Heavy quark potential at non-zero temperature

Péter Petreczky

Brookhaven National Laboratory

Does color screening in high temperature QCD lead to quarkonium melting ?

- Introduction: heavy quark potential and quarkonia
- Lattice calculation of the potential at $T>0$ in $2+1$ flavor QCD on coarse lattices
  Bala et al (HotQCD), PRD 105 (2022) 054513
- Lattice calculation of the potential at $T>0$ in 2+1 flavor QCD on fine lattices
  Bazavov et al (HotQCD), arXiv:2308.16587
- Potential at $T>0$ and bottomonium in deconfined medium
- Summary

23rd Zimányi School, Budapest December 4-8, 2023

HEFTY COLLABORATION

Quarkonia and potential models

$m_b, m_c \gg \Lambda_{QCD}$ $\Rightarrow$ non-relativistic bound states, analogs QED positronium

1-gluon exchange, $\alpha_s \sim 0.4$

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + spin\ dep.$$

Confinement

Eichten et al, PRL 34 (75) 369, PRD 21 (80) 203

Very successful in describing charmonium and bottomonium spectrum below the the open charm and beauty threshold

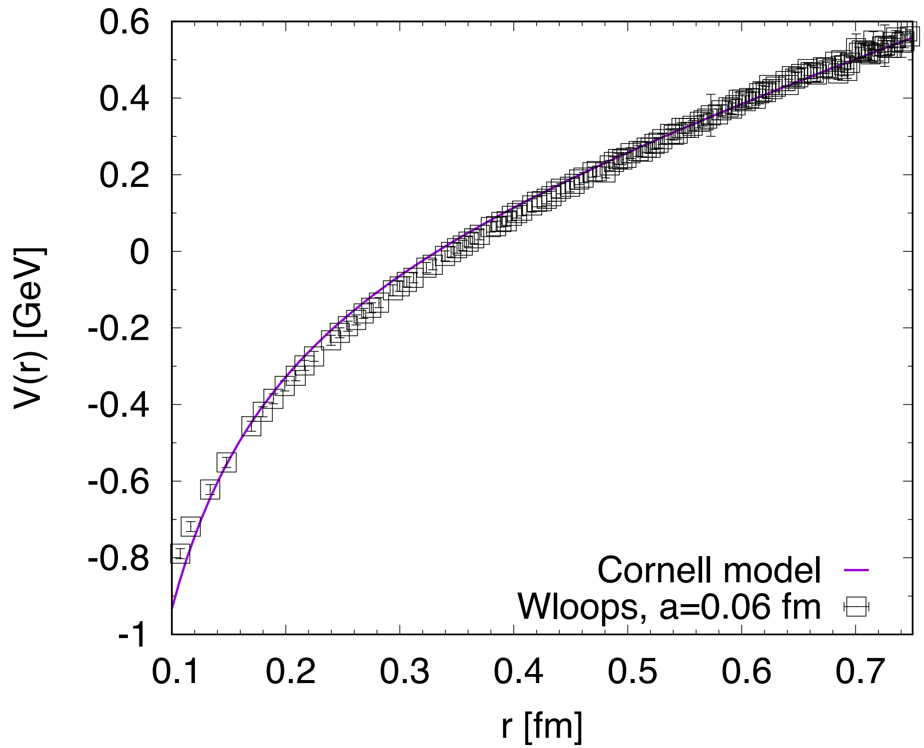
Nevertheless nearly perfect agreement between the phenomenological and lattice potentials

Problems:
Running of α_s ?
Linear potential valid only for $r \gg 1$ fm,

$$V(r) = \sigma r - \frac{\pi}{12r} + \ldots$$

Lüscher term

Soft gluon fields ??



Quarkonia in effective theory approach

$M \gg 1/r \sim Mv \gg Mv^2,\ M = m_{c,b}$ ➡ Effective theory (EFT) approach

Non-relativistic QCD (NRQCD) : EFT at scale $1/r$ (scale M is integrated out):

$$L_{NRQCD} = \psi^\dagger \left(iD_0 - \frac{D_i^2}{2M} \right) \psi + \chi^\dagger \left(iD_0 + \frac{D_i^2}{2M} \right) \chi + ... + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Heavy quark fields are Pauli spinors, heavy pair creation is only present implicitly through higher dimension 4-fermion operators

Caswell, Lepage, PLB 167 (86) 437

potential NRQCD (pNRQCD): EFT at scale $E_{bin} \sim Mv^2$ (scale $1/r \sim Mv$ is integrated out):

$$L_{pNRQCD} = \int d^3\mathbf{r} \mathrm{Tr} \Big[S^\dagger \big[i\partial_0 - (\frac{-\nabla_r^2}{M} + V_s(r) + \dots) \big] S + O^\dagger \big[iD_0 + \frac{-\nabla_r^2}{M} + V_o(r) + \dots) \big] O \Big]$$

$$+ V_A(r) \mathrm{Tr} \Big[O^\dagger \mathbf{r} g \mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O \Big] + V_B(r) \mathrm{Tr} \Big[O^\dagger \mathbf{r} g \mathbf{E} O + O^\dagger O \mathbf{r} g \mathbf{E} \Big] +$$

$$\mathcal{O}(r^2, \frac{1}{M}) + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Brambilla, Pineda, Soto, Vairo, NPB 566 (00) 275

$S = S(\mathbf{r}, \mathbf{R}, t),\ O = O(\mathbf{r}, \mathbf{R}, t),\ E = E(\mathbf{R}, t)$

Potentials are parameters of the EFT Largangian

Tree level $\leftrightarrow$ potential model $(i\partial_0 + \frac{\nabla_r^2}{M} - V_s(r)) S(\mathbf{r}, \mathbf{R}, t) = 0$

Quark anti-quark potential at *T>0*

Conjecture, Matsui and Satz, PLB 178 (86) 416 $-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \rightarrow -\frac{4}{3}\frac{\alpha_s}{r} e^{-m_D r}, T > T_c$

Extending pNRQCD to *T>0*: the potential is complex, the real part can have thermal correction but is not necessarily screened, except when $r \sim 1/m_D$ (HTL regime)

Based on weak coupling

Laine, Philipsen, Romatschke, Tassler, JHEP 03 (06) 054
Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017

Calculate the potential non-perturbatively on the lattice by considering Wislon loops of size *r x τ* at *T>0*

$$W(r,\tau,T) = \int_{-\infty}^{\infty} \rho_r(\omega,T) e^{-\omega\tau}$$

If potential at $T > 0$ exists the $\rho_r(\omega,T)$ should have a well defined peak at $\omega \simeq \mathrm{Re}V(r,T)$, and the width of the peak is $\mathrm{Im}V(r,T)$

Rothkopf, Hatsuda, Sasaki, PRL 108 (2012) 162001

Challenge: reconstruct $\rho_r(\omega,T)$

Hybrid potentials,
pairs of static-light mesons ...

