Charm fluctuations and charm deconfinement from lattice QCD

Sipaz Sharma, F. Karsch, P. Petreczky

HotQCD Collaboration

ZIMÁNYI SCHOOL 2023, Budapest, Hungary
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.
  
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]

- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.  
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]

- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.

- Do charmed hadrons start melting at $T_{pc}$? – compare lattice results with HRG model.
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]
- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.
- Do charmed hadrons start melting at $T_{pc}$? – compare lattice results with HRG model.
- If yes, what are the relevant charmed dofs after the onset of hadron melting? Can we get a signal for the quark appearance above $T_{pc}$?
- Charm fluctuations (cumulants) calculated in the framework of Lattice QCD can receive enhanced contributions due to the existence of not-yet-discovered open-charm states; it is possible to compare this enhancement to the HRG calculations.
Motivation

► Strong interaction matter undergoes a chiral crossover at 
\[ T_{pc} = 156.5 \pm 1.5 \text{ MeV}. \]

[HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]

► In heavy-ion collisions, relevant degrees of freedom change from 
partonic to hadronic in going from high temperature phase to 
temperatures below \[ T_{pc}. \]

► Do charmed hadrons start melting at \[ T_{pc} \]? – compare lattice results 
with HRG model.

► If yes, what are the relevant charmed dofs after the onset of hadron 
melting? Can we get a signal for the quark appearance above \[ T_{pc} \]?

► When do charmed hadrons stop contributing to the total charm 
pressure?
Motivation

- Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV.
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]
- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $T_{pc}$.
- Do charmed hadrons start melting at $T_{pc}$? – compare lattice results with HRG model.
- If yes, what are the relevant charmed dofs after the onset of hadron melting? Can we get a signal for the quark appearance above $T_{pc}$?
- When do charmed hadrons stop contributing to the total charm pressure?
- Charm fluctuations (cumulants) calculated in the framework of Lattice QCD can receive enhanced contributions due the existence of not-yet-discovered open-charm states; it is possible to compare this enhancement to the HRG calculations.
Hadron Resonance Gas (HRG) model

HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.

Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure:

$$P_{C}(T, \mu) = \frac{1}{2} \pi^2 \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2 \left( \frac{m_i}{T} \right) \cosh \left( \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C \right)$$

[A. Bazavov et al., 2014]

For Baryons the argument of $\cosh$ changes to

$$\hat{\mu}_X = \frac{\mu}{T}, X \in \{B, Q, S, C\}.$$
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.
- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure: $P_C(T, \vec{\mu})/T^4 = M_C(T, \vec{\mu}) + B_C(T, \vec{\mu})$. [C. R. Allton et al., 2005]

\[ M_C(T, \vec{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2 \left( \frac{m_i}{T} \right) \cosh \left( \frac{Q_i \hat{\mu}}{T} \right) \]
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.
- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure: $P_C(T, \mu)/T^4 = M_C(T, \mu) + B_C(T, \mu)$. [C. R. Allton et al., 2005]

$$M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)$$

[A. Bazavov et al., 2014]
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$. 

- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure:

$$\frac{P_C(T, \mu)}{T^4} = M_C(T, \mu) + B_C(T, \mu).$$

[C. R. Allton et al., 2005]

$$M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \mu_Q + S_i \mu_S + C_i \mu_C)$$

[A. Bazavov et al., 2014]

- For Baryons the argument of $\cosh$ changes to

$$B_i \mu_B + Q_i \mu_Q + S_i \mu_S + C_i \mu_C$$

- Boltzmann approximation is good in the charm sector not just for mesons and baryons but also for a charm-quark gas.
Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. HRG has been found to be a good approximation to QCD results for $T < T_{pc}$.
- Charmed baryons and mesons contribute separately to the partition function of HRG, which in turn reflects in contributions to the pressure: $P_C(T, \mu)/T^4 = M_C(T, \mu) + B_C(T, \mu)$. [C. R. Allton et al., 2005]

$$M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C)$$

[A. Bazavov et al., 2014]

- For Baryons the argument of $\cosh$ changes to $B_i \hat{\mu}_B + Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C$
- Boltzmann approximation is good in the charm sector not just for mesons and baryons but also for a charm-quark gas.
- $\hat{\mu}_X = \mu/T, \ X \in \{B, Q, S, C\}$. 

Sipaz Sharma  
Bielefeld University  
December 4, 2023
Generalized susceptibilities of the conserved charges

\[
M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \mu + S_i \mu_s + C_i \mu_C)
\]

- \( K_2(x) \sim \sqrt{\pi/2x} e^{-x} [1 + \mathcal{O}(x^{-1})] \). If \( m_i \gg T \), then contribution to \( P_C \) will be exponentially suppressed.

