Mini-jet quenching

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with J. Brewer, A. Mazeliaskas, in preparation
Motivation

- interactions between Quark Gluon plasma (QGP) & hard partons $\rightarrow$ jet quenching
- vacuum radiation $\rightarrow$ seeds mini-jets in medium
- kinetic theory describes the physics in the hard as well as in the soft sector
- serves as bridge between initial state and hydrodynamics

**Goal:** describe parton energy loss in an expanding plasma
Outline

Leading order kinetic theory

Modelling mini-jets
  Thermal background
  Expanding background

Summary & Outlook
Overview

Main features of the evolution of a heavy ion collision, by A. Mazeliauskas

- study pre-equilibrium phase with an effective kinetic theory (EKT)
Framework

- EKT for high temperature gauge theories
  Arnold, Moore, Yaffe (2003)

- weakly coupled quasi-particle picture, $\lambda = 4\pi \alpha_s N_c$

  $\rightarrow$ phase space distribution $f(\tau, x, p)$

  \[ (\partial_\tau + \mathbf{v} \cdot \nabla_x) f(\tau, x, p) = -C[f] \]

Initial distribution $p^2 f_0(x_0, p)$

$t = t_0$

Equilibrium distribution $p^2 f_{\text{eq}}(p)$

$t \rightarrow \infty$

Out-of-equilibrium initial state is transported to equilibrium
Expanding QGP

- longitudinal expansion
- approximate boost invariance
- homogeneity in the transverse plane

\[
\left( \partial_\tau - \frac{p_z}{\tau} \partial_{p_z} \right) f(\tau, \mathbf{p}) = -C[f]
\]

- leading order elastic and inelastic scattering processes

\[
C[f](\mathbf{p}) = C_{2\leftrightarrow2}[f](\mathbf{p}) + C_{1\leftrightarrow2}[f](\mathbf{p})
\]
Collision kernel

\[ C_{2\leftrightarrow 2} \]

\[ C_{1\leftrightarrow 2} \]

- medium induced radiation of gluons
- \( g \rightarrow q\bar{q} \) splittings
- LO: strictly collinear

\[ q = |p' - p| \ll 1 \text{ regulated by} \]

\[ \frac{1}{q^2} \rightarrow \frac{1}{q^2 + m_{\text{eff}}^2} \]
EKT

- Soft sector: EKT describes thermalisation of highly occupied state to QGP → Bottom-up thermalisation, BMSS 2001, Kurkela and Lu 1405.6318, Kurkela and Zhu 1506.06647

- Hard sector: Relaxation of high momentum particles → (mini-)jet quenching, Schlichting and Soudi 2008.04928, Mehtar-Tani, Schlichting, Soudi 2209.10569
1. Initial conditions: thermal, non-expanding

- background + linear perturbation

\[ f(\tau, p) = f_{eq}(\tau, p) + \delta f_{Jet}(\tau, p) \]

→ linearized Boltzmann eq.

\[ \partial_\tau \delta f = -\delta C[\delta f, f_{eq}] \]

Mehtet-Tani, Schlichting, Soudi, 2209.10569

- want to study

\[ \delta f_{Jet}(\tau_0, p) \rightarrow \delta f_{eq}(p) \]
1. Initial conditions: thermal

- initial jet distribution

\[ \delta f_{\text{Jet}}(\tau_0, \mathbf{p}) \propto \delta(\mathbf{p} - \mathbf{p}_0) \]

- Gaussian at \( \mathbf{p}_0 = (E, 0, 0) \)

Initial condition for the jet on top of a thermal background
Inverse energy cascade

1) build up of soft bath 2) transport of energy

Kurkela and Lu, 1405.6318

▶ In thermal equilibrium: isotropic pressure
Pressure equilibration

- energy momentum tensor $\delta T^{\mu\nu} = \int p \frac{p^{\mu}p^{\nu}}{p} \delta f(\tau, p)$
- $\delta T_{eq}^{\mu\nu} = \text{diag}(\delta e, \delta P, \delta P, \delta P)$ with $\delta P = \delta e/3$
- equilibration of transverse pressure $\delta P_T = \frac{1}{2} (\delta T^{xx} + \delta T^{yy})$

Consider different initial jet energies $E$:

(a) Large $E$ equilibrate later

(b) Scaling with $\sqrt{E}$
Pressure equilibration

- Different couplings $\lambda$:

(c) Large $\lambda$ equilibrate faster

(d) Scaling with $\eta/s \sim \lambda^{-2}$

Curves collapse by rescaling time with $\eta/s \sqrt{E}$
Inverse energy cascade

1) build up of soft bath 2) transport of energy

Kurkela and Lu, 1405.6318

How is $\delta f_{eq}(p)$ approached?
Equilibrium distributions

- equilibrated jet $\rightarrow$ increase in temperature by $\delta T$

\[
\delta f_{\text{eq}}(\frac{p}{T}) + \delta f_{\text{Jet}}(\tau_0, p) \xrightarrow{\text{equilibration}} f_{\text{eq}}(\frac{p}{T + \delta T})
\]

- equilibrium distribution

\[
\delta f_{\text{Jet}}(\tau_0, p) \rightarrow \delta f_{\text{eq}}(\frac{p}{T}) = \partial_T f_{\text{eq}}(\frac{p}{T}) \times \delta T
\]

- Look at time evolution of $\delta f_{\text{Jet}}(\tau, p)$ and its angular ($\theta$) structure
Jet distributions

- **equilibrated jet distribution**

\[ \delta f_{eq} \left( \frac{p}{T} \right) = \partial_T f_{eq} \left( \frac{p}{T} \right) \times \delta T \]

\( \delta f \) to \( \delta T \)

- **evolve** \( \delta f_{Jet}(\tau, p) \) for \( \theta \)-slices

![Graphs showing the evolution of jet distributions over time.](image)

**same shape, different magnitude?**
Jet distributions

- equilibrated jet distribution

\[ \delta f_{eq} \left( \frac{p}{T} \right) = \partial_T f_{eq} \left( \frac{p}{T} \right) \times \delta T \]

\[ \text{eq. distr.} \]

\[ \text{magnitude} \]

- Normalise \( \int dpp^2 \delta f(p, \theta) = 1 \) for each \( \theta \)

\[ \rightarrow \text{agreement with eq. distr. while still anisotropic} \]

Equilibrium distributions with temperatures \( \delta T(\theta) \)
Fermions

2 ↔ 2 QCD processes, AMY (2003)

- add quarks to $C[f]$, massless, $N_f = 3$
  - all QCD channels included,
- solve 2 equations for quarks $\delta f_q(\tau, p)$ and gluons $\delta f_g(\tau, p)$
- initialisation: $\delta f_q(\tau_0, p) = 0$, same $\delta f_g(\tau_0, p)$
- gluon evolution similar to YM evolution due to fast radiation
Fermions

▶ early times: depletion of quarks around \( p \sim T \)

→ hard gluon scatters off of the soft background

▶ late times: quark distribution approaches Fermi-Dirac

→ system equilibrates chemically

![Graph showing quark distribution in jet direction](image-url)
2. Initial conditions: anisotropic, expanding

\[ f(\tau, p) = \bar{f}(\tau, p) + \delta f_{\text{Jet}}(\tau, p) \]

- expanding system

\[ \left( \partial_{\tau} - \frac{p_z}{\tau} \partial_{p_z} \right) \delta f(\tau, p) = -\delta C[\delta f, \bar{f}] \]

- non-thermal medium

\[ \bar{f}(\tau_0, p) \propto \exp \left( -\frac{2}{3} \frac{p_{\perp}^2 + \xi^2 p_z^2}{Q^2} \right) \]

- evolving according to

\[ \left( \partial_{\tau} - \frac{p_z}{\tau} \partial_{p_z} \right) \bar{f}(\tau, p) = -C[\bar{f}] \]

Kurkela and Zhu, 1506.06647, Kurkela and Mazeliauskas, 1811.03068

\[ \bar{f}(\tau, p) \]: approach to hydrodynamics! how about mini-jets?
Hydrodynamization of jets

- Now: study hydrodynamization of mini-jets

\[ \delta f_{\text{Jet}}(\tau_0, p) \rightarrow \delta f_{\text{hydro}}(\tau, p) \]

- Thermal, non-expanding background: \( \delta f_{\text{eq}}(p) \rightarrow \) known analytic expression

- Non-equilibrium, expanding background:

  no analytic expression for \( \delta f_{\text{hydro}} \)