$$\rho_r(\omega, T=0) = \delta(\omega - V(r)) + \sum_n \delta(\omega - E_n(r))$$

Lattice calculations on coarse lattices

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied by varying $a \leftrightarrow \beta = 10/g^2$ $T = 151 - 1938$ MeV

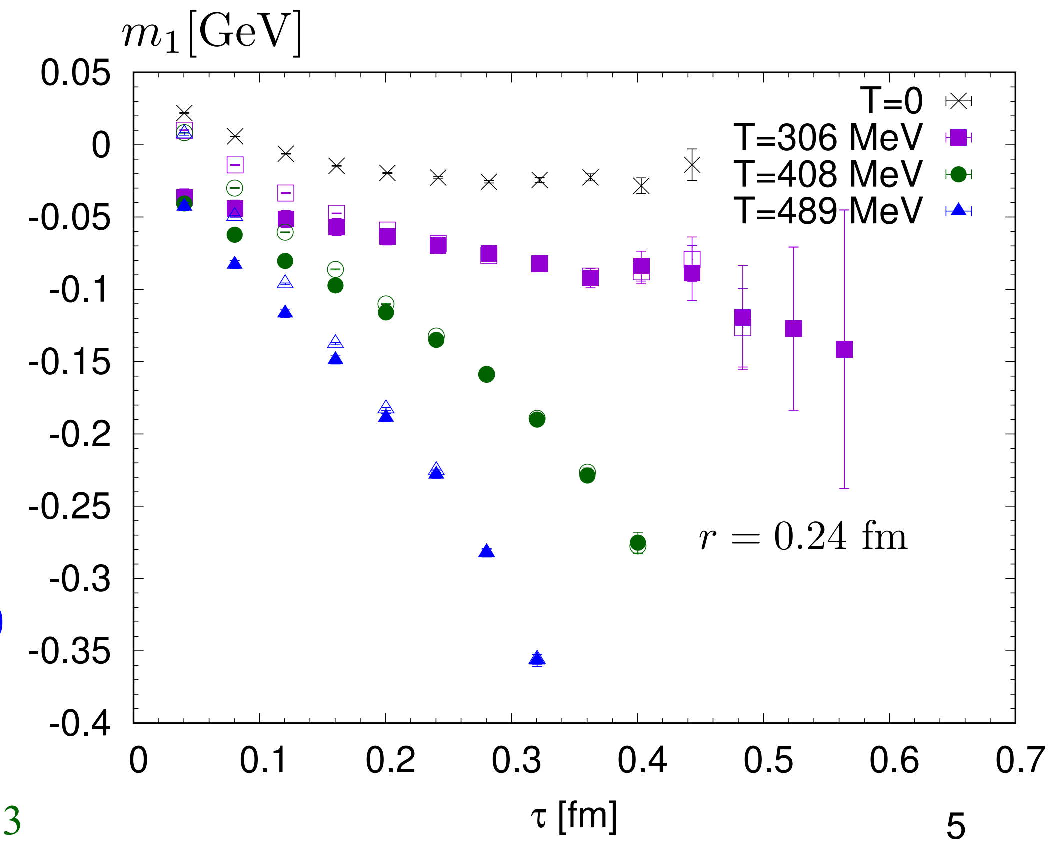
Wilson line correlators in Coulomb gauge for better signal-to-noise ratio (and also Wilson loops with HYP smeared spatial links)

Cumulants:

$$m_1(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T),$$
$$m_n = \partial_\tau m_{n-1}(r,\tau,T), n > 1$$

$$m_1(r,\tau,T) = \frac{1}{a} \ln \frac{W(r,\tau,T)}{W(r,\tau+a,T)}$$

$T = 0$: m_1 (effective mass=$m_{\rm eff}$) reaches a plateau at large τ



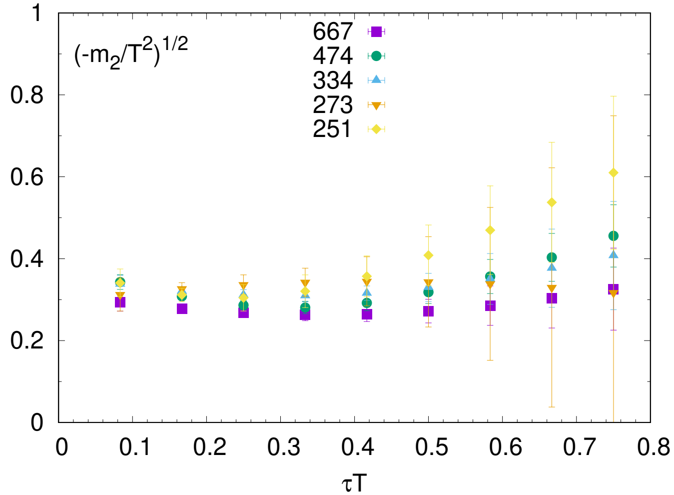
- No plateau at $T > 0$ in m_1 at $T > 0$
- Only tiny T-dependence for small τ

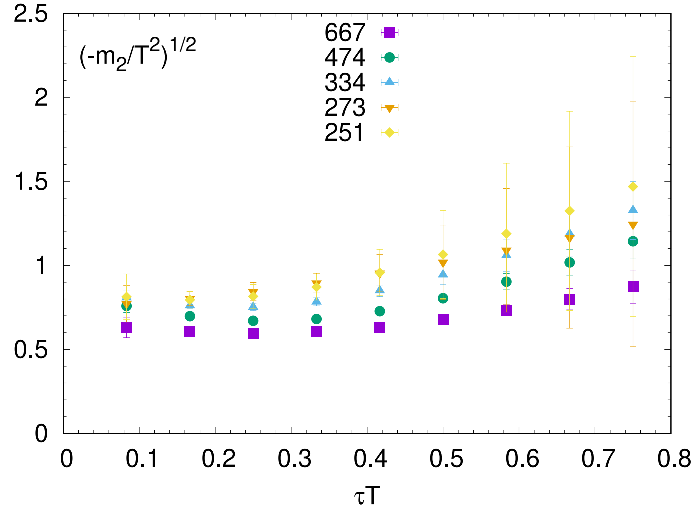
Bala et al (HotQCD), PRD 105 (2022) 054513

