Hybrid star phenomenology from the properties of the special point

Zimányi School

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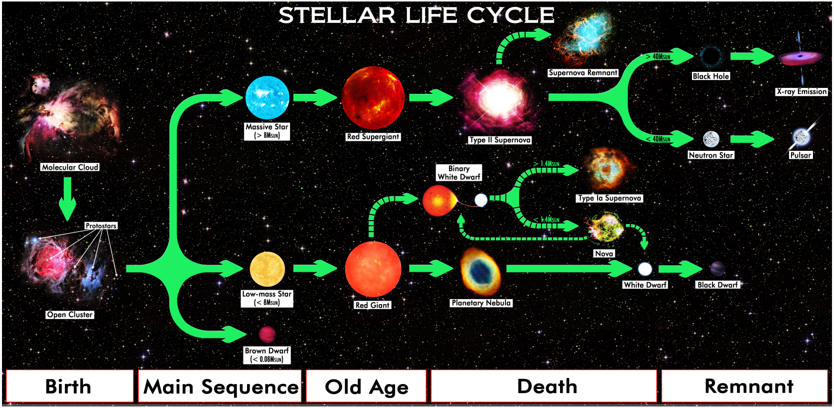
What are NSs?

Evolution of Stars:



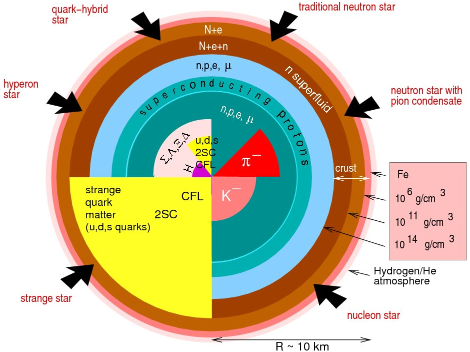
Evolution governed by the mass

	Neutron star	White dwarf	Sun
$M_{max}(M_\odot)$	2	1.44	1
R (km)	11-12	10^4	$7 \cdot 10^5$
n_c (g/cm^3)	$10^{14} - 10^{15}$	10^7	10^2
rotation speed (s)	$10^{-3} - 1$	100	$2 \cdot 10^6$
B (G)	$10^8 - 10^{16}$	100	1
T (K)	$10^6 - 10^{11}$	10^3	10^5

Table including Neutron star properties

NS compared to city

Why are Compact Stars as Neutron Stars (NS) important for this School?



Weber+ 2007

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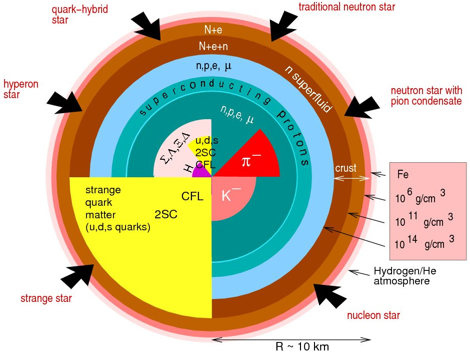

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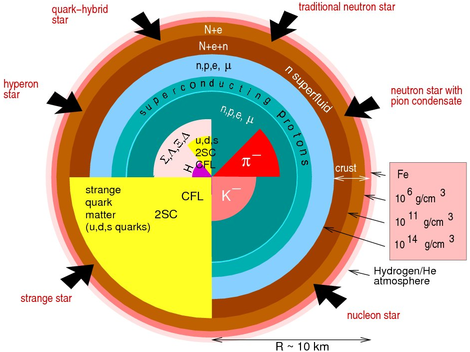

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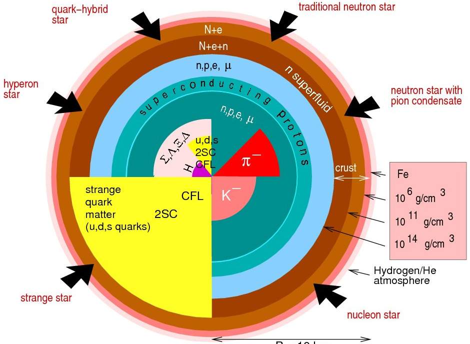
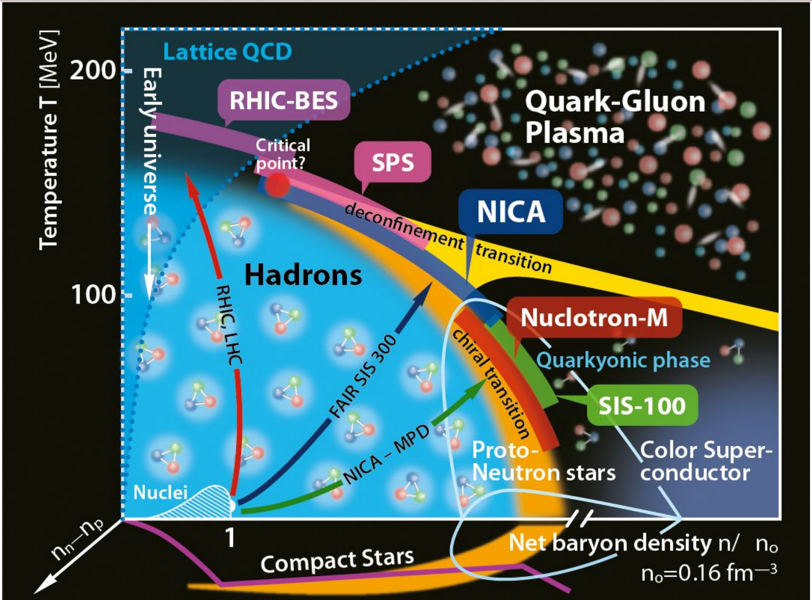


