# Zimányi School (4-8 December, 2023)

## XY Factorization Bias in Luminosity Measurements

Anna Fehérkuti

Eötvös Loránd University, HUN-REN Wigner Research Centre for Physics

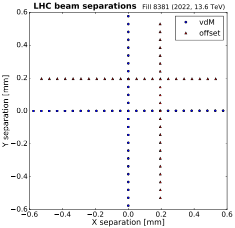
# What Is Luminosity?

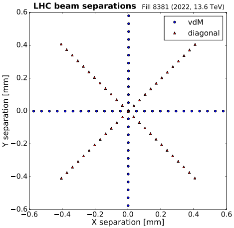
$$\mathcal{L}_{inst} = \frac{R}{\sigma_{vis}} = \frac{fN_1N_2}{2\pi\Sigma_x\Sigma_y}$$

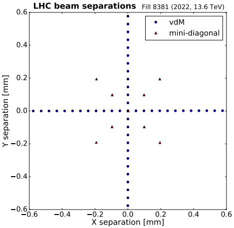
- $\mathcal{L}_{inst}$: instantenous luminosity
- $R$: measured rate of the quantity
- $\sigma_{vis}$: visible cross-section
- Basically the rate of any physical process can be given as $\frac{\mathrm{d}N}{\mathrm{d}t} = \mathcal{L}_{inst} \cdot \sigma_{vis}$
- $f$: machine revolution frequency (11245 Hz)
- $N_{1,2}$: bunch populations
- $\Sigma_{x,y}$: horizontal & vertical beam overlap sizes (widths), measured by vdM scans (next slide)

# About vdM Scans in a Nutshell

- 2 bunches with $\varrho_1(x, y)$ & $\varrho_2(x, y)$ proton density functions collide
- vdM scans sample the convolution function along the axes: $(\varrho_1 \cdot \varrho_2)(0, y)$ & $(\varrho_1 \cdot \varrho_2)(x, 0)$



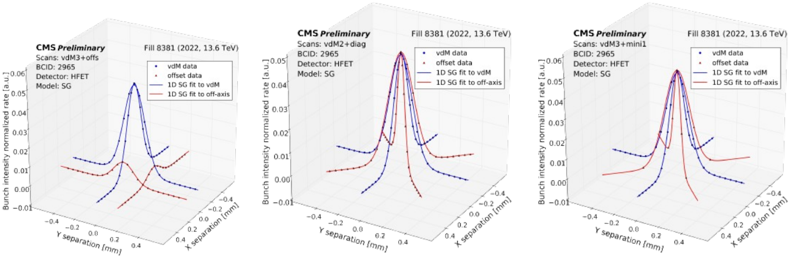




- vdM data separately fitted in the 2 directions with a Double Gaussian
- Width parameter calculated from the fits: $\sigma_{vis} = \int\int(\varrho_1 \cdot \varrho_2)(x, y)\mathrm{d}x\mathrm{d}y \sim 2 \cdot \max(\varrho_1 \cdot \varrho_2) \cdot \Sigma_x \Sigma_y$

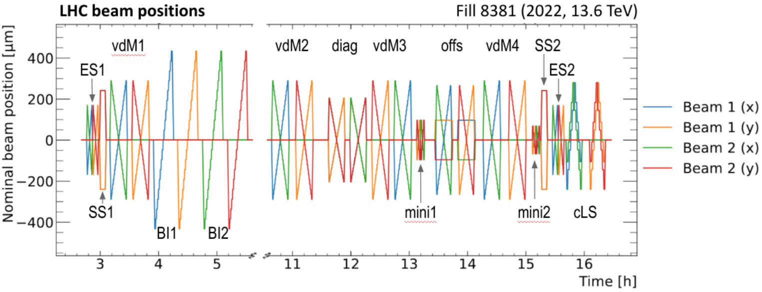
# The XY Factorisation Bias

BUT! $\sigma_{vis} = \int\int(\varrho_1 \cdot \varrho_2)(x, y)\mathrm{d}x\mathrm{d}y \sim 2 \cdot \max(\varrho_1 \cdot \varrho_2) \cdot \Sigma_x\Sigma_y$ approximation is exact only if the convolution shape is factorizable: $(\varrho_1 \cdot \varrho_2)(x, y) = f_1(x) \cdot f_2(y) \rightarrow$ we have to test it!




