A THEORY OF EXOTIC HADRONS?

AD POLOSA, SAPIENZA UNIVERSITY OF ROME

In collaboration with A. Esposito, D. Germani, A. Glioti, B. Grinstein, L. Maiani, R. Rattazzi

BABAR AND EXOTIC SPECTROSCOPY

(From a longer list of contributions)

- $e^+e^- \to \gamma_{\mathrm{ISR}}\,(\pi^+\pi^- J/\psi)$ found a vector at $M = 4.26$ GeV and $\Gamma = 70 \pm 20$ MeV.
  [PRL 95 (2005) 142001 cit. 1013]

- $B^- \to K^- X(3872) \to K^-\,(\pi^+\pi^- J/\psi)$ finding the mass $M = 3873.4 \pm 1.4$ MeV.
  [PRD 71 (2005) 071103 cit. 748]

- $X(3872) \to \psi(2S)\gamma$ and $X(3872) \to \psi(1S)\gamma$ presenting for the first time
  $\mathscr{R} = \mathscr{B}(2S)/\mathscr{B}(1S) = 3.4 \pm 1.4$
  [PRL 102 (2009) 132001 cit. 312]

- $B^+ \to K^+ \underbrace{(\pi^+\pi^-\pi^0 J/\psi)}_{X \to \omega J/\psi}$ but needed $\ell = 1$ giving a preference to $J^P = 2^-$ for the X.
  [PRD 82 (2010) 011101 cit. 302]

THE $X(3872)$ IDENTIKIT TODAY

$$X^0(3872) \qquad J^{PC} = 1^{++}$$

$$M = 3871.65 \pm 0.06 \text{ MeV} \simeq M_D + M_{D^*}$$
$$M = 3871.65 \pm 0.06 \text{ MeV} \simeq M_{J/\psi} + M_\rho$$

$$\Gamma = 1.19 \pm 0.21 \text{ MeV}$$

$\pi^+\pi^- J/\psi$	$\mathcal{B} = 3.8 \pm 1.2\,\%$
$\omega J/\pi$	$\mathcal{B} = 4.3 \pm 2.1\,\%$
$D^0\bar{D}^0\pi^0$	$\mathcal{B} = 49^{+18}_{-20}\,\%$
$D^0\bar{D}^{*0}$	$\mathcal{B} = 37 \pm 9\,\%$
$\pi^0\chi_{c1}(1P)$	$\mathcal{B} = 3.4 \pm 1.6\,\%$

A FEW OTHER RESONANCES

Central mass values and widths in MeV

$X(3872)$	$Z_c^{0\pm}(3900)$	$Z_c^{0\pm}(4020)$	$Z_b^{0\pm}(10610)$	$Z_b^{0\pm}(10650)$
$\Gamma = 1$	$\Gamma = 28$	$\Gamma = 13$	$\Gamma = 18.4$	$\Gamma = 11.5$
$D^0\bar{D}^{*0}$	$D^0\bar{D}^{*0\pm}$	$D^{*0}\bar{D}^{*0\pm}$	$B^0\bar{B}^{*0\pm}$	$B^{*0}\bar{B}^{*0\pm}$
$\delta \approx 0$	$+15$	$+10, +7$	$+3$	$+2$

(MeV)

THE RADIATIVE DECAYS OF $X(3872)$

$$\mathscr{R} \equiv \frac{\mathscr{B}(X \to \gamma\, \psi(2S))}{\mathscr{B}(X \to \gamma\, \psi(1S))} \simeq 2.6 \pm 0.6 \quad \text{(PDG Ave.)}$$

The phase space ratio $\Phi(2S)/\Phi(1S) \simeq 0.26$ would favor a small $\mathscr{R}$.

