$\omega - \rho$ interference in pion induced reactions

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in collaboration with M. Zétényi

Wigner RCP, Budapest

- πN reaction
- Transport equations for spectral functions
- πA reactions and quantum interference in nuclear matter
- Summary

$\pi + N \to N + e^+e^-$

- Coupled-channel approach

  K-matrix, Post, Leupold, Mosel,

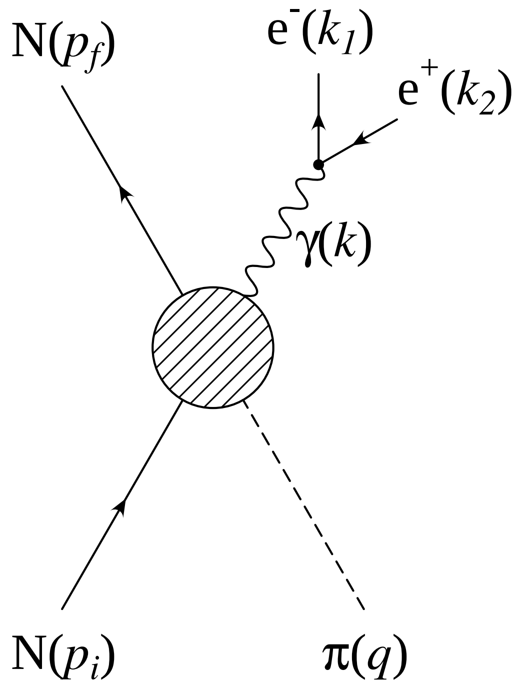
  Nucl. Phys. A689 (2001) 753

  Bethe-Salpeter, Lutz, Wolf, Friman,

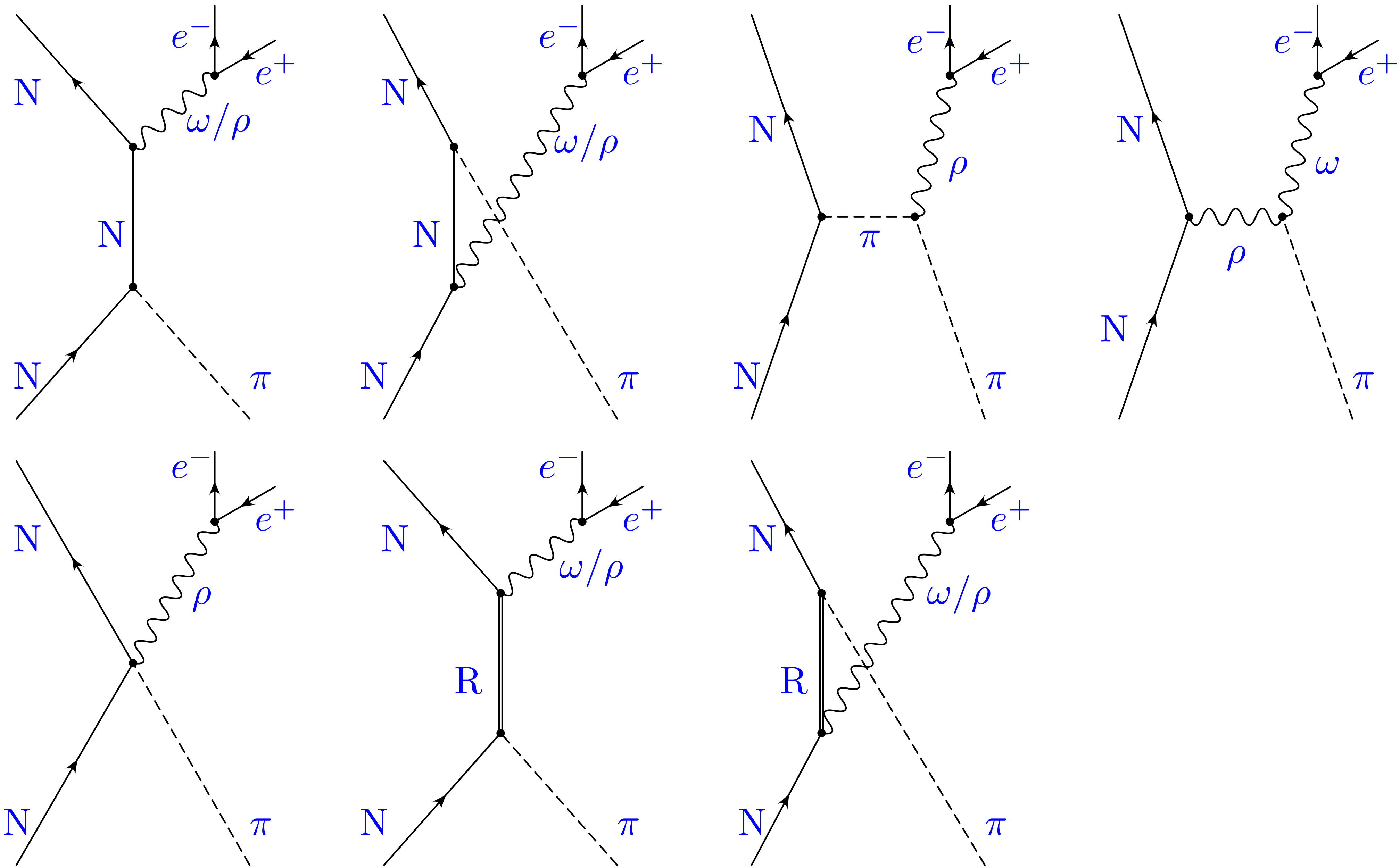
  Nucl.Phys. A706 (2002) 431

- Effective field theory

  Zétényi, Wolf, Phys. Rev. C86 (2012) 065209



Feynman diagrams for $\pi + N \to N + e^+e^-$



Effective Field Theory for $\pi + N \to N + e^+e^-$

$$\mathcal{L}_{\mathcal{N}\mathcal{N}\pi} = -\frac{\mathrm{f_{NN\pi}}}{\mathrm{m_\pi}}\overline{\psi}_{\mathcal{N}}\gamma_5\gamma^\mu\vec{\tau}\psi_{\mathcal{N}}\cdot\partial_\mu\vec{\pi}.$$

$$\mathcal{L}_{\omega\rho\pi} = \frac{\mathrm{g}_{\omega\rho\pi}}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\mathrm{Tr}\left((\partial_\alpha\vec{\rho_\beta}\cdot\vec{\tau})(\vec{\pi}\cdot\vec{\tau})\right)$$

$$D_\mu = \partial_\mu + ieA_\mu Q - ig_\rho\vec{\rho_\mu}\vec{T} - ig_\omega\omega_\mu T_0^{1/2}$$

$$\mathcal{L}_{\mathcal{N}\mathcal{N}\rho} = \mathrm{g}_\rho\overline{\psi}_{\mathrm{N}}\left(\vec{\not\rho} - \kappa_\rho\frac{\sigma_{\mu\nu}}{4\mathrm{m_N}}\vec{\rho}^{\mu\nu}\right)\cdot\vec{\tau}\psi_{\mathrm{N}}, \quad \mathcal{L}_{\mathcal{N}\mathcal{N}\omega} = \mathrm{g}_\omega\overline{\psi}_{\mathrm{N}}\left(\not\omega - \kappa_\omega\frac{\sigma_{\mu\nu}}{4\mathrm{m_N}}\omega^{\mu\nu}\right)\psi_{\mathrm{N}}.$$

