

Modelling choices in multi-fluid-dynamical descriptions of low energy collisions

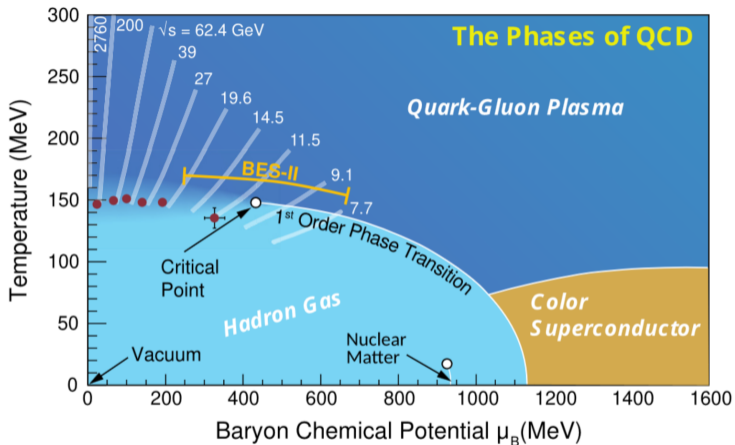
3 fluid code: MUFFIN

Cimerman, Karpenko, Tomášik, Huovinen PRC 107 (2023) 044902

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- goal: probe QCD phase diagram via heavy ion collisions
- expect 1st Order transition line and critical end point
- to probe this region, need higher $\mu_B \Leftrightarrow$ lower $\sqrt{s_{NN}}$

Fluidynamical simulations for low energy collisions

Hydrodynamics works well at high $\sqrt{s_{NN}}$.

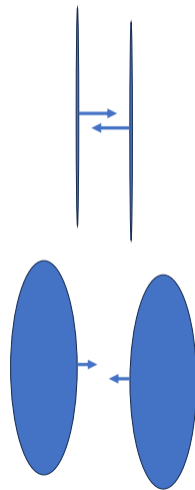
What are the challenges at low $\sqrt{s_{NN}}$?

Lorentz contraction:

- at high energies, nuclei are flat
⇒ timescale separation of:
 1. energy deposition
 2. pre-equilibrium
 3. hydrodynamic stage

many early time descriptions rely on this!

- low energies: extended interpenetration; all 3 processes happen simultaneously!



Fluidynamical simulations for low energy collisions

Hydrodynamics works well at high $\sqrt{s_{NN}}$.

What are the challenges at low $\sqrt{s_{NN}}$?

Baryon stopping vs baryon transparency:

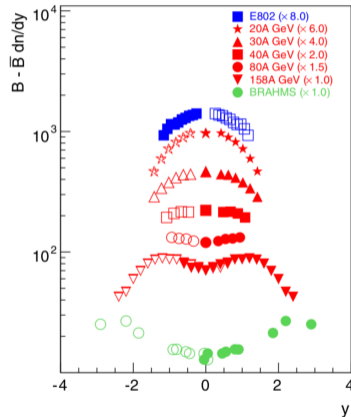
- high beam energies: baryons from nuclei escape collision region
⇒ this is why $\mu_B \approx 0$!
- very low energies: nuclei completely stopped
- intermediate energies: partial baryon transparency causes double peak!

State of the art:

Start hydro at late time with a very complicated initial condition

Shen, Schenke PRC 105 (2022) 064905

net baryon rapidity distribution for low $\sqrt{s_{NN}}$'s:

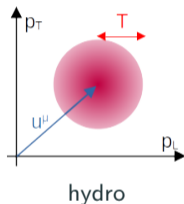


Multi-fluid Hydrodynamics

idea: Hydrodynamics can also describe cold nuclei ("liquid drop model")

⇒ can we use it throughout entire collision evolution?

problem: Hydrodynamics describes a system close to equilibrium, i.e. isotropic in momentum space
not true for colliding nuclei!