- Measurements: convolution shape not factorizable $\rightarrow$ *XY factorization bias*
- 2D fit using multiple functions: **S**ingle **G**aussian, **super** **G**aussian, **D**ouble **G**aussian (with various constraints), **q**-**G**aussian, **poly**nomial **G**aussian

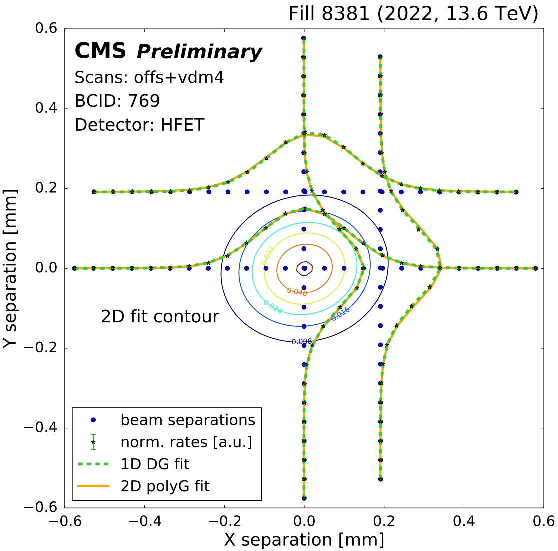
# Input Data




vdM scans used in this analysis:

- vdM(2-4)
- offset
- diagonal
- mini(1-2)

# 2D Fits: Visualization

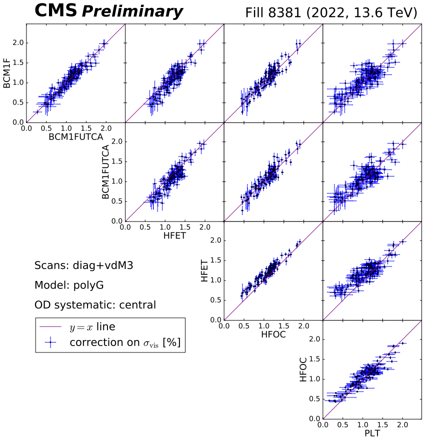


# Simulation

- Given: 2D fit function parameters with uncertainties from measured data (from vdM+diag / vdM+mini / vdM+offs) $\rightarrow$ sample parameter space to get vdM "data"
- Vary these parameters according to the uncertainty $\rightarrow$ random 2D shape (this also lets the uncertainty of the 2D fit propagate onto the correction)
- Calculate rate at pre-defined (nominal) separation points using the randomized shape
- Do (hypothetical) vdM fit (1D) on this artificial data
- Compare $\sigma_{vis}$ from the simulated vdM with the exact $\sigma_{vis}$ of the randomized 2D shape $\rightarrow$ get a factorisation correction
- Standard deviation over many randomized shapes $\rightarrow$ fit uncertainty
- Method cross-checked with closure test

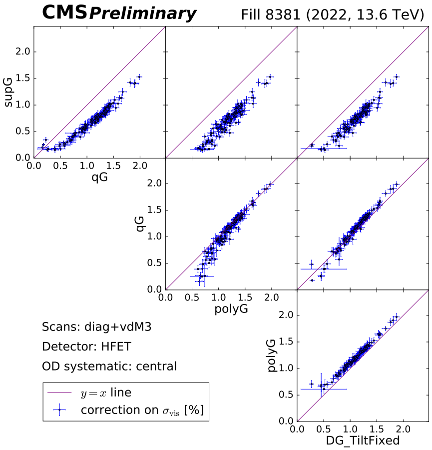
# Correlation Between Detectors

- Strong correlations in case of a stable shape model and well-understood luminometers
- These luminometers are used to define the final correction and its uncertainty