$$\mathcal{L}_{VMD2} = -\frac{e}{2g_\rho}F^{\mu\nu}\rho^0_{\mu\nu}$$

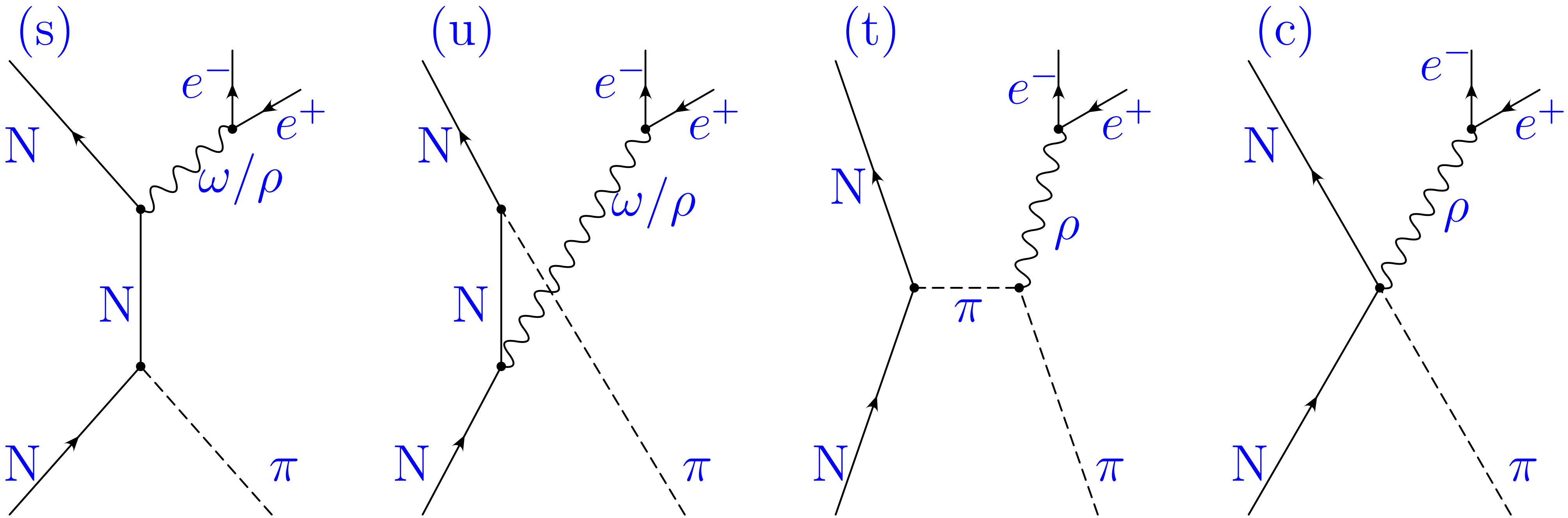
From ρ-width the contribution to the photonic decay can be obtained by multiplying it with $\frac{e}{g_\rho}\frac{k^2}{m_\rho^2 - k^2 - iq\Gamma_\rho(k^2)}$ Decay through ρ does not contribute to the real photonic width.

ρ_0 couples to $\bar{\psi}_N\tau_0\psi_N$ so to p and to n with different signs, while ω with the same sign

Considering $\pi^- p \to ne^+e^-$ and $\pi^+ n \to pe^+e^-$ in one of the channels constructive and in the other channel destructive interference

Current conservation (Gauge invariance)

Photon (even virtual one) couples to conserved current



Only the sum of s, t, u and contact channels is gauge invariant
using different form factors for different diagrams spoils gauge invariance.
With special care one can keep gauge invariance.

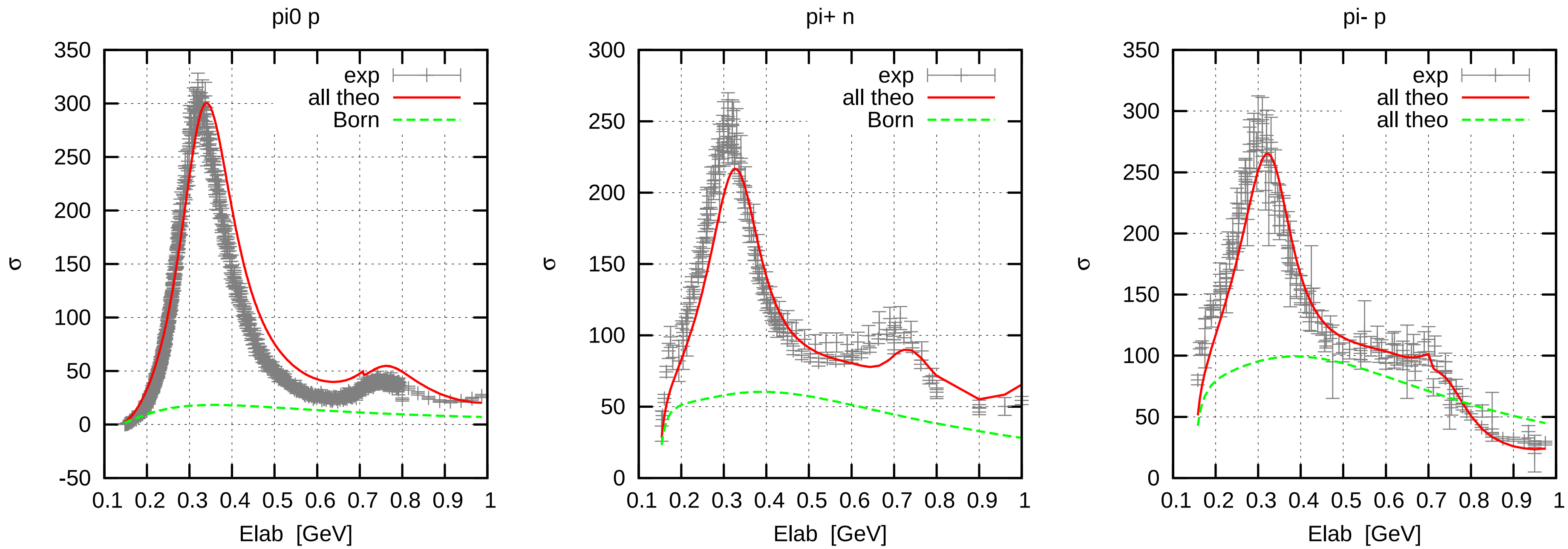
Parameters of the model

$f_{NN\pi}$	0.9702
$f_{NN\omega}$	10.35
$f_{NN\rho}$	2.6
$g_{\rho\omega\pi}$	12.9/GeV
κ_ρ	3.1
κ_ω	0.0
Λ_N	0.5 GeV
Λ_ρ	0.5 GeV
$g_{\rho\pi\pi}$	5.01

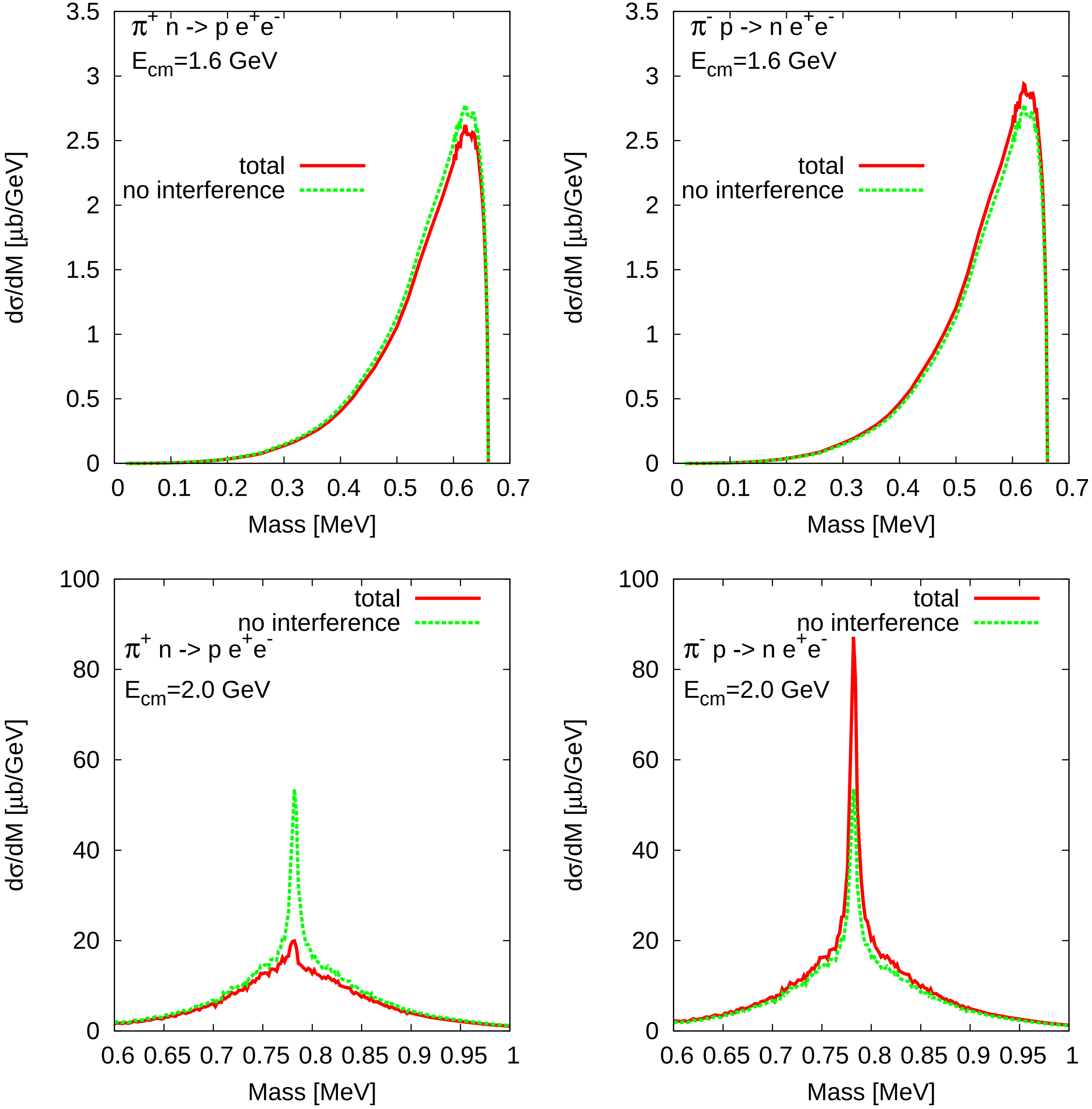
C Fernandez-Ramirez, E. Moya de Guerra, J.M. Udias, Annals of Physics 321 (2006) 1408

H.B. O'Connell et al., Prog. Part. Nucl. Phys. 39 (1997) 201.