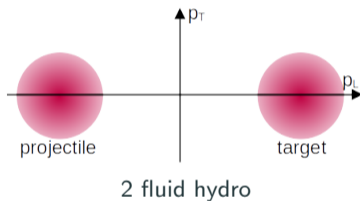
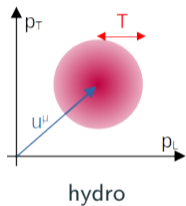


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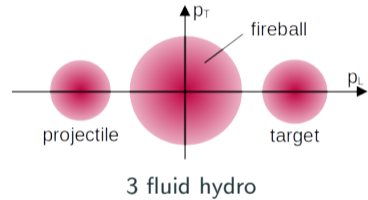
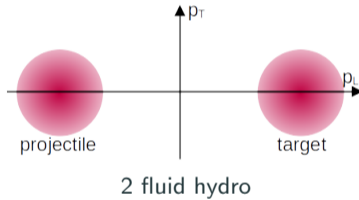
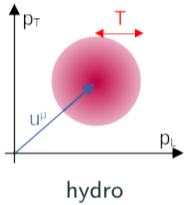


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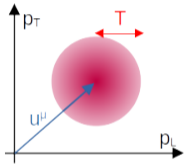


Multi-fluid Hydrodynamics

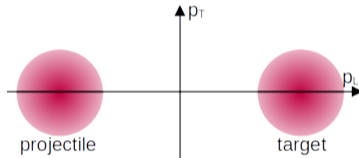
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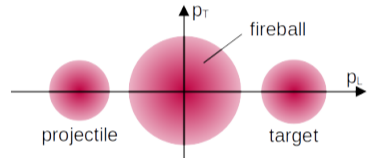
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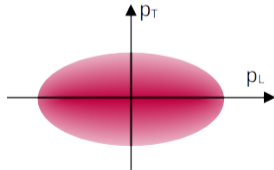
hydro



2 fluid hydro



3 fluid hydro



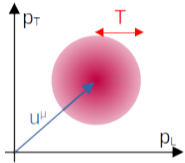
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Multi-fluid Hydrodynamics

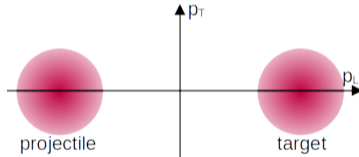
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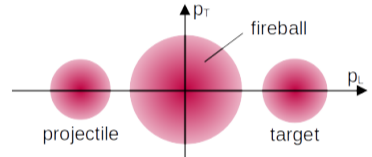
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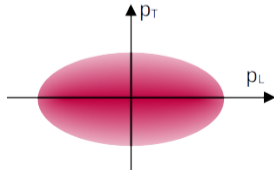
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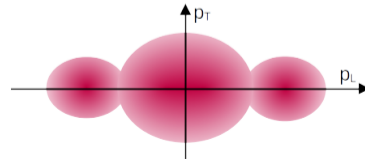
2 fluid hydro



3 fluid hydro



anisotropic hydro



somewhat realistic distribution

Equations governing multi-fluid hydro

3 versions of energy-momentum tensor

$$T_p^{\mu\nu}, T_t^{\mu\nu}, T_f^{\mu\nu}$$

and 3 versions of baryon current

$$J_{B,p}^\mu, J_{B,t}^\mu, J_{B,f}^\mu$$

conservation equations hold only globally:

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0, \quad \partial_\mu (J_{B,p}^\mu + J_{B,t}^\mu + J_{B,f}^\mu) = 0$$

⇒ fluids can exchange energy, momentum and baryon charge via "friction"

$$\partial_\mu T_\alpha^{\mu\nu} = F_\alpha^\nu, \quad \partial_\mu J_{B,\alpha}^\mu = R_{B,\alpha} \quad \text{where} \quad \sum_\alpha F_\alpha^\nu = 0, \quad \sum_\alpha R_{B,\alpha} = 0$$

a priori, any friction is allowed! but:

- system should not deviate too much from modelling assumptions
- needs to reproduce observed behaviour

Example: Csernai's model

"next-to-trivial" model: all that scatters is dumped into the fireball

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26 (1982) 149

⇒ nuclei stay cold!

projectile/target:

$$\partial_\mu T_{p/t}^{\mu\nu} = F_{p/t}^\nu = u_{p/t}^\nu m_N R_{B,p/t}, \quad \partial_\mu J_{B,p/t}^\mu = R_{B,p/t}$$

fireball:

$$\partial_\mu T_f^{\mu\nu} = -F_p^\nu - F_t^\nu, \quad \partial_\mu J_{B,p/t}^\mu = -R_{B,p} - R_{B,t}$$

Only need to model $R_{B,p/t}$: how nuclei lose baryon charge

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Only need to model $R_{B,p/t}$: how nuclei lose baryon charge

problem: no double peak!

