KINETIC-THEORY-WISE FORMULATION OF RELATIVISTIC SPIN HYDRODYNAMICS

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Amaresh Jaiswal *(NISER Bhubaneswar),*

Avdhesh Kumar *(IOP Academia Sinica Taipei)*

based on:

Phys. Lett. B 814 136096 (2021); ***Phys. Rev. D 103***, 014030 (2021); ***Phys. Rev. Lett 129***, 192301 (2022)

23RD ZIMÁNYI SCHOOL WINTER WORKSHOP ON HEAVY ION PHYSICS

DECEMBER 4-8, 2023, BUDAPEST, HUNGARY

NATIONAL SCIENCE CENTRE POLAND

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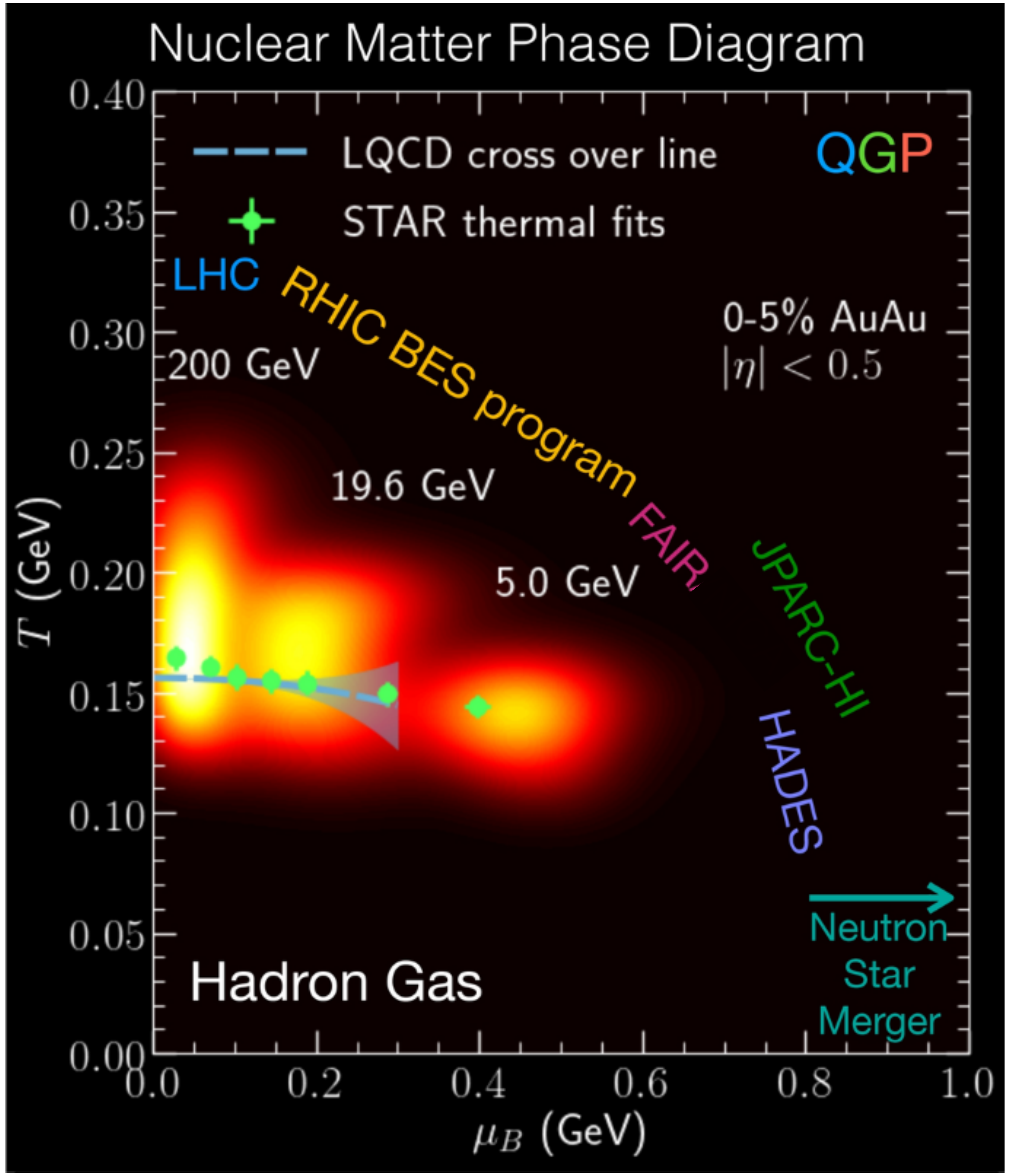
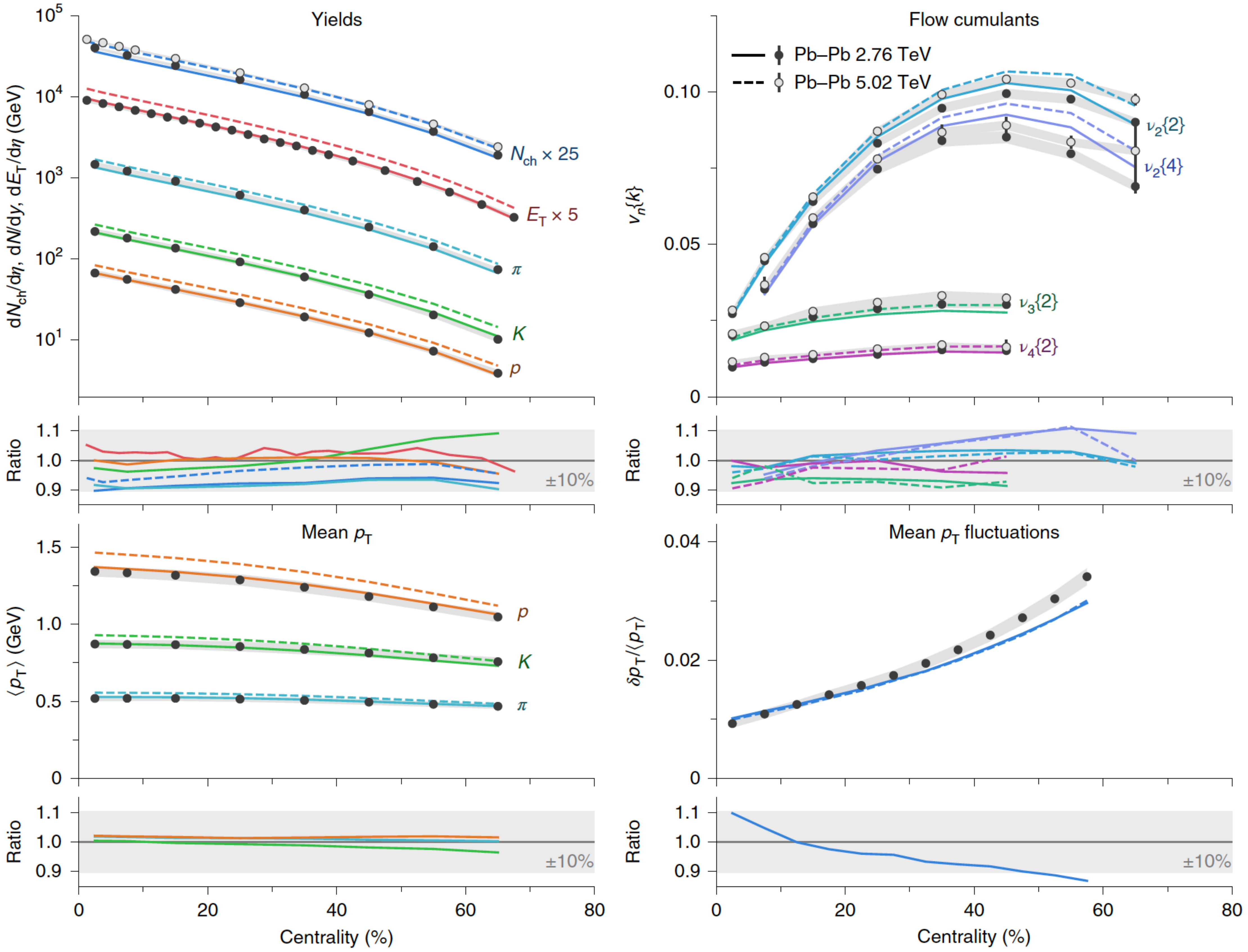
THE HENRYK NIEWODNICZAŃSKI INSTITUTE OF NUCLEAR PHYSICS POLISH ACADEMY OF SCIENCES

RELATIVISTIC HEAVY-ION COLLISIONS PROBE QCD PHASE DIAGRAM

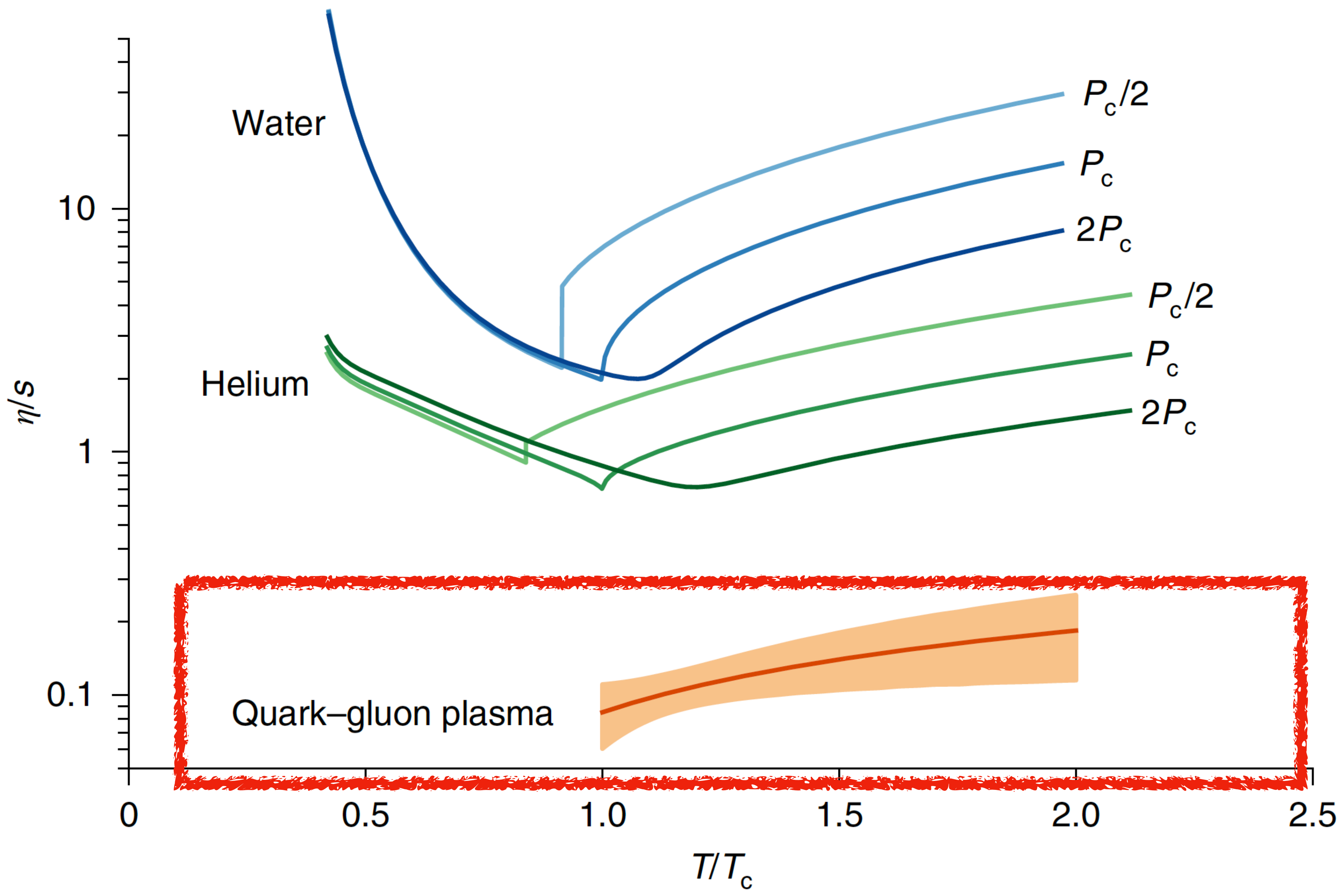


Shen, C., Yan, L. *NUCL SCI TECH* **31**, 122 (2020)

EXPERIMENTAL DATA SUGGESTS THAT QGP IS THE MOST PERFECT FLUID



J. Bernhard, J. Moreland, S. Bass, *Nat. Phys.* **15,** 1113–1117 (2019)



☐ hydrodynamics is applicable

☐ inclusion of dissipative effects is required

SPIN-ORBIT TRANSFER OF ANGULAR MOMENTUM IN HIC

- the low-energy non-central heavy-ion collisions create fireballs with **large global orbital angular momenta**

  F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

$$\boldsymbol{L}_{\mathrm{init}} \sim 10^5 \hbar$$

- part of the angular momentum can be transferred from the orbital to the spin part

- emitted particles are expected to be **globally polarized along the system's angular momentum**

  Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005); 96, 039901(E) (2006)
  Z.-T. Liang and X.-N. Wang, Phys. Lett. B 629, 20 (2005)
  S. A. Voloshin, arXiv:nucl-th/0410089.
  B. Betz, M. Gyulassy, and G. Torrieri, Phys. Rev. C 76, 044901 (2007).
  F. Becattini and F. Piccinini, Ann. Phys. (Amsterdam) 323, 2452 (2008).

