Mini-jet quenching

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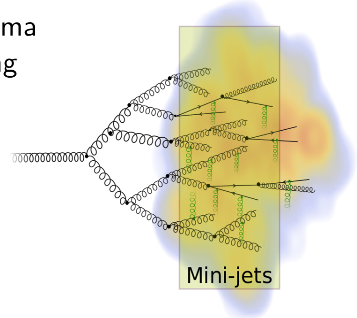
with J. Brewer, A. Mazeliauskas, *in preparation*

Motivation

- interactions between Quark Gluon plasma (QGP) & hard partons $\rightarrow$ jet quenching
- vacuum radiation $\rightarrow$ seeds mini-jets in medium
- kinetic theory describes the physics in the hard as well as in the soft sector

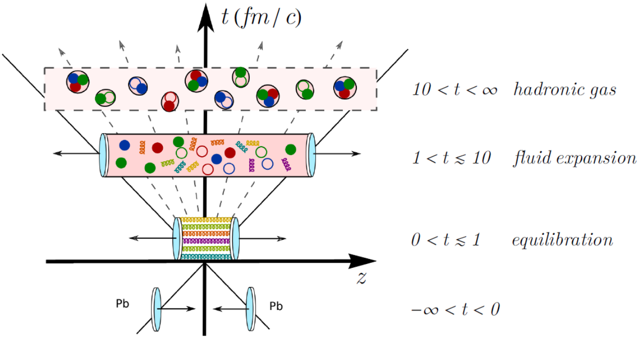


- serves as bridge between initial state and hydrodynamics

Goal: describe parton energy loss in an expanding plasma

Outline

Overview



Main features of the evolution of a heavy ion collision, by A. Mazeliauskas

- study pre-equilibrium phase with an effective kinetic theory (EKT)

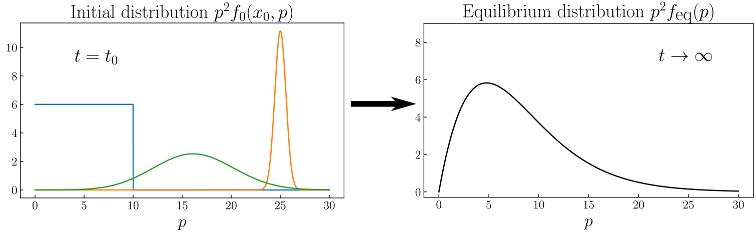
Framework

- EKT for high temperature gauge theories
  Arnold, Moore, Yaffe (2003)

- weakly coupled quasi-particle picture, $\lambda = 4\pi\alpha_s N_c$

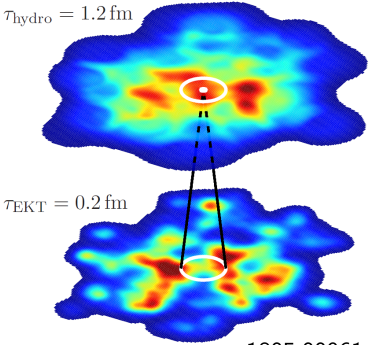
  $\rightarrow$phase space distribution $f(\tau, \mathbf{x}, \mathbf{p})$

$$(\partial_\tau + \mathbf{v}\cdot\nabla_{\mathbf{x}})f(\tau, \mathbf{x}, \mathbf{p}) = -C[f]$$



Out-of-equilibrium initial state is transported to equilibrium

Expanding QGP



1805.00961

- longitudinal expansion
- approximate boost invariance
- homogeneity in the transverse plane

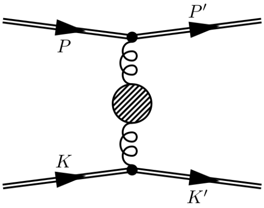
$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\tau, \mathbf{p}) = -C[f]$$

- leading order elastic and inelastic scattering processes

$$C[f](\mathbf{p}) = C_{2\leftrightarrow 2}[f](\mathbf{p}) + C_{1\leftrightarrow 2}[f](\mathbf{p})$$

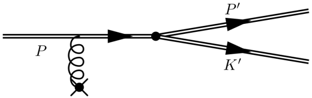
Collision kernel

$C_{2\leftrightarrow2}$



- small momentum transfer $q = |\mathbf{p}' - \mathbf{p}| \ll 1$ regulated by
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2 + m_{\mathrm{eff}}^2}$$

$C_{1\leftrightarrow2}$



- medium induced radiation of gluons
- $g \rightarrow q\bar{q}$ splittings
- LO: strictly collinear

EKT

- Soft sector: EKT describes thermalisation of highly occupied state to QGP $\rightarrow$ Bottom-up thermalisation, BMSS 2001, Kurkela and Lu 1405.6318, Kurkela and Zhu 1506.06647

