Kinematic Self-Similar Solutions with Dynamical Dark Fluid Model

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Motivation

- The properties and existence of dark matter is one of the most fascinating questions in cosmology.
- The scale-free nature of gravitational interaction in both Newtonian gravity and the general theory of relativity gives rise to the concept of self-similarity
- This implies that the governing partial differential equations are invariant under scale transformation if we consider appropriate matter fields.
- Self-similar solutions (SSs) have a wide range of applications in astrophysics
- We studied different kinds of dark fluid models with self-similar solutions.

Self-similarity in General Relativity

- In GR, the concept of SSs is not quite straightforward because GR has general covariance against coordinate transformation.
- Can be seen in two ways: Properties of the space-time, and properties of the matter fields [Cahill and Taub (1971)]
- Self-similarity of the space-time $\Rightarrow$ **Homothetic vector fields (HVF):**

$$\mathcal{L}_\xi g_{\mu\nu} = 2\alpha g_{\mu\nu}$$

The **kinematic self-similar solution** can be defined via a kinematic self-similar vector ξ (KSS). The KSS vector satisfies the following identities:

$$\mathcal{L}_\xi h_{\mu\nu} = 2\delta h_{\mu\nu} \tag{1}$$

$$\mathcal{L}_\xi u_\mu = \alpha u_\mu \tag{2}$$

The definition of the $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projection tensor.

General Spherically Symmetric Space-time

The line element of the general symmetric spacetimes is given by:

$$ds^2 = -e^{2\Phi(t,r)}dt^2 + e^{2\Psi(t,r)}dr^2 + R(t,r)^2[d\theta^2 + \Sigma(k,\theta)^2 d\phi^2] \qquad (3)$$

where,

$$\Sigma(k,\theta) = \begin{cases} \sin(\theta), & k = 1 \\ \theta, & k = 0 \\ \sinh(\theta), & k = -1 \end{cases}$$

We adopt comoving frames:

$$u_\mu = (e^{-\Phi}, 0, 0, 0)$$

Equation of State

We are interested in finding kinematic self-similar solutions for different dark energy models. We are interested to find solution when the following linear equation of state (EOS1) are used:

$$p = w(\xi)\rho, \tag{4}$$

where the w parameter is explicitly depend on the ξ similarity variable. Linear equation of state are widely used in cosmological astrophysics to describe dark matter, dark energy as well as ordinary matter. The other equation of state we used is more restricted (EOS2):

$$p = w(\mathcal{R}(r,t))\rho, \tag{5}$$

Solutions and Summary

From this analysis, we showed that for the first EoS $p = w(\xi)\rho$, the **relevant solutions** yet we find are the following:

Second kind, tilted	Homothetic Static
Second kind, parallel	Flat FRWL
Second kind, orthogonal	Homothetic Static
Zeroth kind, tilted	No solution
Zeroth kind, parallel	No solution
Zeroth kind, orthogonal	All static solution

We are currently working on the solutions, where the EOS2 is used, and we are also interested in those solutions, where bulk viscosity is added to the $T_{\mu\nu}$.