Elliptic flow of deuterons

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The deuteron wave function

Wave function(s) (r is the distance between proton and neutron):

- Hulthen form ($\alpha = 0.23$ fm^{-1}, $\beta = 1.61$ fm^{-1})

$$\varphi_d(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}}\,\frac{e^{-\alpha r} - e^{-\beta r}}{r}$$

- spherical harmonic oscillator ($d = 3.2$ fm)

$$\varphi_d(r) = (\pi d^2)^{-3/4} \exp\left(-\frac{r^2}{2d^2}\right)$$

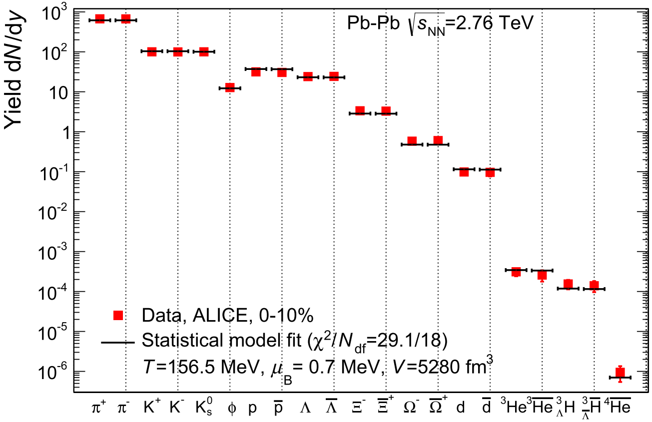
rms radius: 1.96 fm
Binding energy 2.2 MeV
spin 1

Clusters and statistical model: a neat coincidence

Cluster abundancies fit into a universal description with the statistical model

Is this robust feature, or is this a result of fine-tuning?

What does it actually tell us?



[A. Andronic *et al.*, J. Phys: Conf. Ser 779 (2017) 012012]

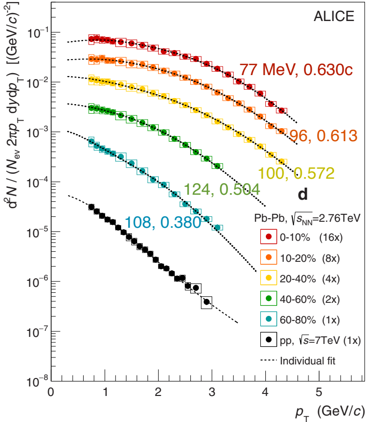
This is (a part of the) motivation to look at clusters,
although clusters actually carry *femtoscopic* information about the freeze-out.

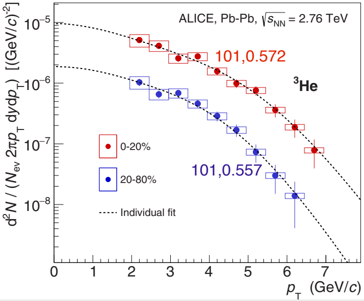
Kinetic freeze-out of clusters: ALICE

[J. Adam *et al.* [ALICE collab], Phys. Rev. C 93 (2016) 024917]

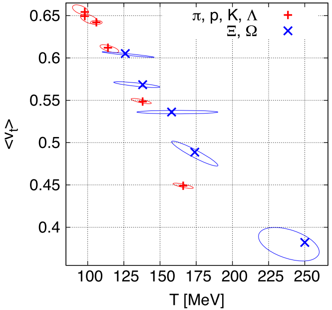
p_t spectra of d and ^{3}He fitted individually with the blast-wave formula

$$\frac{dN}{p_t\,dp_t} \propto \int_0^R r\,dr\,m_t\,I_0\left(\frac{p_t \sinh\rho(r)}{T}\right) K_1\left(\frac{m_t \cosh\rho(r)}{T}\right)\,, \qquad \rho(r) = \tanh^{-1}(\beta_s (r/R)^n)$$





Fit with MC blast-wave:



[I. Melo, B. Tomášik, J. Phys. G. 43 (2016) 015102]