RNπ and RNρ from measured partial decay widths, RNγ is fitted to the pion photoproduction, ω couplings were taken from the literature



$\pi N \to e^+e^-$



π+ n -> p e+e-
Ecm=1.6 GeV
π- p -> n e+e-
Ecm=1.6 GeV
π+ n -> p e+e-
Ecm=2.0 GeV
π- p -> n e+e-
Ecm=2.0 GeV
total
no interference
dσ/dM [μb/GeV]
Mass [MeV]

Dileptons in pion-nucleon collisions

- $\frac{\frac{d\sigma}{dM}^{\pi^- p \to n e^+ e^-}(m_\omega)}{\frac{d\sigma}{dM}^{\pi^+ n \to p e^+ e^-}(m_\omega)} \approx 4$ because of the interference
- The effect is strong if cross secton through ρ and ω are similar
- coupling constants of ω were taken from the literature, we plan to make our fit
- The same problem was studied in M.F.M. Lutz, B. Friman, M. Soyeur, Nucl. Phys. A713 (2003) 97 and A.I. Titov, B Kämpfer, EPJ A 12 (2001) 217. They had smaller ρ cross section, so the effect was strong at lower $\sqrt{s}$
- How much of this coherence survive in a nucleus?
- In a nucleus coherence is lost if one of the vector meson collides
- Will those ω's broaden which will not collide but interfere with ρ?

BUU

- $$\frac{\partial F}{\partial t} + \frac{\partial H}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{x}} - \frac{\partial H}{\partial \mathbf{x}}\frac{\partial F}{\partial \mathbf{p}} = \mathcal{C}, \quad H = \sqrt{(m_0 + U(\mathbf{p},\mathbf{x}))^2 + \mathbf{p}^2}$$

- potential: momentum dependent, soft: K=215 MeV
  $$U^{nr} = A\frac{n}{n_0} + B\left(\frac{n}{n_0}\right)^{\tau} + C\frac{2}{n_0}\int \frac{d^3p'}{(2\pi)^3}\frac{f_N(x,p')}{1+\left(\frac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2},$$

- testparticle method
  $$F = \sum_{i=1}^{N_{test}} \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t))\delta^{(4)}(p - p_i(t)).$$

- Unknown cross sections: Statistical bootstrap:
  G. Balassa, P. Kovács, Gy. Wolf, Eur. Phys. J. A54 (2018) 25,

Gy. Wolf et al., Phys.Atom.Nucl. 75 (2012) 718-720

Gy. Wolf, M. Zetenyi, Eur.Phys.J. A52 (2016) 258

M. Zetenyi, Gy. Wolf, Phys.Lett. B785 (2018) 226

Spectral equilibration

- medium effects on the spectrum of hadrons (vector mesons)
- how they get on-shell (energy-momentum conservation)
- Field theoretical method (Kadanoff-Baym equation)
  B. Schenke, C. Greiner, Phys.Rev.C73:034909,2006
- Off-shell transport
  W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
  S. Leupold, Nucl.Phys. A672 (2000) 475
- Spectral equilibration: Markov or memory effect

Off-shell transport

- Kadanoff-Baym equation for retarded Green-function
  Wigner-transformation, gradient expansion
- transport equation for $F_\alpha = f_\alpha(x,p,t)A_\alpha$
  $A(p) = -2ImG^{ret} = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \mathrm{Re}\Sigma^{ret})^2 + \frac{1}{4}\hat{\Gamma}^2},$

  W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
  S. Leupold, Nucl.Phys. A672 (2000) 475
- testparticle approximation

Transport equations

- $$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \vec{\nabla}_{P_i} Im\Sigma^{ret}_{(i)} \right]$$

  $$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \vec{\nabla}_{X_i} Im\Sigma^{ret}_{(i)} \right]$$

  $$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \frac{\partial Im\Sigma^{ret}_{(i)}}{\partial t} \right]$$

- where $C_{(i)}$ renormalization factor

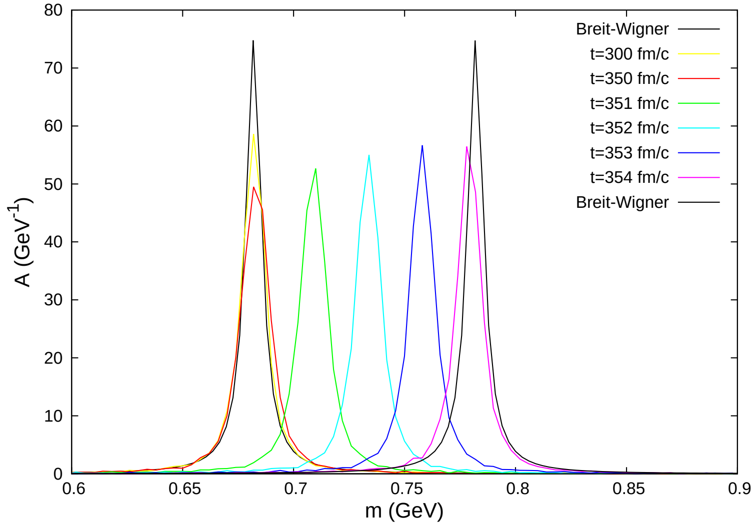
  $$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \frac{\partial}{\partial \epsilon_i} Im\Sigma^{ret}_{(i)} \right]$$

- the last equation for homogenous system can be rewritten as

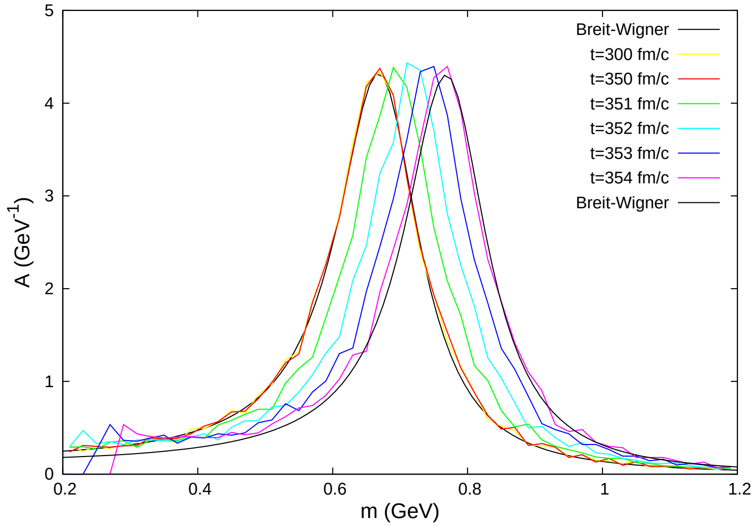
  $$\frac{dM_i^2}{dt} = \frac{d(\epsilon_i^2 - P_i^2)}{dt} = \frac{dRe\Sigma^{ret}_{(i)}}{dt} + \frac{M_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \frac{dIm\Sigma^{ret}_{(i)}}{dt}$$

Evolution of mass distribution in a box

the vector meson masses are shifted linearly with density, and change the density linearly from ρ_0 to 0 in 4 fm/c:

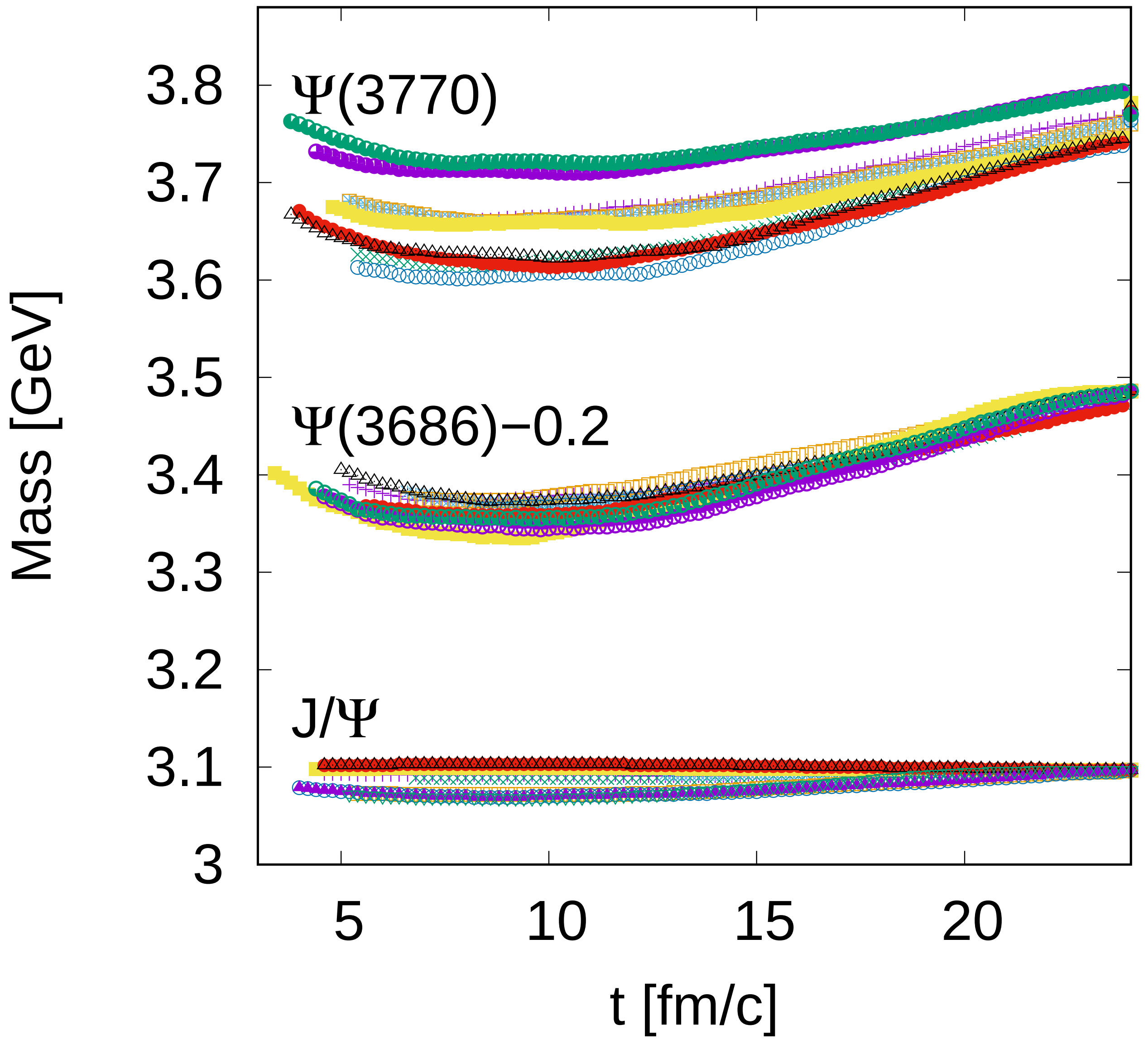


ω



ρ

Evolution of mass charmonium states in πAu 6.5 GeV



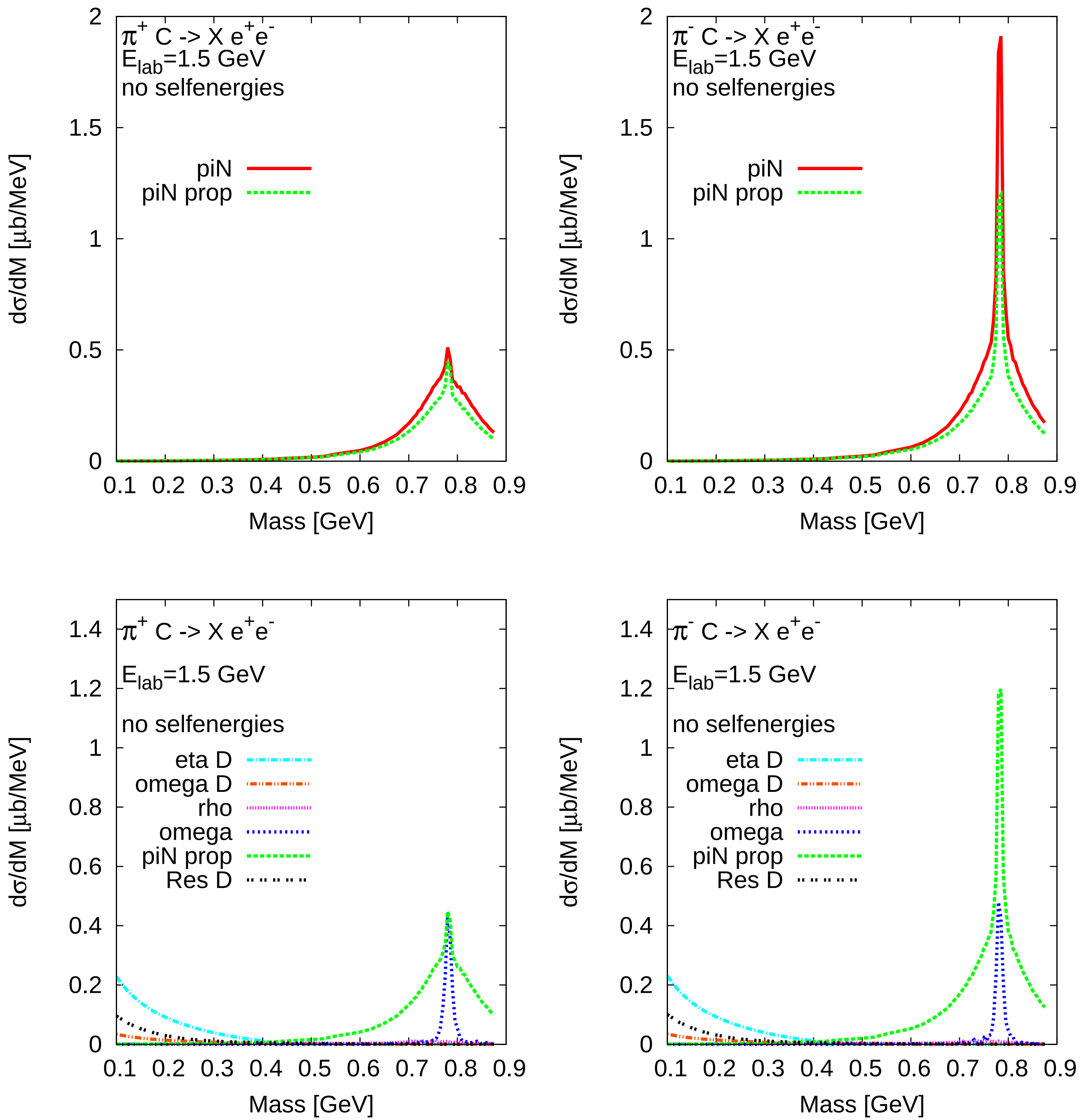
Simulation of π A collisions

- Same as usually except for $\pi N \to N e^+ e^-$
- in case of a πN collision several "doublets" are created. (The original π and N do not change their state.) A doublet consists of 2 perturbative particles ρ and ω with their cross sections and the "cross section" of the interference term. ρ and ω are created with the same position, momentum and mass.
- They propagate, decay and can be absorbed. The interference term contribute to the "decays".
- Propagation: perturbative ρ's and ω's propagate in the surounding medium
- Absorption: ρ's and ω's can be absorbed by a nucleon

