HOLOGRAPHY

IN ELASTIC pp SCATTERING AND BINARY STARS

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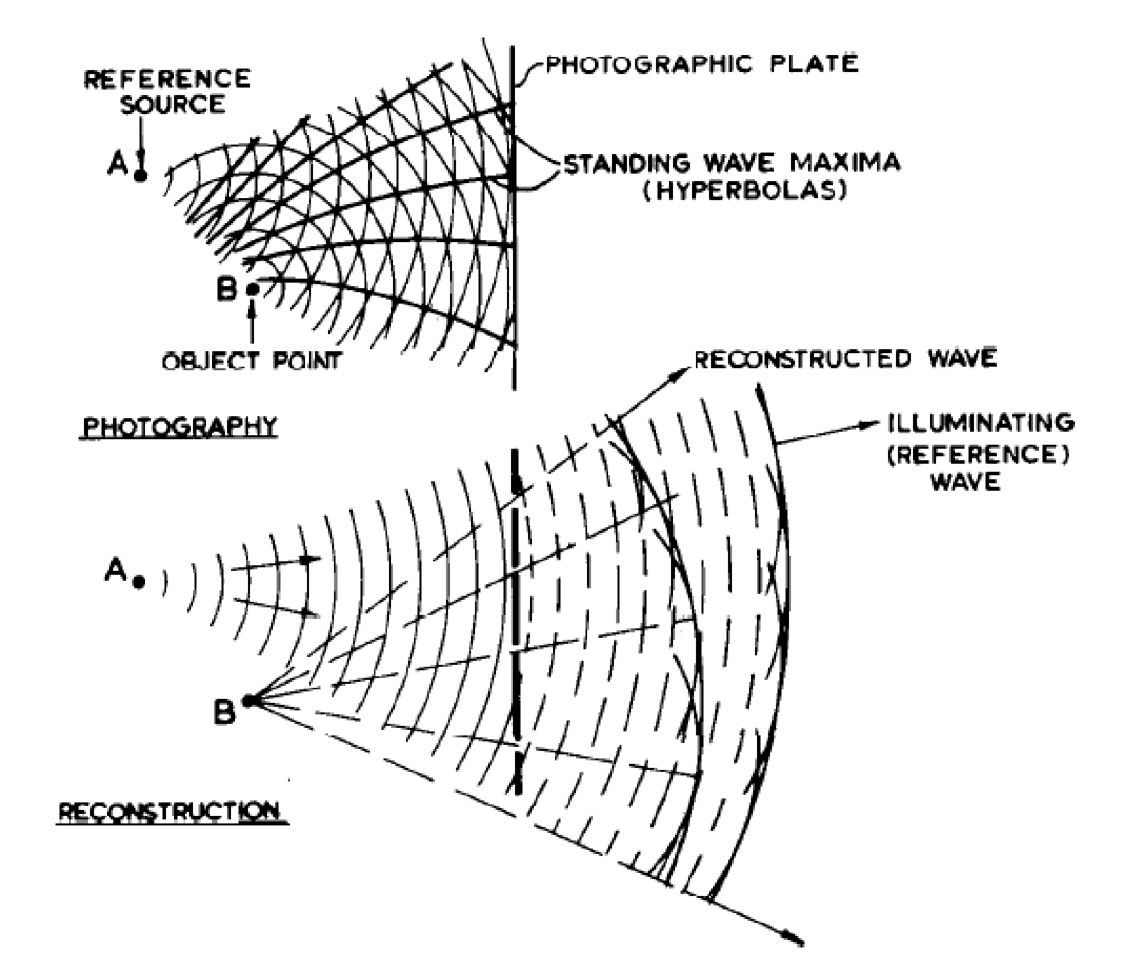
Motivation:
Proton Holography
phase reconstruction
e-Print: 2004.07095 [hep-ph], EPJ Web Conf. 235 (2020) 06002

Stellar Interferometry:
Binary stars
New results, motivated by M. Lisa's and Naomi Vogel's talk at WPCF2023

Conclusion

INTRODUCTION: HOLOGRAPHY



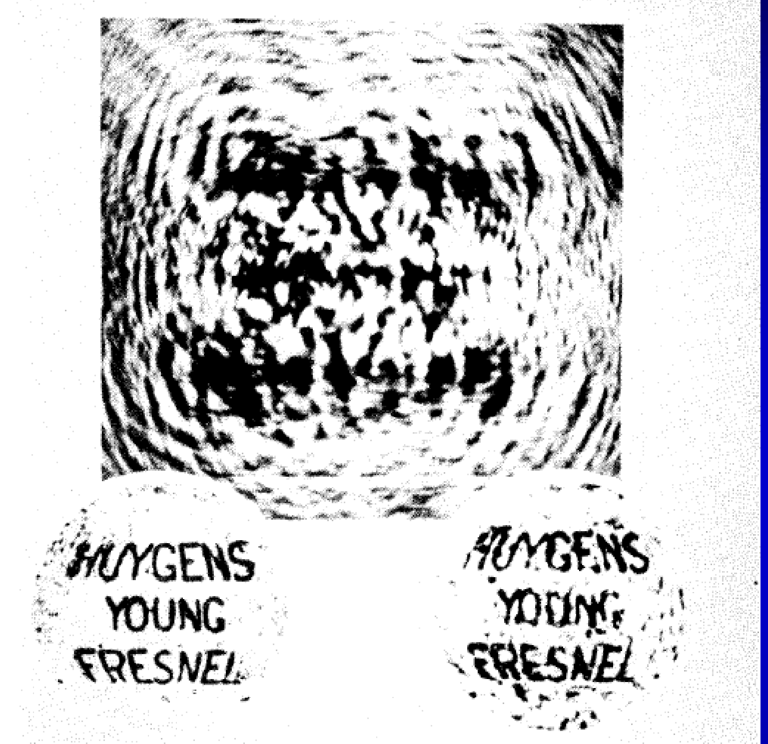


Fig 4.
First Holographic Reconstruction, 1948

Basic idea of holography (1947): amplitude level reconstruction.
First hologram (1948) from D. Gabor's Nobel lecture (1967).
https://www.nobelprize.org/uploads/2018/06/gabor-lecture.pdf

