Partially funded by:
U.S. DEPARTMENT OF ENERGY

ELTE
EÖTVÖS LORÁND
UNIVERSITY

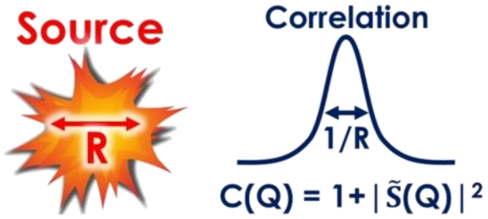
Two-particle interferometry with Lévy-stable sources in Au+Au collisions at STAR

Dániel Kincses for the STAR Collaboration
Eötvös University, Budapest

23rd ZIMÁNYI SCHOOL WINTER WORKSHOP
ON HEAVY ION PHYSICS, December 4-8, 2023,
BUDAPEST

Part I.
Introduction, Motivation

Basic definitions of femtoscopic correlation funtions



phase-space density

- Single particle momentum distribution: $N_1(p) = \int d^4x\, S(x,p)$
- Pair momentum distribution: $N_2(p_1,p_2) = \int d^4x_1 d^4x_2 S(x_1,p_1)S(x_2,p_2)\left|\psi_{p_1,p_2}(x_1,x_2)\right|^2$
- **Correlation function:** $C(p_1,p_2) = \frac{N_2(p_1,p_2)}{N_1(p_1)N_2(p_2)}$

pair separation

- **Pair source/spatial correlation:** $D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2},K\right) S\left(\rho - \frac{r}{2},K\right)$

pair center-of-mass

relative pair momentum

*average pair momentum**

Pair wave function, ***containing FSI!***

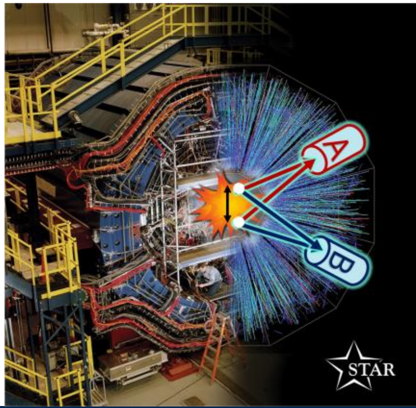
$$C(Q,K) = \int d^4r D(r,K)\left|\psi_Q(\mathrm{r})\right|^2$$

**Instead of K, m_T is often used:*

$$m_T = \sqrt{k_T^2 + m_\pi^2}, k_T = \sqrt{K_x^2 + K_y^2}$$

- **Experiments: measuring C(Q) → information about D(r) and FSI**
- Experimental (and phenomenological) indications:
  power-law tail for pions, **non-Gaussianity?**

D. A. Brown and P. Danielewicz. In: Phys. Lett. B 398 (1997), pp. 252–258.
S. S. Adler et al., PHENIX Coll. In: Phys. Rev. Lett. 98 (2007), p. 132301.



$$S(\boldsymbol{r}) = \mathcal{L}(\alpha, R; \boldsymbol{r}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{r}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T \boldsymbol{R}^2 \boldsymbol{q}\right|^{\alpha/2}}$$

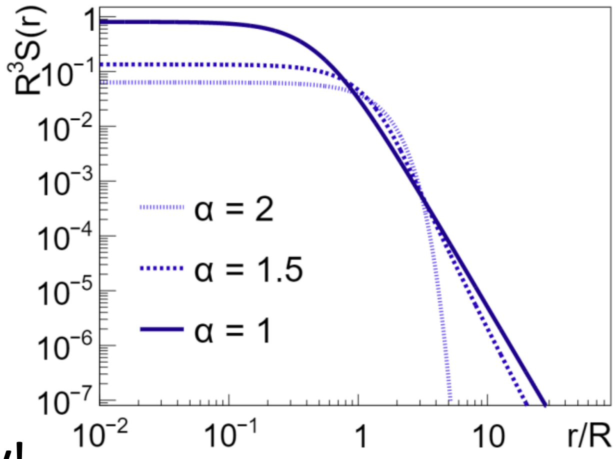
spherical sym.: $R^2_{ij} = R^2\delta_{ij}$

What is the shape of the source? Gaussian vs. Lévy distributions in heavy-ion physics

Csörgő, Hegyi, Zajc, Eur.Phys.J.C 36 (2004) 67

- **Symmetric Lévy-stable distribution**
  - From generalized central limit theorem, power-law tail (if $\alpha < 2$) $\sim r^{-(1+\alpha)}$
  - $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
  - Retains the same α under convolution

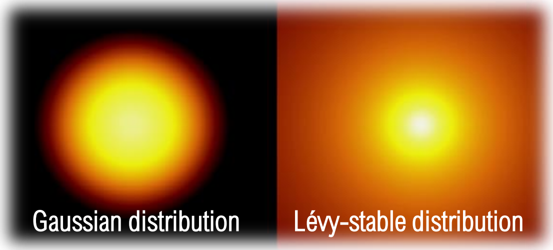
$$S(r) = \mathcal{L}(\alpha, R; r)$$
$$\Downarrow$$
$$D(r) = \mathcal{L}\left(\alpha, 2^{1/\alpha}R; r\right)$$



- **Experimental indications – Lévy source for pion pairs?**
  - **RHIC (PHENIX, STAR)** *Phys.Rev. C97 (2018) no.6, 064911*
  - **LHC (CMS)** *Phys.Part.Nucl. 51 (2020) 3, 267-269*
  - **SPS (NA61/SHINE)** *arXiv:2306.11574 (CMS-HIN-21-011)* *Eur.Phys.J.C 83 (2023) 10, 919*

See talk of M. Csanád and B. Pórfy!

- **Possible interpretations of the α Lévy exponent based on:**
  - Jet fragmentation *Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36*
  - Critical behavior *Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828*
  - Event averaging *Cimerman, Tomasik, Plumberg, Phys.Part.Nucl. 51 (2020) 3, 282*
  - Resonance decays *Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308*
  - Anomalous diffusion *Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;*



- **Lévy in 1D: not because of 3D→1D conversion!** *Kurgyis, Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), 477 (2019)*

New data analyses and phenomenological investigations are needed to gain a better understanding!

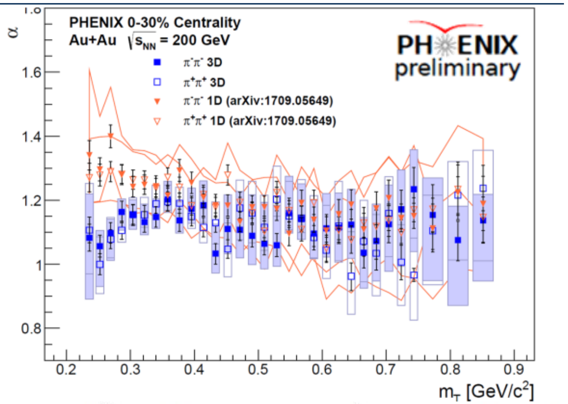
$$S(\boldsymbol{r}) = \mathcal{L}(\alpha, R; \boldsymbol{r}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{r}} e^{-\frac{1}{2}\left|\boldsymbol{q}^{\boldsymbol{T}}\boldsymbol{R}^{\boldsymbol{2}}\boldsymbol{q}\right|^{\alpha/2}}$$

spherical sym.: $R_{ij}^2 = R^2\delta_{ij}$

Lévy source at RHIC energies

- **Not spherically sym. source: 3D vs. 1D α compatible!** **$\alpha < 2$ in 1D analyses not because of angle averaging!**
  Kurgyis, Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), 477 (2019)

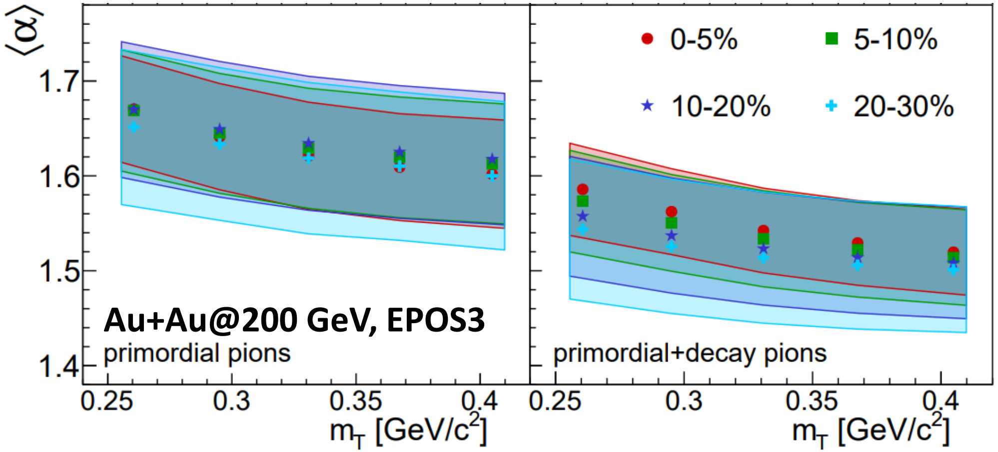
$$\boldsymbol{R}^2 = \begin{pmatrix} R_{out}^2 & 0 & 0 \\ 0 & R_{side}^2 & 0 \\ 0 & 0 & R_{long}^2 \end{pmatrix} \quad \boldsymbol{q} = \begin{pmatrix} q_{out} \\ q_{side} \\ q_{long} \end{pmatrix}$$

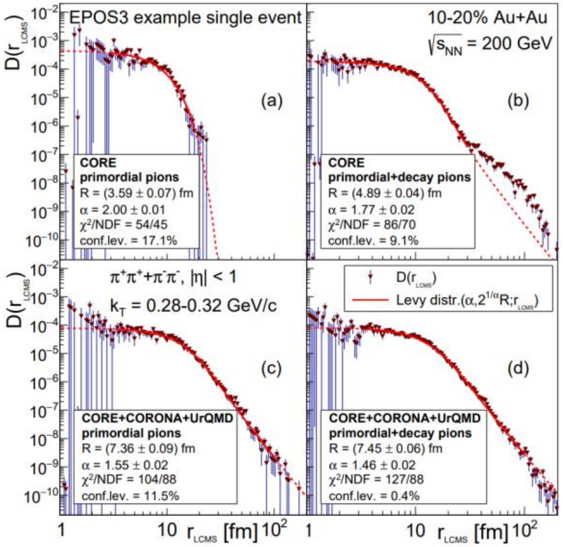


- **EPOS pion pair-source analysis at 200 GeV:**

Decays and hadronic rescattering (UrQMD) play an important role!