Decays

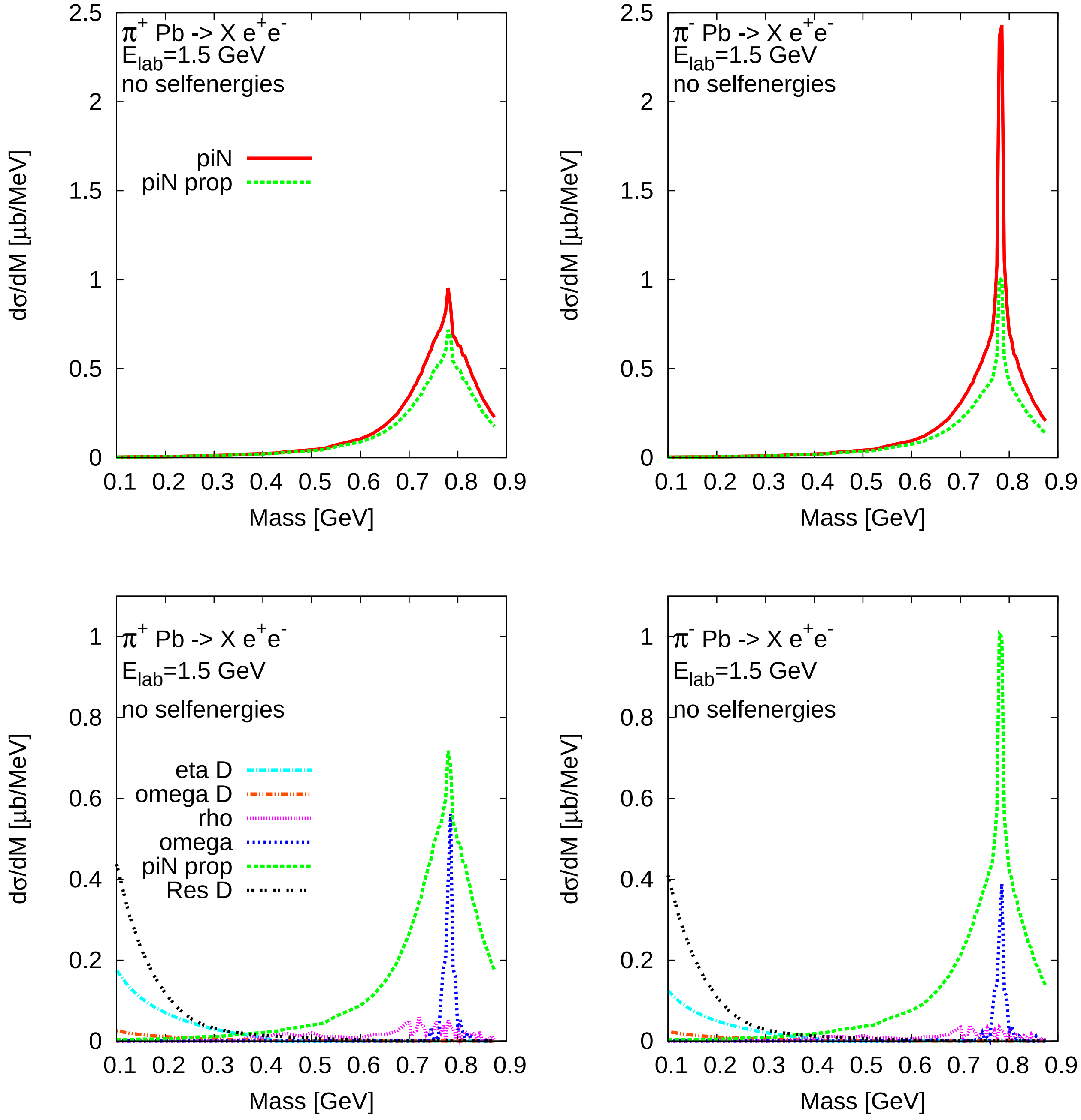
- Denote the probability that the ρ and ω decay in the ith time step as α_i and β_i, respectively.
- $\sum \alpha_i \leq 1$. At the end it is 1 if not absorbed. The same is for ω.
- In the nth timestep the ρ contribution to the dilepton yield: $\alpha_n \, \sigma^{\pi N \to N\rho_0 \to Ne^+e^-} \left(\approx \alpha_n \, \Gamma_\rho^{Ne^+e^-} / \Gamma_\rho^{tot} \, \sigma^{\pi N \to N\rho_0} \right)$. Similarly for ω.
  The contribution of the interference term is:
  $\sum_{ij} \alpha_i \beta_j \, \sigma^{\pi N \to N\rho - \omega \to Ne^+e^-}$.
  In vacuum it reproduces the original cross section.

π C, 1.5 GeV



π+ C -> X e+e-
Elab=1.5 GeV
no selfenergies
piN
piN prop
π- C -> X e+e-
Elab=1.5 GeV
no selfenergies
piN
piN prop
π+ C -> X e+e-
Elab=1.5 GeV
no selfenergies
eta D
omega D
rho
omega
piN prop
Res D
π- C -> X e+e-
Elab=1.5 GeV
no selfenergies
eta D
omega D
rho
omega
piN prop
Res D
dσ/dM [μb/MeV]
Mass [GeV]
0
0.5
1
1.5
2
0.2
0.4
0.6
0.8
1.2
1.4
0.1
0.3
0.5
0.7
0.9

π Pb, 1.5 GeV


π+ Pb -> X e+e-
Elab=1.5 GeV
no selfenergies
piN
piN prop
dσ/dM [μb/MeV]
Mass [GeV]
π- Pb -> X e+e-
Elab=1.5 GeV
no selfenergies
eta D
omega D
rho
omega
piN prop
Res D

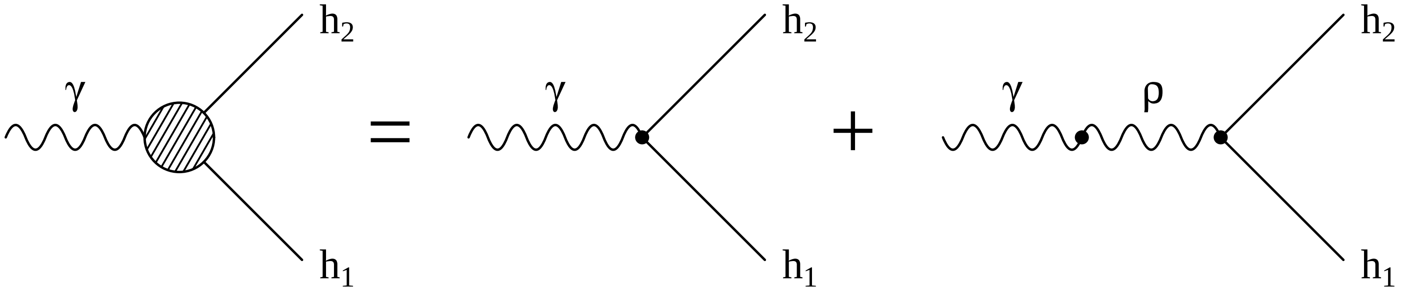
Dileptons in pion-nucleus collisions

- $$\frac{\frac{d\sigma}{dM}^{\pi^- p \to n e^+ e^-}(m_\omega)}{\frac{d\sigma}{dM}^{\pi^+ n \to p e^+ e^-}(m_\omega)} \approx 4$$
  $$\frac{\frac{d\sigma}{dM}^{\pi^- C^{12} \to X e^+ e^-}(m_\omega)}{\frac{d\sigma}{dM}^{\pi^+ C^{12} \to X e^+ e^-}(m_\omega)} \approx 2.9$$
  $$\frac{\frac{d\sigma}{dM}^{\pi^- Pb^{207} \to X e^+ e^-}(m_\omega)/N_p}{\frac{d\sigma}{dM}^{\pi^+ Pb^{207} \to X e^+ e^-}(m_\omega)/N_n} \approx 2.0$$
  In case of complete decoherence these ratios should be 1.
- Experimentally the decoherence can be observed in strongly interacting matter.

