Exact Schwinger mechanism for scalar fields

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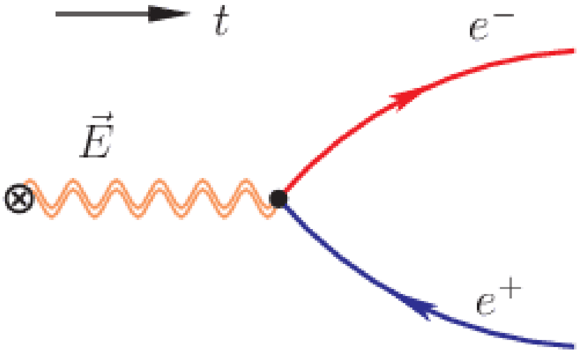
Introduction

- Schwinger mechanism is a nonperturbative effect where electron-positron pairs are created from the vacuum in strong electric field
- We will examine a similar model with scalar fields coupled to a time independent classical source, which is exactly solveable
- Historically believed to not have scattering in this model, but we find scattering due to the vacuum decay



Model and eom

We model a free real scalar field coupled to a static classical source:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + g\rho(\mathbf{x})\,\phi.$$

Solutions to e.o.m. can be computed exactly

$$\phi(x) = \phi_0(x) + ig\int_{t_0}^{t} dy^0 \int d^3y\;\rho(\mathbf{y}) \int \frac{d^3p}{(2\pi)^3 2E_{\mathbf{p}}}\left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right).$$

It has the form "free part + Green's function".

Field and Hamiltonian

$$h(\mathbf{p}) = g \frac{-1}{\sqrt{2E_{\mathbf{p}}^3}} \int d^3y \, e^{-i\mathbf{p}\cdot\mathbf{y}} \rho(\mathbf{y}),$$

$$f(t, \mathbf{p}) = g \frac{e^{iE_{\mathbf{p}}t} - 1}{\sqrt{2E_{\mathbf{p}}^3}} \int d^3y \, e^{-i\mathbf{p}\cdot\mathbf{y}} \rho(\mathbf{y}),$$

The field can be written in the compact way

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}} \left[\Big(a_{\mathbf{p}} + f(t, \mathbf{p}) \Big) e^{-ip\cdot x} + \Big(a_{\mathbf{p}}^\dagger + f^*(t, \mathbf{p}) \Big) e^{ip\cdot x} \right].$$

We can see the time dependent ladder operators $a_{\mathbf{p}}(t) = a_{\mathbf{p}} + f(t, \mathbf{p})$

$$H = \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}} \left[\Big(a_{\mathbf{p}}^\dagger(0) + h^*(\mathbf{p}) \Big) \Big(a_{\mathbf{p}}(0) + h(\mathbf{p}) \Big) - |h(\mathbf{p})|^2 \right] .$$

Different ladder operators in the Hamiltonian and field. No mutual eigenstate of particle number and Hamiltonian exists (for all times).

Vacuum decay probability

Initial vacuum $|0\rangle$ gains a nonzero particle number expectation value after time t

$$N_0(t) = \int \frac{d^3p}{(2\pi)^3} \langle 0| a^\dagger_{\mathbf{p}}(t) a_{\mathbf{p}}(t) |0\rangle = 2 \int \frac{d^3p}{(2\pi)^3} \Big(1 - \cos(E_{\mathbf{p}} t)\Big) |h(\mathbf{p})|^2 \,.$$

Eigenstates of the Hamiltonian form a basis so we can insert them with the identity in the expression

$$P_{0\to n}(t) = \int \frac{d^3p_1 \cdots d^3p_n}{(2\pi)^{3n} 2E_{\mathbf{p}_1} \cdots 2E_{\mathbf{p}_n}} |\langle \mathbf{p}_1, \cdots, \mathbf{p}_n| \, U(t,0) \, |0\rangle|^2$$

Result is a Poisson distribution with time dependent particle number expectation value

$$P_{0\to n}(t) = \frac{e^{-N_0(t)}}{n!} \Big[N_0(t)\Big]^n$$

Small and large time behaviour

Small time behaviour slower than exponential decay.

$$P_{0\to 0}(t) = \exp\left\{-2\int \frac{d^3q}{(2\pi)^3}\Big(1-\cos(E_{\mathbf{q}}t)\Big)\,|h(\mathbf{q})|^2\right\} \sim e^{-\lambda t^2}$$

With N repeated measurements, survival prob. after $T = Nt$

$$P_{0\to 0}(T) = (P_{0\to 0}(t))^N \approx \exp\left[-N\lambda t^2\right] = \exp\left[-\frac{\lambda}{N}T^2\right] \xrightarrow{N\to\infty} 1$$

Zeno effect: initial state is preserved by repeated measurement for as long as we want

