Topology and Axions from Lattice QCD

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23rd Zimányi School Winter Workshop on Heavy Ion Physics, Budapest, Hungary

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

CRC–TR 211
Strong-interaction matter under extreme conditions

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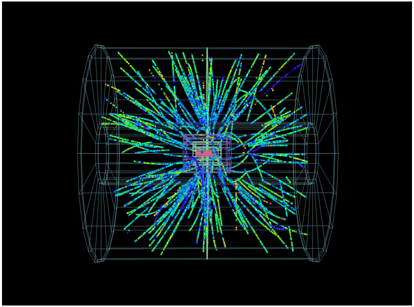
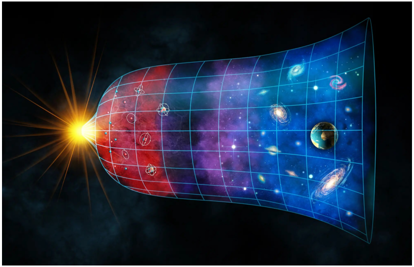
Outline

- Motivation
- Topology in QCD with EM fields
- The topological susceptibility
- The axion-photon coupling
- Conclusions and further work

Motivation





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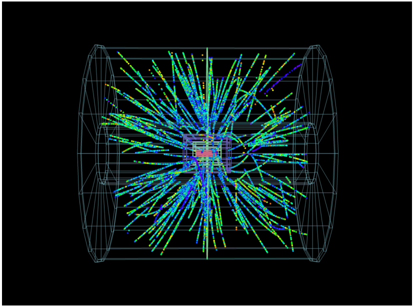

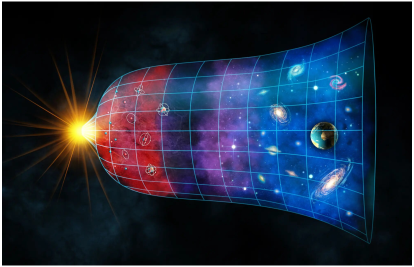

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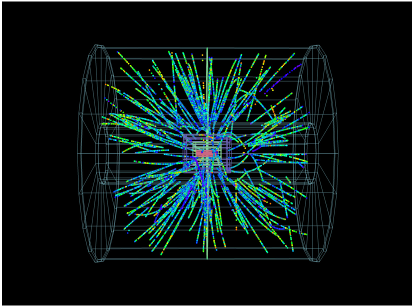

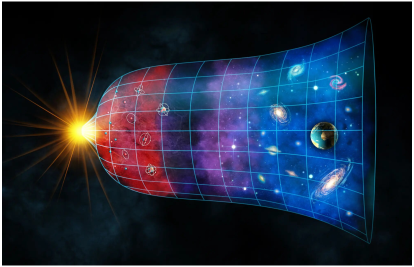

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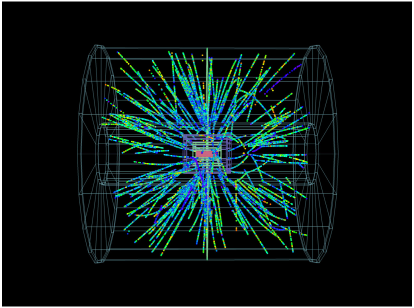
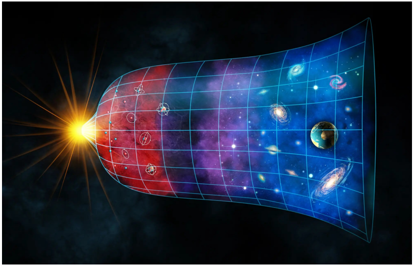
- Very interesting physics when combining strong interactions and electromagnetic fields!

Motivation





$$Q_{\text{top}} = \int d^4x\, q_{\text{top}}(x), \quad q_{\text{top}} = \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\, G_{\mu\nu} G_{\rho\sigma} .$$

Topology in QCD with EM fields

$\langle Q_{\text{top}} \rangle$ *with* EM fields

- In general, the expectation value of any (scalar) operator $\mathcal{O}$ under background electromagnetic fields $F_{\mu\nu}$ is of the form

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$$\langle Q_{\text{top}} \rangle (F_{\mu\nu}) = g \vec{E} \cdot \vec{B}.$$

$\langle Q_{\text{top}} \rangle \neq 0$ only for certain arrangements of the EM fields! $\implies \vec{E} \cdot \vec{B} \neq 0$

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- The leading dependence with $\vec{B}^2$ must be exactly opposite to that with $\vec{E}^2$, at $T = 0$.
- Non-trivial behaviour at finite temperature.
- $\chi_{\text{top}}(0)$ has already been characterised at zero and finite T, both in ChPT Cortona et al 2016 and on the lattice Borsanyi et al 2016. Small enhancement with $\vec{B}$ from ChPT calculations Adhikari 2021.