Higher cumulants

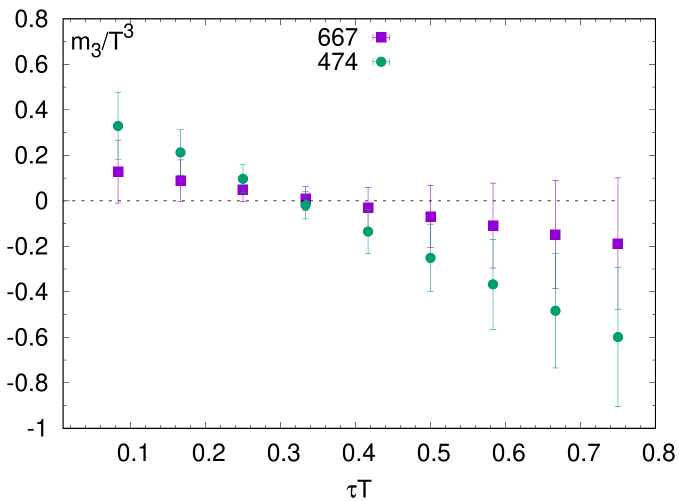
$rT = 1/4$

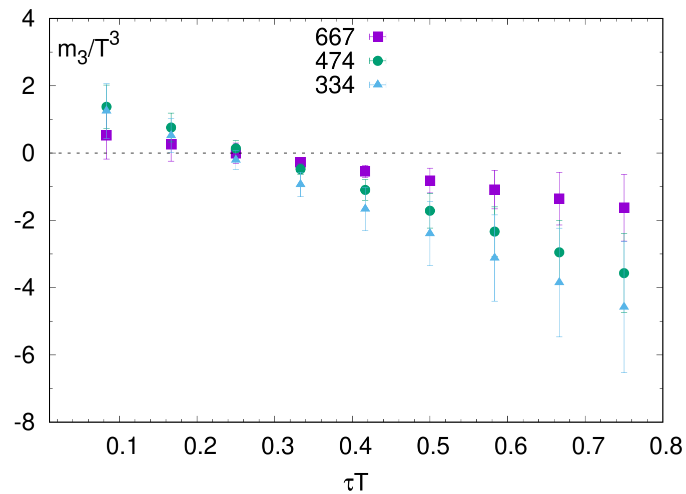
$rT = 1/2$





aprox. constant at small τ





aprox. zero at small τ

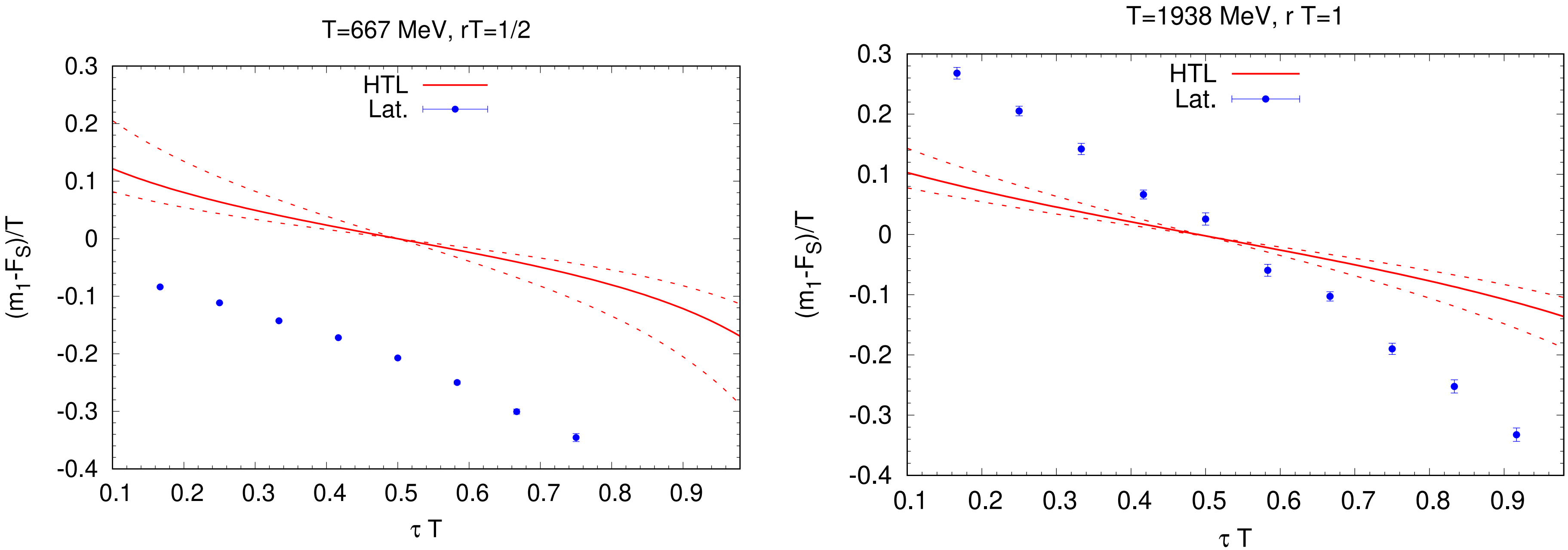
Bala et al (HotQCD), PRD 105 (2022) 054513

Comparison with HTL perturbation theory

In HTL perturbation theory ReV is screened, but HTL approximation is valid only for $r \sim 1/m_D$ and assumes $m_D \ll T$ (problematic in realistic setup)

Lattice results on $W(r, \tau, T)$ can be compared with the HTL calculations

Burnier, Rothkopf, PRD 87 (2013) 114019



In HTL approximation Re$V(r, T) = F_S(r, T)$

No agreement between the lattice and the HTL results even at the highest T

Lattice calculations on fine lattices

HISQ action, $T = 153 - 352$ MeV

$a = 0.028$ fm, $m_l = m_s/5$, $(m_\pi = 320$ MeV$)$, $96^3 \times N_\tau, N_\tau = 56,\ 36,\ 32,\ 28,\ 24,\ 20$

$a = 0.040$ fm, $m_l = m_s/20$, $(m_\pi = 160$ MeV$)$, $64^3 \times N_\tau, N_\tau = 64,\ 32,\ 30,\ 28,\ 26,\ 24,\ 22,\ 20,\ 18,\ 16$

$a = 0.049$ fm, $m_l = m_s/20$, $(m_\pi = 160$ MeV$)$, $64^3 \times N_\tau, N_\tau = 64,\ 26,\ 24,\ 22,\ 20,\ 18,\ 16$

Calculate correlation functions of temporal Wilson line instead of Wilson loops (better signal)

Gradient (Zeuthen) flow for noise reduction:

$$A_\mu(x) \to B_\mu(\tau_F, x) \qquad B_\mu(0,x) = A_\mu(x) \qquad \partial_{\tau_F} B_\mu(\tau_F, x) = -g_0^2 \frac{\delta S_{\mathrm{YM}}[B]}{\delta B_\mu(\tau_F, x)}$$