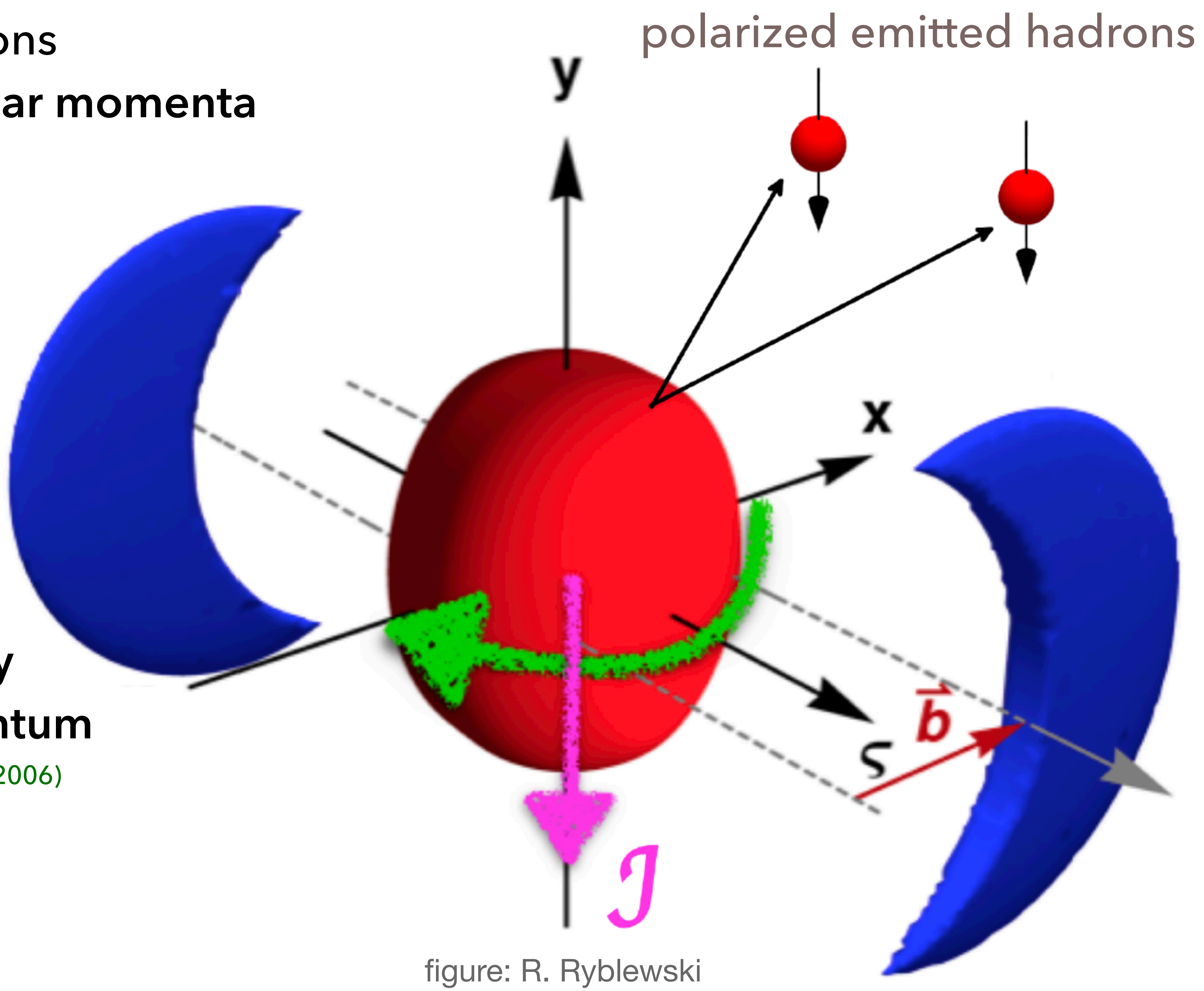
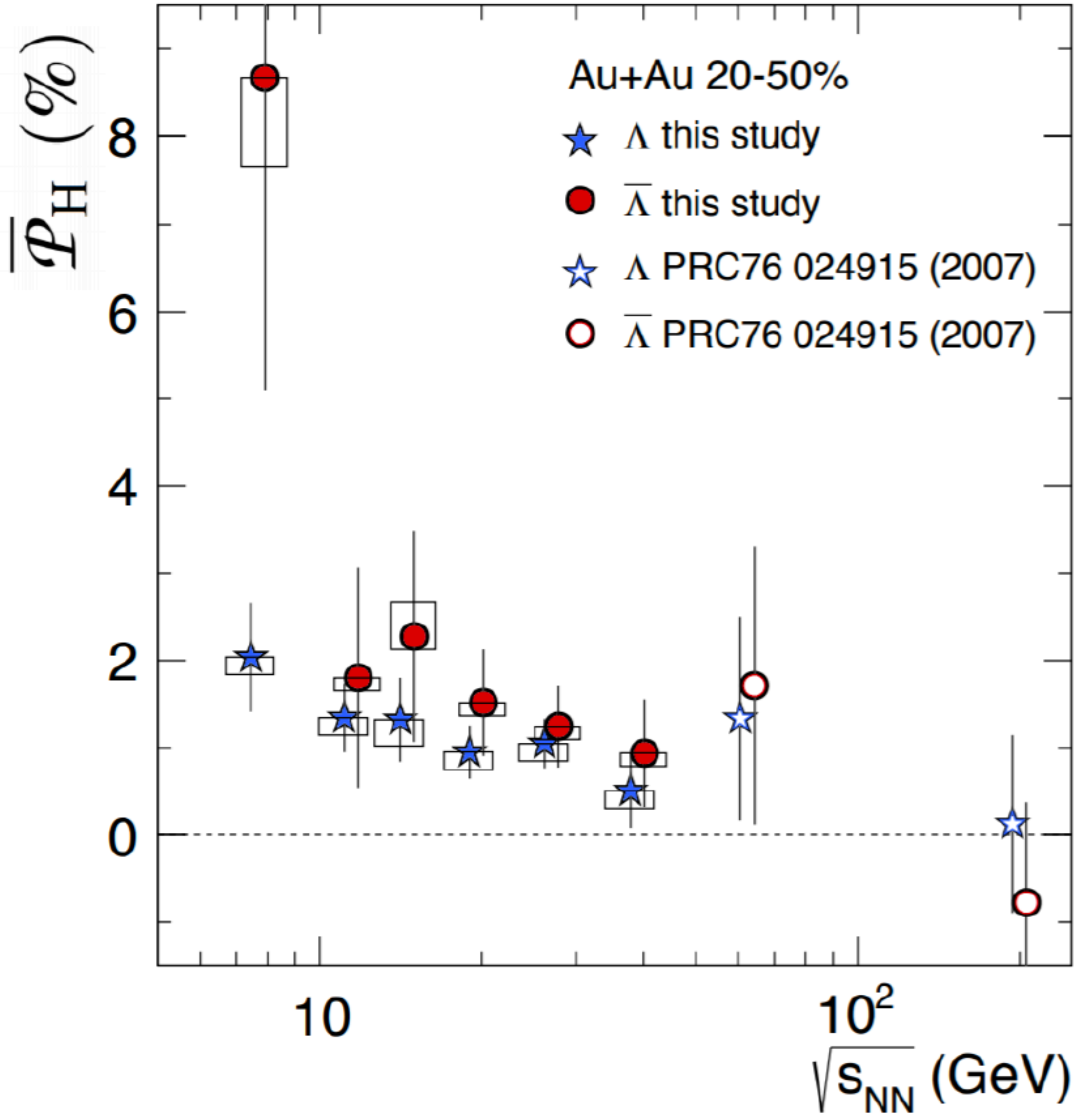


figure: R. Ryblewski

PARTICLES EMERGING FROM HIC ARE POLARIZED

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



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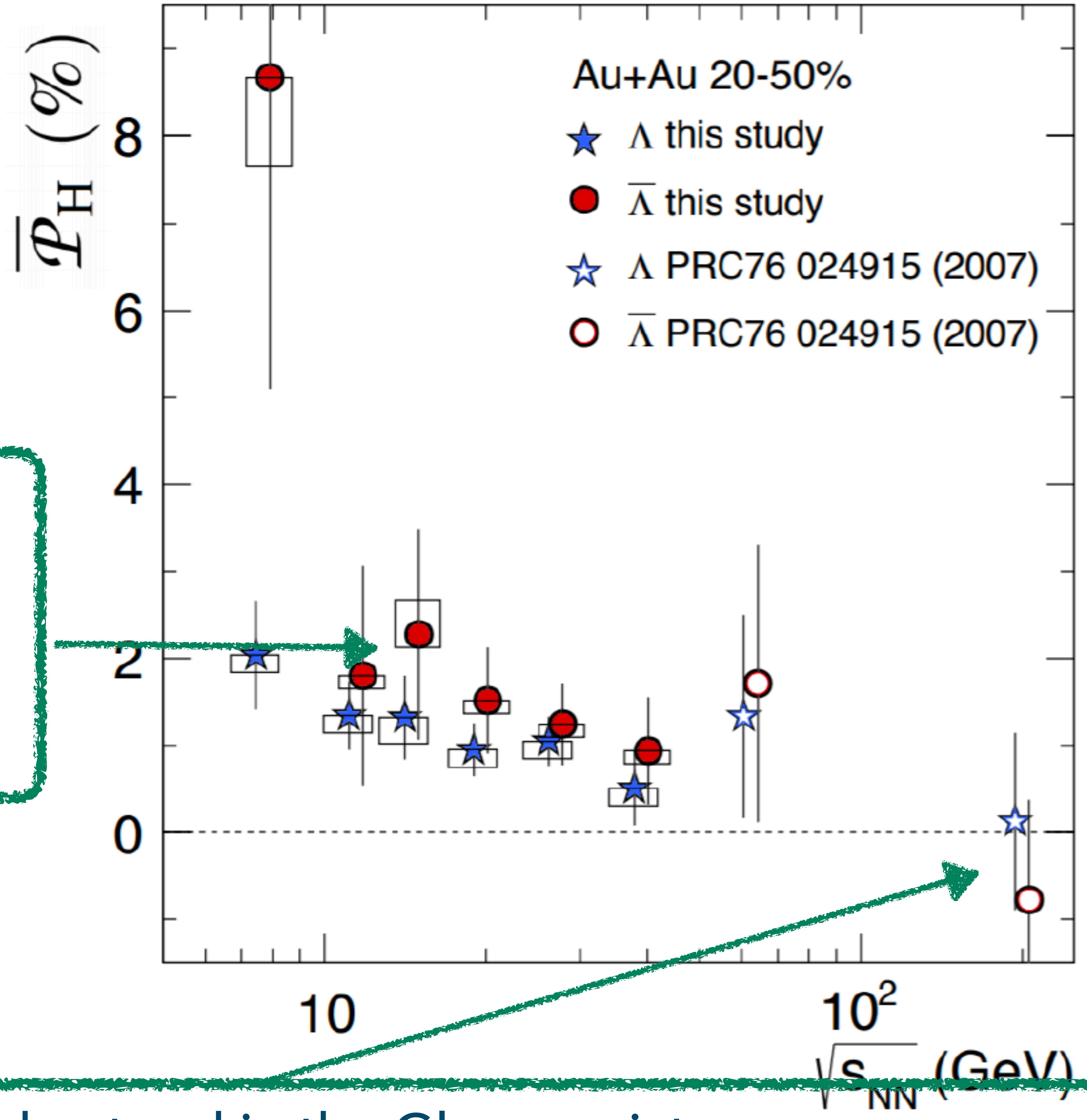


the splitting may be related to the interaction with magnetic field

understood in the Glasma picture

M. E. Carrington, A. Czajka and St. Mrowczynski, Phys. Rev. C 106, 034904 (2022)

SPIN POLARIZATION IN EQUILIBRATED QGP

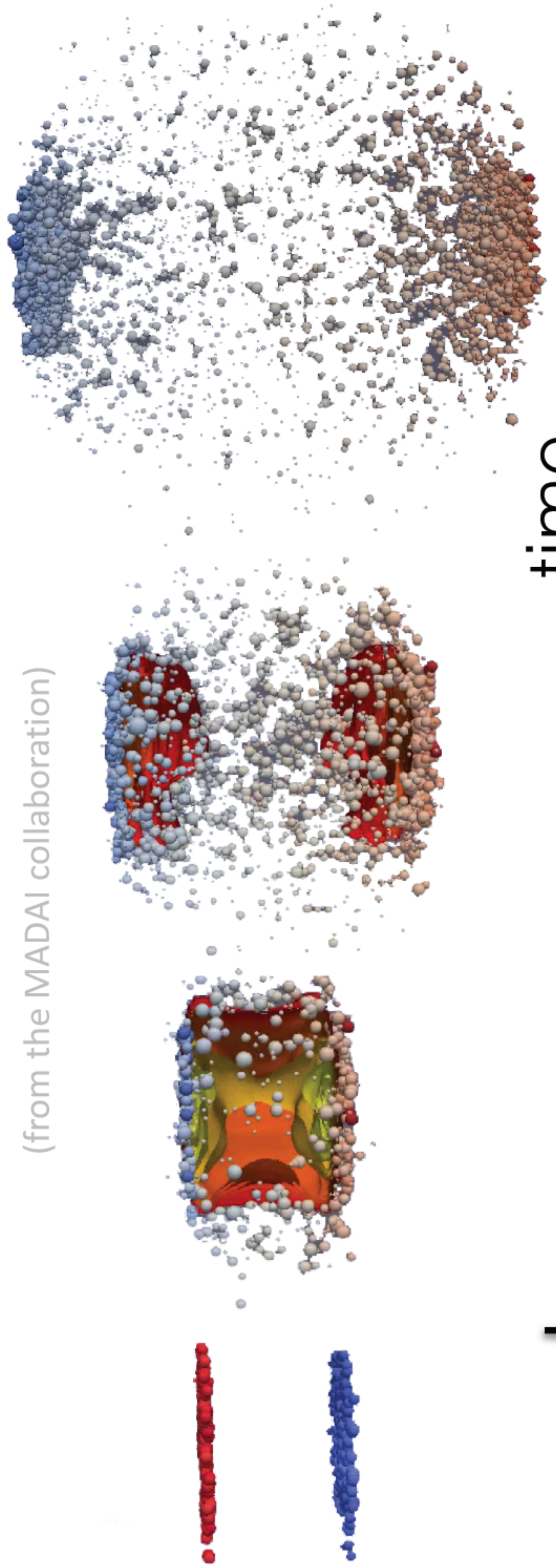
In thermodynamic equilibrium one can establish a link between spin and vorticity

Becattini F, Chandra V, Del Zanna L, Grossi E. AP 338:32 (2013)
F. Becattini, L. Csernai, and D. J. Wang, PRC 88, 034905 (2013)
Fang R, Pang L,Wang Q,Wang X. PRC 94:024904 (2016)
F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin PRC 95, 054902 (2017)

$$S^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int d\Sigma_{\lambda}p^{\lambda}n_F\left(1-n_F\right)\varpi_{\rho\sigma}}{\int d\Sigma_{\lambda}p^{\lambda}n_F}$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_{\mu}\beta_{\nu}-\partial_{\nu}\beta_{\mu}\right) \qquad \beta^{\mu} = \frac{u^{\mu}}{T}$$

Spin is enslaved to thermal vorticity!



