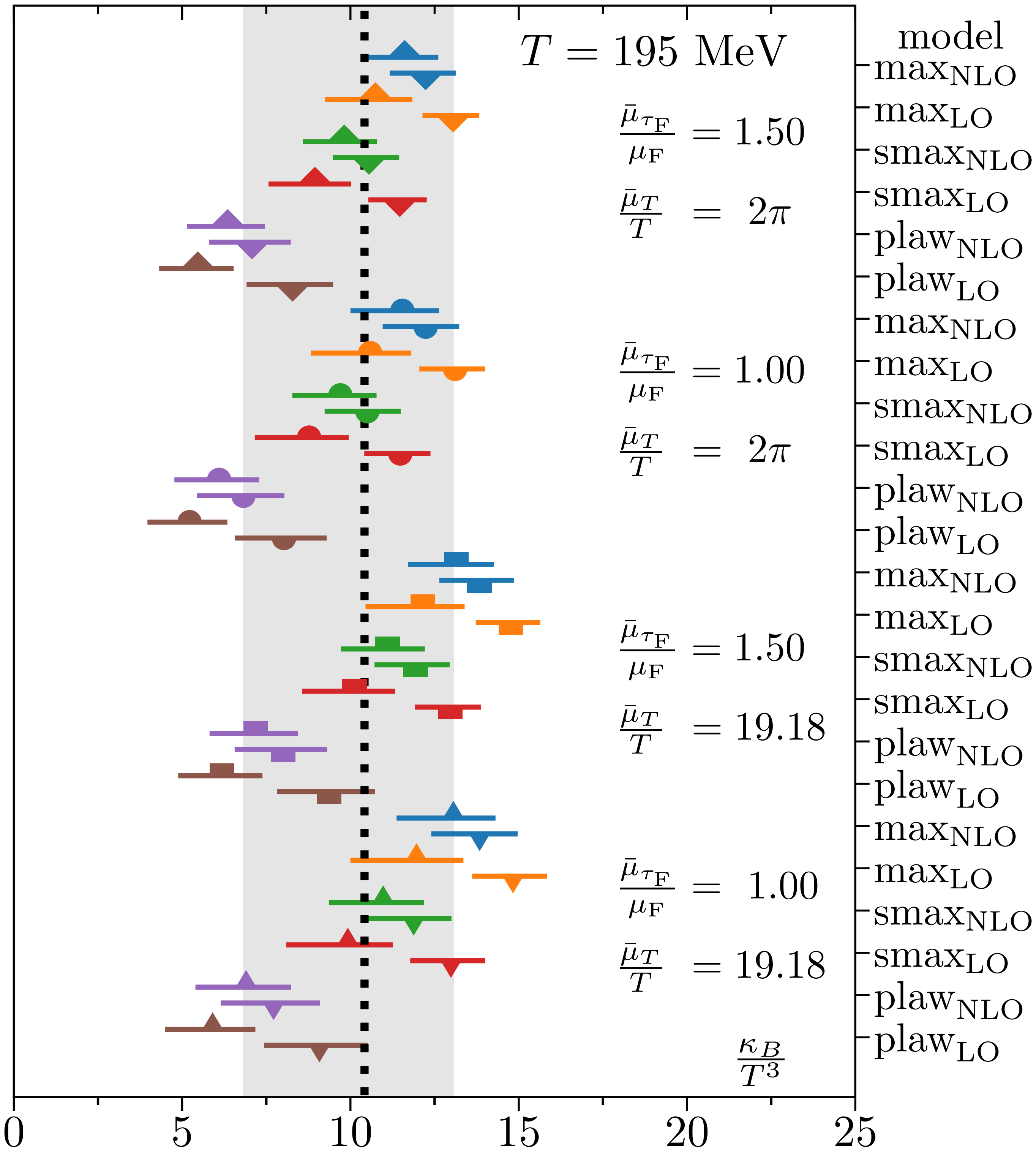
hotQCD, arXiv: 2311.01525

Kappa_B in QCD: spectra modeling

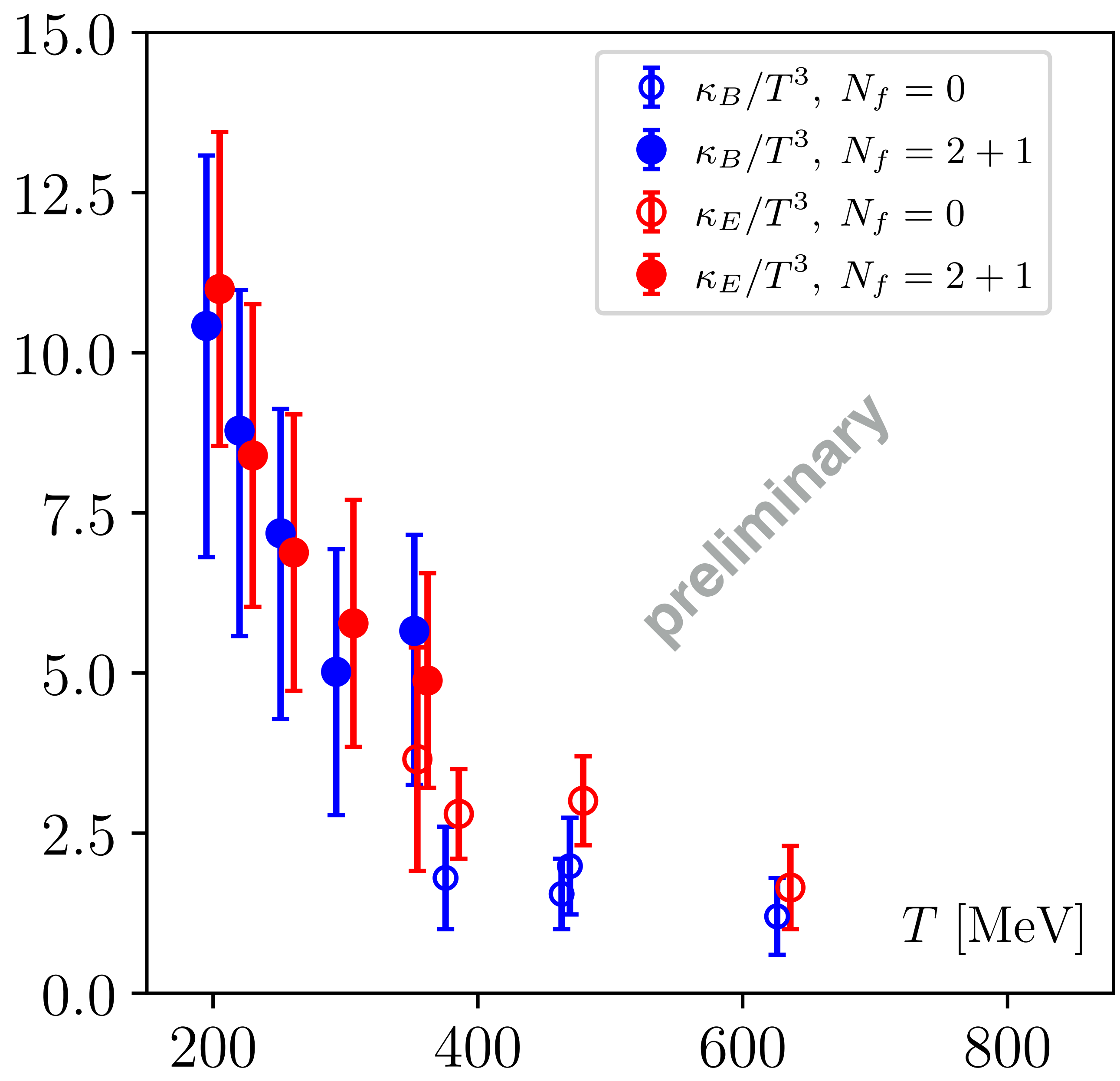


- UV spectral function needs matching to the physical one as well
- Similar modeling methodology as in the color-electric field case
- Good description of lattice data using different models