Formalism: elastic pp scattering

$$\sigma_{el}(s) = \int_0^\infty d|t| \frac{d\sigma(s)}{dt}$$

$$\frac{d\sigma(s)}{dt} = \frac{1}{4\pi} |T_{el}(s,\Delta)|^2, \qquad \Delta = \sqrt{|t|}.$$

$$B(s,t) = \frac{d}{dt} \ln \frac{d\sigma(s)}{dt}$$

$$B(s) \equiv B_0(s) \ = \ \lim_{t\to 0} B(s,t),$$

$$\sigma_{\rm tot}(s) \equiv 2\,{\rm Im}\, T_{el}(\Delta = 0, s)$$

$$\rho(s,t) \equiv \frac{{\rm Re}\, T_{el}(s,\Delta)}{{\rm Im}\, T_{el}(s,\Delta)}$$

$$\rho(s) \equiv \rho_0(s) \ = \ \lim_{t\to 0} \rho(s,t)$$

Basic problem: $d\sigma/dt$ measures an amplitude, *modulus squared*.
Amplitude level reconstruction??? Phase info apparently lost…

Formalism in b space

$$\frac{d\sigma(s)}{dt} = \frac{1}{4\pi}|T_{el}(s,\Delta)|^2, \qquad \Delta = \sqrt{|t|}.$$

$$t_{\rm el}(s,b) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta b}} T_{\rm el}(s,\Delta) =$$
$$= \frac{1}{2\pi}\int J_0(\Delta\, b)\, T_{\rm el}(s,\Delta)\,\Delta\, d\Delta\,,$$
$$\Delta \equiv |\boldsymbol{\Delta}|, \quad b \equiv |\boldsymbol{b}|.$$

$$t_{\rm el}(s,b) = i\left[1 - e^{-\Omega(s,b)}\right]$$

$$P(s,b) = 1 - \left|e^{-\Omega(s,b)}\right|^2$$

Impact parameter or b space:
elastic scattering interferes with propagation w/o collisions: Genuine quantum physics.
Complex opacity function Ω(s,b) (eikonal, from unitarity)
0 ≤ P(s,b) ≤ 1 : *inelastic* scattering has a probabilistic interpretation

MODEL INDEPENDENT LEVY EXPANSION

$$\frac{d\sigma}{dt} = A\, w(z|\alpha) \left| 1 + \sum_{j=1}^{\infty} c_j l_j(z|\alpha) \right|^2 ,$$

$$w(z|\alpha) = \exp(-z^\alpha) ,$$

non-exponential behavior (NEB) in a single parameter

$$z = |t| R^2 \geq 0 ,$$

α

$$c_j = a_j + i b_j ,$$

idea: complete set of orthonormal functions, put NEB to the weight

$$l_j(z\,|\,\alpha) = D_j^{-\frac{1}{2}} D_{j+1}^{-\frac{1}{2}} L_j(z\,|\,\alpha),$$

$$D_0(\alpha) = 1,$$

$$D_1(\alpha) = \mu_{0,\alpha},$$

$$D_2(\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} \end{pmatrix},$$

$$D_3(\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ \mu_{2,\alpha} & \mu_{3,\alpha} & \mu_{4,\alpha} \end{pmatrix},$$

$$\int_0^\infty dz \exp(-z^\alpha) l_n(z\,|\,\alpha) l_m(z\,|\,\alpha) = \delta_{n,m}$$

$$\mu_{n,\alpha} = \int_0^\infty dz\, z^n \exp(-z^\alpha) = \frac{1}{\alpha}\Gamma\left(\frac{n+1}{\alpha}\right)$$