The topological susceptibility

Lattice artifacts

- Index theorem says $\not{D}$ has zero modes when $Q_{\text{top}} \neq 0$.
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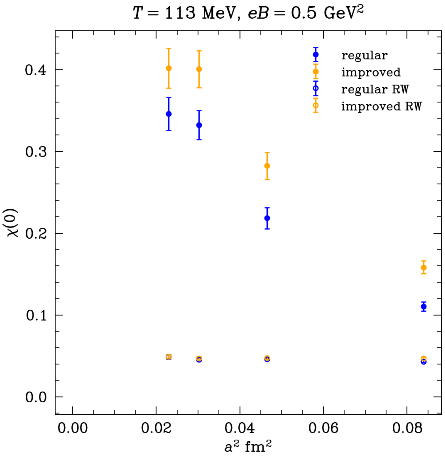
- Index theorem says $\not{D}$ has zero modes when $Q_{\text{top}} \neq 0$.
- Staggered operator lacks these zero modes $\longrightarrow$ huge lattice artifacts, specially for high temperatures (we are talking of several orders of magnitude!).
- One possible solution: substitute the smallest eigenvalues of D_{stagg} with their continuum values Borsanyi et al 2016.
- How? Reweighting each configuration by:

$$\prod_{f} \prod_{i=1}^{2|Q_{\text{top}}|} \prod_{\sigma=\pm} \left(\frac{2m_f}{i\sigma\lambda_i + 2m_f} \right)^{n_f/4}.$$

χ_{top}: preliminary results



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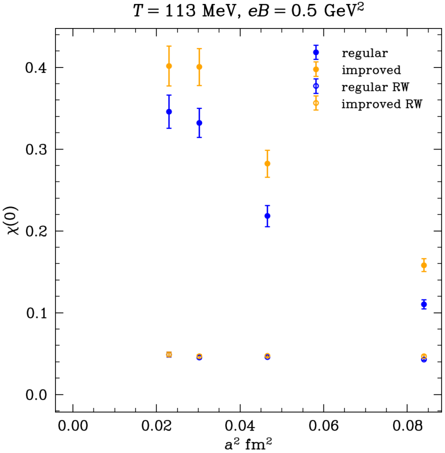

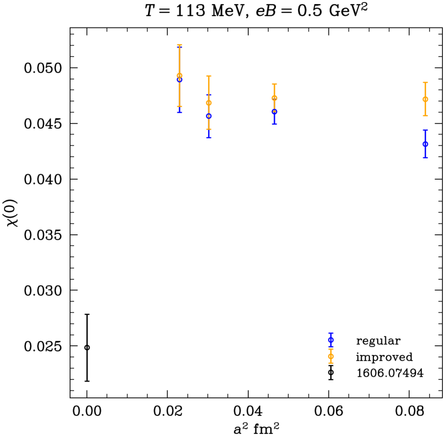

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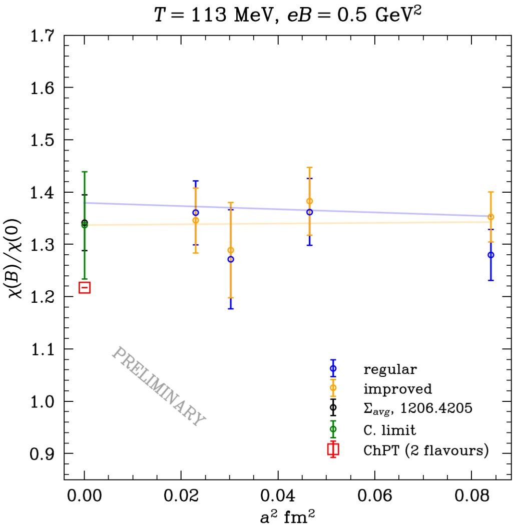

The axion-photon coupling

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- Current estimate from ChPT.: $g_{a\gamma\gamma} = g^0_{a\gamma\gamma} + g^{QCD}_{a\gamma\gamma} = \frac{\alpha_{\text{em}}}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right)$ Cortona et al 2016.

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- We want to compute the QCD dependent part of the coupling $\longrightarrow$ no need to include axions on the lattice!

