Continuum extrapolated high order baryon fluctuations

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in collaboration with

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Introduction: QCD in grand canonical ensemble (GCE)

Partition function of QCD ($N_f = 2+1$):

$$\mathcal{Z}(V, T, \{\mu_B, \mu_Q, \mu_S\}) = \mathrm{Tr}\left[\exp\left\{-\beta\left(H - \mu_B B - \mu_Q Q - \mu_S S\right)\right\}\right]$$

(or μ_u, μ_d, μ_s with $\Delta N_f = N_f - N_{\bar{f}}$)

Conserved charges of QCD:

- baryon number B
- electric charge Q
- strangeness S

Conserved exactly in the whole system,
can **fluctuate** in subsystems!

Introduction: fluctuations of conserved charges of QCD

Observables are derivatives of pressure $p = T \log \mathcal{Z}/V$.

Generalized susceptibilities $(\hat{p} = p/T^4, \hat{\mu} = \mu/T)$:

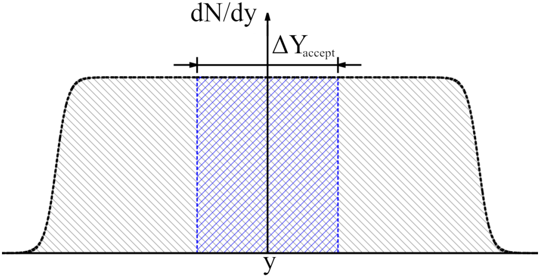
$$\chi_{ij}^{BS} = \frac{\partial^{i+j}\hat{p}(V,T,\{\mu_B,\mu_S\})}{\partial\hat{\mu}_B^i\partial\hat{\mu}_S^j} \qquad \propto \qquad \text{cumulants of } B, S$$

Examples:

$$\langle B\rangle \propto \chi_1^B \qquad \langle B^2\rangle - \langle B\rangle^2 \propto \chi_2^B \qquad \langle BS\rangle - \langle B\rangle\langle S\rangle \propto \chi_{11}^{BS}$$

Introduction: GCE in experiments?

cuts in pseudorapidity $\Longrightarrow$ sub-volume



Caveats:

- cuts in p_{T}
- proxy $\langle \Delta N_p \rangle \neq \langle B \rangle$
- fluctuating volume
- question of thermalization
- final state interactions

[hep-ph:1203.4529],
[nucl-th:2007.02463]

Introduction: importance of fluctuations

1. EoS of hot-and-dense QGP `[hep-lat:2208.05398]`:

$$\hat{p}(\mu_B, \dots) = \hat{p}(0) + \frac{1}{2!}\chi_2^B(T)\hat{\mu}_B^2 + \frac{1}{4!}\chi_4^B(T)\hat{\mu}_B^4 + \frac{1}{6!}\chi_6^B(T)\hat{\mu}_B^6 + \dots$$

2. CEP searches `[nucl-th:2008.04022]`
3. chiral O(4) criticality `[hep-ph:1703.05947]`
4. sensitivity to effective DoFs `[hep-lat:1702.01113]`
5. direct comparison of lattice QCD and experimental data
   (caveats: see previous slide)

History: current continuum results and estimates

Leading order (since 2012) `[hep-lat:1204.6710]`:

$$\chi_2^B \qquad \chi_2^S \qquad \chi_{11}^{BS}$$

Next-to-leading order (since 2015) `[hep-lat:1507.04627, 2212.09043]`:

$$\chi_4^B \qquad \chi_{31}^{BS} \qquad \chi_{22}^{BS} \qquad \chi_{13}^{BS} \qquad \chi_4^S$$

Next-to-next-to-leading order (continuum results now):

$$\chi_6^B \qquad \chi_{51}^{BS} \qquad \chi_{42}^{BS} \qquad \chi_{33}^{BS} \qquad \chi_{24}^{BS} \qquad \chi_{15}^{BS} \qquad \chi_6^S$$

N^{3}LO (results at finite lattice spacing and cont. at $T = 145$ MeV)

$$\chi_8^B \qquad \chi_{71}^{BS} \qquad \chi_{62}^{BS} \qquad \chi_{53}^{BS} \qquad \chi_{44}^{BS} \qquad \chi_{35}^{BS} \qquad \chi_{26}^{BS} \qquad \chi_{17}^{BS} \qquad \chi_8^S$$

Lattice Setup

- $N_f = 2+1+1$ 4HEX staggered action + DBW2 gauge action
- $T = 130 \dots 200$ MeV
- $N_t = 8, 10, 12$ where $T = 1/N_t a$
- aspect ratio $LT = 2$
- physical point: $m_\pi / f_\pi = 1.0337, m_s/m_{ud} = 27.63, m_c/m_s = 11.85$
- statistics: $\mathcal{O}(10^4)$ - $\mathcal{O}(10^5)$ configuration/ensemble

Hadron resonance gas (HRG) model

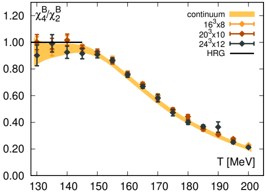
- interacting gas of hadrons ≅ non-interacting gas of hadrons *and* resonances