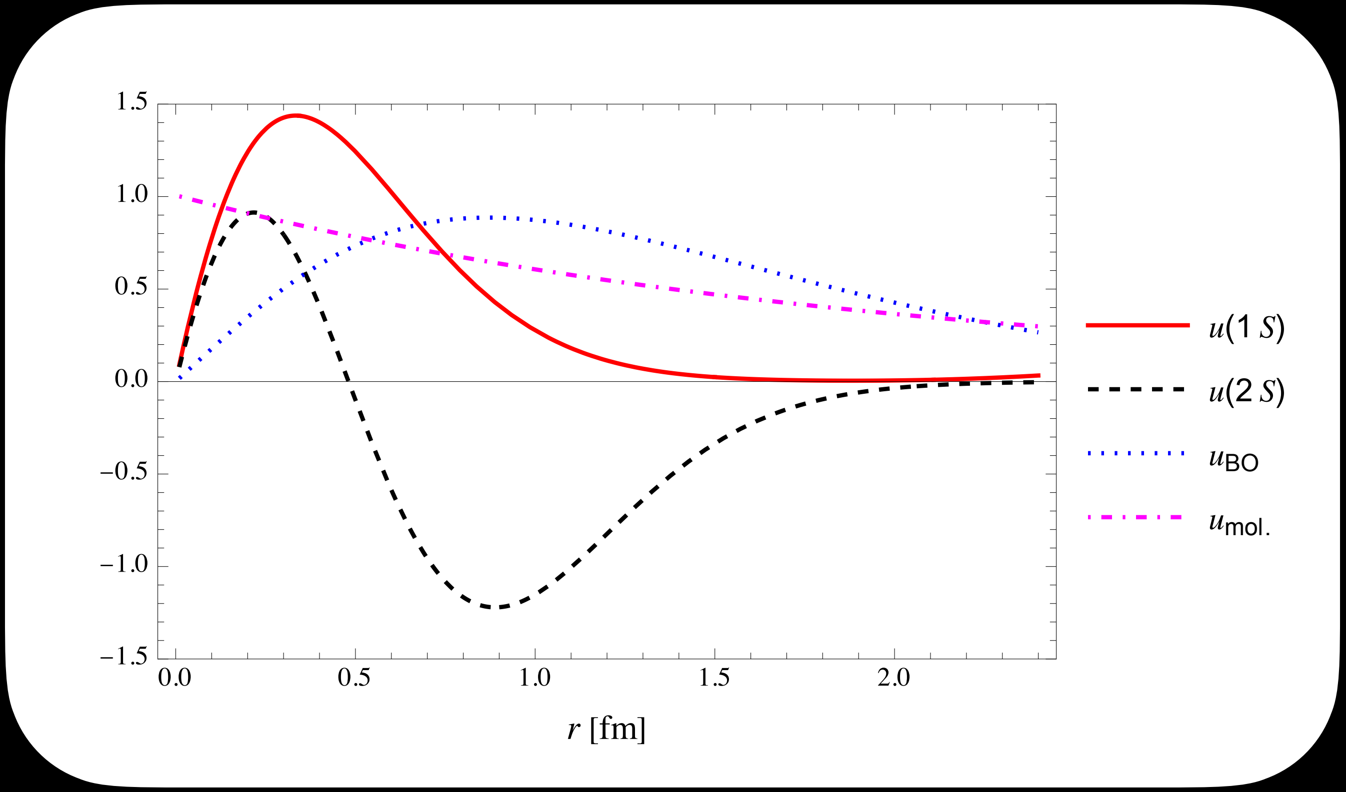
We distinguish between a **compact** $c\bar{c}q\bar{q}$ and a **molecular** $D\bar{D}^*$ interpretation.

We find that $\mathscr{R}$ predicted in the compact case is (at least) 30 times larger, $\mathscr{R} \gtrsim 1$, than that predicted for a molecule, $\simeq 0.04$.

B. Grinstein, L. Maiani, A.D.P., 2401.11623

COMPARING B.O. WF TO CHARMONIA

The universal wavefunction in the molecular picture amplifies small distances enhancing the J/ψ wrt $\psi(2S)$.



Reduced wavefunctions $u(r) = r\,R_{n0}(r)$

B. Grinstein, L. Maiani, A.D.P., 2401.11623

SCATTERING AMPLITUDE POLES

Motion of a particle m in $V(r)$ which vanishes at infinity more rapidly than $\exp(-c\,r)$: there are no singularities of the scattering amplitude as a function of E on the **physical sheet** other than *simple poles* in correspondence of discrete energy levels (bound states) say $E = -B$. It is found that **near the point** $E = -B$ the principal term in f has the form

$$f = -\frac{A_0^2}{2m}\frac{1}{E+B}$$

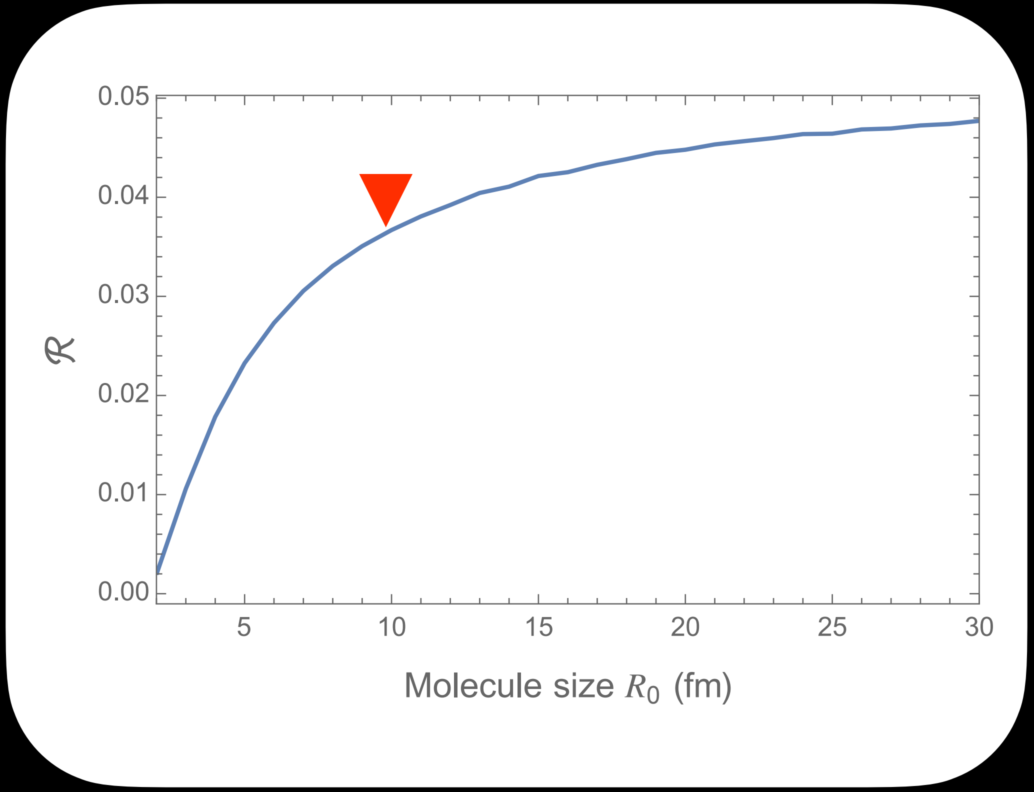
with the **normalized** reduced wf given by