Q_{top} and $g_{a\gamma\gamma}^{QCD}$

- By looking at $\mathcal{Z}$:

$$\left.\frac{\delta \log \mathcal{Z}(a)}{\delta a}\right|_{a=0} = \frac{\langle Q_{\mathsf{top}}\rangle_{E,B}}{f_a} \longrightarrow g_{a\gamma\gamma}^{QCD} f_a = \frac{T}{V}\frac{\partial}{\partial(\mathbf{E}\cdot\mathbf{B})}\langle Q_{\mathsf{top}}\rangle_{E,B}\Big|_{\mathbf{E},\mathbf{B}=0}$$

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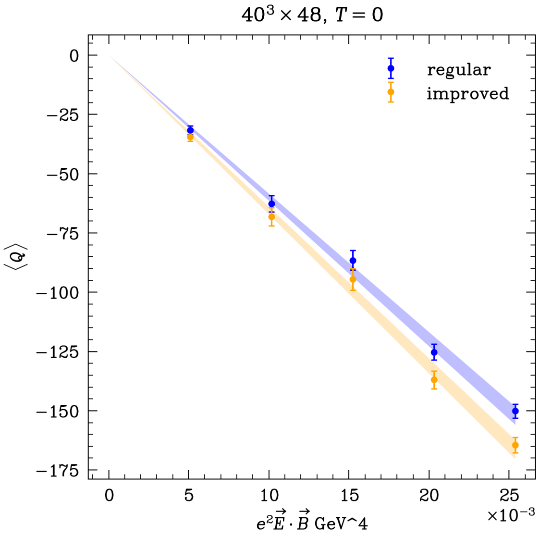
- So for homogeneous, static and weak EM fields

$$\frac{T}{V}\langle \mathcal{Q}_{\mathsf{top}} \rangle_{E,B} \approx \frac{g_{a\gamma\gamma}^{QCD} \cdot f_a}{e^2} e^2 \mathbf{E}\cdot\mathbf{B} \quad \textit{and } g_{a\gamma\gamma}^{QCD} < 0.$$

$g_{a\gamma\gamma}^{QCD}$: preliminary results



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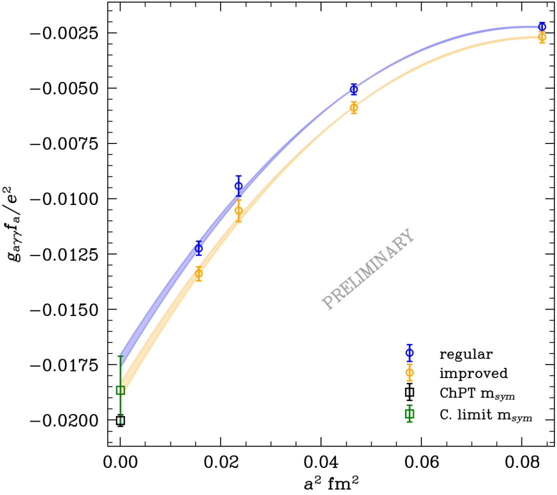

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- Thus, we can take into account isospin violation by rescaling our result by that factor:

$$g_{a\gamma\gamma}^{QCD} = \begin{cases} -0.023(2) & \text{this work,} \\ -0.0243(5) & \text{ChPT.} \end{cases}$$

Conclusions and further work

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- We have shown:
  - that the reweighting is necessary for a controlled continuum limit of $\chi_{top}(B)/\chi_{top}(0)$.
  - First signals of an enhancement of χ_{top} with the magnetic field at low temperatures.
  - a preliminary result for the continuum limit of $g_{a\gamma\gamma}^{QCD}$.
- Further work:
  - generate more statistics for both observables and perform the continuum limit for χ_{top} for the rest of temperatures.
  - implement the reweighting technique for $g_{a\gamma\gamma}^{QCD}$.

Thank you for your attention!

Backup slides

EM and QCD Topology

- EM fields can induce topologies in the gluon sector. But how? $\longrightarrow$ Index theorem.
- The index theorem says (for QCD):

$$\mathrm{Index}(\not{D}) \equiv n_- - n_+ = Q_{top}$$

Since in QCD $\langle Q_{top} \rangle = 0$, we don't see imbalances in chirality.

- But after including electromagnetic fields the situation is different:

$$\mathrm{Index}(\not{D}) \equiv n_- - n_+ = Q_{top} + Q_{U(1)}.$$

We have two different topological contributions to the zero modes.

- Path integral favours as little zero modes as possible: $\det M \uparrow\uparrow$.
- Hence, it selects gluon field configurations such that:

$$Q_{U(1)} \uparrow \iff Q_{top} \downarrow .$$

Simulation setup for χ_{top}

- Improved staggered quarks with 2+1 flavours and physical quark masses.
- $N_s \times N_t = 24^3 \times 6$, $24^3 \times 8$, $28^3 \times 10$, $36^3 \times 12$.
- T = 110-300 MeV, eB = 0, 0.5, 0.8 GeV^2.
- Gradient Flow used to reduce the UV fluctuations and control the topology Lüscher 2010.

Simulation setup for $g_{a\gamma\gamma}^{QCD}$

- Improved staggered quarks with 2+1 flavours and physical quark masses.
- $N_s \times N_t = 24^3 \times 32$, $32^3 \times 48$, $40^3 \times 48$.
- T = 0.
- We keep $\mathbf{E} \cdot \mathbf{B}$ in the linear response region.
- Imaginary electric fields (sign problem).
- Gradient Flow used to reduce the UV fluctuations and control the topology.