Production mechanism: coalescence

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

Projection of the deuteron density matrix onto two-nucleon density matrix

Deuteron spectrum:

$$E_d \frac{dN_d}{d^3 P_d} = \frac{3}{8(2\pi)^3} \int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d)\, f_p\left(R_d, \frac{P_d}{2}\right) f_n\left(R_d, \frac{P_d}{2}\right) \mathcal{C}_d(R_d, P_d)$$

QM correction factor

$$\mathcal{C}_d(R_d, P_d) \approx \int d^3 r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} |\varphi_d(\vec{r})|^2$$

r relative position, R_+, R_-: positions of nucleons

approximation: narrow width of deuteron Wigner function in momentum

Correction factor: limiting cases

$$\mathcal{C}_d(R_d, P_d) \approx \int d^3r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} \left|\varphi_d(\vec{r})\right|^2$$

- Large homogeneity region for nucleon momentum $P_d/2$: $L \gg d$
  (L is the scale on which $f(R, P_d/2)$ changes)

  $$\mathcal{C}_d(R_d, P_d) \approx \int d^3r \left|\varphi_d(\vec{r})\right|^2 = 1$$

  No correction! Just product of nucleon source functions.

- Small homogeneity region for nucleon momentum $P_d/2$: $L \ll d$
  $f(R, P_d/2)$ effectively limits the integration to Ω

  $$\mathcal{C}_d(R_d, P_d) \approx \int_\Omega d^3r \left|\varphi_d(\vec{r})\right|^2 = \mathcal{C} < 1$$

Interesting regime: $L \approx d$

Analytical approximation of the (average) correction factor

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$\langle \mathcal{C}_d \rangle (P_d) = \frac{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2\left(R_d, \frac{P_d}{2}\right) \mathcal{C}_d(R_d, P_d)}{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2\left(R_d, \frac{P_d}{2}\right)}$$

Approximations:

- Gaussian profile in rapidity and in the transverse direction,
- weak transverse expansion ($\Rightarrow$ no p_t dependence)
- saddle point integration

$$\langle \mathcal{C}_d \rangle \approx \left\{ \left(1 + \left(\frac{d}{2\mathcal{R}_\perp(m)} \right)^2 \right) \sqrt{1 + \left(\frac{d}{2\mathcal{R}_\parallel(m)} \right)^2} \right\}^{-1}$$

Homogeneity lengths:

$$\mathcal{R}_\perp = \frac{\Delta\rho}{\sqrt{1 + (m_t/T)\eta_f^2}} \qquad \mathcal{R}_\parallel = \frac{\tau_0\,\Delta\eta}{\sqrt{1 + (m_t/T)(\Delta\eta)^2}}$$

The invariant coalescence factor B_2

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$E_d \frac{dN_d}{d^3P_d} = B_2 E_p \frac{dN_p}{d^3P_p} E_n \frac{dN_n}{d^3P_n}\bigg|_{P_p=P_n=P_d/2}$$

(Approximations with corrections for box profile)

$$\boxed{B_2 = \frac{3\pi^{3/2}\langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t)\mathcal{R}_\parallel(m_t)} e^{2(m_t-m)(1/T_p^*-1/T_d^*)}}$$

where the effective temperatures are

$$T_p^* = T + m_p \eta_f^2 \qquad T_d^* = T + \frac{M_d}{2}\eta_f^2$$

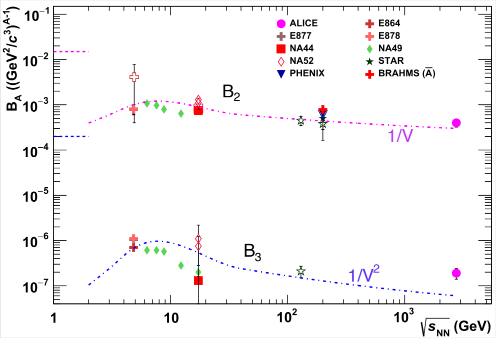
Approximate behaviour: $B_2 \approx 1/volume$

It works!