# Correlation Between Models

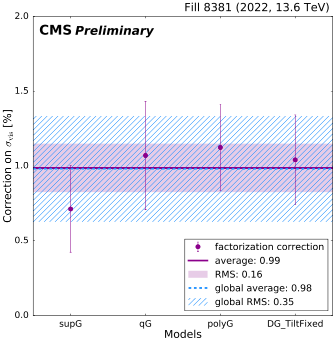
- Strong correlations
- supG: smallest corrections
- These models are used to define the final correction and its uncertainty

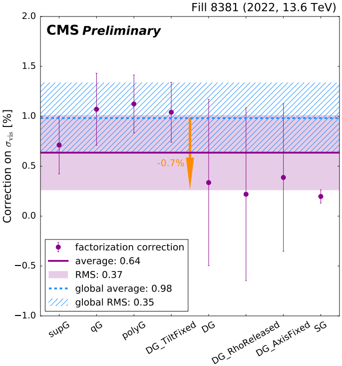


# Model Dependence of Results

Comparison of the results from the chosen vs other models:

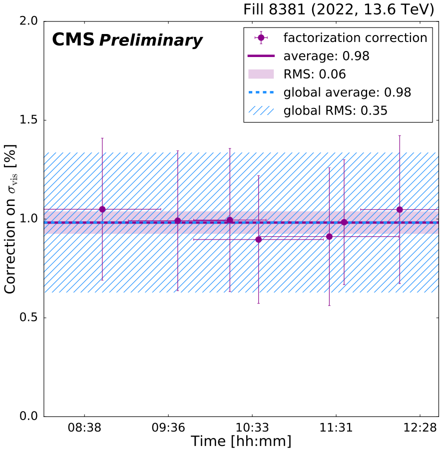
- Global average out of the results of the stable models
- Additional model systematics of 0.7% accounts for the results of the 3 unstable DG models



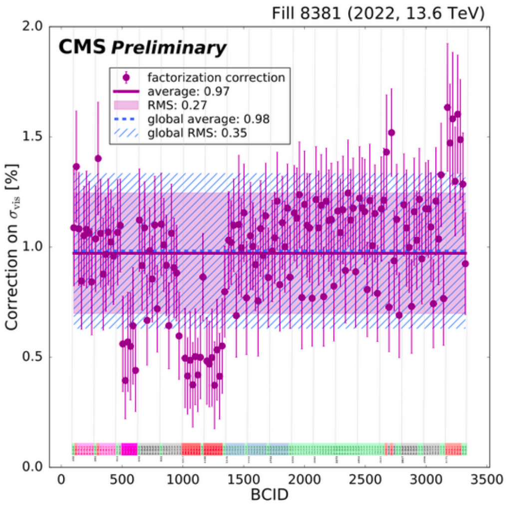
# Checking Time Dependence

- No time dependence during these measurements
- Points refer to: vdM2+diag, diag+vdM3, vdM3+mini1, vdM3+offs, mini1+vdM4, offs+vdM4, vdM4+mini2 combinations



# BCID Dependence of Corrections

- BCID dependence gives the vast majority of the RMS
- Pattern corresponds to the collision scheme in LHC (i.e. in which experiments the participating bunches collide)



# Summary and Outlook

The final result: $(0.98 \pm 0.35 \pm 0.7)\% \rightarrow$ *$(1.0 \pm 0.8)\%$*

- Global RMS: 0.35% (from all $144 \times 4 \times (9 \text{ or } 10) \times 7 \times 5$ measurements)
- Shape uncertainty: 0.7%
- Closure test: negligible

Uncertainties in more detail:

- BCID (144): 0.27% (true dependence: collision pattern at LHC)
- Model (only the 4 well-behaving): 0.16%
  - For all 8: 0.37%
  - Additional model uncertainty to cover the lack of knowledge of true shape: 0.7%
- OD systematics (10 for offset + 9 for diag): 0.05%
- Time (aka. scan pairing, 7 combinations): 0.05%
- Luminometer (5): 0.02%

