Incoherent processes in dileptons production in proton-nucleus scattering at high energies

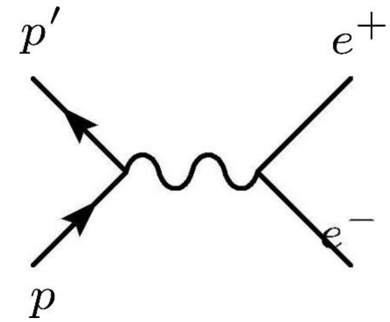
S. P. Maydanyuk, G. Wolf

[1] Wigner Research Centre for Physics, Budapest

Idea of research

Process for study:

Production of electron-positron pairs (dileptons) in nucleon and nuclear scattering.



Question:

Production of dileptons in scattering of nucleon (in external field) via exchange of virtual photon is the simplest example.

Individual nucleons in nucleus-nucleus scattering (collision) play some role in dilepton production. Is many nucleon dynamics important in production of dileptons?

Result from research:

1) Role of many nucleon dynamics is huge.

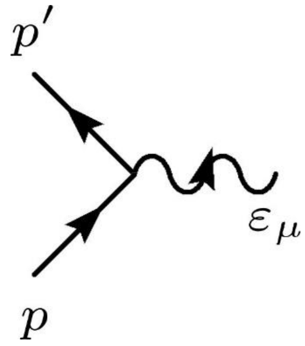
2) Magnetic moments of nucleons in nuclei are essentially more important than electric charges of protons in nuclei.

3) Important role of nuclear forces.

Method

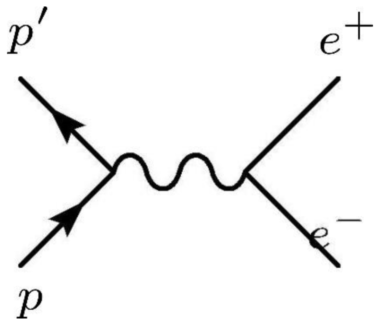
Let us write down two processes:

(1) Emission of photon by nucleon,



$$-i\,M = \bar{u}(p')\,(ie\gamma^{\mu})\,u(p)\,e_{\mu},$$

(2) Production of pair of leptons via exchange of virtual photon emitted by nucleon.



$$-i\,M = \bar{u}(p')\,(ie\gamma^{\mu})\,u(p)\cdot\left(-\frac{ig_{\mu\nu}}{q^{2}}\right)\cdot\bar{u}(k')\,(ie\gamma^{\nu})\,u(k).$$

Question: How to include formalism of proton-nucleus scattering?

Electrically charged particle in field of nucleus

Motion of particle with mass m in field of nucleus with potential U(**r**) can be described by Hamiltonian as

$$\hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m} + U(\mathbf{r}), \quad \hat{\mathbf{p}} = -i\hbar \frac{\mathbf{d}}{\mathbf{dr}}.$$

If particle is electrically charged, then it is under influence of action of electromagnetic field with vector potential **A**(***r***, *t*), formed by this nucleus. Hamiltonian of particle inside field of nucleus can be found from ***Pauli equation***

$$\hat{H} = \hat{H}_0 + \hat{W}, \quad \hat{W} = -Z_{\text{eff}} \frac{e}{2mc}\left(\mathbf{pA} + \mathbf{Ap}\right) + eA_0 - Z_{\text{eff}} \frac{e\hbar}{2mc} \mu \cdot \mathbf{rot}\,\mathbf{A} + Z_{\text{eff}}^2 \frac{e^2}{2mc^2} \mathbf{A}^2.$$

One can neglect by magnetic field (μ = 0) and omit component A_0 (to be enough small). Taking into account ***Coulomb gauge*** (**div A** = 0), and terms proportional to c^{-2}, we obtain

$$\hat{W} = -Z_{\text{eff}} \frac{e}{mc} \mathbf{A}\hat{\mathbf{p}}.$$ Operator of emission of photon

Generalized many nucleons Pauli equation

Way how we obtain it:

1) Starting point is Dirac eq.: $$i\frac{\partial\psi}{\partial t}=\left\{c\,\boldsymbol{\alpha}\,\hat{\mathbf{p}}+\beta\,mc^2\right\}\psi,\quad \hat{\mathbf{p}}=-i\hbar\frac{\mathbf{d}}{\mathbf{dr}}.$$

2) We introduce electromagnetic field via Coulomb gauge:

$$\mathbf{p}\to\mathbf{p}-\frac{z_i e}{c}\mathbf{A},\quad p_0\to p_0-\frac{z_i e}{c}A_0,$$

3) We add interactions via potential:

$$\hat{H}_0=c\,\boldsymbol{\alpha}\,\hat{\mathbf{p}}+\beta\,mc^2,\quad H_0\to H_0+V(\mathbf{r}),$$

4) Pauli eq. is 1-st approximation of 1/c:

$$i\frac{\partial\psi}{\partial t}=\left\{\frac{1}{2m_i}\left(\mathbf{p}_i-\frac{z_i e}{c}\mathbf{A}_i\right)^2-\frac{z_i e\hbar}{2m_i c}\boldsymbol{\sigma}\cdot\mathbf{rot}\,\mathbf{A}_i+z_i e\,A_{i,0}\right\}\psi.$$

z and m are electromagnetic charge and mass of fermion,

V ($\mathbf{r}$) is potential of interactions,

$\mathbf{A}_{i}$ = ($\mathbf{A}_i$, $A_{i,0}$) is potential of electromagnetic field formed by moving fermion,

$\boldsymbol{\sigma}$ are Pauli matrixes.

5) Generalized Pauli eq. on *A* nucleons of nucleus and proton-projectile:

$$\hat{H}=\sum_{i=1}^{A+1}\left\{\frac{1}{2m_i}\left(\mathbf{p}_i-\frac{z_i e}{c}\mathbf{A}_i\right)^2-\frac{z_i e\hbar}{2m_i c}\boldsymbol{\sigma}\cdot\mathbf{rot}\,\mathbf{A}_i+z_i e\,A_{i,0}\right\}+V(\mathbf{r}_1\ldots\mathbf{r}_{A+1}).$$

Anomalous magnetic moments of nucleons

We introduce magnetic momentum of particle with number i (from Dirac's theory for proton [1]) as

$$\mu_i^{(Dirac)} = \frac{z_i \cdot e\hbar}{2m_i c^2}.$$

In order to go to anomalous magnetic momenta of particle, we use change:

$$\mu_i^{(\mathrm{Dirac})} \to \mu_i^{(\mathrm{an})}.$$

$$\mu_{\mathrm{p}}^{(\mathrm{an})} = 2.792847\mu_N,$$
$$\mu_{\mathrm{n}}^{(\mathrm{an})} = -1.913042\mu_N,$$
$$\mu_{\Lambda}^{(\mathrm{an})} = -0.613\mu_N.$$

$\mu_N = e\hbar/(2m_{\mathrm{p}}c)$ is nuclear magneton.

In the first approximation, operator of emission by proton-nucleus system in the laboratory frame is

$$\mathbf{H} = \mathbf{rot\ A}$$

$$\hat{H}_\gamma = -\frac{e\,z_p}{m_p c}\mathbf{A}_p\,\hat{\mathbf{p}}_p - \mu_p^{(\mathrm{an})}\boldsymbol{\sigma}\cdot\hat{\mathbf{H}}_p + \sum_{j=1}^{A}\left\{-\frac{e\,z_j}{m_j c}\mathbf{A}_j\,\hat{\mathbf{p}}_j - \mu_j^{(\mathrm{an})}\boldsymbol{\sigma}\cdot\hat{\mathbf{H}}_j\right\}.$$

[1] A.I.Ahiezer, V.B.Berestetskii, *Quantum electrodynamics* (Nauka, Moskva, 1981).

Formalism in space representation

Principle of uncertainty of quantum mechanics gives relations between space coordinates and corresponding momenta.

By such reason, we construct formalism in space representation.

Potential of electromagnetic field:

μ_N is nuclear magneton.

$$\mathbf{A}(\mathbf{r}_s, t) = \sqrt{\frac{2\pi\hbar c^2}{w_{ph}}} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha),*} e^{-i\mathbf{k}_{ph}\mathbf{r}_s}$$

$$\hat{\mathbf{H}} = \left[\nabla \times \hat{\mathbf{A}}\right] = \sqrt{\frac{2\pi\hbar c^2}{w_{\rm ph}}} \sum_{\alpha=1,2} \left\{-i\, e^{-i\mathbf{k}_{\rm ph}\mathbf{r}} [\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha),*}] + e^{-i\mathbf{k}_{\rm ph}\mathbf{r}} [\nabla \times \mathbf{e}^{(\alpha),*}]\right\}.$$

Operator of emission of photons:

$$\hat{H}_\gamma = \sqrt{\frac{2\pi\hbar c^2}{w_{\rm ph}}} \sum_{\alpha=1,2} e^{-i\mathbf{k}_{\rm ph}\mathbf{r}_i} \left\{ i\mu_N \frac{2z_i m_{\rm p}}{m_p} \mathbf{e}^{(\alpha)} \nabla_p + \mu_p^{(an)} \boldsymbol{\sigma}\left(i[\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha)}] - [\nabla_p \times \mathbf{e}^{(\alpha)}]\right)\right\} +$$

$$+ \sqrt{\frac{2\pi\hbar c^2}{w_{\rm ph}}} \sum_{j=1}^{A} \sum_{\alpha=1,2} e^{-i\mathbf{k}_{\rm ph}\mathbf{r}_j} \left\{ i\mu_N \frac{2z_j m_{\rm p}}{m_{Aj}} \mathbf{e}^{(\alpha)} \nabla_j + \mu_j^{(an)} \boldsymbol{\sigma}\left(i[\mathbf{k}_{\rm ph} \times \mathbf{e}^{(\alpha)}] - [\nabla_j \times \mathbf{e}^{(\alpha)}]\right)\right\}.$$

Transition to relative coordinates

We rewrite formalism in relative coordinates.