- Hard sector: Relaxation of high momentum particles $\rightarrow$ (mini-)jet quenching, Schlichting and Soudi 2008.04928, Mehtar-Tani, Schlichting, Soudi 2209.10569

1. Initial conditions: thermal, non-expanding

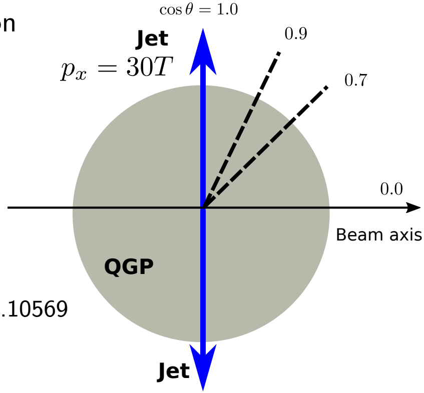
- background + linear perturbation

$$f(\tau, \mathbf{p}) = f_{\text{eq}}(\tau, \mathbf{p}) + \delta f_{\text{Jet}}(\tau, \mathbf{p})$$

$\rightarrow$ linearized Boltzmann eq.

$$\partial_\tau \delta f = -\delta C[\delta f, f_{\text{eq}}]$$

Mehtar-Tani, Schlichting, Soudi, 2209.10569



Momentum space!

- want to study

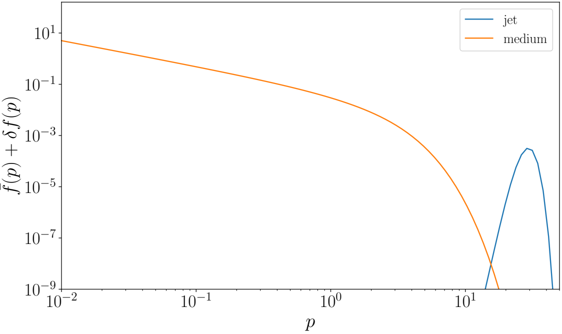
$$\boxed{\delta f_{\text{Jet}}(\tau_0, \mathbf{p}) \rightarrow \delta f_{\text{eq}}(p)}$$

1. Initial conditions: thermal

- initial jet distribution

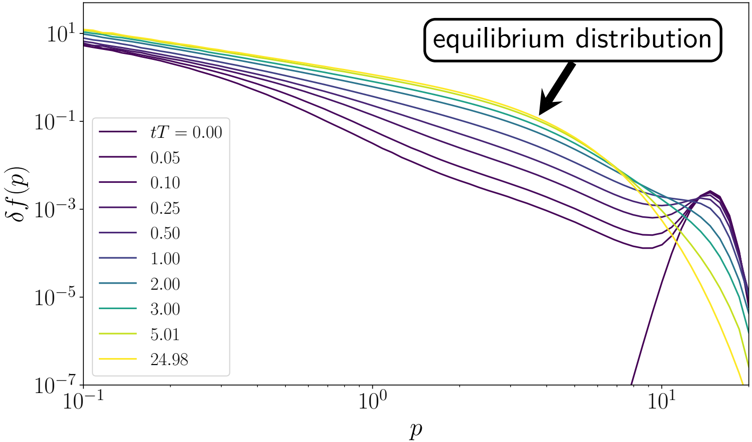
$$\delta f_{\mathrm{Jet}}(\tau_0, \mathbf{p}) \propto \delta(\mathbf{p} - \mathbf{p}_0)$$

- Gaussian at $\mathbf{p}_0 = (E, 0, 0)$



Initial condition for the jet on top of a thermal background

Inverse energy cascade



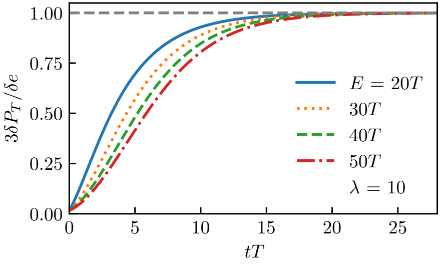
1) build up of soft bath 2) transport of energy
Kurkela and Lu, 1405.6318

- In thermal equilibrium: isotropic pressure

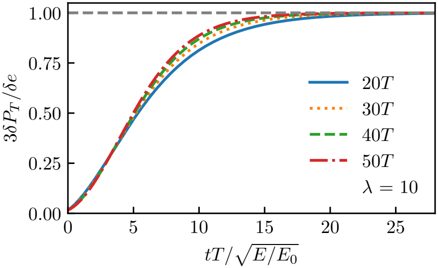
Pressure equilibration

- energy momentum tensor $\delta T^{\mu\nu} = \int_{\mathbf{p}} \frac{p^\mu p^\nu}{p} \delta f(\tau, \mathbf{p})$
- $\delta T^{\mu\nu}_{\mathrm{eq}} = \mathrm{diag}(\delta e, \delta P, \delta P, \delta P)$ with $\delta P = \delta e/3$
- equilibration of transverse pressure $\delta P_T = \frac{1}{2}(\delta T^{xx} + \delta T^{yy})$