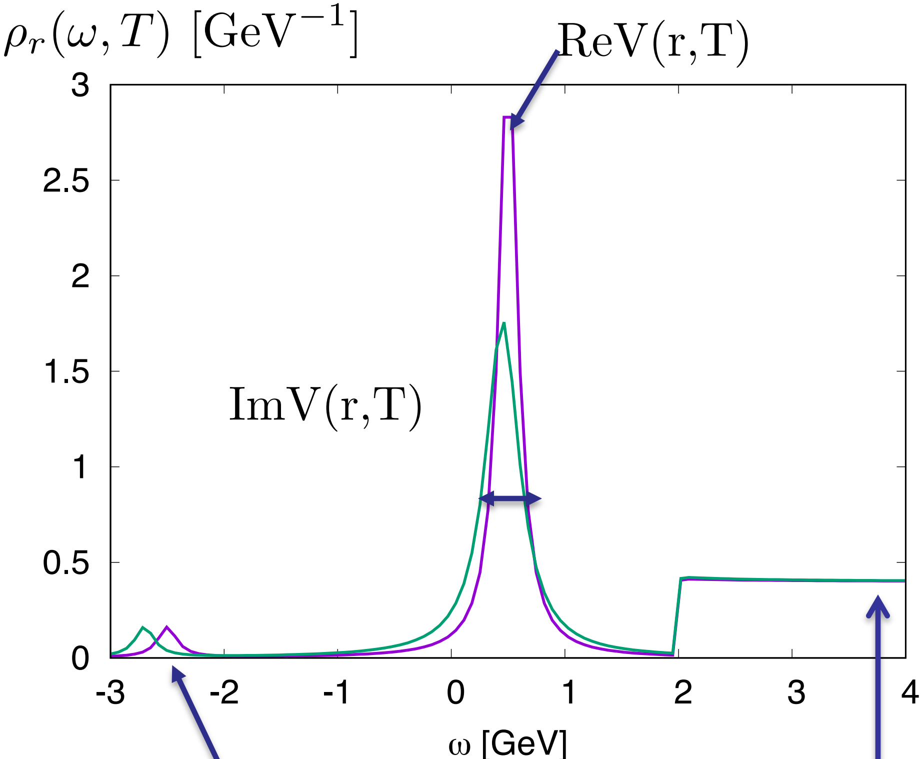
Gauge fields are smeared in the radius $\sqrt{8\tau_F}$

$$\sqrt{8\tau_F}T = 0.04 - 0.05$$

Additional noise reduction from smooth interpolation of the Wilson line correlators in r

Bazavov et al (HotQCD), arXiv:2308.16587

Spectral function and effective masses

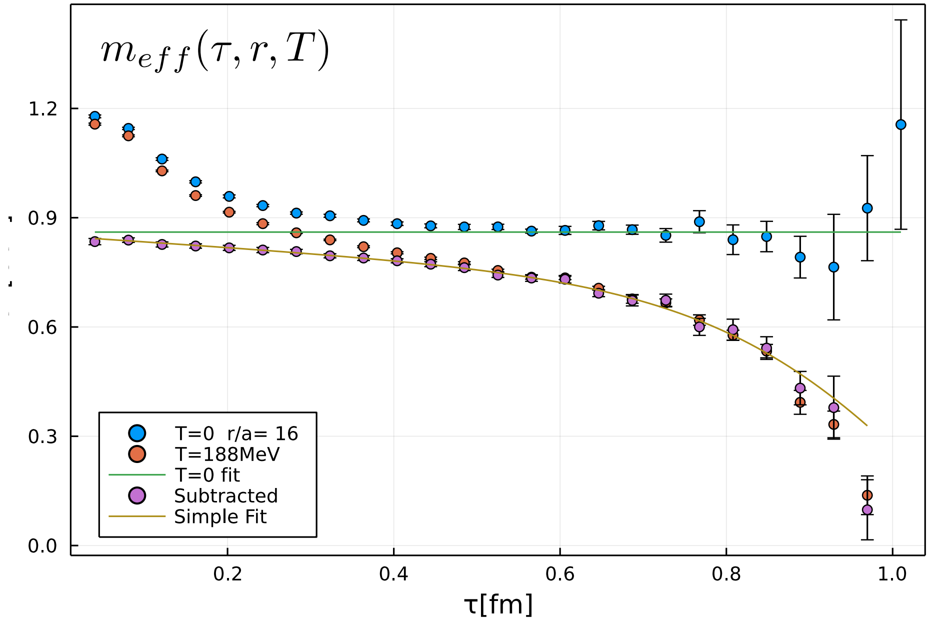


$$\rho_r(\omega, T) = \rho_r^{tail}(\omega, T) + \rho_r^{peak}(\omega, T) + \rho_r^{high}(\omega)$$

See, Bala et al (HotQCD), PRD 105 (2022) 054513

$$W^{high}(r, \tau) = \int_{-\infty}^{\infty} d\omega \rho_r^{high}(\omega) e^{-\omega\tau}$$