We define coordinate of centers of masses for the nucleus as $\mathbf{R}_A$, proton as $\mathbf{r}_p$, complete system as $\mathbf{R}$:

$$\mathbf{R}_A = \frac{1}{m_A} \sum_{j=1}^{A} m_j \mathbf{r}_{Aj}, \quad \mathbf{R} = \frac{m_A \mathbf{R}_A + m_p \mathbf{r}_p}{m_A + m_p}.$$

We introduce new independent variables: $\mathbf{r}_{Aj} = \mathbf{R}_A + \boldsymbol{\rho}_{Aj}, \quad \mathbf{r} = \mathbf{r}_p - \mathbf{R}_A.$

$$\mathbf{r}_p = \mathbf{R} + c_A \mathbf{r}, \quad \mathbf{r}_{Aj} = \boldsymbol{\rho}_{Aj} + \mathbf{R} - c_p \mathbf{r}.$$

$$c_p = \frac{m_p}{m_A + m_p}, \quad c_A = \frac{m_A}{m_A + m_p}.$$

Old momenta: $\mathbf{p}_p = -i\hbar \frac{\mathbf{d}}{\mathbf{dr}_p}, \quad \mathbf{p}_{Aj} = -i\hbar \frac{\mathbf{d}}{\mathbf{dr}_{Aj}}.$

New momenta:

$$\tilde{\mathbf{p}}_{Aj} = -i\hbar \frac{\mathbf{d}}{\mathbf{d}\boldsymbol{\rho}_{Aj}}, \quad \mathbf{P} = -i\hbar \frac{\mathbf{d}}{\mathbf{dR}}, \quad \mathbf{p} = -i\hbar \frac{\mathbf{d}}{\mathbf{dr}}.$$

Also: $\mathbf{p}_p = -i\hbar \frac{\mathbf{d}}{\mathbf{dr}_p}, \quad \mathbf{P}_A = -i\hbar \frac{\mathbf{d}}{\mathbf{dR}_A}.$

Emission operator: coherent terms

Operator of emission of photon is $\hat{H}_{\gamma} = \hat{h}_{P1} + \hat{h}_{p} + \Delta\hat{h}_{E} + \Delta\hat{h}_{M} + \hat{h}_{k}.$

$$\hat{h}_p = -\sqrt{\frac{2\pi c^2}{\hbar w_{\rm ph}}}\, 2\mu_N m_{\rm p}\, e^{-i\mathbf{k}_{\rm ph}\mathbf{R}} \sum_{\alpha=1,2} \left\{ e^{-ic_A \mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{z_p}{m_{\rm p}} - e^{ic_p \mathbf{k}_{\rm ph}\mathbf{r}}\, \frac{1}{m_{\rm A}} \sum_{j=1}^{A} z_j e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}} \right\} \mathbf{e}^{(\alpha)} \cdot \hat{\mathbf{p}} - $$
$$- i\sqrt{\frac{2\pi c^2}{\hbar w_{\rm ph}}}\, e^{-i\mathbf{k}_{\rm ph}\mathbf{R}} \sum_{\alpha=1,2} \left\{ e^{-ic_A \mathbf{k}_{\rm ph}\mathbf{r}}\, \mu_p^{(an)} \boldsymbol{\sigma} - \frac{e^{ic_p \mathbf{k}_{\rm ph}\mathbf{r}}}{m_A} \sum_{j=1}^{A} \mu_j^{(an)} m_{Aj} e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}}\, \boldsymbol{\sigma} \right\} \cdot [\hat{\mathbf{p}} \times \mathbf{e}^{(\alpha)}].$$

z_s and m_s are electric charge and mass of nucleon with number s, m_A and m_α are mass of nucleus and α-particle, $\mathbf{e}^{(\alpha)}$ - unit vectors of polarization of photon (α=1, 2), $\mathbf{k}_{\rm ph}$ - wave vector of the photon, $w_{\rm ph} = |\mathbf{k}_{\rm ph}|$.

[1] S. P. Maydanyuk, Phys. Rev. **C107**, 024618 (2023) + references.

Emission operator: incoherent terms

Operator of emission of photon

$$\hat{H}_{\gamma} = \hat{h}_{P1} + \hat{h}_{p} + \Delta\hat{h}_{E} + \Delta\hat{h}_{M} + \hat{h}_{k},$$

$$\Delta\hat{h}_{E} = -\sqrt{\frac{2\pi c^{2}}{\hbar w_{\mathrm{ph}}}}\, 2\mu_{N} e^{-i\mathbf{k}_{\mathrm{ph}}\mathbf{R}} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)} \Biggl\{ e^{i c_{p} \mathbf{k}_{\mathrm{ph}} \mathbf{r}} \sum_{j=1}^{A-1} \frac{z_{j} m_{\mathrm{p}}}{m_{Aj}} e^{-i\mathbf{k}_{\mathrm{ph}} \boldsymbol{\rho}_{Aj}} \tilde{\mathbf{p}}_{Aj} - \frac{m_{\mathrm{p}}}{m_{A}} e^{i c_{p} \mathbf{k}_{\mathrm{ph}} \mathbf{r}} \sum_{j=1}^{A} z_{j} e^{-i\mathbf{k}_{\mathrm{ph}} \boldsymbol{\rho}_{Aj}} \sum_{k=1}^{A-1} \tilde{\mathbf{p}}_{Ak} \Biggr\},$$

$$\Delta\hat{h}_{M} = -i\sqrt{\frac{2\pi c^{2}}{\hbar w_{\mathrm{ph}}}}\, \mu_{N} e^{-i\mathbf{k}_{\mathrm{ph}}\mathbf{R}} \sum_{\alpha=1,2} \Biggl\{ e^{i c_{p} \mathbf{k}_{\mathrm{ph}} \mathbf{r}} \sum_{j=1}^{A-1} \mu_{j}^{(\mathrm{an})} e^{-i\mathbf{k}_{\mathrm{ph}} \boldsymbol{\rho}_{Aj}} \boldsymbol{\sigma} \cdot \bigl[\tilde{\mathbf{p}}_{Aj} \times \mathbf{e}^{(\alpha)}\bigr] - e^{i c_{p} \mathbf{k}_{\mathrm{ph}} \mathbf{r}} \sum_{j=1}^{A} \mu_{j}^{(\mathrm{an})} \frac{m_{\mathrm{Aj}}}{m_{A}} e^{-i\mathbf{k}_{\mathrm{ph}} \boldsymbol{\rho}_{Aj}} \sum_{k=1}^{A-1} \boldsymbol{\sigma} \cdot \bigl[\tilde{\mathbf{p}}_{Ak} \times \mathbf{e}^{(\alpha)}\bigr] \Biggr\}$$

[1] S. P. Maydanyuk, Phys. Rev. **C107**, 024618 (2023) + references.

Wave function of p-nucleus system

$$\Psi_s = \Phi_s(\mathbf{R}) \cdot \Phi_{\text{p-nucl},\, s}(\mathbf{r}) \cdot \psi_{\text{nucl},\, s}(\beta_A),$$

$$\psi_{\text{nucl},s}(1 \ldots A) = \frac{1}{\sqrt{A!}} \sum_p (-1)^{\varepsilon_p} \psi_{\lambda_1}(1) \cdot \psi_{\lambda_2}(2) \cdot \ldots \cdot \psi_{\lambda_A}(A).$$

Summation is performed over all $A!$ interchanges of coordinates or states of nucleons.

One-nucleon functions is multiplication of space and spin-isospin functions:

$$\psi_{\lambda_s}(s) = \varphi_{n_s}(\mathbf{r}_s) \left| \sigma^{(s)} \tau^{(s)} \right\rangle,$$

φ_n is space function of nucleon with number s, n_s is number of state of the space function of nucleon with number s,

$\left| \sigma^{(s)} \tau^{(s)} \right\rangle$ is spin-isospin function of nucleon with number s.

Matrix element of emission is written via combination of one-nucleon functions as

$$\left\langle \psi_{\text{nucl},\, f}(1 \ldots A) \right| \hat{H}_\gamma \left| \psi_{\text{nucl},i}(1 \ldots A) \right\rangle =$$

$$= \frac{1}{A(A-1)} \sum_{k=1}^{A} \sum_{\substack{m=1, \\ m \neq k}}^{A} \left\{ \left\langle \psi_k(i) \psi_m(j) \right| \hat{H}_\gamma \left| \psi_k(i) \psi_m(j) \right\rangle - \left\langle \psi_k(i) \psi_m(j) \right| \hat{H}_\gamma \left| \psi_m(i) \psi_k(j) \right\rangle \right\}.$$

Probability of emission of photons

$$\hat{H}_\gamma = \hat{h}_{P1} + \hat{h}_p + \Delta\hat{h}_E + \Delta\hat{h}_M + \hat{h}_k.$$

Matrix element of emission:

We define the matrix element of emission, using wave functions of full nuclear system in states before and after emission of photon as

$$\langle \Psi_f | \hat{H}_{\rm ph} | \Psi_i \rangle = \sqrt{\frac{2\pi c^2}{\hbar w_{\rm ph}}} \cdot M_{\rm full}, \quad M_{\rm full} = M_P + M_p + M_{\Delta E} + M_{\Delta M} + M_k.$$

Bremsstrahlung cross-section:

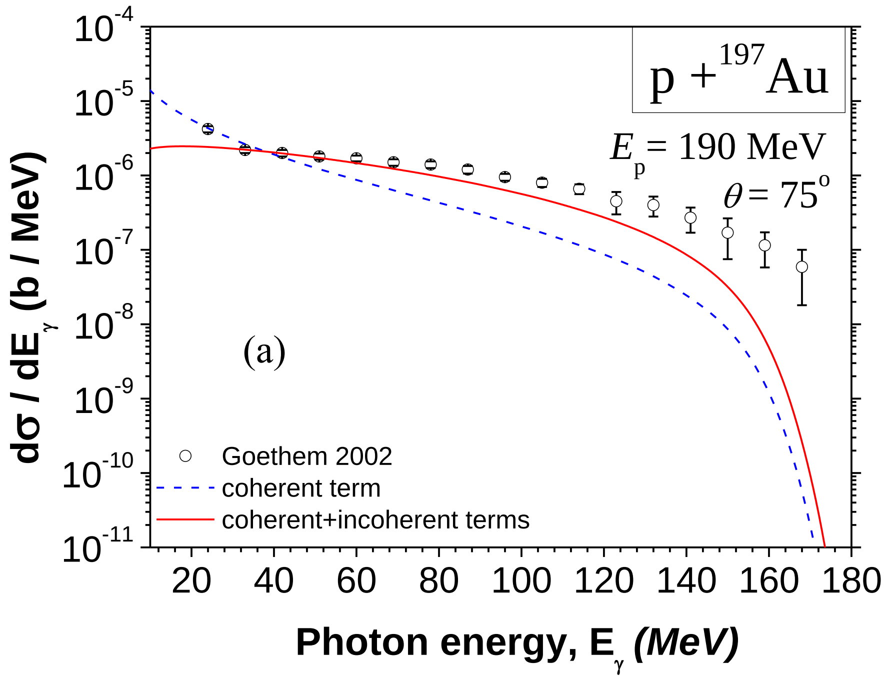
We define the probability of the emitted bremsstrahlung photons on the basis of the full matrix element in frameworks of our formalism [1]:

$$\frac{dP}{dw_{\rm ph}} = N_0 \frac{e^2}{2\pi c^5} \frac{w_{\rm ph} E_i}{m^2 k_i} | p_{\rm full} |^2, \quad M_{\rm full} = -\frac{e}{m_{\rm p}} p_{\rm full}.$$

N_0 is normalization factor (obtained from comparison of calculations and exp. data for scattering).

1. S.P.Maydanyuk, P.-M.Zhang, L.-P.Zou, Phys. Rev. **C98**, 054613 (2018).

Theory & Experiment



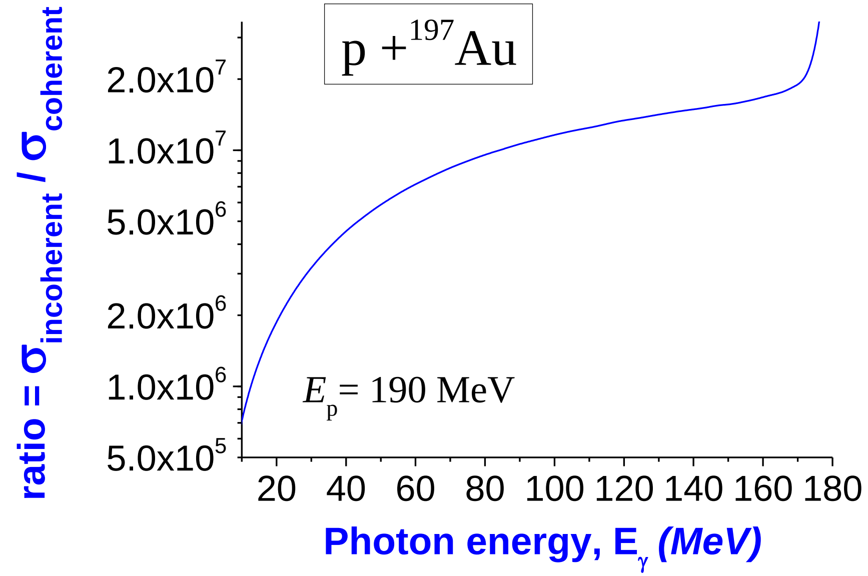


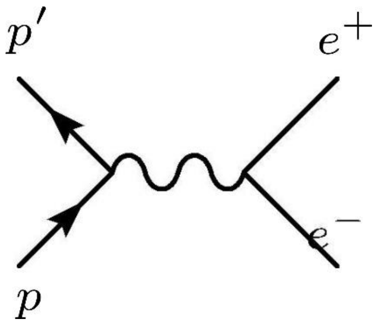
Fig.1. The calculated cross sections (with coherent and incoherent terms) in the scattering of p + ^{197}Au nuclei at energy of proton beam of 190 MeV in comparison with experimental data [1].

Fig.2. Ratio between incoherent and coherent bremsstrahlung contributions (to the full spectrum) for result presented in Fig. 1.

[1] M. J. van Goethem, et al., Phys. Rev. Lett. **88**, 122302 (2002).

[2] S. P. Maydanyuk, Phys. Rev. **C107**, 024618 (2023) + references.

Matrix element of dilepton production



We describe production of dileptons from proton scattering in external field. Simplest matrix element can be written via S-matrix formalism (2-order) as

$$j_\mu(x) = ie\bar{\psi}_f(x)\gamma_\mu\psi_i(x),$$

$$\psi_i(x) = \frac{1}{\sqrt{2\varepsilon}}u\,e^{ipx},$$

$$A_\mu^a(x) = \frac{1}{\sqrt{2w_{ph}}}e_\mu^a e^{ikx}.$$

$$\langle f|S^{(2)}|i\rangle = \frac{1}{2}\int [j_\nu(x_1)A_\nu^a(x_1)]_p[j_\mu(x_2)A_\mu^a(x_2)]_e\, d^4x_1\, d^4x_2.$$

That can be rewritten as

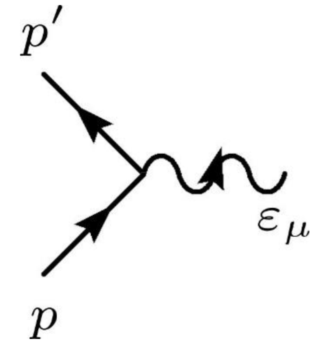
$$\langle f|S^{(2)}|i\rangle = \frac{1}{2}\int [j_\nu(x_1)A_\nu^a(x_1)]_p d^4x_1 \cdot \int [j_\mu(x_2)A_\mu^a(x_2)]_e d^4x_2 = M_p^a \cdot M_e^a,$$

$$M_p^a = \frac{1}{\sqrt{2}}\int [j_\nu(x_1)A_\nu^a(x_1)]_p d^4x_1$$ - hadronic matrix element,

$$M_e^a = \frac{1}{\sqrt{2}}\int [j_\mu(x_2)A_\mu^a(x_2)]_e d^4x_2$$ - leptonic matrix element.

Hadronic matrix element

Different forms of hadronic matrix element are below:



1) Relativistic formulation for one nucleon:

$$M_p^a = \frac{1}{\sqrt{2}} \int [j_\mu(x_1) A_\mu^a(x_1)]_p \, d^4 x_1 = \frac{ie}{\sqrt{2}} \int [\bar{\psi}_f(x_1) \gamma_\mu \psi_i(x_1)] A_\mu^a(x_1)|_p \, d^4 x_1$$

2) Non-Relativistic formulation for one nucleon (without magnetic moment):

$$M_p^a = \frac{1}{\sqrt{2}} \int [j_\mu(x_1) A_\mu^a(x_1)]_p \, d^4 x_1 = \frac{1}{\sqrt{2}} \frac{ie\hbar}{2m_p} \int [\psi_f(x_1) \nabla \psi_i^*(x_1) \cdot A_\mu^a(x_1) - c.c.]_p \, d^3 x_1$$

3) Non-Relativistic formulation for one nucleon in proton-nucleus scattering (with magnetic moments of nucleons, nuclear structure):

$$\langle \Psi_f(\mathbf{r}) | \hat{H}_\gamma | \Psi_i(\mathbf{r}) \rangle^{(a)} = \sqrt{\frac{2\pi c^2}{\hbar w_{ph}}} M_{full}^{(a)}, \quad M_{full}^{(a)} \approx M_p^{(E),a} = i\hbar (2\pi)^3 \frac{2\mu_N m_p}{\mu} Z_{eff}^{(mon,0)} \mathbf{e}^{(a)} \cdot \mathbf{I}_1,$$

$$\mathbf{I}_1 = \langle \Phi_{p-nucl,f}(\mathbf{r}) | e^{-i\mathbf{k}_{ph}\mathbf{r}} \frac{\mathbf{d}}{\mathbf{dr}} | \Phi_{p-nucl,i}(\mathbf{r}) \rangle$$

Nuclear (Hadronic) matrix element

Full nuclear matrix element [1]: $M_{full}^{(a)} = M_p^{(E),a} + M_p^{(M),a} + M_{\Delta M}^{a} + M_k^{a}.$

Coherent terms:

$$M_p^{(E)} = -\hbar\,(2\pi)^3\, \frac{\mu_N}{\sqrt{3}}\, \frac{m_p Z_{eff}}{\mu} \cdot J_1(0,1,0),$$

$$M_p^{(E)} + M_p^{(M)} = M_p^{(E)} \left(1 + i\, \frac{\mu^2}{2 m_p^2 Z_{eff}}\, \alpha \right).$$

Incoherent terms:

$$M_{\Delta M} = \hbar\,(2\pi)^3\, \frac{\sqrt{3}}{2}\, \mu_N k_{ph} f_A \cdot Z_A(\mathbf{k}_{ph}) \cdot J_2(-c_p,0,1,1),$$

$$M_{\Delta M} + M_k = -\hbar\,(2\pi)^3\, \frac{\sqrt{3}}{2}\, \mu_N k_{ph} \times$$

$$\times \left\{ \frac{A+1}{2A}\, \mu_{pn}^{(A)} \cdot Z_A(\mathbf{k}_{ph}) \cdot J_2(-c_p,0,1,1) + \mu_p \cdot J_2(+c_A,0,1,1) \right\}$$

Radial integrals (numerical calc.):

$$J_1(l_i, l_f, n) = \int_0^{+\infty} \frac{dR_i(r,l_i)}{dr}\, R_f^*(r,l_f)\, j_n(k_{ph} r)\, r^2 dr, \quad J_2(c, l_i, l_f, n) = \int_0^{+\infty} R_i(r,l_i) R_f^*(r,l_f)\, j_n(c\, k_{ph} r)\, r^2 dr.$$

[1] S. Maydanyuk, Phys. Rev. **C107,** 024618 (2023) + Supplement.

