QCD in the Early Universe

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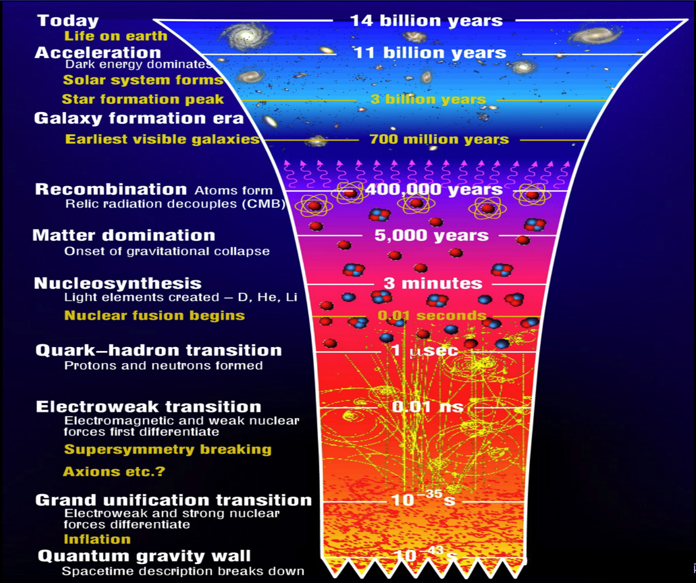
Table of Contents

Introduction: Equation of State of the Early Universe

A Brief History of the Universe



Standard Cosmological Model

Einstein equations of general relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(R - 2\Lambda) = -8\pi G \mathcal{T}_{\mu\nu} \tag{1}$$

$g^{\mu\nu}$ – metric tensor, $R_{\mu\nu} = f(g_{\mu\nu}, \partial_\lambda g_{\mu\nu}, \partial^2_{\lambda,\kappa} g_{\mu\nu})$ – Ricci tensor,
$R = R_{\mu\nu} g^{\mu\nu}$ – scalar curvature, Λ, G – cosmological, gravitational constants,
$\mathcal{T}_{\mu\nu}$ – energy-momentum tensor.

Cosmological principle: the Universe is Homogenous and Isotropic

Solution of (1) preserving space homogeneity and isotropy under its time evolution is spacetime of constant curvature $k = \{+1, 0, -1\}$ with FLRW metrics

$$ds^2 = g^{FLRW}_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2}$$

$a(t)$ = scale factor of the Universe – connects co-moving (Lagrange) and physical (Euler) coordinates $\hat{r}(t) = a(t)r$.

Early Universe Made Simple

Friedman equation: $g_{\mu\nu}^{FLRW} \rightarrow$ Eq. (1) with $\mu = \nu = 0$

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\epsilon - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (3)$$

Perfect fluid: $\mathcal{T}_\nu^\mu = diag(\epsilon, -p, -p, -p) \Rightarrow$ Fluid equation

$$\dot{\epsilon} + 3(\epsilon + p)H(t) = 0 \quad (4)$$

- [Ornik,Weiner:1987] : Early Universe: $\epsilon \gtrsim 1\text{GeV fm}^{-3} \Rightarrow$ can neglect k and Λ in (3)

$$-\frac{d\epsilon}{3\sqrt{\epsilon}(\epsilon + p)} = \sqrt{\frac{8\pi G}{3}}dt \quad \Rightarrow \quad \dot{\epsilon} + \sqrt{\frac{24\pi G}{3}}(\sqrt{\epsilon}(\epsilon + p(\epsilon)) = 0 \quad (5)$$

- Integration of (5) using barotropic form of EoS $p(\epsilon)$ yields $\epsilon(t)$.
- **Example:** Time-independent speed of sound $p/\epsilon = w$ (see also slide #35):

$$\int \frac{d\epsilon}{\epsilon + p(\epsilon)} = -\log a^3 + const., \ \epsilon \sim a^{-3(1+w)} \sim (t(1+w))^{-2}, \ a(t) \sim t^\alpha, \ \alpha = \frac{2}{3(1+w)} \quad (6)$$

Evolution of the Early Universe

Cooling of the Universe: The Insight from the Asymptotic Freedom

Universe cooled down via series of first- or second-order phase transitions (PT) associated with the various spontaneous symmetry breakings (SSBs) of the basic non-Abelian gauge fields, see e.g. the classical textbooks [Linde:1978], [Bailin and Love:2004], [Boyanovsky et al:2006].

Standard Model Predicts Two Phase Transitions

1. The electroweak (EW) PT at $T \sim m_H$ provides masses to elementary particles. LQFT calculations show that for $m_H \geq 67$ GeV this PT is an analytic crossover [Kajantie et al.:1996], [Csikor et al.:1999].

2. At $T < 200$ MeV the SSB of the chiral symmetry of the $SU(3)_c$ color group, the QCD, takes place. In a strong first-order PT scenario, the de-confined matter supercools before bubbles of hadron gas are formed. $\Rightarrow$ Produced inhomogeneities in this phase could have a strong effect on the nucleosynthesis epoch.