T. Csörgő, R. Pasechnik, A. Ster, arxiv.org:1807.02897 *Eur.Phys.J.C* 79 (2019) 1, 62

$$L_0(z\,|\,\alpha) = 1,$$

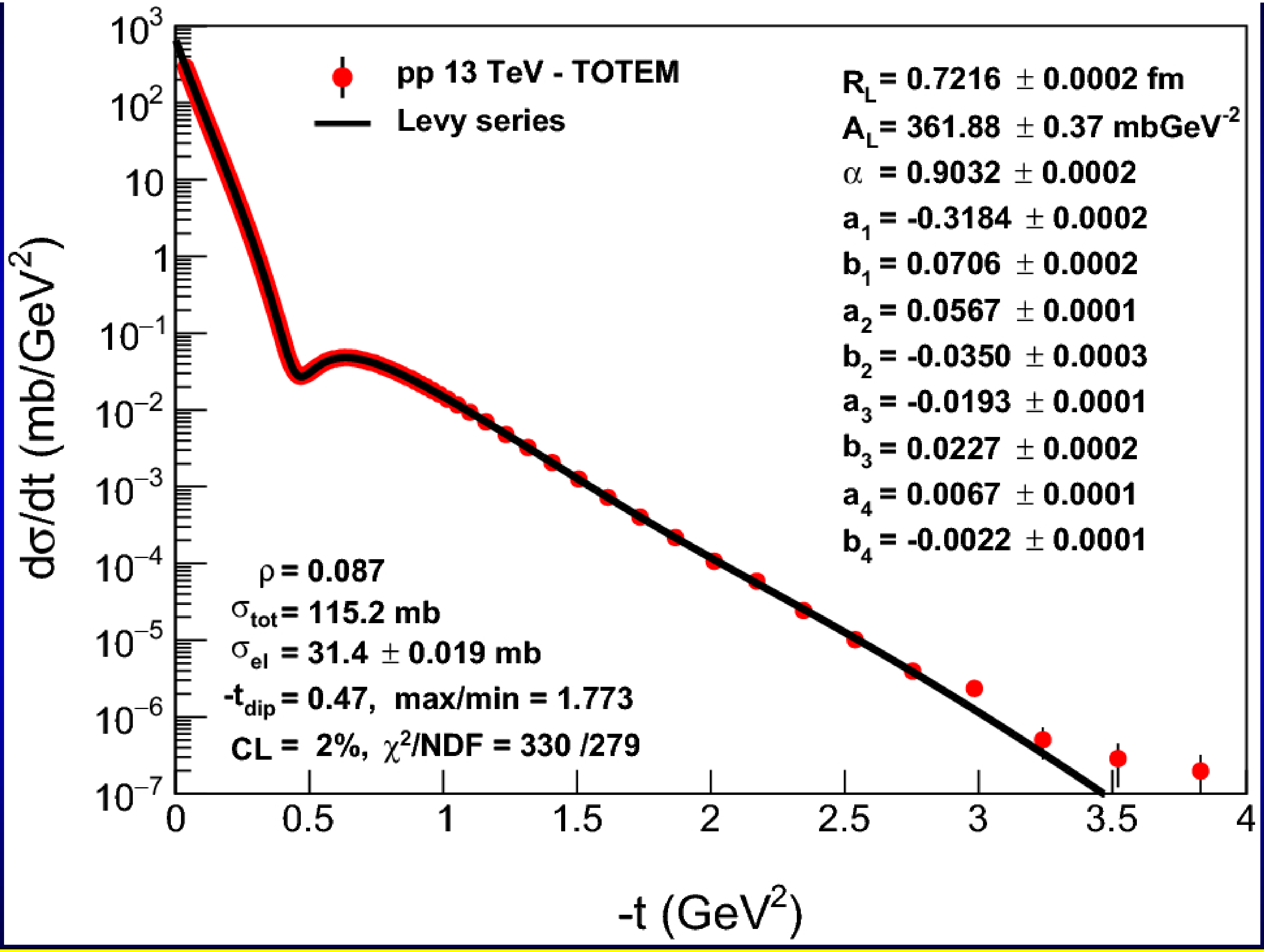
$$L_1(z\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & z \end{pmatrix},$$

$$L_2(z\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & z & z^2 \end{pmatrix},$$

$$L_3(z\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} & \mu_{4,\alpha} \\ \mu_{2,\alpha} & \mu_{3,\alpha} & \mu_{4,\alpha} & \mu_{5,\alpha} \\ 1 & z & z^2 & z^3 \end{pmatrix},$$

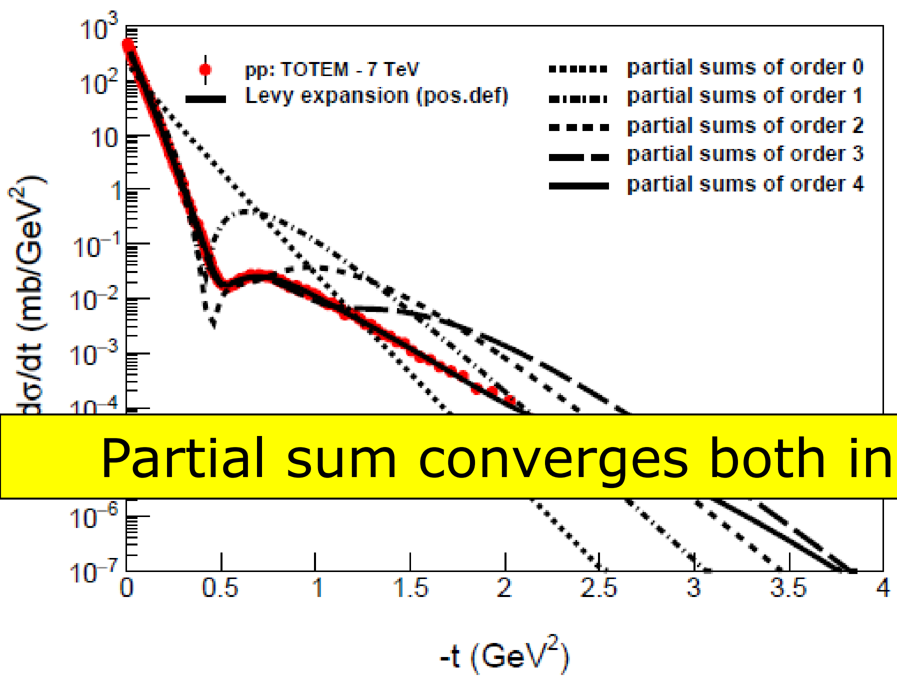
Levy series ~ Taylor series: orthonormal wrt w(z) = exp(-z^α). T. Cs., R. Pasechnik, A. Ster, arXiv:1807.02897, arXiv:1811.08913, arXiv:1902.00109, arXiv:1903.08235