Deriving friction from kinetic theory

consider fluids as phase space distributions: $f_\alpha = \frac{dN_\alpha}{d^3x d^3p}$,

can obtain hydro d.o.f. as:

$$T_\alpha^{\mu\nu} = \int \frac{d^3p}{p^0} p^\mu p^\nu f_\alpha, \quad J_{B,\alpha}^\mu = B_\alpha \int \frac{d^3p}{p^0} p^\mu f_\alpha$$

time evolution is described via Boltzmann equation

$$p^\mu \partial_\mu f_\alpha = C_\alpha[f_p, f_t, f_f] = \sum_{\beta,\gamma} C_\alpha^{\beta\gamma}[f_\beta, f_\gamma]$$

for given $C_\alpha^{\beta\gamma}$, hydrodynamic equations obtained as

$$\partial_\mu T_\alpha^{\mu\nu} = \int \frac{d^3p}{p^0} p^\nu C_\alpha = F_\alpha^\nu, \quad \partial_\mu J_{B,\alpha}^\mu = B_\alpha \int \frac{d^3p}{p^0} C_\alpha = R_{B,\alpha}$$

Wait, how many different terms do we need to model now?

$$\partial_\mu T_\alpha^{\mu\nu} = \sum_{\beta,\gamma} F_\alpha^{\beta\gamma,\nu}$$

How many unique $F_\alpha^{\beta\gamma}$ are there?

a priori

$$3 \cdot 3 \cdot 3 = 27$$

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$\beta = \gamma$ is hydro	$3 \cdot 3 \cdot 2 = 18$

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symmetry $\alpha = t \leftrightarrow \alpha = p$	$2 \cdot 3 = 6$

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conservation	$4 - 2 = 2$

have to model

- how p-t interaction affects p/f
- how p-f interaction affects p/f

collision integrals are given in terms of scattering crosssections

$$C_{\alpha}^{\alpha\beta}[f_{\alpha}, f_{\beta}](p_{\alpha}) = \int d^3 p_{\beta} p_{\alpha}^0 \left[\underbrace{-f_{\alpha}(p_{\alpha}) f_{\beta}(p_{\beta}) v_{\text{rel}} \sigma_{\alpha\beta \rightarrow X}}_{\text{loss}} + \underbrace{\int d^3 q_{\alpha} f_{\alpha}(q_{\alpha}) f_{\beta}(p_{\beta}) v_{\text{rel}} \frac{d\sigma_{\alpha\beta \rightarrow \alpha X}}{d^3 p_{\alpha}}}_{\text{gain}} \right]$$

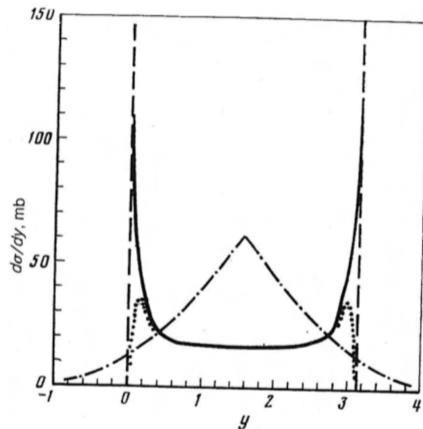
problems:

- crosssections may not be fully measured in experiment
- d.o.f. change in deconfinement transition

- N+N scattering: N strongly peaked at ingoing rapidities, π at midrapidity
⇒ in p-t friction: N stay in p/t, π go to f
- $\pi + N$ mostly resonance formation
⇒ all outgoing particles from p-f friction go to p
- uncertainty in deconfined phase: densities multiplied with \sqrt{s} -dependent prefactor

can describe the double peak!