Summary

- Dilepton production in πN and πA an unique way to study quantum interference inside strongly interacting matter by measuring on nucleon, on light and on heavy nuclei.
- Make own fit to vector meson production (including the resonances and their interference)

Vector meson photon coupling



- $\mathcal{L}_{VDM1} = -\frac{em_\rho^2}{g_\rho}\rho^0_\mu A^\mu$
  The width of $R \to N\gamma$ and $R \to N\rho$ are not independent
  photons from ρ (ρ-width taken from PDG) overestimate the γ-width (also photon gets a mass, not gauge invariant; can be cured)

- $\mathcal{L}_{VMD2} = -\frac{e}{2g_\rho}F^{\mu\nu}\rho^0_{\mu\nu}$
  From ρ-width the contribution to the photonic decay can be obtained by multiplying it with $\frac{e}{g_\rho}\frac{k^2}{m_\rho^2 - k^2 - iq\Gamma_\rho(k^2)}$
  Decay through ρ does not contribute to the real photonic width.
  We use VMD2. The final result depend on the choice, the ratio: M_{dil}^2/m_ρ^2

Form factors

$$\mathcal{M}_i^\mu \longrightarrow \mathcal{M}_i^\mu \frac{\Lambda_i^4}{\Lambda_i^4 + (i - m_i^2)^2} \equiv \mathcal{M}_i^\mu F_i \qquad i = s, u, t$$

Add an extra term (combination of the 3 form factors) to keep gauge invariance

$$\hat{F} = F_s + F_u + F_t - F_s F_u - F_s F_t - F_u F_t + F_s F_u F_t$$

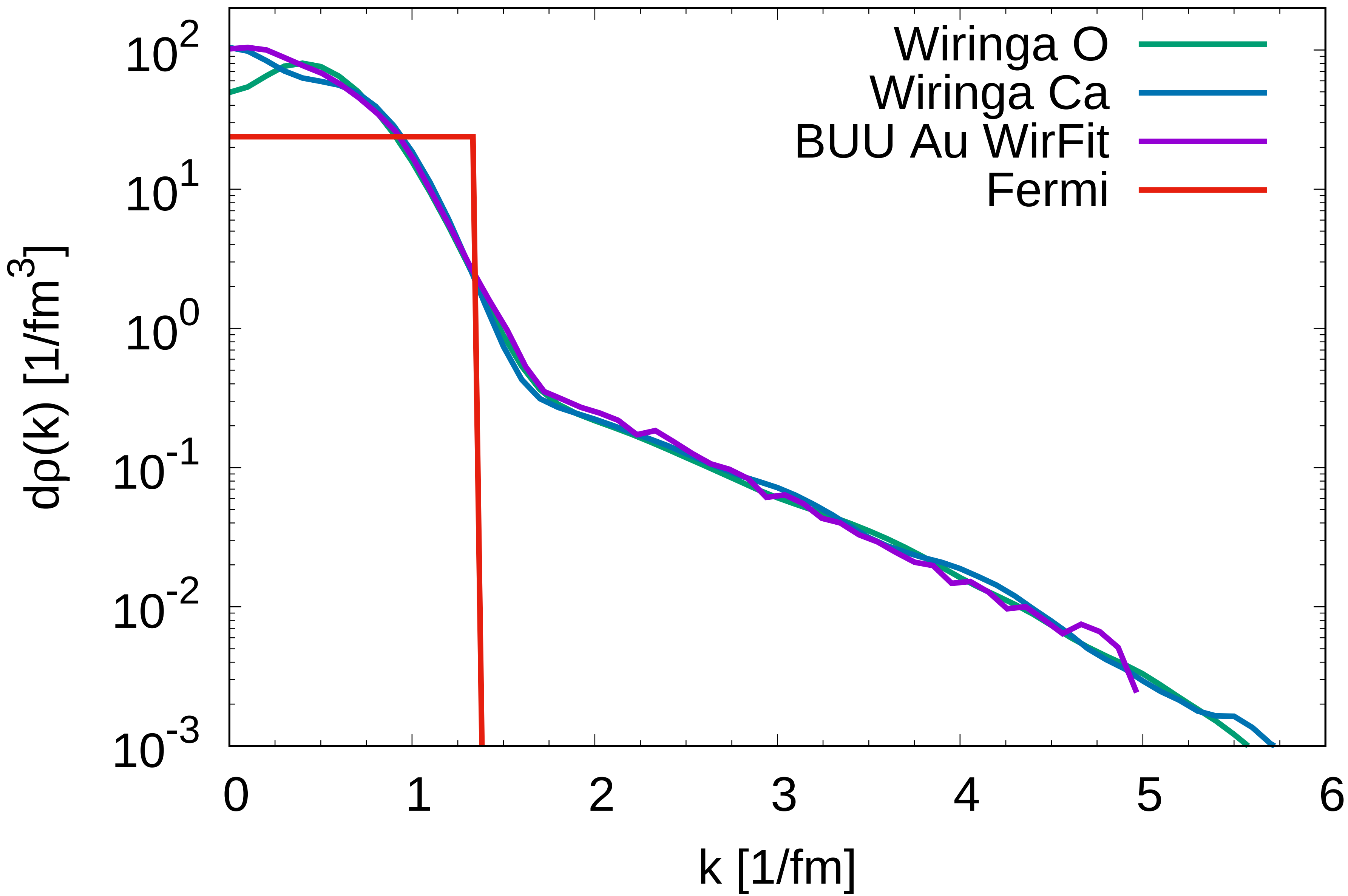
$$\Delta\mathcal{M}_s^\mu = (\hat{F} - F_s) C_s \gamma_5 \frac{2p_f^\mu + k^\mu}{s - m_N^2}$$

The coefficient of $\hat{F} - F_s$ is chosen so, that multiplied with k_μ it should be equal to $\mathcal{M}_s^\mu k_\mu$

$\Delta\mathcal{M}_s^\mu$ has no pole, it can be generated by a contact term

$$(\hat{F} - F_s) \sim (1 - F_s) \sim s - m_N^2$$

Initial momentum distribution



Cross sections

Elastic baryon-baryon cross section is fitted to the elastic pp data

Meson absorption cross sections are given by

$$\sigma_{\pi N \to R} = \frac{4\pi}{p^2} (spin factors) \frac{\Gamma_{in} \Gamma_{tot}}{(s - m_R^2) + s\Gamma_{tot}^2}$$

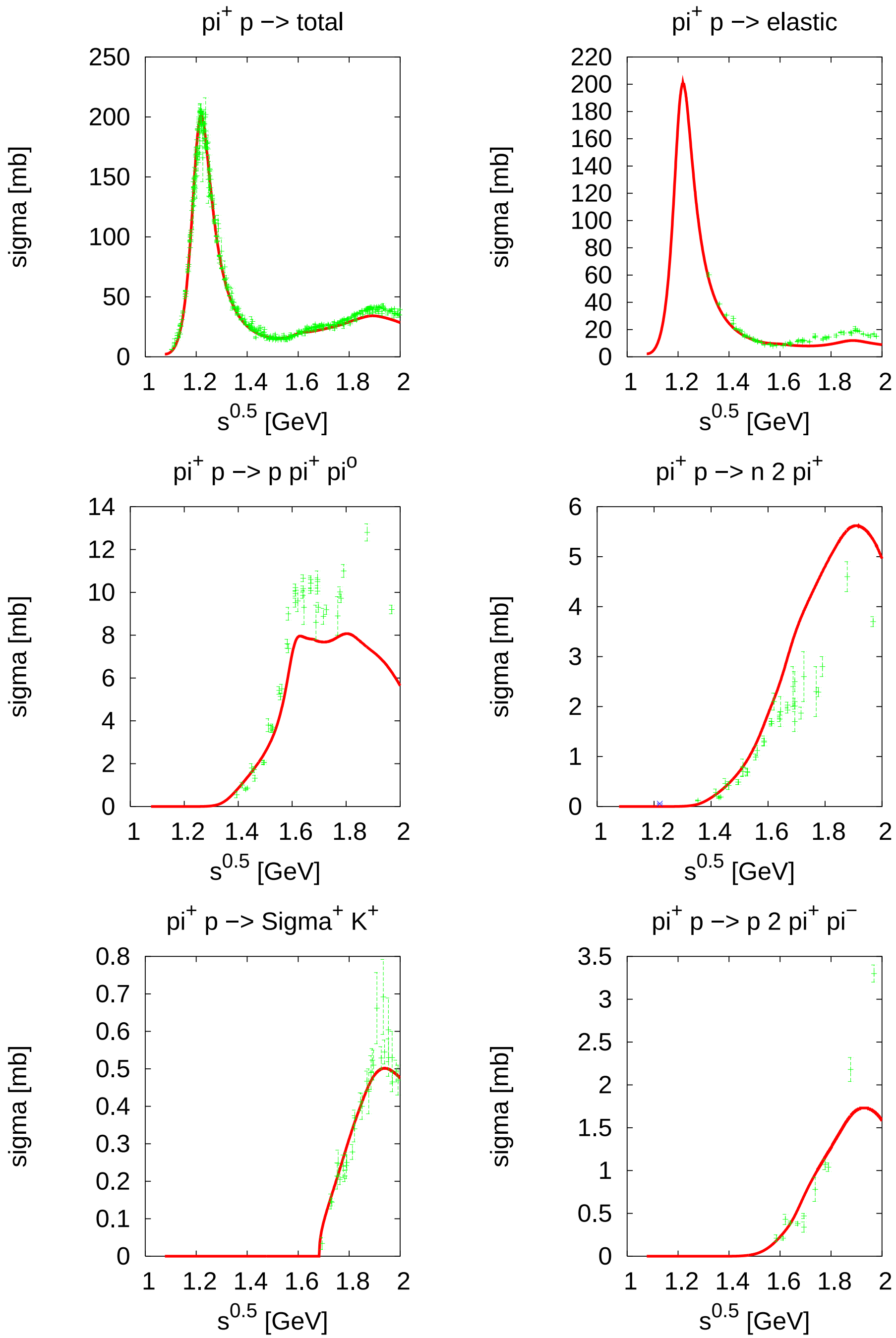
Baryon resonance parameters: mass, width, branching ratios are fitted by describing the meson production channels in πN collisions:

$$\sigma_{\pi N \to NM} = \sum_R \sigma_{\pi N \to R} \frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