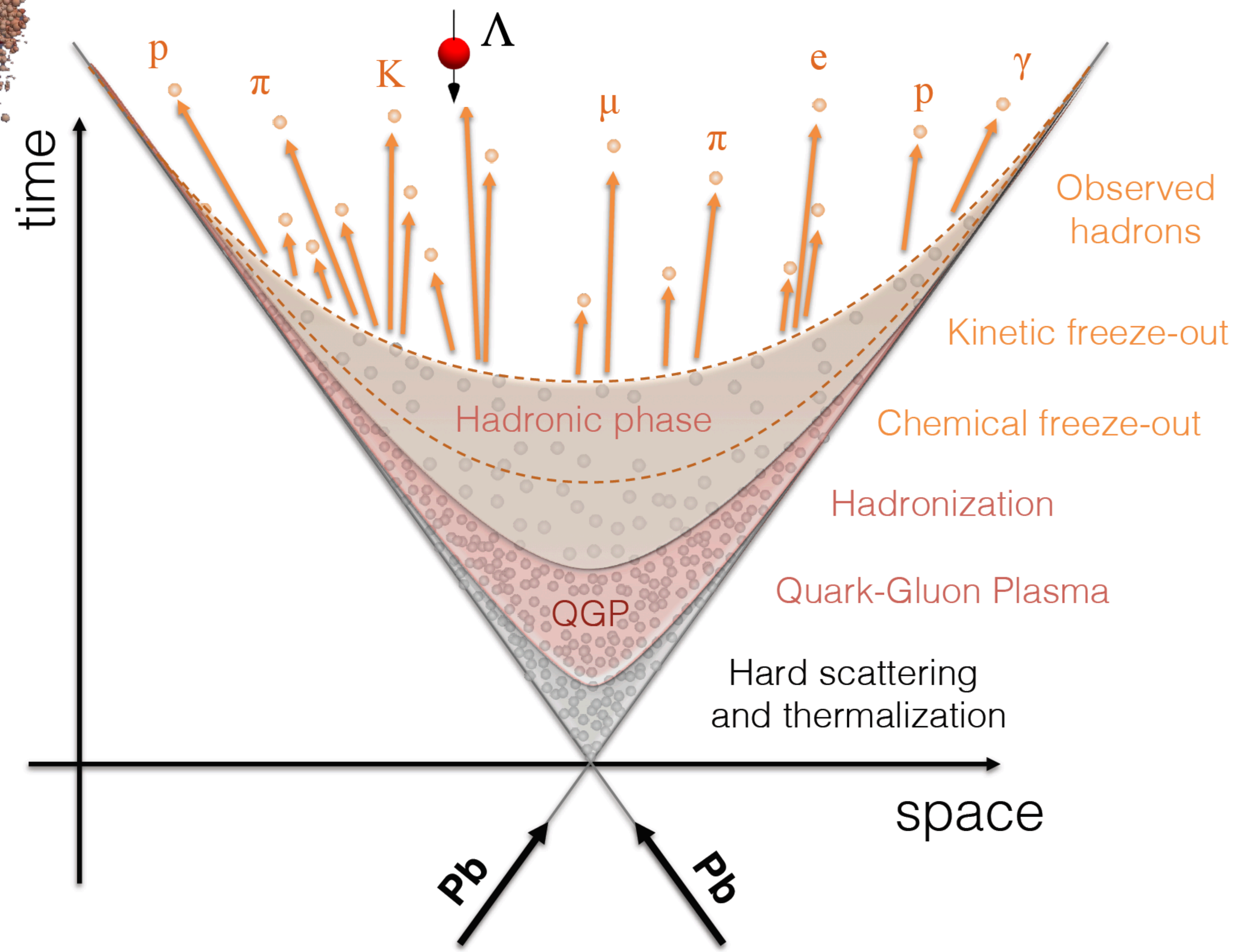


figure: D.D. Chinellato

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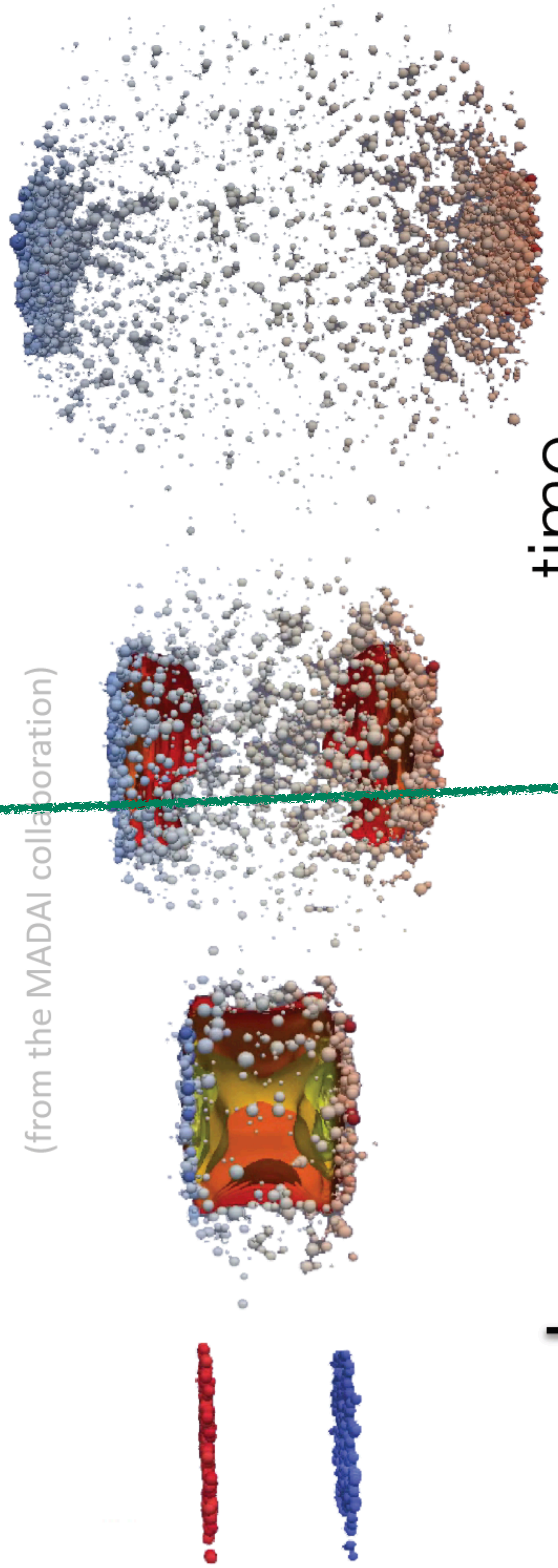

(from the MADAI collaboration)

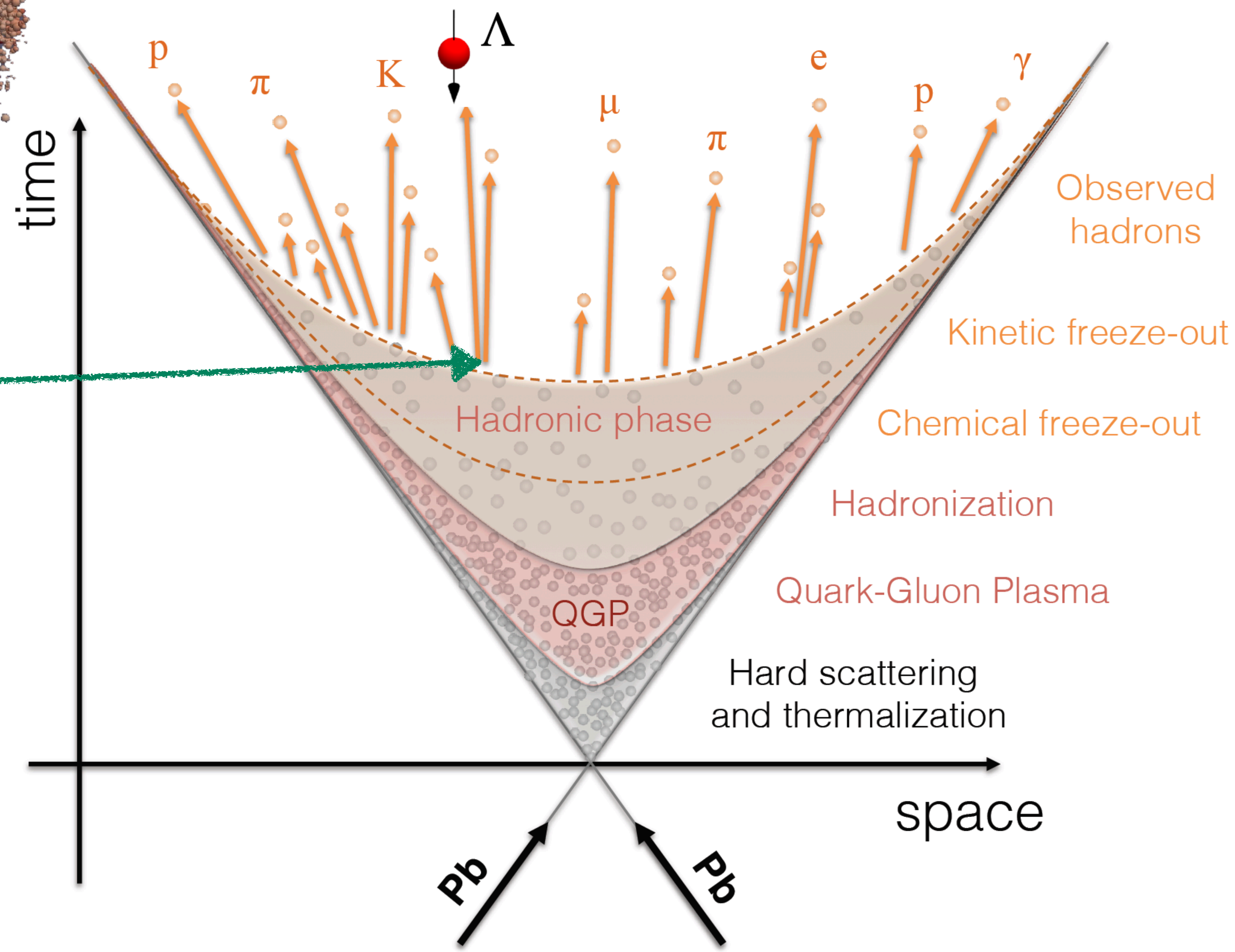

figure: D.D. Chinellato

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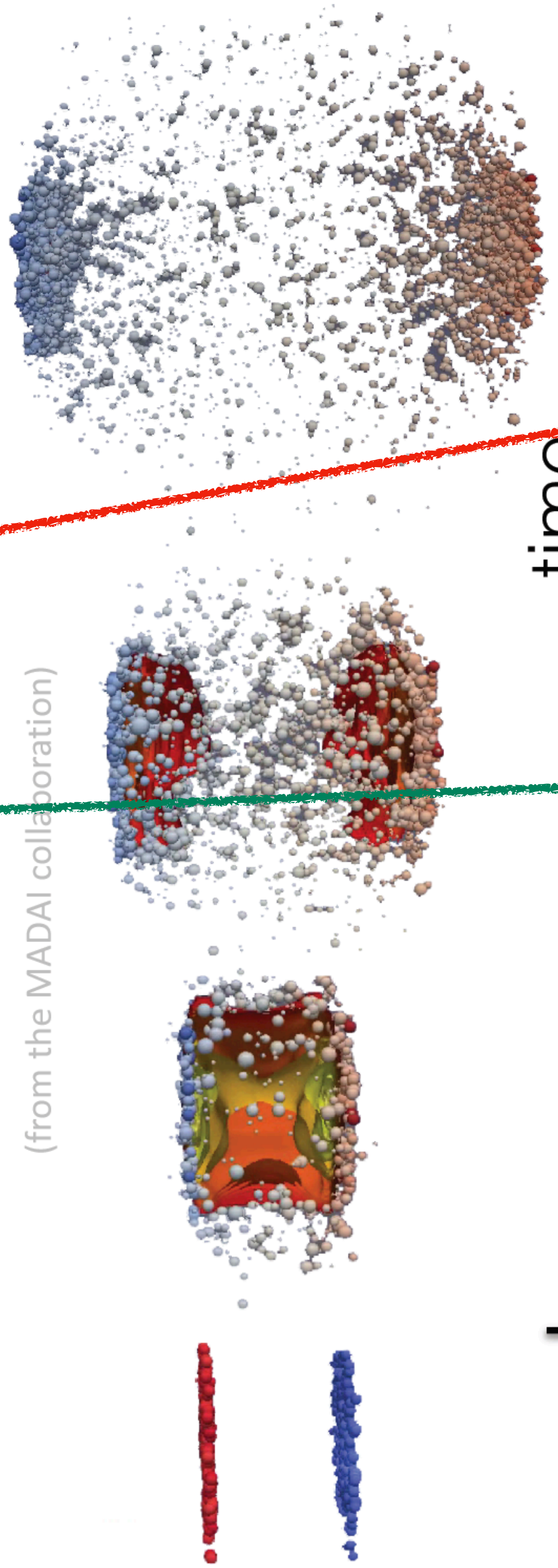