$$\chi = A_0 \exp(-r\sqrt{2mB})$$

So the residue of the scattering amplitude f at some discrete level is $\propto$

$$A_0 = (4 \cdot 2mB)^{1/4}$$

We find in any case $\mathscr{R}_{\text{compact}} > \mathscr{R}_{\text{mol.}}$ with $\mathscr{R}_{\text{compact}} \gtrsim 1$

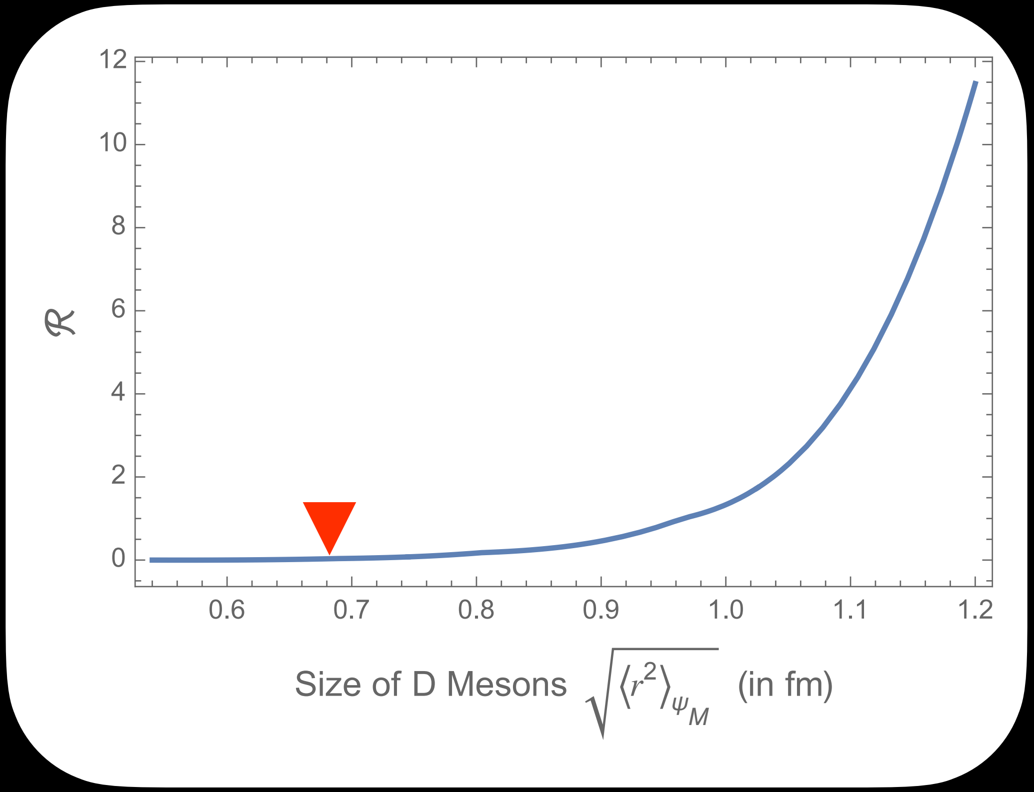


$$R_0 = 1/\sqrt{2mB}$$

B. Grinstein, L. Maiani, A.D.P., to appear soon.

THE RADIATIVE DECAYS OF $X(3872)$

How to overturn the molecule situation: $\mathscr{R}$ is a rapidly increasing function of the size of $D^{(*)}$ (here $\Psi_{\text{mol.}}$ is used).

