Continuum extrapolated high order baryon fluctuations

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Introduction: QCD in grand canonical ensemble (GCE)

Partition function of QCD ($N_f = 2 + 1$):

$$Z(V, T, \{\mu_B, \mu_Q, \mu_S\}) = \text{Tr} \left[ \exp \left\{ -\beta \left( H - \mu_B B - \mu_Q Q - \mu_S S \right) \right\} \right]$$

(or $\mu_u, \mu_d, \mu_s$ with $\Delta N_f = N_f - N_{\bar{f}}$)

Conserved charges of QCD:

- baryon number $B$
- electric charge $Q$
- strangeness $S$

Conserved exactly in the whole system, can fluctuate in subsystems!
**Introduction:** fluctuations of conserved charges of QCD

Observables are derivatives of pressure $p = T \log \mathcal{Z}/V$.

**Generalized susceptibilities** ($\hat{p} = p/T^4$, $\hat{\mu} = \mu/T$):

$$
\chi_{ij}^{BS} = \frac{\partial^i + j \hat{p}(V, T, \{\mu_B, \mu_S\})}{\partial \hat{\mu}_B^i \partial \hat{\mu}_S^j} \propto \text{cumulants of } B, S
$$

Examples:

$$
\langle B \rangle \propto \chi_1^B \quad \langle B^2 \rangle - \langle B \rangle^2 \propto \chi_2^B \quad \langle BS \rangle - \langle B \rangle \langle S \rangle \propto \chi_{11}^{BS}
$$
Introduction: GCE in experiments?

cuts in pseudorapidity $\Rightarrow$ sub-volume

Caveats:
- cuts in $p_T$
- proxy $\langle \Delta N_p \rangle \neq \langle B \rangle$
- fluctuating volume
- question of thermalization
- final state interactions

Introduction: importance of fluctuations

1. EoS of hot-and-dense QGP \cite{hep-lat:2208.05398}:

\[ \hat{p}(\mu_B, \ldots) = \hat{p}(0) + \frac{1}{2!} \chi^B_2(T) \hat{\mu}_B^2 + \frac{1}{4!} \chi^B_4(T) \hat{\mu}_B^4 + \frac{1}{6!} \chi^B_6(T) \hat{\mu}_B^6 + \ldots \]

2. CEP searches \cite{nucl-th:2008.04022}

3. chiral O(4) criticality \cite{hep-ph:1703.05947}

4. sensitivity to effective DoFs \cite{hep-lat:1702.01113}

5. direct comparison of lattice QCD and experimental data (caveats: see previous slide)
**History:** current continuum results and estimates

**Leading order** (since 2012) [hep-lat:1204.6710]:
\[
\chi^B_2 \quad \chi^S_2 \quad \chi^{BS}_{11}
\]

**Next-to-leading order** (since 2015) [hep-lat:1507.04627, 2212.09043]:
\[
\chi^B_4 \quad \chi^{BS}_{31} \quad \chi^{BS}_{22} \quad \chi^{BS}_{13} \quad \chi^S_4
\]

**Next-to-next-to-leading order** (continuum results now):
\[
\chi^B_6 \quad \chi^{BS}_{51} \quad \chi^{BS}_{42} \quad \chi^{BS}_{33} \quad \chi^{BS}_{24} \quad \chi^{BS}_{15} \quad \chi^S_6
\]

**N^3LO** (results at finite lattice spacing and cont. at \(T = 145\) MeV)
\[
\chi^B_8 \quad \chi^{BS}_{71} \quad \chi^{BS}_{62} \quad \chi^{BS}_{53} \quad \chi^{BS}_{44} \quad \chi^{BS}_{35} \quad \chi^{BS}_{26} \quad \chi^{BS}_{17} \quad \chi^S_8
\]
Lattice Setup

- $N_f = 2 + 1 + 1$ 4HEX staggered action + DBW2 gauge action
- $T = 130 \ldots 200$ MeV
- $N_t = 8, 10, 12$ where $T = 1/N_t a$
- aspect ratio $LT = 2$
- physical point: $m_\pi/f_\pi = 1.0337, m_s/m_{ud} = 27.63, m_c/m_s = 11.85$
- statistics: $\mathcal{O}(10^4) - \mathcal{O}(10^5)$ configuration/ensemble
Hadron resonance gas (HRG) model

- interacting gas of hadrons \( \cong \) non-interacting gas of hadrons and resonances

\[
p_{\text{QCD}} = \sum_h p_{\text{h}}^\text{free}
\]

- \( O(10^3) \) hadrons

- non-critical baseline [nucl-th:2007.02463]

- uses GCE (just like lattice QCD)

Can HRG describe lattice data?
Results - 4HEX continuum

- Agreement with HRG for $T < 145$ MeV

- 4HEX: small cut-off effects due to smaller taste-breaking
Strangeness neutrality

so far: \( \mu_S = 0 \)

\[ \chi_1^S(T, \mu_B, \mu_S) \propto \langle S(T, \mu_B, \mu_S) \rangle = 0 \] phenomenonologically more relevant

- tuning of \([\text{hep-lat:1701.04325}]\)

\[ \mu_S \equiv \mu_S^*(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + s_7(T)\mu_B^7 + \mathcal{O}(\mu_B^9) \]

- \( s_1, s_3, s_5, s_7 \) from Taylor coefficients order-by-order

- e.g. to first order:

\[ \frac{n_S}{T^3} = \frac{\partial \hat{p}}{\partial \hat{\mu}_S} = \sum_{i,j} \frac{1}{i!j!} \chi_{ij}^{BS}(T) \hat{\mu}_B^i \hat{\mu}_S^{j-1} = \chi_1^S + \chi_{11}^{BS} \hat{\mu}_B + \chi_2^S \hat{\mu}_S \]

\[ \implies \hat{\mu}_S = s_1 \hat{\mu}_B = -\frac{\chi_{11}^{BS}}{\chi_2^S} \hat{\mu}_B \]

(also \( \mu_Q = 0 \) vs. \( \langle Q \rangle = r\langle B \rangle \))
Results - $n_S = 0$

Once $s_1, s_3, s_5, s_7$ known

$$p_n = \left. \frac{\partial^n \hat{p}}{\partial \mu_B^n} \right|_{n_S=0} \quad \text{of} \quad \hat{p}_{n_S=0} = \sum_n p_n \hat{\mu}_B^n$$

can be computed on strangeness neutral line.
Results - comparing with literature

Previous results:

- Pisa (not shown)
- HotQCD: HISQ, $N_t = 8$, $LT = 4$
- WB: 4stout, $N_t = 12$, $LT = 4$
- WB: 4HEX, cont. & $N_t = 10$, $LT = 2$

- 4stout $N_t = 12 \approx 4$HEX cont.
  for $T < 145$ MeV
  $\implies$ small finite-volume effects
  $\implies$ agreement with HRG
- HISQ $N_t = 8 \not\approx 4$stout $N_t = 12$
  $\implies$ large cut-off effects
- finite-volume effects for larger $T$
The End

Grateful for your time and engagement.

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Fig.: QCD, heavy ion collision, etc. Made with dream.ai.