Standard Model Couplings in the Hot Big Bang Era

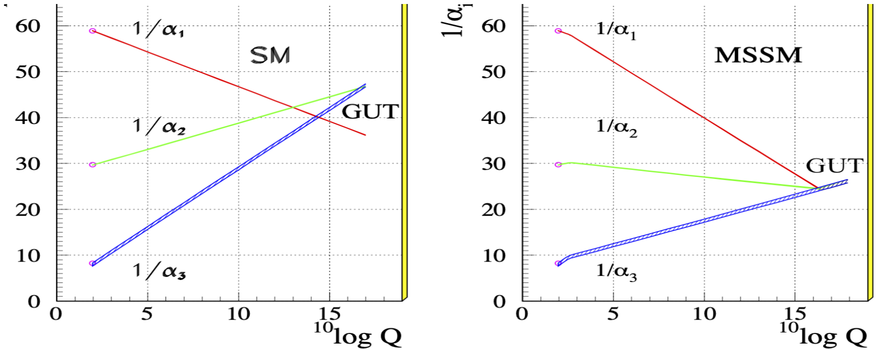


Figure 1: Evolution of the inverse of the three coupling constants $\alpha_1 = \alpha_{EM}, \alpha_2 = \alpha_W, \alpha_3 = \alpha_S$ in the Standard Model $\mathrm{U}(1)_\mathrm{Y} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{C}$ (left) and in its supersymmetric extension MSSM (right).

- Thermodynamics is applicable if the Universe is in global equilibrium.
- Hydrodynamical description needs only local thermal equilibrium (LTE).

The Early Universe in Local Thermal Equilibrium

Q: *How do we know that the Early Universe was in the state of LTE?*
A: [Mukhanov: Physical Foundations of Cosmology, CUP, 2005]

- Collision time among the constituents $t_c = 1/(\sigma n v)$
- LTE $\Leftrightarrow$ Local equilibrium must be reached well before expansion becomes relevant.

$\Rightarrow$ At the cosmic time $t_H \sim 1/H(t)$: $t_c \ll t_H$.

- At $T > T_{\mathrm{EW}}$ all (most of) particles of the SM are ultra-relativistic ($k^2 \gg m^2$) and gauge bosons are massless. $\Rightarrow \sigma \approx \mathcal{O}(1)\alpha^2\lambda^2 \sim \alpha^2/k^2 \sim \alpha^2/E^2 \sim \alpha^2/T^2$,
- For $n \sim T^3, v = 1$ and $\alpha \simeq 10^{-1} - 10^{-2}, \alpha = \alpha_{\mathrm{EM}}, \alpha_{\mathrm{W}}, \alpha_{\mathrm{S}}$:

$$t_c \sim \frac{1}{\alpha^2 T} \ll t_H \sim \frac{1}{H} \sim \frac{1}{\sqrt{\epsilon}} \sim \frac{1}{T^2}\,.$$

$\Rightarrow$ For $10^{15} - 10^{17}$ GeV $\gtrsim T \gtrsim T_{\mathrm{EW}}$ LTE in expanding fluid persists.

- For $T_{\mathrm{EW}} > T > T_c^{\mathrm{QCD}} \approx 160$ MeV the LTE continues due to large effective cross-section among the particles forming the QGP medium. For details see slides #32 and #33.

$$g_{\rm eff}(T) \equiv \frac{\epsilon(T)}{\epsilon_0(T)}\,, \quad \epsilon_0(T) = \frac{\pi^2}{30}T^4 \qquad (7)$$

$$h_{\rm eff}(T) \equiv \frac{s(T)}{s_0(T)}\,, \quad s_0(T) = \frac{2\pi^2}{45}T^3 \qquad (8)$$

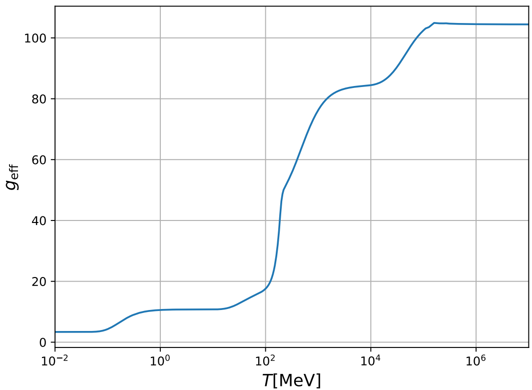
$$g_{\rm eff}^{id}(T) = h_{\rm eff}^{id}(T) = \frac{7}{8}4N_{\rm F} + 3N_{\rm V} + 2N_{\rm V0} + N_{\rm S} \qquad (9)$$

- For an adiabatic process

$$s(T) = \frac{\epsilon(T) + p(T)}{T} \qquad (10)$$

- EoS in cosmology $p = w\epsilon$

$$w(T) = \frac{sT}{\epsilon} - 1 = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1 \qquad (11)$$



$g_{\rm eff}(T)$ in the SM taking into account interactions between particles, obtained with both perturbative and lattice methods. [Hindmarsh et al.:2020].

Cosmological Parametrization of EoS: from Particle Dust to Stiff Fluid

- Causality:

$$w(T) = \frac{sT}{\epsilon} - 1 = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1 \leq 1 \quad \Rightarrow \quad \frac{h_{\rm eff}(T)}{g_{\rm eff}(T)} \leq \frac{3}{2} \tag{12}$$

  where the upper bound of $w = 1$ is reached for absolutely stiff fluid e.g. baryons interacting via massive vector particles [Zeldovich:1961] or free time-like massless scalar field.