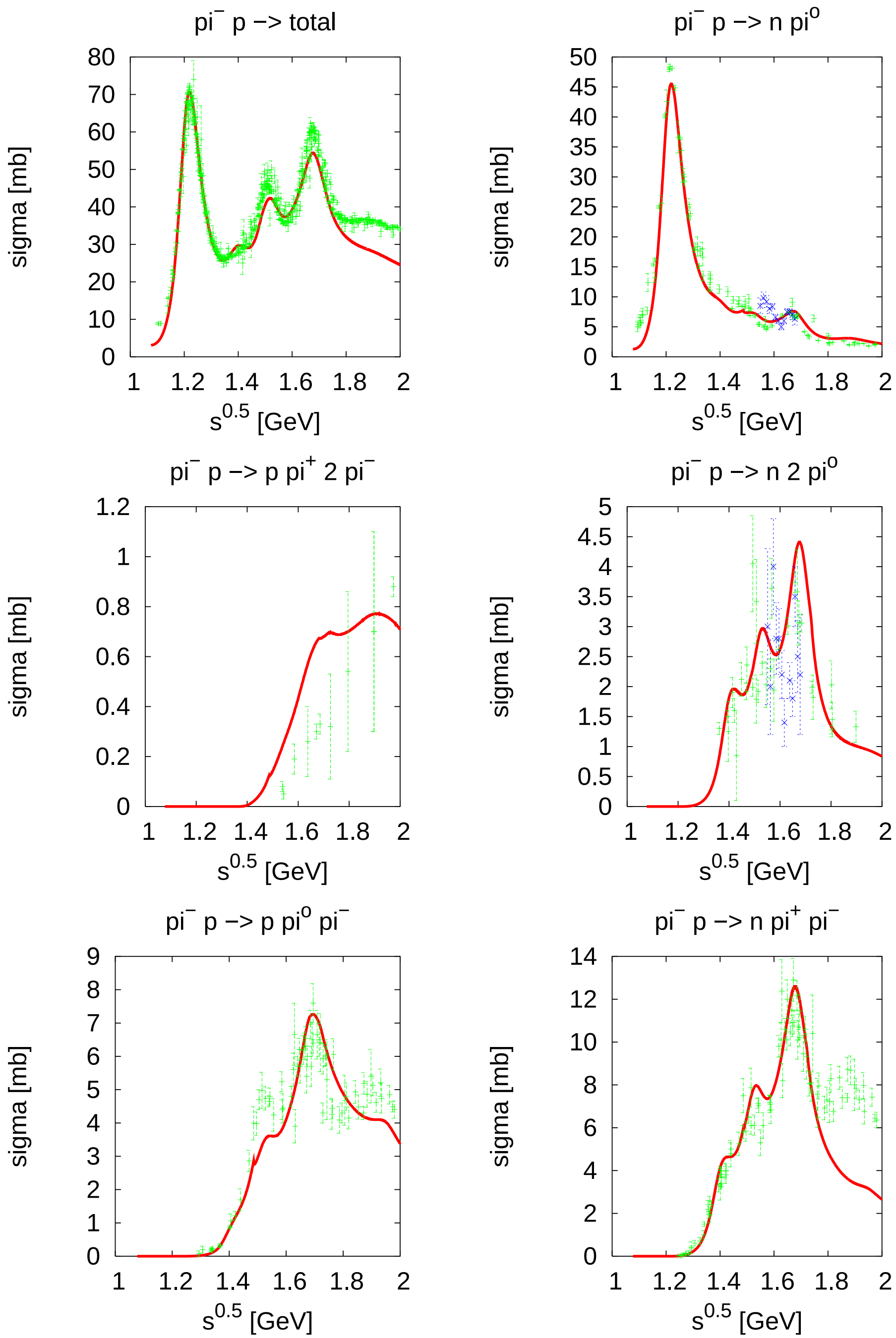
Resonance production cross section $NN \to NR$ is given by the fit of

$$\sigma_{NN \to NM} = \sum_R \sigma_{NN \to NR} \frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

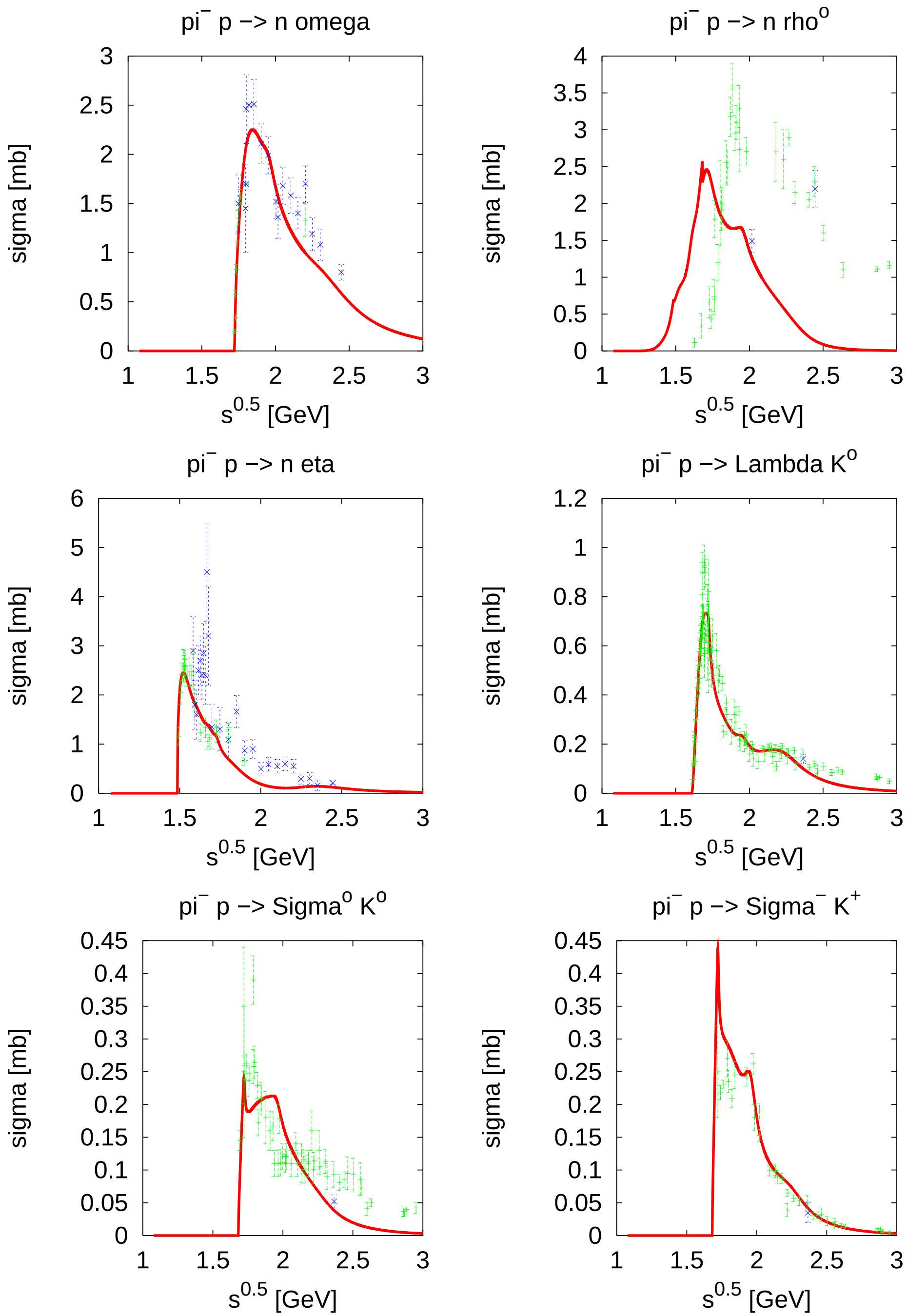
27 baryons, 6 mesons. Fit is done by the Minuit package (CERN)



