Two-particle interferometry with Lévy-stable sources in Au+Au collisions at STAR

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Part I.
Introduction, Motivation
Basic definitions of femtoscopic correlation functions

- Single particle momentum distribution:
  \[ N_1(p) = \int d^4x S(x,p) \]

- Pair momentum distribution:
  \[ N_2(p_1, p_2) = \int d^4x_1 d^4x_2 S(x_1, p_1) S(x_2, p_2) |\psi_{p_1, p_2}(x_1, x_2)|^2 \]

- Correlation function:
  \[ C(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_2(p_2)} \]

- Pair source/spatial correlation:
  \[ D(r, K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right) \]

- Experiments: measuring \( C(Q) \) → information about \( D(r) \) and FSI
- Experimental (and phenomenological) indications:
  power-law tail for pions, non-Gaussianity?

What is the shape of the source?
Gaussian vs. Lévy distributions in heavy-ion physics


• Symmetric Lévy-stable distribution
  - From generalized central limit theorem, power-law tail (if $\alpha < 2 \sim r^{-(1+\alpha)}$)
  - $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
  - Retains the same $\alpha$ under convolution

• Experimental indications – Lévy source for pion pairs?
  - RHIC (PHENIX, STAR)
  - LHC (CMS)
  - SPS (NA61/SHINE)

See talk of M. Csanád and B. Pórfy!

• Possible interpretations of the $\alpha$ Lévy exponent based on:
  - Jet fragmentation
  - Critical behavior
  - Event averaging
  - Resonance decays
  - Anomalous diffusion

• Lévy in 1D: not because of 3D $\rightarrow$ 1D conversion!

New data analyses and phenomenological investigations are needed to gain a better understanding!
\[ S(r) = L(\alpha, R; r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2} q^T R^2 q} |\alpha/2 \]

**Lévy source at RHIC energies**

- Not spherically sym. source: 3D vs. 1D \( \alpha \) compatible!
  \( \alpha < 2 \) in 1D analyses not because of angle averaging!


\[ R^2 = \begin{pmatrix} R_{out}^2 & 0 & 0 \\ 0 & R_{side}^2 & 0 \\ 0 & 0 & R_{long}^2 \end{pmatrix}, \quad q = \begin{pmatrix} q_{out} \\ q_{side} \\ q_{long} \end{pmatrix} \]

- EPOS pion pair-source analysis at 200 GeV:
  Decays and hadronic rescattering (UrQMD) play an important role!

  Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308
Recent phenomenological developments

- Coulomb Corrections for Bose–Einstein Correlations from One- and Three-Dimensional Lévy-Type Source Functions  
  Kurgyis, Kincses, Nagy, Csanád, Universe 9 (2023) 7, 328

- Event-by-Event Investigation of the Two-Particle Source Function in Heavy-Ion Collisions with EPOS  
  Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308  
  Kórodi, Kincses, Csanád, Phys. Lett. B 847 (2023) 138295

- A novel method for calculating Bose-Einstein correlation functions with Coulomb final-state interaction  
  See talk of M. Csanád!

See talk of M. Nagy!
Part II.
Measurement and fitting of correlation functions
Lévy HBT analysis at STAR, Au+Au @ 200 GeV

- **STAR Run-11 data analyzed**
  After trigger cuts and bad run cuts: **550M events**

- **Detectors used for the analysis:**
  - BBC, TPC, VPD: centrality, vertex position
  - TPC: tracking, dE/dx Particle Identification (PID)
  - TOF: time-of-flight PID

- **Event selection:**
  - Pile-up cuts using TOF vs. TPC multiplicity
  - Vertex cuts:
Measurement of two-pion correlation functions

- **Track-selection criteria**
  - Combined PID using TPC $N\sigma$ (based on dE/dx) and TOF $N\sigma$ (based on time-of-flight)
  - Further single-track cuts on TPC number of hits, $p_T$, $\eta$, Distance of Closest Approach (DCA)

- **Pair-selection criteria**
  - Splitting level (SL) $< 0.6$
  - Fraction of Merged Hits (FMH) $< 5$
  - Average pair-separation (on TPC pad rows) $\Delta r > 3$ cm

- **Event mixing**
  - Similarly to *PHENIX Coll., Phys.Rev. C 97 (2018) no.6, 064911*
  - 2 cm wide z vertex bins, 5% wide centrality bins
  - A(Q): pions from the same event
  - B(Q): pions from different events
  - C(Q)=A(Q)/B(Q), appropriately normalized

