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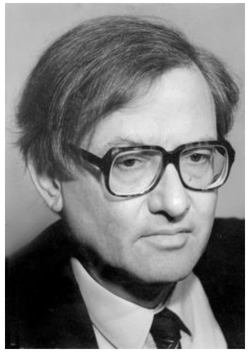
Preprints are available: arXiv:2311.18678 arXiv:2311.03568

Describing hadronic and thermal photon distributions with an analytic solution of hydrodynamics

GÁBOR KASZA, TAMÁS CSÖRGŐ

ZIMÁNYI SCHOOL 2023

BUDAPEST, 12/07/2023

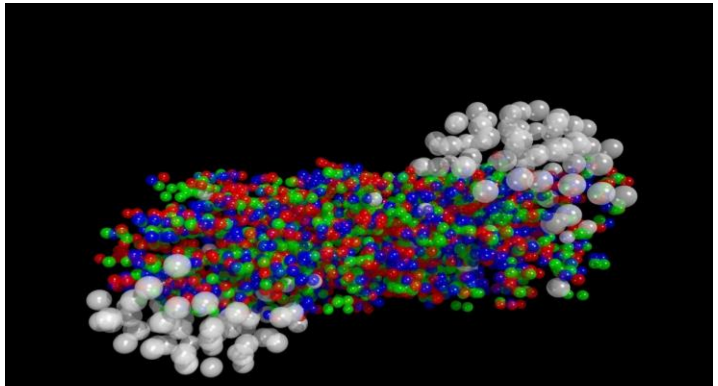


Various application of hydrodynamics

- **Why is hydrodynamics so effective?**
- Works well on …
  - … *microscopic scales*
  - … *macroscopic scales*
  - … *cosmic scales*
- ***Hydrodynamics has no internal scale***
- Averages the microscopic degrees of freedom
- Analytic hydro: with relatively simple equations we can …

  … describe the scaling behaviour of experimental data

  … provide a tool for checking more complex calculations



Heavy-ion collision: in the overlap area, the sQGP is created

Today's presentation:

→ ***scaling of dN/dy***

→ ***describe the thermal radiation***

Relativistic perfect fluid solution with accelerating velocity field

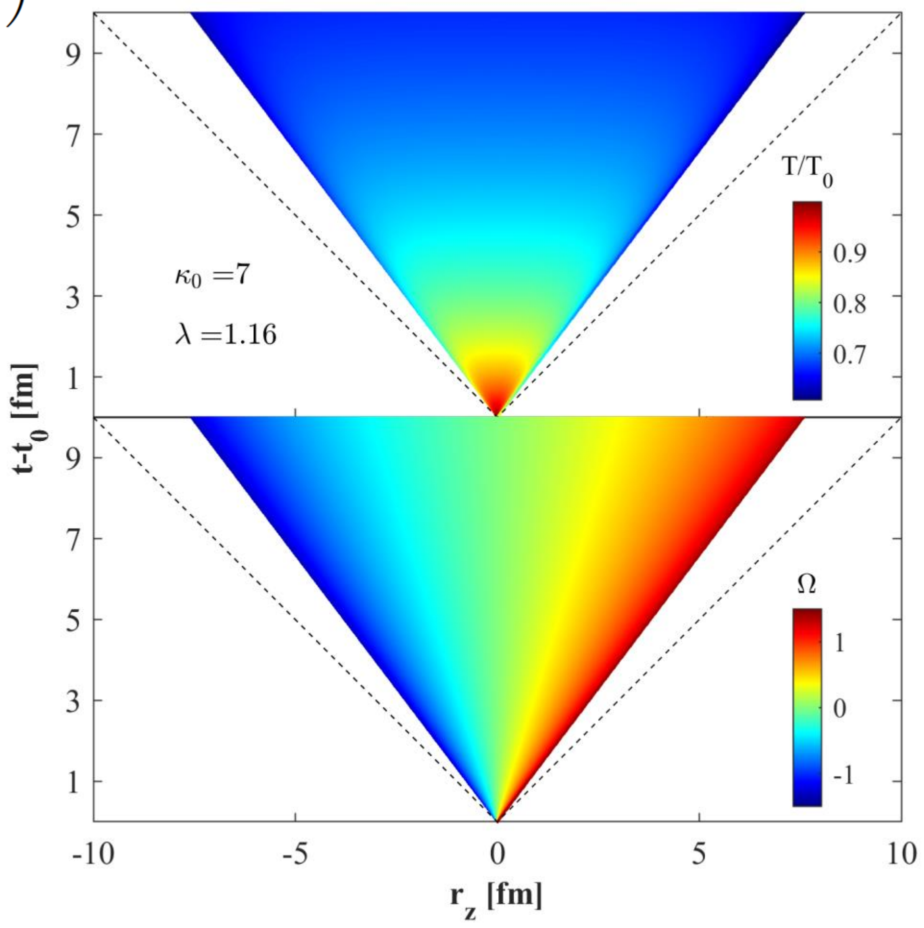
- Equation of state: $\varepsilon = \kappa_0 p \quad (\mu=0)$
- Rindler coordinates: $\tau = \sqrt{t^2 - r_z^2} \qquad \eta_z = \frac{1}{2}\ln\left(\frac{t+r_z}{t-r_z}\right)$
- Velocity field: $u^\mu = \begin{pmatrix}\cosh(\Omega)\\ \sinh(\Omega)\end{pmatrix} \qquad \Omega \equiv \Omega(\eta_z)$
- 1+1 dimensional, parametric, almost self-similar, finite solution

λ: rate of acceleration
(Hwa-Bjorken: $\lambda=1$)

accelerating expansion

realistic $dN/d\eta_p$

T. Csörgő, G. Kasza, M. Csanád and Z-F. Jiang: Universe 4 (2018) 69



Scaling of dN/dy

arXiv:2311.18678

Pseudorapidity distribution

- Starting from the rapidity distribution, we calculated the pseudorapidity distribution
- Parametric curve:

$$\left(\eta_{\mathrm{p}}(y), \frac{dN}{d\eta_{\mathrm{p}}}(y)\right) = \left(\frac{1}{2}\ln\left[\frac{\langle|p(y)|\rangle + \langle p_z(y)\rangle}{\langle|p(y)|\rangle - \langle p_z(y)\rangle}\right], \frac{\langle|p(y)|\rangle}{\langle E(y)\rangle}\frac{dN}{dy}\right)$$