- Consider different initial jet energies E:



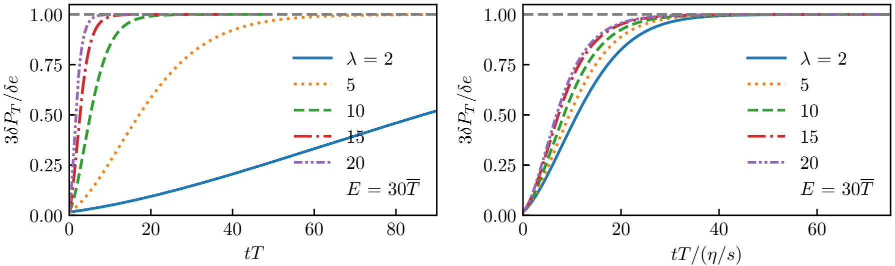
(a) Large E equilibrate later



(b) Scaling with $\sqrt{E}$

Pressure equilibration

- Different couplings λ :

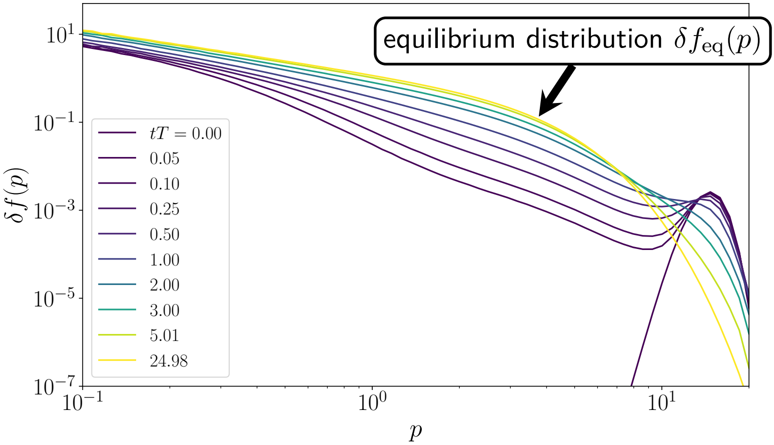


(c) Large λ equilibrate faster

(d) Scaling with $\eta/s \sim \lambda^{-2}$

Curves collapse by rescaling time with $\eta/s\sqrt{E}$

Inverse energy cascade



1) build up of soft bath 2) transport of energy
Kurkela and Lu, 1405.6318

- How is $\delta f_{\text{eq}}(p)$ approached?

Equilibrium distributions

- equilibrated jet $\rightarrow$ increase in temperature by δT

$$f_{\text{eq}}\left(\frac{p}{T}\right) + \delta f_{\text{Jet}}(\tau_0, \mathbf{p}) \xrightarrow{\text{equilibration}} f_{\text{eq}}\left(\frac{p}{T+\delta T}\right)$$

- equilibrium distribution

$$\delta f_{\text{Jet}}(\tau_0, \mathbf{p}) \rightarrow \delta f_{\text{eq}}\left(\frac{p}{T}\right) = \underbrace{\partial_T f_{\text{eq}}\left(\frac{p}{T}\right)}_{\text{eq. distr.}} \times \underbrace{\delta T}_{\text{magnitude}}$$

- Look at time evolution of $\delta f_{\text{Jet}}(\tau, \mathbf{p})$ and its angular (θ) structure

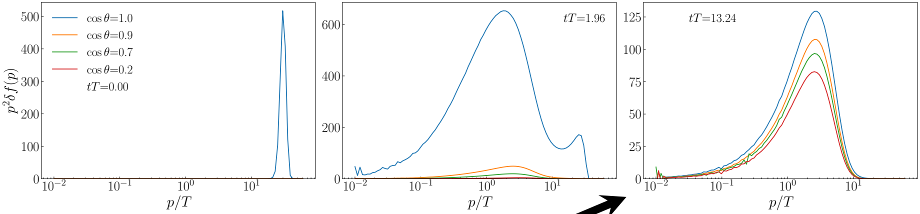
Jet distributions

- equilibrated jet distribution

$$\delta f_{\text{eq}}\left(\frac{p}{T}\right) = \underbrace{\partial_T f_{\text{eq}}\left(\frac{p}{T}\right)}_{\text{eq. distr.}} \times \underbrace{\delta T}_{\text{magnitude}}$$

- evolve $\delta f_{\text{Jet}}(\tau, \mathbf{p})$ for θ-slices

time $\longrightarrow$



same shape, different magnitude?