On the lattice: $W^{high}(r, \tau) = W(r, \tau, T = 0) - A_0 \exp(-V(r)\tau)$



No plateau at $T > 0$ in m_{eff} at $T > 0$

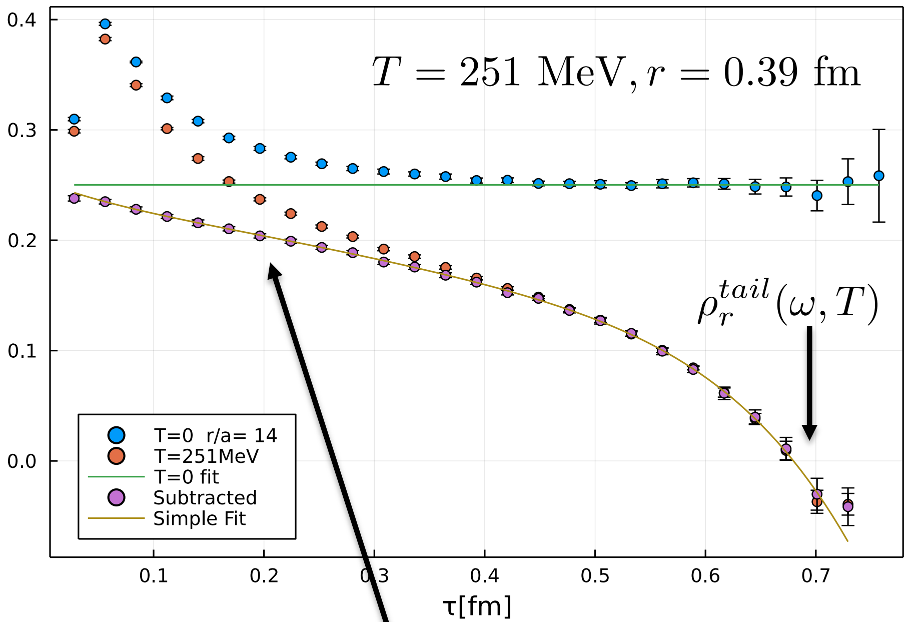
Only tiny T-dependence for small τ

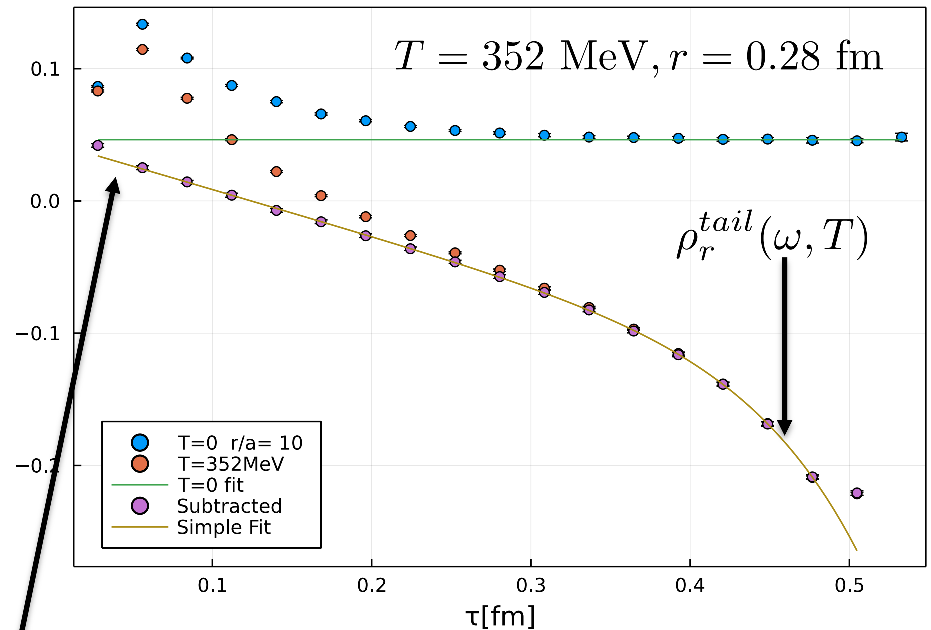
Distortions at small τ due to flow

$$W^{sub}(r, \tau, T) = W(r, \tau, T) - W^{high}(r, \tau)$$

Spectral function and effective masses

- No plateau in m_{eff} at $T > 0$ and only small T-dependence for small τ
- Distortions for small τ are largely removed by subtraction





m_{eff} for the subtracted correlator has milder τ-dependence, which is approximately linear

Thermal width

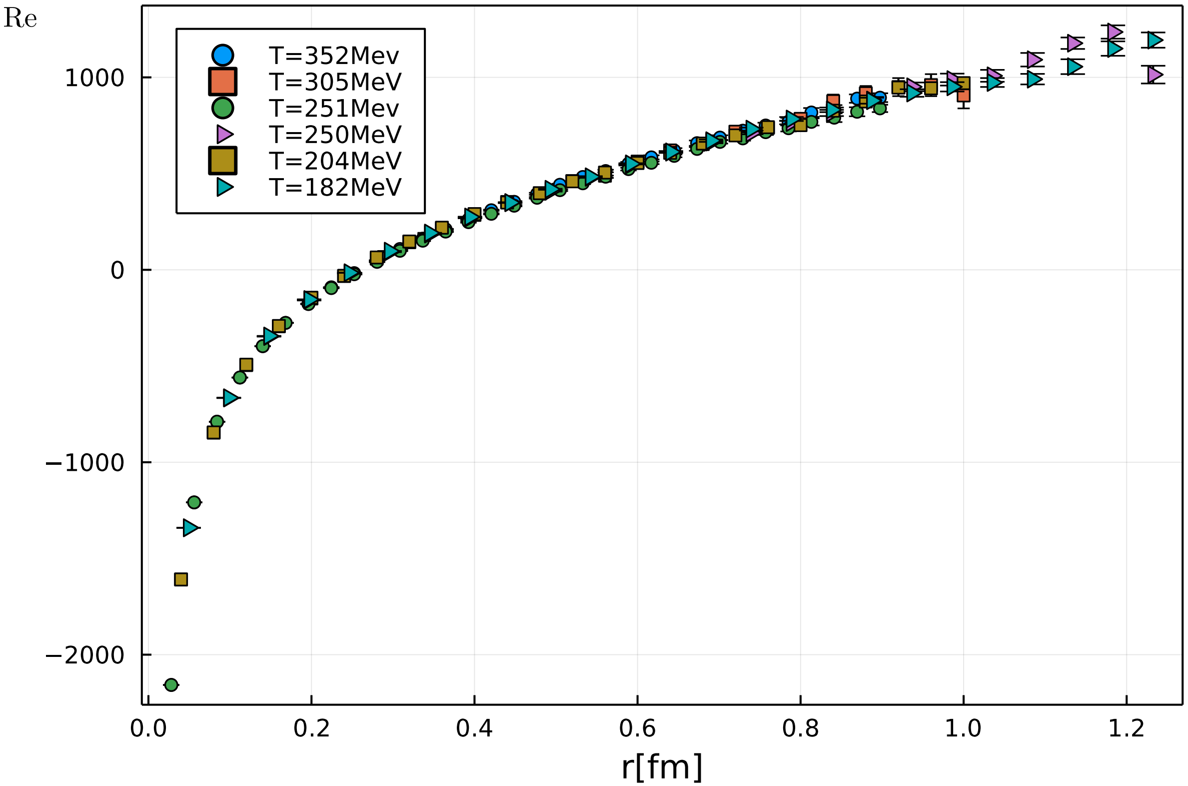
If $\rho_r(\omega, T)$ is Gaussian m_{eff} is linear in τ

$m_{eff}(\tau \simeq 0, r, T) \simeq V(r, T = 0)$

Model spectral function and the complex potential

$$\rho_r^{peak}(\omega,T) = \frac{A}{\pi}\frac{\Gamma(\omega,r,T)}{(\omega-\mathrm{Re}V(r,T))^2+\Gamma^2(\omega,r,T)}. \qquad \rho_r^{tail}(\omega,T) = A^{tail}\delta(\omega-E^{tail})$$

$$\Gamma(\omega,r,T) = \begin{cases} \Gamma_0(r,T) & -2\Gamma_0 < \omega < 2\Gamma_0 \\ 0 & n \text{ otherwise} \end{cases}$$