Dilepton production: Coherent spectrum

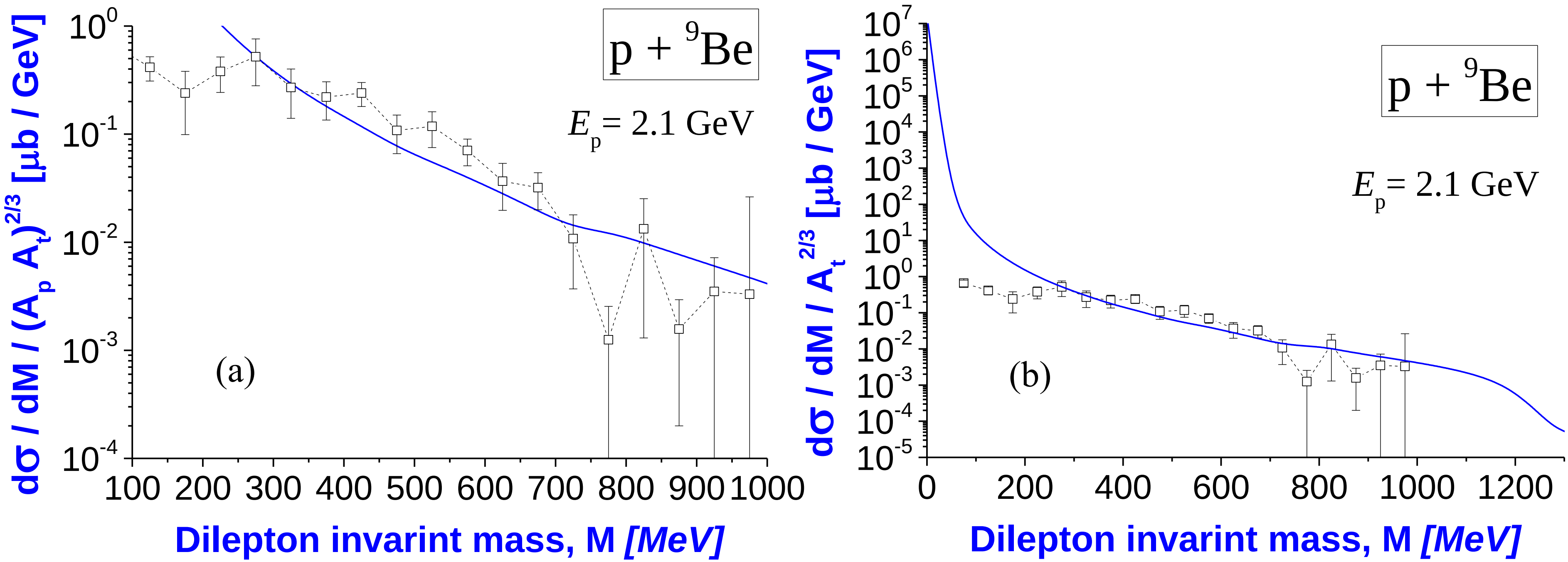


Fig.1. The calculated coherent cross section of production of leptons pair in the scattering of protons off nuclei ^{9}Be at energy of proton beam of E_p = 2.1 GeV in comparison with experimental data [1]

[we normalize calculated spectrum on one point of experimental data; time of computer calculations is 26-30 min for 40 points of each calculated spectrum].

[1] C. Naudet, et al. (DLS Collab.),
Phys. Rev. Lett. **62,** 2652 (1989).

Dilepton production: Coherent spectrum

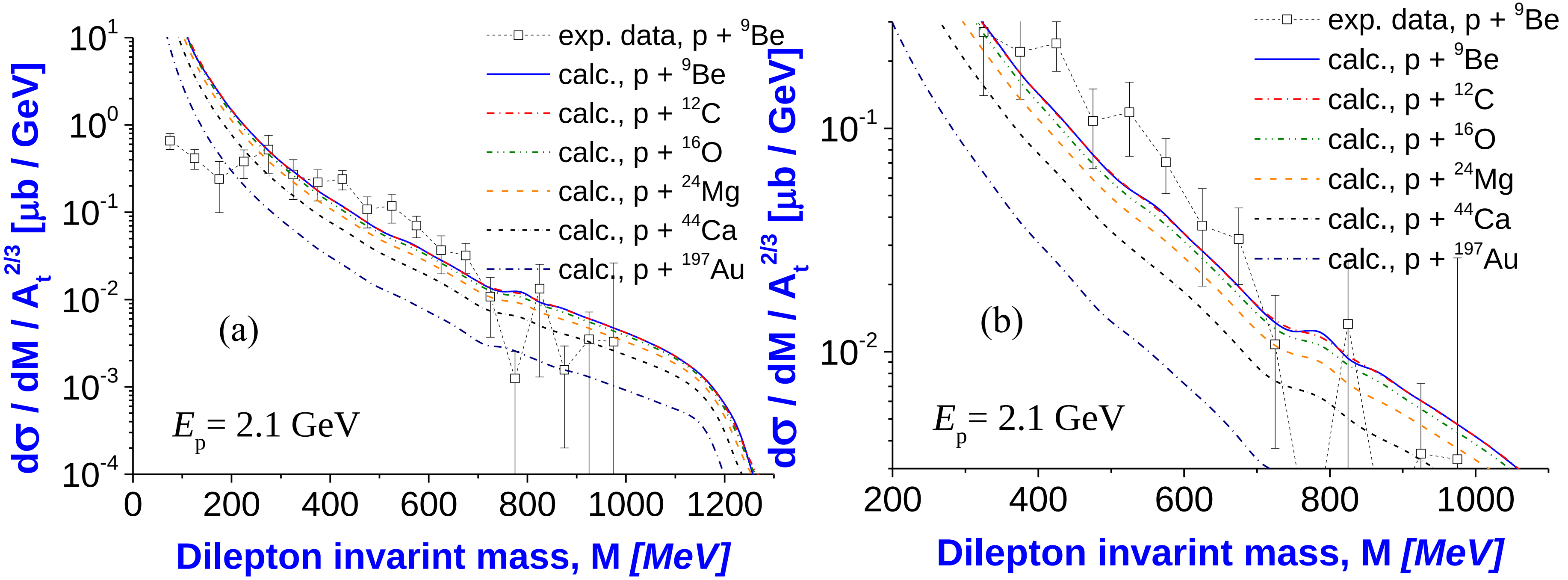


Fig.2. The calculated coherent cross section of production of leptons pair in the scattering of protons off ^{9}Be, ^{12}C, ^{16}O, ^{24}Mg, ^{44}Ca, ^{197}Au at energy of proton beam of E_p = 2.1 GeV in comparison with experimental data [1]

[all calculated spectra have the same normalization factor found in previous Fig. 1].

[1] C. Naudet, et al. (DLS Collab.),
Phys. Rev. Lett. **62,** 2652 (1989).

Coherent spectrum: dependence on beam energy

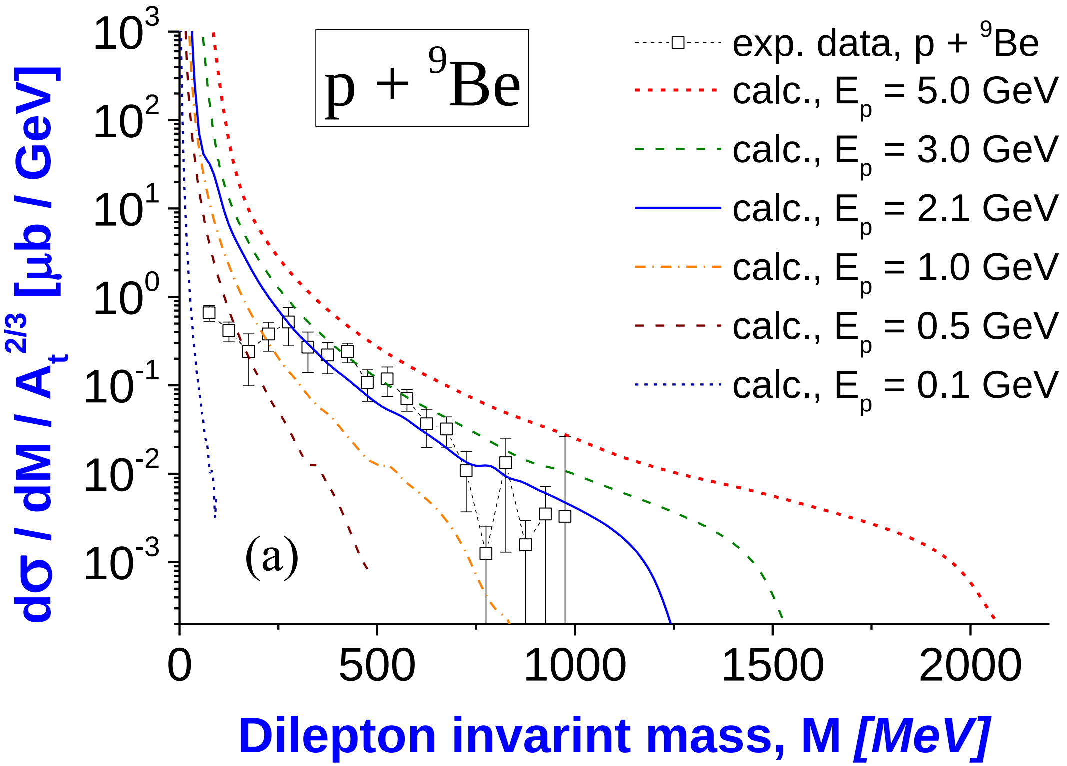


Fig.3. The calculated coherent cross sections of production of leptons pair in the scattering of protons off the ^{9}Be nuclei at different energies of proton beam of E_p [all calculated spectra have the same normalization factor found in Fig.1].