- **Centrality and $k_T$ selection:**
  - 21 $k_T$ bins, (0.175 - 0.750) GeV/c
  - 4 centrality bins (0-10%, 10-20%, 20-30%, 30-40%)
Fitting process with Lévy parametrization

- Lévy parametrization without final state effects:

\[ C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^\alpha} \]

LCMS three-momentum difference

\[ Q = |q_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2} \]

Lévy exponent

Lévy scale parameter

Intercept parameter (correlation strength)

- Formula used for fitting procedure:

\[ C(Q) = \left(1 - \lambda + \lambda \cdot K(Q; \alpha, R) \cdot (1 + e^{-|RQ|^\alpha})\right) \cdot N \cdot (1 + \varepsilon Q) \]

Coulomb correction

Possible linear background (usually negligible)

- Coulomb-correction:

\[ K(Q; \alpha, R) = \frac{\int D(r)|\psi^{Coul}(r)|^2 dr}{\int D(r)|\psi^{(0)}(r)|^2 dr} \]

Spatial correlations

Two-particle wave function (with Coulomb interaction)

Two-particle wave function (plane wave)

\[ \text{calculated numerically} \]

Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 6, 064912
An example fit to a two-pion correlation function

\[ C(Q) = \left( 1 - \lambda + \lambda \cdot K(Q; \alpha, R) \cdot (1 + e^{-|RQ|^\alpha}) \right) \cdot N \cdot (1 + \varepsilon Q) \]

- Example fit in the 10-20% centrality class, at \( k_T = (0.250-0.275) \text{ GeV/c} \)

- Iterative fitting method, Coulomb FSI + Lévy-source

- Track and pair syst. uncertainties illustrated with boxes

- Low-Q points left out from fit, fit range study included in total systematic uncertainties

- Fits converged, conf.level > 0.1%

- Confidence levels approx. uniformly distributed

- Similar fits done in all 4 centrality class and 21 \( k_T \) bins, centrality and \( m_T \) dependence of the source parameters investigated
Part III.
Preliminary results at 200 GeV (centrality, $m_T$ dependence)
Lévy exponent $\alpha(m_T, \text{centrality})$

- Non-gaussian values ($\alpha \ll 2$)
- No dependence on $m_T$, slight centrality dep.
- $\alpha_0$ vs $N_{\text{part}}$ ($m_T$ average values from constant fit)
  - Decreasing trend due to anti-correlation with $\lambda$ and $R$?
  - CMS observed opposite trend, see talk of M. Csanád
- EPOS model: slightly higher $\alpha$ values

*D. Kincses, M. Stefaniak, M. Csanád, Entropy 24 (2022) 3, 308*
Lévy scale $R(m_T, \text{centrality})$

- Decreasing trend with $m_T$, also with centrality
- Connection to flow and initial geometry?
- Fits with $R = R_0(Am_T + B)^{-1/\xi}$, $R_0 = 1$ fm
  - Hydro calculations ($R_{Gauss}$): $\xi = 2$


- Our case ($R_{Levy}$): $\xi$ close to 1
Correlation strength $\lambda(m_T, centrality)$

- Without FSI: $C_2^{(0)}(Q = 0) = 2 \rightarrow \lambda \equiv \lim_{Q \to 0} C_2^{(0)}(Q) - 1$, experimentally often $\lambda < 1$
- **Core-halo picture** - two-component source: $S = S_{core} + S_{halo} \Rightarrow D = D_{(c,c)} + D_{(c,h)} + D_{(h,h)}$
  - Core: primordial + decays of short-lived resonances
  - Halo: decays of long-lived resonances $\rightarrow$ large $R$ $\rightarrow$ small $Q$ $\rightarrow$ measurement limited

\[
\lambda = \frac{N_{core}^2}{(N_{core} + N_{halo})^2}
\]


- For power-law sources, more complicated picture!
- Increase from low to high $m_T$
  - More decay products at low $m_T$?
  - In-medium mass modification of $\eta'$?
  - Partially coherent particle emission?