- The observation that the stiff fluid saturates the holographic covariant entropy bound was used to describe a cosmology of the very early universe [Banks:2001].

- Can the stiff fluid represent some form of QCD matter?

- Approaching the stiff fluid limit (stiff fluid d.o.f. do not depend on the temperature)

$$c_s^2(T) = \frac{dp}{d\epsilon} = T\frac{ds}{d\epsilon} + s\frac{dT}{d\epsilon} - 1 = \frac{4}{3}\left[\frac{4h_{\rm eff}(T) + T \cdot h'_{\rm eff}(T)}{4g_{\rm eff}(T) + T \cdot g'_{\rm eff}(T)}\right] - 1 \tag{13}$$

  where the prime indicates differentiation with respect to T.

Energy Density of Ideal Massless Gas in Early Universe

- Simultaneous presence of the EW and the QCD matter in LTE is one of the remarkable differences between the QGP produced in accelerator experiments and the deconfined QCD matter in the Early Universe.
- Including only particles which can be at $T \lesssim T_{\rm EW} = 160$ GeV considered massless: $g_{\rm eff}^{\rm EW} = \frac{7}{8}(12+6)+2 = 17.75$ and $g_{\rm eff}^{\rm QCD} = 2\times 8 + \frac{7}{8}(3\times N_F \times 2\times 2)$.
- At $T \lesssim T_{\rm EW}$ and for N_F active quark flavors, the QGP contains a factor of $g_{\rm eff}^{\rm QCD}/g_{\rm eff}^{\rm EW} \simeq 2 \div 4$ more energy and pressure than the EW matter.
- Even for $T \gg T_c^{\rm EW}$ wih $g_{\rm eff}^{\rm QCD} = 79$ and $g_{\rm eff}^{\rm EW} = 26.75$ the QCD matter has a factor of $g_{\rm eff}^{\rm QCD}/g_{\rm eff}^{\rm EW} \simeq 3$ larger energy density and pressure than the EW matter.
- QCD matter represents the densest form of matter filling the Early Universe during QCD and EW epochs and beyond.

The Bare-bones EoS with the 1^{st} Order Phase Transition

- Simplest EoS incorporating confinement – the MIT bag model [Chodos et al.:1974]

$$\epsilon_q = \sigma_q T^4 + \mathcal{B},\ p_q = \frac{\sigma_q}{3} T^4 - \mathcal{B} \Rightarrow p_q(\epsilon_q) = \frac{1}{3}(\epsilon_q - 4\mathcal{B}),\ \sigma_q = \frac{\pi^2}{30} g_{\rm eff}^{\rm QCD},\ \sigma_h = \frac{3\pi^2}{30} \quad (14)$$

$$\Theta(T) = \frac{\epsilon_q - 3p_q}{T^4} = \frac{4\sigma_q \mathcal{B}}{\epsilon_q - \mathcal{B}},\quad p_q(T_c) = p_h(T_c) = \frac{\sigma_h}{3} T_c^4 \Rightarrow T_c = \left(\frac{3\mathcal{B}}{\sigma_q - \sigma_h}\right)^{1/4} \approx 150 MeV \quad (15)$$

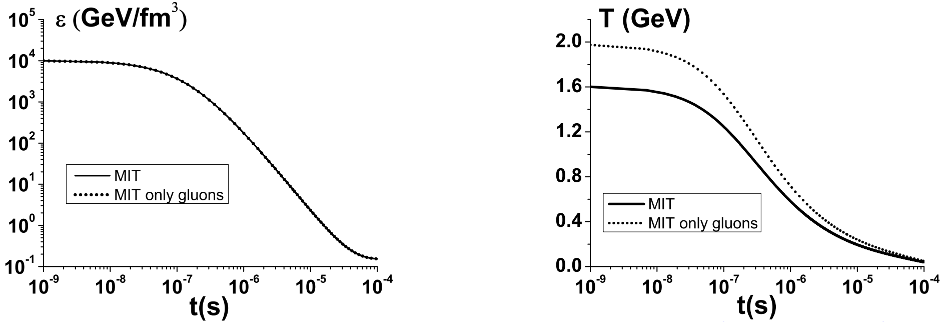


Figure 2: Solution of Eq. (5) using EoS (14) with $\epsilon_0(t_0) = 10^4$ GeVfm^{-3} and $t_0 = 10^{-9}$s. **Left**: $\epsilon(t)$. [Sanches et al.:2014] **Right**: $T(t)$.