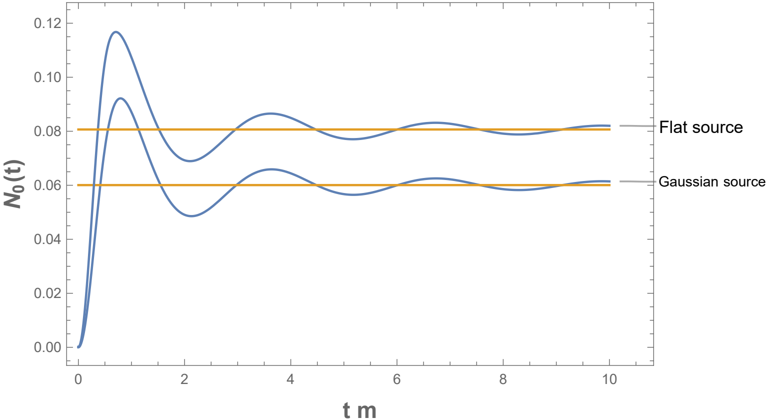
For large time, the oscillation term dies out (Riemann-Lebesgue):

$$\lim_{t\to\infty} N_0(t) = 2\int \frac{d^3p}{(2\pi)^3}\,|h(\mathbf{p})|^2 < \infty.$$

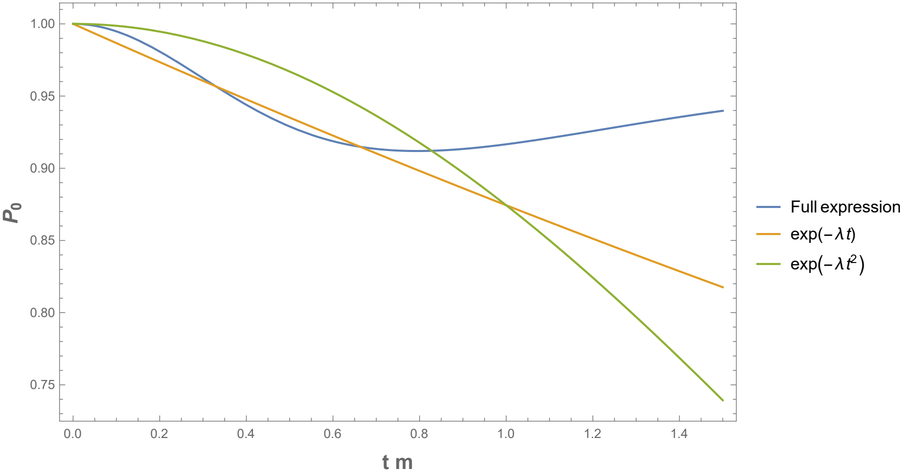
Numerical examples

$$\rho_1(\mathbf{x}) = \exp\left(-\pi \frac{\mathbf{x}^2}{L^2}\right),$$

$$\rho_2(\mathbf{x}) = \theta\left(\frac{L}{2} - x\right)\theta\left(\frac{L}{2} + x\right)\dots$$



Small time behaviour II



- Vacuum survival probability and some approximations for the Gaussian shaped source

Large volume limit

The particle density distribution is well-defined for large volume

$$\lim_{L\to\infty}\lim_{t\to\infty}\frac{1}{L^3}N_0(t) = 2\frac{g^2}{m^3} \qquad \text{(flat source)},$$

$$\lim_{L\to\infty}\lim_{t\to\infty}\frac{1}{L^3}N_0(t) = \frac{1}{\sqrt{2}}\frac{g^2}{m^3} \qquad \text{(Gaussian source)}.$$

Different prefactor depending on shape; the tails matter (somewhat)

One particle Scattering

With initial condition

$$|\psi_1\rangle = \int \frac{d^3k}{(2\pi)^3 2E_{\mathbf{k}}}\, \psi_1(\mathbf{k})\, |\mathbf{k}\rangle,$$

and the n-particle partial wave of the time evolving vacuum

$$\psi_0(t; \mathbf{p}_1, \cdots, \mathbf{p}_n) = \langle \mathbf{p}_1, \cdots, \mathbf{p}_n |\, U(t,0)\, |0\rangle$$

Vacuum decay causes dynamics in scattering

$$\langle \mathbf{p}_1 \cdots \mathbf{p}_n |\, U(t,0)\, |\psi_1\rangle = -\psi_0(t; \mathbf{p}_1 \cdots \mathbf{p}_n) \int \frac{d^3k}{(2\pi)^3 \sqrt{2E_{\mathbf{k}}}}\, f^*(t,\mathbf{k})\, \psi_1(\mathbf{k})$$

$$+ \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \left[\psi_1(\mathbf{p}_j)\, e^{-iE_{\mathbf{p}_j} t}\, \psi_0(t; \mathbf{p}_1, \cdots \mathbf{p}_{j-1}, \mathbf{p}_{j+1}, \cdots, \mathbf{p}_n) \right].$$

First term: absorption and vacuum decay into n particles,
Second term: free propagation and vacuum decay into $n-1$ particles

Conclusion

- Contrary to what was thought historically, we find scattering of which a key component is the vacuum decay
- Past arguments rely on the adiabatic switching off. This changes the algebra of the operators, even if done at infinity, which gives different results than the exact calculation
- The vacuum decay is conceptually similar to the Schwinger mechanism. The exact calculations show qualitative differences to the expected result, such as a saturation of particle number, non-exponential decay and Zeno effect
- Vacuum decay might be a relevant non-perturbative production mechanism of the dilaton, whose production is suppressed at the tree level

Thank you for your attention!