Jet distributions

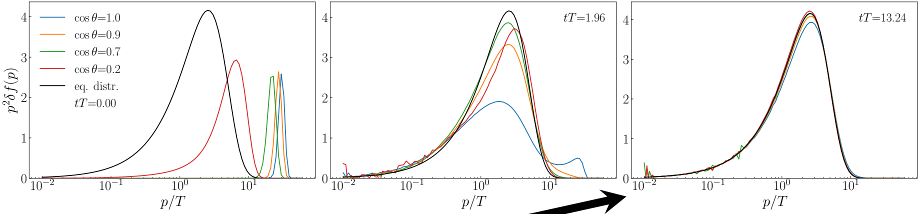
- equilibrated jet distribution

$$\delta f_{\text{eq}}\left(\frac{p}{T}\right) = \underbrace{\partial_T f_{\text{eq}}\left(\frac{p}{T}\right)}_{\text{eq. distr.}} \times \underbrace{\delta T}_{\text{magnitude}}$$

- Normalise $\int dp p^2 \delta f(p,\theta) = 1$ for each θ

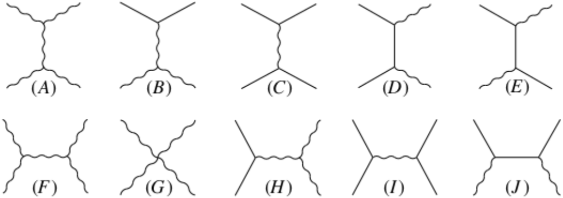
  $\rightarrow$ agreement with eq. distr. while still anisotropic

time $\longrightarrow$



Equilibrium distributions with temperatures $\delta T(\theta)$

Fermions



$2 \leftrightarrow 2$ QCD processes, AMY (2003)

- add quarks to $C[f]$, massless, $N_f = 3$

  $\rightarrow$ all QCD channels included,

- solve 2 equations for quarks $\delta f_q(\tau, \mathbf{p})$ and gluons $\delta f_g(\tau, \mathbf{p})$
- initialisation: $\delta f_q(\tau_0, \mathbf{p}) = 0$, same $\delta f_g(\tau_0, \mathbf{p})$
- gluon evolution similar to YM evolution due to fast radiation

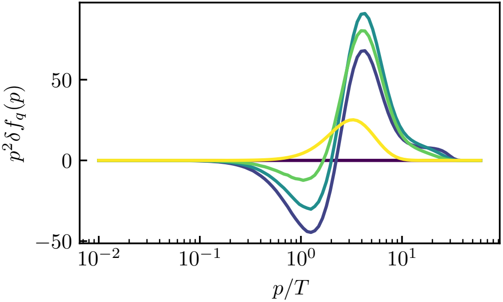
Fermions

- early times: depletion of quarks around $p \sim T$

  $\rightarrow$ hard gluon scatters off of the soft background

- late times: quark distribution approaches Fermi-Dirac

  $\rightarrow$ system equilibrates chemically



Quark distribution in jet direction

2. Initial conditions: anisotropic, expanding

$$f(\tau,\mathbf{p}) = \bar{f}(\tau,\mathbf{p}) + \delta f_{\mathrm{Jet}}(\tau,\mathbf{p})$$

- expanding system

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\delta f(\tau,\mathbf{p}) = -\delta C[\delta f, \bar{f}]$$

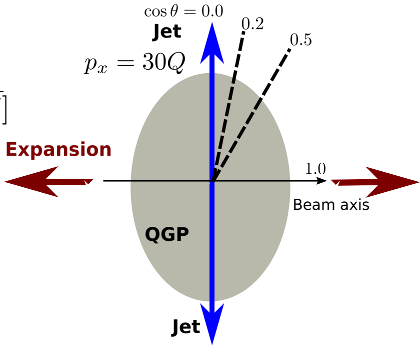
- non-thermal medium

$$\bar{f}(\tau_0,\mathbf{p}) \propto \exp\left(-\frac{2}{3}\frac{p_\perp^2 + \xi^2 p_z^2}{Q^2}\right)$$

- evolving according to

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\bar{f}(\tau,\mathbf{p}) = -C[\bar{f}]$$

Kurkela and Zhu, 1506.06647, Kurkela and Mazeliauskas, 1811.03068



Momentum space!

$\bar{f}(\tau,\mathbf{p})$: approach to hydrodynamics! how about mini-jets?