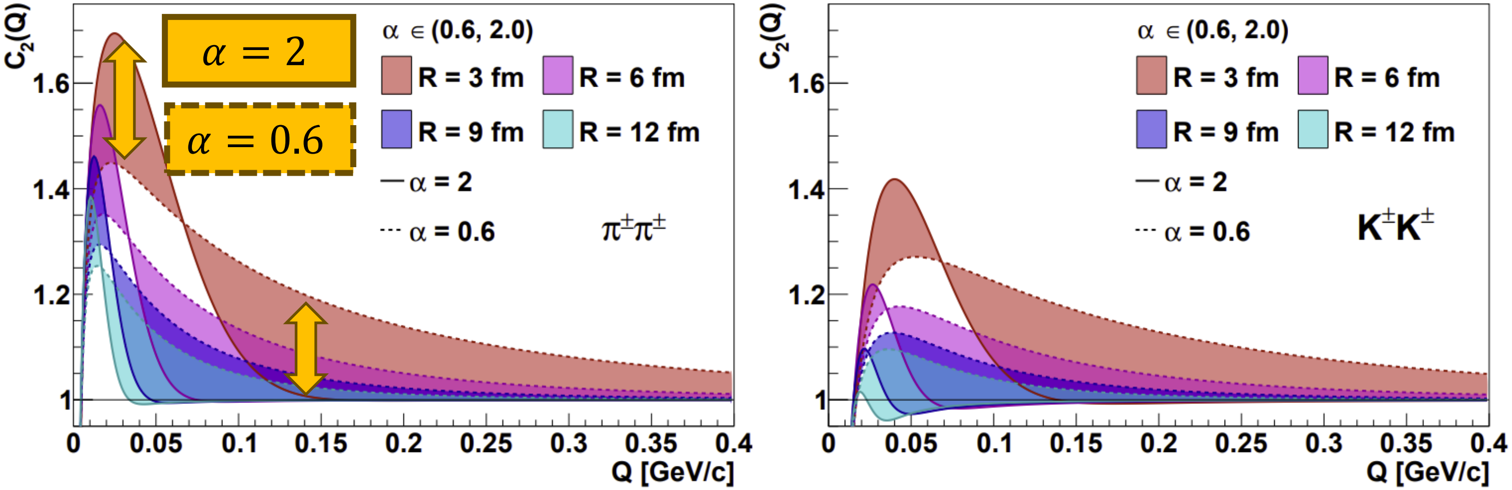
Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308





Recent phenomenological developments

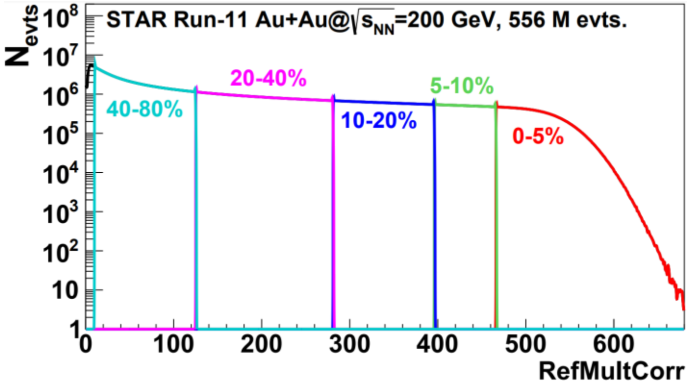
- **Coulomb Corrections for Bose–Einstein Correlations from One- and Three-Dimensional Lévy-Type Source Functions** *Kurgyis, Kincses, Nagy, Csanád, Universe 9 (2023) 7, 328*
- **Event-by-Event Investigation of the Two-Particle Source Function in Heavy-Ion Collisions with EPOS** *Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308*
  Kórodi, Kincses, Csanád, Phys. Lett. B 847 (2023) 138295
  See talk of M. Csanád!
- **A novel method for calculating Bose-Einstein correlation functions with Coulomb final-state interaction** *Nagy, Purzsa, Csanád, Kincses, Eur. Phys. J. C 83, 1015 (2023)*
  See talk of M. Nagy!

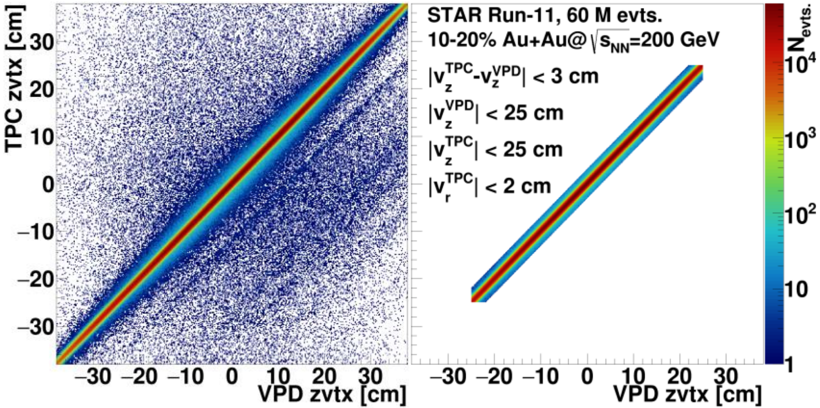


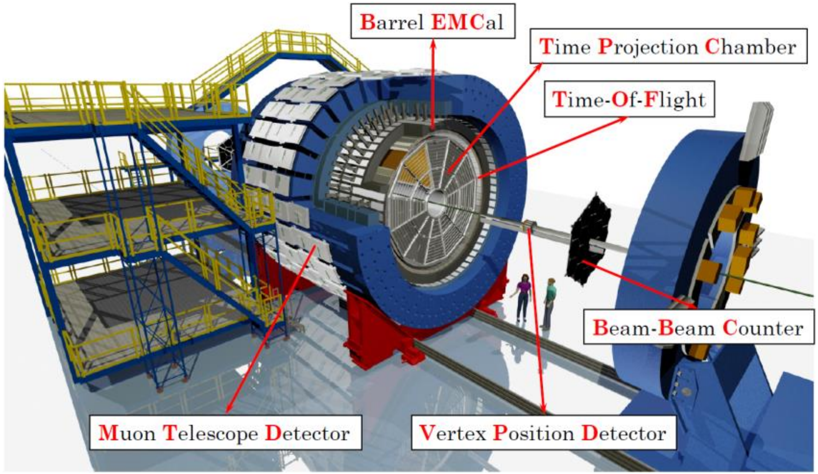
Part II.
Measurement and fitting of correlation functions

Lévy HBT analysis at STAR, Au+Au @ 200 GeV

- **STAR Run-11 data analyzed**
  After trigger cuts and bad run cuts: **550M events**
- **Detectors used for the analysis:**
  - **BBC, TPC, VPD**: centrality, vertex position
  - **TPC**: tracking, dE/dx Particle Identification (PID)
  - **TOF**: time-of-flight PID
- **Event selection:**
  - Pile-up cuts using TOF vs. TPC multiplicity
  - Vertex cuts:

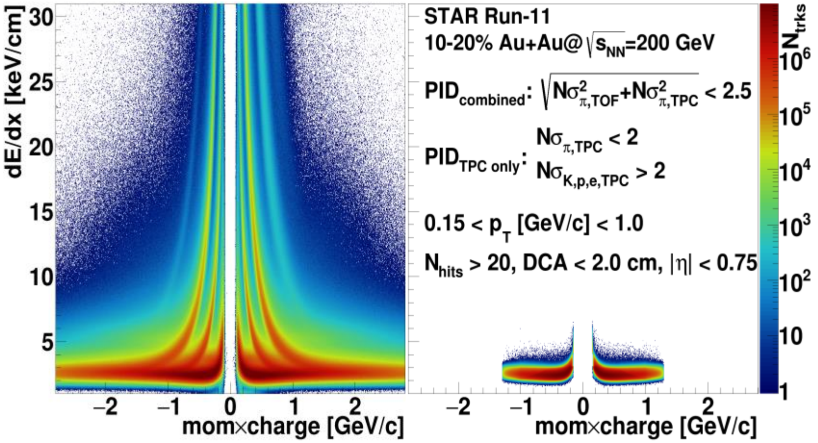


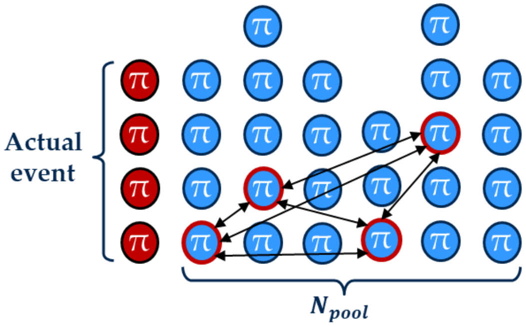




Measurement of two-pion correlation functions

- **Track-selection criteria**
  - Combined PID using TPC $N\sigma$ (based on dE/dx) and TOF $N\sigma$ (based on time-of-flight)
  - Further single-track cuts on TPC number of hits, p_T, η, Distance of Closest Approach (DCA)
- **Pair-selection criteria** *J. Adams et al. (STAR Coll.), Phys. Rev. C 71, 044906 (2005)*
  - Splitting level (SL) < 0.6
  - Fraction of Merged Hits (FMH) < 5%
  - Average pair-separation (on TPC pad rows) Δr > 3 cm
- **Event mixing**
  - Similarly to *PHENIX Coll., Phys.Rev. C97 (2018) no.6, 064911*
  - 2 cm wide z vertex bins, 5% wide centrality bins
  - A(Q): pions from the same event
  - B(Q): pions from different events
  - C(Q)=A(Q)/B(Q), appropriately normalized
- **Centrality and k_T selection:**
  - 21 k_T bins, (0.175 - 0.750) GeV/c
  - 4 centrality bins (0-10%, 10-20%, 20-30%, 30-40%)





Fitting process with Lévy parametrization

- **Lévy parametrization without final state effects:**

Lévy exponent

$$\boldsymbol{C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}}$$

LCMS three-momentum difference

Lévy scale parameter

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + \left(p_{1y} - p_{2y}\right)^2 + q_{long,LCMS}^2}$$

Intercept parameter (correlation strength)