B_2 as function of $\sqrt{s_{NN}}$

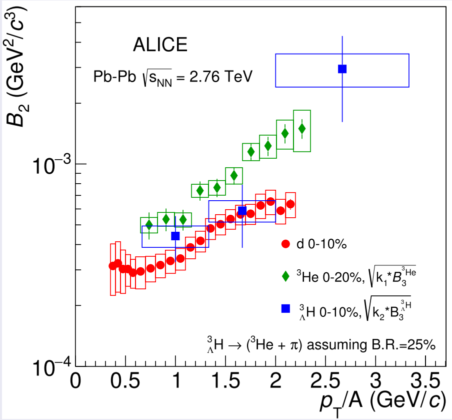
[P. Braun-Munzinger, B. Dönigus, Nucl. Phys. A987 (2019) 144]

Compared with $\propto 1/V$



B_2 as function of p_t

[J. Adam, *et al.* [ALICE coll.], Phys. Lett. B754 (2016) 360]



consistent with decreasing homogeneity volume

Difference between coalescence and thermal production

[F. Bellini, A. Kahlweit, Phys. Rev. C 99 (2019) 054905]

For coalescence use

$$B_2 = \frac{3\pi^{3/2}\langle \mathcal{C}_d \rangle}{2 m_t R^3(m_T)}$$

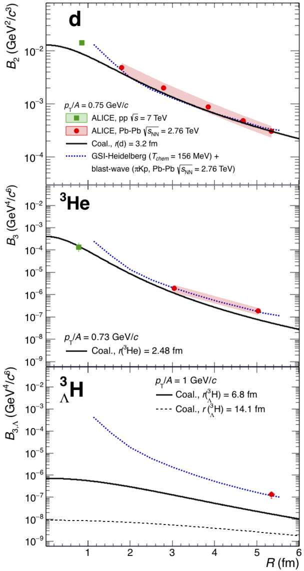
generalized

$$B_A = \frac{2J_A + 1}{2^A} \frac{1}{\sqrt{A}} \frac{1}{m_T^{A-1}} \left(\frac{2\pi}{R^2 + (r_A/2)^2} \right)^{\frac{3}{2}(A-1)}$$

with

$$R = (0.473\,\mathrm{fm}) \langle dN_{ch}/d\eta \rangle$$

Difference between coalescence and blast-wave for small source sizes.



How to get the yields consistent with the statistical model?

Assume thermal source function (Boltzmann)

$$f_N(p_N, x) = 2 \exp\left(-\frac{p_n \cdot u + \mu_N}{T}\right) H(r, \phi, \eta)$$

coalescence:

$$E_d \frac{dN_d}{d^3P_d} = \frac{3}{4} \int_{\Sigma_f} \frac{P_d \cdot d\Sigma_f(R_d)}{(2\pi)^3} \left(2 \exp\left(-\frac{p_n \cdot u + \mu_N}{T}\right)\right)^2 (H(r, \phi, \eta))^2 \mathcal{C}_d(R_d, P_d)$$

thermal production:

$$E_d \frac{dN_d}{d^3P_d} = 3 \int_{\Sigma_f} \frac{P_d \cdot d\Sigma_f(R_d)}{(2\pi)^3} \exp\left(-\frac{P_d \cdot u + \mu_d}{T}\right) H(r, \phi, \eta)$$

they are equal if

- volume is large, i.e. $\mathcal{C}_d(R_d, P_d) = 1$
- $\mu_d = 2\mu_N$, and μ_N guarantees right number of nucleons - Partial Chemical Equilibrium
- $H^2(r, \phi, \eta) = H(r, \phi, \eta)$, fulfilled for box profile

see also [X. Xu, R. Rapp, Eur. Phys. J. 55 (2019) 68]

Lesson from coalescence

- deuteron spectrum sensitive to the shape of the density profile, through $(H(r,\phi,\eta))^2$
- proton spectrum sensitive to $H(r,\phi,\eta)$
- effects for homogeneity lengths comparable with the size of the cluster