Kappa_B in QCD: scattering from various models



Kappa_E v.s. Kappa_B

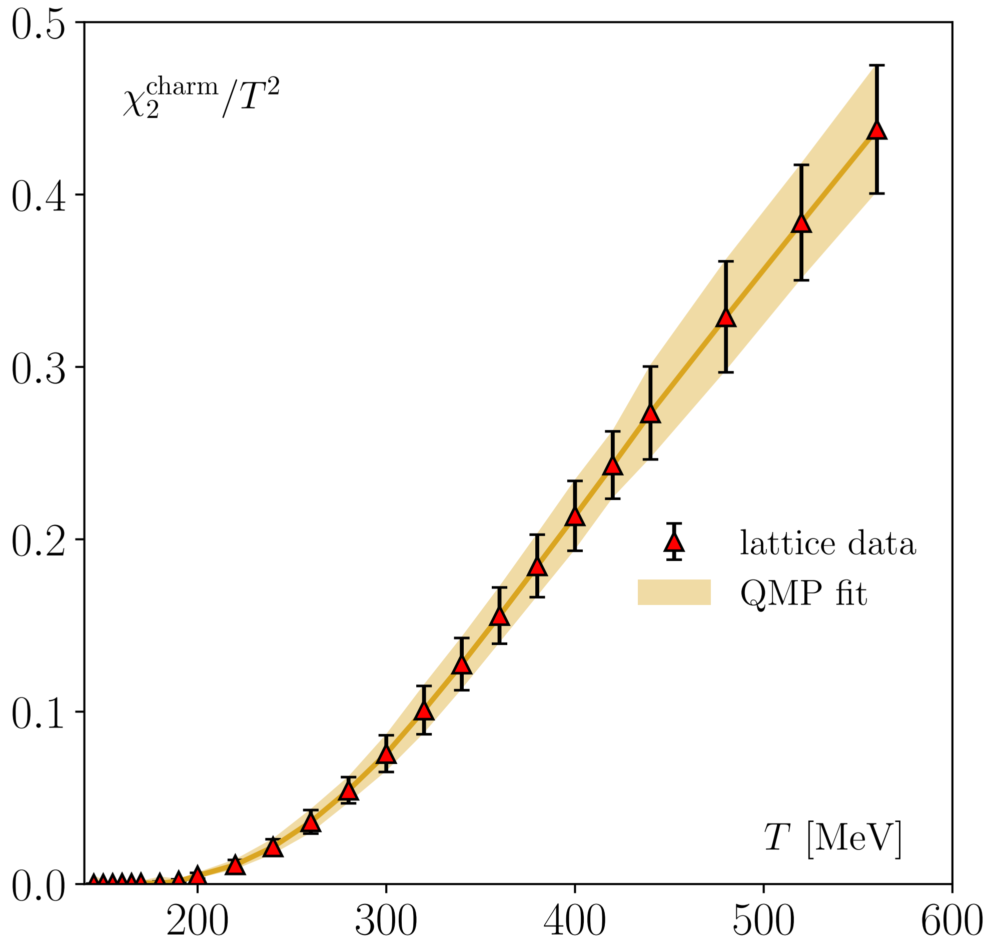


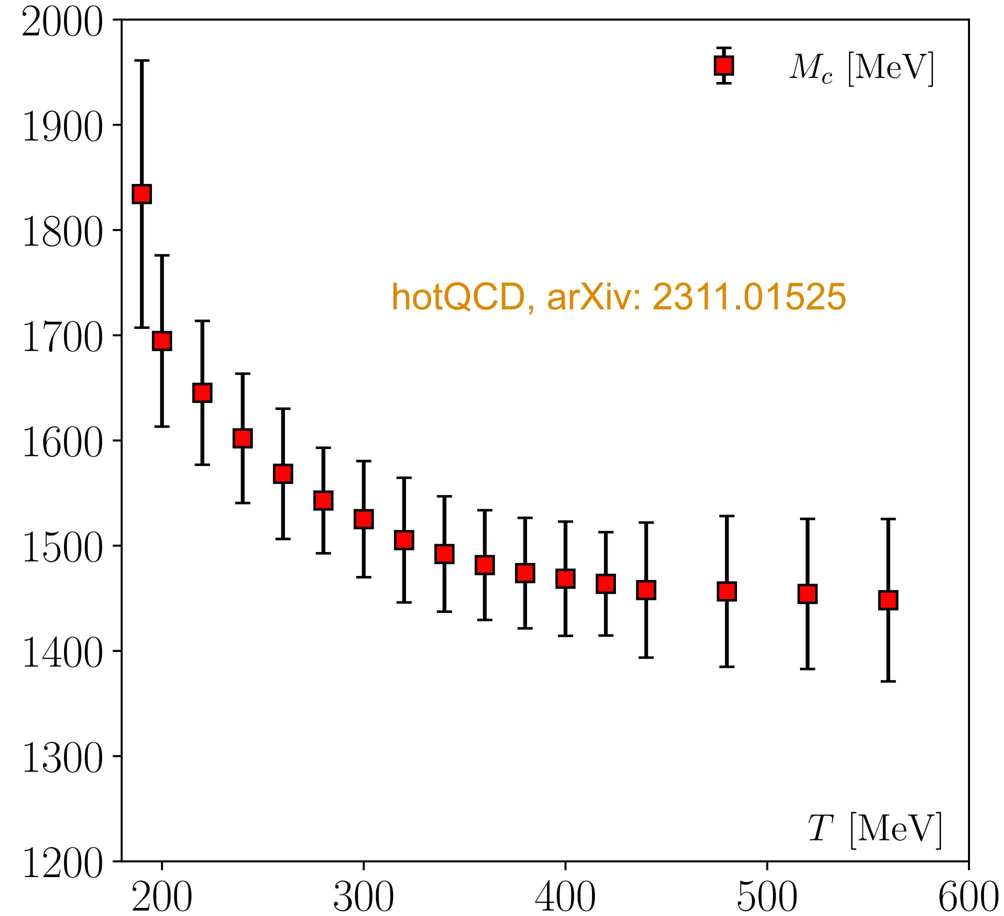
Quenched results from:

A. Francis, et al., PRD92, 116003
B. L. Altenkort, et al., PRD103,014511
D. Banerjee, et al., Nucl.Phys.A.2023.122721
D. Banerjee, et al., JHEP 08 (2022) 128
N. Brambilla, et al., PRD107, 054508

- Similar magnitude for Kappa_E and Kappa_B in full QCD & quenched
- Smooth connection between quenched and full QCD in temperature

Temperature-dependent charm quark mass





$$\frac{\chi_2^{\rm charm}}{T^2} = \frac{4N_c}{(2\pi T)^3}\int d^3p\; e^{-E_p/T}$$