EoS Based on the Fundamental Theory: the SM and GUT

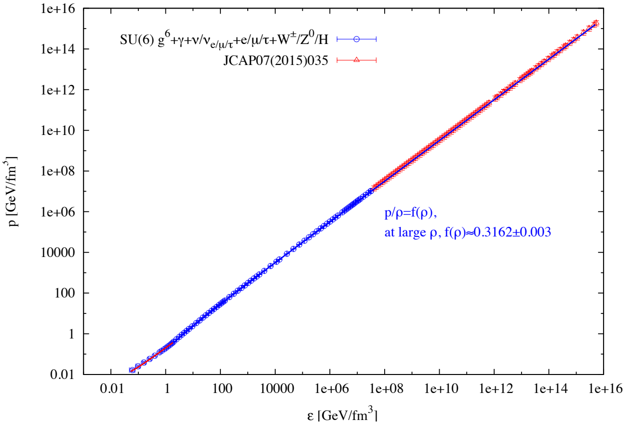


Figure 3: Combined EoS $p(\epsilon), \epsilon \equiv \rho$ of QCD and EW matter, using lattice results of [Borsanyi et al:2016] extended to include other DoFs such as γ, neutrinos, leptons, EW, and Higgs bosons as well as perturbative results from [Laine, Mayer:2015]. Adapted from [Tawfik,Mishustin:2019].

- For details on calculations of the EoS in the SM see slide #31

The EoS parametrization

- GUT EoS (△ in Fig. 3) $p_{\rm GUT} = (0.330 \pm 0.024)\epsilon$ valid for $10^8 \lesssim \epsilon \leq 10^{16}$ GeV·fm^{-3} corresponds to ideal massless gas.
- Combined EoS for QCD and EW eras has two independent contributions $p_1(\epsilon)$ and $p_2(\epsilon)$.

$$p_{\rm SM} = p_1(\epsilon) + p_2(\epsilon)\,, \quad p_1(\epsilon) = b\epsilon\,, \quad p_2(\epsilon) = a + c\epsilon^d \tag{16}$$

$$a = 0.048 \pm 0.016,\, b = 0.316 \pm 0.031,\, c = -0.21 \pm 0.014,\, d = -0.576 \pm 0.034$$

- $p_1(\epsilon) \approx \frac{1}{3}\epsilon$ – ideal gas of massles particles EoS.
- $p_2(\epsilon) = -\mathcal{B}(\epsilon) = a + c\epsilon^d$ – density-dependent bag function (instanton liquid?)
- The sound velocities of both components are positive:

$$c_{s,1}^2(\epsilon) = \frac{dp_1}{d\epsilon} = b > 0\,, \qquad c_{s,2}^2(\epsilon) = \frac{dp_2}{d\epsilon} = c \cdot d \cdot \epsilon^{d-1} = \frac{d \cdot (p_2 - a)}{\epsilon} > 0 \tag{17}$$

- $a \approx 0 \Rightarrow$ EoS $p_2(\epsilon) \approx c\epsilon^d, c < 0, d < 0 \Rightarrow$ must have $p_2 < 0$! Coincides with generalized Chaplygin EoS often applied as a model of DE, see e.g. [Kamenshchik et al:2001].

Saturated QCD matter in the Early Universe

The Weakly Interacting QCD: DGLAP and BFKL

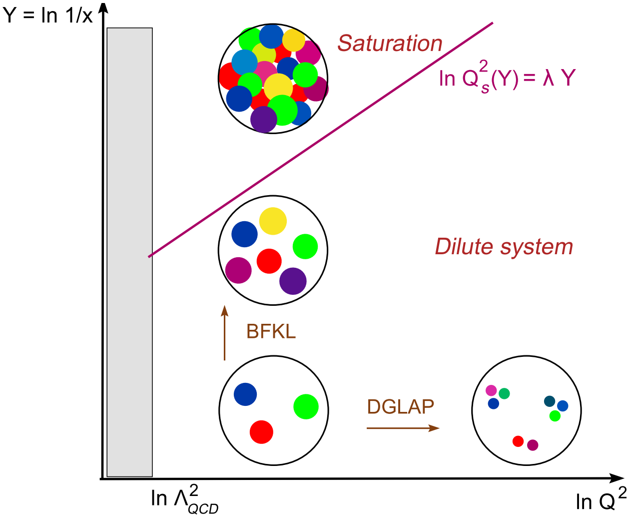


Figure 4: Parton density and size as a function $Y = \ln(1/x)$ and $\ln Q^2$. From [Gelis et al:2010].

Q: What's missing from previous EoS?

A: The physics of QCD saturation.

- In the Bjorken limit of QCD $Q^2 \to \infty$ (impulse approximation) hadron viewed, in the infinite momentum frame (IMF), represents a large number of gluons and sea quark pairs with very small phase-space density.
- At large, fixed $Q^2 \gg \Lambda^2_{QCD}$ gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing x .

QCD at High Parton Densities and Saturation

[Gribov jr.,Levin,Ryskin:1983, McLerran,Venugopalan:1993]

- Partons "overlap" when $\sigma_{gg} \sim \alpha_S/Q^2$ times $xG_A(x, Q^2)$ – the probability to find at fixed Q a parton carrying a fraction x of the parent parton momentum – becomes comparable to the geometrical cross section πR_A^2 of the object A occupied by the gluons.

$$Q_s^2(x) = \frac{\alpha_S(Q_s)}{2(N_c^2-1)} \frac{xG_A(x, Q_s^2)}{\pi R_A^2} \sim \frac{1}{x^\lambda} \Rightarrow \ln Q_s^2(x) = \lambda Y \tag{18}$$

- $Q_s(x) \equiv$ Fixed point of the PDF evolution in $x \Rightarrow$ the emergent "close packing" scale.
- Repulsive gg interactions $\Rightarrow$ occupation number f_g (# of gluons with a given x times the area each gluon fills up divided by the transverse size of the object) saturates at $f_g \sim 1/\alpha_S$.
- The same scaling as for the Higgs condensate, superconductivity, or the inflaton field.
- Saturated gluonic matter is weakly coupled. $\Rightarrow$ weakly interacting means semi-classical. (For details see slide #34).