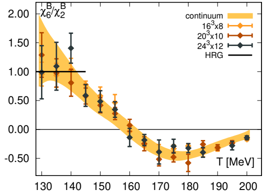
$$p_{\mathrm{QCD}} = \sum_{\mathrm{h}} p_{\mathrm{h}}^{\mathrm{free}}$$

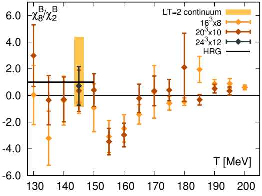
- $\mathcal{O}(10^3)$ hadrons
- non-critical baseline `[nucl-th:2007.02463]`
- uses GCE (just like lattice QCD)

Can HRG describe lattice data?

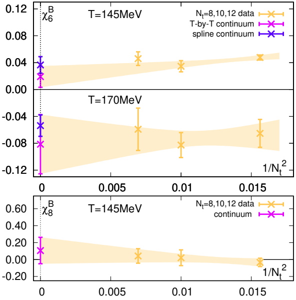
Results - 4HEX continuum







χ_4^B/χ_2^B χ_6^B/χ_2^B χ_8^B/χ_2^B



- agreement with HRG for $T < 145$ MeV
- 4HEX: small cut-off effects due to smaller taste-breaking

Strangeness neutrality

so far: $\mu_S = 0$

$$\Downarrow$$

$\chi_1^S(T, \mu_B, \mu_S) \propto \langle S(T, \mu_B, \mu_S)\rangle = 0$ phenomenologically more relevant

- tuning of [hep-lat:1701.04325]

$$\mu_S \equiv \mu_S^*(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + s_7(T)\mu_B^7 + \mathcal{O}(\mu_B^9)$$

- s_1, s_3, s_5, s_7 from Taylor coefficients order-by-order
- e.g. to first order:

$$\frac{n_S}{T^3} = \frac{\partial \hat{p}}{\partial \hat{\mu}_S} = \sum_{i,j} \frac{1}{i!j!}\chi_{ij}^{BS}(T)\hat{\mu}_B^i j \hat{\mu}_S^{j-1} = \chi_1^S + \chi_{11}^{BS}\hat{\mu}_B + \chi_2^S\hat{\mu}_S \stackrel{!}{=} 0$$

$$\Longrightarrow \hat{\mu}_S = s_1\hat{\mu}_B = -\frac{\chi_{11}^{BS}}{\chi_2^S}\hat{\mu}_B$$

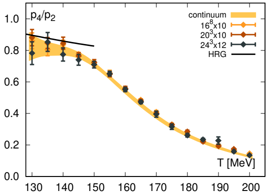
(also $\mu_Q = 0$ vs. $\langle Q\rangle = r\langle B\rangle$)

Results - $n_S = 0$

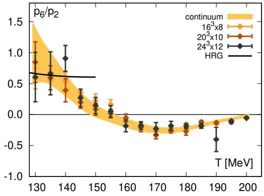
Once s_1, s_3, s_5, s_7 known

$$p_n = \left.\frac{\partial^n \hat{p}}{\partial \hat{\mu}_B^n}\right|_{n_s=0} \qquad \text{of} \qquad \hat{p}_{n_S=0} = \sum_n p_n \hat{\mu}_B^n$$

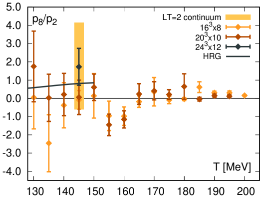
can be computed on strangeness neutral line.



p_4/p_2

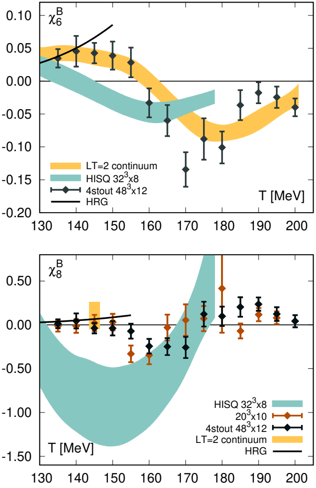


p_6/p_2



p_8/p_2

Results - comparing with literature



Previous results:

- Pisa (not shown)
- HotQCD: HISQ, $N_t = 8$, $LT = 4$
- WB: 4stout, $N_t = 12$, $LT = 4$
- WB: 4HEX, cont. & $N_t = 10$, $LT = 2$

- 4stout $N_t = 12 \approx$ 4HEX cont. for $T < 145$ MeV
  $\Longrightarrow$ small finite-volume effects
  $\Longrightarrow$ agreement with HRG
- HISQ $N_t = 8 \not\approx$ 4stout $N_t = 12$
  $\Longrightarrow$ large cut-off effects
- finite-volume effects for larger T

The End

Grateful for your time and engagement.

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Fig.: QCD, heavy ion collision, etc. Made with `dream.ai`.