- **Formula used for fitting procedure:**

$$\boldsymbol{C(Q) = \left(1 - \lambda + \lambda \cdot \overbrace{K(Q;\alpha,R)}^{\text{Coulomb correction}} \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot \overbrace{N \cdot (1 + \varepsilon Q)}^{\text{Possible linear background (usually negligible)}}}$$

- **Coulomb-correction:**

Two-particle wave function (with Coulomb interaction)

$$\boldsymbol{K(Q;\alpha,R) = \frac{\int D(r)\left|\psi^{Coul}(r)\right|^2 dr}{\int D(r)|\psi^{(0)}(r)|^2 dr}}$$ → **calculated numerically**

Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 6, 064912

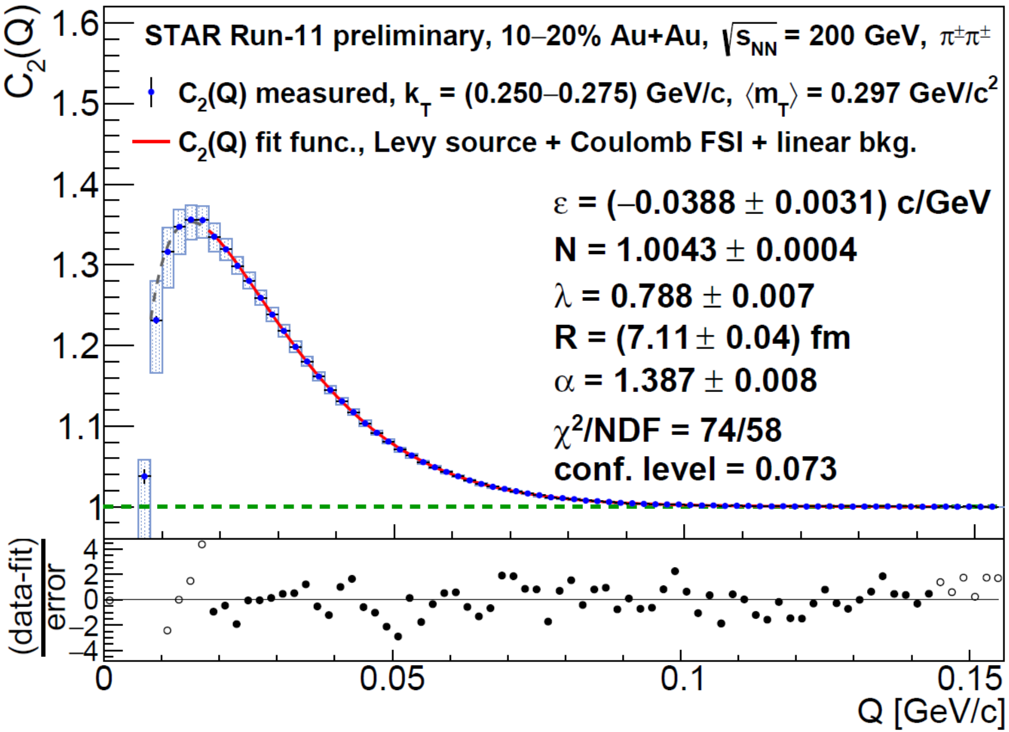
Spatial correlations

Two-particle wave function (plane wave)

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$

An example fit to a two-pion correlation function

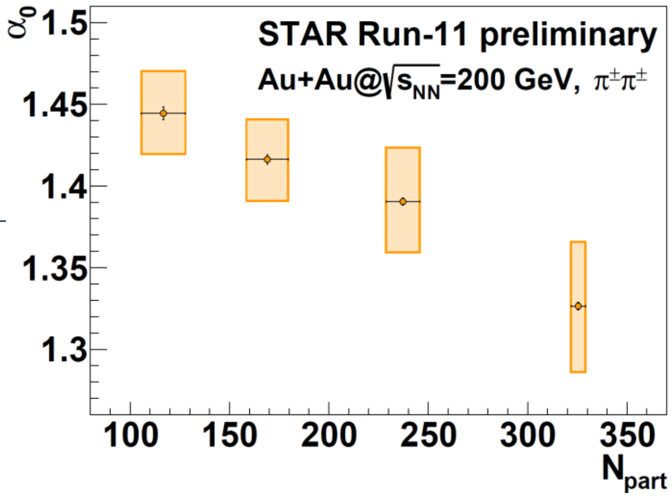
- Example fit in the 10-20% centrality class, at k_T = (0.250-0.275) GeV/c
- **Iterative fitting method,** Coulomb FSI + Lévy-source
- Track and pair **syst. uncertainties illustrated with boxes**
- Low-Q points left out from fit, **fit range study included in total systematic uncertainties**
- **Fits converged, conf.level > 0.1%**



- Confidence levels approx. uniformly distributed
- Similar fits done in all 4 centrality class and 21 k_T bins, **centrality and m_T dependence of the source parameters investigated**

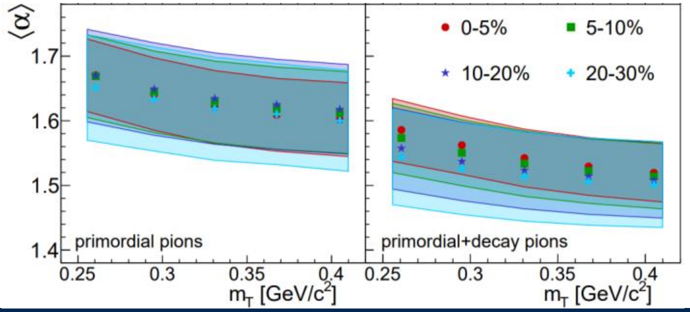
Part III.
Preliminary results at 200 GeV (centrality, m_T dependence)

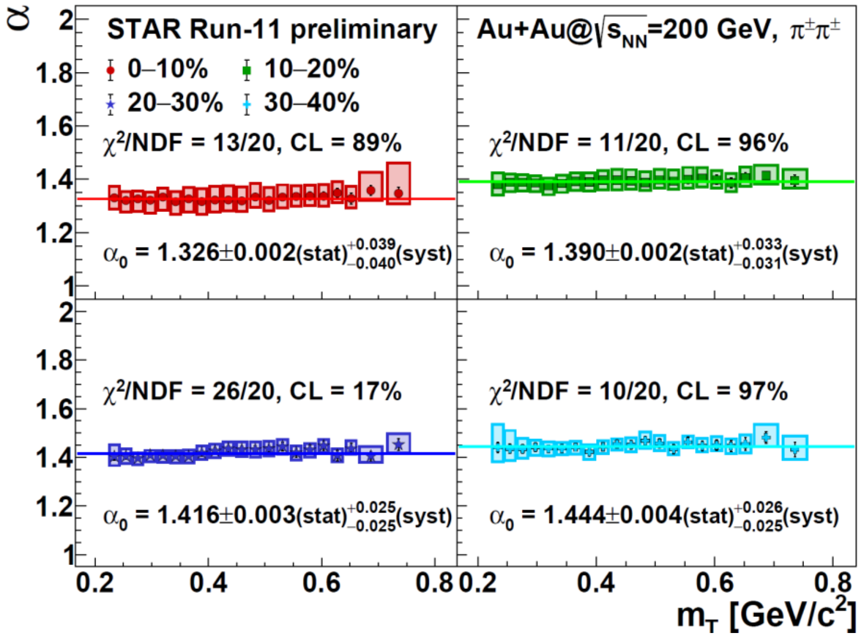
Lévy exponent $\alpha(m_T, \text{centrality})$



- **Non-gaussian values ($\alpha \ll 2$)**
- No dependence on m_T, slight centrality dep.
- $\boldsymbol{\alpha_0}$ vs N_{part} (m_T average values from constant fit)
    - Decreasing trend due to anti-correlation with λ and R?
    - CMS observed opposite trend, see talk of M. Csanád
- EPOS model: slightly higher α values

D. Kincses, M. Stefaniak, M. Csanád, Entropy 24 (2022) 3, 308



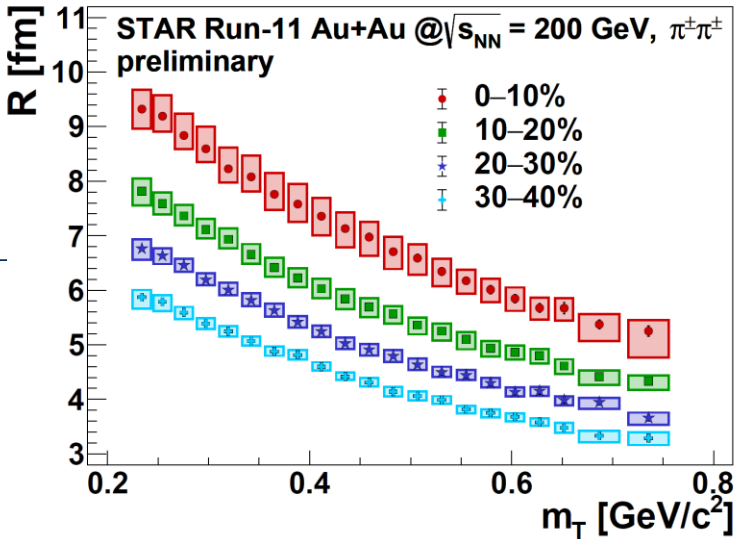


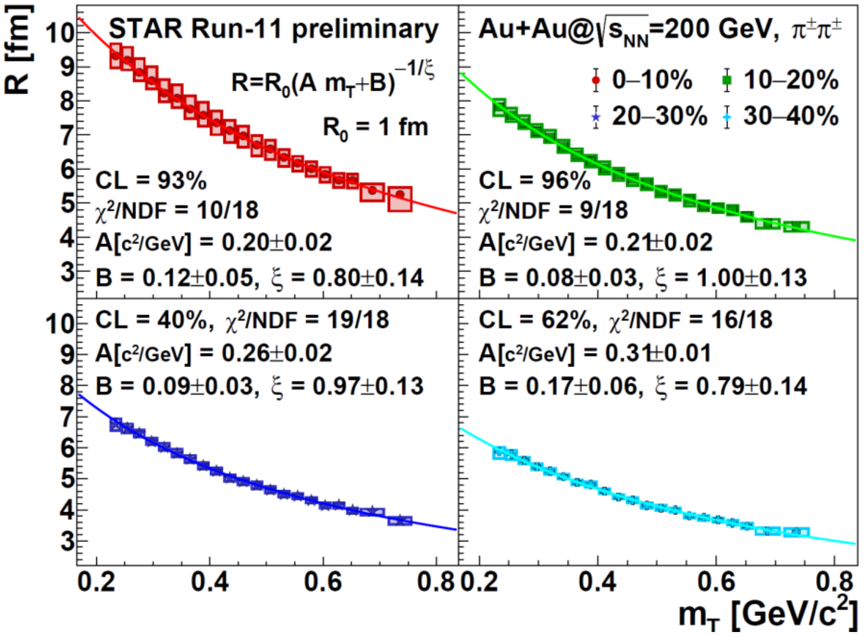
Lévy scale $R(m_T, \text{centrality})$

- **Decreasing trend with m_T, also with centrality**
  - Connection to flow and initial geometry?
- Fits with $\boldsymbol{R = R_0(Am_T + B)^{-1/\xi}}, R_0 = 1$ fm
  - Hydro calculations (R_{Gauss}): $\xi = 2$