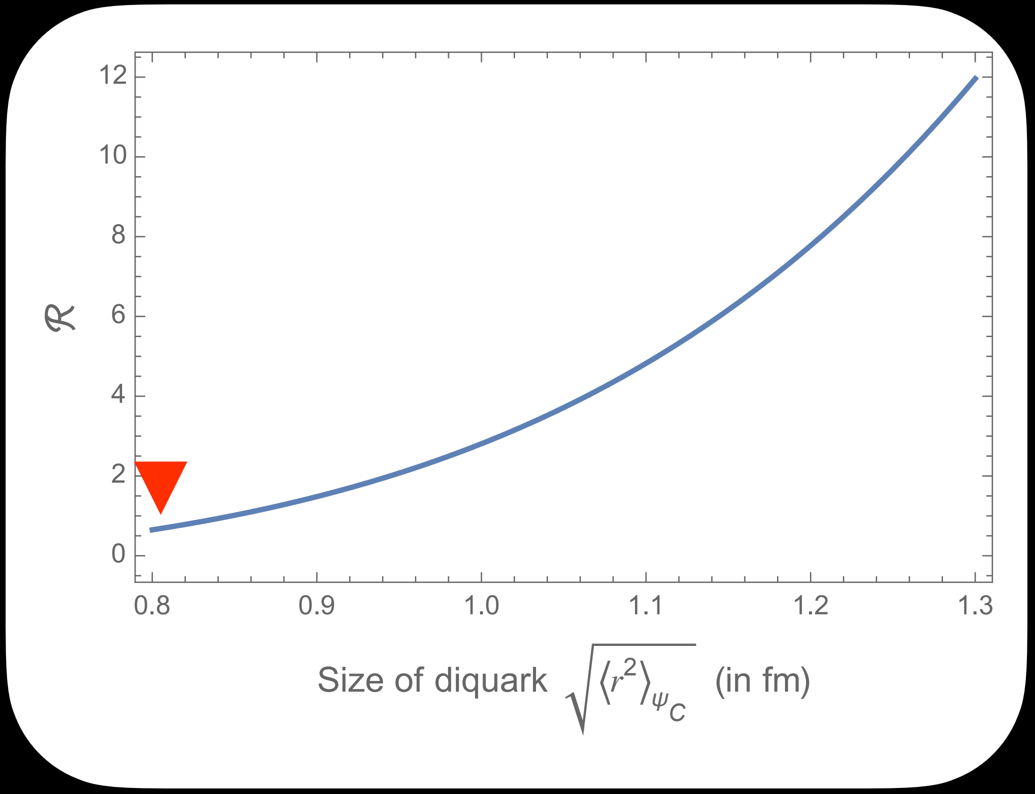


$$\chi_{\text{M}}(r) = \frac{b^{3/2}}{\pi^{3/4}} e^{-\frac{1}{2}b^2 r^2}$$

B. Grinstein, L. Maiani, A.D.P., 2401.11623
Isgur, Scora, Grinstein, Wise (ISGW-model)

THE RADIATIVE DECAYS OF $X(3872)$

On the other hand the diquark in the compact tetraquarks tends to be larger than a D or a $\bar{D}^*$ meson since the binding force in the diquark is weaker! (here $\Psi_{\rm BO}(r)$ is used).



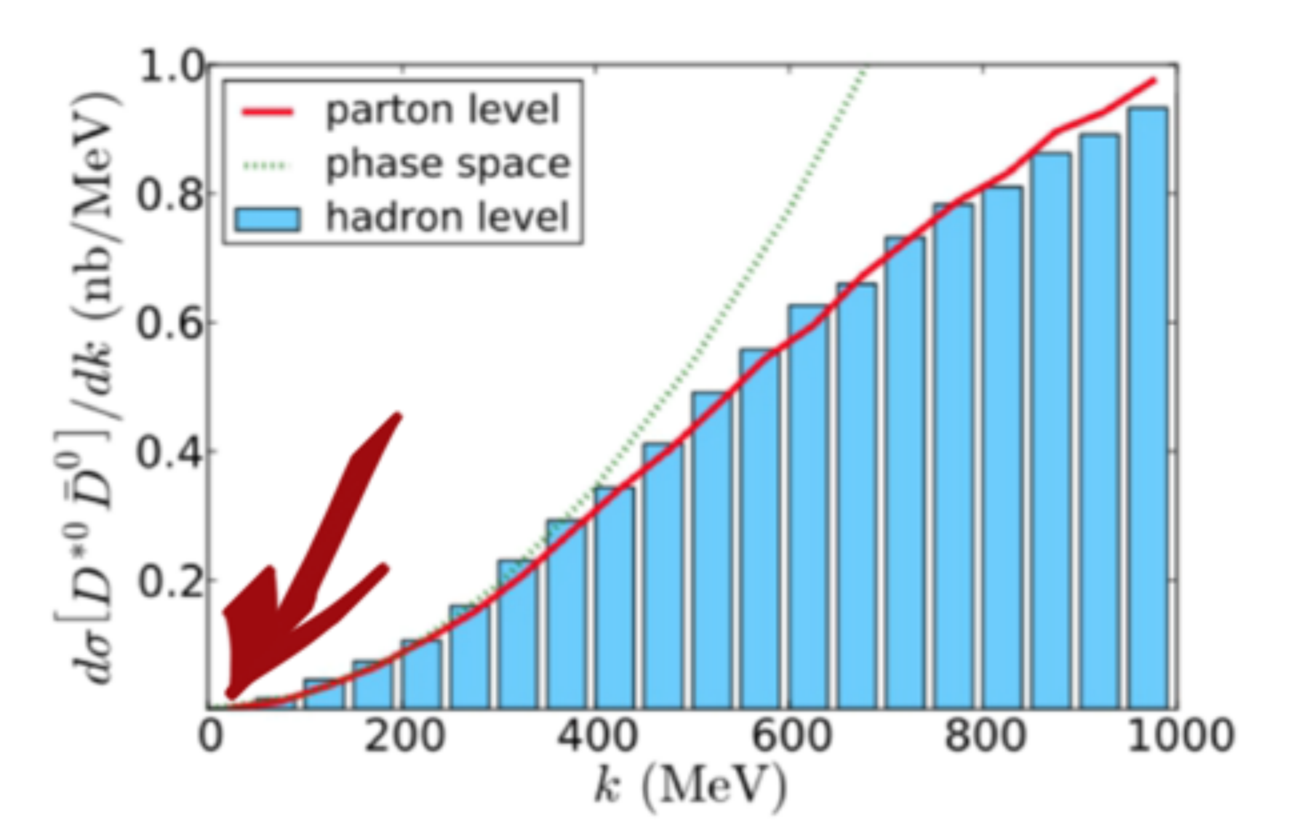
$$\chi_{\rm c}(\xi) = \frac{2\mathscr{C}^{3/2}}{\sqrt{4\pi}} e^{-\mathscr{C}\xi}$$

$$\mathscr{R}_{\rm min} = \frac{\mathscr{B}(X \to \psi'\gamma)}{\mathscr{B}(X \to \psi\gamma)} = 0.95^{+0.01}_{-0.07}$$