Glass Properties of Glasma and CGC–Black Hole Correspondence

- **In condensed matter** glass is a non-equilibrium, disordered state of matter acting like solids on short time scales but liquids on long time scales [Mauro:2014,Sethna:2021].
- Two scales of glasma:
$$\tau_{\rm wee} = \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} = \frac{2xP^+}{k_\perp^2} \ll \frac{2P^+}{k_\perp^2} \approx \tau_{\rm valence} \, . \quad (19)$$
$\Rightarrow$ Valence modes are static over the time scales of wee modes [Berges et al.:2020].
- Glasses are formed when liquids are cooled too fast to form the crystalline equilibrium state. This leads to an enormous # of possible configurations $N_{\rm gl}(T)$ into which the glasses can freeze $\Rightarrow$ large entropy $S = \ln N_{\rm gl}(T)$, such that $S(T=0) > 0$, [Sethna:1988].
- Correspondence between Black Holes as highly occupied condensates of N weakly interacting gravitons and CGCs as highly occupied gluon states [Dvali, Venugopalan:2021].
- Both BH and CGC attain a maximal entropy S_{max} permitted by unitarity when the occupation number f and the coupling α of the respective constituents (gravitons, gluons) satisfy $f = 1/\alpha(Q_s)$, where Q_s represents the point of optimal balance between the kinetic energies of the individual constituents and their potential energies.

Early Universe Occupied by Massless Particles

- Expanding sphere $V = (4/3)\pi R^3,\ R \sim a(T)$ occupied by interacting massless particles.
- Particle with thermal wavelength $\lambda_B = \pi^{2/3}/T$ occupies the fraction $F(T)$ of V

$$F(T) = \frac{\left(\frac{\lambda_B(T)}{2}\right)^3}{a^3(T)} = \frac{\pi^2}{8}(T \cdot a(T))^{-3} \sim T^{-3}(\epsilon)\epsilon^{1/(1+w)} \tag{20}$$

- **Example:** $T^{-3}(\epsilon) \sim \epsilon^{-3/4}$ & $w = (\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \mathbf{1}) \Rightarrow F(T) = (const., \epsilon^{-1/12}, \epsilon^{-3/20}, \boldsymbol{\epsilon^{-1/4}})$.
- Particle carries a fraction $x = k^+/P^+$ of the total light-cone momentum $P^+ \sim 1/a(T)$ of the matter contained in V.
- Local thermal equilibrium $\Rightarrow$ particle momenta are concentrated around $k^+ \approx Q = 2\pi T$, i.e. around $x \approx k^+/P^+ \approx 2\pi T \cdot a(T)$.
- Saturation is relevant for $T \leq T_s$ i.e. when $\left.\frac{xG_A(x, T_s^2)F(T_s)}{2(N_c^2 - 1)}\right|_{x \approx T_s \cdot a(T_s)} \approx \frac{1}{\alpha_S(T_s)}$ (21)

Thermal Evolution of the Strong Coupling Constant $\alpha_s(2\pi T)$

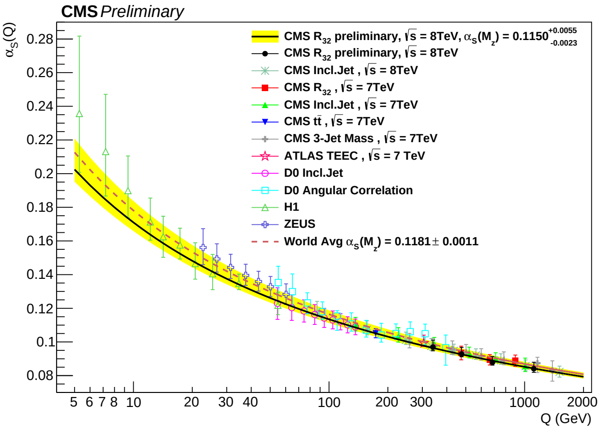


Figure 5: $\alpha_S(Q)$ obtained from MSTW2008 NLO PDF set. [CMS:2017].

- "Temperature" dependence of the running coupling $0.2 \lesssim \alpha_S(T) \lesssim 0.08$ for $T \approx Q/(2\pi) \in (1, 220)$ GeV.

The Glasma Phase of the Early Universe

- The QCD saturation scale $R_S = 1/Q_S$ is now given by the thermal de Broglie wavelength of massless gluons $\lambda_S = \pi^{2/3}/T_S$, where T_S is the saturation temperature.
- Saturated matter is important in a period of cosmological evolution when $T \gtrsim T_S \gg \Lambda_{\rm QCD}/(2\pi)$. Already for $T_S \gtrsim 1$ GeV we have $\alpha_S(2\pi T_S) \lesssim 0.2$, see slide #21.
- At $T \approx T_{EW}$ for YM bosons $g_{\rm eff}^{\rm QCD}/g_{\rm eff}^{\rm EW} \simeq 8/3 \Rightarrow$
  The Glasma might have been a prevalent form of matter also during the EW era.
- For expansion times $R_s\alpha_S^{-1} < t < R_s\alpha_S^{-3/2}$ the semiclassical picture of the Glasma as a high occupancy state with $f \sim \alpha_S^{-1}$ gluons breaks down due to their rescattering.
- This is followed by the quantum kinetic $2 \to 3$ process among liberated gluons at $t \sim R_s\alpha_S^{13/5}$ which leads to bottom-up thermalization [Baier:2001] and the emergence of a strongly interacting QGP*. Cosmological expansion is restored from $a(t) \sim t^{1/3} \to t^{1/2}$.