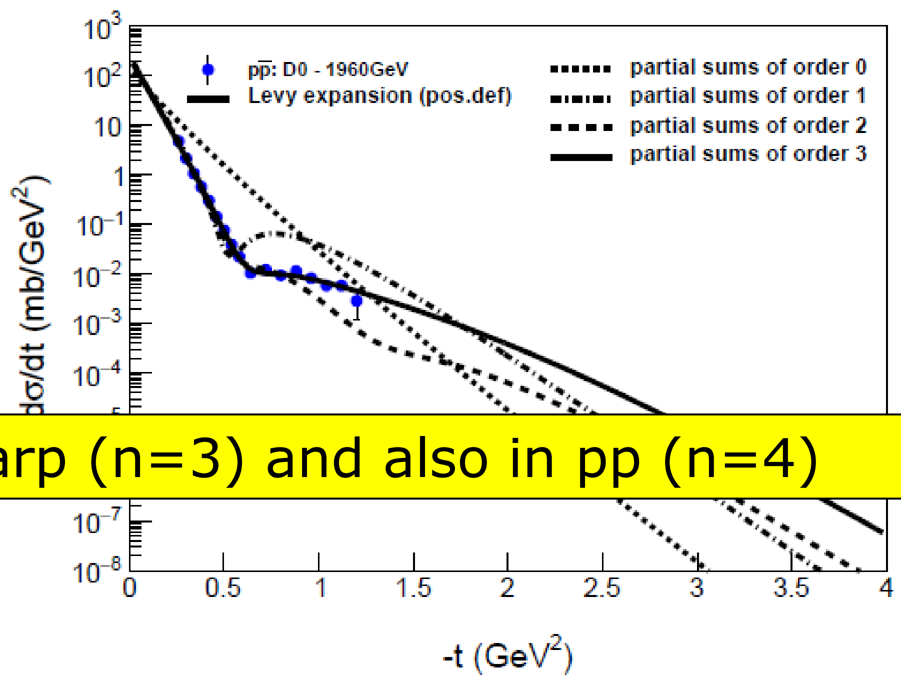
ABILITIES: CONVERGES TO pp dσ/dt @ 13 TeV



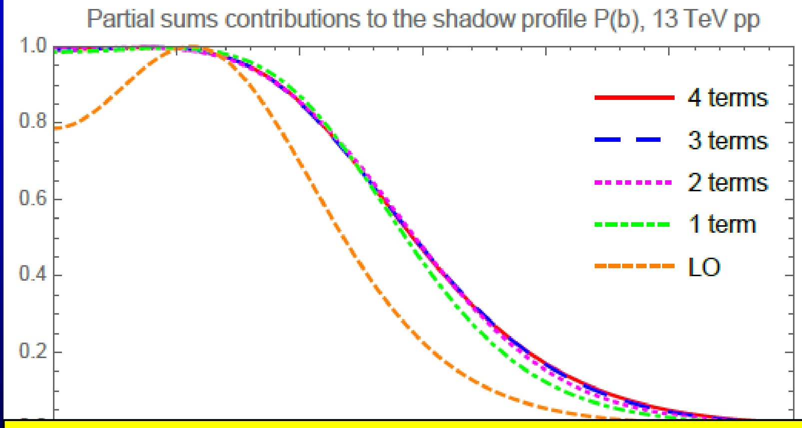
Levy series ~ Taylor series: orthonormal wrt $w(z) = \exp(-z^{\alpha})$. T. Cs., R. Pasechnik, A. Ster, arXiv:1807.02897, arXiv:1811.08913, arXiv:1902.00109, arXiv:1903.08235

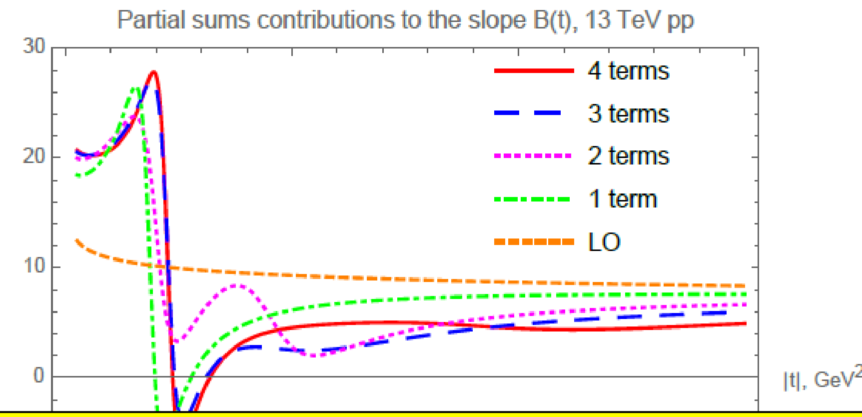
CONVERGENCE PROPERTIES OF LEVY SERIES





Partial sum converges both in pbarp (n=3) and also in pp (n=4)