[1] C. Naudet, et al. (DLS Collab.),
Phys. Rev. Lett. **62,** 2652 (1989).

Incoherent & coherent processes

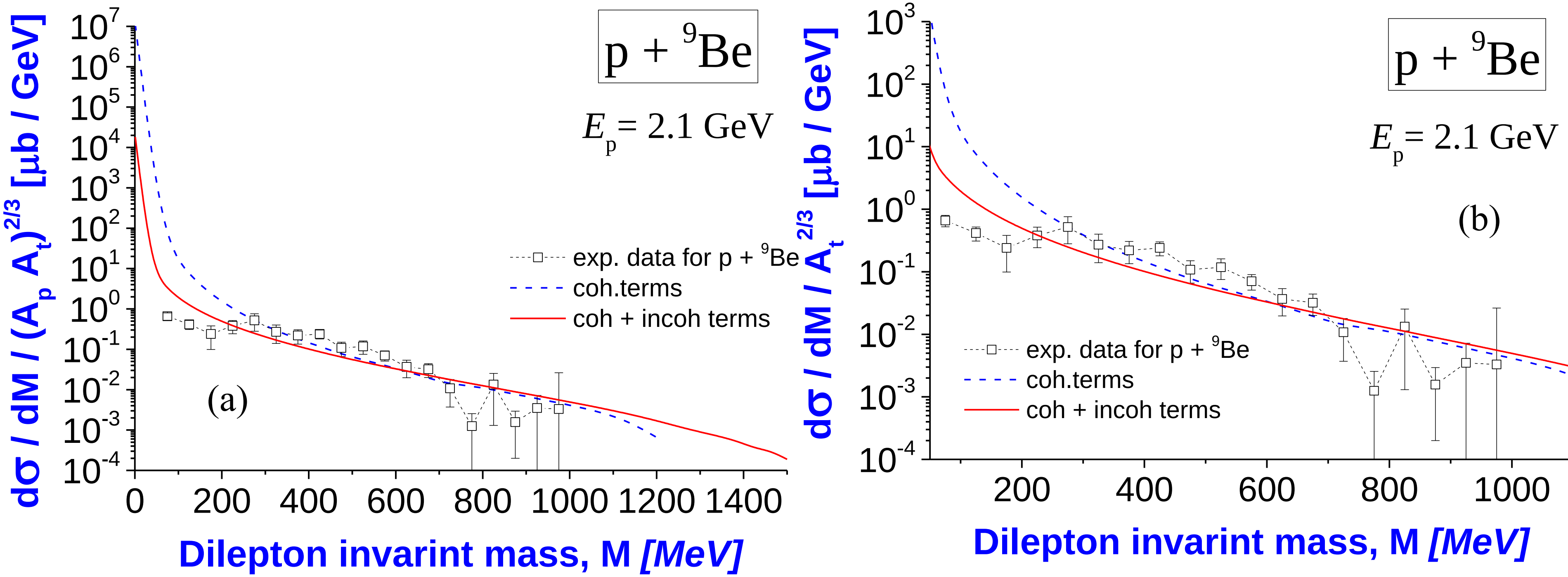


Fig.4. The calculated full cross sections of production of leptons pair (with coherent and incoherent terms) in the scattering of protons off the ^{9}Be nuclei at energy of proton beam of E_p = 2.1 GeV in comparison with experimental data [1].

[1] C. Naudet, et al. (DLS Collab.),
Phys. Rev. Lett. **62,** 2652 (1989).

Incoherent processes : ratio

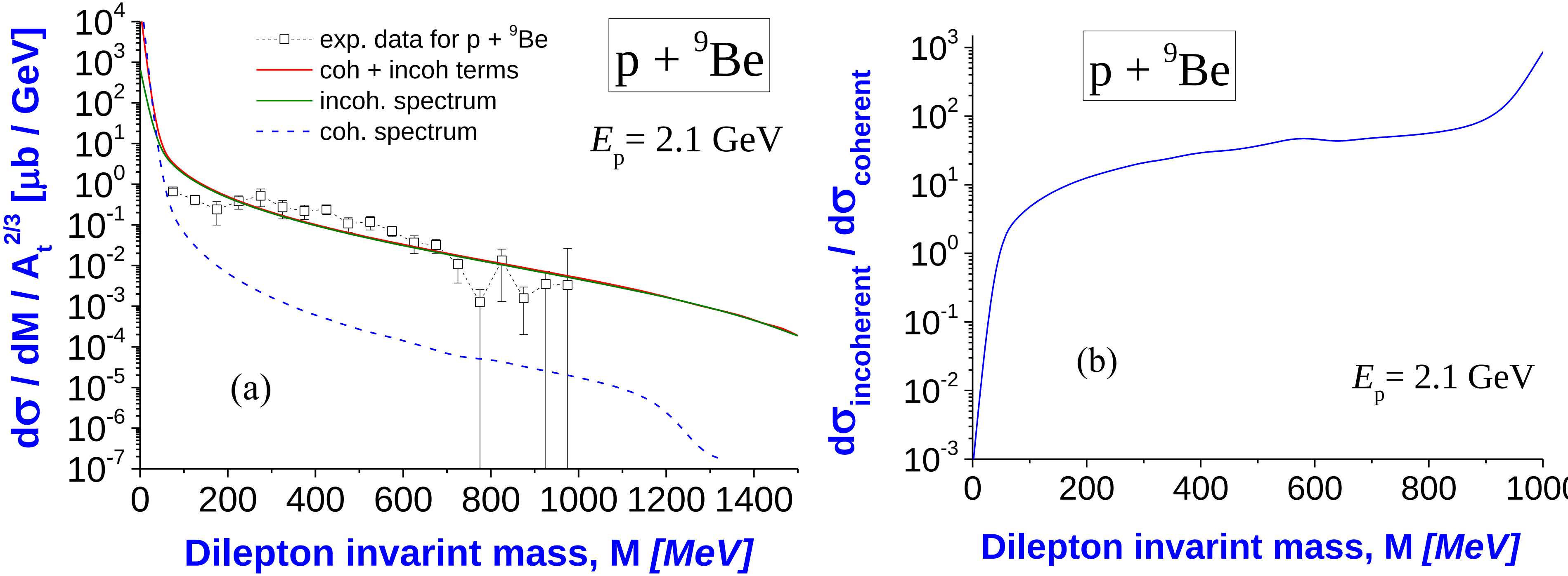


Fig.5. The calculated incoherent and coherent contributions to the full cross sections of production of leptons pair in the scattering of protons off the ^{9}Be nuclei at energy of proton beam of E_p = 2.1 GeV in comparison with experimental data [1] (a) and ratio between the incoherent and coherent contributions for this reaction (b).

[1] C. Naudet, et al. (DLS Collab.),
Phys. Rev. Lett. **62,** 2652 (1989).

Incoherent processes for different nuclei

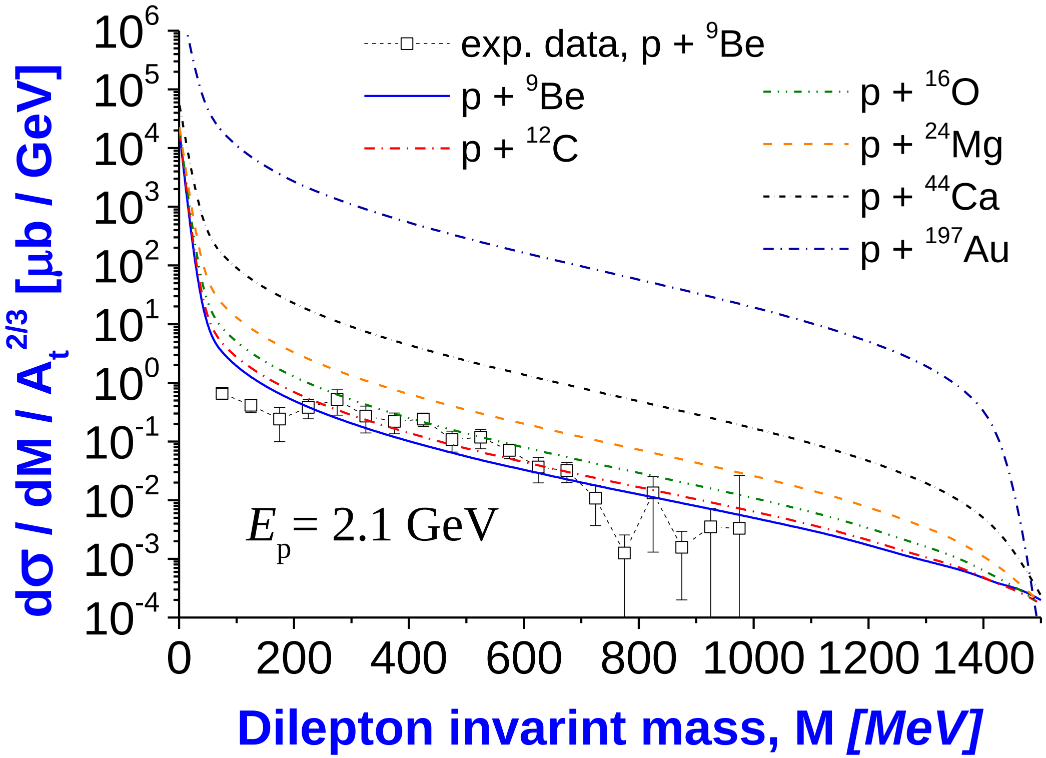


Fig.6. The calculated full cross sections of production of leptons pair in the scattering of protons off ^{9}Be nuclei at energy of proton beam of E_p = 2.1 GeV in comparison with experimental data [1] for ^{9}Be

[time of computer calculations is 26-40 min for 40 points of each calculated spectrum].