Weber+ 2007

- different possible composition of Neutron Stars (NS)
- one possibility: Quark Gluon Plasma in the core (including phase transition)
  ⇒ chance to probe QCD phase diagram with NS



Sahoo,Nayak 2022

How to build up an NS with Equations of State?

What do we need to obtain possible Neutron Star configurations?

⇒ Tolmann-Oppenheimer-Volkoff Equations (spherical symmetric and gravitational equilibrated objects)

$$\frac{dp}{dr} = -(\varepsilon + p)\frac{m + 4\pi r^3}{r^2 - 2rm},$$
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

⇒ need $p(\varepsilon)$ ⇒ need full EOS ⇒ Hybrid EOS

How to build up an NS with Equations of State?

Assumption: We work with a **First order Phase Transition** between Hadronic and Quark Gluon Phase

- Hadronic Phase: **DD2npY-T model** including neutrons, protons and hyperonic degrees of freedom Shahrbaf,Blaschke+(2022)
- Quark matter: confining relativistic density functional approach Ivanytskyi,Blaschke (2022)

  $\Rightarrow$ encoded in underlying Lagrangian

Relativistic density functional for quark matter EOS

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \hat{m})q + \mathcal{L}_{PS} + \mathcal{L}_V + \mathcal{L}_D$$

- **Pseudoscalar interaction ⇒ chiral dynamics**

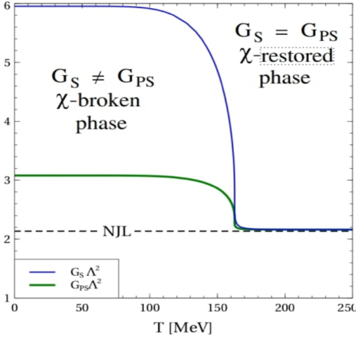
$$\mathcal{L}_{PS} = G_0 \left[(1+\alpha)\langle\bar{q}q\rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2\right]^{\frac{1}{3}}$$

- **Vector interaction ⇒ repulsion**

$$\mathcal{L}_V = -G_V(\bar{q}\gamma_\mu q)^2$$

- **Diquark pairing ⇒ color superconductivity**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\bar{q}i\gamma_5\tau_2\lambda_A q^c)(\bar{q}^c i\gamma_5\tau_2\lambda_A q)$$

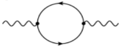


Details in:

Ivanytskyi, Blaschke, PRD (2022)

Ivanytskyi, Blaschke, Particles (2022)

- **Comparison to NJL model**
  - medium dependent scalar G_S and pseudoscalar G_{PS} couplings
  - high vacuum quark mass ⇒ phenomenological confinement
  - quark correlations ⇒ mesons: π, σ =



Model parameters

- **Pseudoscalar interaction channel $\mathcal{L}_{PS}$**

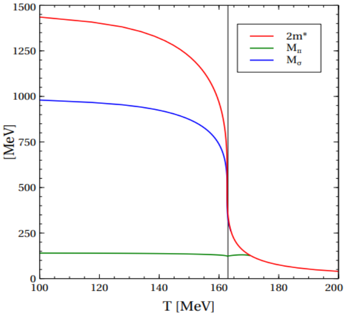
  relevant to vacuum phenomenology

  (chiral condensate & meson properties)

m [MeV]	Λ [MeV]	α	$D_0\Lambda^{-2}$
4.2	573	1.43	1.39
M_π [MeV]	F_π [MeV]	M_σ [MeV]	$\langle \bar{l}l \rangle_0^{1/3}$ [MeV]
140	92	980	-267

Pseudocritical temperature

$$T_c = 163 \text{ MeV}$$



- low T: $2m_{quark} > M_\pi, M_\sigma$
  (stable mesons, confined quarks)
- high T: $2m_{quark} < M_\pi, M_\sigma$
  (unstable mesons, deconfined quarks)