*) For more on sQGP see slides #.

Yang-Mills Field EoS in Cosmology

- Abelian vector field can not be isotropic $\Rightarrow$ at least triplet of vector fields is needed to ensure the isotropy.
- Early studies: [Galtsov, Volkov:1991]: coupled Einstein-Yang-Mills equations with gauge group SU(2) in the FLRW universes.
- Basic features of the Einstein-Yang-Mills homogeneous and isotropic cosmological solutions can be attributed to the conformal nature of the YM field.
- QCD has two emergent scales both originating from the breakdown of translation invariance due to the extended geometry of the object: Λ_{QCD} and Q_s.
- Both violations of conformal invariance can be modeled by appropriate EoS.

Confinement in Modified Bag Model EoS

$$\epsilon(T) = \sigma T^4 - CT^2 + \mathcal{B}\,, \qquad p(T) = \frac{\sigma}{3}T^4 - DT^2 - \mathcal{B} \tag{22}$$

$$\sigma = \frac{\pi^2}{30} g_{\rm eff}^{\rm QCD}\,, \quad g_{\rm eff}^{\rm QCD} = 2\times 8 + \tfrac{7}{8}(3\times N_F \times 2\times 2) \tag{23}$$

$$\sigma(N_F = 0,\ 2,\ 3,\ 4,\ 5) \approx 5,\ 12,\ 16,\ 18,\ 23\,;\quad \mathcal{B}^{1/4} \approx 220\ \text{MeV}\,.$$

- In pure gauge theory up to $T \approx (2 \div 5)T_c$, the dominant power-like correction to pQCD behavior is $\mathcal{O}(T^{-2})$ rather than $\mathcal{O}(T^{-4})$ [Pisarski:2006].
- Quadratic thermal terms in the deconfined phase can also be obtained from gauge/string duality [Zuo:2014vga].

1. $C = D > 0$: LQCD motivated "fuzzy" bag model EoS [Pisarski:2006, Megias et al.:2007].
2. $C = -D < 0$: Gluonic q-particle EoS with $\mathcal{B}(T) = -CT^2 + \mathcal{B}$ [Schneider, Weise:2001].

- In both cases the trace anomaly $\Theta(T) = (\epsilon - 3p)T^{-4} > 0 \Rightarrow$ Quadratic terms lead to $dp/d\epsilon < 1/3$ and hence to the softening of the EoS close to the critical temperature T_c.

Modified Bag Model EoS with the Negative Trace Anomaly

- Consider the same EoS but with $\Theta(T) < 0^*$ valid for $T \gg \Lambda_{QCD}$

$$\Theta(T) = \frac{\epsilon - 3p}{T^4} = \frac{4\mathcal{B}}{T^4} + \frac{3D - C}{T^2} \approx \frac{3D - C}{T^2} \equiv -\frac{A}{T^2} < 0 \; ; \quad A > 0 \tag{24}$$

$\Rightarrow$ with decreasing temperature, the fluid moves away from the ideal massless gas limit.

- The new EoS thus reads: $\epsilon(T) = \sigma T^4 - CT^2 , \;\; p(T) = \frac{1}{3}\left(\sigma T^4 + (A - C)T^2\right)$ (25)

$$p(\epsilon) = \frac{1}{3}\left(\epsilon - \Theta(T)T^4\right) = \frac{1}{3}\left(1 - AT^2(\epsilon)\right) \, , \quad T^2(\epsilon) \equiv \frac{C + \sqrt{C^2 + 4\sigma\epsilon}}{2\sigma} > 0 \, . \tag{26}$$

- Entropy density must be positive:

$$s(T) = \frac{4}{3}\sigma T^3 + \frac{2}{3}(A - C)T = s_0(T) + \frac{2}{3}(A - C)T > s_0(T) > 0 \;\Rightarrow A > C \, . \tag{27}$$

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*) The EoS with $\epsilon - 3p < 0$ was first discussed in [Zeldovich:1961].

The Emergence of a Stiff Fluid in the Early Universe

- With decreasing ϵ speed of sound c_s increases. But it can't exceed the speed of light

$$c_s^2(\epsilon) = \frac{dp(\epsilon)}{d\epsilon} = \frac{1}{3}\left(1 + \frac{2A\sigma}{\sqrt{C^2 + 4\sigma\epsilon}}\right) \leq 1 \tag{28}$$

$$\Longleftrightarrow \quad \epsilon \geq \epsilon_c = \frac{(A^2\sigma^2 - C^2)}{4\sigma}\,, \quad T_c = \sqrt{\frac{A + 2C}{4\sigma}}\,, \quad s_c = AT_c\,. \tag{29}$$

$\Rightarrow$ For $\epsilon \leq \epsilon_c$ the EoS (25) must be replaced by the stiff EoS with $p = \epsilon$.