Hydrodynamization of jets

- Now: study hydrodynamization of mini-jets

$$\boxed{\delta f_{\mathrm{Jet}}(\tau_0, \mathbf{p}) \to \delta f_{\mathrm{hydro}}(\tau, \mathbf{p})}$$

- thermal, non-expanding background: $\delta f_{\mathrm{eq}}(p) \to$ known analytic expression

- non-equilibrium, expanding background:

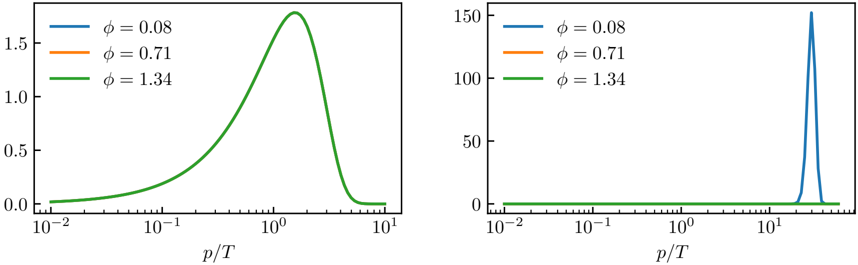
  no analytic expression for $\delta f_{\mathrm{hydro}}$

- compare with an arbitrary perturbation!

Comparison with background-like perturbation

- azimuthal symmetric (ϕ): $\delta f^{\rm az}_{\rm sym}(\tau_0, \mathbf{p}) = \epsilon \bar{f}(\tau_0, \mathbf{p})$

$$\bar{f} + \delta f^{\rm az}_{\rm sym} = (1+\epsilon)\bar{f}$$

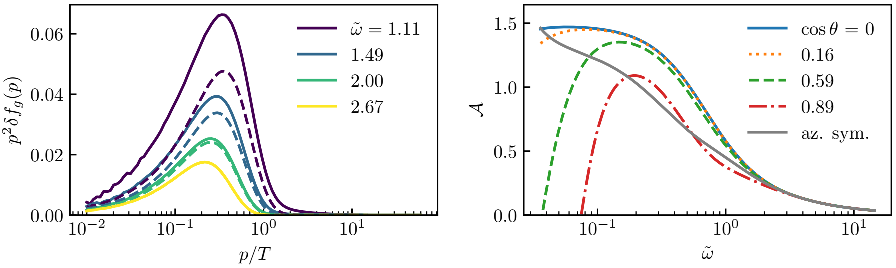


(a) $p^2 \delta f^{\rm az}_{\rm sym}(\tau_0, p, \phi)$ (b) $p^2 \delta f_{\rm Jet}(\tau_0, p, \phi)$

- Hydrodynamization occurs, if both perturbations are indistinguishable

Hydrodynamization

- rescaled time variable $\tilde{\omega} = \tau T(\tau)/(4\pi\eta/s)$
- collapse of $\delta f_g(p)$ and $\delta f_{\mathrm{sym}}^{\mathrm{az}}$ around $\tilde{\omega} \approx 2$
- anisotropy $\mathcal{A} = \frac{P_T - P_L}{3e}$ agrees for different initial conditions



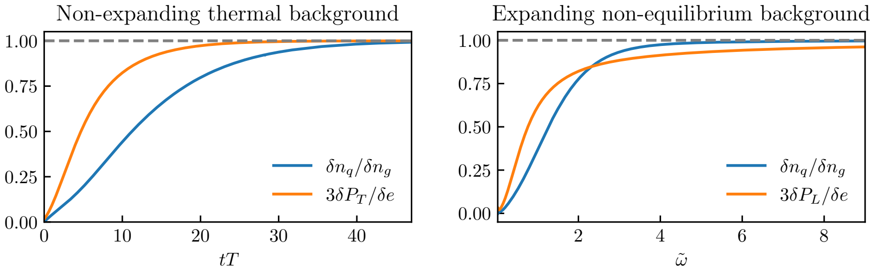
(a) $\delta f_g(p)$ vs. $\delta f_{\mathrm{sym}}^{\mathrm{az}}$ (b) different initial orientations $\cos\theta$

- loss of memory about the initial conditions

→ Hydrodynamization

Chemical equilibration

- compare with kinetic equilibrium (isotropy of pressure)



- in chemical equilibrium more fermionic degrees of freedom
- chem. equilibration not affected by expansion

Summary & Outlook

- modelled mini-jets as perturbations on top of a background

static QGP:

- time scale of thermalisation scales with E and λ
- thermal distribution with $T(\theta)$

expanding QGP:

- mini-jets hydrodynamise (later than the background)
- chemical equilibration before the system isotropises

Outlook:

- extract jet response functions $\rightarrow$ phenomenology
- include transverse dynamics $\rightarrow$ small systems

Backup

Equations of motion from 2PI effective action

$$\left[iG_{0,ac}^{-1,\mu\gamma}(x;\mathcal{A})+\Pi_{ac}^{(0),\mu\gamma}(x)\right]\rho_{\gamma\nu}^{cb}(x,y) = -\int_{y^0}^{x^0} dz\Pi_{ac}^{(\rho),\mu\gamma}(x,z)\rho_{\gamma\nu}^{cb}(z,y) \quad (1)$$

$$\left[iG_{0,ac}^{-1,\mu\gamma}(x;\mathcal{A})+\Pi_{ac}^{(0),\mu\gamma}(x)\right]F_{\gamma\nu}^{cb}(x,y) = -\int_{t_0}^{x^0} dz\Pi_{ac}^{(\rho),\mu\gamma}(x,z)F_{\gamma\nu}^{cb}(z,y)+\int_{t_0}^{y^0} dz\Pi_{ac}^{(F),\mu\gamma}(x,z)\rho_{\gamma\nu}^{cb}(z,y) \quad (2)$$