Makhlin, Sinyukov, Z. Phys. C 39, 69 (1988)
Csörgő, Lörstad, Phys. Rev. C 54, 1390 (1996)
Chapman, Scotto, Heinz, Acta Phys.Hung.A 1 (1995) 1-31
Csanád, Csörgő, Lörstad, Ster, J. Phys. G 30, S1079 (2004)

- **Our case (R_{Levy}): ξ close to 1**





Correlation strength $\lambda(m_T, \text{centrality})$

- Without FSI: $C_2^{(0)}(Q=0) = 2 \longrightarrow \lambda \equiv \lim_{Q\to 0} C_2^{(0)}(Q) - 1$, experimentally often $\lambda < 1$
- **Core-halo picture** - two-component source: $S = S_{core} + S_{halo} \Rightarrow D = D_{(c,c)} + D_{(c,h)} + D_{(h,h)}$
  - Core: primordial + decays of short-lived resonances
  - Halo: decays of long-lived resonances → large R → small Q → measurement limited

$$\lambda = N_{core}^2 / (N_{core} + N_{halo})^2$$

Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497

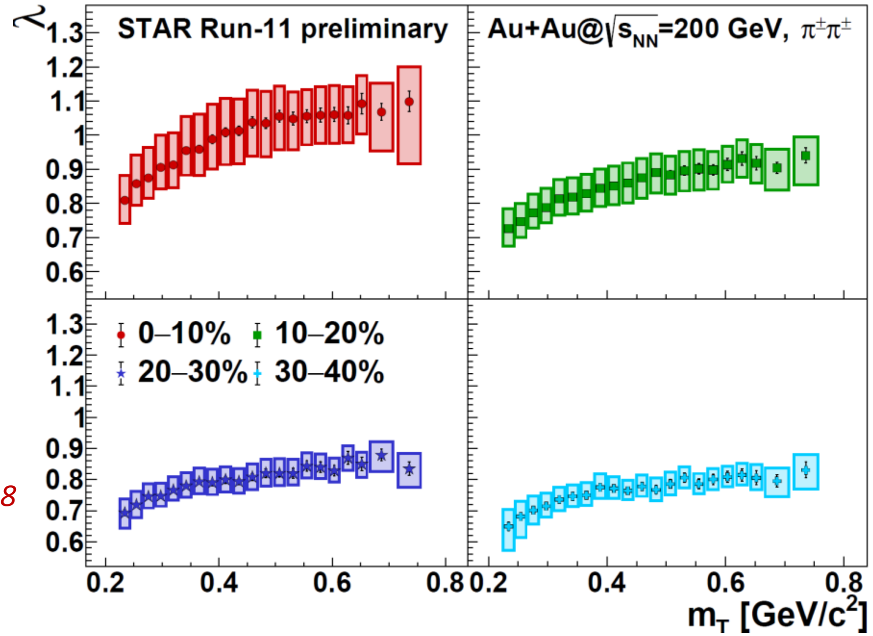
Bolz et al, Phys.Rev. D47 (1993) 3860-3870;

- **For power-law sources, more complicated picture!**
- **Increase from low to high m_T**
  - More decay products at low m_T?
  - In-medium mass modification of η'?
  - Partially coherent particle emission?

Vance, Csörgő, Kharzeev, Phys. Rev. Lett.81 (1998), pp. 2205–2208

Bolz et al., Phys. Rev. D47 (1993), pp. 3860–3870

- **Decrease from central to peripheral**

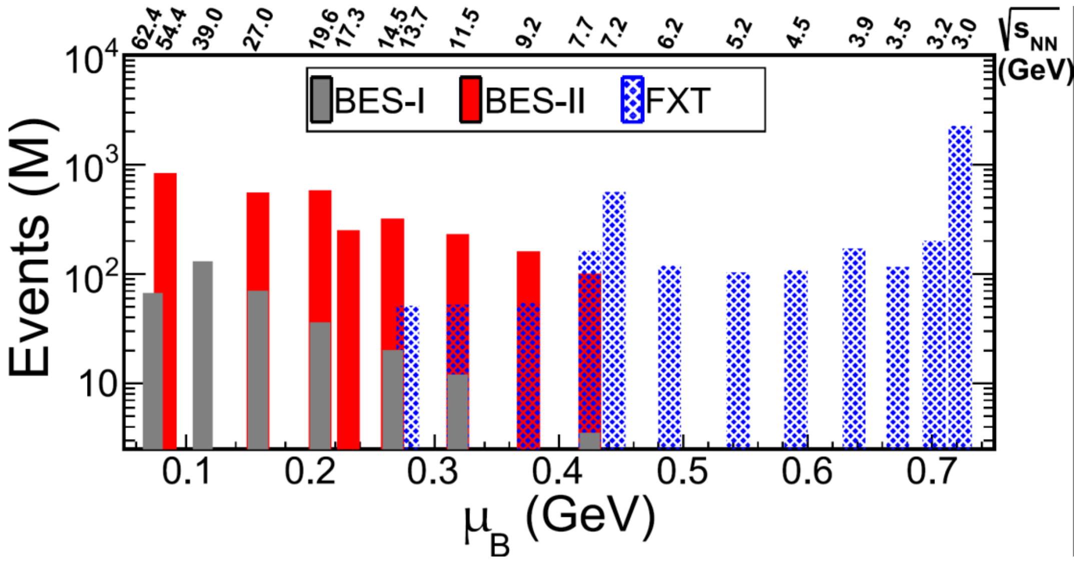


Part IV.
Preliminary results at BES-II
($\sqrt{s_{NN}}$, m_T dependence)

Beam energy dependent analysis in Au+Au collisions

- **BES-II data taking** recently conducted at STAR
  - Increased luminosity
  - Many detector improvements

- Next step: similar analysis with the same settings at lower Au+Au energies
  - Run-17, **54.4 GeV**
  - Run-18, **27 GeV**
  - Run-19, **19.6 GeV**
  - Run-19, **14.5 GeV**
  - Run-21, **7.7 GeV**



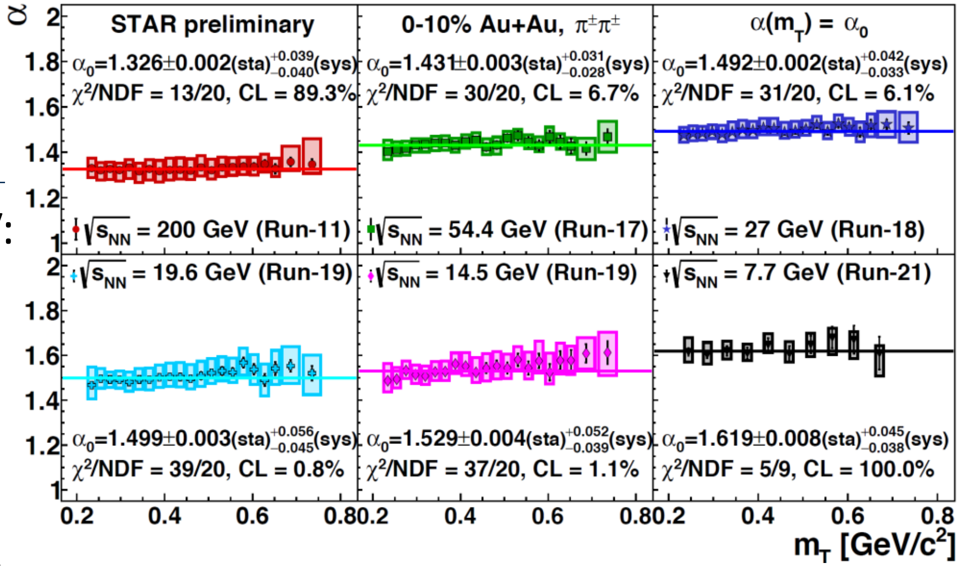
$\sqrt{s_{NN}}$ and m_T dependence of the source parameters

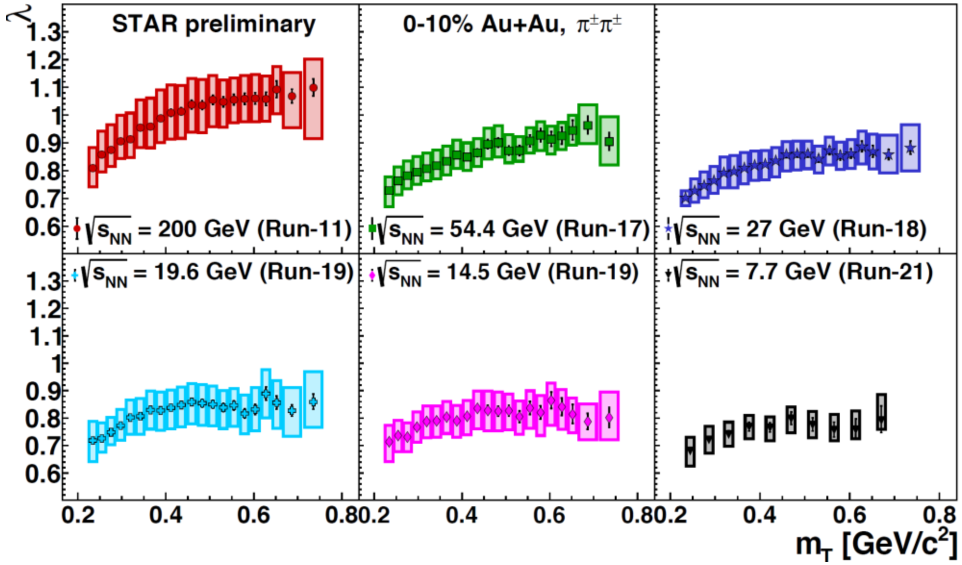
- $\sqrt{s_{NN}}$ dependence from 200 to 7.7 GeV:

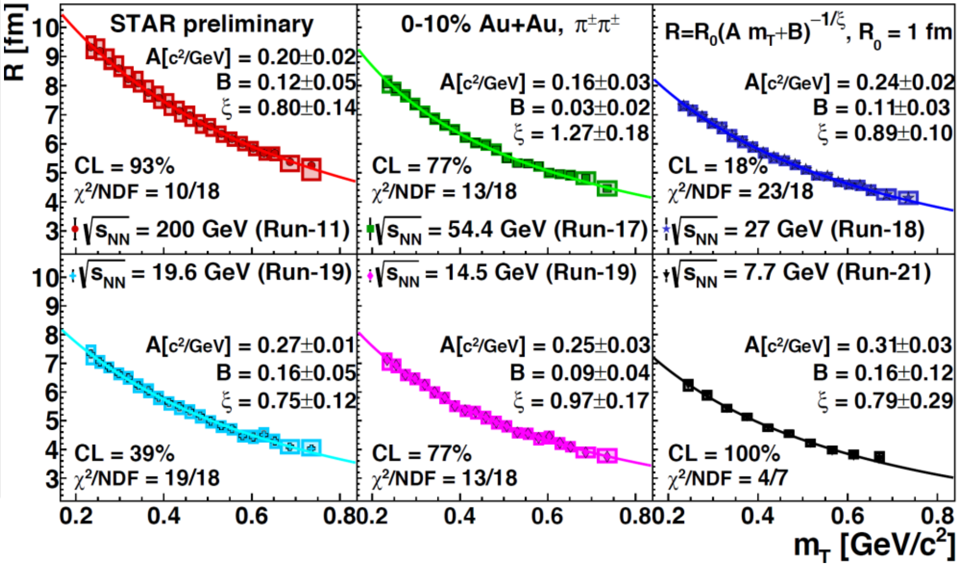
  $$\alpha\uparrow, R\downarrow, \lambda\downarrow$$

- m_T dependent trends at all energies:

  $$\alpha \text{ const.}, R\downarrow, \lambda\uparrow$$







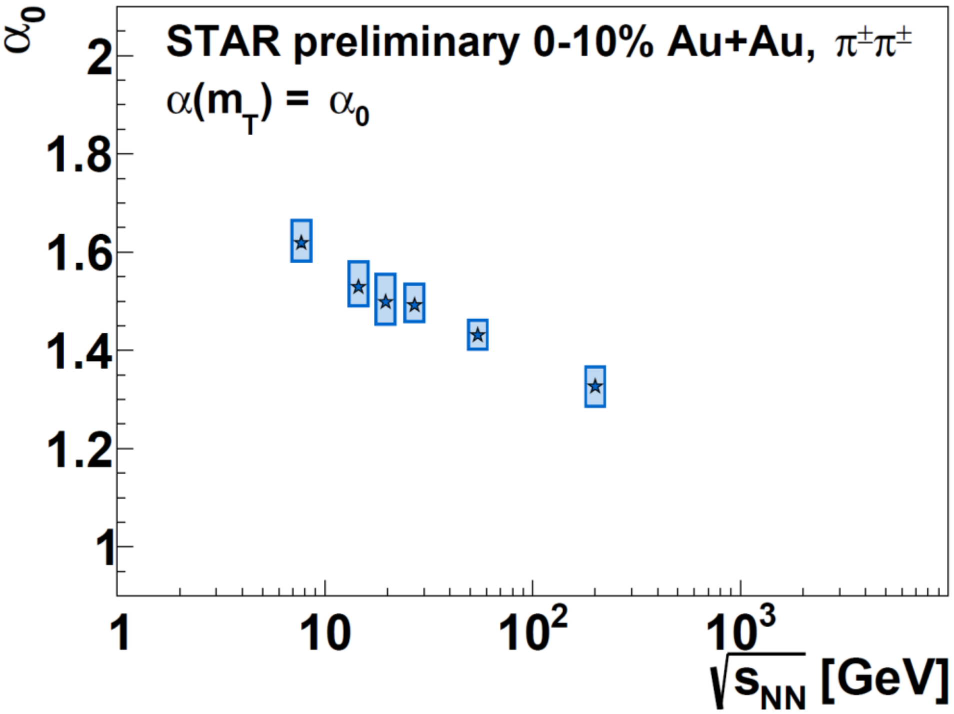
Excitation function of the Lévy exponent

- Non-gaussian values ($\alpha \ll 2$)

- Monotonic decrease from 7.7 GeV to 200 GeV

- CMS 0-10% PbPb result at 5.02 TeV:
  arXiv:2306.11574 $\alpha_0 \approx 1.86$
  (note that the kinematic range is different, see talk of M. Csanád)

- Interpretation of α still an open question, possibilities include:
  - Jet fragmentation
  - Critical behavior
  - Event averaging
  - Resonance decays
  - Anomalous diffusion

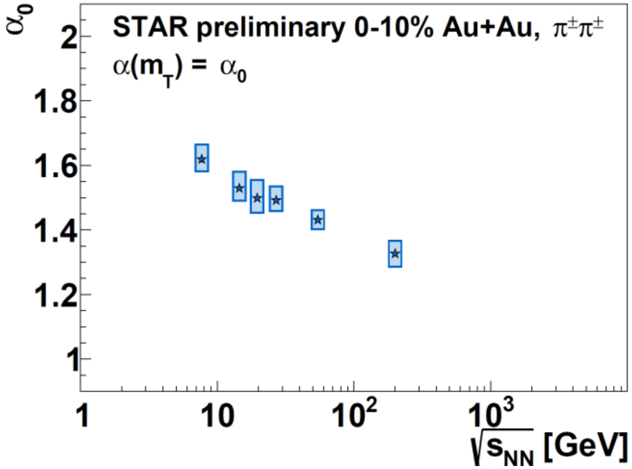


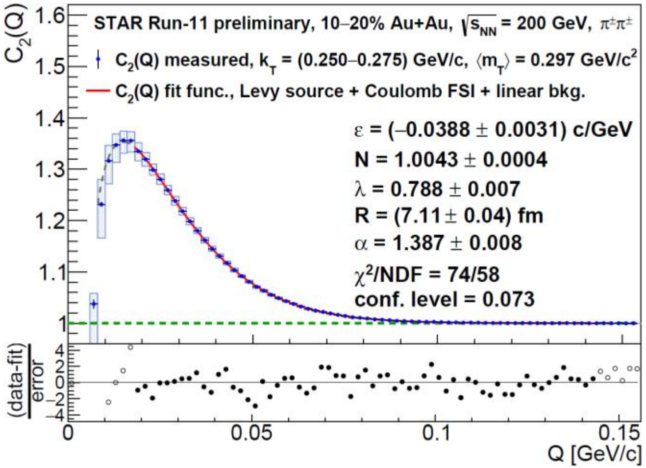
Summary

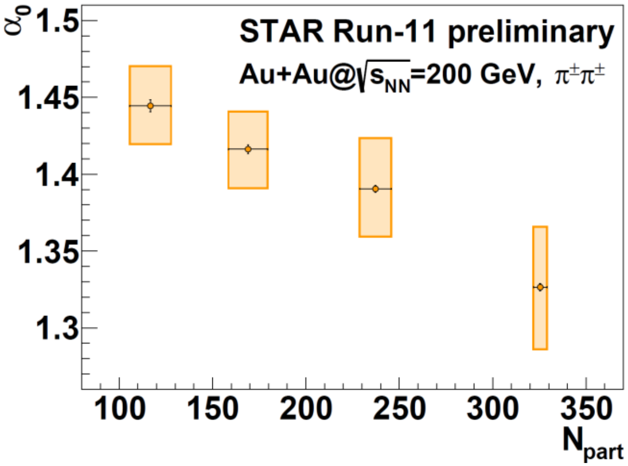
- 1-dim. two-pion correlation functions investigated
- Lévy-source + Coulomb FSI → good description
- Further syst. uncertainty investigations underway
- 0-10% Au+Au: **200 GeV → 7.7 GeV**
  200 GeV Au+Au: **central → peripheral**

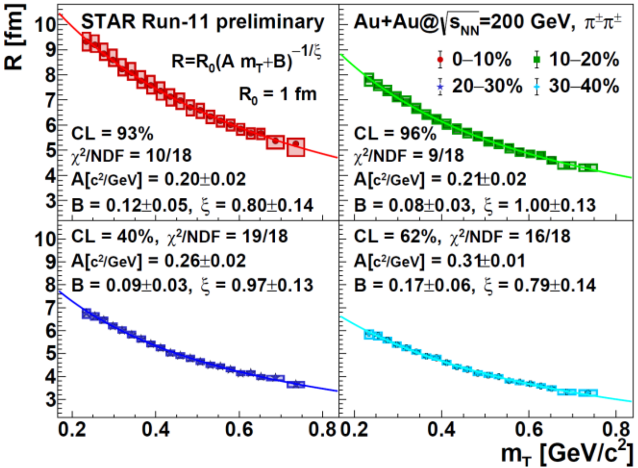
$\boldsymbol{\alpha \uparrow, R \downarrow, \lambda \downarrow}$

- **Next steps: even lower energies (fxt), 3D analysis!**









Thank you!