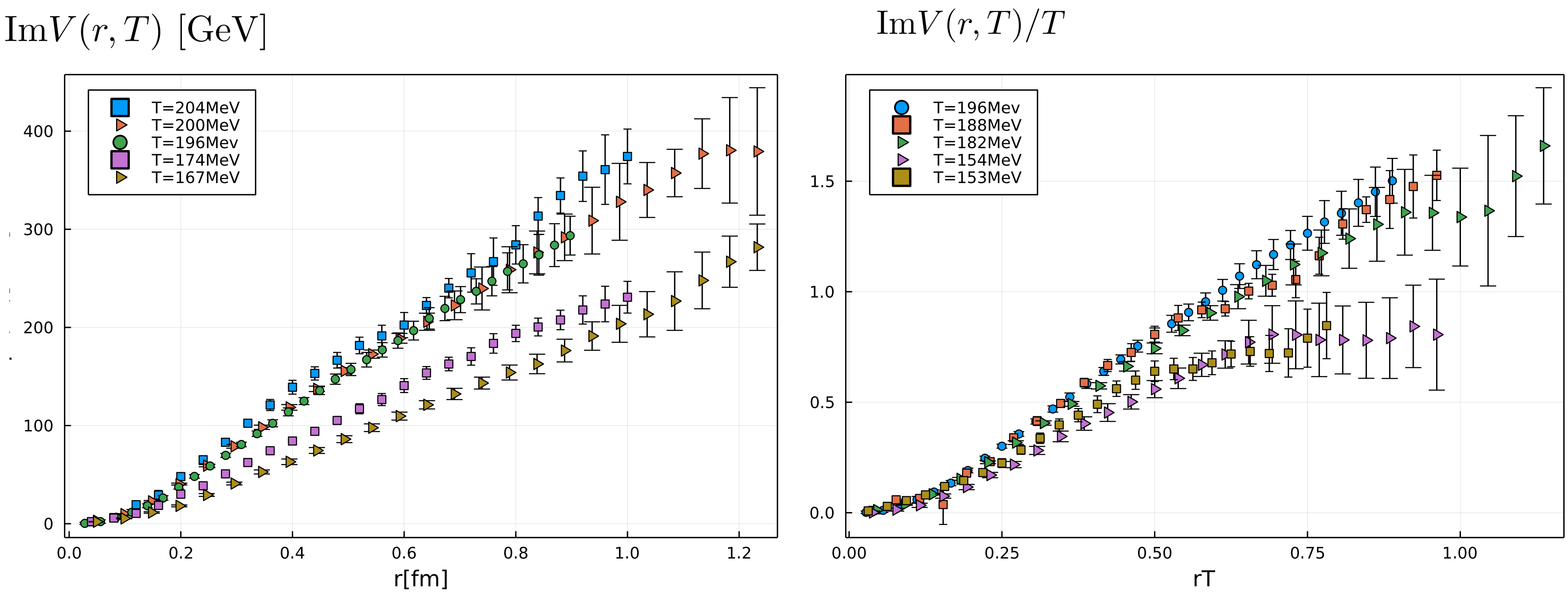


Re$V(r,T)$ shows only tiny temperature dependence and no hint of screening !

The same result if $\rho_r^{peak}(\omega,T) \sim \exp(-(\omega-\mathrm{Re}V(r,T))^2/(\mathrm{Im}V(r,T))^2)$

Bala et al (HotQCD), PRD 105 (2022) 054513

Imaginary part of the potential



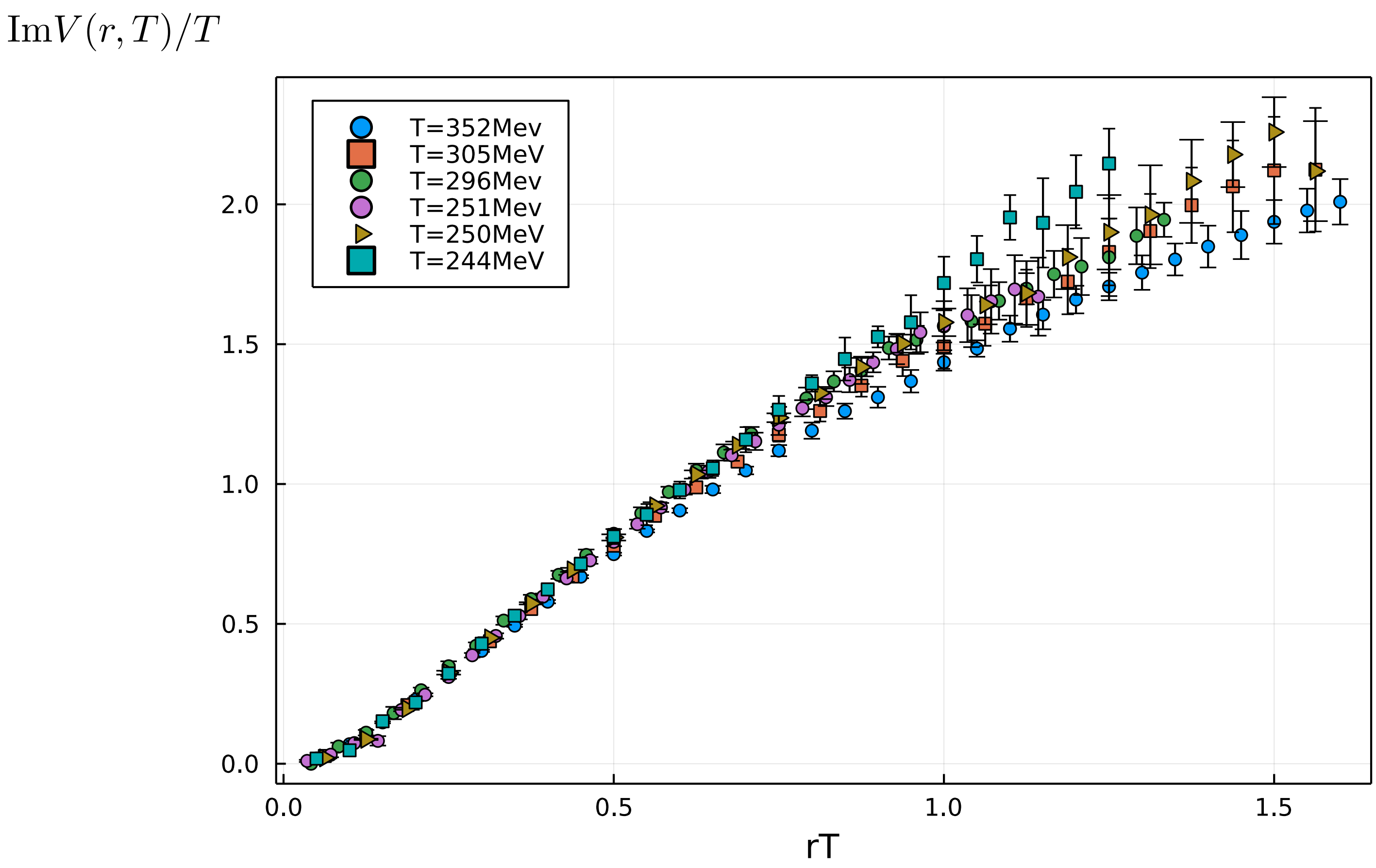
circles: $a = 0.028$ fm, squares: $a = 0.040$ fm, triangles: $a = 0.049$ fm