[1] C. Naudet, et al. (DLS Collab.),
Phys. Rev. Lett. **62,** 2652 (1989).

Role of longitudinal part of virtual photon in production of dileptons

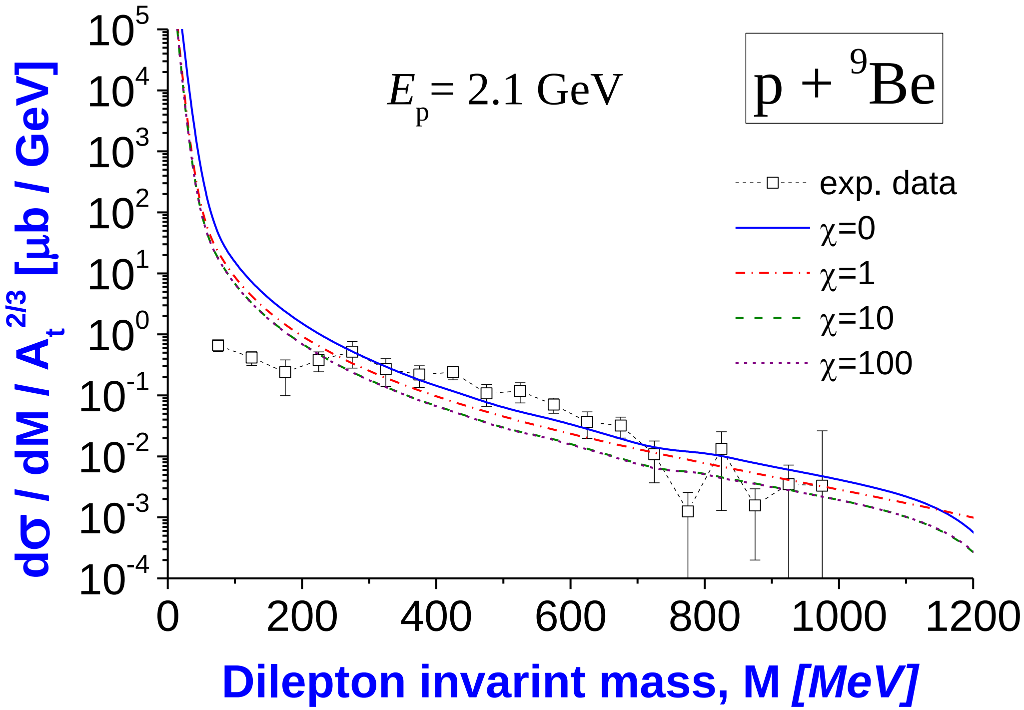
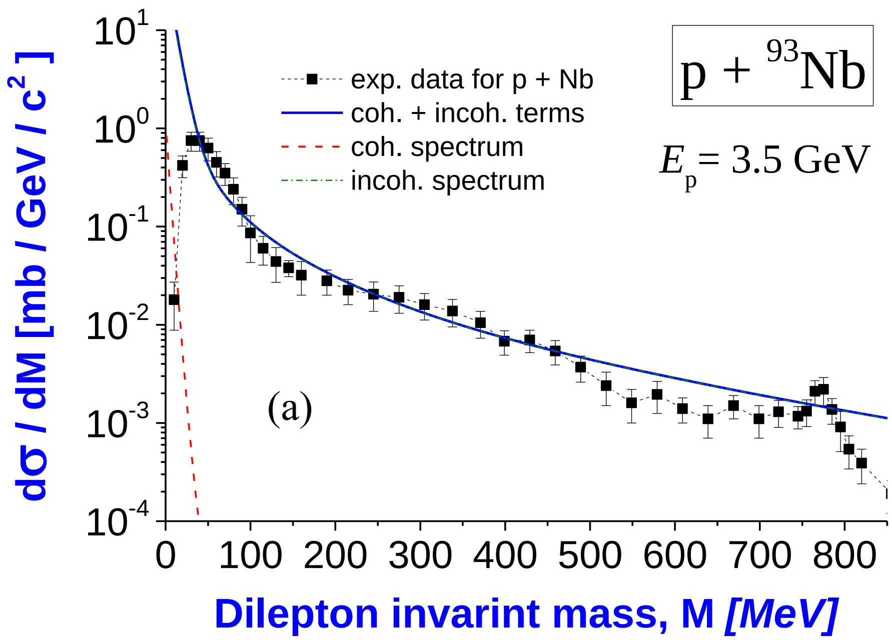


Fig.7. The calculated cross sections of production of leptons pair (with coherent terms) in the scattering of protons off ^{9}Be nuclei at energy of proton beam of E_p = 2.1 GeV for different virtualities of photon in comparison with experimental data [1]

[all calculated spectra have the same normalization factors on experimental data found from results in Fig. 1].

[1] C. Naudet, et al. (DLS Collab.), Phys. Rev. Lett. **62,** 2652 (1989).

Dileptons in p + ^{93}Nb: theory & experiment



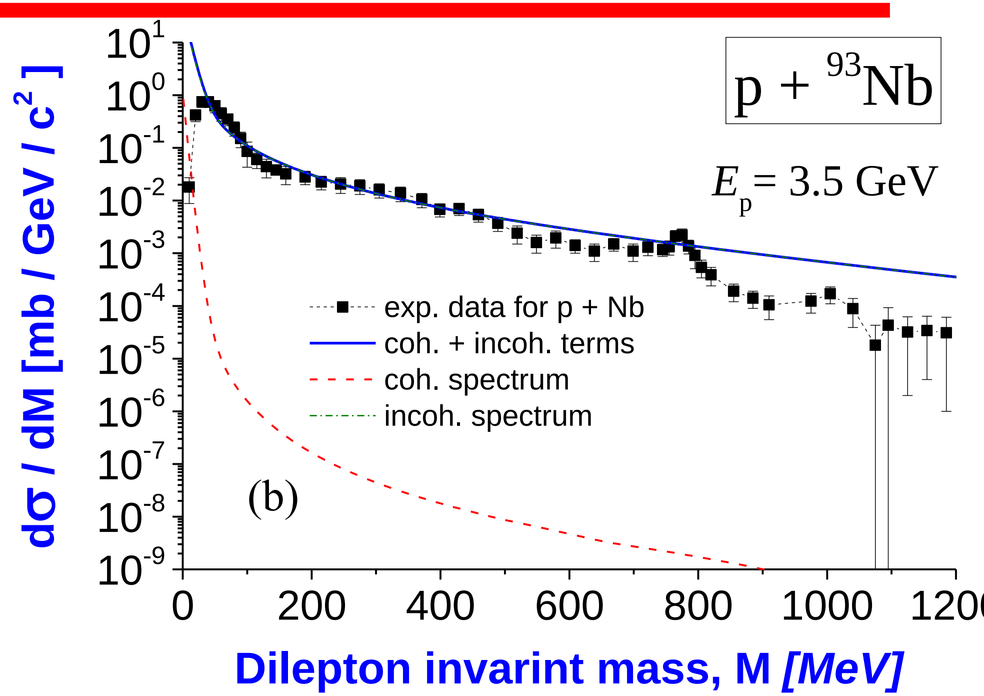


Fig.8. The calculated cross section of production of leptons pair in the scattering of protons off ^{93}Nb nuclei at energy of proton beam of E_p = 3.5 GeV in comparison with experimental data obtained by HADES collaboration [2]

[we normalize the full calculated spectrum on one point of experimental data, obtain factor of normalization, then renormalize the coherent and incoherent contributions on this one factor].

Here, blue solid line is the calculated full cross section (which includes coherent and incoherent terms), red dashed line is coherent contribution, green dash-dotted line is incoherent contribution (it is almost coincides with blue solid line of the full spectrum).

[2] G. Agakishiev, et al. (HADES Collab.), Phys. Lett. **B 715,** 304 (2012).