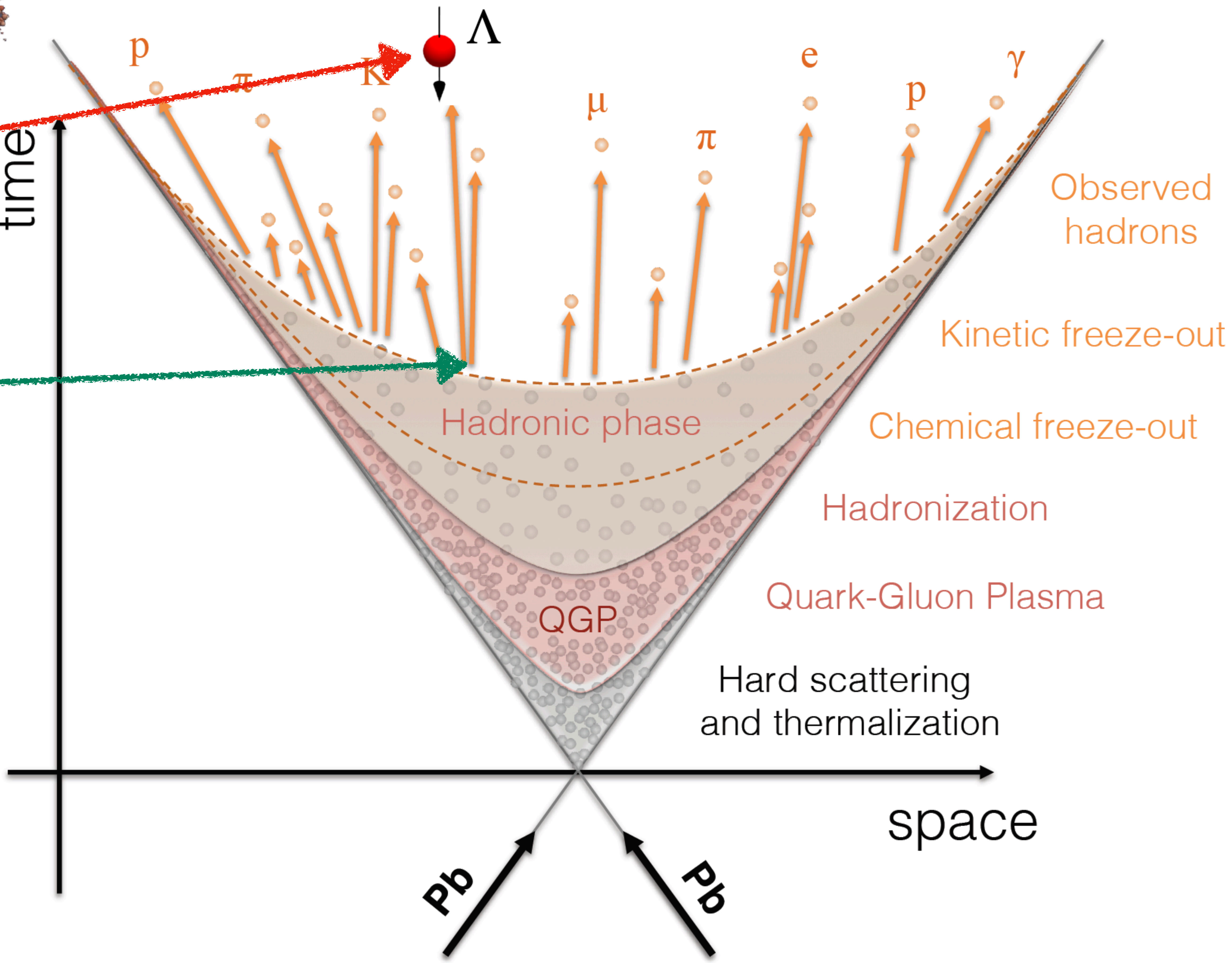


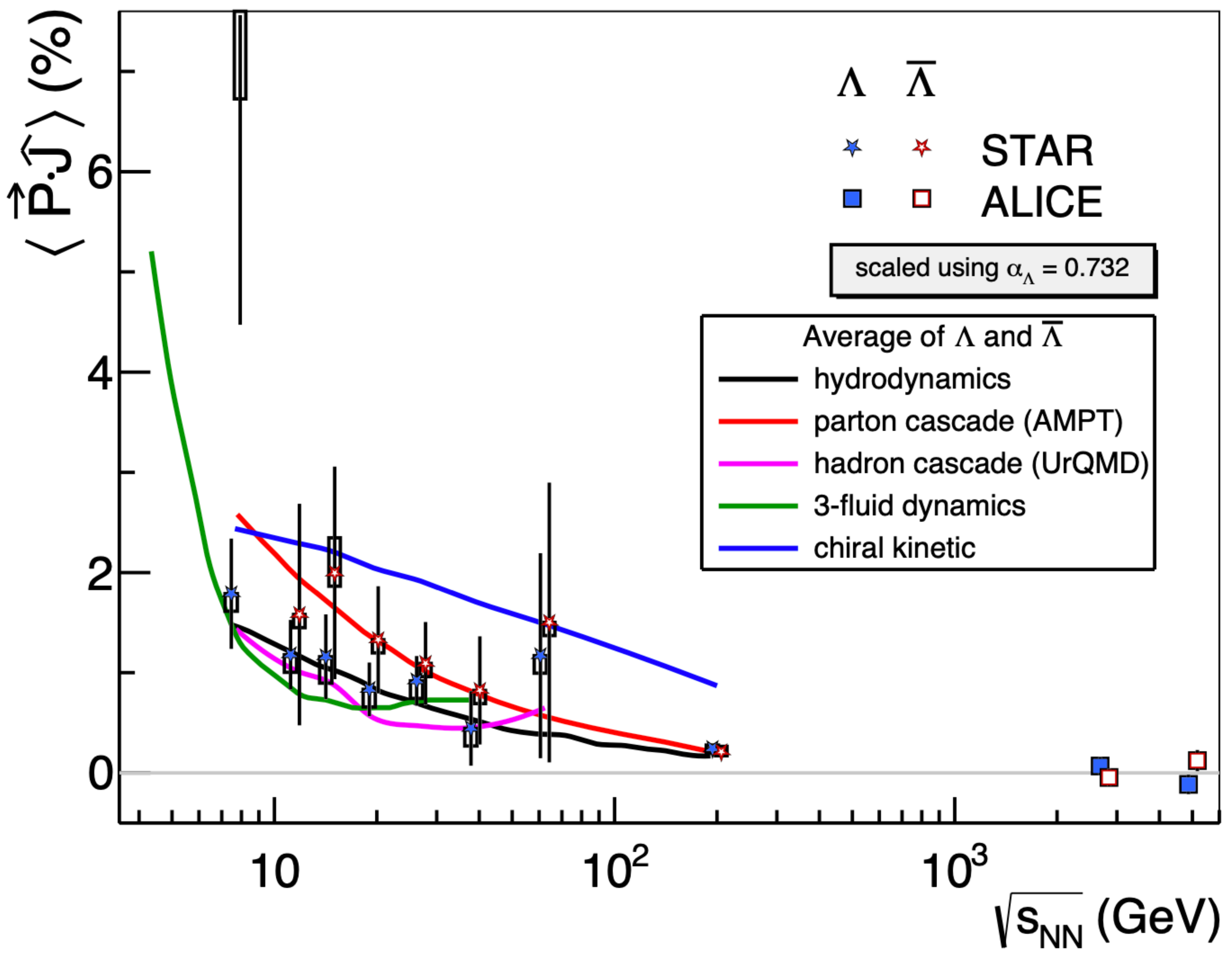
figure: D.D. Chinellato

MEASUREMENT VS SPIN-THERMAL APPROACH: GLOBAL POLARIZATION

☺

Global polarization data supports
the spin-thermal approach

F. Becattini, J. Liao, M. Lisa Lect.Notes Phys. 987 (2021) 1-14

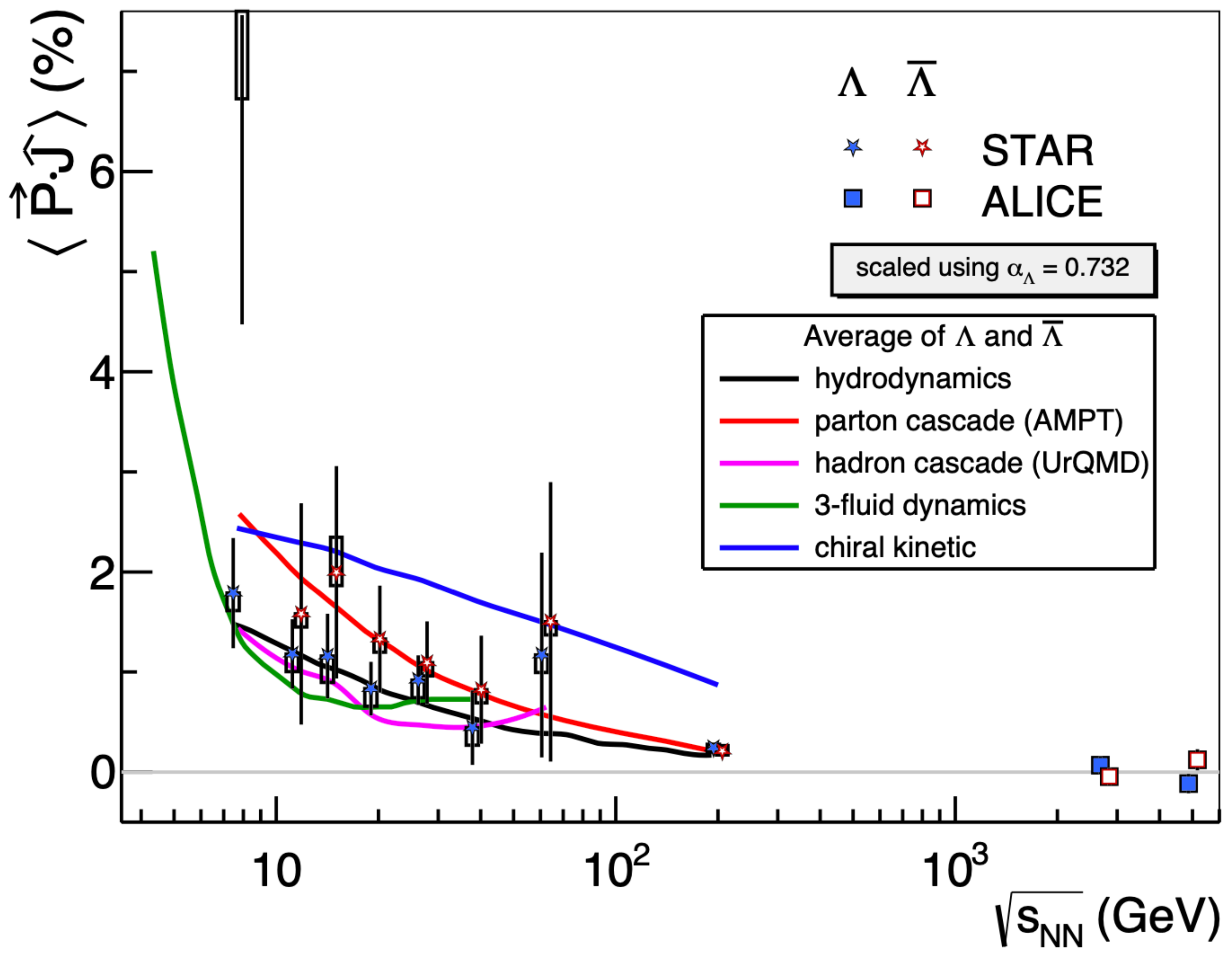


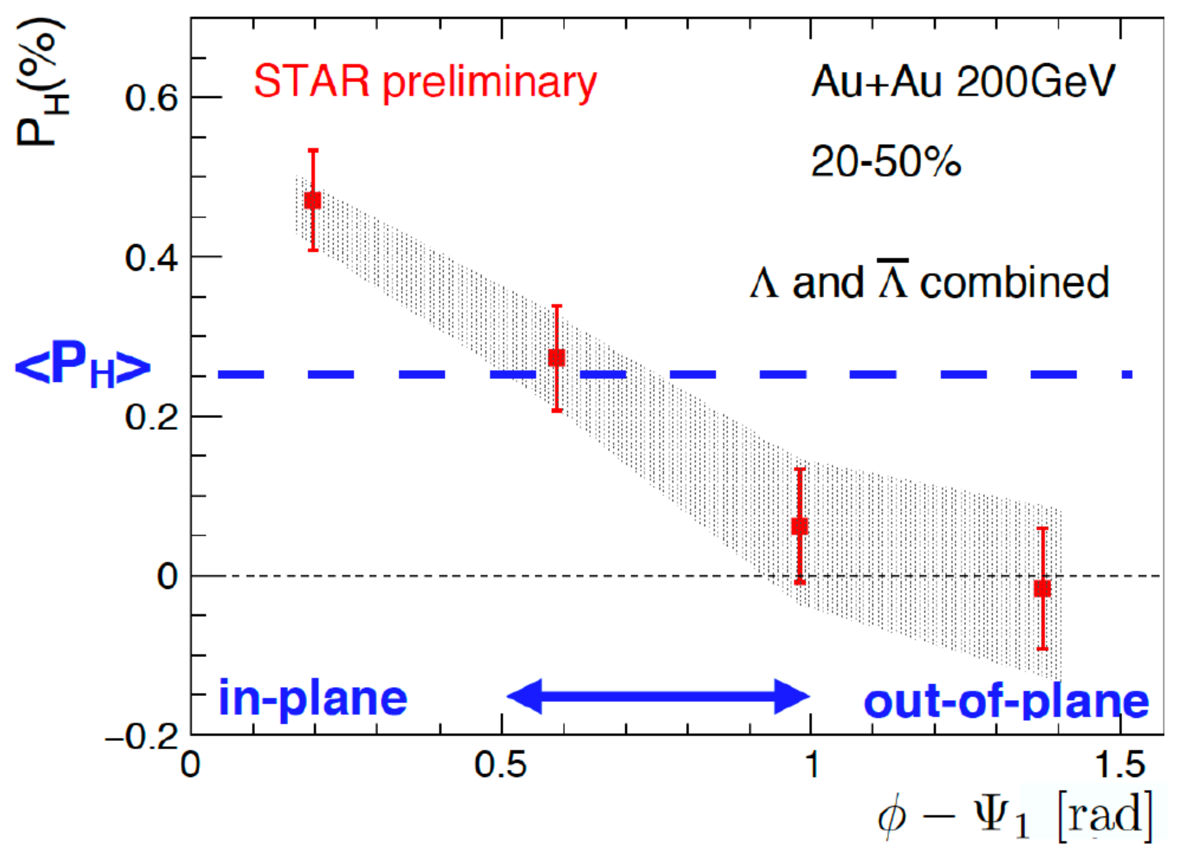
MEASUREMENT VS SPIN-THERMAL APPROACH: GLOBAL POLARIZATION

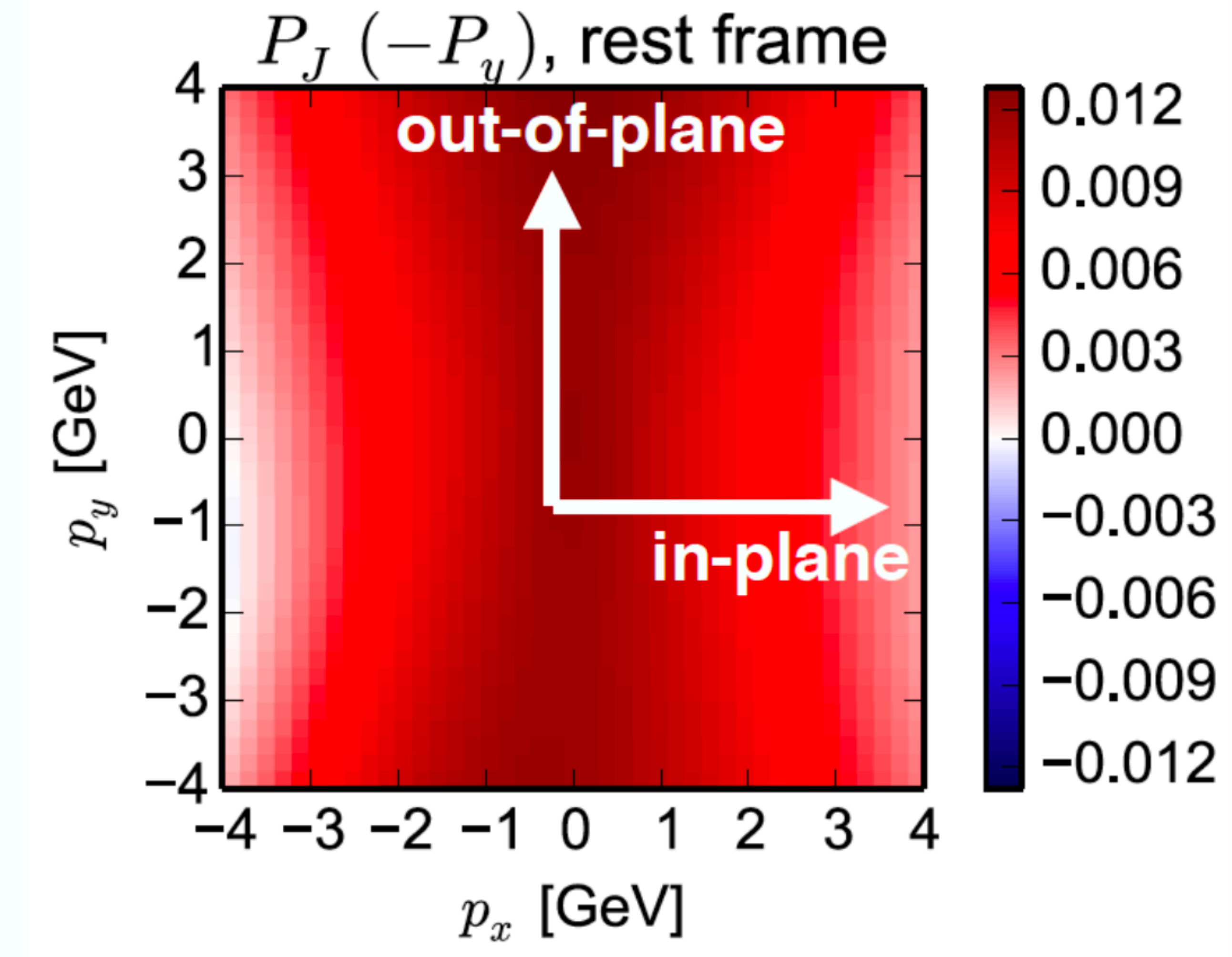
Global polarization data supports the spin-thermal approach

Azimuthal modulation is not captured

F. Becattini, J. Liao, M. Lisa Lect.Notes Phys. 987 (2021) 1-14



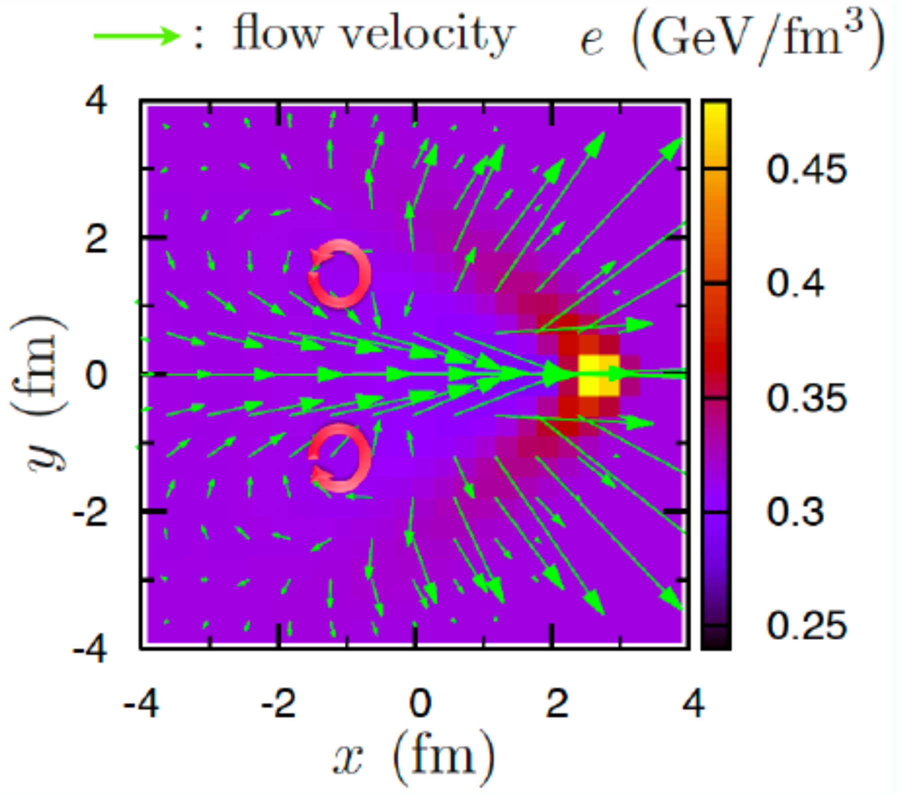




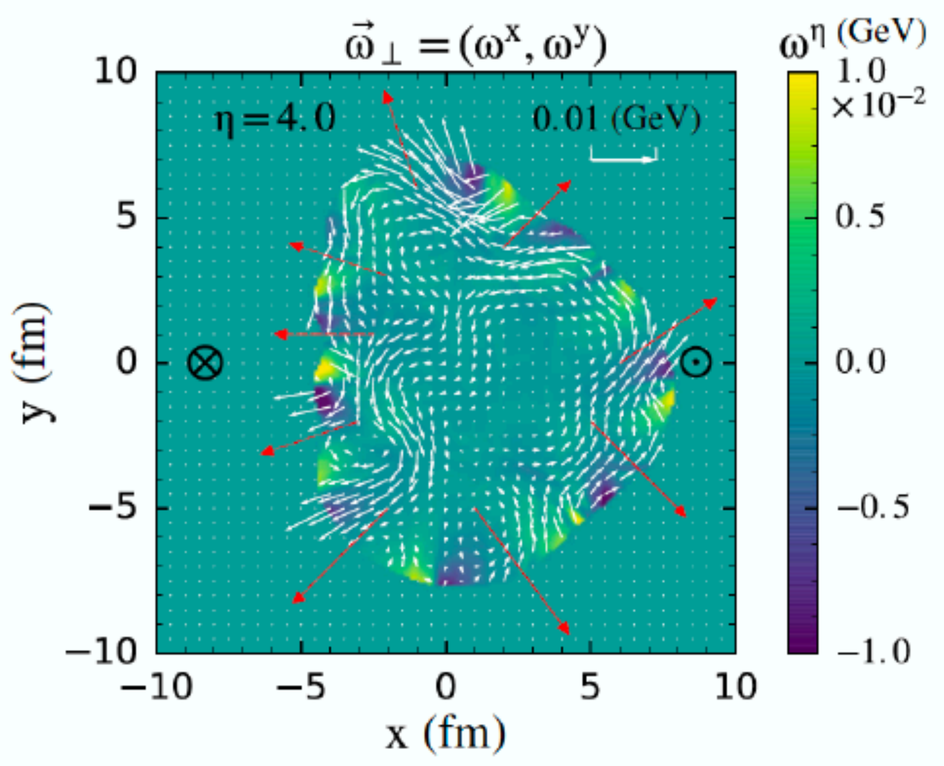
Credit: T.Niida, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 2019