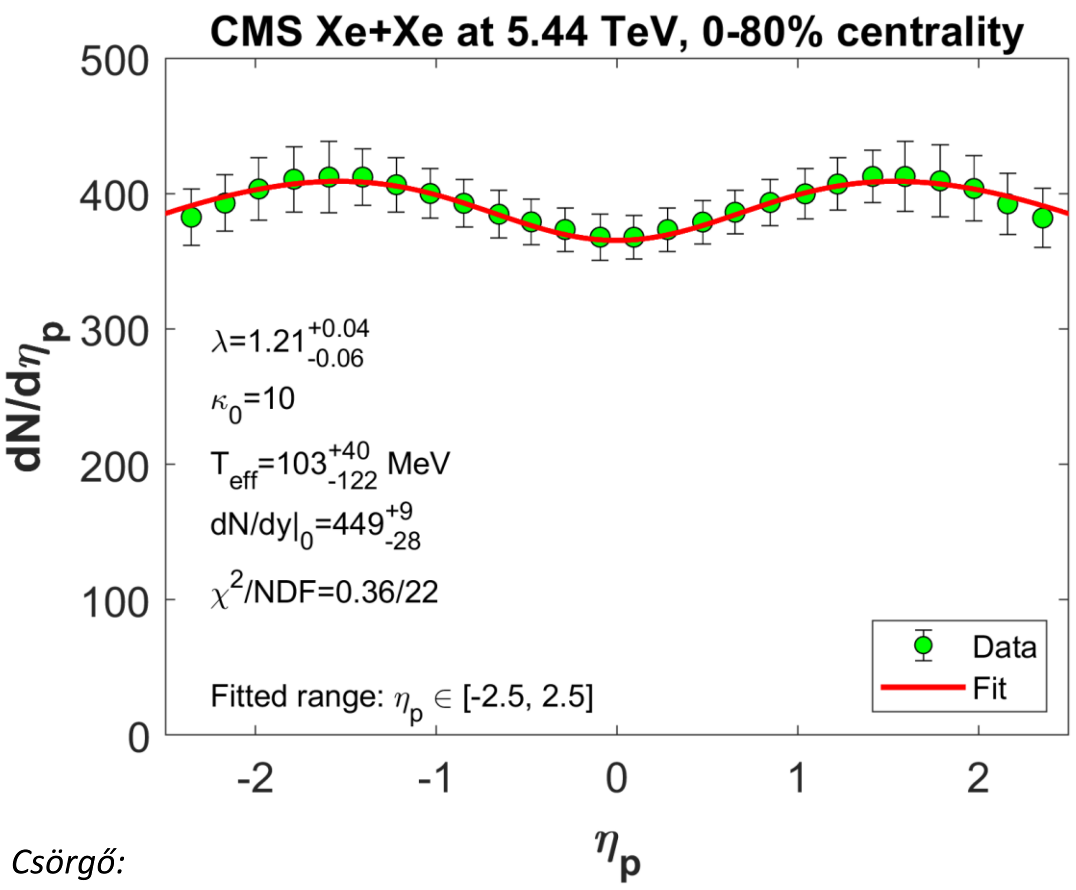
- We compared this curve with experimental data :
  - PHOBOS Au+Au 130 GeV, 200 GeV
  - ALICE Pb+Pb 5.02 TeV
  - CMS p+p 7 TeV, 8 TeV, 13 TeV → ***described very well, collectivity in p+p collisions?***
  - CMS Xe+Xe 5.44 TeV

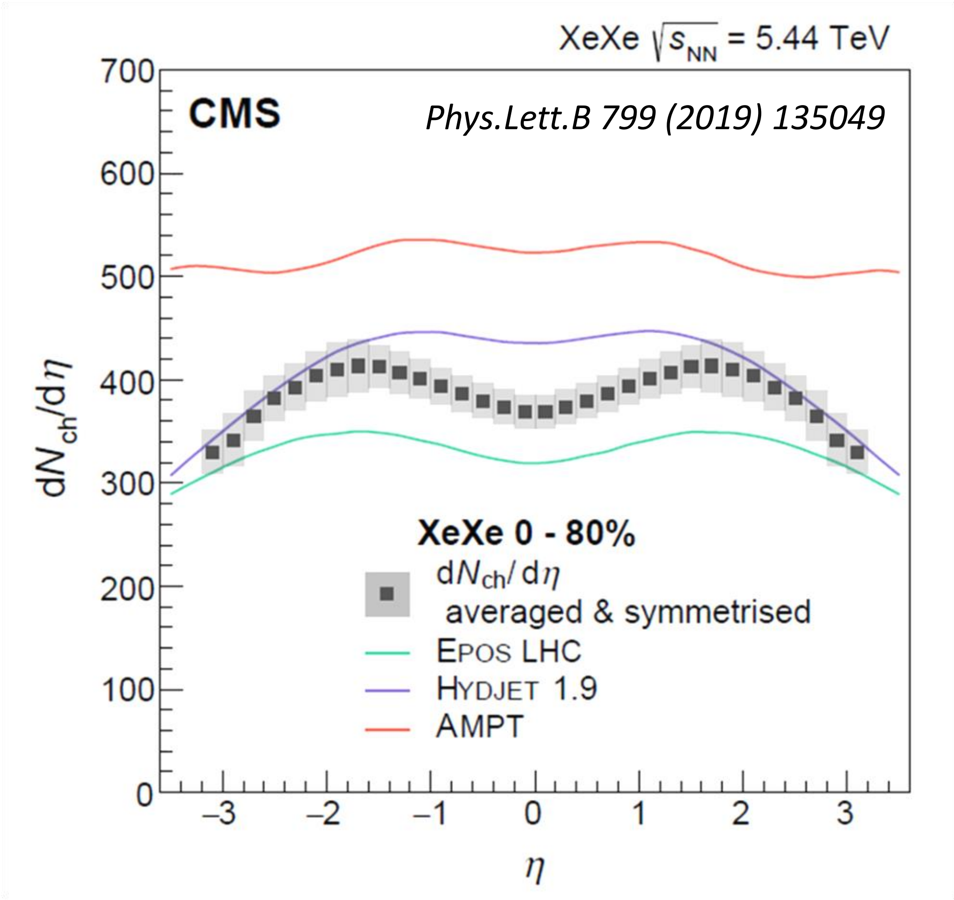
G. Kasza, T. Csörgő:
Int.J.Mod.Phys. A34 no.26, 1950147 (2019)

- Different sizes (p+p, A+A), different centralities, different collision energies

Fits to CMS data (Xe+Xe)

- The parametric formula works well in such cases where other models fail:





G. Kasza, T. Csörgő:
Int.J.Mod.Phys. A34 no.26, 1950147 (2019)

A reasonable approximation of the rapidity distribution

- If **|y| << 2+1/(λ-1)**, then the rapidity distribution becomes Gaussian:

$$\frac{dN}{dy} \approx \frac{\langle N \rangle}{(2\pi \Delta y^2)^{1/2}} \exp\left(-\frac{y^2}{2\Delta y^2}\right)$$