- Decrease from central to peripheral
Part IV.
Preliminary results at BES-II
($\sqrt{s_{NN}}, m_T$ dependence)
Beam energy dependent analysis in Au+Au collisions

- **BES-II data taking** recently conducted at STAR
  - Increased luminosity
  - Many detector improvements

- Next step: similar analysis with the same settings at lower Au+Au energies
  - Run-17, **54.4 GeV**
  - Run-18, **27 GeV**
  - Run-19, **19.6 GeV**
  - Run-19, **14.5 GeV**
  - Run-21, **7.7 GeV**
$\sqrt{S_{NN}}$ and $m_T$ dependence of the source parameters

- $\sqrt{S_{NN}}$ dependence from 200 to 7.7 GeV:
  \[ \alpha \uparrow, R \downarrow, \lambda \downarrow \]

- $m_T$ dependent trends at all energies:
  \[ \alpha \text{ const.}, R \downarrow, \lambda \uparrow \]
Excitation function of the Lévy exponent

- Non-gaussian values ($\alpha \ll 2$)

- Monotonic decrease from 7.7 GeV to 200 GeV

- CMS 0-10% PbPb result at 5.02 TeV: 
  \[ \alpha_0 \approx 1.86 \]
  (note that the kinematic range is different, see talk of M. Csanád)

- Interpretation of $\alpha$ still an open question, possibilities include:
  - Jet fragmentation
  - Critical behavior
  - Event averaging
  - Resonance decays
  - Anomalous diffusion
Summary

- 1-dim. two-pion correlation functions investigated
- Lévy-source + Coulomb FSI $\rightarrow$ good description
- Further syst. uncertainty investigations underway

- 0-10% Au+Au: $200$ GeV $\rightarrow$ $7.7$ GeV $\quad \alpha \uparrow, R \downarrow, \lambda \downarrow$
- 200 GeV Au+Au: central $\rightarrow$ peripheral

- Next steps: even lower energies (fxt), 3D analysis!
Further details, backup slides
Systematic uncertainties

- **Systematic uncertainties already investigated:**
  - Single track- and pair-cut variations, fit limits

- **Systematic uncertainties to be investigated:**
  - Purity correction, momentum smearing, more detailed fit limit study, strong interaction

### Table II. Sources of systematic uncertainties.

<table>
<thead>
<tr>
<th>$n$</th>
<th>source of uncertainty</th>
<th>settings ($j = 0, 1, \ldots$)</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>NHitsFit cut</td>
<td>18, 20, 22</td>
</tr>
<tr>
<td>1</td>
<td>DCA cut</td>
<td>1.5 cm, 2 cm, 2.5 cm</td>
</tr>
<tr>
<td>2</td>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>3</td>
<td>PID $N\sigma$</td>
<td>default, loose, strict</td>
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<tr>
<td>4</td>
<td>SL cut</td>
<td>0.5, 0.6, 0.7</td>
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<tr>
<td>5</td>
<td>FMH cut</td>
<td>0%, 5%, 10%</td>
</tr>
<tr>
<td>6</td>
<td>$\Delta u$, $\Delta z$ limits</td>
<td>default, loose, strict</td>
</tr>
<tr>
<td>7</td>
<td>Lower fit limit in $Q$</td>
<td>default, +1 bin, -1 bin</td>
</tr>
<tr>
<td>8</td>
<td>Higher fit limit in $Q$</td>
<td>default, +5 bin, -5 bin</td>
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</table>

### Table III. $m_T$ averaged asymmetric systematic uncertainties [%].

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<th>$n$</th>
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<th>$10-20%$</th>
<th>$20-30%$</th>
<th>$30-40%$</th>
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<td>0.5</td>
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<td>0.7</td>
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<td>0.2</td>
<td>0.2</td>
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<td>1.1</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
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<tr>
<td>$\Sigma$</td>
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<td>5.2</td>
<td>4.7</td>
<td>4.9</td>
<td>4.6</td>
</tr>
</tbody>
</table>
Properties of univariate stable distributions

- **Univariate stable distribution:** \( f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq \), where the characteristic function:
  - \( \varphi(q; \alpha, \beta, R, \mu) = \exp(\text{i}q\mu - |qR|^\alpha (1 - i\beta \text{sgn}(q) \Phi)) \)
  - \( \alpha \): index of stability
  - \( \beta \): skewness, symmetric if \( \beta = 0 \)
  - \( R \): scale parameter
  - \( \mu \): location, equals the median, if \( \alpha > 1 \): \( \mu = \text{mean} \)

- **Important characteristics of stable distributions:**
  - Retains same \( \alpha \) and \( \beta \) under convolution of random variables
  - Any moment greater than \( \alpha \) isn’t defined

In 3D:

\[
\mathcal{L}(r; \alpha, R) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha/2}
\]

\[
\begin{pmatrix}
R^2_{\sigma\nu} \\
R^2_{\text{out}} & 0 & 0 \\
0 & R^2_{\text{side}} & 0 \\
0 & 0 & R^2_{\text{long}}
\end{pmatrix}
\]
Second order phase transition?