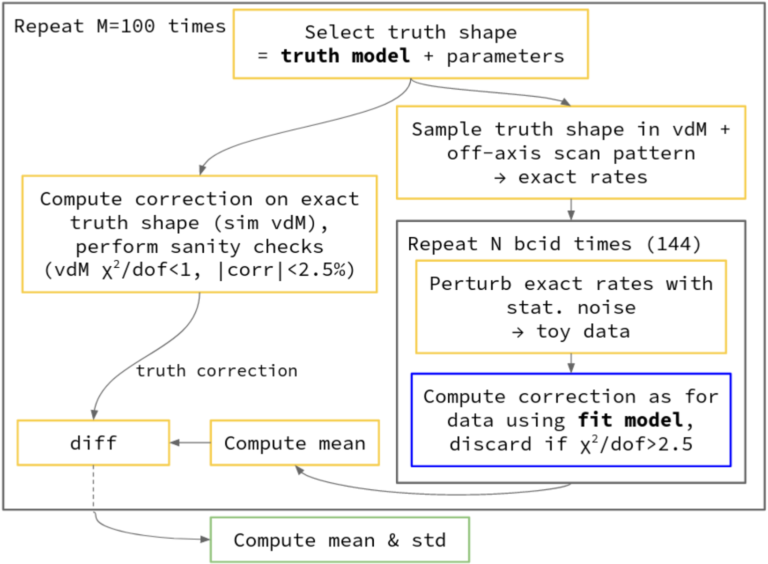
# Thank You for Your Time!

Backups »

# Workline

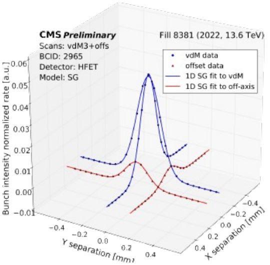
- 1D fits
- Rate matching for offset $\rightarrow$ more precise OD correction
- $\langle$rate matching vs lin OD correction separation$\rangle$ = 0.003 mm (used as systematic variation)
- 2D fits - further tunes of DG shape considered:
    - RhoFixed: fitting with $\varrho = 0$ requirement
    - RhoReleased: on the basis of RhoFixed results, refit releasing $\varrho$
    - Reparametrizations: aiming to avoid oscillation between 2 minima in the prediction of the correction in DG case
        - TiltFixed: fixing the sigmas in the ≈same directions between the 2 DGs (in 2D) to be the same ($X$ and $Y$ direction still can be different), also fixing $\mathrm{d}\varrho = 0$
        - AxisFixed: fixing the angle between the axes of the DG's SG components to 0 (requiring having the same orientation)
- Simulation
- Closure test (next slide)

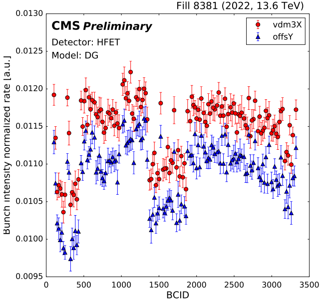
# Closure Test

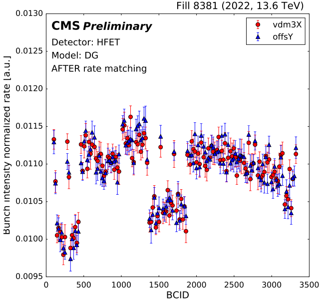


# Rate Matching

Fast change of the rate with separation → offset scans: even a small change in the separation can have a significant effect → additional correction on offset data ensuring that rates match at the “common” scan point:

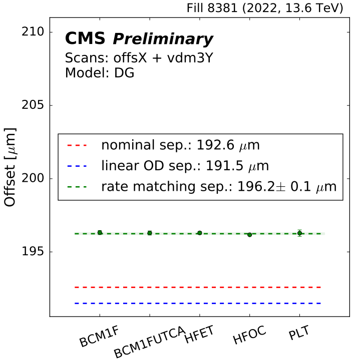


# Additional Orbit Drift Correction

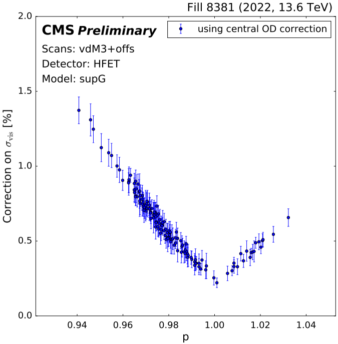
- Linear orbit-drift (OD) correction applied beforehand on input data (based on beam position measurements by DOROS: *Diode ORbit and OScillation* system)
- Rate matching ≈additional OD correction in the non-scanning direction of the offset scan
  - Largest shift shown
  - Luminometers agree well
  - Value also used as a ± variation estimating possible unknown OD