- **Vector & diquark interaction channels $\mathcal{L}_V$ & $\mathcal{L}_D$**

  parameterized by the dimensionless couplings $\eta_V \equiv G_{0V}/G_{S0}$ & $\eta_D \equiv G_{0D}/G_{S0}$

Couplings

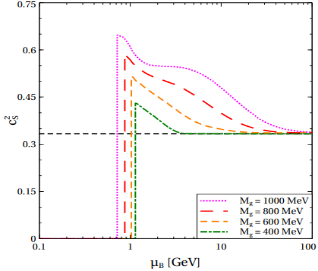
$$G_V = \frac{G_V^{vacuum}}{1 + \frac{8}{9M_{gluon}^2}\left(\frac{\pi^2\langle q^+ q\rangle}{2}\right)^{2/3}} \qquad G_D = \frac{G_D^{vacuum}}{1 + \frac{8}{9M_{gluon}^2}\left(\frac{\pi^2\langle \overline{q}^c i\tau_2\gamma^5\lambda_2 q\rangle}{2}\right)^{2/3}}$$

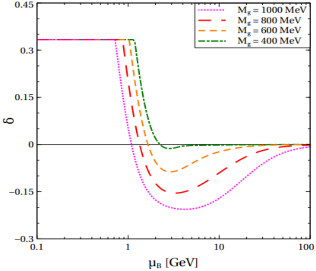
O. Ivanytskyi, D. Blaschke, Particles 5 (4), 514-534 (2022)

- **Motivated by non-perturbative massive gluon exchange**

Y. Song, G. Baym, T. Hatsuda, and T. Kojo Phys. Rev. D 100, 034018 (2019)

- **Provide asymptotic conformal behavior ($c_S^2 \to 1/3, \quad \delta = 1/3 - p/\varepsilon \to 0$)**





- **Rearrangement terms in pressure ensure thermodynamic consistency**

ABPR parametrization

- Extention of the bag pressure model accounting for the perturbative QCD correction to pressure and effects of quark pairing M. Alford, M. Braby, M. W. Paris, and S. Reddy, Astrophys. J. 629, 969 (2005), arXiv:nucl-th/0411016.

$$p = \frac{3A_4\mu^4}{4\pi^2} + \frac{3\Delta^2\mu^2}{\pi^2} - B \tag{1}$$

i	units	a_i	b_i	c_i	d_i	e_i
1		0.757	-1.955	1.799	-0.063	0.046
2	[MeV]	300.7	8.534	-308.2	-0.235	1.458
3	[MeV/fm^3]	72.018	170.8	-241.0	512.7	-626.6

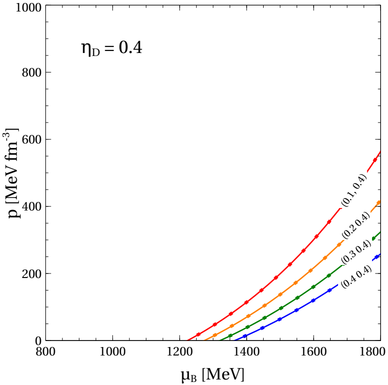
$$A_4 = a_1 + b_1\eta_V + c_1\eta_V^2 + \left(d_1 + \frac{e_1}{\eta_V}\right)\eta_D, \tag{2}$$

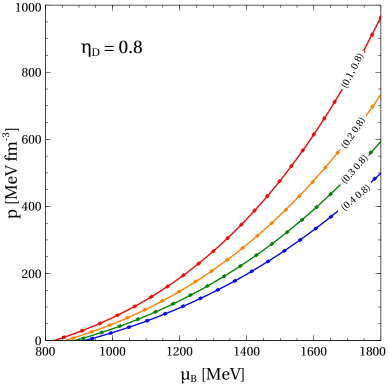
$$\Delta = (a_2 + b_2\eta_V + c_2\eta_V^2)\sqrt{d_2 + e_2\eta_V + \eta_D}, \tag{3}$$

$$B = a_3 + b_3\eta_V + c_3\eta_V^2 + d_3\eta_D + e_3\eta_D^2. \tag{4}$$

ABPR parametrization

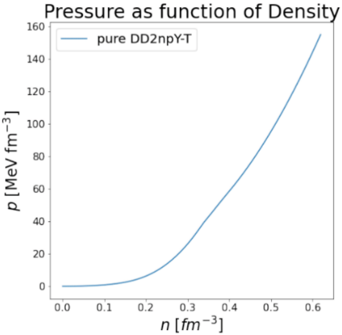
Fitting couplings



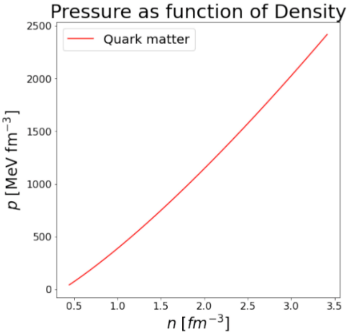


- remarkable agreement between RDF approach (solid lines) and the ABPR parametrization (dots) !!!