I. Karpenko, F. Becattini, EPJC 77, 213 (2017)

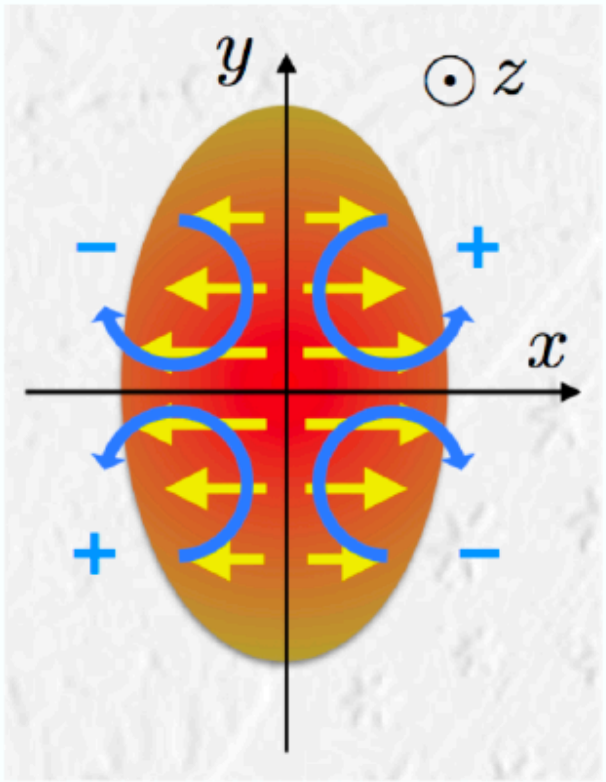
LONGITUDINAL (BEAM-AXIS) POLARIZATION



Y. Tachibana and T. Hirano,
NPA904-905 (2013) 1023

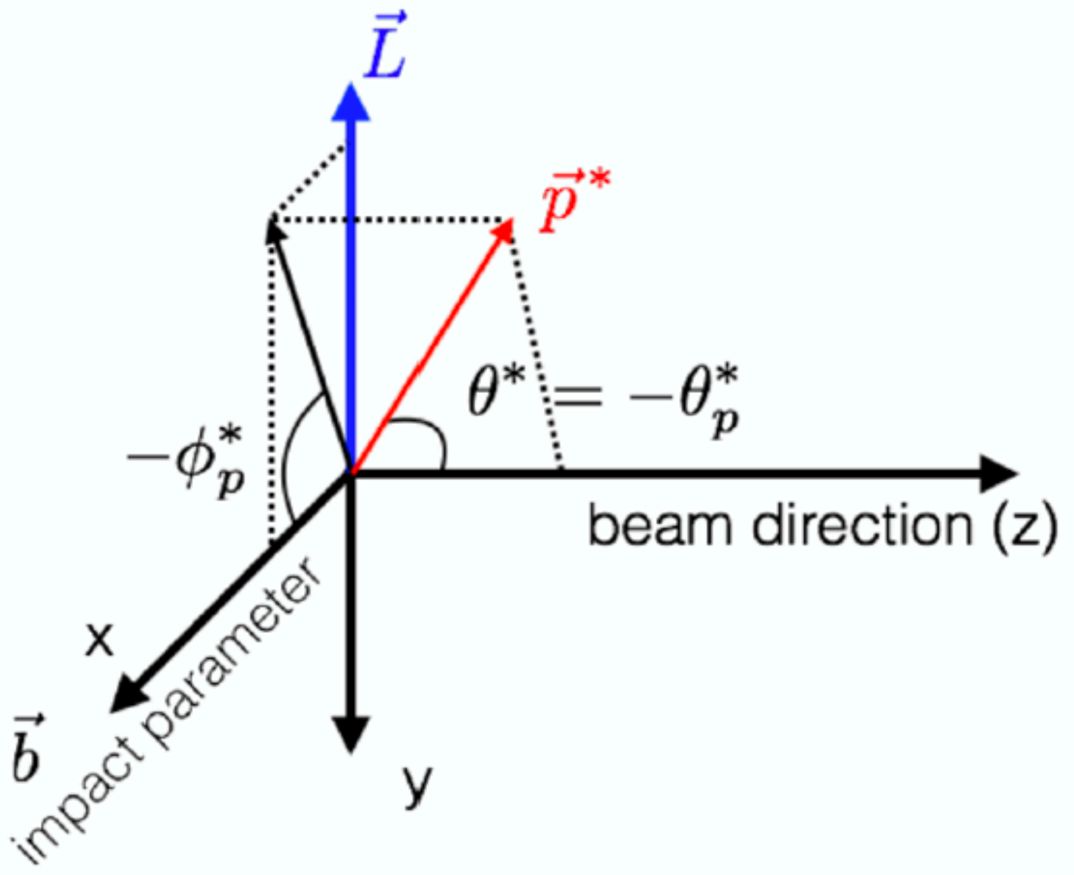


L.-G. Pang, H. Peterson, Q. Wang, and X.-N. Wang
PRL117, 192301 (2016)



F. Becattini and I. Karpenko, PRL120.012302 (2018)
S. Voloshin, EPJ Web Conf.171, 07002 (2018)

Flow structures in the plane transverse to beam (jet, ebe fluctuations etc.) may generate longitudinal polarization



$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\rm H}\mathbf{P_H}\cdot\mathbf{p}^*_p)$$

$$\langle\cos\theta^*_p\rangle = \int \frac{dN}{d\Omega^*}\cos\theta^*_p d\Omega^*$$

$$= \alpha_{\rm H} P_z \langle(\cos\theta^*_p)^2\rangle$$

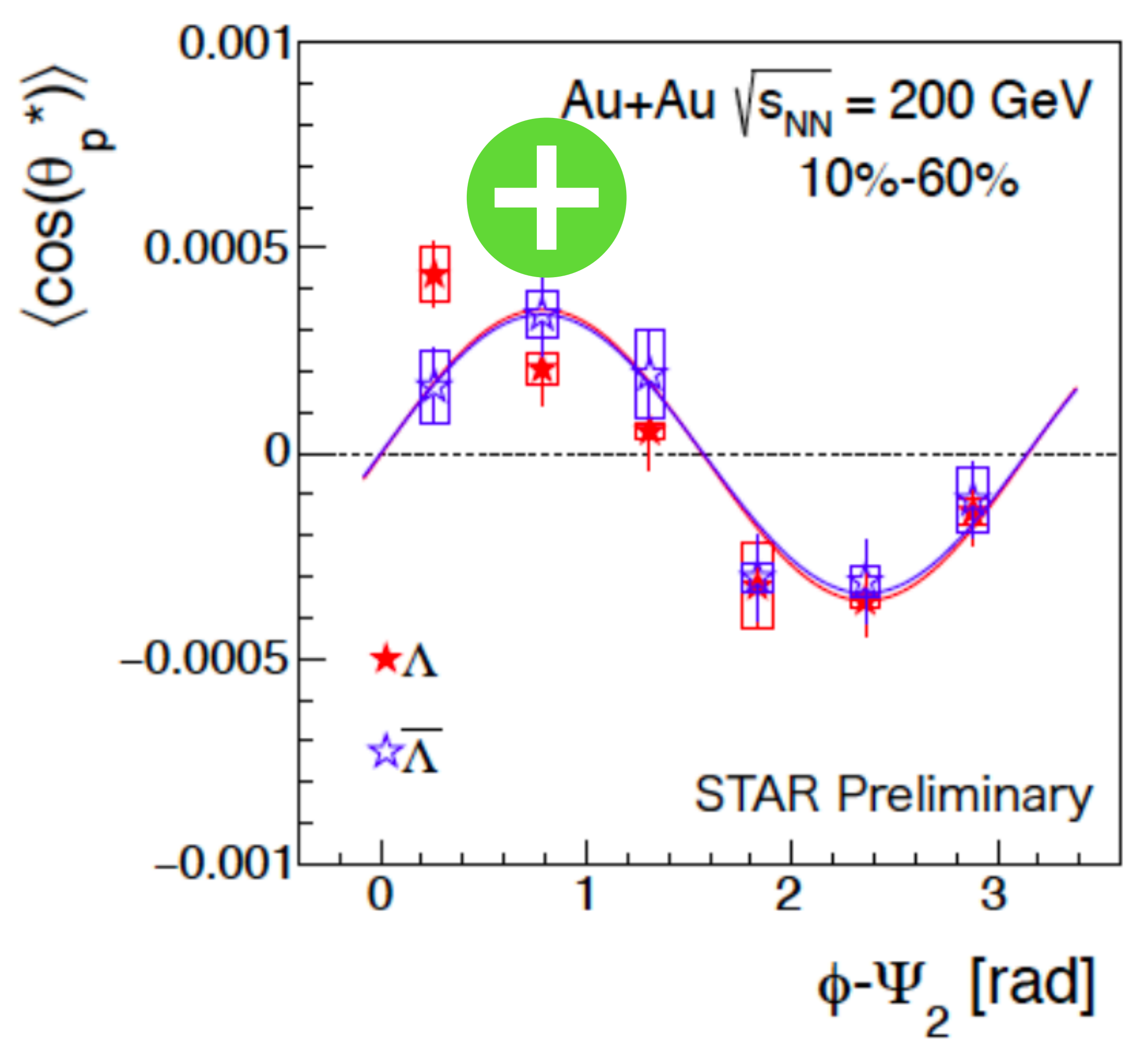
$$\therefore\ P_z = \frac{\langle\cos\theta^*_p\rangle}{\alpha_{\rm H}\langle(\cos\theta^*_p)^2\rangle}$$

$$= \frac{3\langle\cos\theta^*_p\rangle}{\alpha_{\rm H}} \quad \text{(if perfect detector)}$$