p + ^{93}Nb: Role of incoherent processes

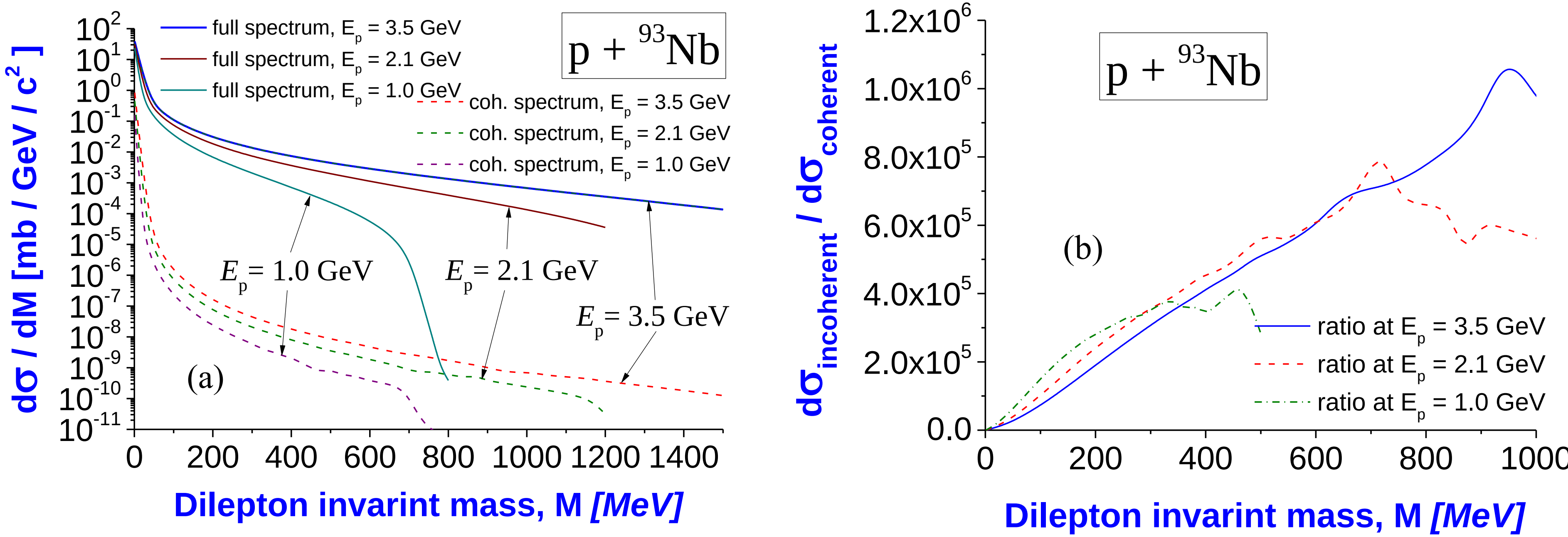


Fig.9. The calculated cross section of production of leptons pair in the scattering of protons off ^{93}Nb nuclei at energies of proton beam 1.0 GeV, 2.1 GeV and 3.5 GeV.

Panel (a): The coherent contribution and full spectrum at different energies of proton beam. The incoherent contribution almost coincides with the full spectrum for each energy E_p.

Panel (b): Ratio between incoherent contribution and coherent contribution for different energies of proton beam. One can see that role of incoherent contribution for this reaction is essentially larger than for p + ^{9}Be

Role of nuclear potential in dilepton production

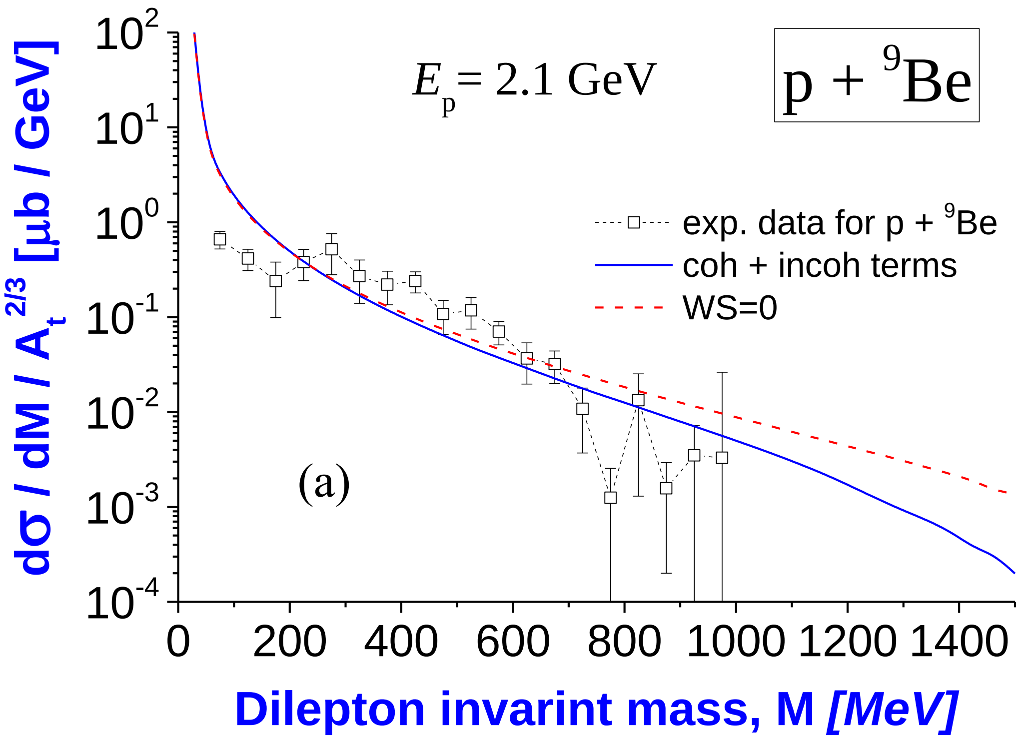


Fig.10. The calculated cross section of production of leptons pair in the scattering of protons off ^{9}Be nuclei with included nuclear potential and without this potential.

Proton-nucleus potential:

$$V(r) = v_C(r) + v_N(r) + v_{so}(r) + v_l(r),$$

$$v_N(r) = -\frac{V_R}{1+\exp\frac{r-R_R}{a_R}},$$

$$v_l(r) = \frac{l(l+1)}{2mr^2},$$

$$v_{so}(r) = V_{so}\,\mathbf{q}\cdot\mathbf{l}\,\frac{\lambda_\pi^2}{r}\frac{d}{dr}\left[1+\exp\left(\frac{r-R_{so}}{a_{so}}\right)\right]^{-1},$$

Parameters:

$$R_R = r_R A^{1/3}, \quad R_C = r_C A^{1/3}, \quad R_{so} = r_{so} A^{1/3},$$

$$r_{so} = 1.01\,\text{fm}, \quad a_R = 0.75\,\text{fm}, \quad a_{so} = 0.75\,\text{fm}.$$

Conclusions

We investigated production of lepton pairs in the scattering of protons off nuclei at intermediate energy region. For that, we constructed a new model, on the basis of such a model we conclude the following.

1)Calculated cross sections of dilepton production in scattering p + ^{9}Be at E_p = 2.1 GeV and p + ^{93}Nb at E_p = 3.5 GeV are in good agreement with trends of experimental data.

2)We analyzed full cross sections of dilepton productions, their coherent and incoherent contributions in dependence on different nuclei-targets (^{9}Be, ^{12}C, ^{16}O, ^{24}Mg, ^{44}Ca, ^{197}Au), different energies of proton beam.

	Energy of proton beam		
	1.0 GeV	2,1 GeV	3.5 GeV
p + ^{9}Be	5 - 50	5 - 100	
p + ^{93}Nb	$10^5 – 4 \times 10^5$	$10^5 – 7 \times 10^5$	$10^5 – 10^6$

Tabl.1. Ratio between incoherent contribution and coherent contribution for different reactions at different energies of proton beam

Thank you for attention!

Cross-section of α-capture: method MIR & WKB

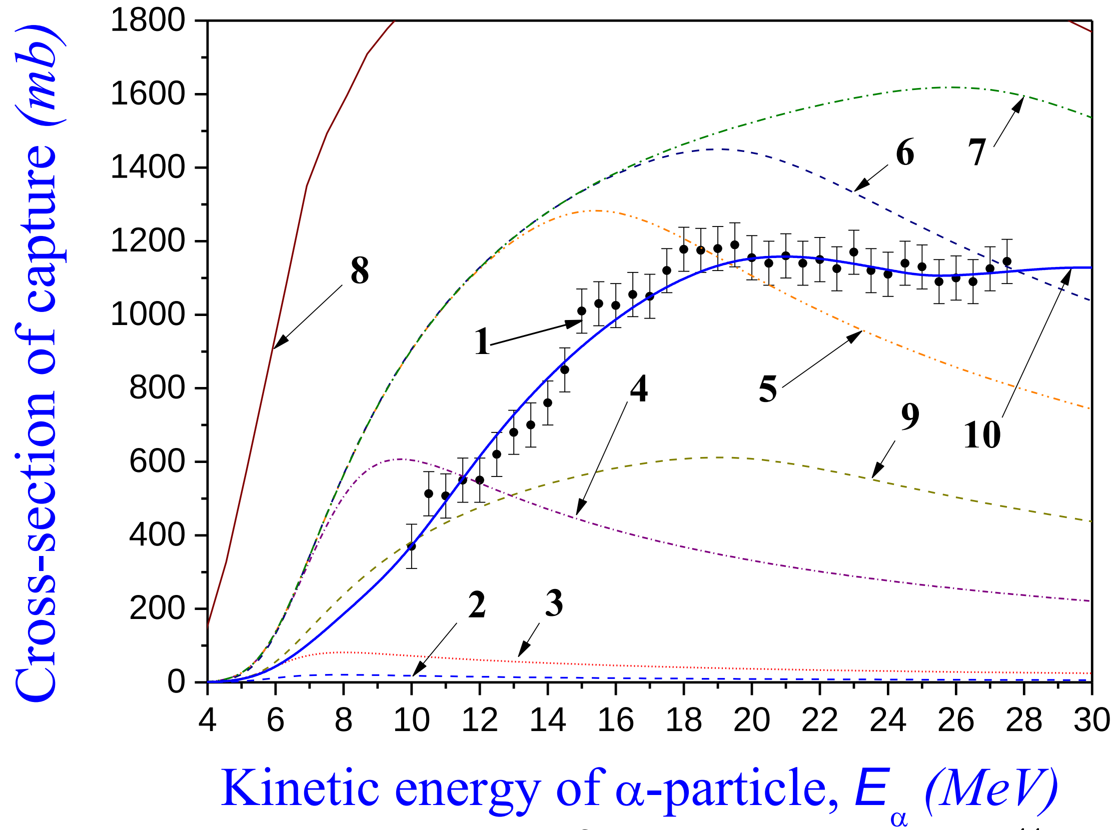


Fig.2. Capture cross-sections of α-particle by nucleus ^{44}Ca, obtained by method MIR (lines 2-7, 9-10) and WKB-approach (line 8). *Line 10 is obtained at inclusion of probabilities of fusion, lines 2-9 are without fusion prob.* [1].