Maxwell construction



Hadronic EOS

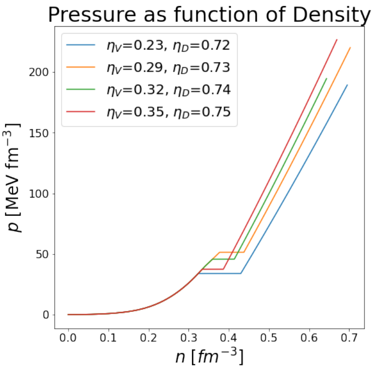


Quark EOS

- Maxwell construction: intersection of both functions $p(\mu)$

$$p_{Hadron}(\mu_c) = p_{Quark}(\mu_c) \Rightarrow \mu_c \tag{5}$$

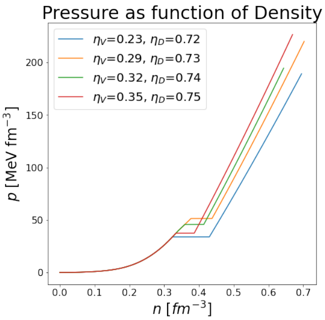
Maxwell construction



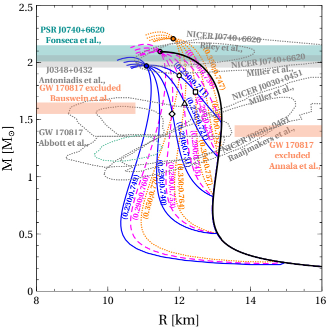
Hybrid EOS

- typical plateau of first order phase transition
- **But:** inside star, no shell of mixed phase ⇒ narrow

From EOS to M-R curves



TOV



TOV equations:

$$\frac{dp}{dr} = -(\varepsilon + p)\frac{m + 4\pi r^3}{r^2 - 2rm},$$

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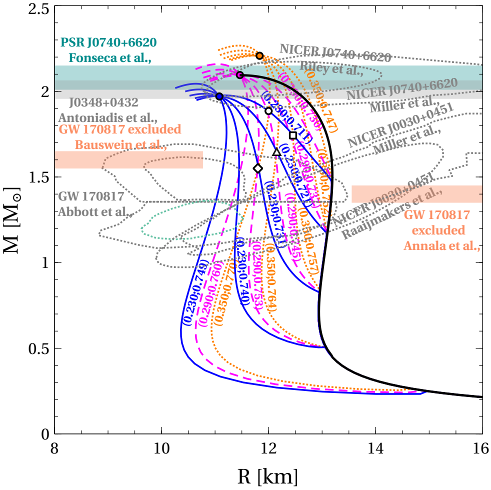
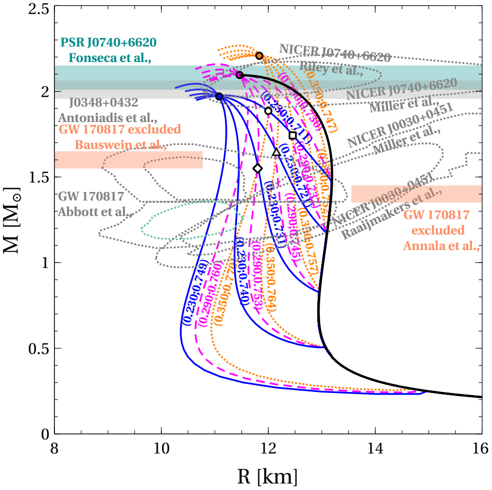
Mass-Radius curves and its properties



M-R diagram for η_V, η_D-combinations, Gärtlein+ 2023

- each point is a NS configuration

Mass-Radius curves and its properties



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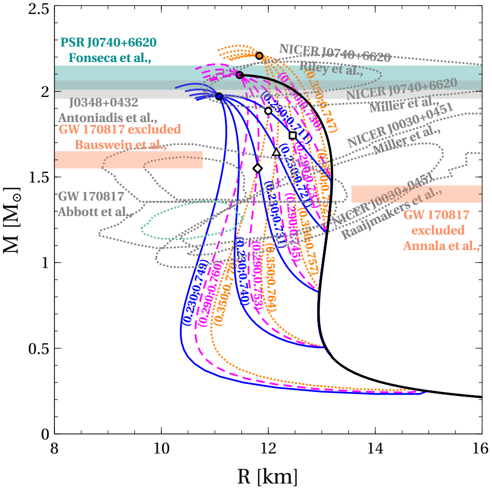