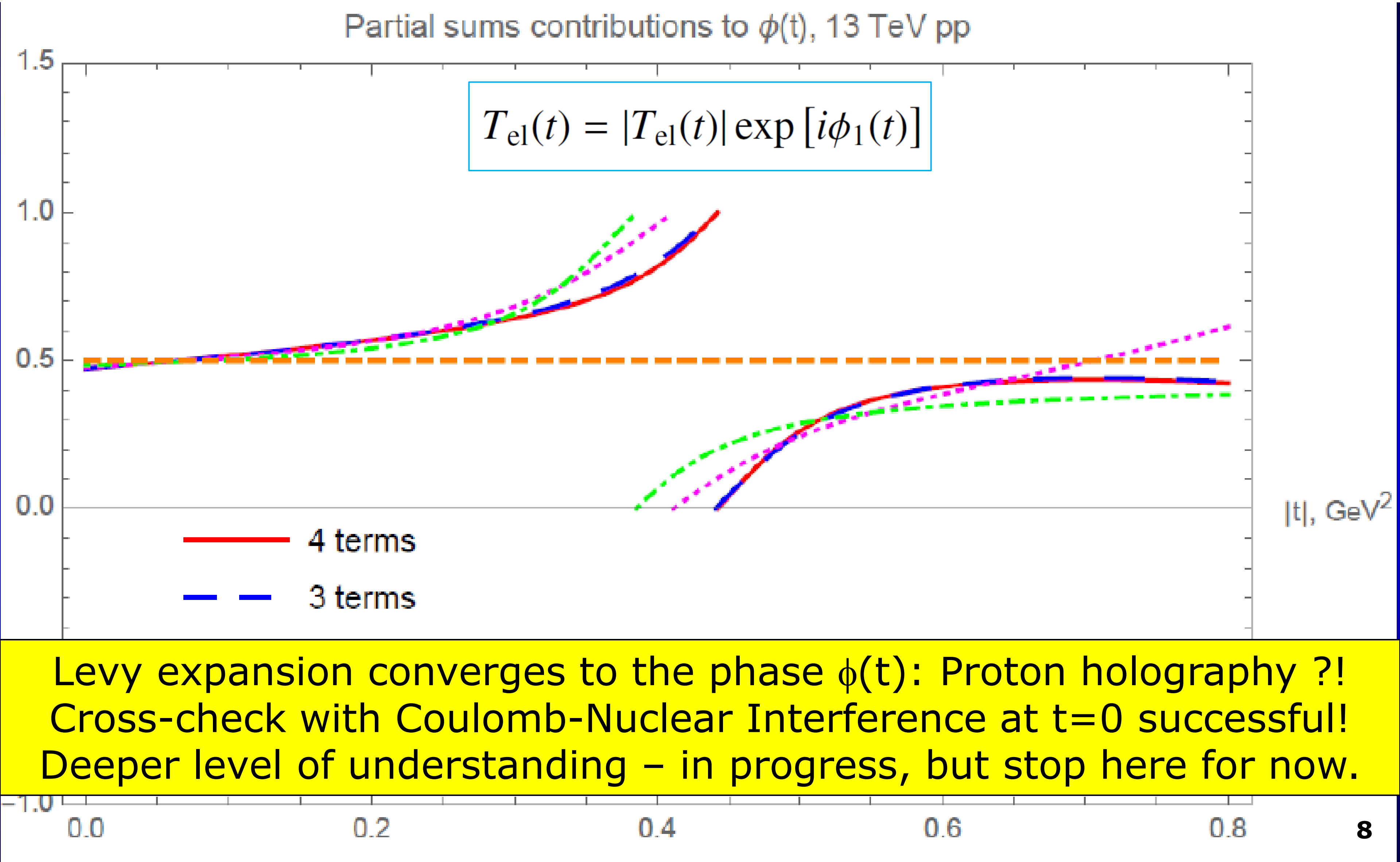




Partial sum converges to profile function P(b) and slope B(t)

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235

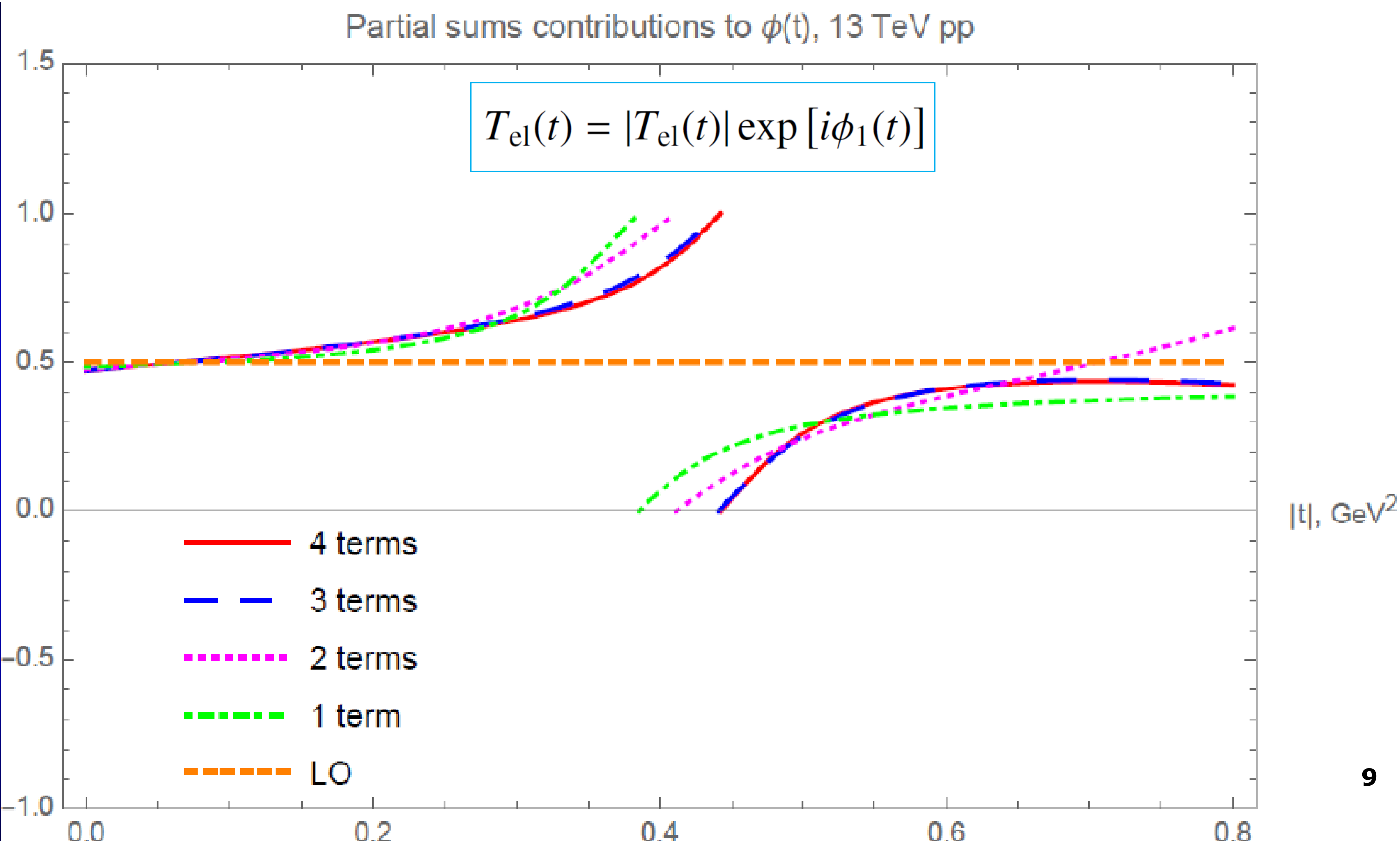
CONVERGENCE OF PHASE RECONSTRUCTION



Levy expansion converges to the phase $\phi(t)$: Proton holography ?!
Cross-check with Coulomb-Nuclear Interference at t=0 successful!
Deeper level of understanding – in progress, but stop here for now.