- Compare with an arbitrary perturbation!
Comparison with background-like perturbation

- azimuthal symmetric ($\phi$): $\delta f_{\text{sym}}(\tau_0, p) = \epsilon \bar{f}(\tau_0, p)$

$$\bar{f} + \delta f_{\text{sym}} = (1 + \epsilon)\bar{f}$$

(a) $p^2 \delta f_{\text{sym}}(\tau_0, p, \phi)$

(b) $p^2 \delta f_{\text{Jet}}(\tau_0, p, \phi)$

- Hydrodynamization occurs, if both perturbations are indistinguishable
Hydrodynamization

- rescaled time variable $\tilde{\omega} = \tau T(\tau)/(4\pi \eta/s)$
- collapse of $\delta f_g(p)$ and $\delta f_{az sym}$ around $\tilde{\omega} \approx 2$
- anisotropy $A = \frac{P_T - P_L}{3e}$ agrees for different initial conditions

(a) $\delta f_g(p)$ vs. $\delta f_{az sym}$

(b) different initial orientations $\cos \theta$

- loss of memory about the initial conditions

$\rightarrow$ Hydrodynamization
Chemical equilibration

▶ compare with kinetic equilibrium (isotropy of pressure)

▶ in chemical equilibrium more fermionic degrees of freedom

▶ chem. equilibration not affected by expansion
Summary & Outlook

- modelled mini-jets as perturbations on top of a background static QGP:
  - time scale of thermalisation scales with $E$ and $\lambda$
  - thermal distribution with $T(\theta)$

expanding QGP:

- mini-jets hydrodynamise (later than the background)
- chemical equilibration before the system isotropises

Outlook:

- extract jet response functions $\rightarrow$ phenomenology
- include transverse dynamics $\rightarrow$ small systems
Equations of motion from 2PI effective action

\[
\left[ iG_{0,ac}^{-1,\mu\gamma}(x; A) + \Pi^{(0),\mu\gamma}_{ac}(x) \right] \rho_{\gamma\nu}^{cb}(x, y) \\
= - \int_{y^0}^{x^0} dz \Pi^{(\rho),\mu\gamma}_{ac}(x, z) \rho_{\gamma\nu}^{cb}(z, y) \\
\left[ iG_{0,ac}^{-1,\mu\gamma}(x; A) + \Pi^{(0),\mu\gamma}_{ac}(x) \right] F_{\gamma\nu}^{cb}(x, y) \\
= - \int_{t_0}^{y^0} dz \Pi^{(F),\mu\gamma}_{ac}(x, z) F_{\gamma\nu}^{cb}(z, y) + \int_{t_0}^{y^0} dz \Pi^{(F),\mu\gamma}_{ac}(x, z) \rho_{\gamma\nu}^{cb}(z, y)
\]
local homogeneity $\rightarrow$ relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$

gradient expansion in $X^\mu$

to lowest order, spectral function $\rho$ is on shell $\rightarrow$ quasi-particle picture

non-equilibrium distribution function $f(X,p)$:

$$F(X,p) = -i \left[ \frac{1}{2} \pm f(X,p) \right] \rho(X,p)$$

$\Rightarrow p^\mu \partial_\mu f(X,p) = -C[f]$
$C_{2\leftrightarrow 2}[f](p) = \frac{1}{4p\nu} \int \frac{(2\pi)^4 \delta^{(4)}(p^\mu + k^\mu - p'^\mu - k'^\mu)}{k, p', k'}$

$\times |\mathcal{M}|^2 \left\{ f_p f_k \left( 1 \pm f_{p'} \right) \left( 1 \pm f_{k'} \right) - f_{p'} f_{k'} \left( 1 \pm f_p \right) \left( 1 \pm f_k \right) \right\}$ (3)
Hard (left) and soft (right) medium regulated scattering

\[ |\mathcal{M}|^2 = 2\lambda^2\nu \left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2}\right) \]

\[ \text{small momentum transfer } q = |\mathbf{p}' - \mathbf{p}| \ll 1 \text{ regulated by } \]

\[ \frac{1}{q^2} \rightarrow \frac{1}{q^2 + m_{\text{eff}}^2} \]