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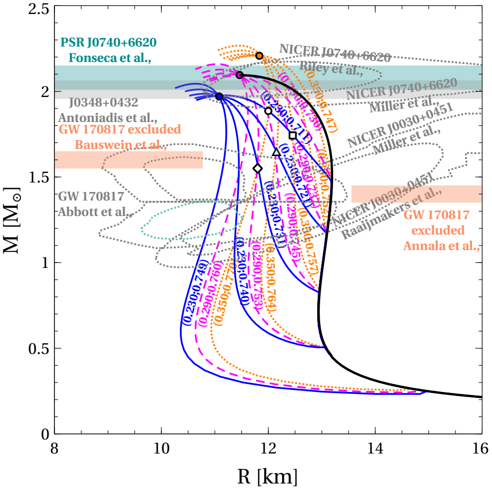

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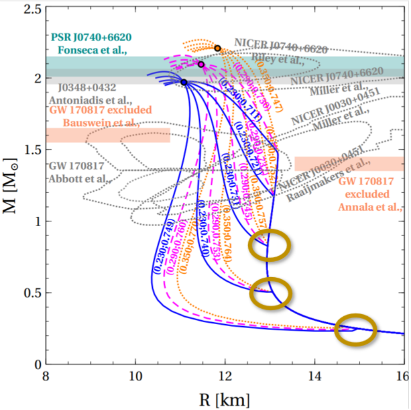
- point of leaving the black curve ⇒ deconfinement phase transition

Deconfinement phase transition



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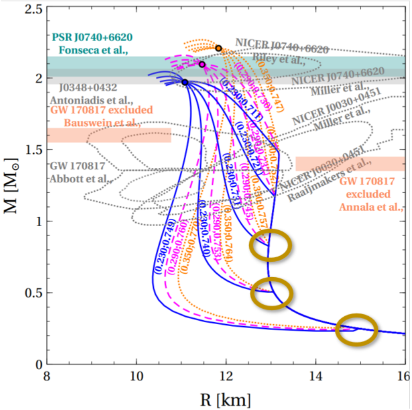
Deconfinement phase transition



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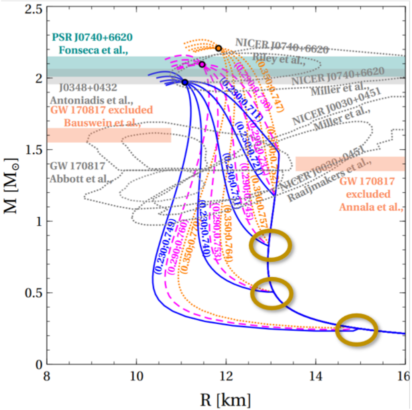
- fixed η_V: smaller diquark coupling ⇒ later deconfinement

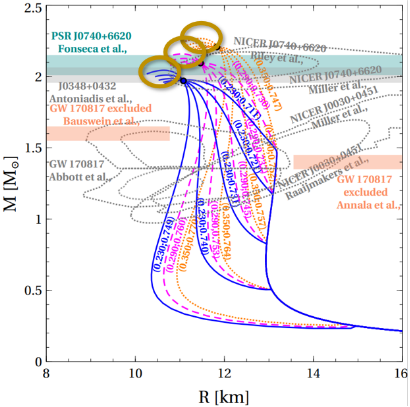
Deconfinement phase transition



M-R diagram for η_V, η_D-combinations, Gärtlein+ 2023

- certain combination of (η_V, η_D) give point of phase transition
- fixed η_V: smaller diquark coupling ⇒ later deconfinement
- larger η_D ⇒ earlier phase transition but greater maximum mass