# Parameter Dependence

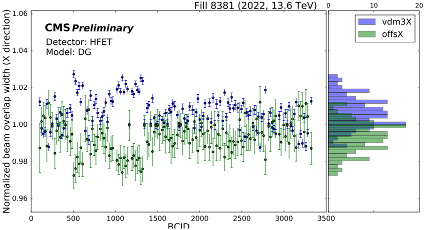
$$supG_{2D}(x, y; V, \mu_x, \mu_y, \sigma_x, \sigma_y, \varrho, p) = \frac{V}{N}\exp\left(-\frac{r^{2p}}{2}\right)$$

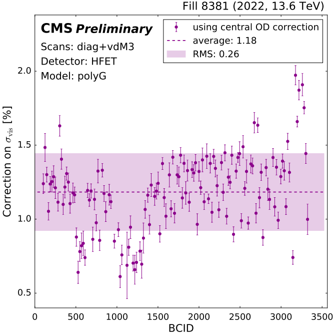
- $p$ parameter of supG (responsible for the "peakiness" of the supG shape: $p > 1$ means flatter than Gaussian shape) divides obtained correction value to 2 trends
- Phenomena present in other models as well (with the corresponding parameters)



# BCID-Dependence Pattern

- Plotting the correction values wrt. the BCID: same pattern present as in 1D
- Pattern corresponds to the collision scheme in LHC (i.e. in which experiments the participating bunches collide)



# LHC Collision Pattern

2+2L0: bunch in beam 1 and bunch beam 2 collides only at IP1 and IP5
3+2L0: bunch in beam 1 also collides at IP2 (ALICE)
3+2L1: bunch in beam 1 also collides at IP8 (LHCb)
2+3L0: bunch in beam 2 also collides at IP2 (ALICE)
2+3L1: bunch in beam 2 also collides at IP8 (LHCb)
4+2L1: bunch in beam 1 collides at all for IPs
2+4L1: bunch in beam 2 collides at all for IPs

LHC filling scheme: 525ns_146b_144_35_22_8bpi_20inj_nocloseLR.csv

# 2D Functions in Use I/IV.

Short-hand notation: $r^2 = \frac{\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 - 2\varrho \frac{x-\mu_x}{\sigma_x} \frac{y-\mu_y}{\sigma_y}}{1-\varrho^2}$

2D fitting functions used in the analyses:

- Single Gaussian:
  $SG_{2D}(x, y; V, \mu_x, \mu_y, \sigma_x, \sigma_y, \varrho) = \frac{V}{2\pi\sigma_x\sigma_y\sqrt{1-\varrho^2}} \exp\left(-\frac{r^2}{2}\right)$
- Super Gaussian:
  $supG_{2D}(x, y; V, \mu_x, \mu_y, \sigma_x, \sigma_y, \varrho, p) = \frac{V}{N}\exp\left(-\frac{r^{2p}}{2}\right)$
  - Where the normalization: $N = 2\pi\sigma_x\sigma_y\sqrt{1-\varrho^2} \cdot \frac{\Gamma(1/p)}{p} 2^{1/p}$
- ... (next slides)