$\Rightarrow$ femtoscopy probe

- elliptic flow of deuterons - probes finer changes in homogeneity lengths

see also [A. Polleri, *et al.*, Phys. Lett. B 473 (2000) 193]

Simulate v_2 of deuterons—The Strategy

- set-up Blast Wave model with azimuthal anisotropy
- assume Partial Chemical Equilibrium (lower FO temperature than T_{ch})
- the model must reproduce p_t-spectra and $v_2(p_t)$ of protons and pions
- simulate p_t spectra and $v_2(p_t)$ of deuterons in blast-wave model and in coalescence, and look for differences
- features of the model:
  - includes resonance decays
  - Monte Carlo simulation (SMASH: modified HadronSampler and decays)
  - built-in anisotropy in expansion flow and in fireball shape
  - includes modification of distribution function due to viscosity
  - freeze-out time depending on radial coordinate
- obtain T and transverse expansion from fitting p_t spectra of p and π (and K, Λ)
- then obtain anisotropy parameters from $v_2(p_t)$
- simulate thermal production of deuterons
- simulate coalescence of deuterons (by proximity in phase-space)

Enhanced Monte Carlo Blast-Wave model: freeze-out hypersurface

The Cooper-Frye formula:

$$E\frac{d^3N_i}{dp^3} = \int_\Sigma d^3\sigma_\mu p^\mu f(x,p)\,,$$

The freeze-out hypersurface:

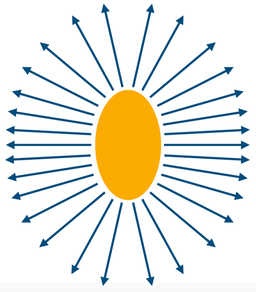
$$x^\mu = (\tau(r)\cosh\eta_s, r\cos\Theta, r\sin\Theta, \tau(r)\sinh\eta_s)$$

$$\tau(r) = s_0 + s_2 r^2\,, \qquad \eta_s = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

$$d^3\sigma^\mu = (\cosh\eta_s, 2s_2 r\cos\Theta, 2s_2 r\sin\Theta, \sinh\eta_s)\, r\tau_f(r) d\eta_s dr d\Theta\,,$$

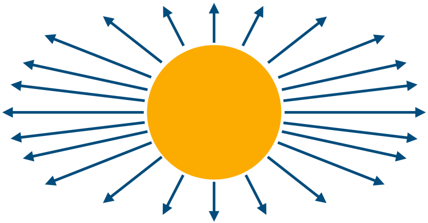
Enhanced Monte Carlo Blast-Wave model: azimuthal anisotropies

Shape anisotropy:



$$R(\Theta) = R_0\,(1 - a_2 \cos(2\Theta))$$

Flow anisotropy:



$$u^\mu = (\cosh\eta_s \cosh\rho(r), \sinh\rho(r)\cos\Theta_b, \sinh\rho(r)\sin\Theta_b, \sinh\eta_s\cosh\rho(r))$$

$$\bar{r} = r/R(\Theta)$$

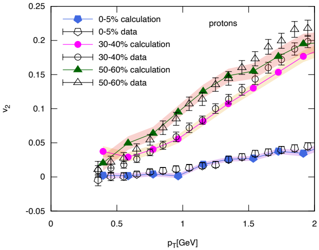
$$\rho(\bar{r},\Theta_b) = \bar{r}\rho_0\,(1 + 2\rho_2\cos(2\Theta_b))$$

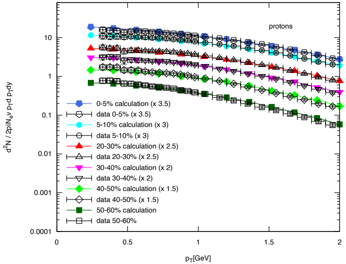
Identified $v_2(p_t)$ for different species allows resolving them.