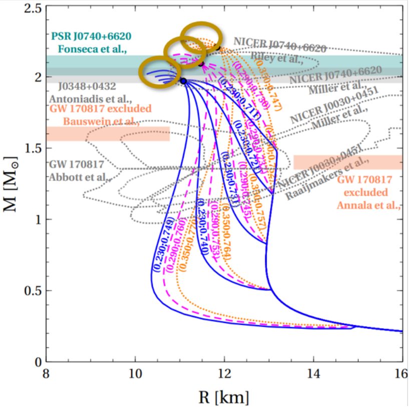
Maximum mass



M-R diagram for η_V, η_D-combinations, Gärtlein+ 2023

- curves greater vector repulsion
  ⇒ higher maximum masses

Maximum mass



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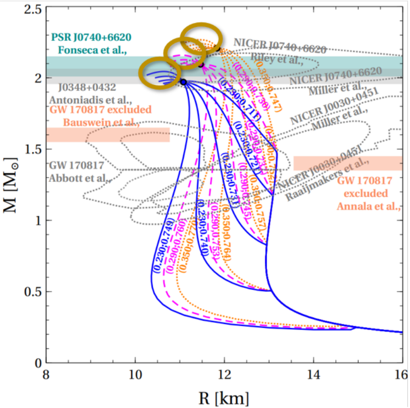
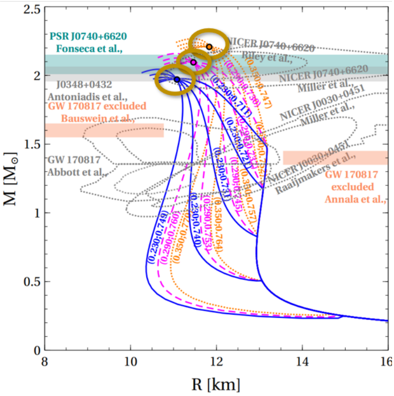
Maximum mass



M-R diagram for η_V, η_D-combinations, Gärtlein+ 2023

- curves greater vector repulsion ⇒ higher maximum masses
- in general combination fixes maximum mass
- higher vector repulsion ⇒ stiffer EOS ⇒ higher masses

The Special points



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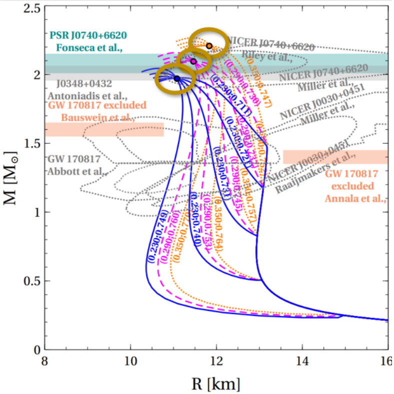

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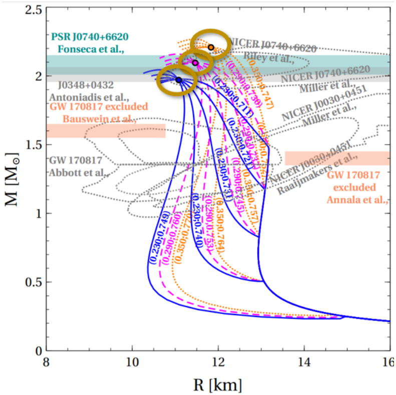

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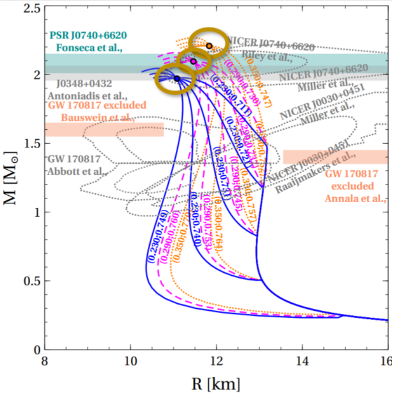

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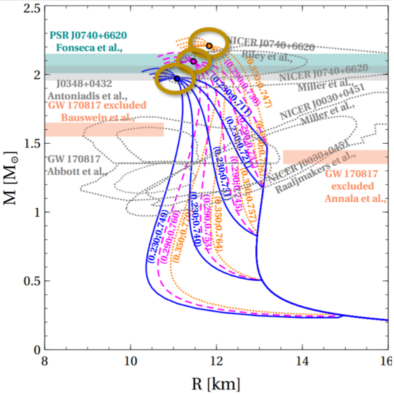

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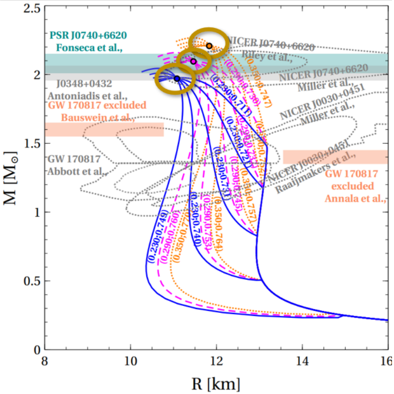

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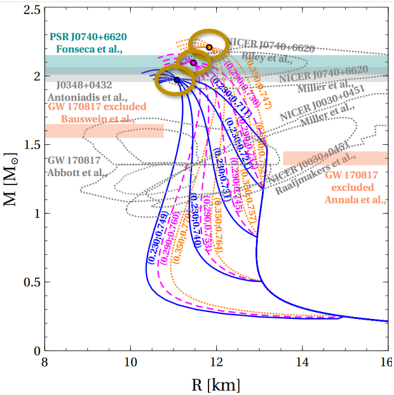

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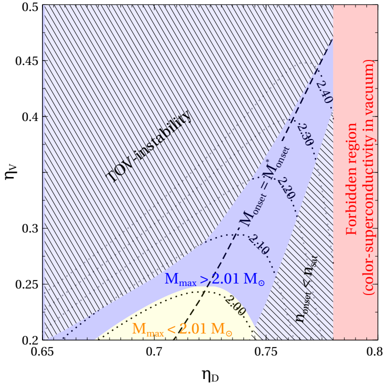
$\Rightarrow$ fixing parameters by data fit

$$M^*_{onset} = 1.254 M_\odot, \quad \delta = k_\delta \eta_V + b_\delta$$

where $k_\delta = -0.096\ \mathrm{M}_\odot^{-1}$ and $b_\delta = 0.093\ \mathrm{M}_\odot^{-1}$.

