Baryon number fluctuations at finite temperature and magnetic field

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Why study QCD in strong magnetic field?

May be important for phenomenology:
- Non-central heavy-ion collisions
- Magnetars
- Early Universe

Additional parameter to study QCD under extreme conditions
- Can be probed directly in LQCD simulations
  - Quark condensate
  - Fluctuations of conserved charges
- Possibility to test effective models
Schematic behavior of the quark condensate from first-principle numerical simulations

Magnetic catalysis

Well understood

Inverse magnetic catalysis

Most chiral models predict the opposite trend

Opposite $T_c(B)$ for models and LQCD $\rightarrow$ Possible missing interactions

**Our work:** We explore the effect of screening of four-quark interactions in an effective chiral quark model\(^1\)

\(^1\)2107.05521, 2109.04439
Starting point → Chiral model inspired by Coulomb gauge QCD$^2$

$$\mathcal{L} = \overline{\psi}(x)(i\slashed{D} - m_0)\psi(x) + \int d^4y \rho^{a}(x)V^{ab}(x-y)\rho^{b}(y)$$

with

- $\rho^{a}(x) = \overline{\psi}(x)\gamma^0T^{a}\psi(x) \rightarrow$ color quark current
- $V^{ab}(x-y) \rightarrow$ Interaction potential

This work → Contact interaction, gap equation:

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - N_{th}(E, \mu) - \tilde{N}_{th}(E, \mu)\right)$$

The same form as the NJL model if $C_F V_0 \rightarrow 4N_c N_f (2G_{NJL})$

- NJL → Scalar-scalar interaction

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + G_{NJL} \left[ (\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2 \right]$$

- Current model → Vector-vector interaction
  - Systematic improvements possible → dressing by polarization

$^2$See e.g. P. M. Lo, E. S. Swanson Phys. Rev. D 81 034030 (2010)
Dressing by polarization, ring diagram approximation

\[
\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}
\]

Static limit

\[
m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \rightarrow 0)
\]

Screening \(\rightarrow\) Medium-dependent coupling

Contact interaction \(\rightarrow\) No confinement

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- Polyakov loop \(\rightarrow\) Statistical confinement

- \(\Pi_{00}(M) \rightarrow \Pi_{00}(M, \ell, \bar{\ell})\)

- Regulates the screening strength

Vacuum term \(\rightarrow\) Proper-time regularization
LQCD → First-order transition for $4 \text{ GeV}^2 < eB_{\text{LQCD}}^{\text{crit.}} < 9 \text{ GeV}^2$

Current model → $eB_{\text{crit.}} \approx 0.5 \text{ GeV}^2 \ll eB_{\text{LQCD}}^{\text{crit.}}$

Approximation: $\Pi(M, \ell, \bar{\ell}) \rightarrow \xi \times \Pi(\tilde{M} \approx 0.130 \text{ MeV}, \ell, \bar{\ell})$

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3M. D’Elia et al., PoS(LATTICE2022)184
$$\langle \bar{\psi}\psi \rangle (T, B) / \langle \bar{\psi}\psi \rangle_0$$

With screening
No screening

Screening $\rightarrow$ MC and IMC captured
\begin{align*}
\text{G.S. Bali et al., JHEP02(2012)044} & \quad \text{Current model} \\
\text{G. Endr"odi, JHEP07(2015)173} & \quad \text{Crossover} \\
\text{M. D'Elia et al., PoS(LATTICE2022)184} & \quad \text{First-order}
\end{align*}
Current model
PNJL
Crossover
First-order
This talk → Baryon number fluctuations

\[ \chi_n = \frac{\partial^n P}{\partial \mu_B^n} \]

\[ \chi^2(T) = \frac{\chi^2(eB, T_c(eB))}{\chi^2(0, T_c(0))} \]

- \( eB = 0.0 \text{ GeV}^2 \)
- \( eB = 0.6 \text{ GeV}^2 \)
- \( eB = 2.0 \text{ GeV}^2 \)

With screening

No screening

H.-T. Ding et al., arXiv:2208.07285
χ₄ \( T(\text{GeV}) \)

- \( eB = 0.0 \text{ GeV}^2 \)
- \( eB = 0.6 \text{ GeV}^2 \)
- \( eB = 2.0 \text{ GeV}^2 \)

- With screening
- No screening

Graph showing the behavior of \( \chi_4 \) as a function of \( T(\text{GeV}) \) for different values of \( eB \) with and without screening.
Conclusions and outlook

- Effect of the screening of the 4-quark interaction
  - $B = 0$: Reduction of $T_C$
  - $B \neq 0$: MC and IMC captured

- Baryon number fluctuations $\rightarrow$ strongly enhanced

- Future plans:
  - Higher-order fluctuations
  - Charge fluctuations
  - Going beyond the contact interaction