$\alpha_{\rm H}$: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE

Experiment



T. Niida, NPA 982 (2019) 511514

LONGITUDINAL POLARIZATION – 'SPIN SIGN' PUZZLE

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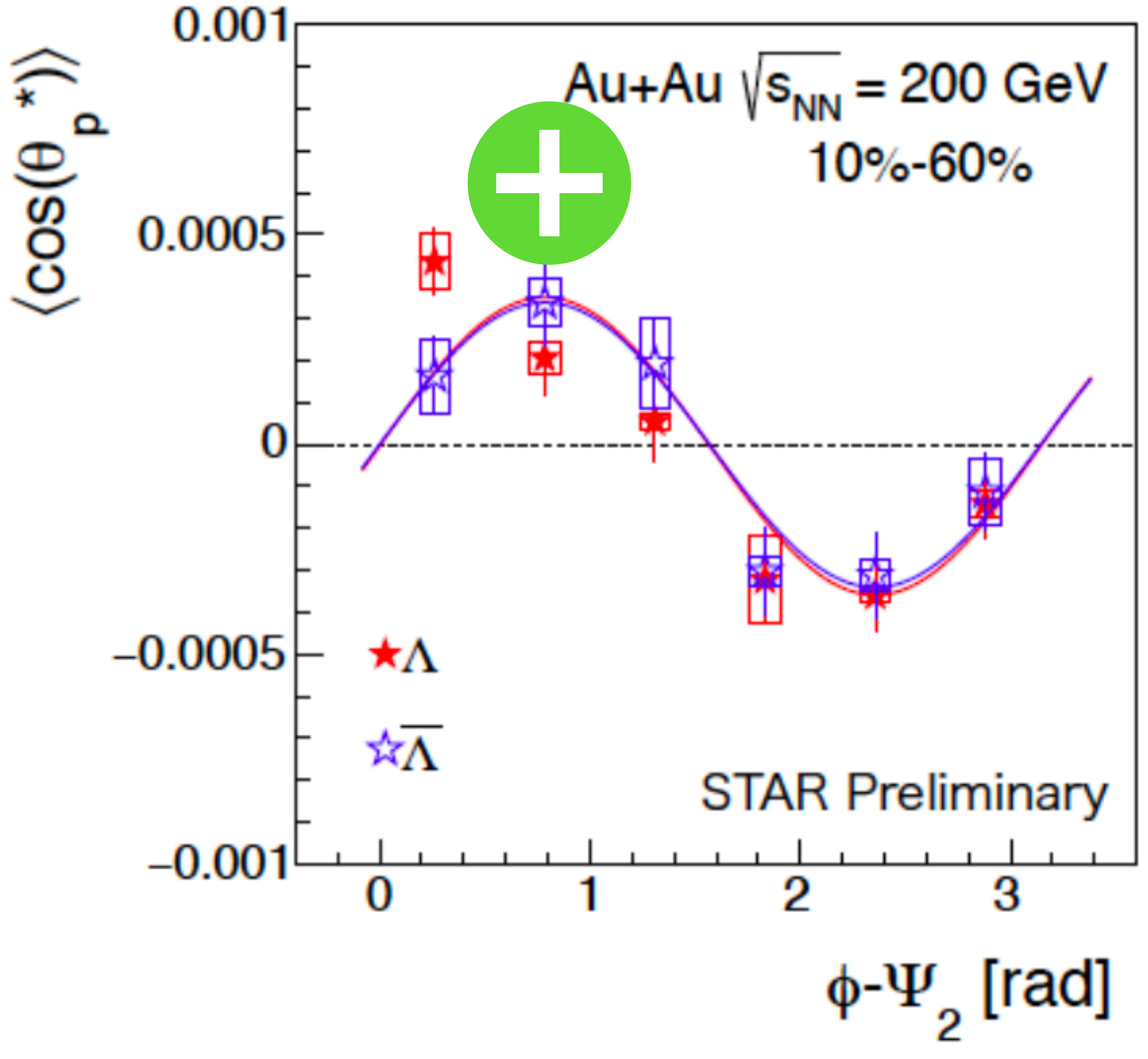
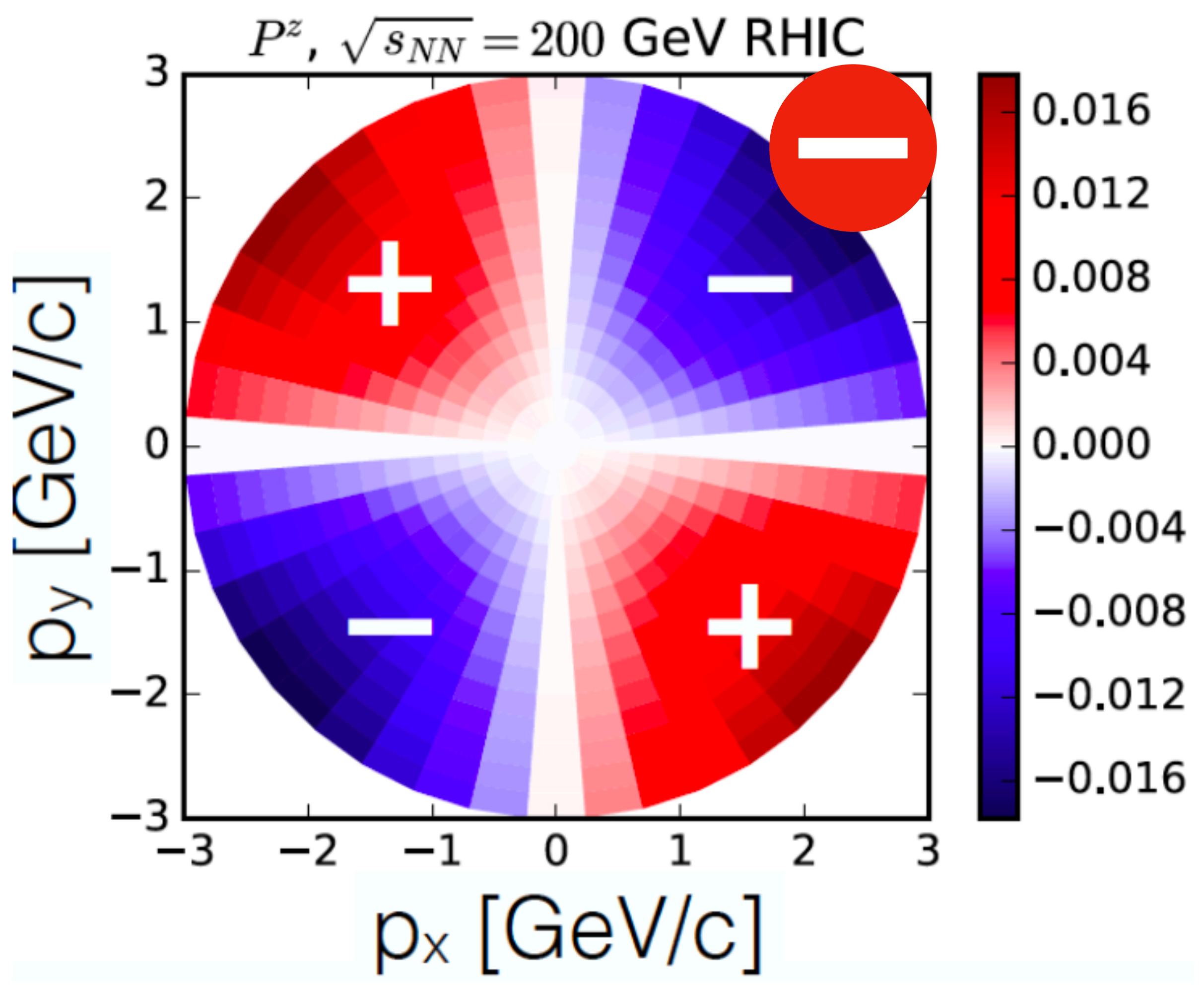
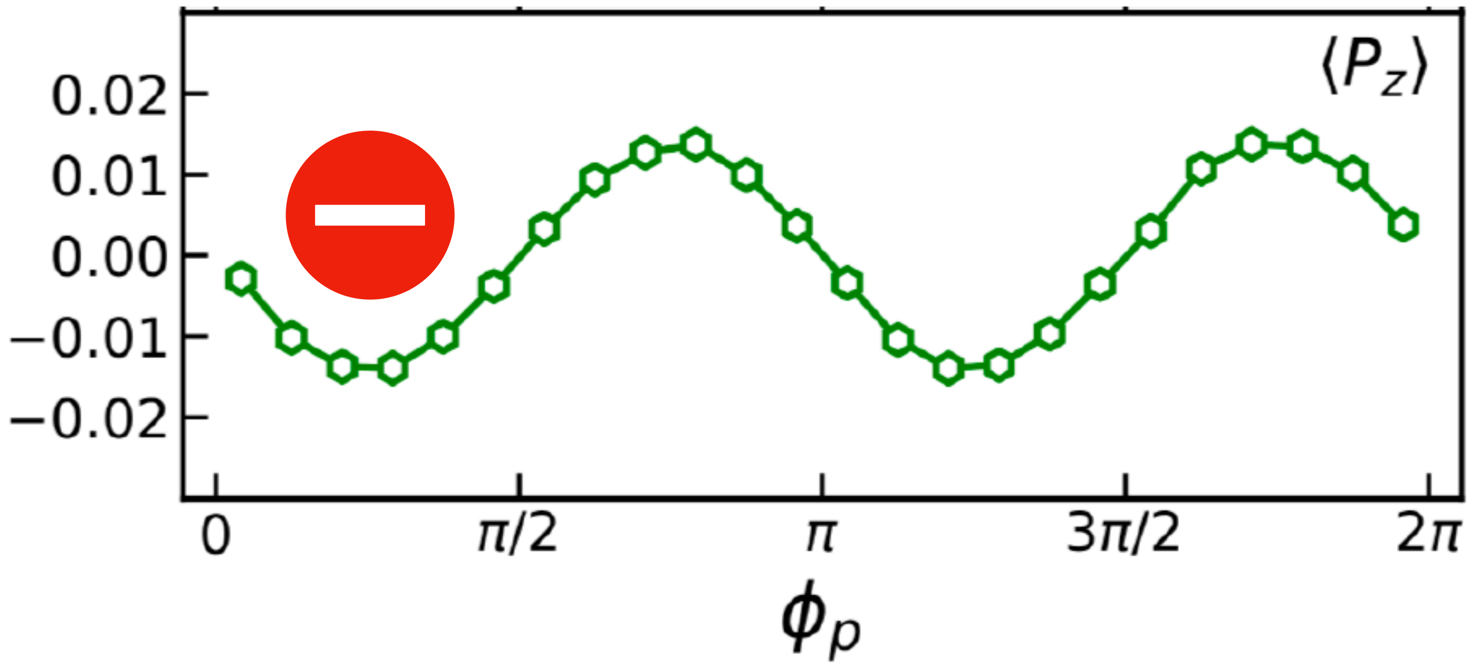


T. Niida, NPA 982 (2019) 511514



UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,



AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

Theory

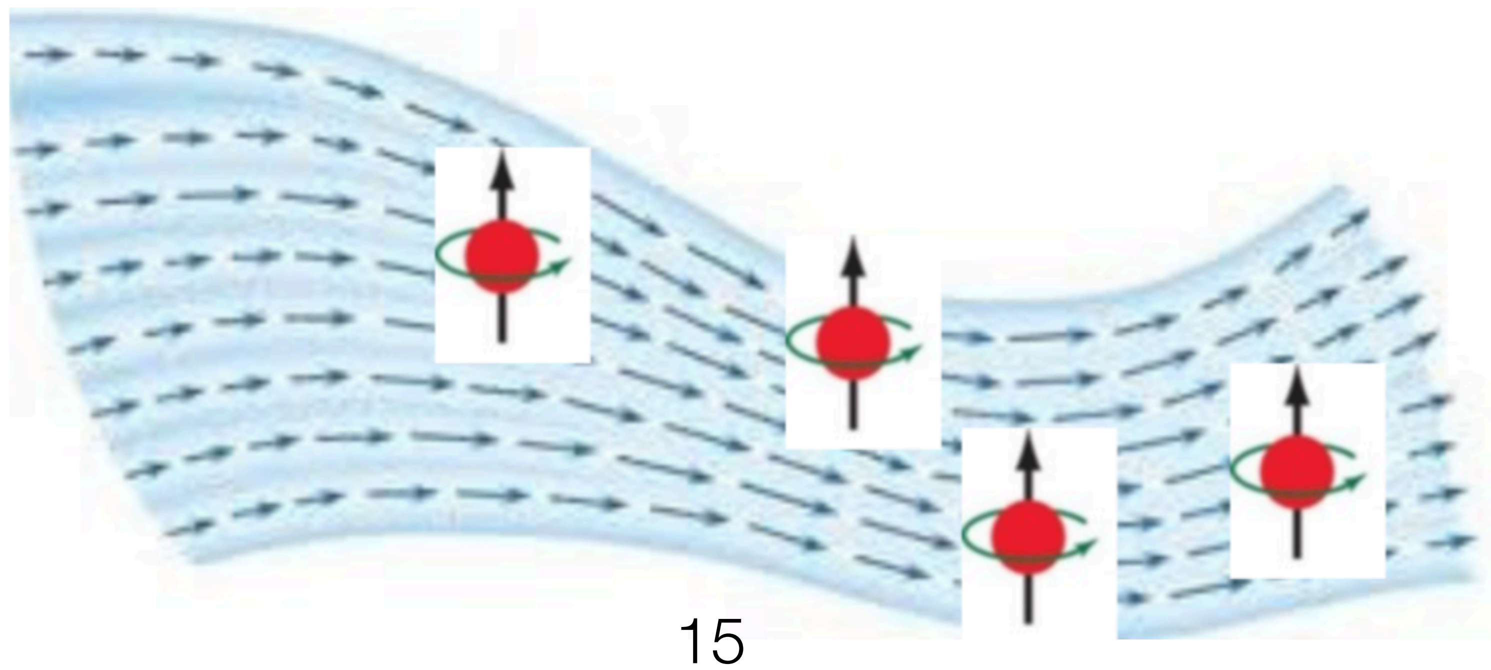
FLUID DYNAMICS OF SPIN?!

Spin-thermal approach does not capture properly phenomena seen in experiment.

If spin polarization is trully hydrodynamic quantity it should not be enslaved to thermal vorticity.

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901

Fluid dynamics with spin



CONSERVED CURRENTS IN QUANTUM FIELD THEORY

Noether's theorem:

for each continuous symmetry of the action there is a corresponding **conserved (canonical) current**

S. Weinberg, The Quantum Theory Of Fields Vol. 1 Cambridge University Press (1995)

- invariance under space-time translation ➡ conservation of energy and linear momentum

$$\widehat{T}_C^{\mu\nu} = \sum_a \frac{\partial \mathcal{L}}{\partial(\partial_\mu \widehat{\psi}^a)} \partial^\nu \widehat{\psi}^a - g^{\mu\nu} \mathcal{L}$$

$$\nabla_\mu \widehat{T}_C^{\mu\nu} = 0,$$

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$$\nabla_\mu \widehat{T}_C^{\mu\nu} = 0,$$

- invariance under rotations and boosts ➡ conservation of total angular momentum

$$\widehat{\mathcal{J}}_C^{\mu,\lambda\nu} = \underbrace{x^\lambda \widehat{T}_C^{\mu\nu} - x^\nu \widehat{T}_C^{\mu\lambda}}_{\text{orbital part}} + \underbrace{\widehat{\mathcal{S}}_C^{\mu,\lambda\nu}}_{\text{spin part}}$$

$$\nabla_\mu \widehat{\mathcal{J}}_C^{\mu,\lambda\nu} = \underbrace{\widehat{T}_C^{\lambda\nu} - \widehat{T}_C^{\nu\lambda}}_{\text{asymmetric part}} + \nabla_\mu \widehat{\mathcal{S}}_C^{\mu,\lambda\nu} = 0$$

$$\widehat{\mathcal{S}}_C^{\mu,\lambda\nu} = -i \sum_{a,b} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \widehat{\psi}^a)} D(J^{\lambda\nu})^a_b \widehat{\psi}^b$$

spin is sourced by antisymmetric part of stress-energy tensor

CONSERVATION LAWS AND LAGRANGE MULTIPLIERS

conservation laws + local equilibrium ➡ hydrodynamics

- conservation of charge (baryon number, electric charge, …)

$$\partial_\mu N^\mu(x) = 0 \quad \rightarrow \quad \mu \equiv \xi T$$

(1 eq / charge)

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \rightarrow \quad T, u^\nu$$

(4 eqs)

- conservation of angular momentum

$$\partial_\lambda J^{\lambda\mu\nu}(x) = 0 \quad \rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu}$$

(6 eqs)

spin chemical potential

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, Phys. Rev. D97 (2018) 116017
F. Becattini, W. Florkowski, E. Speranza, Phys. Lett. B 789 (2019) 419-425

CONSERVED CURRENTS AND CONSTITUTIVE RELATIONS

If the **energy-momentum tensor is symmetric** the spin tensor is conserved

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (2018) 116017
F. Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419-425
W. Florkowski, A. Kumar, R. R., PPNP 108 (2019) 103709

$$\nabla_\mu \widehat{\mathcal{J}}_C^{\mu,\lambda\nu} = \widehat{T}_C^{\lambda\nu} - \widehat{T}_C^{\nu\lambda} + \nabla_\mu \widehat{\mathcal{S}}_C^{\mu,\lambda\nu} = 0$$

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

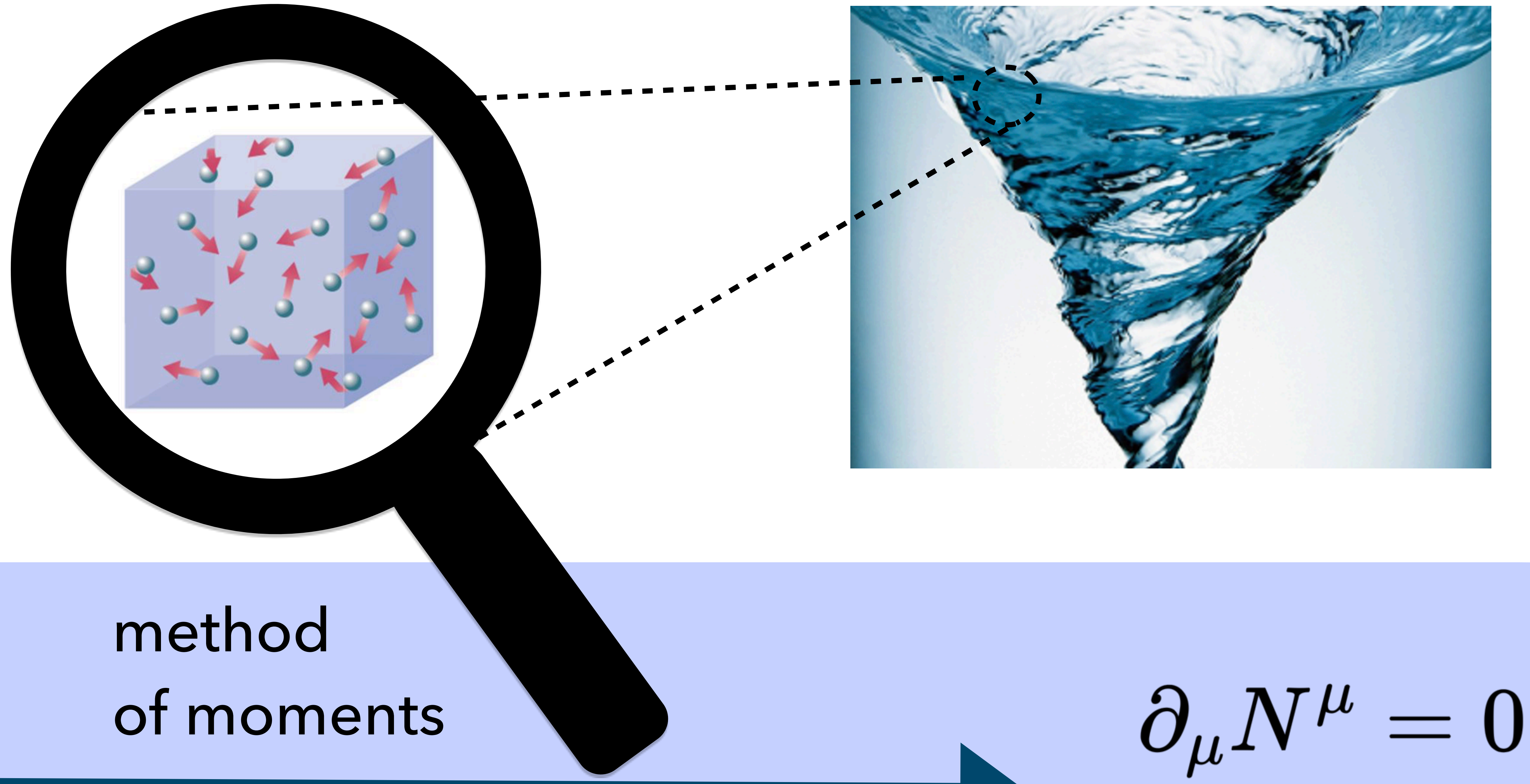
What are the **constitutive relations** which enter **equations of motion**?

$$N^\mu = N^\mu[T,\mu,\omega] \quad T^{\mu\nu} = T^{\mu\nu}[T,\mu,\omega], \quad S^{\lambda\mu\nu} = S^{\lambda\mu\nu}[T,\mu,\omega]$$

Coarse-graining of underlying microscopic theory is required!