- Manifest of the hydrodynamic scaling behaviour:

$$\frac{1}{\Delta y^2} = (\lambda - 1)^2 \left[1 + \left(1 + \frac{1}{\kappa_0}\right)\left(\frac{1}{2} + \frac{m}{T_{\mathrm{eff}}}\right)\right]$$

$$\langle N \rangle = (2\pi \Delta y^2)^{1/2} \left.\frac{dN}{dy}\right|_{y=0}$$

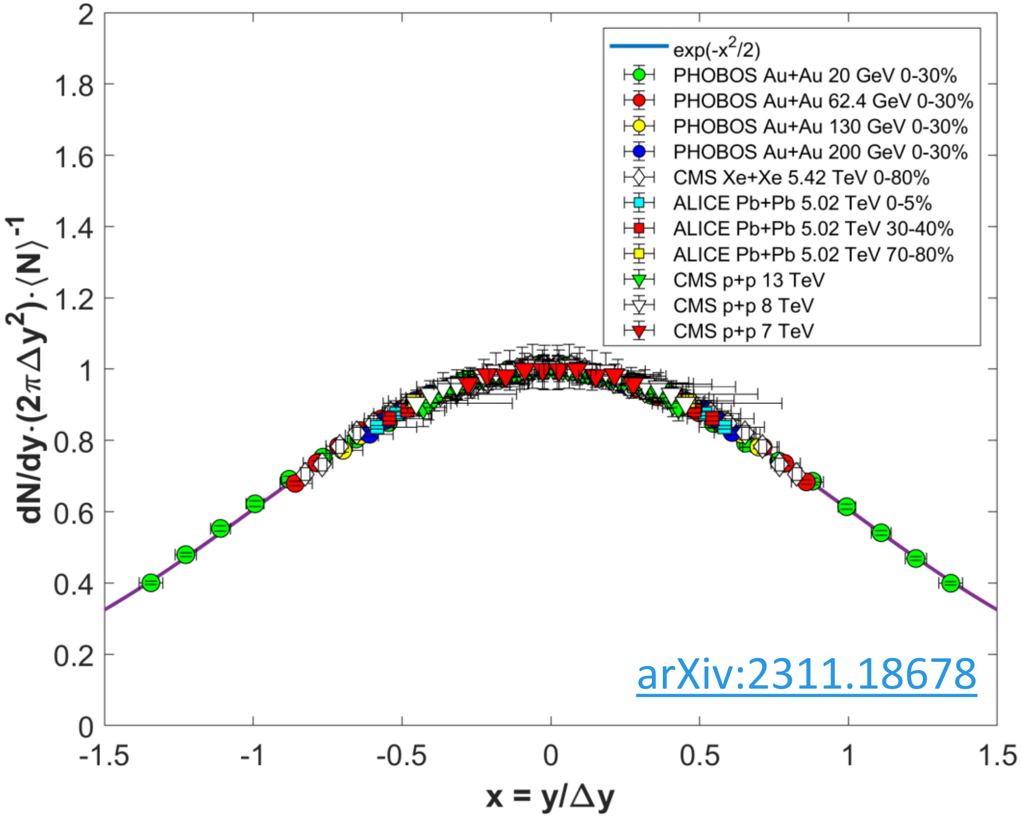
- The physical properties of different collisions (√s, centrality, size) are scaled out:

$$\frac{dN}{dy} = \left.\frac{dN}{dy}\right|_{y=0} \exp\left(-\frac{y^2}{2\Delta y^2}\right) \longrightarrow f(x) = \exp\left(-\frac{x^2}{2}\right)$$

arXiv:2311.18678

Data collapsing

- Pseudorapidity distributions were transformed into rapidity distributions → fits to the *dN/dy* data series



For each dataset, the fit range satisfies:
$|y| << 2+1/(\lambda-1)$

This condition is the strictest in the case of *PHOBOS Au+Au@20 GeV* → $|y| << 3$, $|x| << 1.8$

- Data collapsing on the $f(x)=exp(-x^2/2)$ curve of the scale function

Another prediction for Gaussian dN/dy from Landau hydrodynamics:
C-Y. Wong: Phys. Rev. C 78, 054902
C-Y. Wong et al.: Phys.Rev.C 90 (2014) 6, 064907
A. Sen et al.: J.Phys.Conf.Ser. 630 (2015) 1, 012042

Thermal photon radiation

arXiv:2311.03568

Derivation of the thermal radiation

Source function:

$$S\left(x^{\mu},p^{\mu}\right)d^4x=\frac{g}{(2\pi\hbar)^3}\frac{H(\tau)}{\tau_R}\frac{p_\mu d\Sigma^\mu}{\exp\left(\frac{p^\mu u_\mu}{T}\right)-1}$$

Using the **1+1 dimensional** CKCJ solution:

$$d\Sigma^\mu=\frac{u^\mu\tau d\tau d\eta_z dr_x dr_y}{\cosh\left(\Omega\left(\eta_z\right)-\eta_z\right)}$$

Assuming **homogeneous transverse distribution** of temperature

Window function: sQGP is transparent for photons

$$H(\tau)=\Theta\left(\tau-\tau_{\mathrm{f}}\right)-\Theta\left(\tau-\tau_0\right)$$

arXiv:2311.03568

The new analytic formula for the thermal radiation

The source function is integrated over space and time

- Motivated by earlier results: ***λ* was assumed to be close to 1**
- The integral was perfromed by **saddle point approximation**
- The result is evaluated at **midrapidity (*y≈0*)**

$$\left.\frac{d^2N}{2\pi p_T dp_T dy}\right|_{y=0} = N_0 \frac{2\alpha}{3\pi^{3/2}} \left[\frac{1}{T_\mathrm{f}^\alpha} - \frac{1}{T_0^\alpha}\right]^{-1} p_T^{-\alpha-2} \left[\Gamma\left(\alpha + \frac{5}{2}, \frac{p_T}{T_0}\right) - \Gamma\left(\alpha + \frac{5}{2}, \frac{p_T}{T_\mathrm{f}}\right)\right]$$

λ and κ are collapsed into α (typical behaviour of hydro):