Use of Emperical Relation

This pretty accurate relation allows us to constrain the couplings of the quark matter !!



Constraints, Gärtlein+ 2023

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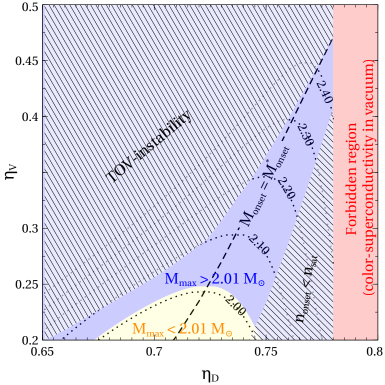

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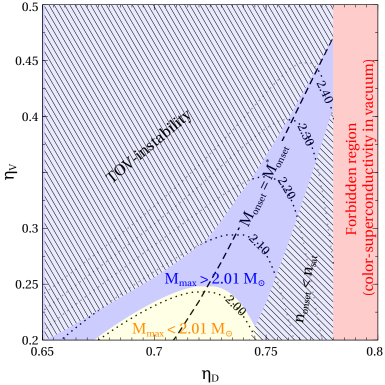

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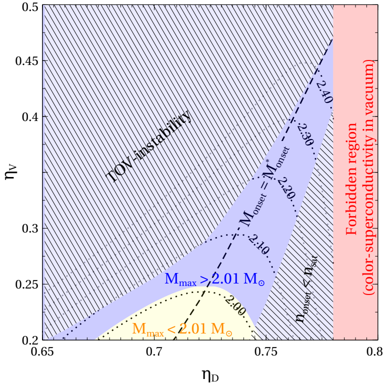

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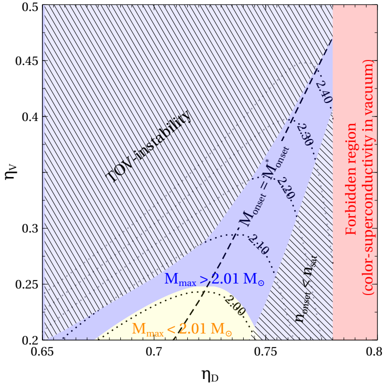

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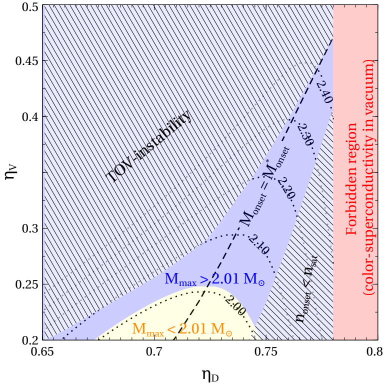
Use of Emperical Relation

This pretty accurate relation allows us to constrain the couplings of the quark matter !!



Constraints, Gärtlein+ 2023

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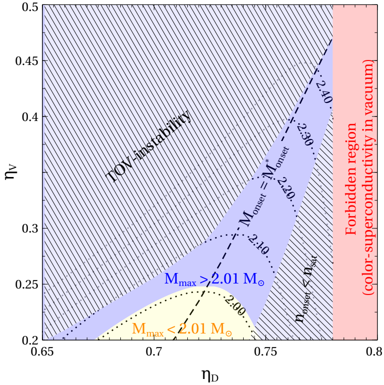

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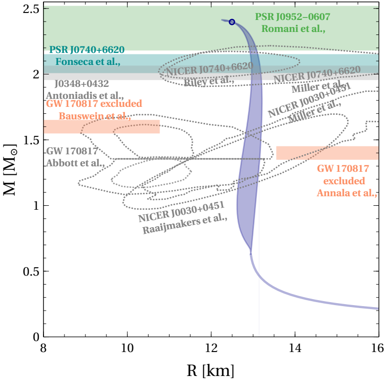
⇒ narrow range for η_D $\approx (0.775 - 0.78)$