Conclusion: ***Method MIR with included probabilities of fusion (line 10) is in higher agreement with experimental data, than WKB-approach without fusion (line 8).***

Black circles 1 is experimental data, dashed blue line 2 is cross-section at l_{max}=0, short dashed red line 3 is cross-section at l_{max}=1, short dash-dotted purple line 4 is cross-section at l_{max}=5, dash-double dotted orange line 5 is cross-section at l_{max}=10, dashed dark blue line 6 is cross-section at l_{max}=12, dash-dotted green line 7 is cross-section at l_{max}=15, solid brown line 8 is cross-section at l_{max}=20, dashed dark yellow line 9 is renormalized cross-section at l_{max}=17, solid blue line 10 is cross-section at l_{max}=17.

- Cross-section of capture:

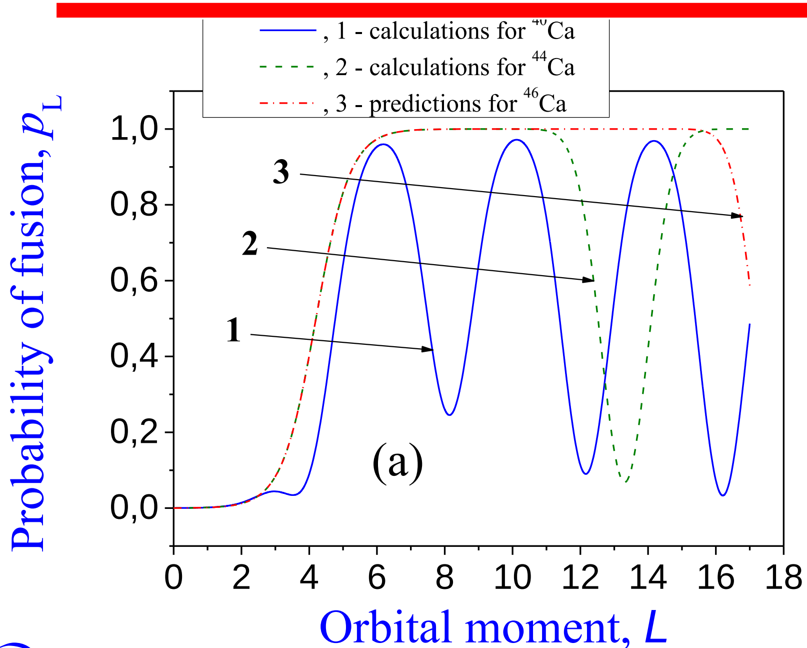
$$\sigma_{\text{capture}}(E) = \frac{\pi \hbar^2}{2mE} \sum_{l=0}^{l_{\max}} (2l+1) T_l P_l.$$

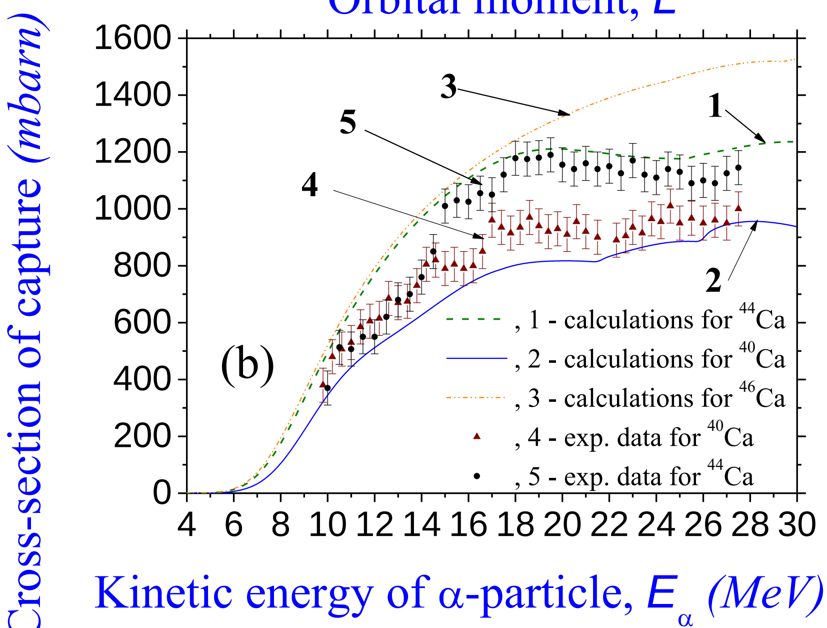
Here, E is kinetic energy of α-particle in lab. frame, E_1 is kinetic energy of relative motion of α-particle and nucleus, m is reduced mass of α-particle and nucleus, P_l is probability of fusion of α-particle and nucleus, T_l is penetrability of barrier.

- Test of method: $T_{MIR} + R_{MIR} = 1.$

[1] Maydanyuk S. P., Zhang P.-M., et al. Nucl. Phys. **A940,** 89-118 (2015).

Formula for probability of fusion





Using fitting procedure, we found probabilities of fusion and described them as

$$\sigma_{capture}(E) = \frac{\pi\hbar^2}{2mE}\sum_{l=0}^{+\infty}(2l+1)T_l P_l.$$

$$p_{full}(L) = 1 - p_1(L) - p_2(L),$$

$$p_1(L) = \frac{c_1}{1+\exp\left[\frac{(L-c_2)}{c_3}\right]}, \quad p_2(L) = f_2(L)\cdot\sum_{n=1}\exp\left\{-\frac{(L-n\cdot\Delta)^2}{c_{4n}}\right\},$$

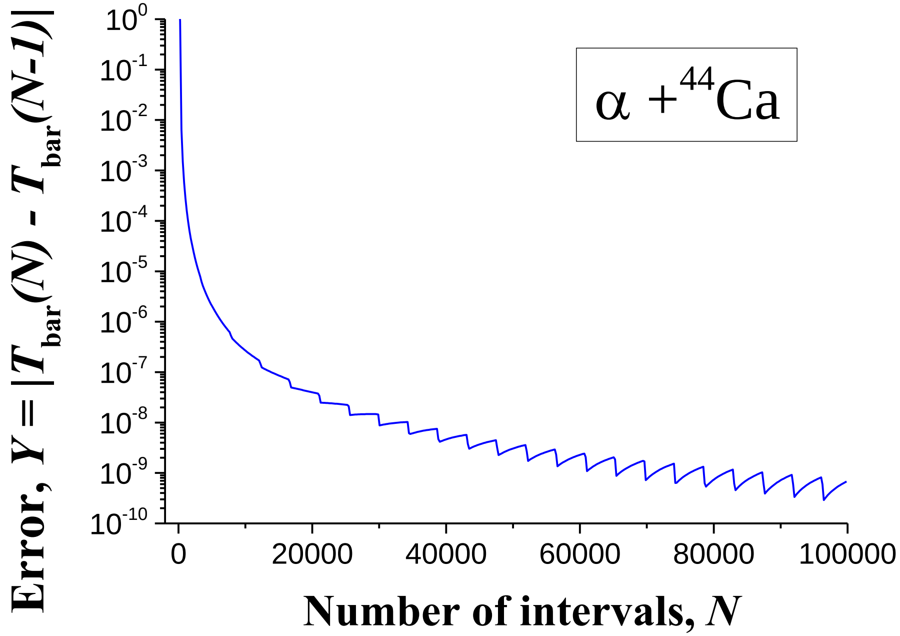
$$f_2(L) = 1-\exp\{-c_5\cdot(L-c_6)\}, \quad \Delta = a\cdot(N-N_{magic})+b,$$

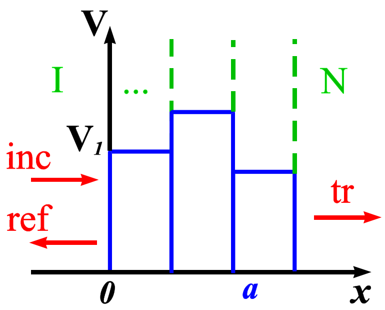
$$a = 2.31, \quad b = 4.05, \quad c_1 = 1, \quad c_2 = 4.2, \quad c_3 = 0.5.$$

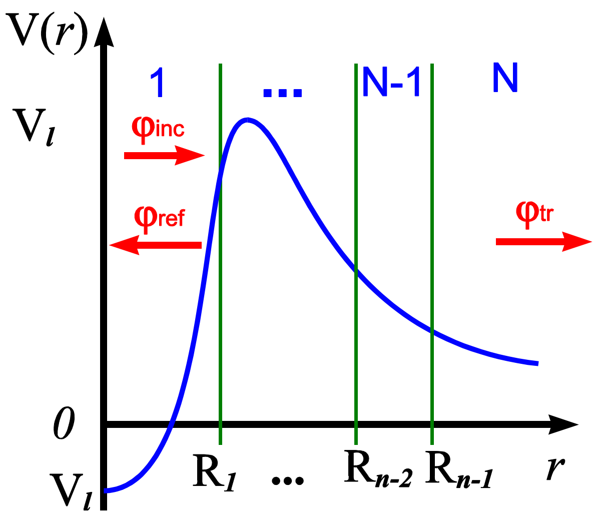
Fig.7. Probabilities of fusion (a) calculated by formulas above and cross-sections (b) for capture of α-particles by ^{40}Ca, ^{44}Ca, ^{46}Ca, obtained by method MIR [1].

[1] Maydanyuk S. P., Zhang P.-M., Belchikov S. V. Nucl. Phys. A. - 2015. - Vol. 940. - P. 89-118.

Accuracy of MIR method in capture task







Test of method:

$$T_{\text{bar}} + R_{\text{bar}} = 1.$$

Accuracy of method:

- Our method of Mult. Int. Refl.: 10^{-15} ;
- WKB-method (semiclassical, 1 order): 10^{-3}.