$$E_p^2(T) = m^2(T) + p^2$$

→ $m(T)$

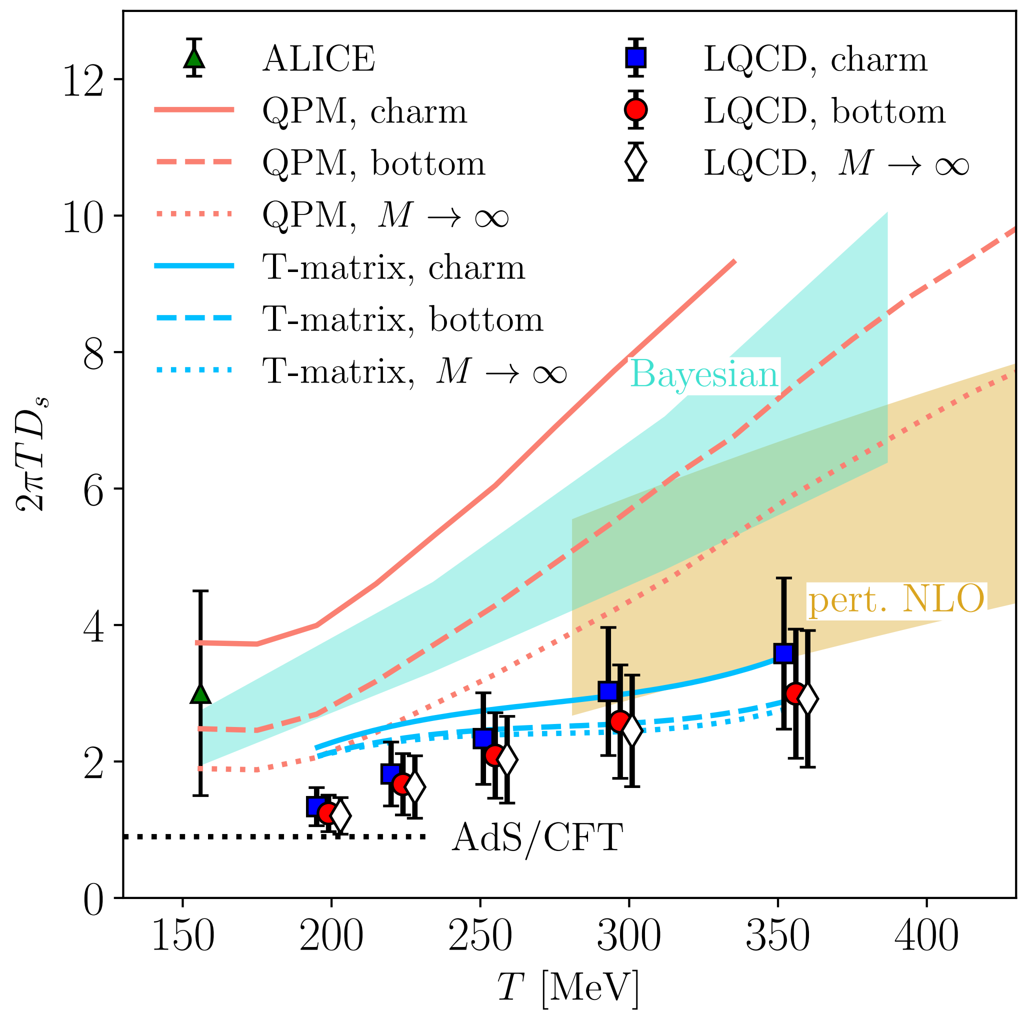
$$\langle v^2\rangle = \left(\int d^3p\; \frac{p^2}{E_p^2} e^{-E_p/T}\right) / \left(\int d^3p\; e^{-E_p/T}\right)$$