- Second order phase transitions: **critical exponents**
  - Near the critical point
    - Specific heat \( \sim (\frac{T - T_c}{T_c})^{-\alpha} \)
    - Order parameter \( \sim (\frac{T - T_c}{T_c})^{-\beta} \)
    - Susceptibility/compressibility \( \sim (\frac{T - T_c}{T_c})^{-\gamma} \)
    - Correlation length \( \sim (\frac{T - T_c}{T_c})^{-\nu} \)
  - At the critical point
    - Order parameter \( \sim (\text{source field})^{1/\delta} \)
    - **Spatial correlation function** \( \sim r^{-d+2-\eta} \)
      - Ginzburg-Landau: \( \alpha = 0, \beta = 0.5, \gamma = 1, \eta = 0.5, \delta = 3, \eta = 0 \)
- QCD ↔ 3D Ising model
- Can we measure the \( \eta \) power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- **What distribution has a power-law exponent?** Levy-stable distribution!
Lévy index as critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  

- QCD universality class $\leftrightarrow$ 3D Ising
  

- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
  - 3D Ising: $\eta = 0.03631(3)$

- Motivation for precise Lévy HBT!
- Change in $\alpha_{\text{Lévy}}$ - proximity of CEP?

- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distributions: anomalous diffusion, QCD jets, …
Kinematic variables of the correlation function I.

- Smoothness approximation \((p_1 \approx p_2 \approx K)\): \(S(x_1, K - q/2) S(x_2, K + q/2) \approx S(x_1, K) S(x_2, K)\)

- \(C_2(q, K) = \int d^4 r D(r, K) \left| \psi_q^{(2)}(r) \right|^2\)

\[ C_2^{(0)}(q, K) \approx 1 + \frac{\tilde{D}(q, K)}{\tilde{D}(0, K)}, \text{ where } \tilde{D}(q, K) = \int D(x, K)e^{iqr}d^4x \]

- Without any FSI \(\left| \psi_q^{(2)}(r) \right|^2 = 1 + \cos(qr)\)

- **HBT correlation function in direct connection with Fourier transform of the pair-source function**

- Important to determine the nature and dimensionality of the correlation function

- Lorentz-product of \(q = (q_0, \vec{q})\) and \(K = (K_0, \vec{K})\) is zero, i.e.: \(qK = q_0K_0 - \vec{q}\vec{K} = 0\)

- Energy component of \(q\) can be expressed as \(q_0 = q \frac{K}{K_0}\)

- If the energy of the particles are similar, \(K\) is approximately on shell

- **Correlation function can be measured as a function of three-momentum variables**
Kinematic variables of the correlation function II.

- $C_2(q, K)$ as a function of three-momentum variables
- $K$ dependence is smoother, $q$ is the main kinematic variable
- Close to mid-rapidity one can use $k_T = \sqrt{K_x^2 + K_y^2}$, or $m_T = \sqrt{k_T^2 + m^2}$
- For any fixed value of $m_T$, the correlation function can be measured as a function of $q$ only
- Usual decomposition: **out-side-long or Bertsch-Pratt (BP) coordinate-system**
  - $q \equiv (q_{out}, q_{side}, q_{long})$
  - long: beam direction
  - out: $k_T$ direction
  - side: orthogonal to the others
  - Essentially a rotation in the transverse plane
- Customary to use a Lorentz-boost in the long direction and change to the **Longitudinal Co-Moving System (LCMS)** where the average longitudinal momentum of the pair is zero
Kinematic variables of the correlation function III.

- Drawback of a 3D measurement: lack of statistics, difficulties of a precise shape analysis
- What is the appropriate one-dimensional variable?
- Lorentz-invariant relative momentum: $q_{\text{inv}} \equiv \sqrt{-q^\mu q_\mu} = \sqrt{q_x^2 + q_y^2 + q_z^2 - (E_1 - E_2)^2}$
- Equivalent to three-mom. diff. in Pair Co-Moving System (PCMS), where $E_1 = E_2$: $q_{\text{inv}} = |q_{PCMS}|$
- In LCMS using BP variables: $q_{\text{inv}} = \sqrt{(1 - \beta_T)^2 q_{\text{out}}^2 + q_{\text{side}}^2 + q_{\text{long}}^2}$, $\beta_T = 2k_T/(E_1 + E_2)$
- Value of $q_{\text{inv}}$ can be relatively small even when $q_{\text{out}}$ is large!
- Experimental indications: in LCMS source is $\approx$ spherically symmetric
- Correlation function boosted to PCMS will not be spherically symmetric
- Let us introduce the following variable invariant to Lorentz boosts in the beam direction:

$$Q \equiv |q_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{z,LCMS}^2}$$

where $q_{z,LCMS}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}$. 