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235

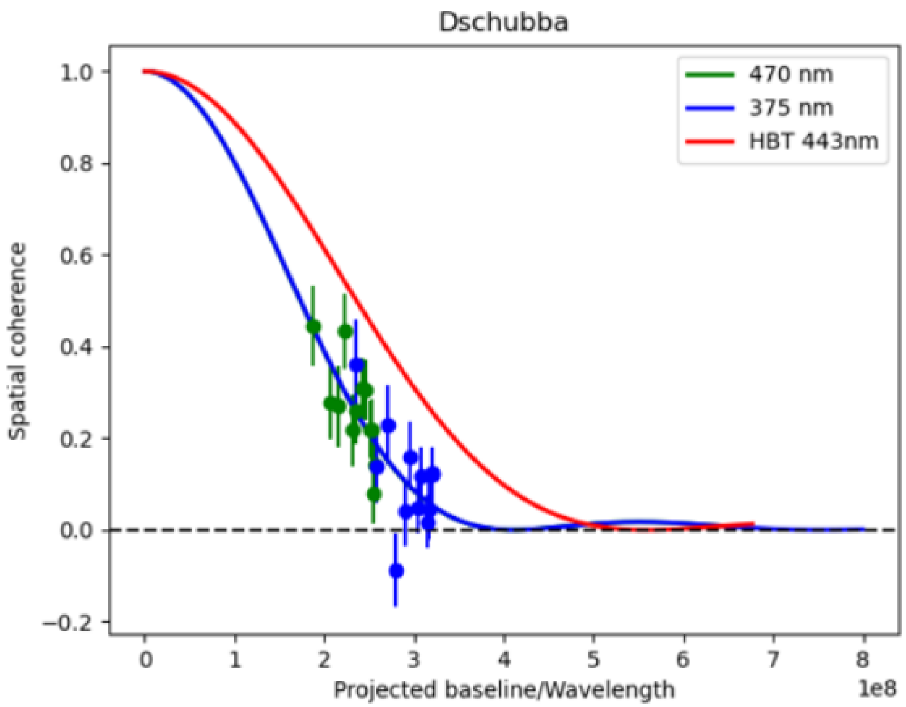
SUMMARY 1: PROTON HOLOGRAPHY



Levy expansion converges to the phase ϕ(t): Proton holography !

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235, ISMD 2018, *EPJ Web Conf.* 206 (2019) 06007

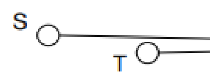
INTENSITY INTERFEROMETRY FOR BINARIES



Source	Dschubba (Delta Scorpii)
Magnitude (mag)	2.2
Spectral type	B0.3IV
System	Binary star
HBT time (h)	115.1
HBT diameter (mas)	0.45 ± 0.04
Time (h)	5.1
Diameter (mas)	470nm: 0.613 +/- 0.072 *375nm: 0.612 +/- 0.081*

Stellar interferometry results, HESS, talk of Naomi Vogel@ WPCF23:
Fits a **BINARY** star with intercept parameter **λ = 1**
But **this is a puzzle**, see R. Hanbury Brown et al, MNRAS (1974) **167**, 121

HBT FOR BINARY AND MULTIPLE STARS



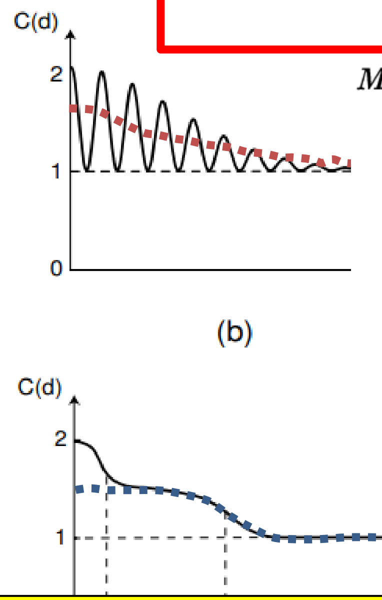
The normalized correlation also depends upon whether a star is single or multiple. It was shown in Paper II that, if a star is binary and the angular separation of the two components is completely resolved by the interferometer at the shortest baseline, then the normalized zero-baseline correlation $\overline{c_N(0)'}$ averaged over a range of position angles is reduced relative to a single star $\overline{c_N(0)}$ by the factor,

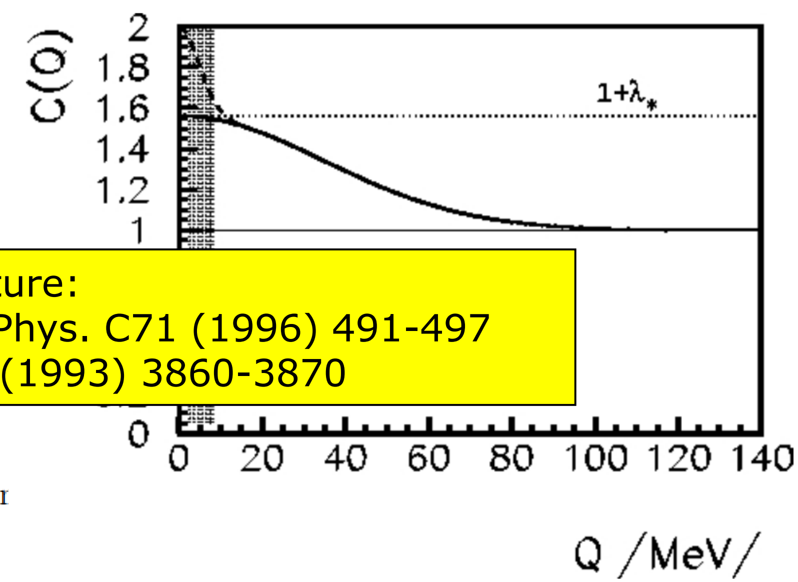
$$\overline{c_N(0)'}/\overline{c_N(0)} = (I_1^2 + I_2^2)/(I_1 + I_2)^2 \qquad (9)$$

Mon. Not. R. astr. Soc. (1974) **167**, 121–136.