$\mathrm{Im}V(r,T)$ increases with increasing temperature and distance

No apparent quark mass effects for $T > 196$ MeV

No apparent cutoff effects

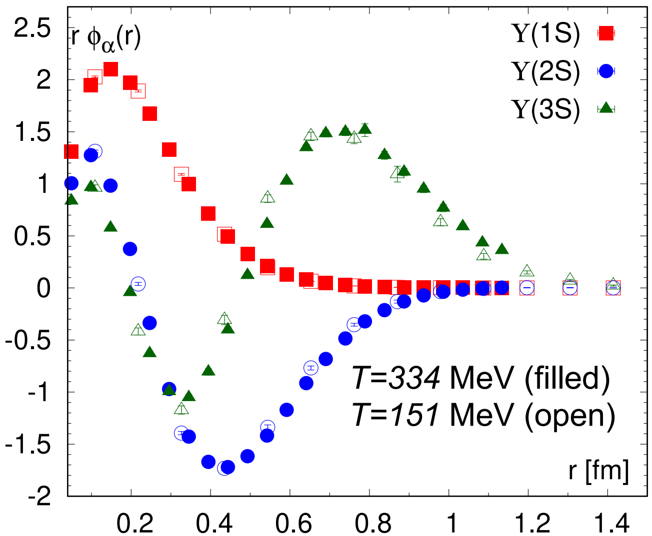
Imaginary part of the potential (cont'd)

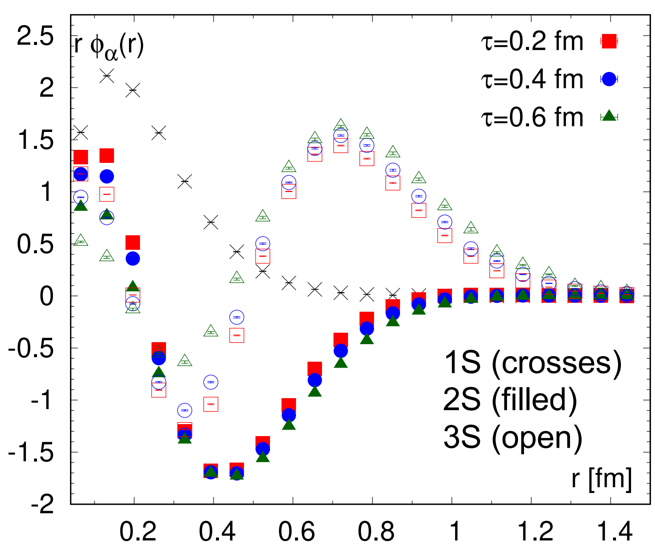


For 244 MeV $< T <$ 352 MeV $\mathrm{Im}V(r,T)/T$ approximately scales with rT as one would expect based on weak coupling calculations

Bethe-Salpeter amplitude at *T>0* and potential model

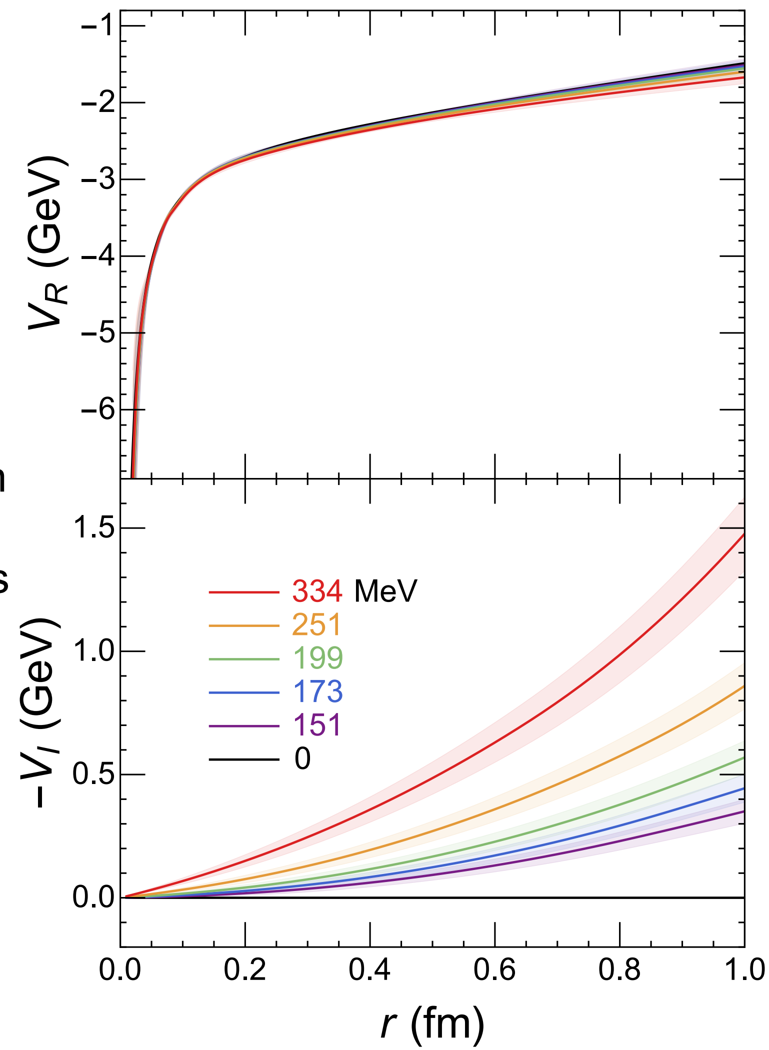
Larsen, Meinel, Mukherjee, PP, PRD 102 (20)114508





potential model
with inverse problem
+
Thermal width /mass
from lattice
+
Machine learning

Shi et al, PRD 105 (2022) 014017



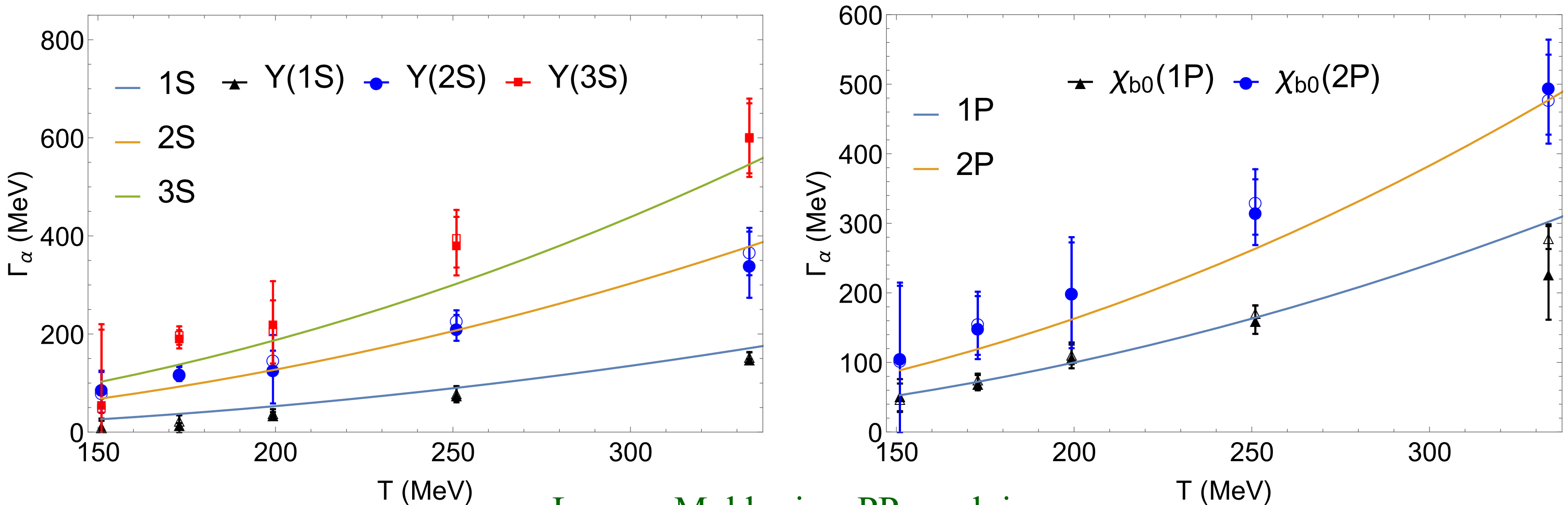
Thermal width of bottomonium in deconfined medium