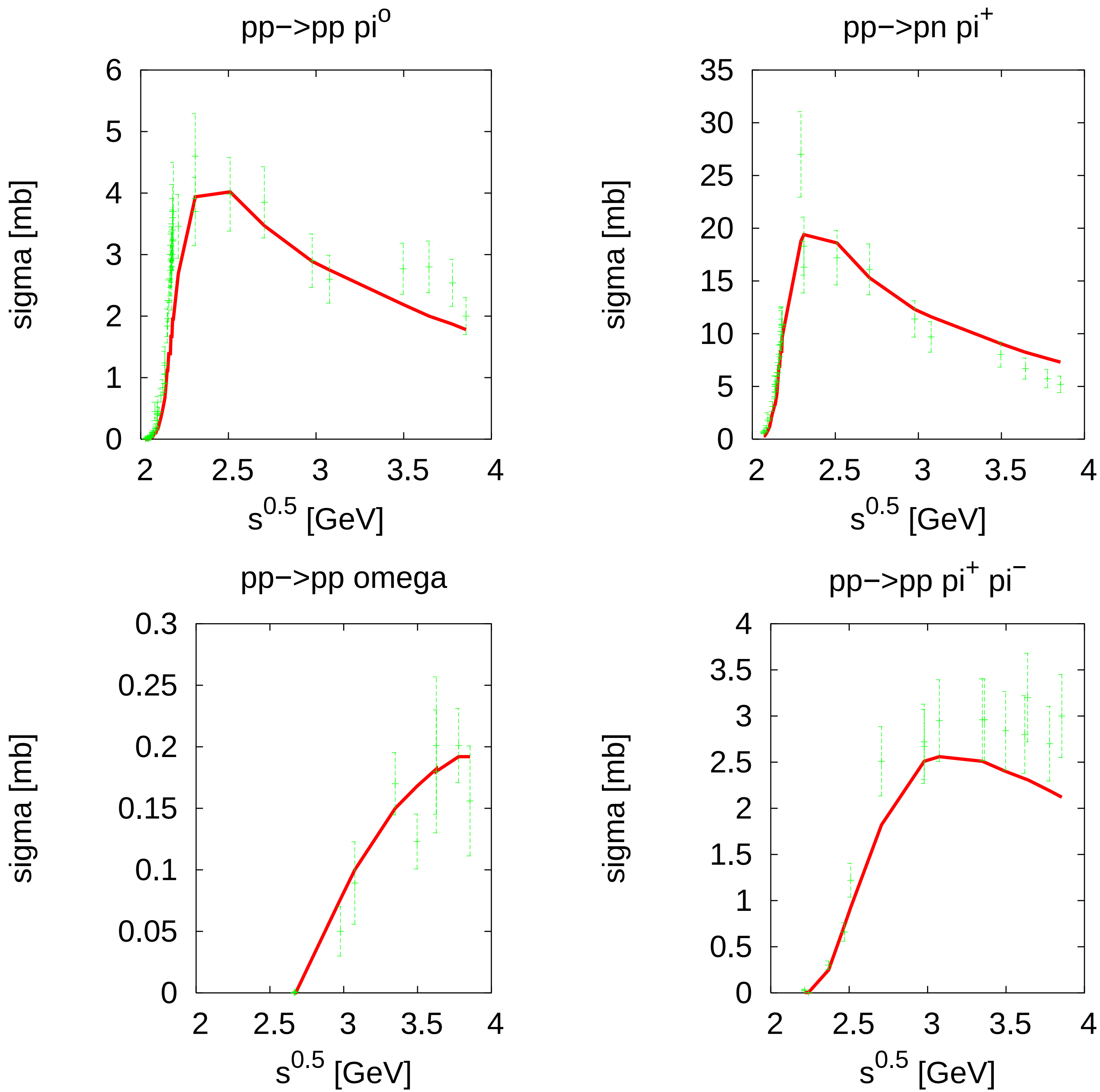
pi+ p −> total
250
200
150
100
50
0
sigma [mb]
1 1.2 1.4 1.6 1.8 2
s0.5 [GeV]
pi+ p −> elastic
220
200
180
160
140
120
100
80
60
40
20
0
sigma [mb]
1 1.2 1.4 1.6 1.8 2
s0.5 [GeV]
pi+ p −> p pi+ pio
14
12
10
8
6
4
2
0
sigma [mb]
1 1.2 1.4 1.6 1.8 2
s0.5 [GeV]
pi+ p −> n 2 pi+
6
5
4
3
2
1
0
sigma [mb]
1 1.2 1.4 1.6 1.8 2
s0.5 [GeV]
pi+ p −> Sigma+ K+
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
sigma [mb]
1 1.2 1.4 1.6 1.8 2
s0.5 [GeV]
pi+ p −> p 2 pi+ pi−
3.5
3
2.5
2
1.5
1
0.5
0
sigma [mb]
1 1.2 1.4 1.6 1.8 2
s0.5 [GeV]



pi⁻ p −> total
pi⁻ p −> n pi⁰
pi⁻ p −> p pi⁺ 2 pi⁻
pi⁻ p −> n 2 pi⁰
pi⁻ p −> p pi⁰ pi⁻
pi⁻ p −> n pi⁺ pi⁻
sigma [mb]
s^0.5 [GeV]



pi⁻ p -> n omega
pi⁻ p -> n rho⁰
pi⁻ p -> n eta
pi⁻ p -> Lambda K⁰
pi⁻ p -> Sigma⁰ K⁰
pi⁻ p -> Sigma⁻ K⁺
sigma [mb]
s^0.5 [GeV]



pp−>pp pi^o
sigma [mb]
s^0.5 [GeV]
pp−>pn pi^+
sigma [mb]
s^0.5 [GeV]
pp−>pp omega
sigma [mb]
s^0.5 [GeV]
pp−>pp pi^+ pi^−
sigma [mb]
s^0.5 [GeV]

Kadanoff-Baym Equation

Schwinger-Dyson equation:

$$G = G_0 + G_0 \Sigma G$$

$$\begin{aligned}
G^{11}(1,2) &= G^{T}(1,2) = \langle T(\phi(1)\phi(2))\rangle \\
G^{22}(1,2) &= G^{AT}(1,2) = \langle \tilde{T}(\phi(1)\phi(2))\rangle \\
G^{21}(1,2) &= G^{<}(1,2) = \langle \phi(2)\phi(1)\rangle \\
G^{12}(1,2) &= G^{>}(1,2) = \langle \phi(1)\phi(2)\rangle \\
G^{r}(1,2) &= \theta(t1-t2)(G^{>}(x1,t1;x2,t2) - G^{<}(x1,t1;x2,t2)) \\
G^{a}(1,2) &= \theta(t2-t1)(G^{<}(x1,t1;x2,t2) - G^{>}(x1,t1;x2,t2))
\end{aligned}$$

After some manipulation: *Kadanoff-Baym equation*:

$$\begin{aligned}
(i\hbar\partial_{t1} - H_0(1))G^{<}(1,2) &= \int d3\, \Sigma^{r}(1,3)G^{<}(3,2) + \int d3\, \Sigma^{<}(1,3)G^{a}(3,2) \\
(i\hbar\partial_{t1} - H_0(1))G^{r}(1,2) &= \delta^{4}(1,2) + \int d3\, \Sigma^{r}(1,3)G^{r}(3,2)
\end{aligned}$$

Wigner-transformation

- Retarded propagator is not a distribution function
- Wigner transform:

$$r = x1 - x2 \qquad , \qquad R = x1 + x2$$

R (center of mass) dependence of propagators and selfenergies are weaker than the r dependence

$$G^r(R,P) = \int d^4r\, G^r(X+r, X-r)$$

- Gradient expansion in r. Neglect all terms with more than one derivative in R
- transport equation for $F_\alpha = iG^<(R,P) = f_\alpha(x,p,t)A_\alpha$

$A(p) = -2ImG^r = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \mathrm{Re}\Sigma^r)^2 + \frac{1}{4}\hat{\Gamma}^2},$

Cassing, Juchem (2000) and Leupold (2000)

- testparticle approximation