B. Grinstein, L. Maiani, A.D.P., 2401.11623

$X(3872)$ PROMPT PRODUCTION IN $pp(\bar{p})$ COLLISIONS

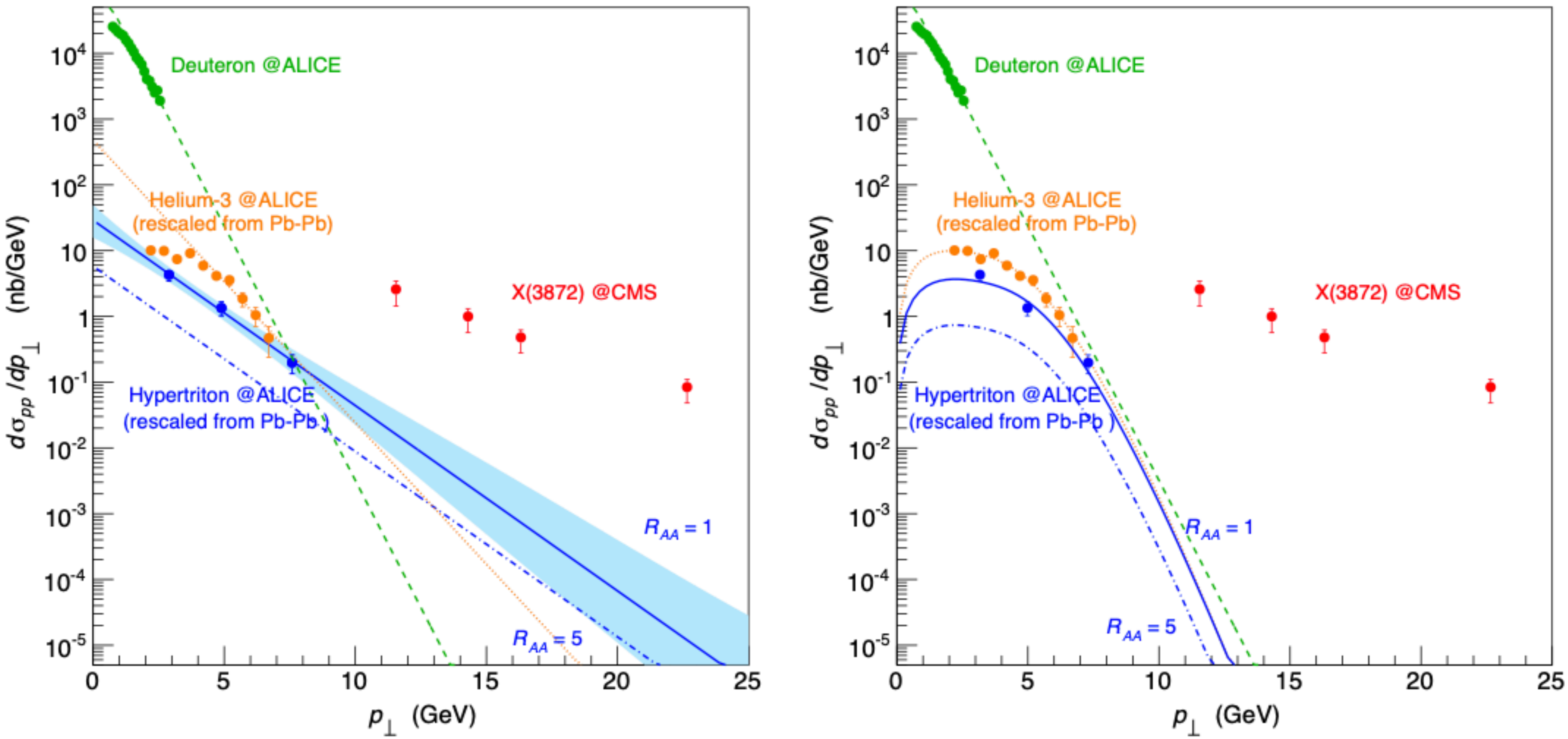
About 300 times smaller than observed (CDF), same in CMS, ATLAS.