- Expansion of the universe stiffens the EoS from $w = 1/3$ to $w = 1$.
- Substitution for the $p + \epsilon$ from Eq.(25) into Eq.(4) and integrating $\delta = \int d\epsilon/(\epsilon + p)$

$$\delta(\epsilon) = \frac{3\left((A - 2C)\log\left(A + 2\sqrt{C^2 + 4\sigma\epsilon} - 2C\right) - 2C\log\left(\sqrt{C^2 + 4\sigma\epsilon} + C\right)\right)}{2(A - 4C)}\,. \tag{30}$$

- Solving the eq. $\delta(\epsilon) - \delta(\epsilon_c) = \log a^{-3}$, cf. (6), yields $(a(\epsilon))^{-3} = \exp(\delta(\epsilon) - \delta(\epsilon_c))$.

Approaching the Saturation Temperature

- Substitution $\epsilon \to \sigma T^4 - CT^2$ in $(a(\epsilon))^{-3}$ allows us to obtain $F(T) = (T \cdot a(T))^{-3}$
- Solution with $C = 0$: $$F(T) = T^{-3}\left(\frac{A + 4\sigma T^2}{A(2\sigma+1)}\right)^{3/2} \xrightarrow[T^2 \ll A/2]{} T^{-3} \tag{31}$$
- Solution with $C = A/2$: $$F(T) = T^{-3}\left(\frac{A + \sqrt{(A - 4\sigma T^2)^2}}{A(2\sigma+1)}\right)^{3/2} \xrightarrow[T^2 \ll A/(4\sigma)]{} (\sqrt{2\sigma+1}\,T)^{-3} \tag{32}$$
- $\epsilon_c = (A^2\sigma^2 - C^2)/(4\sigma) > 0 \Rightarrow$ General solution: $C \in (-\sigma A, \sigma A)$: $\lim_{T\to 0} a(T) < \infty$.
- The steep increase in $F(T) \sim T^{-3}$, the fraction of volume occupied by the gluons, with decreasing T must be compensated by the increased rate of gluon fusion $\Rightarrow$ saturation.

Summary

Take-Home Message

- QCD represents a fruitful theory when applied to the early history of our Universe.
- Breaking of its scale invariance by quantum effects leads to the emergence of two scales $\Lambda_{\rm QCD}$ and $T_s \gg \Lambda_{\rm QCD}$. First associated with confinement and second with saturation.
- Application of saturation to the thermal evolution of the early Universe implies the existence of the new state of matter, the precursory of the sQGP, which might have played an essential role over a broad range of temperatures $T_{\rm GUT} \gtrsim T \gtrsim T_{\rm EW} > T_{\rm QGP}$.
- A solvable model modifying the EoS of non-interacting massless gas by the inclusion of the quadratic thermal terms leads to an increase in the fraction of volume occupied by thermal gluons with decreasing temperature $F(T) \sim T^{-3}$ and hence to the saturation.
- The speed of sound c_s in the matter described by this new EoS increases with decreasing energy density leading ultimately to its transformation into stiff matter.
- TBD: Solution of matter-antimatter asymmetry using saturated QCD matter.

Thank you for your attention!

Backup Slides

Equation of State Based on the Fundamental Theory

- In the SM $\quad p_B(T) = \lim_{V\to\infty} \frac{T}{V} \ln \mathcal{Z}(T,V)$

$$\mathcal{Z}(T,V) = \exp\left[\frac{p_B(T)V}{T}\right], \quad p_B(T) = p_E(T) + p_M(T) + p_G(T) \qquad (33)$$

- p_B is the "bare" result related to the physical (renormalized) pressure as $p(T) = p_B(T) - p_B(0)$.
- $p_E(T), p_M(T), p_G(T)$ collect the contributions from the momentum scales $k \sim \pi T$, $k \sim gT$, and $k \sim g^2 T/\pi$, respectively.
- Couplings of SM are $g \in \{h_t, g_1, g_2, g_3\}$, where h_t is the Yukawa coupling between the top quark and the Higgs boson, and g_1, g_2, g_3 are related to $U_Y(1)$, $\mathrm{SU_L}(2)$ and $\mathrm{SU}_c(3)$ gauge groups, respectively.
- Calculations of the dimensionless function $p(T)/T^4$ and of the trace anomaly $\Theta(T)$ up to $\mathcal{O}(g^5)$ were performed in [Laine,Mayer:2015].

The Plasma of Charged Particles

- Plasma = system of mobile charged particles [Ichimaru:1982].
- Electrically neutral gas (liquid, crystal) at high temperatures T turns into a system of charged particles with the long-range $U(1)$ interaction.
- Plasma interaction parameter

$$\Gamma_{\mathrm{EM}} = \frac{q^2}{r_s k_B T} \sim \frac{U_{int}}{E_{th}}, \qquad r_s = \left(\frac{3V}{4\pi N}\right)^{1/3} \approx 0.62 n^{-1/3}, \tag{34}$$

  q - particle charge
  r_s - average inter-particle distance (Wigner-Seitz radius)
- Strongly-coupled (SC) plasma: $\Gamma_{\mathrm{EM}} > 1$, i.e. when $U_{int} > E_{th} \sim k_B T$, interaction energy prevails over thermal energy of the plasma particles.
- **Example** Table salt – crystalline plasma made of permanently charged Na^+ and $C\ell^-$ ions. $\Gamma_{\mathrm{EM}} \approx 60$ at $T \approx 10^3$K. [Shuryak:2008].