For more details see 2107.05521, 2109.04439, 2309.03124
Polykaov loop increases with $B \rightarrow$ Consistent with LQCD observations

Final set of gap equations:

\[ M = m_0 + C_F \tilde{V}_0(M, \ell) \left[ I_{\text{vac}} - \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} (N_{th}(E, \ell, \ell, \mu) + \bar{N}_{th}(E, \ell, \ell, \mu)) \right] \]

\[ \tilde{V}_0(M, \ell, \ell) = \frac{1}{V_0^{-1} + m_{el}^2(T, M, \ell, \ell)} \]

\[ \frac{\partial}{\partial \ell} (U_G + U_Q) = 0 \]

\[ \frac{\partial}{\partial \ell} (U_G + U_Q) = 0 \]

Non-consistent approximation:

\[ m_{el}^2(T, M, \ell, \ell) \rightarrow \xi m_{el}^2(T, \bar{M}, \ell, \ell) \]

\(\xi \) – controls the ring strength, \( \bar{M} \approx 0.14 \text{ GeV} \)

Regularization:

\[ I_{\text{vac}} = \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \rightarrow \int_{1/\Lambda^2}^{\infty} ds \frac{1}{16\pi^2} \frac{1}{s^2} e^{-M^2s} \]
Coupling to the Polyakov loop → Statistical confinement

- Pure gluon system → Deconfinement order parameter
- Effective models → Accounts for non-preturbative gluon dynamics

\[
N_{th}(E, \mu) \rightarrow N_{th}(E, \ell, \bar{\ell}, \mu) = \frac{\ell e^{-\beta(E-\mu)} + 2\bar{\ell}e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}{1 + 3\ell e^{-\beta(E-\mu)} + 3\bar{\ell}e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}
\]

\[
= \begin{cases} 
\frac{1}{1 + e^{3\beta(E-\mu)}}, & \ell = \bar{\ell} = 0, \text{ baryon-like} \\
\frac{1}{1 + e^{\beta(E-\mu)}}, & \ell = \bar{\ell} = 1, \text{ quark-like}
\end{cases}
\]

Two additional gap equations

\[
\frac{\partial}{\partial \ell} (U_G + U_Q) = 0 \quad \frac{\partial}{\partial \bar{\ell}} (U_G + U_Q) = 0
\]

- \(U_G\) – pure gauge potential\(^5\)
- \(U_Q\) – quark-gluon interaction

Screening

- Reduction of pseudocritical temperature
- No additional modification of model parameters necessary
- Role of Polyakov loop → suppression of the screening strength
Pure gauge part $\rightarrow$ Polyakov loop potential$^1$

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2} a(T)\bar{\ell}\ell + b(T) \ln M_H(\ell, \bar{\ell}) + \frac{1}{2} c(T)(\ell^3 + \bar{\ell}^3) + d(T)(\ell\bar{\ell})^2$$

- Polyakov loop & fluctuations determined from LQCD

Quark-gluon interaction

$$\mathcal{U}_Q = -2T \int \frac{d^3q}{(2\pi)^3} 2 \ln \left( 1 + 3\ell e^{-\beta E} + 3\ell e^{-2\beta E} + e^{-3\beta E} \right)$$

Electric mass

\[ m_{el}^2 = -\frac{1}{2} \mathcal{N}_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0) = \frac{1}{2} \mathcal{N}_f \times \int \frac{d^3 q}{(2\pi)^3} 4\beta N_{th}(1 - N_{th}) \]

Constant, uniform magnetic field → Landau quantization

\[ E^2 = m^2 + \vec{p}^2 \quad \rightarrow \quad E_{k,s}^2 = m^2 + p_z^2 + 2|q_f B|(k + \frac{1}{2} - s) , \]

\[ 2\mathcal{N}_f \int \frac{d^3 p}{(2\pi)^3} \quad \rightarrow \quad \sum_f \frac{|q_f B|}{2\pi} \sum_{s = \pm 1/2} \sum_{k = 0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \]

Electric mass (per flavor)

\[ m_{el}^2 = \frac{1}{2} \frac{|q_f B|}{2\pi} \sum_{k = 0}^{\infty} (2 - \delta_{k,0}) \int \frac{dq_z}{2\pi} 4\beta N_{th}(E_k)(1 - N_{th}(E_k)) \]

\[ \approx \frac{1}{2} \frac{|q_f B|}{4\pi} \int \frac{dq_z}{2\pi} \frac{4\beta e^{\beta \sqrt{(q_z)^2 + m^2}}}{(e^{\beta \sqrt{(q_z)^2 + m^2}} + 1)^2} , \quad |q_f B| \gg T^2 \]