Botottomium in deconfined medium shows no mass shift as expected in color screening picture but large thermal width

Larsen, Meinel, Mukherjee, PP, PRD 100 (2019) 094510; PLB 800 (2020) 135119

Simple model for thermal width of bottomonium state:

$$\Gamma(T) = \int d^3r \mathrm{Im V(r,T)} |\phi_\alpha(r)|^2, \quad \alpha = \Upsilon(1S), \Upsilon(2S), \Upsilon(3S), \chi_b(1P), \chi_b(2P)$$



Larsen, Mukherjee, PP, work in progress

The thermal width can be understood in terns of imaginary potential and no screening

⇒ <u>different mechanism for quarkonium melting</u>

Summary

- We calculated Wilson line correlators at T>0 using HISQ action at many lattice spacings thus, controlling discretization effects

- The complex potential at *T>0* was obtaine from the Wilson line correlators using a simple parametrization of the corresponding spectral function

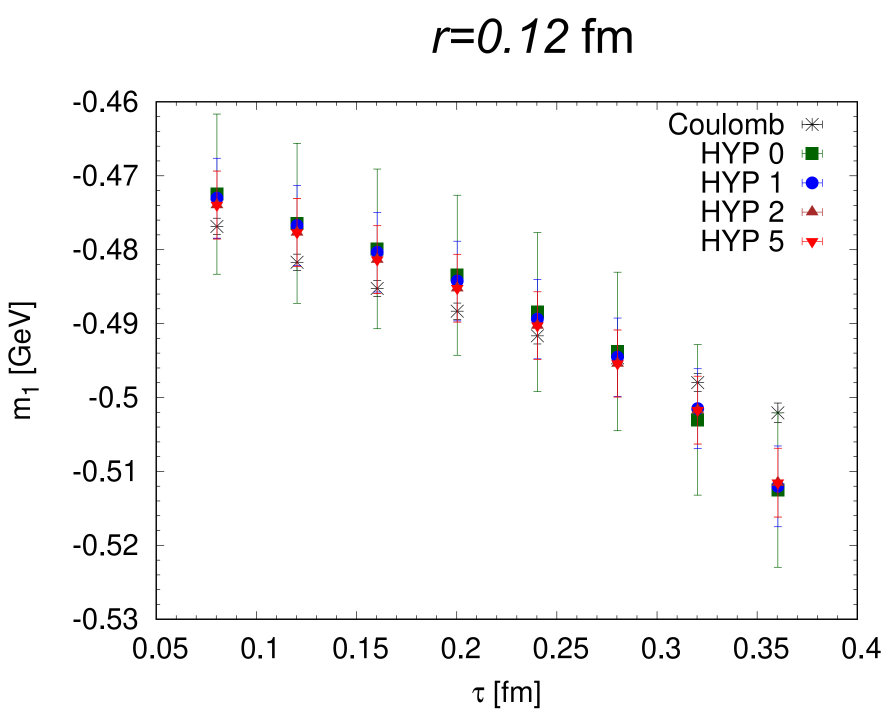
- The real part of the potential is NOT screened, while the imaginary part of the potential is large and increases with distance

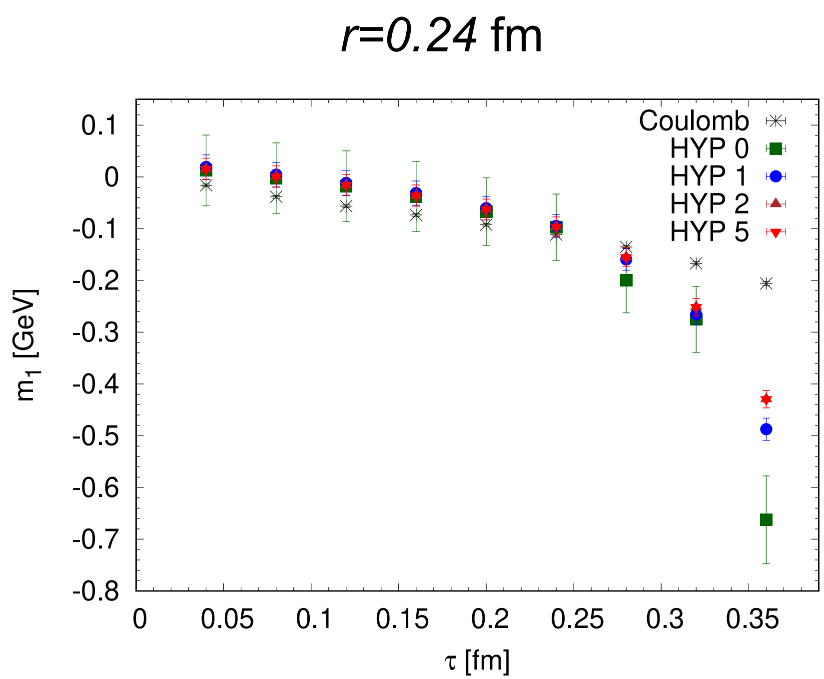
- The HTL perturbative calculation for the Wilson line correlator which incorporates screening does not agree with the lattice results even at *T=1938* MeV

- The results on the complex potential are corroborated by the results on the bottomonium Bethe-Salpeter amplitude obtained on the lattice as well as by the in-medium bottomonium masses

The thermal width of bottomonium states can be understood in terms of imaginary part of the potential
$\Rightarrow$ melting of quarkonium is not related to color screening

Back-up:


r=0.12 fm
Coulomb
HYP 0
HYP 1
HYP 2
HYP 5
-0.46
-0.47
-0.48
-0.49
-0.5
-0.51
-0.52
-0.53
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
m_1 [GeV]
τ [fm]


r=0.24 fm
Coulomb
HYP 0
HYP 1
HYP 2
HYP 5
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
m_1 [GeV]
τ [fm]