THE ANGULAR DIAMETERS OF 32 STARS

R. Hanbury Brown, J. Davis and L. R. Allen



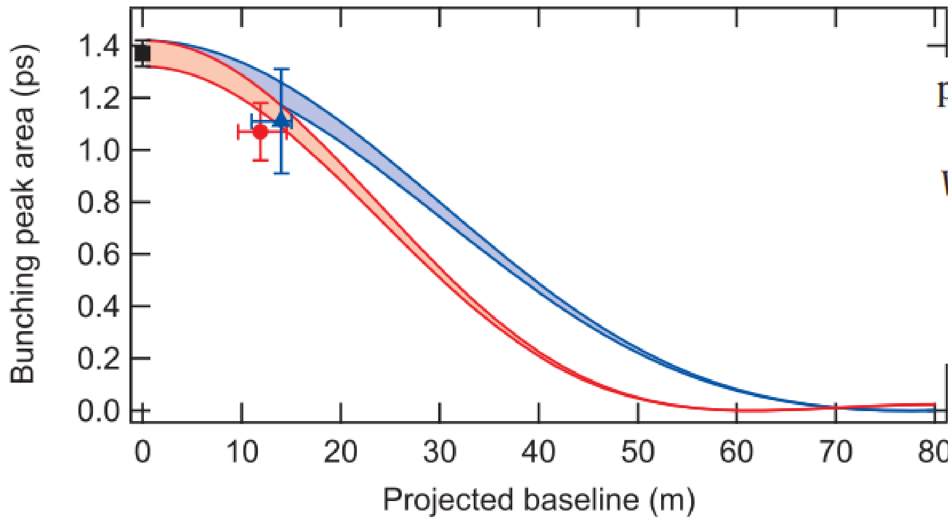


Core-halo picture:
T. Cs, B. Lörstad and J. Zimányi, Z.Phys. C71 (1996) 491-497
J. Bolz et al, Phys.Rev.D 47 (1993) 3860-3870

Fig. 7. a) Two stars, S, and T, along nearby lines of sight from the earth; b) of correlated intensity from the two stars; c) schematic of HBT measuremei halo of dim stars.

Fig. 7 from G. Baym's review paper: arXiv:nucl-th/9804026
Schematics of HBT for multiple stars, but no formula. Note: intercept!

HBT FOR MARGINALLY RESOLVED STARS



projection of the baseline (Hanbury Brown et al. 1967) :

$$V^2(r) = \frac{1}{(I_a + I_b)^2} \times \left[I_a^2 V_a^2(r) + I_b^2 V_b^2(r) + 2 I_a I_b V_a(r) V_b(r) \cos\left(\frac{2\pi r \overline{\theta} \cos\psi}{\lambda_0} \right) \right],$$

Partial resolution of the components from W. Guerin et al,
MNRAS 480, 245–250 (2018)

Reduction of average measured intensity reduces:
R. H. Brown and R. Q. Twiss, Brown, R. Hanbury, and R. Q. Twiss: in *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* (1958): 291-319.

HBT EFFECT FOR „IDENTICAL TWIN" STARS

$$\rho(x) = f_+ s(x - x_+) + f_- s(x - x_-),$$

$$f_+ + f_- = 1$$

Normalization: FT(s|q = 0) = 1

$$C(q) \quad = \quad 1 \pm |\tilde{\rho}(q)|^2 = 1 \pm \Omega(q)|\tilde{s}(q)|^2,$$

Stellar model: uniformly illuminates sphere $|\tilde{s}(q)|^2 = \left[\frac{2J_1(\pi r\theta/\lambda_0)}{\pi r\theta/\lambda_0}\right]^2 .$

Normalization: C(q = 0) = 1 +1 but with an oscillating prefactor

$$\Omega(q) = \left[(f_+^2 + f_-^2) + 2 f_+ f_- \cos[q(x_+ - x_-)]\right]$$

Binary source formalism: binary sources in hydro (Cooper-Frye)
Schematics worked out for identical stars, e-Print: hep-ph/0011320
Detailed **review** including hydro results with two saddle points in hep-ph/0001233

HBT FOR BINARY AND MULTIPLE STARS

Mon. Not. R. astr. Soc. (1974) **167**, 121–136.

THE ANGULAR DIAMETERS OF 32 STARS

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…ds upon whether a star is single or …tar is binary and the angular separation of the two components is completely resolved by the interferometer at the shortest baseline, then the normalized zero-baseline correlation $\overline{c_N(0)'}$ averaged over a range of position angles is reduced relative to a single star $\overline{c_N(0)}$ by the factor,

$$\overline{c_N(0)'}/\overline{c_N(0)} = (I_1^2 + I_2^2)/(I_1 + I_2)^2 \qquad (9)$$

where I_1, I_2 are the brightness of the two components. It is simple to extend this analysis to a multiple star with n components and to show that, if the angular separation between all the components is resolved, the zero-baseline correlation is reduced relative to a single star by the factor,

$$\overline{c_N(0)'}/\overline{c_N(0)} = \sum_n I^2 \Big/ \left(\sum_n I\right)^2. \qquad (10)$$

It follows that if a star yields a correlation which is significantly less than that expected from a single star, then it must be multiple.