Further details, backup slides

Systematic uncertainties

- **Systematic uncertainties already investigated:**
  - Single track- and pair-cut variations, fit limits
- **Systematic uncertainties to be investigated:**
  - Purity correction, momentum smearing, more detailed fit limit study, strong interaction

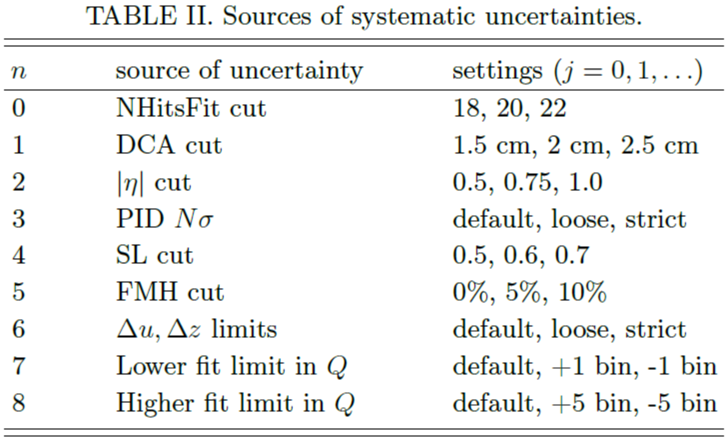


TABLE II. Sources of systematic uncertainties.

n	source of uncertainty	settings ($j = 0, 1, \ldots$)
0	NHitsFit cut	18, 20, 22
1	DCA cut	1.5 cm, 2 cm, 2.5 cm
2	$\|\eta\|$ cut	0.5, 0.75, 1.0
3	PID $N\sigma$	default, loose, strict
4	SL cut	0.5, 0.6, 0.7
5	FMH cut	0%, 5%, 10%
6	Δu, Δz limits	default, loose, strict
7	Lower fit limit in Q	default, +1 bin, -1 bin
8	Higher fit limit in Q	default, +5 bin, -5 bin

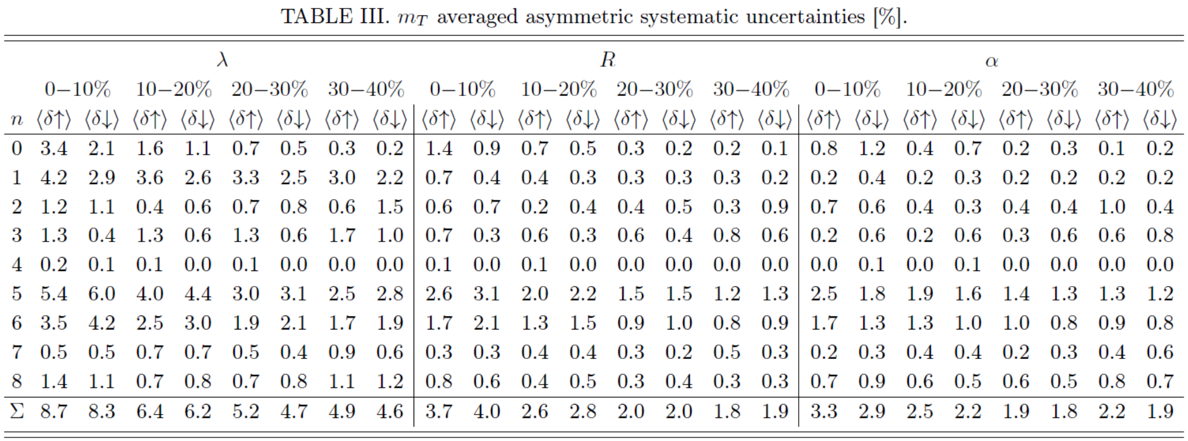


TABLE III. m_T averaged asymmetric systematic uncertainties [%].

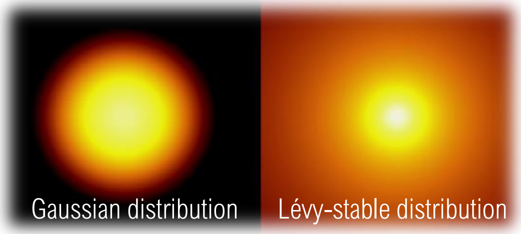
	λ								R								α							
	0–10%		10–20%		20–30%		30–40%		0–10%		10–20%		20–30%		30–40%		0–10%		10–20%		20–30%		30–40%	
n	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$
0	3.4	2.1	1.6	1.1	0.7	0.5	0.3	0.2	1.4	0.9	0.7	0.5	0.3	0.2	0.2	0.1	0.8	1.2	0.4	0.7	0.2	0.3	0.1	0.2
1	4.2	2.9	3.6	2.6	3.3	2.5	3.0	2.2	0.7	0.4	0.4	0.3	0.3	0.3	0.3	0.2	0.2	0.4	0.2	0.3	0.2	0.2	0.2	0.2
2	1.2	1.1	0.4	0.6	0.7	0.8	0.6	1.5	0.6	0.7	0.2	0.4	0.4	0.5	0.3	0.9	0.7	0.6	0.4	0.3	0.4	0.4	1.0	0.4
3	1.3	0.4	1.3	0.6	1.3	0.6	1.7	1.0	0.7	0.3	0.6	0.3	0.6	0.4	0.8	0.6	0.2	0.6	0.2	0.6	0.3	0.6	0.6	0.8
4	0.2	0.1	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0
5	5.4	6.0	4.0	4.4	3.0	3.1	2.5	2.8	2.6	3.1	2.0	2.2	1.5	1.5	1.2	1.3	2.5	1.8	1.9	1.6	1.4	1.3	1.3	1.2
6	3.5	4.2	2.5	3.0	1.9	2.1	1.7	1.9	1.7	2.1	1.3	1.5	0.9	1.0	0.8	0.9	1.7	1.3	1.3	1.0	1.0	0.8	0.9	0.8
7	0.5	0.5	0.7	0.7	0.5	0.4	0.9	0.6	0.3	0.3	0.4	0.4	0.3	0.2	0.5	0.3	0.2	0.3	0.4	0.4	0.2	0.3	0.4	0.6
8	1.4	1.1	0.7	0.8	0.7	0.8	1.1	1.2	0.8	0.6	0.4	0.5	0.3	0.4	0.3	0.3	0.7	0.9	0.6	0.5	0.6	0.5	0.8	0.7
Σ	8.7	8.3	6.4	6.2	5.2	4.7	4.9	4.6	3.7	4.0	2.6	2.8	2.0	2.0	1.8	1.9	3.3	2.9	2.5	2.2	1.9	1.8	2.2	1.9

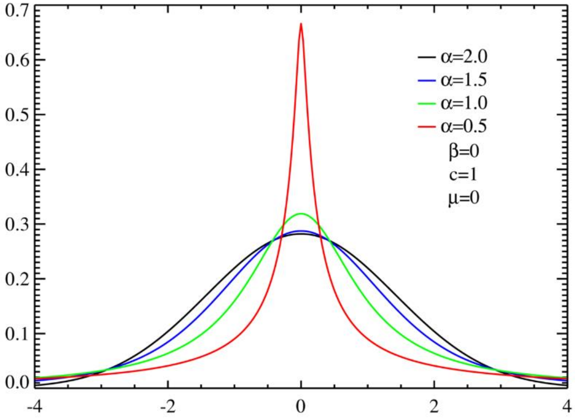
Properties of univariate stable distributions

- **Univariate stable distribution:** $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq$, where the characteristic function:
- $\varphi(q;\alpha,\beta,R,\mu) = \exp(iq\mu - |qR|^{\alpha}(1 - i\beta \mathrm{sgn}(q)\Phi))$
- α: index of stability
- β: skewness, symmetric if $\beta = 0$
- R: scale parameter
- μ: location, equals the median,

  if $\alpha > 1$: $\mu =$ mean

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), \alpha \neq 1 \\ -\frac{2}{\pi}\log|q|, \alpha = 1 \end{cases}$$





In 3D: $$\mathcal{L}(\boldsymbol{r};\alpha,R) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q} e^{i\boldsymbol{q}\boldsymbol{r}} e^{-\frac{1}{2}|\boldsymbol{q}\mathbf{R}\boldsymbol{q}|^{\alpha/2}}$$

- **Important characteristics of stable distributions:**
  - Retains same α and β under convolution of random variables
  - Any moment greater than α isn't defined

$$R^2_{\sigma\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & R^2_{\text{out}} & 0 & 0 \\ 0 & 0 & R^2_{\text{side}} & 0 \\ 0 & 0 & 0 & R^2_{\text{long}} \end{pmatrix}$$

Second order phase transition?

- Second order phase transitions: **critical exponents**
  - **Near the critical point**
    - Specific heat ~ $((T - T_c)/T_c)^{-\alpha}$
    - Order parameter ~ $((T - T_c)/T_c)^{-\beta}$
    - Susceptibility/compressibility ~ $((T - T_c)/T_c)^{-\gamma}$
    - Correlation length ~ $((T - T_c)/T_c)^{-\nu}$
  - **At the critical point**
    - Order parameter ~ (source field)$^{1/\delta}$
    - **Spatial correlation function** ~ $r^{-d+2-\eta}$
  - Ginzburg-Landau: $\alpha = 0, \beta = 0.5, \gamma = 1, \eta = 0.5, \delta = 3, \eta = 0$
- QCD ↔ 3D Ising model
- Can we measure the η power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- **What distribution has a power-law exponent? Levy-stable distribution!**

Lévy index as critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;

  Lévy source: $\sim r^{-(1+\alpha)}$; $\boldsymbol{\alpha \Leftrightarrow \eta}$?

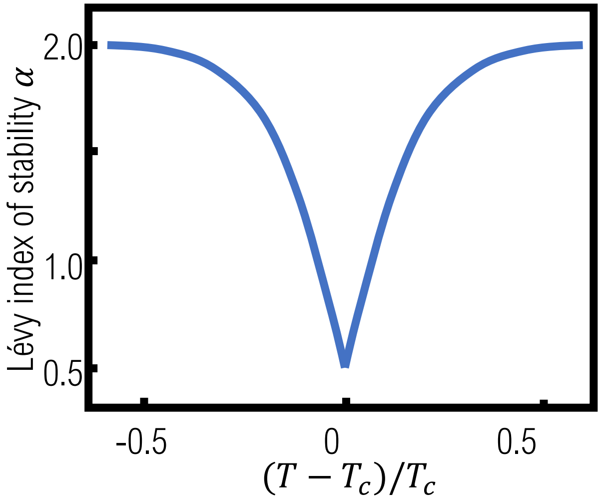
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising

  Halasz et al., Phys.Rev.D58 (1998) 096007

  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$

    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)

    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- **Change in α_{Levy} - proximity of CEP?**



- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distributions: anomalous diffusion, QCD jets, …

Kinematic variables of the correlation function I.