Implications



Including ω-mass, Gärtlein+ 2023

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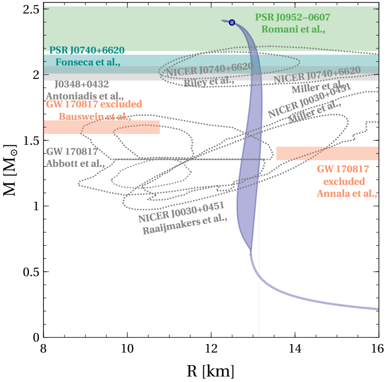

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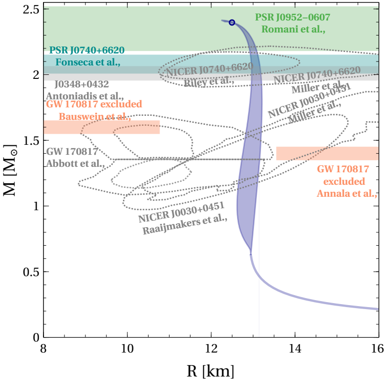

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   ⇒ Family of Hybrid EOS
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4. microscopic parameters govern special point (independent of hadronic EOS, due to First Order Phase Transition)

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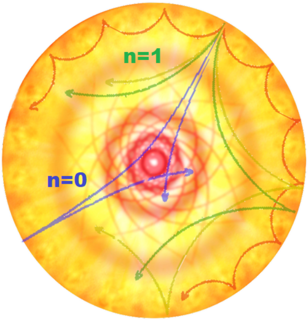
8. radial oscillations may give insights as well

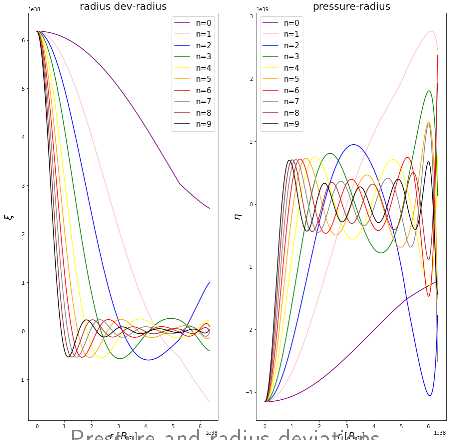
Thank you for your attention.
Please ask your questions.

Radial Oscillations

Different sources of forces on stars/NS can cause oscillations ⇒ radial oscillations ⇒ gravity as pullback force



Radial modes inside a star



Pressure and radius deviations

Solve differential equations:

$$\xi \equiv \frac{\Delta r}{r}, \quad \eta \equiv \frac{\Delta p}{p}$$

$$\frac{d\xi}{dr} = -\left(\frac{3}{r} + \frac{1}{\epsilon + p}\frac{dp}{dr}\right)\xi - \frac{\eta}{r\gamma}, \tag{6}$$

$$\begin{aligned}\frac{d\eta}{dr} &= \omega^2\left[\frac{\epsilon + p}{p} r e^{(\lambda-\nu)}\right]\xi \\ &\quad -\left[\frac{4}{p}\frac{dp}{dr} + 8\pi(\epsilon + p) r e^{\lambda} - \frac{r}{p(\epsilon + p)}\left(\frac{dp}{dr}\right)^2\right]\xi \\ &\quad -\left[\frac{\epsilon}{p(\epsilon + p)}\frac{dp}{dr} + 4\pi\zeta r e^{\lambda}\right]\eta,\end{aligned} \tag{7}$$

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  $\Rightarrow$ lowest frequency modes $\Rightarrow$ easy to excite

SP	M_{SP} [$M_\odot$]	R_{SP} [km]	η_V	η_D	M_{onset} [$M_\odot$]	[M_{max}] [$M_\odot$]	f_{new} [kHz]
blue	1.973	11.06	0.23	0.749	0.251	2.044	2.120
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				0.731	0.826	1.986	1.445
				0.721	1.169	1.974	1.096
				0.711	1.483	1.976	imaginary
magenta	2.092	11.46	0.29	0.760	0.251	2.159	2.008
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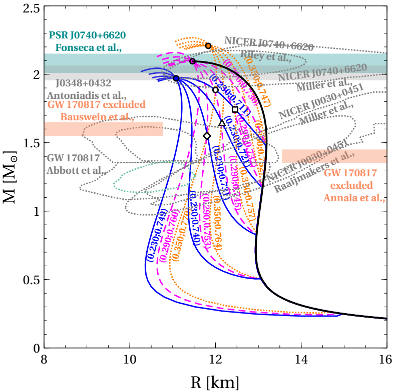
Table including important quantities



- smaller frequencies ⇒ closer to maximum mass
- smaller f ⇔ smaller η_D ⇔ later deconfinement
- f-mode can tell us how close we are to the maximum mass of certain curve ⇔ empirical relation ⇔ position of deconfinement phase transition