RELATIVISTIC KINETIC THEORY DERIVATION OF SPIN HYDRODYNAMICS

For dilute systems, the derivation of fluid dynamics can be done starting from the underlying kinetic theory



classical RKT

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)]$$

method of moments

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

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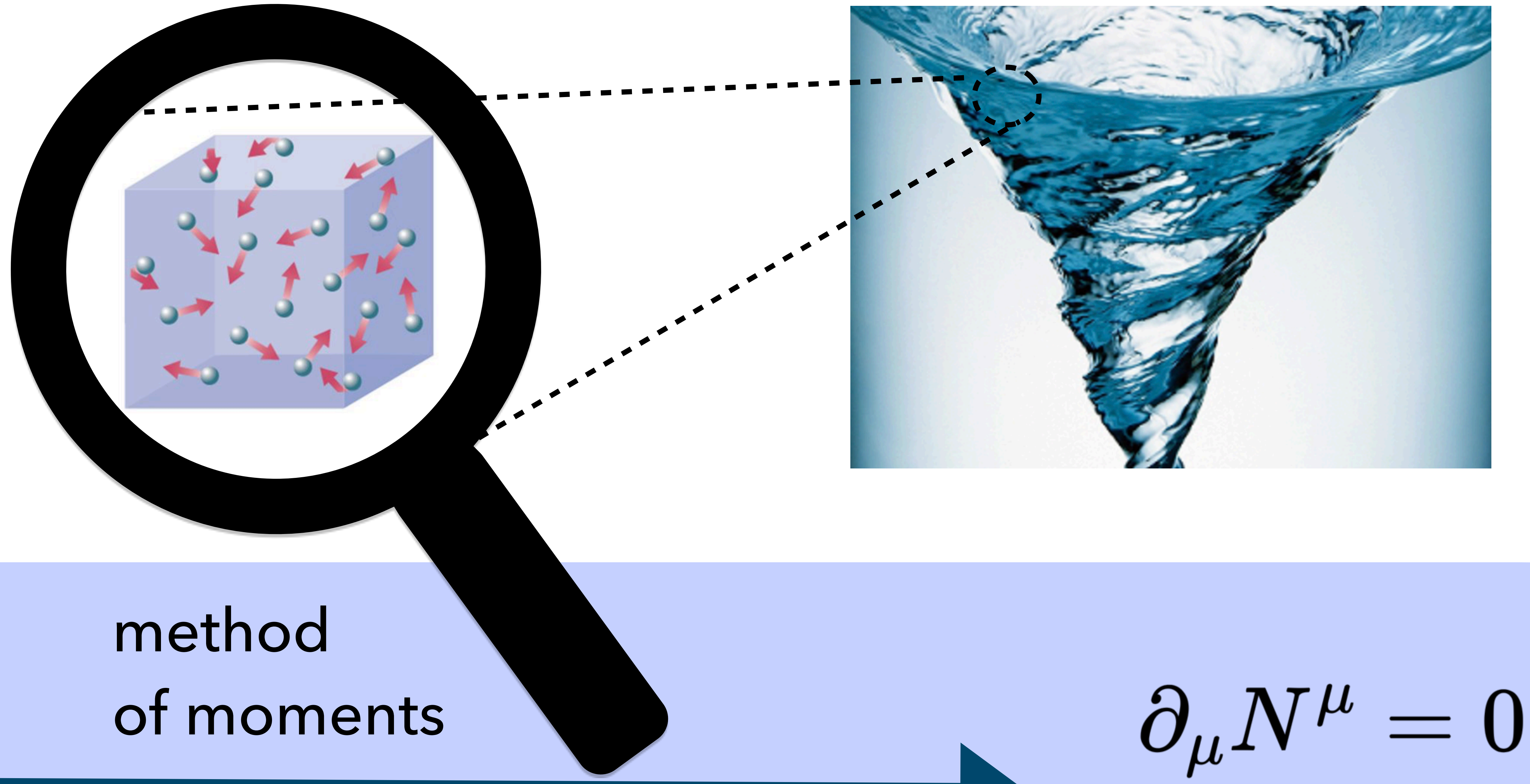

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quantum field theory

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

RELATIVISTIC KINETIC THEORY WITH SPIN

To include spin in RKT, we start from the **Wigner function (WF)** that bridges the gap between QFT and RKT

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta} \qquad \Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$$

For spin-1/2 particles the WF satisfies the **quantum kinetic equation**

D. Vasak, M. Gyulassy, H.T. Elze, Ann. Phys. 173 (1987) 462–492,

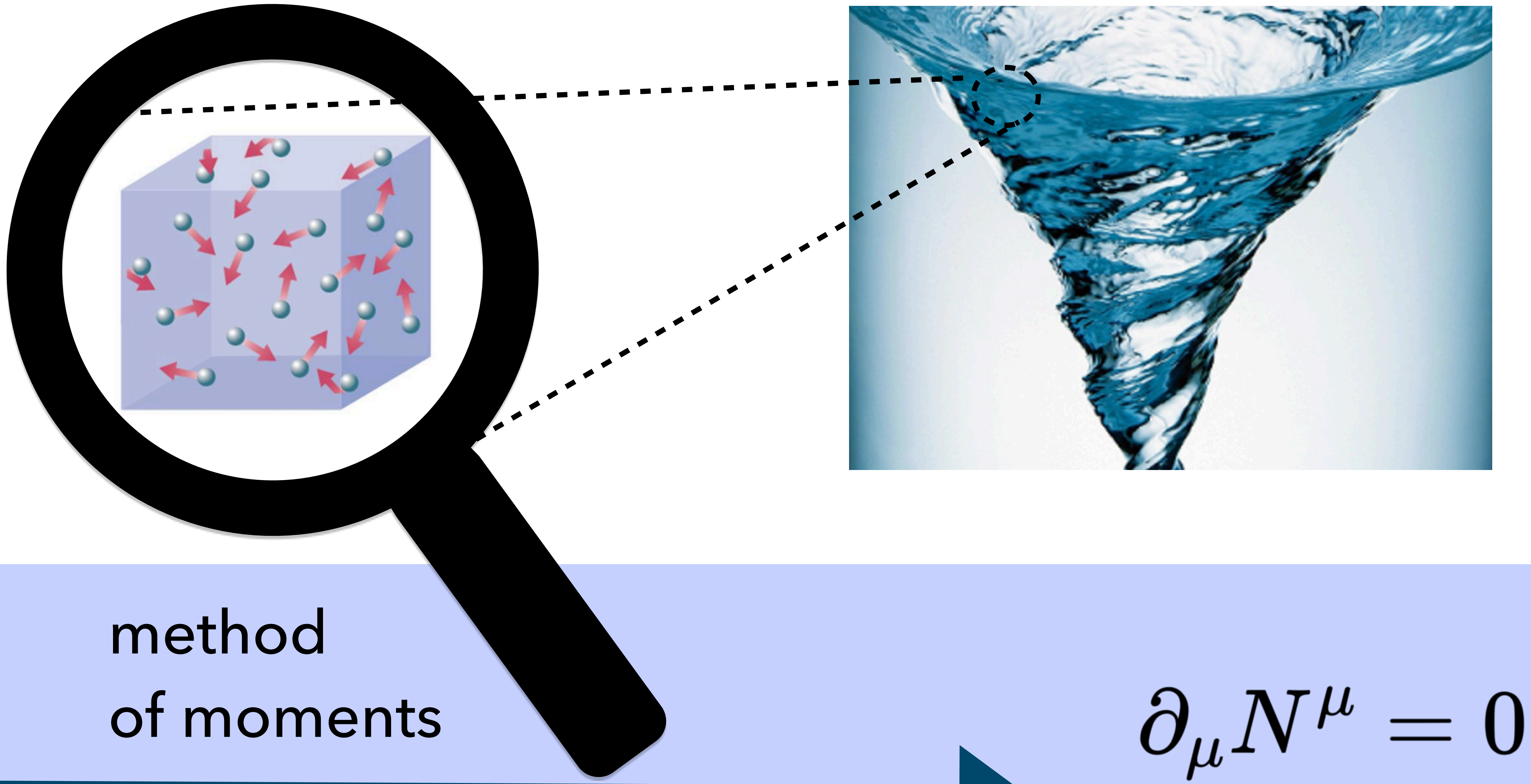
$$\left[\gamma\cdot\left(p + \frac{i}{2}\partial\right) - m\right]\mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

From the **LO and NLO of the semi-classical expansion of the WF in powers of Planck's constant,** one obtains two independent kinetic equations

$k^\mu\partial_\mu\mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$	$k^\mu\partial_\mu\mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$

RELATIVISTIC KINETIC THEORY DERIVATION OF SPIN HYDRODYNAMICS

For dilute systems, the derivation of fluid dynamics can be done starting from the underlying kinetic theory



classical RKT

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)]$$

method of moments →

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

quantum RKT

$$(\gamma_\mu K^\mu - m)\mathscr{W}(x,k) = C[\mathscr{W}(x,k)]$$

semi-classical expansion →

$$k^\mu \partial_\mu \mathscr{F}_{\text{eq}}(x,k) = 0$$
$$k^\mu \partial_\mu \mathscr{A}^\nu_{\text{eq}}(x,k) = 0$$

method of moments →

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

CLASSICAL APPROACH TO SPIN HYDRODYNAMICS

In the classical treatments of particles with spin-half one introduces **internal angular momentum tensor of particles**

M. Mathisson, APPB 6 (1937) 163-2900
W. Florkowski, A. Kumar, R. R., PPNP 108 (2019) 103709

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta}p_\gamma s_\delta$$

Satisfies Frenkel condition

$$p_\alpha s^{\alpha\beta} = 0$$

In particle rest frame (PRF)

$$p^\mu = (m, 0, 0, 0),\, s^\alpha = (0, \mathbf{s}_*)$$

$$-s^2 = -s^\alpha s_\alpha = |\mathbf{s}_*|^2 = \mathfrak{z}^2 = \tfrac{1}{2}\left(1 + \tfrac{1}{2}\right) = \tfrac{3}{4}$$

CLASSICAL APPROACH TO SPIN HYDRODYNAMICS

scalar	axial vector
$k^\mu \partial_\mu \mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$	$k^\mu \partial_\mu \mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$
$C_\mathcal{F} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{F}_{\rm eq}(x,k) - \mathcal{F}(x,k)\Big]$	$C^\nu_\mathcal{A} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{A}^\nu_{\rm eq}(x,k) - \mathcal{A}^\nu(x,k)\Big]$
$\mathcal{F}^\pm(x,k) = 2m \int_{p,s} f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$	$\mathcal{A}^\mu_\pm(x,k) = 2m \int_{p,s} s^\mu f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$

Momentum measure $\rightarrow \quad \int_p(\cdots) \rightarrow \int d^3p/\,(2\pi)^3\,p^0.$

Spin measure $\rightarrow \quad \int_s(\cdots) \rightarrow (m/\pi\mathfrak{s}) \int d^4s\,\delta(s\cdot s + \mathfrak{s}^2)\delta(p\cdot s).$

$\mathfrak{s}^2 = \frac{1}{2}(1+\frac{1}{2})$

RELATIVISTIC KINETIC THEORY WITH SPIN

The **distribution function** in the **extended phase-space** is a function of **spacetime**, **momentum**, and **internal angular momentum** of the particles

$$f^{\pm}(x,p,s) \qquad x \equiv x^{\mu} \quad p \equiv p^{\mu} \quad s \equiv s^{\mu\nu}$$

The **kinetic equation (KE)** governing the evolution of the distribution function can be written as

W. G. Dixon, Nuovo Cimento (1955–1965) 34, 317 (1964).
L. Suttorp and S. De Groot, Il Nuovo Cimento A (1965–1970) 65, 245 (1970)
C.G. van Weert, thesis, The University of Amsterdam, 1970.

$$p^{\mu}\partial_{\mu}^{(x)} f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)} f^{\pm} + m\mathcal{S}^{\mu\nu}\partial_{\mu\nu}^{(s)} f^{\pm} = \mathcal{C}[f^{\pm}]$$

where

$$\partial_{\mu}^{(x)} \equiv \frac{\partial}{\partial x^{\mu}}, \qquad \partial_{\mu}^{(p)} \equiv \frac{\partial}{\partial p^{\mu}}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}}, \qquad \mathcal{F}^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau} \qquad \mathcal{S}^{\alpha\beta} \equiv \frac{ds^{\alpha\beta}}{d\tau}$$

Using the Frenkel condition, one can derive the **force (Lorentz and Mathisson)** and **torque**

I. Bailey and W. Israel, Commun. Math. Phys. 42, 65 (1975).

$$\mathcal{F}^{\alpha} = \frac{\mathfrak{q}}{m} F^{\alpha\beta} p_{\beta} + \frac{1}{2}\left(\partial^{\alpha} F^{\beta\gamma}\right) m_{\beta\gamma} \qquad \mathcal{S}^{\alpha\beta} = 2\,F^{\gamma[\alpha}\, m^{\beta]}{}_{\gamma} - \frac{1}{m^2}\left(\chi - \frac{\mathfrak{q}}{m}\right) F_{\phi\gamma}\, s^{\phi[\alpha}\, p^{\beta]} p^{\gamma}$$

where **magnetic dipole moment** is $m^{\alpha\beta} = \chi\, s^{\alpha\beta}$

INFINITE CONDUCTIVITY LIMIT

In the **limit of infinite conductivity**, field strength tensor is

$$F^{\mu\nu} \to B^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta$$

$$u_\mu B^\mu = 0 \qquad\qquad B_\mu B^\mu \leq 0$$

If the **medium is magnetizable**, then the Maxwell's equations are given by

$$\partial_\mu H^{\mu\nu} = J^\nu, \qquad\qquad \partial_\mu \widetilde{F}^{\mu\nu} = 0,$$

$$\left(\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\right)$$

$$H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$$

FROM KT TO SPIN MHD

The **particle current**, **energy-momentum tensor**, and **spin tensor** of the fluid can be expressed as

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. R., Phys. Lett. B 814, 136096 (2021); Phys. Rev. D 103, 014030 (2021).

$$N^\mu = \int_{p,s} p^\mu \left(f^+ - f^-\right),$$

$$T_f^{\mu\nu} = \int_{p,s} p^\mu p^\nu \left(f^+ + f^-\right),$$

$$S^{\lambda,\mu\nu} = \int_{p,s} p^\lambda s^{\mu\nu} \left(f^+ + f^-\right)$$

where we use the notation

$$\int_p(\cdots) \to \int d^3p/\,(2\pi)^3\, p^0.$$

$$\int_s(\cdots) \to (m/\pi\mathfrak{s}) \int d^4s\,\delta(s\cdot s + \mathfrak{s}^2)\delta(p\cdot s).$$

$$\mathfrak{s}^2 = \tfrac{1}{2}\left(1 + \tfrac{1}{2}\right)$$

The **polarization-magnetization tensor** is

$$M^{\alpha\beta} = m \int_{p,s} m^{\alpha\beta} \left(f^+ - f^-\right)$$

FROM KT TO SPIN MHD

Assuming that the microscopic interactions **preserve fundamental conservation laws** one requires

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. R., Phys. Lett. B 814, 136096 (2021); Phys. Rev. D 103, 014030 (2021).

$$\int_{p,s} \mathcal{C}\left[f\right] = 0,$$

$$\int_{p,s} p^{\mu}\mathcal{C}\left[f\right] = 0,$$

$$\int_{p,s} s^{\mu\nu}\mathcal{C}\left[f\right] = 0$$

Zeroth, first, and 'spin' moment of the KE (**in absence of the torque**) then lead to equations defining **relativistic magnetohydrodynamics for fluid with spin**

$$p^{\mu}\partial_{\mu}^{(x)} f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)} f^{\pm} = \mathcal{C}[f^{\pm}]$$