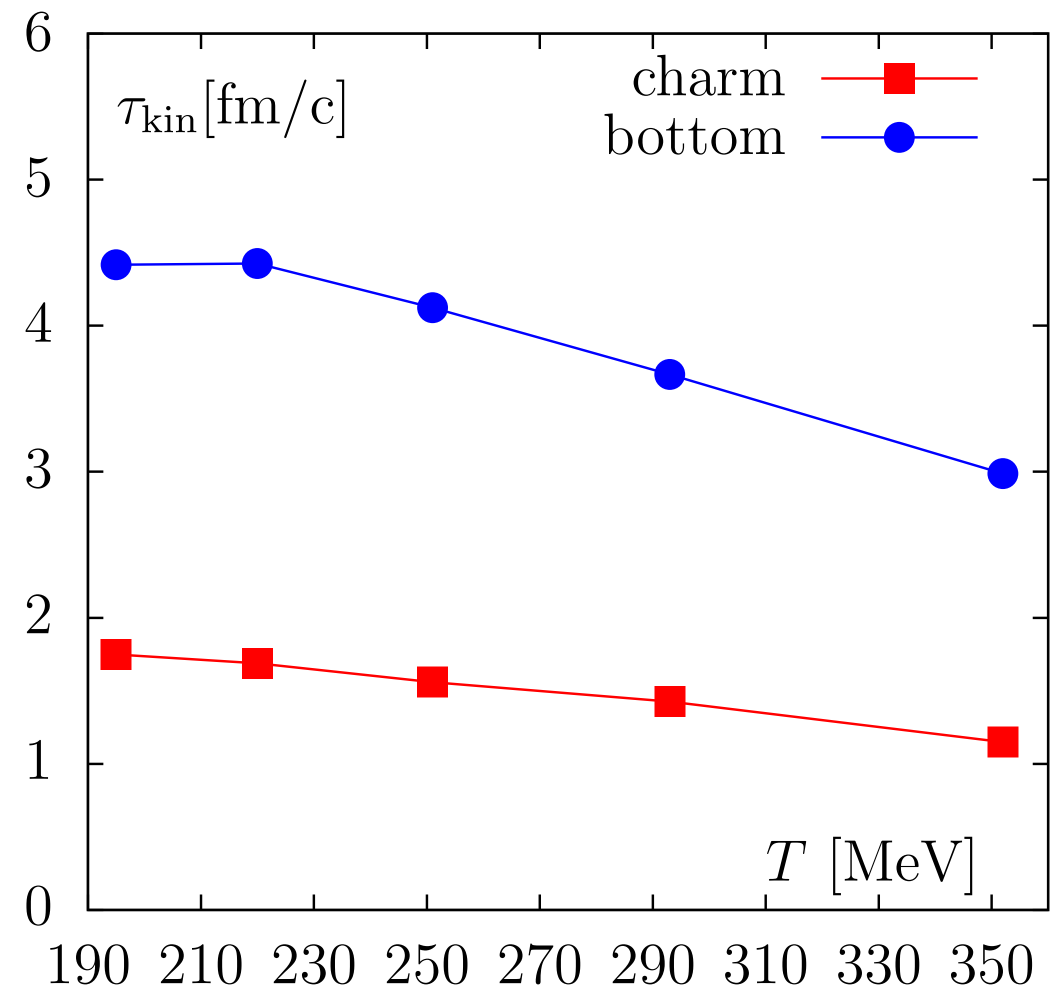
$$\langle p^2\rangle = \left(\int d^3p\; p^2 e^{-E_p/T}\right) / \left(\int d^3p\; e^{-E_p/T}\right)$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}}$$