Braaten and Artoisenet, PRD81103 (2010) 114018

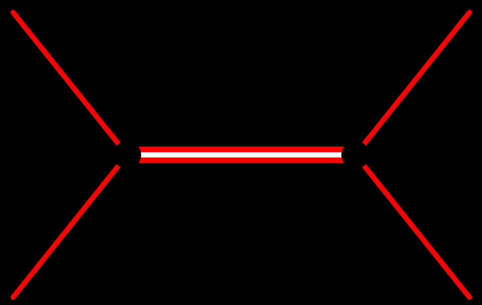
Bignamini, Grinstein, Piccinini, ADP, Sabelli, PRL103 (2009) 162001

THE X BY A $c\bar{c}$ CORE



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

LANDAU ARGUMENT (BOUND STATES IN QFT)



Consider the $D\bar{D}^*$ scattering. The X **molecule (a multiparticle intermediate state)** couples to the asymptotic 2-particle states $|\alpha\rangle, |\beta\rangle = |D\bar{D}^*\rangle$ with some coupling g – the coupling to its constituents. How does g depend on B?

$$f(\alpha \to \beta) = \frac{1}{8\pi E} g^2 \Delta'(p) = \frac{1}{8\pi E} \frac{g^2}{p^2 + m_X^2 - i\epsilon}$$

It is like using the complete propagator (for bare fields) in the Källén-Lehmann representation with

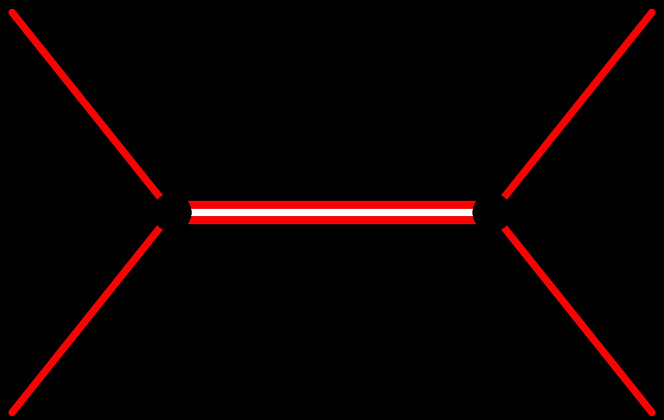
$$\sigma(\mu^2) = \delta(\mu^2 - (m_D + m_{D*})^2) \simeq \delta(\mu^2 - m_X^2)$$

due to the exceptional fact that $m_X \simeq (m_D + m_{D*})$.

L.D. Landau, JETP 39, 1865 (1960)

LANDAU ARGUMENT

Introduce the coupling to X



Neglecting terms of order B^2 and E^2 ($E = k^2/2m$) one finds in the case of the X

$$f(\alpha \to \beta) = \frac{1}{8\pi m_X} \frac{g^2}{(p_D + p_{D^*})^2 + m_X^2 - i\epsilon} \simeq -\frac{1}{16\pi m_X^2} \frac{g^2}{E + B}$$

ADP Phys. Lett. B746, 248 (2015)

LANDAU ARGUMENT

In the presence of a shallow bound state the polar amplitude gives a relation between the coupling g and the B as follows

$$f(\alpha \to \beta) = -\frac{1}{16\pi m_X^2}\frac{(8mm_X^2 g_0^2)}{E+B} = -\sqrt{\frac{2B}{m}}\frac{1}{E+B}$$

$$\Rightarrow g_0^2 = \frac{2\pi}{m}\sqrt{\frac{2B}{m}}$$

This very same formula was found by Weinberg, rescaled by the $(1-Z)$ factor – as from solving Lehman identity with $Z \neq 0$.

$$g_Z^2 = \frac{2\pi}{m}\sqrt{\frac{2B}{m}}(1-Z)$$

WEINBERG ARGUMENT

Weinberg does not go through the K.L. argument but rather uses a NRQM approach in which the Hilbert space is enlarged to include $\mathfrak{X}$

$$|\langle D\bar{D}^*|V|X\rangle|^2 = \frac{2\pi}{m}\sqrt{\frac{2B}{m}}(1-Z)$$

and $$|X\rangle = \sqrt{Z}\,|\mathfrak{X}\rangle + \int_k C_k \underbrace{|D\bar{D}^*(k)\rangle}_{|\alpha\rangle}$$

The NR quantum mechanics treatment of Weinberg suggests the idea of a mixing between compact and molecular/continuum which generates conflicting interpretations. Are there two X mixed states (compact + shallow bound state)?

S. Weinberg Phys. Rev. 137, B672 (1965)

EFFECTIVE RANGE EXPANSION

The NR low energy scattering formula is

$$\frac{A_0^2}{2m} = \frac{1}{mR_0} = \sqrt{\frac{2B}{m}} \qquad R_0 = \frac{1}{\sqrt{2mB}}$$

It corresponds to setting $r_0 = 0$ in the effective range expansion. Keeping a finite r_0 gives instead

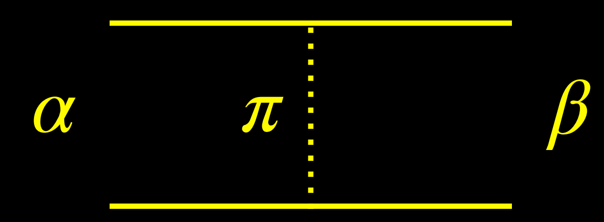
$$\frac{A_0^2}{2m} = \frac{1}{m(R_0 - r_0)}$$

$$\underbrace{\frac{1}{mR_0}(1 - Z)}_{g_0^2 \to g_Z^2} = \frac{1}{m(R_0 - r_0)} \Rightarrow r_0 = -\frac{Z}{1 - Z}R_0$$

r_0 AND a FORMULAE

Solving the previous formula for r_0

$$r_0 = -\frac{Z}{1-Z} R_0 + O\left(\frac{1}{\Lambda}\right)$$



$$R_0 = \frac{1}{\varkappa} = \frac{1}{\sqrt{2mB}}$$

The (positive!) scattering length is obtained using the expression of r_0 given above into the pole condition $\left(-\varkappa_0 + \frac{1}{2} r_0 k^2 - ik\right)_{k=i\varkappa} = 0$

$$a = \frac{2(1-Z)}{2-Z} R_0 + O\left(\frac{1}{\Lambda}\right)$$

(scattering length > 0)