The Plasma of Quarks and Gluons

- Generalization $U(1) \to SU(3)_c$ [Thoma:2005], see also [Bannur:2005]

$$\Gamma_{\rm QCD} \simeq 2\frac{C_{q,g}\alpha_{\rm S}}{r_s T}\,, \quad C_q = \frac{N_c^2-1}{2N_c} = \frac{4}{3}\,, \quad C_g = N_c = 3 \tag{35}$$

N.B. In relativity chromo-electric $\approx$ chromo-magnetic $\Rightarrow$ 2 in (35).

- For ideal massless QCD gas with N_F active quarks and $d_F = g_{\rm eff}^{\rm QCD}$

$$n = d_F \frac{\zeta(3)}{\pi^2} T^3 \approx d_F \left(\frac{T}{2}\right)^3, d_F = 2\times 8 + \tfrac{7}{8}(3 \times N_F \times 2 \times 2) \tag{36}$$

$$r_s \simeq 1.24 d_F^{-1/3} T^{-1} \Rightarrow r_s T = f(N_F(T)) \tag{37}$$

1. $T \approx 200$ MeV: $\alpha_{\rm S} = 0.3-0.5$, $N_F = 2$, $d_F = 37 \Rightarrow \Gamma_{\rm QCD} \simeq 2-8$.
2. $T \simeq T_{\rm EW}$: $\alpha_{\rm S} = 0.08$, $N_F = 5$, $d_F = 52.5$ and $\Gamma_{\rm QCD} \simeq 0.5-1.5$.
3. $T \gg T_{\rm EW}$: $N_F = 6$, $\Gamma_{\rm QCD}(T)$ is solely driven by $\alpha_{\rm S}(T) \sim -\ln T$.

The Glasma as a (Semi-)Classical Matter

QCD in the classical regime [Kharzeev:2002]

- Introduce coupling-independent field tensors

$$A^a_\mu \to \mathcal{A}^a_\mu \equiv g_S A^a_\mu, \quad g_S^2 = 4\pi\alpha_S$$

$$F^a_{\mu\nu} \to g_S F^a_{\mu\nu} \equiv \mathcal{F}^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + f^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu\,, \tag{38}$$

- Calculate action of the gluon field

$$S_g = -\frac{1}{4}\int F^a_{\mu\nu}F^{\mu\nu,a}d^4x = -\frac{1}{4g_S^2}\int \mathcal{F}^{\mu\nu,a}\mathcal{F}^a_{\mu\nu}d^4x \tag{39}$$

- Gluon occupation number
$$f_g \sim \frac{S_g}{\hbar} = \frac{1}{\hbar g_S^2}\rho_4 V_4 \tag{40}$$

  where $\rho_4 \sim \langle \mathcal{F}^{\mu\nu,a}\mathcal{F}^a_{\mu\nu}\rangle$ is four-dimensional gluon condensate density.

- Saturated gluon matter is weakly coupled.
- The limits $g_S^2 \to 0$ and $\hbar \to 0$ are equivalent! $\Rightarrow$ weakly interacting means semi-classical.

Medium with $c_s = const.$ and Ideal Gas of Quasi-particles

- Consider an ideal gas of free particles in D-dimensional space expressed in terms of single-particle statistical sum $f(E, T, \mu)$ (see e.g. [Kapusta:1989]):

$$n = \gamma \int f \cdot d^D P\,, \quad f(E, T, \mu) = \frac{1}{\exp[(E-\mu)/T] \pm 1}\,, \quad \gamma \equiv \frac{2s+1}{2\pi)^D}\,, \tag{41}$$

$$\epsilon = \gamma \int f \cdot E(P) d^D P = \gamma S(D) \int_0^\infty f \cdot E(P) P^{D-1} dP\,, \quad S(D) = \frac{D\pi^{D/2}}{\Gamma(D/2+1)}\,, \tag{42}$$

$$p = -\frac{T}{V} \ln Z = -\gamma T \int \ln f \cdot d^D P = \gamma \frac{S(D)}{D} \int_0^\infty f \cdot \frac{\partial E(P)}{\partial P} P^D dP\,, \tag{43}$$

- Substitution of (42) and (43) into Equation $p - w\epsilon = 0$ gives

$$\frac{P}{D} \frac{\partial E(P)}{\partial P} = wE(P) \quad \Rightarrow \quad E(P) = \xi P^{wD}, \xi = const. \tag{44}$$

$\Rightarrow$ Medium with $c_s^2 = w = const.$ in $D = 3$ space is equivalent to ideal gas of quasi-particles with dispersion relation $E(P)$ (44) in D-dimensional space [Trojan:2011ma].

- $wD = 1$ corresponds to massless particles in D-dimensional space with the EoS $p = \epsilon / D$.