$$D_s = \frac{2T^2}{\kappa}\cdot\frac{\langle p^2\rangle}{3MT}$$

Summary





hotQCD, arXiv: 2311.01525

- 2piTD decreases with increasing quark mass, same for LQCD, QPM & T-matrix
- Quark mass dependence is small in LQCD & T-matrix
- Weaker quark mass dependence than QMP calculations
- Agree with AdS/CFT at ~Tc (rapid equilibrium)
- Agree with T-matrix estimate at moderate and high T
- Lattice results favor the experimental estimate for the charm quark equilibration time

Backup: full QCD setup

$$N_f = 2+1, \text{ HISQ}, \ m_\pi = 320 \text{ MeV}$$

T [MeV]	β	am_s	am_l	N_σ	N_τ	# conf.
195	7.570	0.01973	0.003946	64	20	5899
	7.777	0.01601	0.003202	64	24	3435
	8.249	0.01011	0.002022	96	36	2256
220	7.704	0.01723	0.003446	64	20	7923
	7.913	0.01400	0.002800	64	24	2715
	8.249	0.01011	0.002022	96	32	912
251	7.857	0.01479	0.002958	64	20	6786
	8.068	0.01204	0.002408	64	24	5325
	8.249	0.01011	0.002022	96	28	1680
293	8.036	0.01241	0.002482	64	20	6534
	8.147	0.01115	0.002230	64	22	9101
	8.249	0.01011	0.002022	96	24	688
352	8.249	0.01011	0.002022	96	20	2488

- Wide temperature range
- Different lattice spacings
- Large lattices towards thermodynamic limit

Backup: matching factor

$$\ln Z_{\text{match}} = \int_{\bar{\mu}_T^2}^{\bar{\mu}_{\tau_F}^2} \gamma_0 g^2_{\overline{\text{MS}}}(\bar{\mu}) \frac{d\bar{\mu}^2}{\bar{\mu}^2} + \gamma_0 g^2_{\overline{\text{MS}}}(\bar{\mu}_T) \left[\ln \frac{\bar{\mu}_T^2}{(4\pi T)^2} - 2 + 2\gamma_{\text{E}}\right] - \gamma_0 g^2_{\overline{\text{MS}}}(\bar{\mu}_{\tau_F}) \left[\ln \frac{\bar{\mu}_{\tau_F}^2}{4\,\mu_F^2} + \gamma_{\text{E}}\right]$$