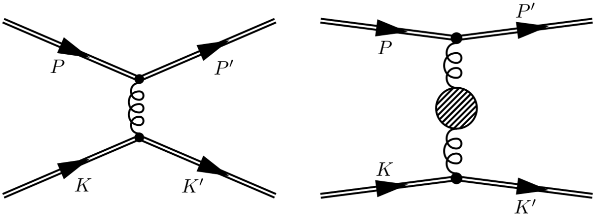
Backup

- local homogeneity $\rightarrow$ relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$
- gradient expansion in X^μ
- to lowest order, spectral function ρ is on shell
  $\rightarrow$ quasi-particle picture
- non-equilibrium distribution function $f(X, p)$:

$$F(X, p) = -i \left[\frac{1}{2} \pm f(X, p) \right] \rho(X, p)$$

$$\Rightarrow p^\mu \partial_\mu f(X, p) = -C[f]$$

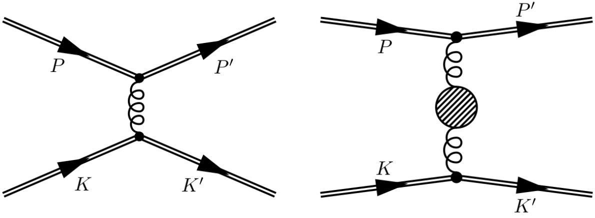
$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

$$C_{2\leftrightarrow 2}[f](\mathbf{p}) = \frac{1}{4p\nu} \int_{\mathbf{k},\mathbf{p}',\mathbf{k}'} (2\pi)^4 \delta^{(4)} \left(p^\mu + k^\mu - p'^\mu - k'^\mu\right)$$
$$\times |\mathcal{M}|^2 \{\underbrace{f_{\mathbf{p}} f_{\mathbf{k}} \left(1 \pm f_{\mathbf{p}'}\right) (1 \pm f_{\mathbf{k}'})}_{\text{loss}} - \underbrace{f_{\mathbf{p}'} f_{\mathbf{k}'} (1 \pm f_{\mathbf{p}}) (1 \pm f_{\mathbf{k}})}_{\text{gain}}\} \quad (3)$$

$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

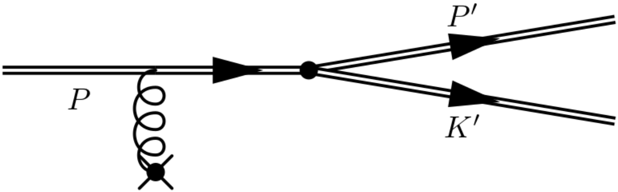
$$|\mathcal{M}|^2 = 2\lambda^2\nu\left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2}\right)$$

- small momentum transfer $q = |\mathbf{p}' - \mathbf{p}| \ll 1$ regulated by

$$\frac{1}{q^2} \to \frac{1}{q^2 + m_{\rm eff}^2}$$

$$m_{\rm eff}^2 = 2g^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3 p}\left[N_c f^g_{\mathbf{p}} + N_f f^q_{\mathbf{p}}\right]$$

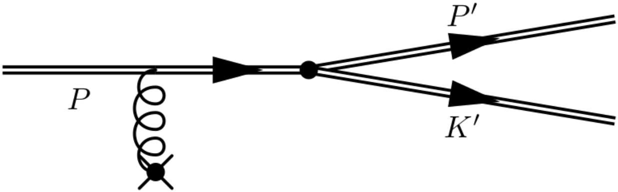
$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

$$C_{1\leftrightarrow 2}[f](\mathbf{p}) = \frac{1}{2}\frac{1}{\nu}(2\pi)^3 \int_{\tilde{\mathbf{p}},\mathbf{p}',\mathbf{k}'} (2\pi)^4 \delta^{(4)}\left(\tilde{p}^\mu - p'^\mu - k'^\mu\right)$$
$$\times \left[\delta^{(3)}(\mathbf{p}-\tilde{\mathbf{p}}) - \delta^{(3)}\left(\mathbf{p}-\mathbf{p}'\right) - \delta^{(3)}\left(\mathbf{p}-\mathbf{k}'\right)\right]$$
$$\times\, \gamma\{\underbrace{f_{\mathbf{p}}\left(1 \pm f_{\tilde{\mathbf{p}}'}\right)(1 \pm f_{\mathbf{k}'})}_{\text{loss}} - \underbrace{f_{\mathbf{p}'} f_{\mathbf{k}'}\left(1 \pm f_{\tilde{\mathbf{p}}}\right)}_{\text{gain}}\} \quad (4)$$