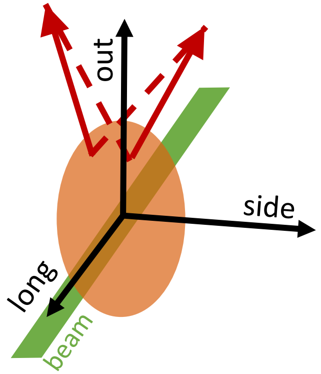
- Smoothness approximation ($p_1 \approx p_2 \approx K$): $S(x_1, K - q/2)\, S(x_2, K + q/2) \;\approx\; S(x_1, K)\, S(x_2, K)$
- $C_2(q, K) = \int d^4 r D(r, K) \left|\psi_q^{(2)}(\mathrm{r})\right|^2$
- Without any FSI $\left|\psi_q^{(2)}(\mathrm{r})\right|^2 = 1 + \cos(\mathrm{qr})$

$C_2^{(0)}(q, K) \simeq 1 + \frac{\widetilde{D}(q,K)}{\widetilde{D}(0,K)}$, where $\widetilde{D}(q, K) = \int D(x, K) e^{iqx} d^4 x$

- **HBT correlation function in direct connection with Fourier transform of the pair-source function**
- Important to determine the nature and dimensionality of the correlation function
- Lorentz-product of $q = (q_0, \boldsymbol{q})$ and $K = (K_0, \boldsymbol{K})$ is zero, i.e.: $qK = q_0 K_0 - \boldsymbol{qK} = 0$
- Energy component of q can be expressed as $q_0 = \boldsymbol{q}\frac{\boldsymbol{K}}{K_0}$
- If the energy of the particles are similar, K is approximately on shell
- **Correlation function can be measured as a function of three-momentum variables**

Kinematic variables of the correlation function II.

- $C_2(\boldsymbol{q},\boldsymbol{K})$ as a function of three-momentum variables
- $\boldsymbol{K}$ dependence is smoother, $\boldsymbol{q}$ is the main kinematic variable
- Close to mid-rapidity one can use $k_T = \sqrt{K_x^2 + K_y^2}$, or $m_T = \sqrt{k_T^2 + m^2}$
- For any fixed value of m_T, the correlation function can be measured as a function of $\boldsymbol{q}$ only
- Usual decomposition: **out-side-long or Bertsch-Pratt (BP) coordinate-system**
  - $\boldsymbol{q} \equiv (q_{out}, q_{side}, q_{long})$
  - long: beam direction
  - out: k_T direction
  - side: orthogonal to the others
  - Essentially a rotation in the transverse plane
- Customary to use a Lorentz-boost in the long directon and change to the **Longitudnal Co-Moving System (LCMS)** where the average longitudinal momentum of the pair is zero



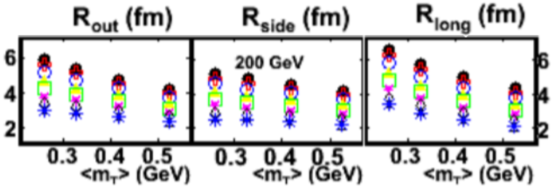
Kinematic variables of the correlation function III.

- Drawback of a 3D measurement: lack of statistics, difficulties of a precise shape analysis
- **What is the appropriate one-dimensional variable?**
- Lorentz-invariant relative momentum: $q_{inv} \equiv \sqrt{-q^{\mu}q_{\mu}} = \sqrt{q_x^2 + q_y^2 + q_z^2 - (E_1 - E_2)^2}$
- Equivalent to three-mom. diff. in Pair Co-Moving System (PCMS), where $E_1 = E_2$: $q_{inv} = |\boldsymbol{q}_{PCMS}|$
- In LCMS using BP variables: $q_{inv} = \sqrt{(1-\beta_T)^2 q_{out}^2 + q_{side}^2 + q_{long}^2}$ $\beta_T = 2k_T/(E_1 + E_2)$
- **Value of q_{inv} can be relatively small even when q_{out} is large!**
- Experimental indications: **in LCMS source is $\approx$ spherically symmetric**
- Correlation function boosted to PCMS will not be spherically symmetric
- Let us introduce the following variable invariant to Lorentz boosts in the beam direction:

$$Q \equiv |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{z,LCMS}^2},$$

$$\text{where } q_{z,LCMS}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}.$$

STAR, Phys.Rev.C 92 (2015) 1, 014904



Kinematic variables of the correlation function IV.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

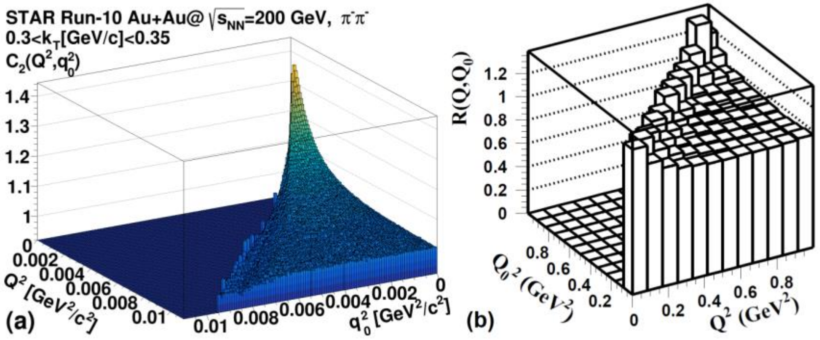


Figure 3.4: Example two-dimensional pion correlation functions for $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions (a) and $\sqrt{s} = 91$ GeV e^+e^- collisions (b). The latter figure is taken from the thesis of Tamás Novák [161].

Q dep. corr.func.

$\boldsymbol{q_{inv}}$ dep. corr.func.

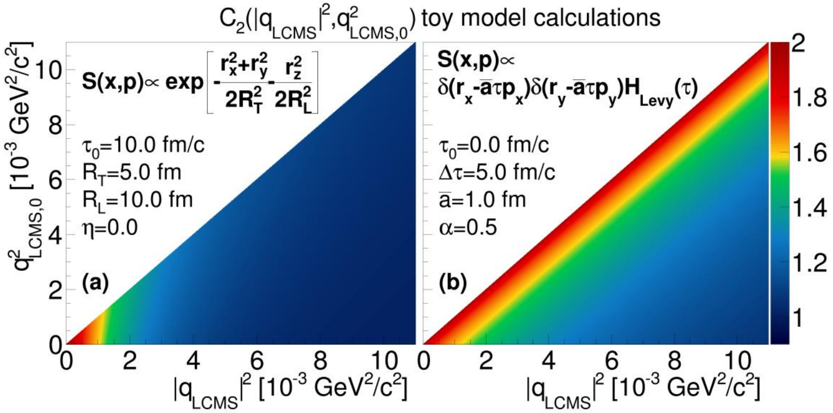
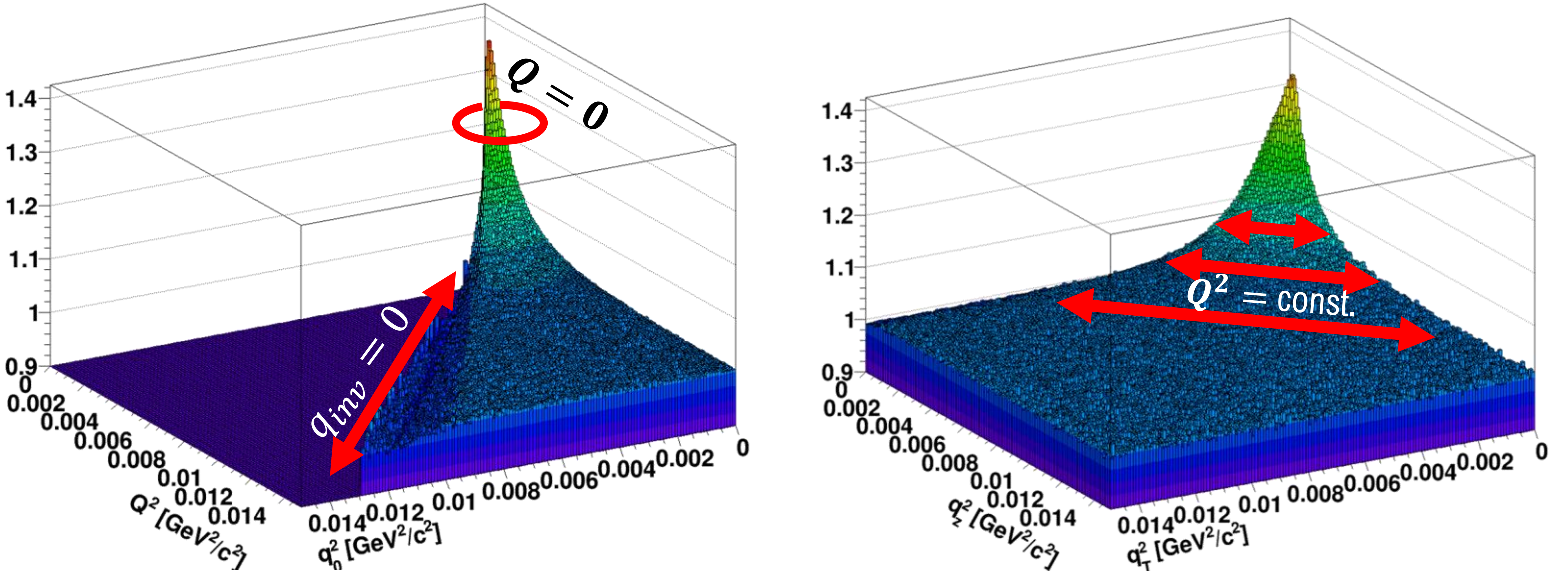


Figure 3.5: Toy model calculation for two different types of source functions. Taking a Gaussian source in both space and time leads to a correlation function that depends mostly on $|q_{LCMS}|$ (a), while a source that shows strong space-time and momentum space correlation leads to a q_{inv} dependent correlation function (b).

Kinematic variables of the correlation function V.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + \left(p_{1y} - p_{2y}\right)^2 + q_{long,LCMS}^2}$$



D. Kincses, Ph.D. thesis, 10.15476/ELTE.2022.164

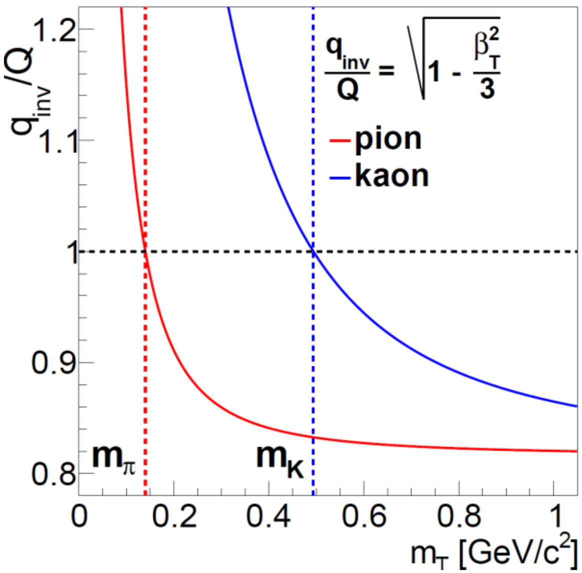
Kinematic variables of the correlation function VI.