$$\alpha = \frac{2\kappa}{\lambda} - 3$$

T_f: freeze-out temperature

T_0: initial temperature

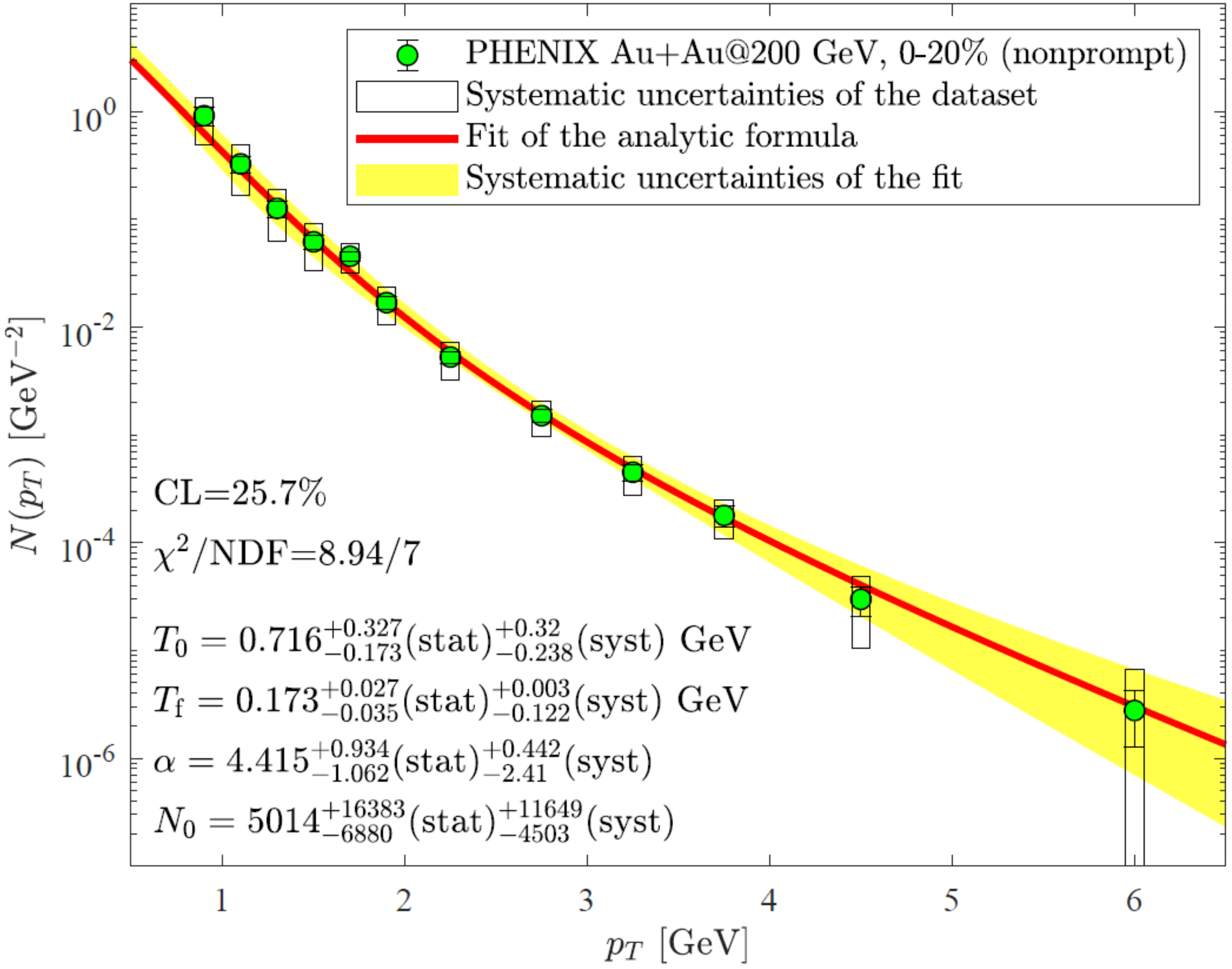
N_0: multiplicity at midrapidity

arXiv:2311.03568

Fit to experimental data

Good confidence level with realistic values of physical parameters

Intermediate p_T regime → ***T_0* can be determined more precisely**



Data is from: arXiv:2203.17187

In conclusion…

Thank you for your attention!

- Scaling behaviour of *dN/dy*

  → p+p collisions can be described as collective systems

  → Our fits indicate low c_s value (≈0.35) → indicate the presence of fluid

  → p+p and A+A collisions: *self-similar* systems

- New formula for thermal radiation

  → lacks of radial flow and viscous effects

  → but describes the data very well

  } *Message?*

  → compare the obtained initial temperature to the Hagedorn temperature (< 350 MeV):

$$T_H \ll T_0 = 0.7^{+0.3}_{-0.2}(\mathrm{stat})^{+0.3}_{-0.2}(\mathrm{syst})$$

Backup slides

Csörgő-Kasza-Csanád-Jiang (CKCJ) hydro solution

- Rindler coordinates, velocity field:

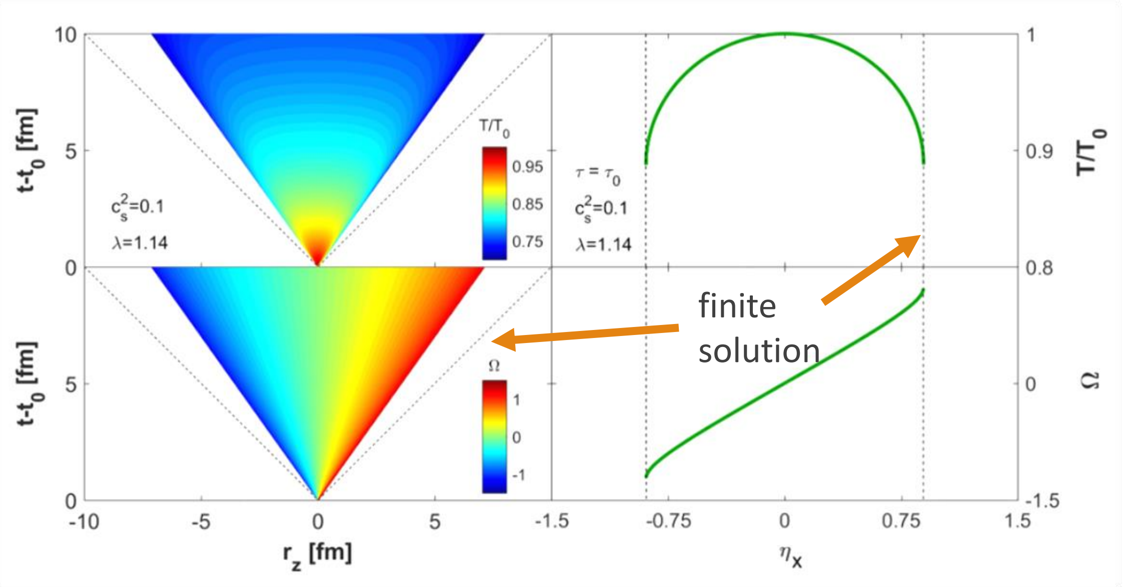
$$(\tau, \eta_x) = \left(\sqrt{t^2 - r_z^2}\,, \frac{1}{2}\ln\left[\frac{t + r_z}{t - r_z}\right] \right)$$

$$u^\mu = (\cosh(\Omega), \sinh(\Omega))$$

- 1+1 dimensional perfect fluid solution:

$$\begin{aligned}
\eta_x(H) &= \Omega(H) - H, \\
\Omega(H) &= \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}}\arctan\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh(H)\right) \\
\sigma(\tau, H) &= \sigma_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \mathcal{V}_\sigma(s) \left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2}}, \\
T(\tau, H) &= T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \mathcal{T}(s) \left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2\kappa}}, \\
\mathcal{T}(s) &= \frac{1}{\mathcal{V}_\sigma(s)}, \\
s(\tau, H) &= \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \sinh(H) \left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\lambda/2}
\end{aligned}$$

Universe 4 (2018) 6, 69



Equation of State:

$$\varepsilon = \kappa p$$

(with μ=0)

λ: acceleration parameter

accelerating solution

realistic $dN/d\eta_p$

Rapidity distribution

- We applied the Cooper-Frye formula
- Temperature is determined on the freeze-out hypersurface
- Integrals were calculated by saddle-point approximation
- Fluid rapidity could be well approximated by a linear function: $\Omega \approx \lambda \eta_z$
- The 1+1 dimensional rapidity distribution was embedded in 1+3 dimension

$$\frac{dN}{dy} \approx \left.\frac{dN}{dy}\right|_{y=0} \cosh^{-\frac{\alpha(\kappa_0)}{2}-1}\left(\frac{y}{\alpha}\right) \exp\left(-\frac{m}{T_{\mathrm{eff}}}\left[\cosh^{\alpha(\kappa_0)}\left(\frac{y}{\alpha}\right)-1\right]\right)$$