➡

$$\partial_{\mu} N^{\mu} = 0$$

$$\partial_{\nu} T_f^{\mu\nu} = F^{\mu}{}_{\alpha} J_f^{\alpha} + \frac{1}{2}\left(\partial^{\mu} F^{\nu\alpha}\right) M_{\nu\alpha}$$

$$\partial_{\lambda} S^{\lambda,\mu\nu} = 0$$

$$J_{\mathrm{f}}^{\mu} = \mathfrak{q} N^{\mu}$$

RELATIVISTIC MHD WITH SPIN IN RTA APPROXIMATION

Kinetic equation with collision kernel in the **relaxation-time approximation (RTA)** reads

J. L. Anderson and H. Witting, Physica (Utrecht) 74, 466 (1974)

$$p^\mu \partial_\mu^{(x)} f^\pm + m\mathcal{F}^\mu \partial_\mu^{(p)} f^\pm = -\frac{(u\cdot p)}{\tau_R}\delta f^\pm$$

$$\delta f^\pm(x,p,s) = f^\pm(x,p,s) - f^\pm_{\rm eq}(x,p,s)$$

Using RTA kinetic equation we can write the **first-order gradient correction** as

$$p^\mu \partial_\mu^{(x)} f^\pm + m\mathcal{F}^\mu \partial_\mu^{(p)} f^\pm = -\frac{(u\cdot p)}{\tau_R}\delta f^\pm \quad \Rightarrow \quad \delta f^\pm_{(1)} = -\mathcal{D} f^\pm_{\rm eq},$$

$$\mathcal{D} = \frac{\tau_R}{(u\cdot p)}\left(p^\alpha \frac{\partial}{\partial x^\alpha} + \mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha}\right)$$

The **equilbrium distribution function** has the form

$$f^\pm_{\rm eq} = \frac{1}{1+\exp\left[\beta(u\cdot p) \mp \xi - \frac{1}{2}\omega : s\right]} \quad \Rightarrow \quad f_{\rm eq} = f_0 + \frac{1}{2}(\omega : s)\, f_0 \tilde{f}_0,$$

$$f_0 \equiv \{1+\exp\left[\beta(u\cdot p) - \xi\right]\}^{-1} \qquad \tilde{f}_0 \equiv 1 - f_0$$

RELATIVISTIC MHD WITH SPIN

The expressions for **dissipative currents** in terms of the nonequilibrium correction to the distribution function are

$$N^\mu = nu^\mu + n^\mu$$

$$T_{\rm f}^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$S^{\lambda,\mu\nu} = S_{\rm eq}^{\lambda,\mu\nu} + \delta S^{\lambda,\mu\nu}$$

$$\Pi = -\frac{\Delta_{\alpha\beta}}{3}\int_{p,s} p^\alpha p^\beta \left(\delta f^+ + \delta f^-\right)$$

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int_{p,s} p^\alpha p^\beta \left(\delta f^+ + \delta f^-\right)$$

$$n^\mu = \Delta^\mu_\alpha \int_{p,s} p^\alpha \left(\delta f^+ - \delta f^-\right)$$

$$\delta S^{\lambda,\mu\nu} = \int_{p,s} p^\lambda s^{\mu\nu} \left(\delta f^+ + \delta f^-\right)$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$$

EMERGENCE OF BARNETT EFFECT

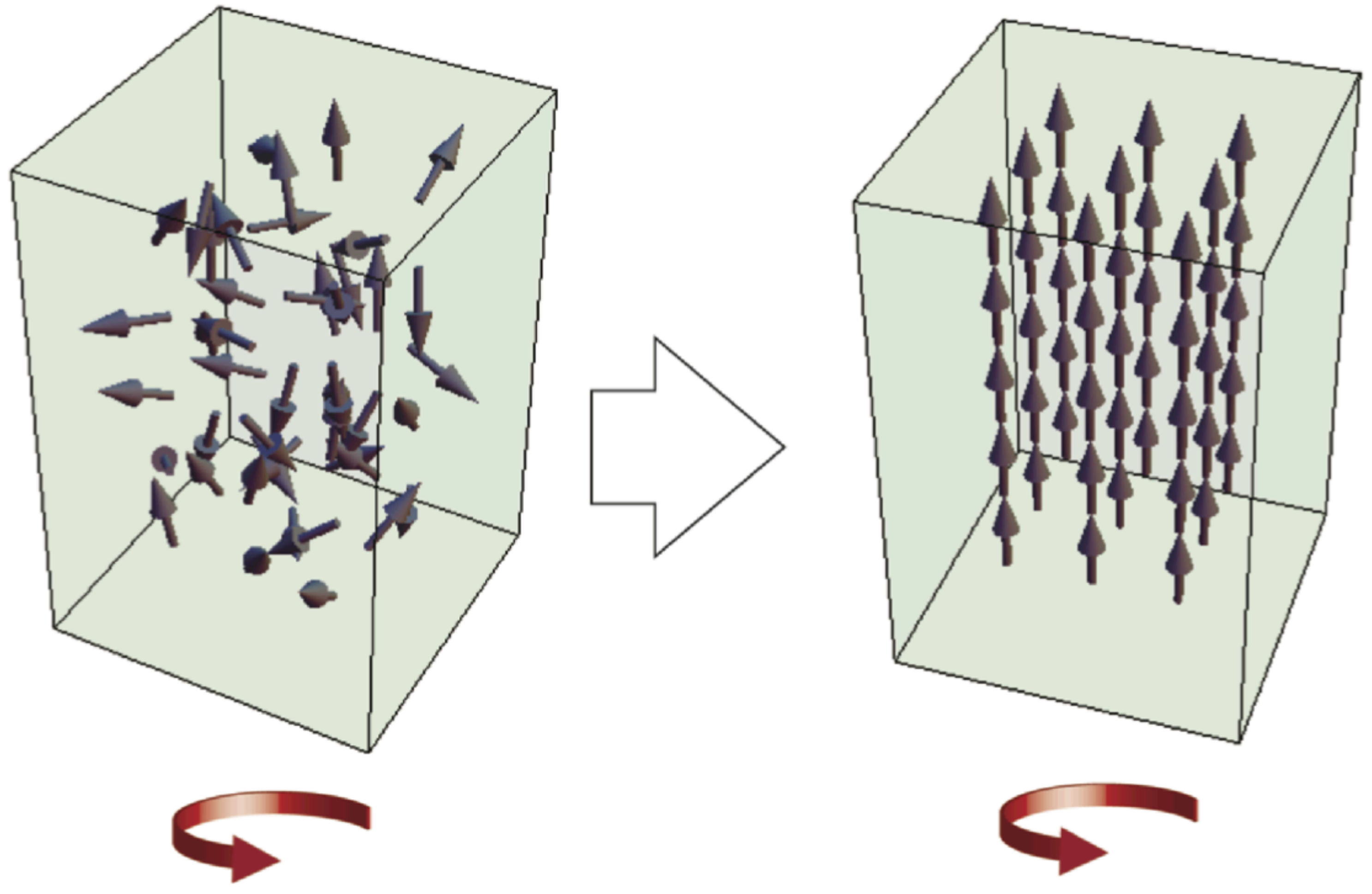
Equilibrium polarization-magnetization tensor is

$$M^{\mu\nu}_{\rm eq} = a_1(T,\mu)\omega^{\mu\nu} + a_2(T,\mu)u^{[\mu}u_\gamma\omega^{\nu]\gamma}$$

In global equilibrium, spin chemical potential corresponds to rotation of the fluid

F. Becattini and F. Piccinini, Ann. Phys. (Amsterdam) 323, 2452 (2008)
F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019).

$$\omega^{\mu\nu}|_{\rm geq} \propto \varpi^{\mu\nu} = \left(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu\right)/2$$



We conclude that rotation of the fluid produces magnetization, which is precisely the physics of Barnett effect.

S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)
A. Einstein and W. de Haas, Deutsch. Phys. Ges., Verh. 17, 152 (1915)

figure: Journal of the Physical Society of Japan 90, 081003 (2021)

CONVERSION BETWEEN VORTICITY AND SPIN

Using the **spin matching condition** we obtain the **evolution equation for the spin polarization tensor**

$$\dot{\omega}^{\mu\nu} = \mathcal{D}_{\Pi}^{\mu\nu}\theta + \mathcal{D}_{\mathrm{n}}^{\mu\nu\gamma}(\nabla_\gamma \xi) + \mathcal{D}_{\mathbf{a}}^{\mu\nu\gamma}\dot{u}_\gamma + \mathcal{D}_{\pi}^{\mu\nu\rho\kappa}\sigma_{\rho\kappa} + \mathcal{D}_{\Omega}^{\mu\nu\rho\kappa}\Omega_{\rho\kappa} + \mathcal{D}_{\Sigma}^{\mu\nu\phi\rho\kappa}(\nabla_\phi \omega_{\rho\kappa})$$

$$\Omega_{\mu\nu} \equiv (\partial_\mu u_\nu - \partial_\nu u_\mu)/2$$

We observe that the above equation contains information about the **connection between evolution of spin polarization tensor** and **fluid vorticity.**

$\mathcal{D}_{\Omega}^{\mu\nu\rho\kappa}$ vanishes in absence of electromagnetic field which leads us to conclusion that the **conversion between spin-polarization and vorticity proceeds via coupling with electromagnetic field**.

FIRST-ORDER DISSIPATIVE CURRENTS IN SMHD

The expressions for **dissipative currents** in terms of the nonequilibrium corrections to the DF are

$$X = \tau_{\rm eq}\left[\beta_{X\Pi}\theta + \beta^{\alpha}_{Xn}(\nabla_\alpha \xi) + \beta^{\alpha}_{Xa}\dot{u}_\alpha + \beta^{\alpha\beta}_{X\pi}\sigma_{\alpha\beta} + \beta^{\alpha\beta}_{X\Omega}\Omega_{\alpha\beta} + \beta^{\alpha\beta}_{XF}(\nabla_\alpha B_\beta) + \beta^{\alpha\beta\gamma}_{X\Sigma}(\nabla_\alpha \omega_{\beta\gamma})\right]$$

where $X \equiv n^\mu,\ \Pi,\ \pi^{\mu\nu},\ \delta S^{\lambda,\mu\nu}$

These expressions contain gradients of magnetic field.

Demanding that the divergence of the above entropy current is positive definite we identify **first-order dissipative gradient terms**

$$\Pi = -\zeta\theta, \quad n^\mu = \kappa^{\mu\alpha}\left(\nabla_\alpha \xi\right), \quad \pi^{\mu\nu} = \eta^{\mu\nu\alpha\beta}\sigma_{\alpha\beta},$$
$$\delta S^{\mu,\alpha\beta} = \Sigma^{\mu\alpha\beta\lambda\gamma\rho}\left(\nabla_\lambda \omega_{\gamma\rho}\right).$$

CONCLUSIONS

We presented the first kinetic theory formulation of **relativistic dissipative nonresistive MHD with spin** in the limit of small polarization.

We demonstrated that multiple transport coefficients, dissipative as well as non-dissipative, are present.

We showed that our framework naturally leads to the **emergence of the relativistic analog of Barnett effect**.

We show that the **coupling between the magnetic field and spin polarization appears at gradient order**.

Simulation based on our unified framework has the potential of **explaining the difference of Λ and anti-Λ polarization.**

THANK YOU FOR YOUR ATTENTION.

BACKUP SLIDES

PSEUDOGAUGE FREEDOM AND PROBLEM OF LOCALIZATION

- **Pseudogauge transformation:** densities are not uniquely defined
  F.W. Hehl, Rep. Math. Phys. 9 (1976) 55.

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda\left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}\right)$$
$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu},$$

The new tensors satisfy conservation equations and preserve Poincare algebra generators

$$\widehat{P}^\nu = \int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}, \qquad \widehat{J}^{\lambda\nu} = \int_\Sigma \mathrm{d}\Sigma_\mu \widehat{\mathcal{J}}^{\mu,\lambda\nu}$$

- Inequivalence of different pseudogauge pairs was shown.
  F. Becattini, L. Tinti, Phys. Rev. D 84 (2011) 025013; F. Becattini, L. Tinti, Phys. Rev. D 87 (2) (2013) 025029
- Canonical currents act as sources for Einstein-Cartan theory.
  F. W. Hehl, P. von der Heyde, and G. D. Kerlick, Rev. Mod. Phys. 48, 393 (1976)

- **Belinfante pseudogauge** $\widehat{\Phi} = \widehat{\mathcal{S}}$
  F. Belinfante, Physica 7 (5) (1940) 449–474
- Belinfante energy-momentum tensor gives the Einstein-Hilbert one.

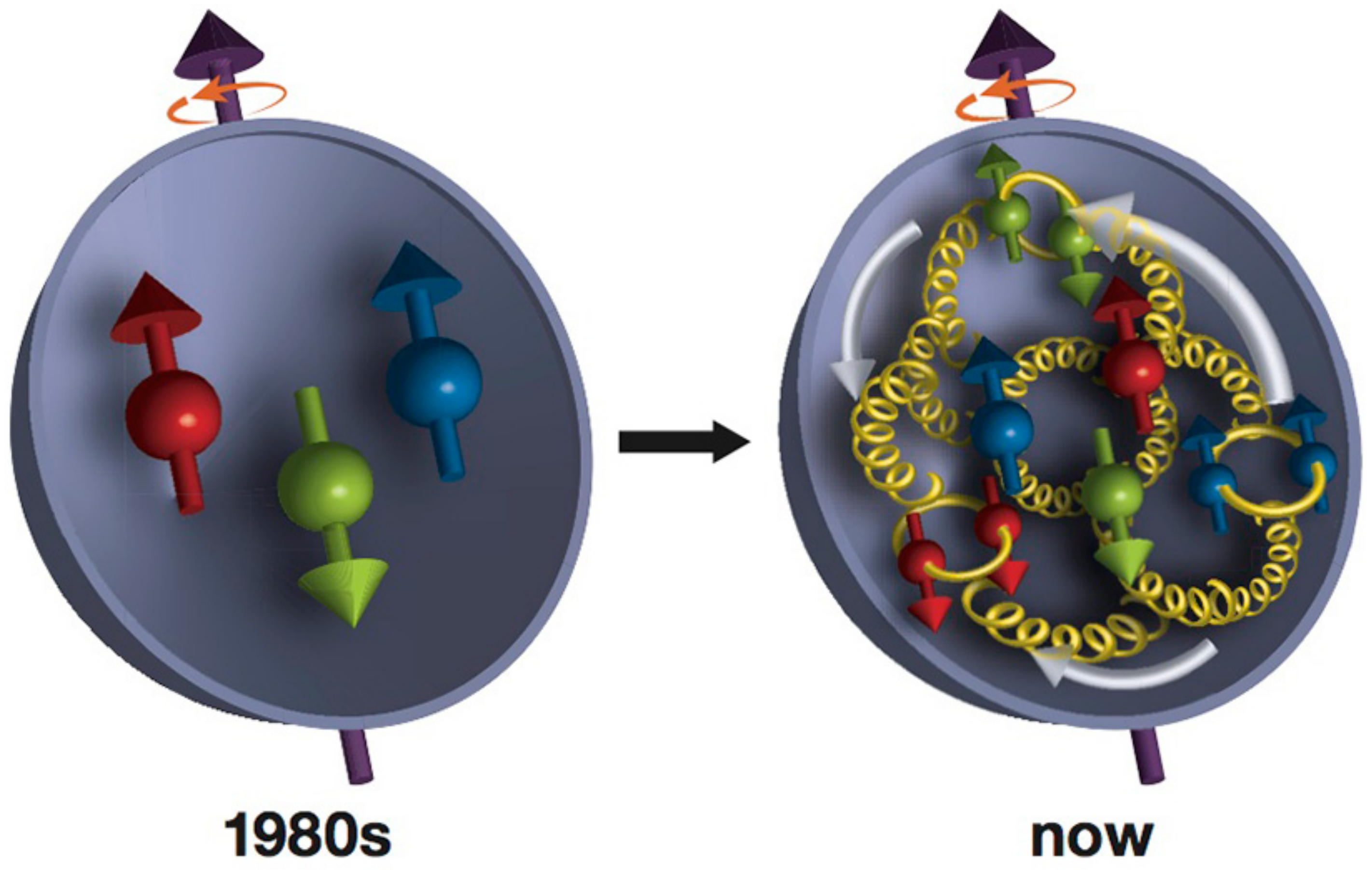


figure: Physics World