R. Hanbury Brown, J. Davis and L. R. Allen, MNRAS (1974) **167** 121
Note: $f_1 = I_1/(I_1+I_2)$, $f_2 = I_2/(I_1+I_2)$, etc → HEP connection

HBT FOR „NON-IDENTICAL TWIN" STARS

Similar, but two sources have different sizes, but with

$$f_+ + f_- = 1$$

Stellar model: uniformly illuminates sphere or any unknown shape

An oscillating prefactor remains, and averages to

$$\overline{\Omega(q)} = [(f_+^2 + f_-^2) + 2f_+f_-\cos[q(x_+ - x_-)]]$$

Binary source formalism: effective reduction of intercept
for well separated compact sources
Effectively $\frac{1}{2} \leq \lambda = \overline{\Omega(q)} \leq 1$
Schematics works even for out non-identical stars,
observations of N. Vogel explained

Multiple n source, well separated compact sources
Easy to show: $1/n \leq \lambda = \overline{\Omega_n(q)} \leq 1$

STRENGTH OF HBT FOR BINARY STARS

$$\lambda = \frac{\overline{c_2(0)'}}{c_2(0)'} = \frac{I_1^2 + I_2^2}{(I_1 + I_2)^2}$$

$$f_1 = \frac{I_1}{(I_1 + I_2)}$$

$$f_2 = \frac{I_2}{(I_1 + I_2)}$$

$$1 = f_1 + f_2,$$

$$\lambda = f_1^2 + f_2^2.$$

$$\langle f \rangle = \frac{f_1 + f_2}{2} = \frac{1}{2},$$

$$\delta_1 = f_1 - \langle f \rangle,$$

$$\delta_2 = f_2 - \langle f \rangle,$$

$$0 = \delta_1 + \delta_2,$$

C(q = 0) = 1 +1 but oscillating prefactor REDUCES average intercept

$$\lambda = f_1^2 + f_2^2 = (\langle f \rangle + \delta_1)^2 + (\langle f \rangle + \delta_2)^2,$$

$$\lambda = f_1^2 + f_2^2 = 2(\langle f \rangle)^2 + \delta_1^2 + \delta_2^2,$$

$$\frac{1}{2} \le \lambda \le 1.$$

Lower limit if and only if $\delta_1 = \delta_2 = 0$, $f_1 = f_2 = \langle f \rangle$

STRENGTH OF HBT FOR MULTIPLE STARS

$$\lambda = \frac{\overline{c_N(0)'}}{c_N(0)'} = \frac{\sum_{j=1}^{N} I_j^2}{(\sum_{j=1}^{N} I_j)^2}$$

$$f_j = \frac{I_j}{(\sum_{j=1}^{N} I_j)}$$

$$1 = \sum_{j=1}^{N} f_j,$$

$$\lambda = \sum_{j=1}^{N} f_j^2$$

$$\langle f \rangle = \frac{(\sum_{j=1}^{N} f_i)}{N} = \frac{1}{N},$$

$$\delta_j = f_j - \langle f \rangle,$$

$$0 = \sum_{j=1}^{N} \delta_j,$$

C(q = 0) = 1 +1 but oscillating prefactors REDUCE average intercept

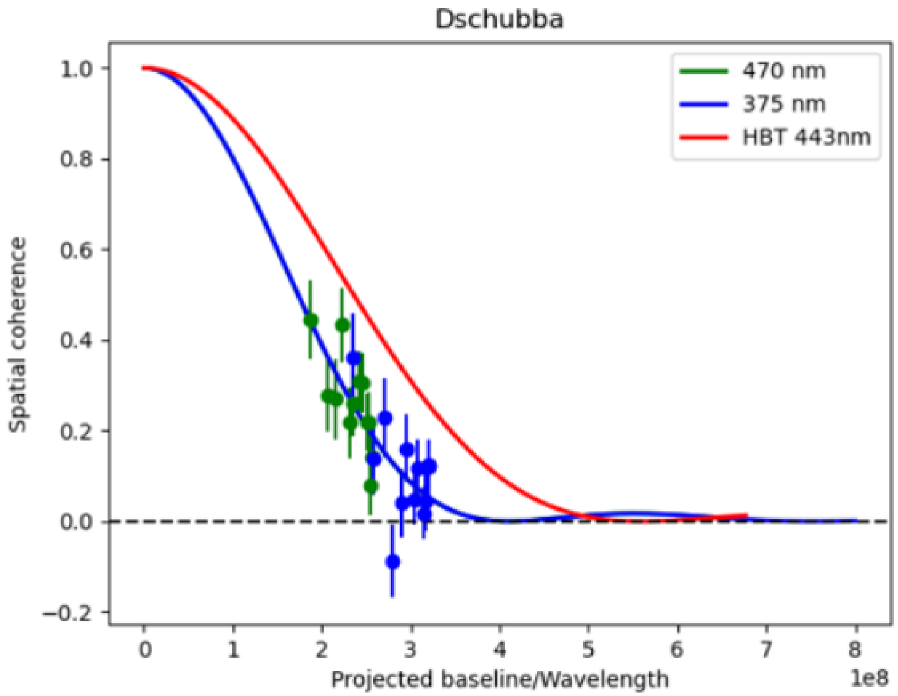
$$\lambda = \sum_{j=1}^{N} f_j^2 = \sum_{j=1}^{N} (\langle f \rangle + \delta_j)^2,$$

$$\lambda = N(\langle f \rangle)^2 + \sum_{j=1}^{N} \delta_j^2,$$

$$\frac{1}{N} \le \lambda \le 1.$$

Lower limit if and only if for all j = 1,…N, $\delta_j = 0$, f_j = <f>

SUMMARY FOR BINARY STARS



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Stellar interferometry results, HESS, from N. Vogel’s talk @ WPCF23: Fits with intercept parameter ½ ≤ λ ≤ 1 possible, **puzzle resolved**. Looks to be a new result in astrophysics