- \( \Lambda_c^+ \) mass \( \sim 2286 \) MeV, \( \Xi_{cc}^{++} \) mass \( \sim 3621 \) MeV. At \( T_{pc} \), contribution to \( B_C \) from \( \Xi_{cc}^{++} \) will be suppressed by a factor of \( 10^{-4} \) in relation to \( \Lambda_c^+ \).
Generalized susceptibilities of the conserved charges

\[ M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C) \]

- \( K_2(x) \sim \sqrt{\pi/2x} \ e^{-x} \ [1 + O(x^{-1})] \). If \( m_i \gg T \), then contribution to \( P_C \) will be exponentially suppressed.

- \( \Lambda^+ \) mass \( \sim 2286 \) MeV, \( \Xi^{++} \) mass \( \sim 3621 \) MeV. At \( T_{pc} \), contribution to \( B_C \) from \( \Xi^{++} \) will be suppressed by a factor of \( 10^{-4} \) in relation to \( \Lambda^+ \).

- Dimensionless generalized susceptibilities of conserved charges:

\[ \chi_{BQSC}^{klmn} = \left. \frac{\partial^{(k+l+m+n)} \left[ P (\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C) / T^4 \right]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \right|_{\hat{\mu} = 0} \]
Generalized susceptibilities of the conserved charges

\[ M_C(T, \mu) = \frac{1}{2\pi^2} \sum_i g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \mu_Q + S_i \mu_S + C_i \mu_C) \]

- Dimensionless generalized susceptibilities of conserved charges are given by,
  \[ \chi_{BQSC}^{klmn} = \frac{\partial^{(k+1+m+n)} [P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C) / T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \bigg|_{\vec{\mu} = 0} \]

\[ \chi_{BC}^{mn} = B_{C,1} + 2^n B_{C,2} + 3^n B_{C,3} \approx B_{C,1} \]

- At present, we have gone up to fourth order in calculating various cumulants.
Ratios independent of the hadron spectrum

Irrespective of the details of the baryon mass spectrum, in the validity range of HRG, $\chi_{mn}^{BC}/\chi_{kl}^{BC} = 1$, $\forall(m + n), (k + l) \in \text{even}$. 
Ratios independent of the hadron spectrum

- Irrespective of the details of the baryon mass spectrum, in the validity range of HRG, $\chi_{mn}^{BC}/\chi_{kl}^{BC} = 1$, $\forall (m + n), (k + l) \in$ even.
- $\chi_{1n}^{BC}/\chi_{1l}^{BC} = 1$, $\forall n, l \in$ odd, for the entire temperature range.
States with fractional $B$ start appearing near $T_{pc}$. Is it possible to determine this fractional $B$?
Approach to free charm-quark gas limit

\[ P_c(T, \mu) = \frac{3}{\pi^2} \left( \frac{m_c}{T} \right)^2 K_2 \left( \frac{m_c}{T} \right) \cosh \left( \frac{2}{3} \mu_Q + \frac{1}{3} \mu_B + \mu_C \right) \]

\[ m_c = 1.27 \text{ GeV}. \]
Charm degrees of freedom in the intermediate T range

Quasi-particle model:

\[ P^C(T, \hat{\mu}_C, \hat{\mu}_B)/T^4 = P^C_M(T) \cosh(\hat{\mu}_C) + P^C_B(T) \cosh(\hat{\mu}_C + \hat{\mu}_B) + P^C_q(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3) \]

\[ P^C_q = 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2 \]
\[ P^C_B = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2 \]
\[ P^C_M = \chi_4^C + 3\chi_{22}^{BC} - 4\chi_{13}^{BC} \]

Constraint on cumulants in a simple quasi-particle model:

\[ c = \chi_{13}^{BC} + 3\chi_{31}^{BC} - 4\chi_{22}^{BC} = 0 \]

[S. Mukherjee et al., 2016]
The constraint holds true \( \implies \) quasi-particle states with \( |B| = 0, 1 \) or \( 1/3 \) exist in the intermediate temperature range.
Charm-quark-like excitations in QGP

Right after $T_{pc}$, $P_q$ starts contributing to $P_C$, which is compensated by a reduction (and deviation from HRG) in the fractional contribution of the hadron-like states to $P_C$. 
Quantum numbers of the charm-quark like excitations in QGP?

Our data suggests only the existence of $|B| = 0, 1$ or $1/3$, which implies four possibilities for $|Q|$: $0, 1, 2, \text{and } 2/3$.

We can use four fourth-order QC correlations to determine partial pressures of the four possible electrically-charged-charm subsectors.

For the $BQC$ sector there are three possibilities: i) $\{ |B| = 1, |Q| = 1 \}$; ii) $\{ |B| = 1, |Q| = 2 \}$; iii) $\{ |B| = 1/3, |Q| = 2/3 \}$.

$$P_B = 1/3, Q = 2/3, C = 27/4 \left[ \chi_{BQC11} - \chi_{BQC21} \right]$$
Charm-quark-like excitations in QGP

- Quantum numbers of the charm-quark like excitations in QGP?
- Our data suggests only the existence of $|B| = 0, 1$ or $1/3$
  $\implies$ four possibilities for $|Q| : 0, 1, 2$ and $2/3$. 
Quantum numbers of the charm-quark like excitations in QGP?