G. Kasza, T. Csörgő:
Int.J.Mod.Phys. A34 no.26, 1950147 (2019)

Pseudorapidity distribution

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- Parametric curve:

$$\left(\eta_{\mathrm{p}}(y), \frac{dN}{d\eta_{\mathrm{p}}}(y)\right) = \left(\frac{1}{2}\ln\left[\frac{\langle|p(y)|\rangle + \langle p_z(y)\rangle}{\langle|p(y)|\rangle - \langle p_z(y)\rangle}\right], \frac{\langle|p(y)|\rangle}{\langle E(y)\rangle}\frac{dN}{dy}\right)$$

$$\eta_{\mathrm{p}}(y) = \tanh^{-1}\left(\mathcal{J}^{-1}\tanh(y)\right) = \tanh^{-1}\left(\frac{\tanh(y)}{\sqrt{1 - \frac{m^2}{\langle m_{\mathrm{T}}(y)\rangle^2 \cosh^2(y)}}}\right)$$

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$$\frac{dN}{d\eta_{\rm p}}(y) \approx \left.\frac{dN}{dy}\right|_{y=0} \sqrt{1 - \frac{m^2}{\langle m_{\rm T}(y)\rangle^2 \cosh^2(y)}} \cosh^{-\frac{\alpha(\kappa_0)}{2}-1}\left(\frac{y}{\alpha}\right) \exp\left(-\frac{m}{T_{\rm eff}}\left[\cosh^{\alpha(\kappa_0)}\left(\frac{y}{\alpha}\right) - 1\right]\right)$$

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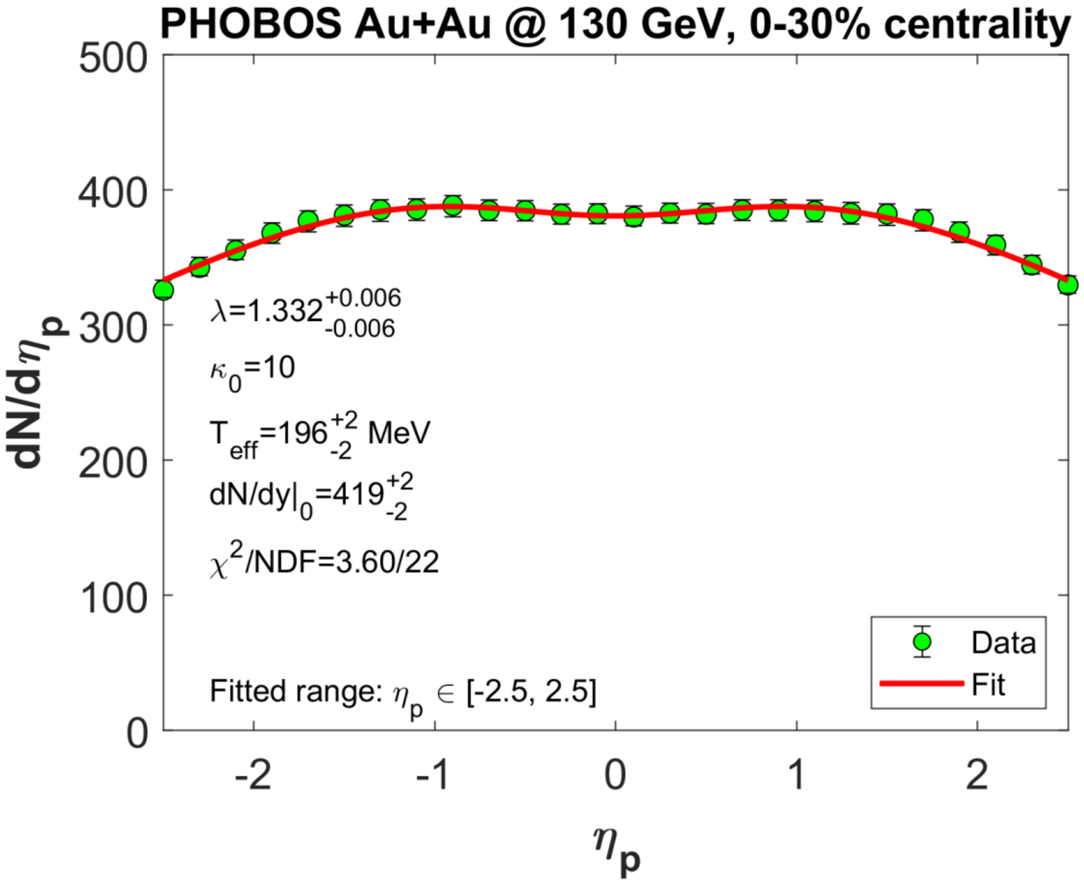
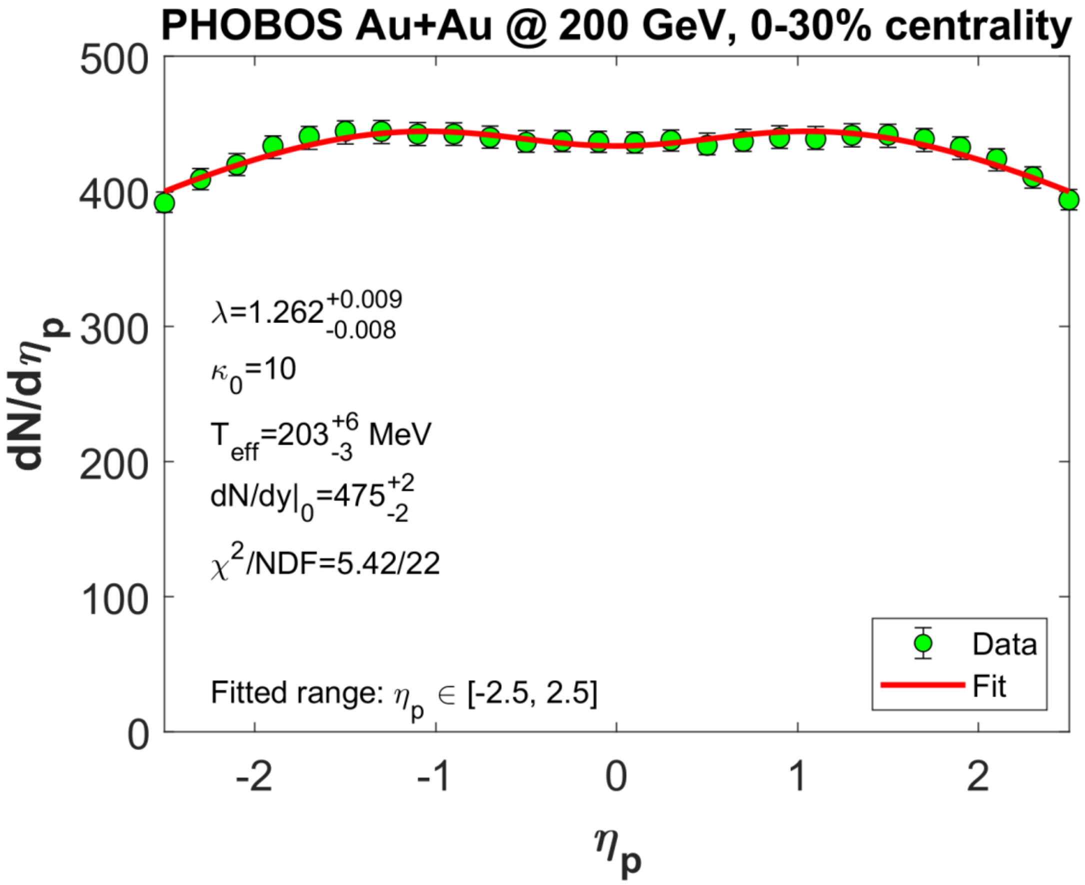
Fits to PHOBOS data (Au+Au)