\[ m_{\text{eff}}^2 = 2g^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[ N_c f_{\mathbf{p}}^g + N_f f_{\mathbf{p}}^q \right] \]
\[ C_{1 \leftrightarrow 2}[f](p) = \frac{1}{2} \frac{1}{\nu} (2\pi)^3 \int_{\tilde{p}, p', k'} (2\pi)^4 \delta^{(4)}(\tilde{p}^\mu - p'^\mu - k'^\mu) \]

\[
\times \left[ \delta^{(3)}(p - \tilde{p}) - \delta^{(3)}(p - p') - \delta^{(3)}(p - k') \right]
\times \gamma \left\{ f_p \left( 1 \pm f_{\tilde{p}'} \right) \left( 1 \pm f_{k'} \right) - f_{p'} f_{k'} \left( 1 \pm f_{\tilde{p}} \right) \right\} 
\]

(4)
1 ↔ 2

effective 1 ↔ 2 process

- LO → strictly collinear
- medium induced radiation of gluons
- $N + 1 \leftrightarrow N + 2$ effectively 1 ↔ 2
Parton going through the medium, Figure from [1]

- hard parton receiving multiple kicks
- formation time $\tau_f \sim E$
- BH: $l_f \ll l_{\text{mfp}}$, independent emissions
- LPM: $l_f \sim l_{\text{mfp}}$, destructive interference $\rightarrow$ suppression
Radiation vs. elastic scattering

- **Particle number**
  \[
  n = \int p f(\tau, p) \rightarrow C_{1\leftrightarrow2}[f]
  \]

- **Transverse pressure**
  \[
  P_T = \frac{1}{2} \int p p^2_\perp / p f(\tau, p) \rightarrow C_{2\leftrightarrow2}[f]
  \]
Equilibrium distribution

- equilibrated jet $\rightarrow$ change in temperature $\delta T$ and velocity $\delta u^z$

$$
\delta f_{eq}(p) = (\delta T \partial_T + \delta u^z \partial_{u^z}) n_{BE} (p_\mu u^\mu / T) \bigg|_{u^z=0}
$$

$$
\delta f_{eq}(p, \theta) = \left( \delta u^z \cos \theta + \frac{\delta T}{T} \right) F(p/T)
$$

- both contributions can be disentangled
Equilibrium distribution

- back-to-back jet conserves net momentum:

\[ \delta f_{eq}(p, \theta) = \left( \delta u \cos \theta + \frac{\delta T}{T} \right) F(p/T) \]

Initial condition: back-to-back jet
Only $1 \leftrightarrow 2$

- similar timescales of equilibration
  
  $\Rightarrow 2 \leftrightarrow 2$ contribute more to equilibration of the anti-jet
Moments of $\delta f$

- angular effective temperature

\[ I_n(\theta) \equiv 4\pi \int \frac{p^2 dp}{(2\pi)^3} p^n f(p, \theta) = \mathcal{N}_n \times T(\theta)^{n+3} \]  \hspace{1cm} (5)

\[ T(\theta) = \bar{T} + \delta T(\theta) \]

- temperature perturbation

\[ \frac{\delta T(\theta)}{\bar{T}} = \frac{\delta I_n(\theta)}{(n+3)\bar{I}_n(\theta)} \]

- look at time evolution!
Moments of $\delta f$

Moments of the back-to-back jet

- different moments agree before different angles do!
Anti-jet

- introduce anti-jet → no energy deposited, **only** net momentum

![Graph showing particle and "hole" function](image)

\[
\delta f_{eq}(p, \theta) = \left( \delta u^z \cos \theta + \frac{\delta T}{T} \right) F\left(\frac{p}{T}\right)
\]

- allows us to study the build up of \(\delta u^z\)
Moments of $\delta f_{\text{anti}}$

Collapse of different $\theta$ much earlier!

$$\delta f_{\text{eq}}(p, \theta) = (\delta u^z \cos \theta) F(p/T)$$

- $\theta$-dependence $\rightarrow$ faster reached by elastic processes
- single jet: velocity builds up faster than temperature
Pressure equilibration

- scaled time $\tilde{\omega} = \tau / \tau_R$ with $\tau_R = \frac{4\pi\eta}{T(\tau)}$

- effective temperature from $e(\tau) = \nu_{\text{eff}}\frac{\pi^2}{30} T(\tau)^4$

(c) Scaling with jet energy $E$

(d) coupling $\lambda$