# 2D Functions in Use II/IV.

- … (previous slide)
- q-Gaussian: $qG_{2D}(x,y) = \frac{V}{C(q)\cdot\sigma_x\sigma_y\sqrt{1-\varrho^2}} \cdot e_q\left(-\frac{r^2}{(4-2q)}\right)$
  - Where: $e_q(x) = \left[1+(1-q)x\right]_+^{\frac{1}{1-q}}$
  - And: $C(q) = \begin{cases} \frac{4-2q}{1-q}\pi\frac{\Gamma\left(\frac{2-q}{1-q}\right)}{\Gamma\left(\frac{2-q}{1-q}+1\right)} & q<1 \\ \frac{4-2q}{q-1}\pi\frac{\Gamma\left(\frac{1}{q-1}-1\right)}{\Gamma\left(\frac{1}{q-1}\right)} & \text{otherwise.} \end{cases}$
- Polynomial Gaussian: $polyG_{2D}(x,y;V,\mu_x,\mu_y,\sigma_x,\sigma_y,\varrho,a_2,\bar{a}_4) = \frac{V}{N}(1+a_2r^2+a_4r^4)\cdot\exp(-0.5r^2)$
  - Where the normalization: $N = 2\pi\sigma_x\sigma_y\sqrt{1-\varrho^2}\cdot(1+2a_2+8a_4)$
  - And the other parameters: $a_4 = \begin{cases} \bar{a}_4 & \text{if } a_2>0 \\ \bar{a}_4 + a_2^2/4 & \text{otherwise.} \end{cases}$
  - For positivity: $\bar{a}_4 > 0$ is required
- … (next slides)

# 2D Functions in Use III/IV.

- ... (previous slides)
- Double Gaussian:

$$DG_{2D}(x,y) = V \cdot \left( v_R \cdot SG_{2D}\left(x,y;1,\mu_x,\mu_y,\sigma_x^{(1)},\sigma_y^{(1)},\varrho^{(1)}\right) + (1 - v_R) \cdot SG_{2D}\left(x,y;1,\mu_x,\mu_y,\sigma_x^{(2)},\sigma_y^{(2)},\varrho^{(2)}\right)\right)$$

- Reparametrizations of the DG 2D fitting function:
  - $DG\_TiltFixed(x,y) = V \cdot \left( v_R \cdot SG_{2D}\left(x,y;1,\mu_x,\mu_y,\sigma_x^{(1)},\sigma_y^{(1)},\varrho + \mathrm{d}\varrho\right) + (1 - v_R) \cdot SG_{2D}\left(x,y;1,\mu_x,\mu_y,\sigma_x^{(2)},\sigma_y^{(2)},\varrho - \mathrm{d}\varrho\right)\right)$
    - Where: $\sigma_x^{(1)} = \sigma_x\sqrt{S_R}\sqrt{S_A}, \quad \sigma_y^{(1)} = \sigma_y\sqrt{S_R}/\sqrt{S_A},$ $\sigma_x^{(2)} = \sigma_x/\sqrt{S_R}/\sqrt{S_A}, \quad \sigma_y^{(2)} = \sigma_y/\sqrt{S_R}\sqrt{S_A}$
  - ... (next slide)

# 2D Functions in Use IV/IV.

- … (previous slides)
- Another reparametrization of the DG 2D fitting function also needs for reparametrization of the 2D single Gaussian function:
  $SG_{2D}^{(\alpha)}(x, y; V, \mu_x, \mu_y, \sigma_x, \sigma_y, \alpha) = \frac{V}{2\pi\sigma_x\sigma_y\sqrt{1-\varrho^2}} \cdot \exp\left(-\frac{r_\alpha^2}{2}\right)$
  - Where: $r_\alpha^2 = \begin{pmatrix} x-\mu_x \\ y-\mu_y \end{pmatrix}^T R(-\alpha) \begin{pmatrix} \sigma_x^{-2} & 0 \\ 0 & \sigma_x^{-2} \end{pmatrix} R(\alpha) \begin{pmatrix} x-\mu_x \\ y-\mu_y \end{pmatrix}$
    - Where: $R(\alpha)$ is a $2 \times 2$ rotation matrix
  - With these the reparametrized DG:
    $DG\_AxisFixed(x, y) = V \cdot \left(v_R \cdot SG_{2D}^{(\alpha)}\left(x, y; 1, \mu_x, \mu_y, \sigma_x^{(1)}, \sigma_y^{(1)}, \alpha + \mathrm{d}\alpha\right) + (1 - v_R) \cdot SG_{2D}^{(\alpha)}\left(x, y; 1, \mu_x, \mu_y, \sigma_x^{(2)}, \sigma_y^{(2)}, \alpha - \mathrm{d}\alpha\right)\right)$