G. Kasza, T. Csörgő:
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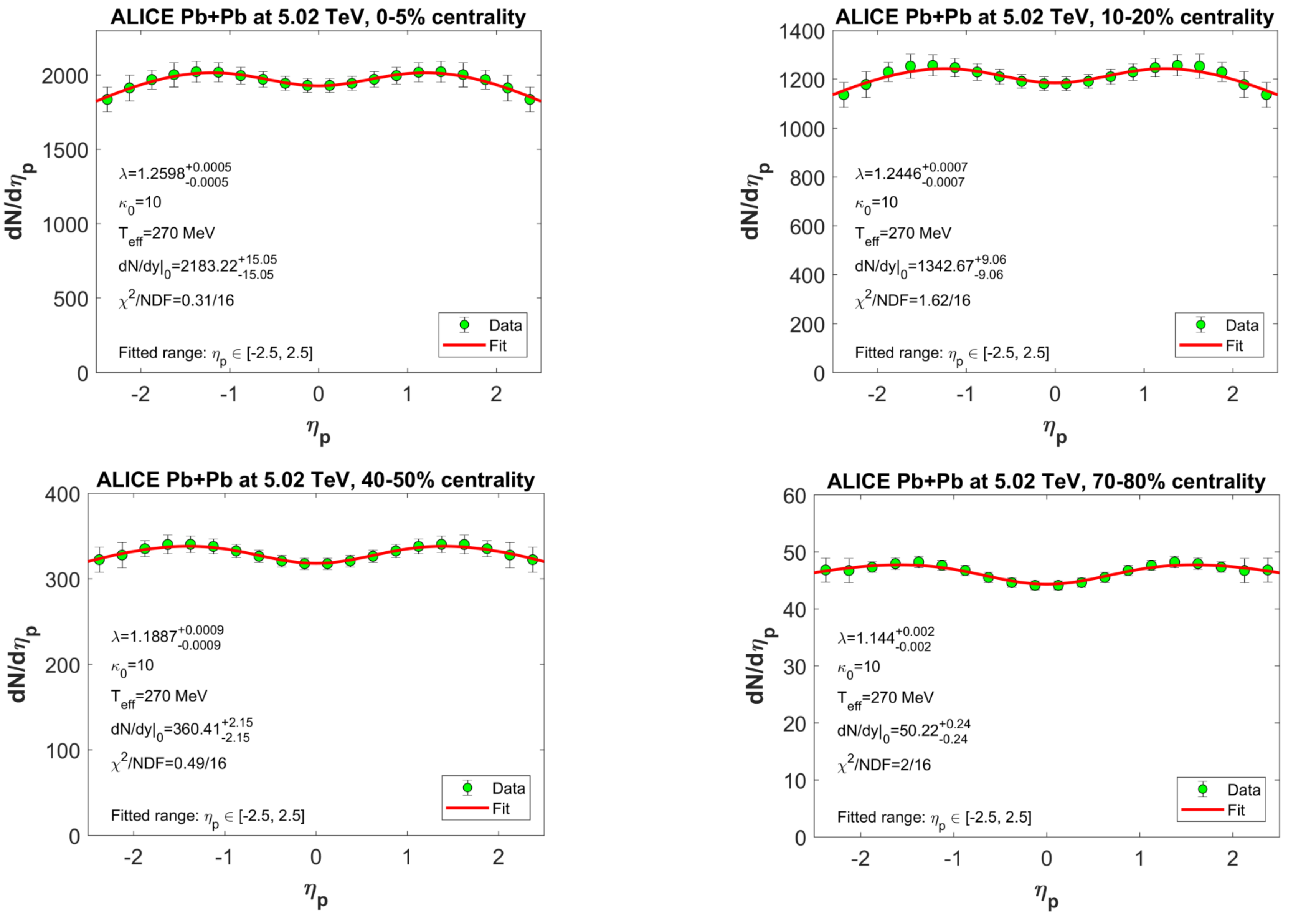
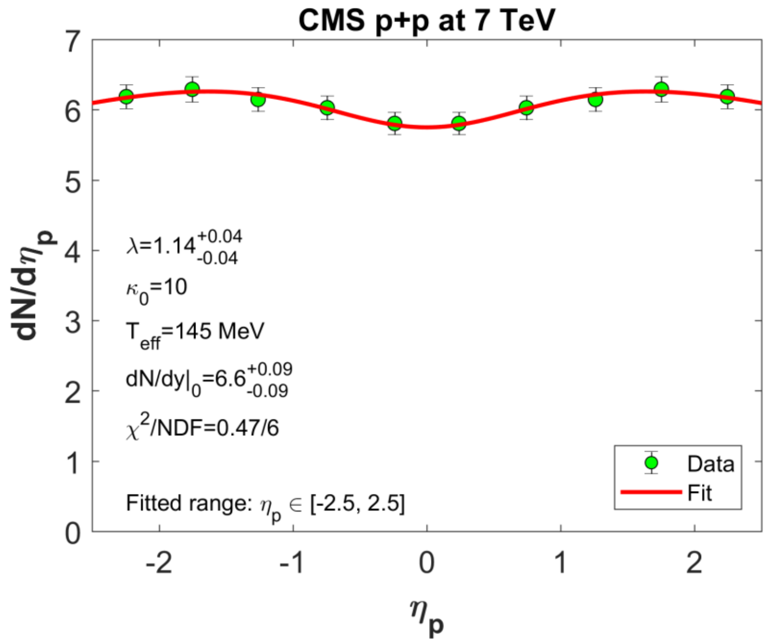
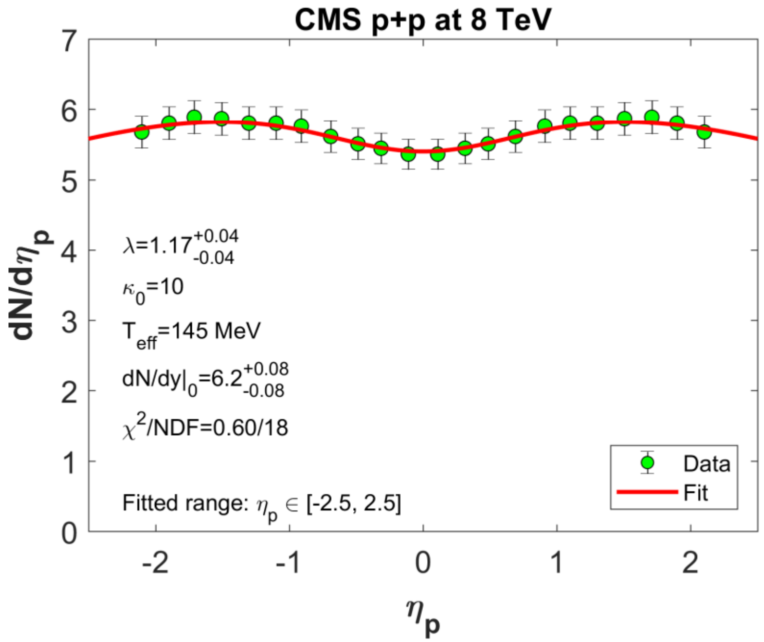
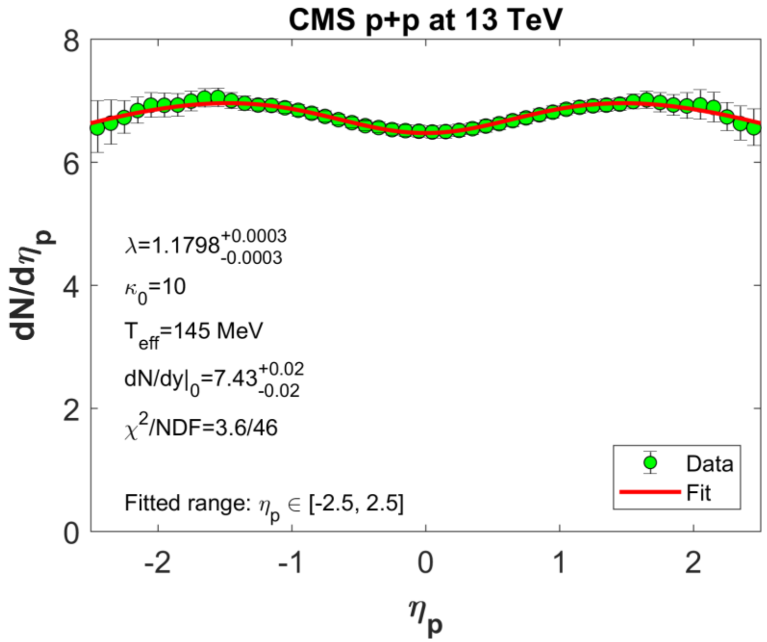
Fits to ALICE data (centrality dependence)