*STAR, Phys.Rev.C 92 (2015) 1, 014904*
Kinematic variables of the correlation function IV.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

Figure 3.4: Example two-dimensional pion correlation functions for $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions (a) and $\sqrt{s} = 91$ GeV $e^+e^-$ collisions (b). The latter figure is taken from the thesis of Tamás Novák [161].

$Q$ dep. corr.func.  $q_{\text{inv}}$ dep. corr.func.

Figure 3.5: Toy model calculation for two different types of source functions. Taking a Gaussian source in both space and time leads to a correlation function that depends mostly on $|q_{\text{CMS}}|$ (a), while a source that shows strong space-time and momentum space correlation leads to a $q_{\text{inv}}$ dependent correlation function (b).
Kinematic variables of the correlation function $V$.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

$$Q = |q_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

Kinematic variables of the correlation function VI.

- Correlation function measured in LCMS, Coulomb effect calculated in PCMS
- Approximation:
  \[ q_{\text{inv}} \equiv q_{\text{PCMS}} \approx q_{\text{LCMS}} \cdot \sqrt{1 - \frac{\beta_T^2}{3}} \]
  \[ R_{\text{PCMS}} \approx R_{\text{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}} \]
- (Note \( m_T < m \) not physical of course)
Coulomb correction and fitting of the corr. function

- Core-Halo model, Bowler-Sinyukov method: \( C_2(Q, k_T) = 1 - \lambda + \lambda \int d^3r D_{(c,c)}(r, k_T) |\Psi_Q^{(2)}(r)|^2 \)

- Neglecting FSI and using a Lévy-stable source function: \( C_2^{(0)}(Q, k_T) = 1 + \lambda e^{-|RQ|^\alpha} \)

- Using numerical integral calculation as fit function results in numerically fluctuating \( \chi^2 \) landscape

- Treat FSI as correction factor: \( K(Q, k_T) = \frac{C_2(Q, k_T)}{C_2^{(0)}(Q, k_T)} \)

- An iterative method can be used: \( C_2^{(fit)}(Q; \lambda, R, \alpha) = C_2^{(0)}(Q; \lambda, R, \alpha) \cdot K(Q; \lambda_0, R_0, \alpha_0) \)

- Procedure continued until \( \Delta_{\text{iteration}} = \sqrt{\frac{(\lambda_{n+1} - \lambda_n)^2}{\lambda_n^2} + \frac{(R_{n+1} - R_n)^2}{R_n^2} + \frac{(\alpha_{n+1} - \alpha_n)^2}{\alpha_n^2}} < 0.01 \)

- Iterations usually converge within 2-3 rounds, fit parameters can be reliably extracted
Coulomb correction and fitting of the corr. function

- Lévy-type correlation function without final state effects: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|\alpha}$

- Bowler-Sinyukov method:
  
  $C(Q_{LCMS}; \lambda, R_{LCMS}, \alpha) = (1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{PCMS}) \cdot (1 + e^{-|R_{LCMS}Q_{LCMS}|\alpha})) \cdot N \cdot (1 + \varepsilon Q_{LCMS})$

- Coulomb-correction calculated numerically (in PCMS)

  $q_{inv} \equiv q_{PCMS} \approx q_{LCMS} \cdot \sqrt{1 - \beta_T^2 / 3}$

  $R_{PCMS} \approx R_{LCMS} \cdot \sqrt{\frac{1 - 2\beta_T^2 / 3}{1 - \beta_T^2}}$
Cross-check with 3D analysis – PHENIX preliminary

\[ C(Q) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{inv}) \cdot \left(1 + e^{-|R_0 q_0^2 + R_5 q_5^2 + R_i q_i^2|^{\alpha/2}}\right)\right) \cdot N \cdot (1 + \epsilon Q) \]
Cross-check with 3D analysis – PHENIX preliminary

- Compatible with 1D ($Q_{LCMS}$) measurement of Phys. Rev. C 97, 064911 (2018)
- Small discrepancy at small $m_T$: due to large $R_{long}$ at small $m_T$?
3D Gaussian vs 1D Levy

- 3D Gaussian does not result in 1D Levy
  - Difference: several percent
  - Available experimental precision: much better than this difference