$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

- LO $\rightarrow$ strictly collinear
- medium induced radiation of gluons
- $N+1 \leftrightarrow N+2$ effectively $1 \leftrightarrow 2$

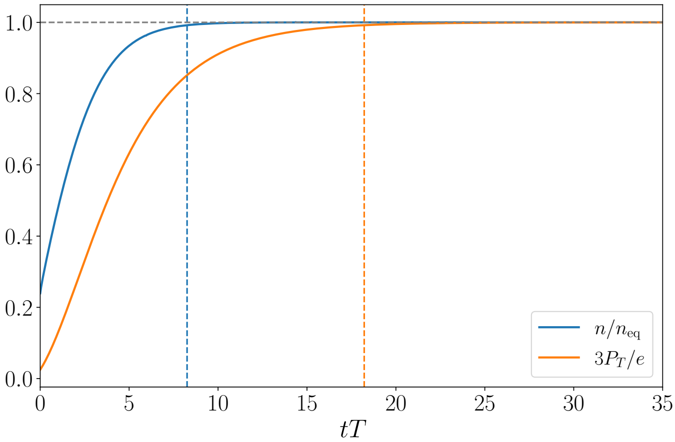
$1 \leftrightarrow 2$

Parton going through the medium, Figure from [1]

- hard parton receiving multiple kicks
- formation time $\tau_f \sim E$
- BH: $l_f \ll l_{\mathrm{mfp}}$, independent emissions
- LPM: $l_f \sim l_{\mathrm{mfp}}$, destructive interference $\rightarrow$ suppression

Radiation vs. elastic scattering

- particle number $n = \int_{\mathbf{p}} f(\tau, \mathbf{p}) \to C_{1\leftrightarrow 2}[f]$
- transverse pressure $P_T = \frac{1}{2}\int_{\mathbf{p}} p_\perp^2/p f(\tau, \mathbf{p}) \to C_{2\leftrightarrow 2}[f]$



Equilibration along each θ-slice

Equilibrium distribution

- equilibrated jet $\rightarrow$ change in temperature δT and velocity δu^z

$$\delta f_{\rm eq}(\mathbf{p}) = (\delta T \partial_T + \delta u^z \partial_{u^z})\, n_{\rm BE}\left(p_\mu u^\mu / T\right)\Big|_{u^z=0}$$

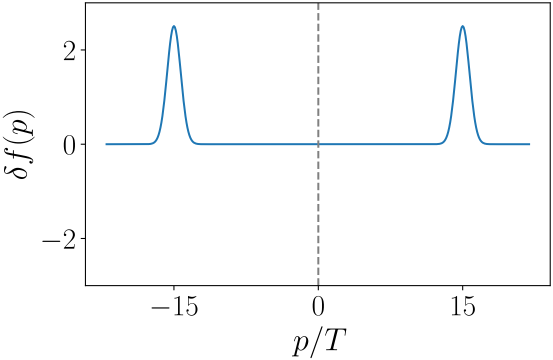
$$\delta f_{\rm eq}(p,\theta) = \left(\delta u^z \cos\theta + \frac{\delta T}{T}\right) F(p/T)$$

- both contributions can be disentangled

Equilibrium distribution

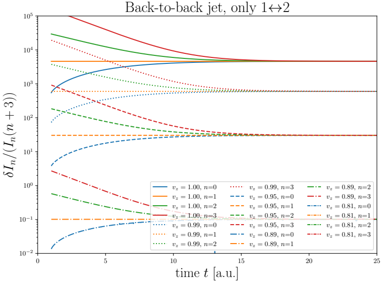
- back-to-back jet conserves net momentum:

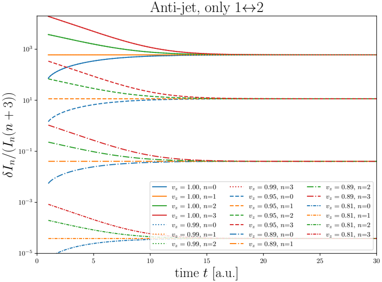
$$\delta f_{\mathrm{eq}}(p,\theta) = \left(\cancel{\delta u^z \cos\theta} + \frac{\delta T}{T} \right) F(p/T)$$



Initial condition: back-to-back jet

Only $1 \leftrightarrow 2$





- similar timescales of equilibration
  $\Rightarrow$ 2↔2 contribute more to equilibration of the anti-jet

Moments of δf

- angular effective temperature

$$I_n(\theta) \equiv 4\pi \int \frac{p^2 dp}{(2\pi)^3} p^n f(p,\theta) = \mathcal{N}_n \times T(\theta)^{n+3} \tag{5}$$