Fits to CMS data (p+p)







Collectivity in p+p collisions?

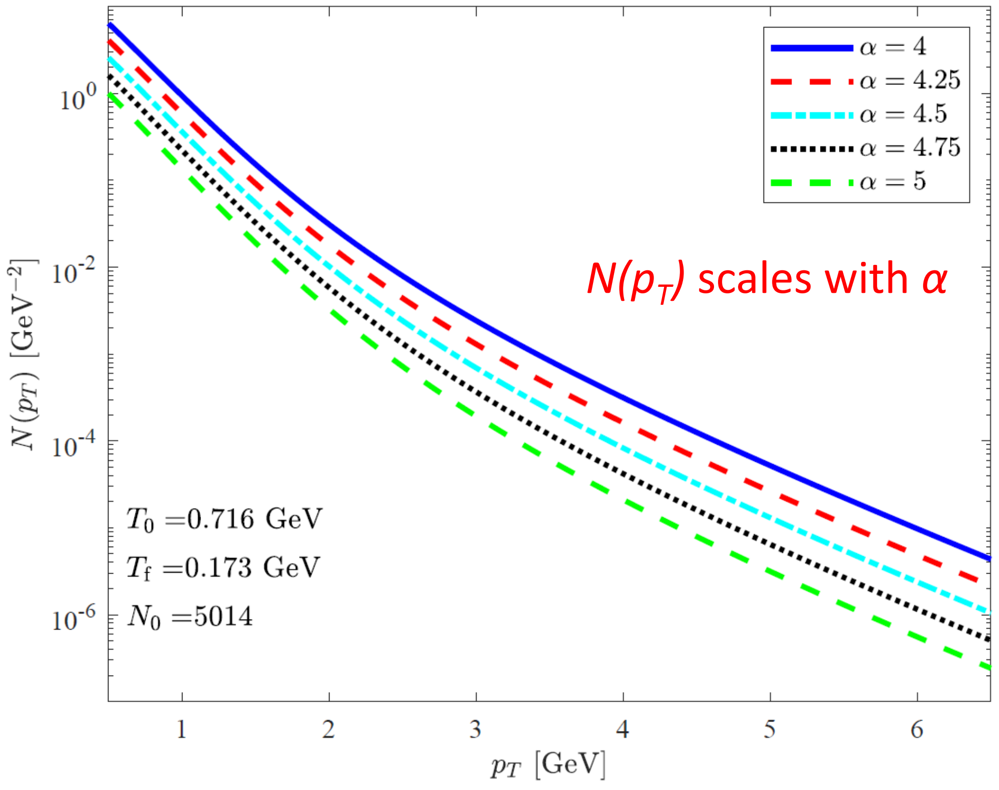
The new analytic formula for the thermal radiation

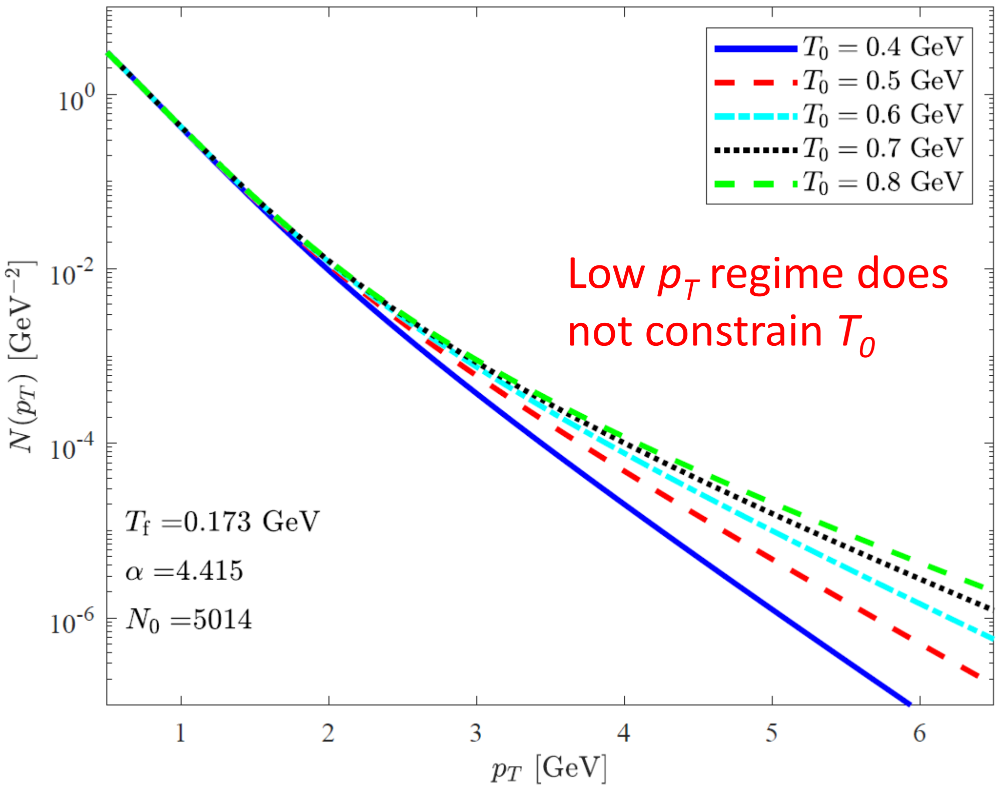
$$\left.\frac{d^2N}{2\pi p_T dp_T dy}\right|_{y=0} = N_0 \, \frac{2\alpha}{3\pi^{3/2}} \left[\frac{1}{T_\mathrm{f}^\alpha} - \frac{1}{T_0^\alpha}\right]^{-1} p_T^{-\alpha-2} \left[\Gamma\left(\alpha+\frac{5}{2}, \frac{p_T}{T_0}\right) - \Gamma\left(\alpha+\frac{5}{2}, \frac{p_T}{T_\mathrm{f}}\right)\right]$$