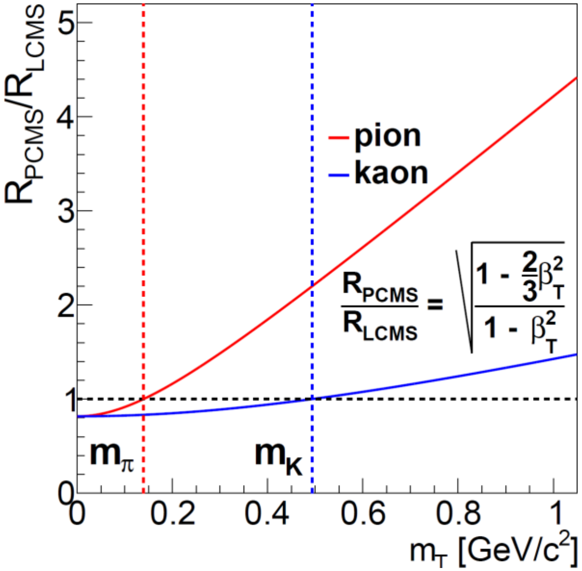
- Correlation function measured in LCMS, Coulomb effect calculated in PCMS
- Approximation:
- (Note $m_T < m$

  not physical of course)

$$q_{\text{inv}} \equiv q_{\text{PCMS}} \approx q_{\text{LCMS}} \cdot \sqrt{1 - \beta_T^2/3}$$

$$R_{\text{PCMS}} \approx R_{\text{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$





Coulomb correction and fitting of the corr. function

- Core-Halo model, Bowler-Sinyukov method: $C_2(Q,k_T) = 1 - \lambda + \lambda \int d^3\boldsymbol{r} D_{(c,c)}(\boldsymbol{r},k_T) |\Psi_Q^{(2)}(\boldsymbol{r})|^2$
- Neglecting FSI and using a Lévy-stable source function: $C_2^{(0)}(Q,k_T) = 1 + \lambda e^{-|RQ|^\alpha}$
- **Using numerical integral calculation as fit function results in numerically fluctuating χ^2 landscape**
- Treat FSI as correction factor: $K(Q,k_T) = \dfrac{C_2(Q,k_T)}{C_2^{(0)}(Q,k_T)}$
- An iterative method can be used: $C_2^{(fit)}(Q;\lambda,R,\alpha) = C_2^{(0)}(Q;\lambda,R,\alpha) \cdot K(Q;\lambda_0,R_0,\alpha_0)$
- Procedure continued until $\Delta_{\mathrm{iteration}} = \sqrt{\frac{(\lambda_{\mathrm{n}+1}-\lambda_{\mathrm{n}})^2}{\lambda_{\mathrm{n}}^2} + \frac{(R_{\mathrm{n}+1}-R_{\mathrm{n}})^2}{R_{\mathrm{n}}^2} + \frac{(\alpha_{\mathrm{n}+1}-\alpha_{\mathrm{n}})^2}{\alpha_{\mathrm{n}}^2}} < 0.01$
- **Iterations usually converge within 2-3 rounds, fit parameters can be reliably extracted**

Coulomb correction and fitting of the corr. function

- Lévy-type correlation function without final state effects: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- Bowler-Sinyukov method:

Intercept parameter (correlation strength)

Lévy scale parameter

$$C(Q_{\mathrm{LCMS}}; \lambda, R_{LCMS}, \alpha) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{\mathrm{PCMS}}) \cdot \left(1 + e^{-|R_{\mathrm{LCMS}} Q_{\mathrm{LCMS}}|^{\alpha}}\right)\right) \cdot \overbrace{N \cdot (1 + \varepsilon Q_{\mathrm{LCMS}})}^{\text{Possible linear background (usually negligible)}}$$

Coulomb correction

Lévy exponent

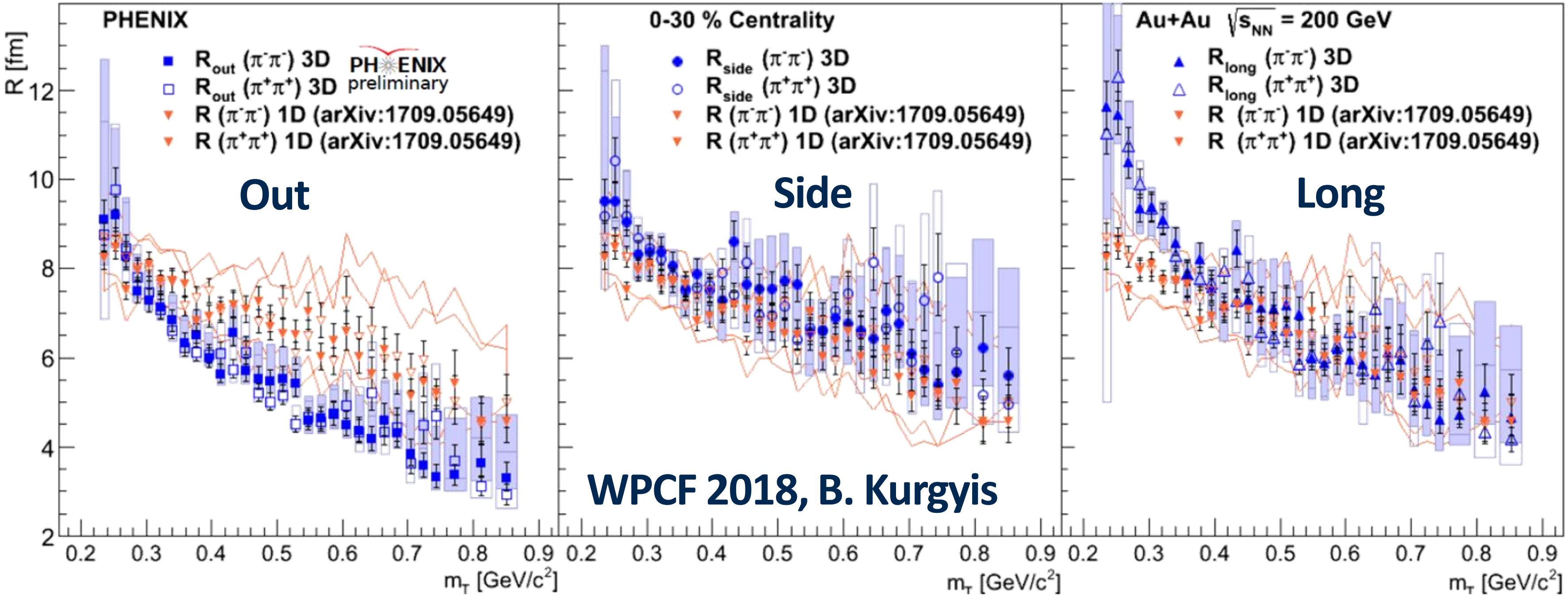
- Coulomb-correction calculated numerically (in PCMS)

$$q_{\mathrm{inv}} \equiv q_{\mathrm{PCMS}} \approx q_{\mathrm{LCMS}} \cdot \sqrt{1 - \beta_T^2/3} \qquad R_{\mathrm{PCMS}} \approx R_{\mathrm{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$

B. Kurgyis for the PHENIX Coll.,
Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), pp. 477 - 482 (2019)

Cross-check with 3D analysis – PHENIX preliminary

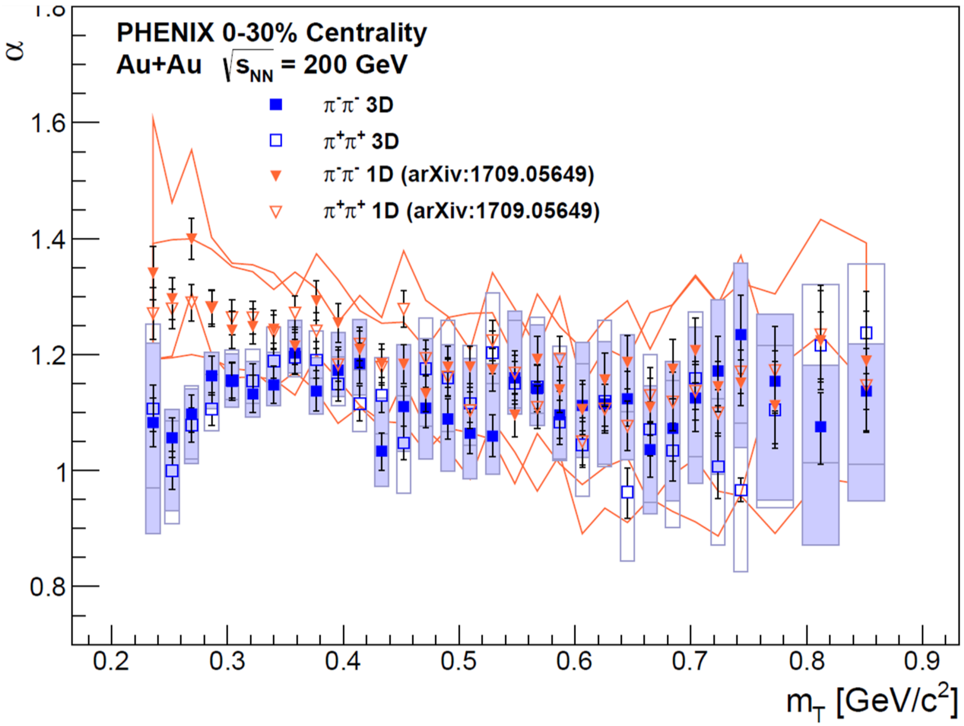
$$C(Q) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{inv}) \cdot \left(1 + e^{-\left|R_o^2 q_o^2 + R_s^2 q_s^2 + R_l^2 q_l^2\right|^{\alpha/2}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$



WPCF 2018, B. Kurgyis

B. Kurgyis for the PHENIX Coll.,
Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), pp. 477 - 482 (2019)

Cross-check with 3D analysis – PHENIX preliminary



- **Compatible with 1D (Q_{LCMS}) measurement** of Phys. Rev. C 97, 064911 (2018)
- Small discrepancy at small m_T: due to large R_{long} at small m_T?

3D Gaussian vs 1D Levy

- **3D Gaussian does not result in 1D Levy**
  - Difference: several percent
  - Available experimental precision: much better than this difference