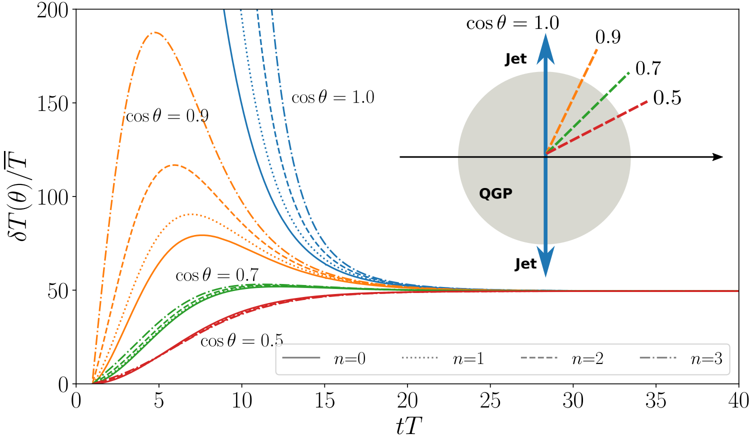
$$T(\theta) = \overline{T} + \delta T(\theta)$$

- temperature perturbation

$$\frac{\delta T(\theta)}{\overline{T}} = \frac{\delta I_n(\theta)}{(n+3)\bar{I}_n(\theta)}$$

- look at time evolution!

Moments of δf

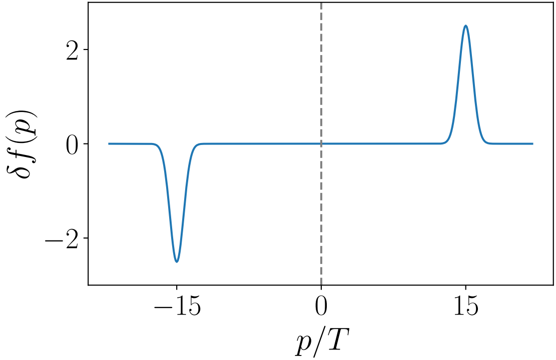


Moments of the back-to-back jet

- different moments agree before different angles do!

Anti-jet

- introduce anti-jet $\rightarrow$ no energy deposited, **only** net momentum

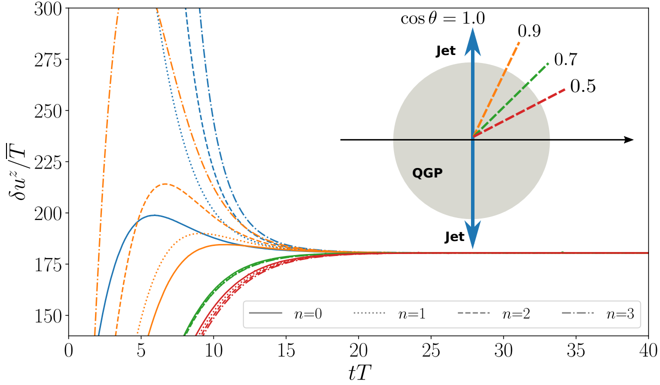


Particle and "hole"

$$\delta f_{\text{eq}}(p,\theta) = \left(\delta u^z \cos\theta + \cancel{\frac{\delta T}{T}}\right) F(p/T)$$

- allows us to study the build up of δu^z

Moments of δf_{anti}



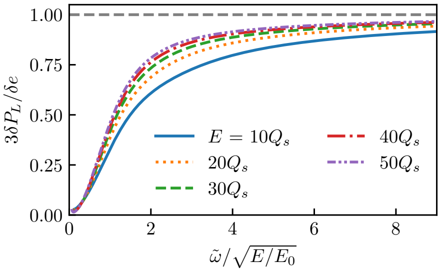
Collapse of different θ much earlier!

$$\delta f_{\mathrm{eq}}(p,\theta) = (\delta u^z \cos\theta)\, F(p/T)$$

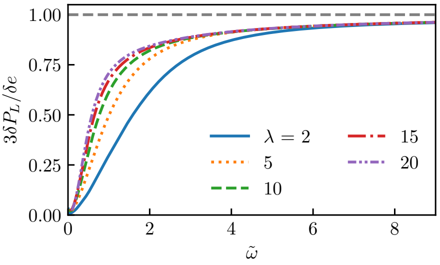
- θ-dependence $\rightarrow$ faster reached by elastic processes
- single jet: velocity builds up faster than temperature

Pressure equilibration

- scaled time $\tilde{\omega} = \tau/\tau_R$ with $\tau_R = \frac{4\pi\eta/s}{T(\tau)}$
- effective temperature from $e(\tau) = \nu_{\text{eff}} \frac{\pi^2}{30} T(\tau)^4$



(c) Scaling with jet energy E



(d) coupling λ