$$N_0 = \left.\frac{dN}{dy}\right|_{y=0} = \frac{gA_T}{(2\pi\hbar)^3} \frac{\tau_0}{\tau_R} \frac{T_0^{\alpha+3}}{\alpha} \left[\frac{1}{T_\mathrm{f}^\alpha} - \frac{1}{T_0^\alpha}\right] \frac{3\pi^{3/2}\kappa}{2\lambda} \left(\frac{2\pi\kappa}{\lambda^2(2\kappa-1) - \lambda(\kappa-1)}\right)^{1/2}$$

The new analytic formula for the thermal radiation

$$\left.\frac{d^2N}{2\pi p_T dp_T dy}\right|_{y=0} = N_0 \frac{2\alpha}{3\pi^{3/2}} \left[\frac{1}{T_f^\alpha} - \frac{1}{T_0^\alpha}\right]^{-1} p_T^{-\alpha-2} \left[\Gamma\left(\alpha+\frac{5}{2}, \frac{p_T}{T_0}\right) - \Gamma\left(\alpha+\frac{5}{2}, \frac{p_T}{T_f}\right)\right]$$



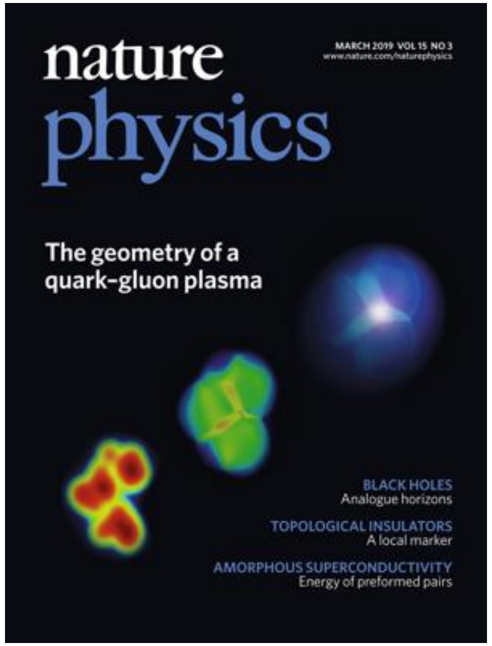


In conclusion…

- p+p collisions can be described as collective systems
- Our fits indicate low c_s value (≈0.35)
- Low c_s value indicate the presence of fluid, so the presence of QGP
- p+p and A+A collisions: *self-similar* systems

Is the hydrodynamic description well-accepted?

- A+A collisions: become a major trend since 2005
- p+A, d+A and He+A collisions: accepted since 2019
- p+p collisions: not widely accepted yet



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- A+A collisions: become a major trend since 2005
- p+A, d+A and He+A collisions: accepted since 2019
- p+p collisions: not widely accepted yet
- However, describing H+H systems by hydro is *not* a recent idea

arXiv:hep-ex/9711009v2 19 Dec 1997

Nijmegen preprint
HEN-405
Dec. 97

ESTIMATION OF HYDRODYNAMICAL MODEL PARAMETERS FROM THE INVARIANT SPECTRUM AND THE BOSE-EINSTEIN CORRELATIONS OF π^- MESONS PRODUCED IN (π^+/K^+)p INTERACTIONS AT 250 GeV/c

EHS/NA22 Collaboration

N.M. Agababyan[g], M.R. Atayan[g], T. Csörgő[h], E.A. De Wolf[a,1], K. Dziunikowska[b,2], A.M.F. Endler[e], Z.Sh. Garutchava[f], H.R. Gulkanyan[g], R.Sh. Hakobyan[g], J.K. Karamyan[g], D. Kisielewska[b,2], W. Kittel[d], S.S. Mehrabyan[g], Z.V. Metreveli[f], K. Olkiewicz[b,2], F.K. Rizatdinova[c], E.K. Shabalina[c], L.N. Smirnova[c], M.D. Tabidze[f], L.A. Tikhonova[c], A.V. Tkabladze[f], A.G. Tomaradze[f], F. Verbeure[a], S.A. Zotkin[c]

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[b] Institute of Physics and Nuclear Techniques of Academy of Mining and Metallurgy and Institute of Nuclear Physics, PL-30055 Krakow, Poland
[c] Nuclear Physics Institute, Moscow State University, RU-119899 Moscow, Russia
[d] High Energy Physics Institute Nijmegen (HEFIN), University of Nijmegen/NIKHEF, NL-6525 ED Nijmegen, The Netherlands
[e] Centro Brasileiro de Pesquisas Fisicas, BR-22290 Rio de Janeiro, Brazil
[f] Institute for High Energy Physics of Tbilisi State University, GE-380086 Tbilisi, Georgia
[g] Institute of Physics, AM-375036 Yerevan, Armenia
[h] KFKI, Hungarian Academy of Sciences, H-1525 Budapest 114, Hungary

Abstract: The invariant spectra of π^- mesons produced in (π^+/K^+)p interactions at 250 GeV/c are analysed in the framework of the hydrodynamical model of three-dimensionally expanding cylindrically symmetric finite systems. A satisfactory description of experimental data is achieved. The data favour the pattern according to which the hadron matter undergoes predominantly longitudinal expansion and non-relativistic transverse expansion with mean transverse velocity $\langle u_t \rangle = 0.20 \pm 0.07$, and is characterized by a large temperature inhomogeneity in the transverse direction: the extracted freeze-out temperature at the center of the tube and at the transverse rms radius are 140 ± 3 MeV and 82 ± 7 MeV, respectively. The width of the (longitudinal) space-time rapidity distribution of the pion source is found to be $\Delta\eta = 1.36 \pm 0.02$. Combining this estimate with results of the Bose-Einstein correlation analysis in the same experiment, one extracts a mean freeze-out time of the source of $\langle \tau_f \rangle = 1.4 \pm 0.1$ fm/c and its transverse geometrical rms radius, $R_G(\text{rms}) = 1.2 \pm 0.2$ fm.