S. Weinberg Phys. Rev. 137, B672 (1965)

INTERPRETATION OF $r_0 < 0$

A negative r_0 implies a finite Z, provided that $1/\Lambda$ corrections are not negative and sizeable. In QFT the meaning of Z is

$$\langle 0 | \Phi(0) | \boldsymbol{k} \rangle = \frac{N}{\sqrt{2E}} \qquad E = \sqrt{\boldsymbol{k}^2 + m^2}$$

$$Z = |N|^2$$

This number has no probabilistic interpretation: it is not the probabilistic weight of the compact component to be compared to the one of the bound state component.

THE Λ SCALE

In the case of the deuteron $\boldsymbol{d}$

$$\Lambda = m_\pi \Rightarrow \frac{1}{\Lambda} \simeq 1 \text{ fm}$$

because the pion can be integrated out given that

$$m_n - m_p \ll m_\pi$$

In the case of the $\boldsymbol{X}$, pion interactions between $\boldsymbol{D}$ and $\boldsymbol{\bar{D}^*}$ (u-channel)

$$\Lambda^2 = m_\pi^2 - \underbrace{(m_{D^*} - m_D)^2}_{q_0^2} \simeq (44 \text{ MeV})^2$$

giving

$$\frac{1}{\Lambda} \simeq 4.5 \text{ fm}$$

THE SIGN OF r_0 IN A ATTRACTIVE V

Scattering in the presence of shallow bound states generated by *purely attractive potentials* in NRQM are characterized by

$$r_0 \geq 0$$

even if there is a repulsive core, but in a *very* narrow region around the origin. Therefore the 1 fm estimated above is +1 fm

$$r_0 \simeq -\frac{Z}{1-Z}R_0 + 1 \text{ fm} = r_0^{\text{exp.}} = +1.74 \text{ fm}$$

So we conclude that $Z \simeq 0$. The deuteron is a molecule! Only a "large" (wrt 1 fm) and negative r_0 would have been the token of the elementary deuteron.

Esposito, Maiani, Pilloni, ADP, Riquer, 2108.11413, *Phys. Rev. D*105 (2022) 3, L031503

DATA ON X: LHCB ANALYSIS

arXiv:2005.13419

For small kinetic energies – Flatté parametrization for the coupled channel decay $X \to D^0\bar{D}^{*0}$ and $X \to D^+\bar{D}^{*-}$.

$$f(X \to J/\psi\pi\pi) = -\frac{N}{(E - m_X^0) + \frac{i}{2}g(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)}) + \frac{i}{2}(\Gamma_\rho + \Gamma_\omega + \dots)}$$

$$\delta = (m_{D^{*-}} + m_{D^+}) - (m_{\bar{D}^{*0}} + m_{D^0}) = \text{Isospin splitting}$$

$$E = m_{J/\psi\pi\pi} - m_D - m_{\bar{D}^*}$$

$\mu_+, \mu =$ reduced mass of the charged/neutal $D\bar{D}^*$ pair

$m_X^0 =$ a bare parameter for the mass of the X (stable determination)

If the fit to the Flatté lineshape works, it means only that the compact hypothesis for the X works!

The r_0 can only be found negative here!

DATA ON X: LHCB ANALYSIS

arXiv:2005.13419

The single channel analysis is obtained by $\Gamma_V, \ldots = 0$

$$f(X \to J/\psi\pi\pi) = -\frac{(2N/g)}{(2/g)(E - m_X^0) - \sqrt{2\mu_+\delta} + E\sqrt{\mu_+/2\delta} + ik}$$

$$-\frac{1}{a} = \frac{2m_X^0}{g} + \sqrt{2\mu_+\delta} \simeq -6.92 \text{ MeV}$$ positive a

$$r_0 = -\frac{2}{\mu g} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}$$ negative r_0

using $E = k^2/2\mu$, μ being the reduced mass of the neutral $D\bar{D}^*$ pair, and taking g (LHCb) and m_X^0 (stable determination) from the experimental analysis. Since g can be larger, $r_0 \leq -2$ fm.

DETERMINATION OF Z

Neglect for the moment $O(1/\Lambda)$ corrections

$$r_0 = -\frac{Z}{1-Z}R_0 = -5.34 \text{ fm}$$
$$a = \frac{2(1-Z)}{2-Z}R_0 = 197/6.92 \text{ fm}$$