Ivanov, Russkikh, Toneev PRC 73 (2006) 044904



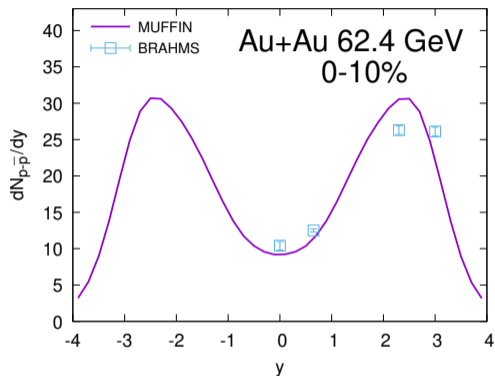
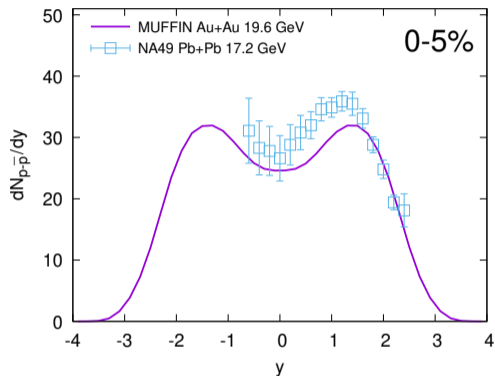
- implementation of 3-fluid (ideal) hydrodynamics in Ivanov's formulation
 - hybrid with SMASH

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- allows easy modification of e.o.s.
- all 3 fluids heat up, p and t slow down a lot

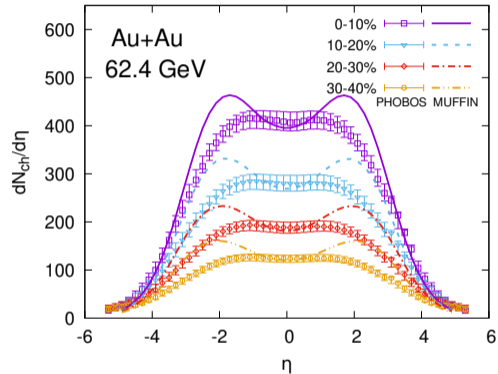
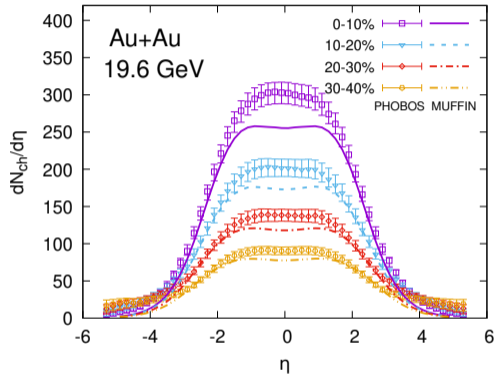
Au+Au 7.7 GeV $b = 5$ fm

Baryon transparency/stopping



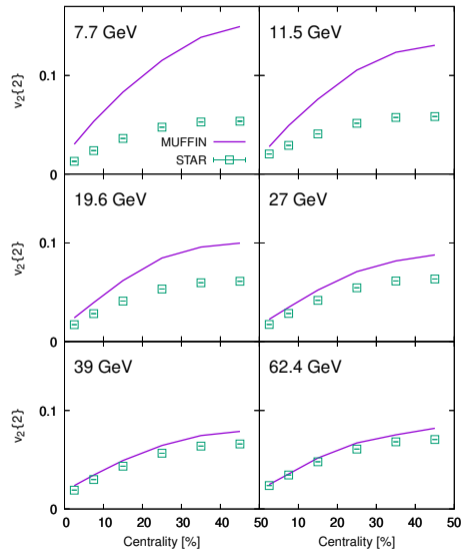
net proton rapidity distributions:
simulated results agree well with experiment
 \Rightarrow correct baryon stopping!

Charged hadron yield



reasonable agreement also in $dN_{ch}/d\eta$
⇒ correct entropy production

v_2 is strongly overestimated at small $\sqrt{s_{NN}}$
 \Rightarrow probably need viscosity



thermal spread:

- full friction computation requires integration over thermal distribution
- current implementation neglects thermal spread!

thermalization:

- 3 versions of $u^\mu, T \Rightarrow$ triple d.o.f.!
- e.o.s. evaluated for each fluid $\Rightarrow \mu_B = 0$ in f!

freezeout:

- what is the combined freezeout criterion?
- merge fluids before or after freezeout?

- low $\sqrt{s_{NN}}$ collisions allow to examine QCD phase diagram
- 3 fluid hydrodynamics models 3 momentum space components of the collision:
projectile, target, fireball
- fluid friction obtained from kinetic theory; requires crosssection input
- can describe full time evolution, correct results for baryon stopping and entropy production