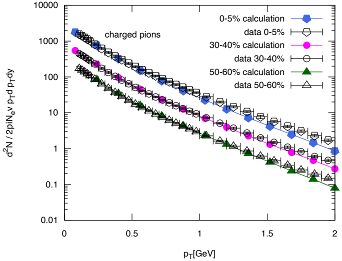
Model calibration

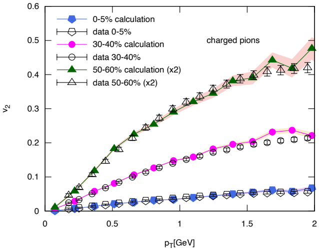
centrality	T[MeV]	ρ_0	R_0[fm]	s_0[fm/c]	a_2	ρ_2
0-5%	95	0.98	15.0	21 ± 2	0.016	0.008
30-40%	106	0.91	10.0	9 ± 1	0.085	0.03
50-60%	118	0.80	6.0	6 ± 0.5	0.15	0.02

$s_2 = -0.02\ \mathrm{fm}^{-1}$

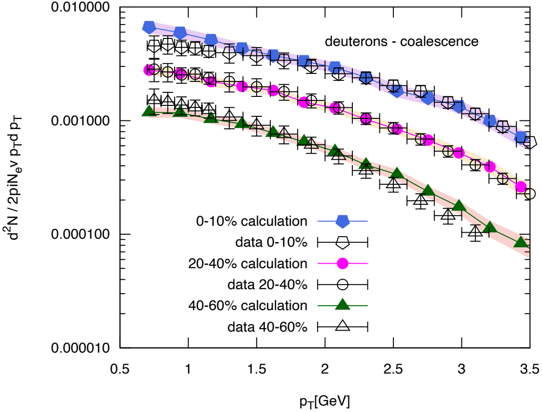


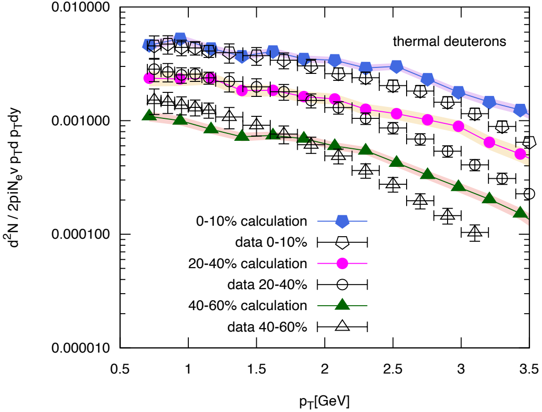




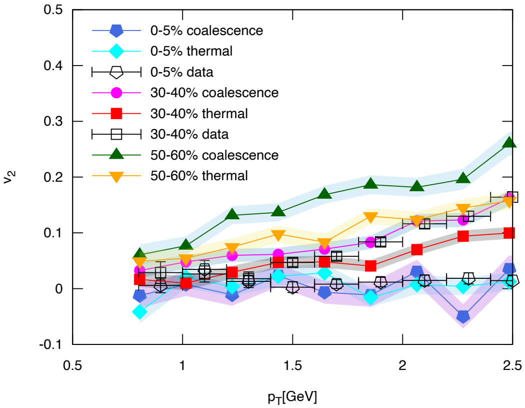


Results for deuterons: p_t spectra



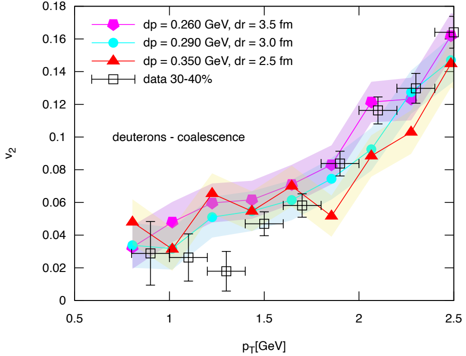


Results for deuterons: v_2



Coalescence for

$\Delta p < \Delta p_{max}$ and $\Delta r < \Delta r_{max}$.



Conclusions

- Deuteron (and cluster) production is a femtoscopic probe

- Elliptic flow in more peripheral collisions can help resolving the mechanism of deuteron production: coalescence vs. thermal production