Gives $Z = 0.15 \neq 0!$ and $B = 20$ keV

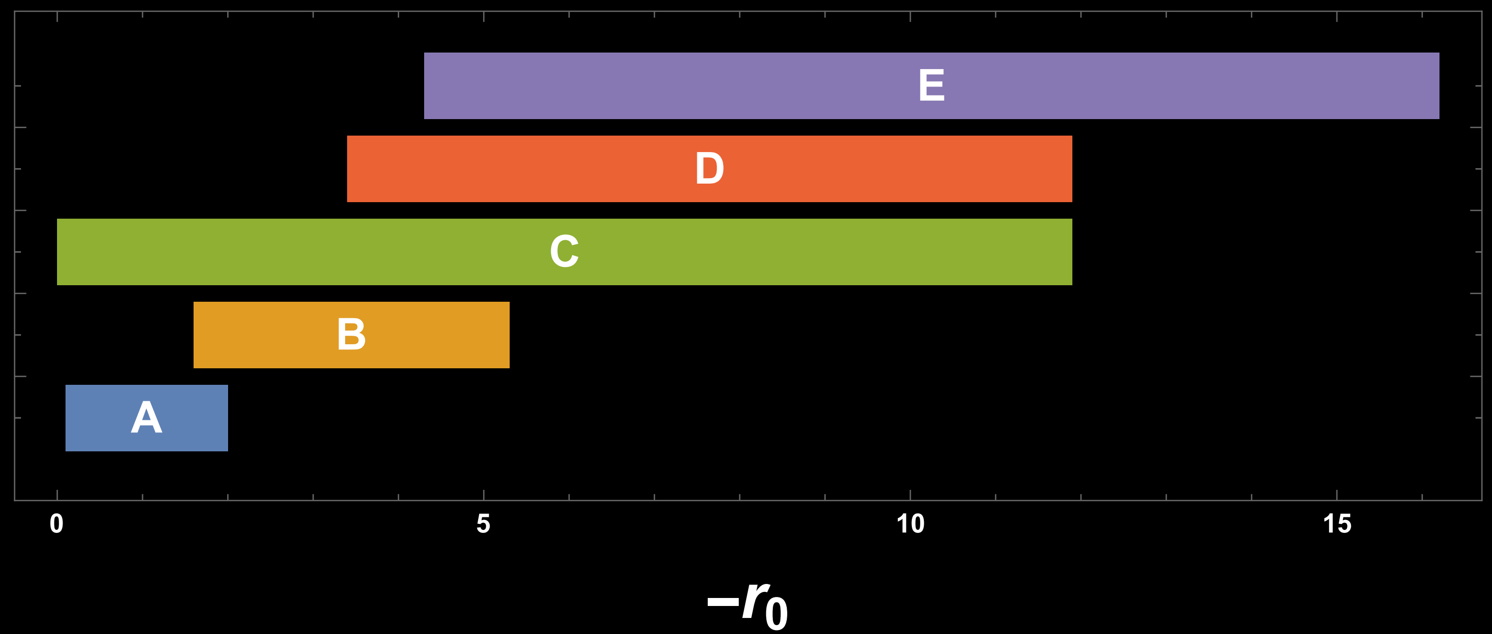
Including ± 5 fm makes quite a difference depending on the sign.

In the case of -5 fm we might have $Z = 0$ even with $r_0^{\text{exp}} = -5.32$ fm!

In the case of $+5$ fm, a negative experimental r_0 is the proof of the compact state.

However we shall see that in the molecular case $O(1/\Lambda) \to -0.2$ MeV

$(-r_0)$ ACCORDING TO SOME ESTIMATES



A: Baru et al., 2110.07484

B: Esposito et al., 2108.11413

C: LHCb, 2109.01056

D: Maiani & Pilloni GGI-Lects

E: Mikhasenko $T_{cc'}$, 2203.04622

H. Xu, N. Yu and Z. Zhang 2401.00411: $r_0 \approx -14$ fm combining LHCb and Belle data (for the X).

r_0 FROM LATTICE

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 2202.10110

Applying the lattice Lüscher method, the authors study the $D\bar{D}^*$ scattering amplitude and make a determination of the scattering length and of the effective range for $\mathcal{T}_{cc}$

$$a = -1.04(29) \text{ fm}$$
$$r_0 = +0.96^{+0.18}_{-0.20} \text{ fm}$$

The mass of the pion is $m_\pi = 280$ MeV, to keep the D^* stable. This result, for the moment, is compatible with a *virtual state* because of the negative a – like the singlet deuteron.
As for LHCb (2109.01056 p.12)

$$a = +7.16 \text{ fm}$$
$$-11.9 \leq r_0 \leq 0 \text{ fm}$$

DATA ON X: LINESHAPE ANALYSIS

What should be done: Flatté can be derived from an EFT with the field of the X and those of D^0, D^{*0}, D^+, D^{*-}. If the X field is in $\mathscr{L}$ the particle is elementary.

If the fit "works", there is an elementary X!

Write an EFT with only D^0, D^{*0}, D^+, D^{*-}. Find the f, do another fit!

This would test the molecular hypotheis.

There are no half ways!

EXOTIC HADRONS

The first to show up was the $X(3872)$, which could have been another state of charmonium (discovered in the $J/\psi\pi^+\pi^-$ channel), **but it's not**

"could have been a compact tetraquark, **but it's not**" (E. Braaten)

"could have been a loosely bound molecule of hadrons, **but it's not**"

and then hadrocharmonium (M. Voloshin), deuson (N. Tornqvist)….

but it's not

CONCLUSIONS

- It would be useful to have two fits to the lineshape: the genuine "molecular" parametrization to be compared to the Flatté fit.
- Learn more, on the experimental side, about deuteron production at high p_T .
- Some states are produced promptly in pp collisions, some are not. There is no clear reason why!
- Are there loosely bound molecules $B\bar{B}^*$? Can we formulate more stringient bounds on $X^{\pm}$ particles, or…discover them?