Our data suggests only the existence of $|B| = 0, 1$ or $1/3$  
$\implies$ four possibilities for $|Q| : 0, 1, 2$ and $2/3$.

We can use four fourth-order QC correlations to determine partial pressures of the four possible electrically-charged-charm subsectors.

$$P_{C}^{Q=2/3} = \frac{1}{8} \left[ 54 \chi_{13}^{QC} - 81 \chi_{22}^{QC} + 27 \chi_{31}^{QC} \right]$$
Charm-quark-like excitations in QGP

- Quantum numbers of the charm-quark like excitations in QGP?
- Our data suggests only the existence of $|B| = 0, 1$ or $1/3$ 
  $\implies$ four possibilities for $|Q| : 0, 1, 2$ and $2/3$.
- We can use four fourth-order QC correlations to determine partial pressures of the four possible electrically-charged-charm subsectors.

$$P_{C}^{Q=2/3} = \frac{1}{8} \left[ 54 \chi_{13}^{QC} - 81 \chi_{22}^{QC} + 27 \chi_{31}^{QC} \right]$$

- For the BQC sector there are three possibilities: i) $\{ |B| = 1, |Q| = 1 \}$; ii) $\{ |B| = 1, |Q| = 2 \}$; iii) $\{ |B| = 1/3, |Q| = 2/3 \}$.

$$P_{C}^{B=1/3, Q=2/3} = \frac{27}{4} \left[ \chi_{112}^{BQC} - \chi_{211}^{BQC} \right]$$
Charm-quark-like excitations in QGP

Clear agreement between three independent observables which correspond to the partial pressures of
i) $B = \frac{1}{3}$, ii) $Q = \frac{2}{3}$, and iii) $B = \frac{1}{3}$ and $Q = \frac{2}{3}$ charm subsectors.
Baryonic and mesonic contributions to $P_C$

In the low temperature range, where HRG works,

- $\chi_{13}^{BC}$ is the partial pressure from the charmed-baryonic subsector.
- $\chi_{4}^{C} - \chi_{13}^{BC}$ can be interpreted as the partial pressure from the charmed-mesonic subsector.
Baryonic and mesonic contributions to $P_C$

In the low temperature range, where HRG works,

- $\chi^{BC}_{13}$ is the partial pressure from the charmed-baryonic subsector.
- $\chi^C_4 - \chi^{BC}_{13}$ can be interpreted as the partial pressure from the charmed-mesonic subsector.
- Unlike the previous quantities shown, ratios such as $\frac{\chi^{BC}_{13}}{(\chi^C_4 - \chi^{BC}_{13})}$ will partially depend upon the hadron spectrum.
Ratios of baryonic and mesonic contributions to $P_C$

\[ \Delta = 93 \text{ percent} \]

\[ \Delta = 15 \text{ percent} \]

\[ \Delta = \left( |1 - \text{QM-HRG/PDG-HRG}| \right) |T_{pc}| \]

Missing charmed-baryonic states below $T_{pc}$.
With now available statistics, possibility of distinguishing $|Q| = 0, 1, 2$ charm subsectors in the hadronic phase.
Electrically-charged-charm subsector

Ratio of QM-HRG/PDG-HRG increases with increasing $Q$-moments

$|Q| = 2$ sector more sensitive to ‘missing resonances’.

$\chi^{QC}_{22}$ and $\chi^{QC}_{31}$ give evidence for ‘missing resonances’.
Electrically-charged-charm subsector

- Ratio of QM-HRG/PDG-HRG increases with increasing $Q$-moments.
- Close to freeze-out, an enhancement over the PDG expectation in the fractional contribution of the $|Q| = 2$ ($\Sigma_c^{++}$) charm subsector to the total charm partial pressure.
Conclusions & Outlook

- Onset of hadron melting at $T_{pc}$.
- Evidence of deconfinement in terms of presence of charm-quark-like excitations in QGP.
- No evidence for the existence of charmed diquarks above $T_{pc}$.
- Analysis shows that there are missing states in the PDG record.
Conclusions & Outlook

- Onset of hadron melting at $T_{pc}$.
- Evidence of deconfinement in terms of presence of charm-quark-like excitations in QGP.
- No evidence for the existence of charmed diquarks above $T_{pc}$.
- Analysis shows that there are missing states in the PDG record.
- Continuum limit with two different LCPs is in progress; it will enable us to make a statement based on the absolute cumulants.
No strange-charm diquarks.
Only $|B| = 1$ sector contributes to partial pressure from $|Q| = 2$ charmed subsectors.
$|Q| = 2$

$x = 1.8994$