Unitarity effects in elastic scattering at the LHC

Emerson Luna

Instituto de Física

Universidade Federal do Rio Grande do Sul

New Trends in High-Energy and Low-x Physics

Sfantu Gheorghe, Romania, 2024

* in collaboration with M. Maneyro and M. Peláez

Outline

- Unitarization Schemes

  $\Rightarrow$ Born amplitudes: Pomeron and Odderon inputs

- The tension between the TOTEM and ALFA/ATLAS measurements

- Results

- Conclusion and Perspectives

■ In order to describe the observed increase of $\sigma_{tot}(s)$, the Pomeron should have a supercritical intercept given by $\alpha_{\mathbb{P}}(0) = 1 + \epsilon$ with $\epsilon > 0$

$\Rightarrow$ the behavior of the total cross section for $\alpha_{\mathbb{P}}(0) > 1$ betokens the violation of the Froissart-Martin limit at some energy scale

■ It is expected that unitarity can be enforced in high-energy hadron-hadron interactions by the inclusion of the exchange series $\mathbb{P} + \mathbb{P}\mathbb{P} + \mathbb{P}\mathbb{P}\mathbb{P} + \ldots$

$\Rightarrow$ $\alpha_{\mathbb{P}}(0)$ is an effective power representing n-Pomeron exchange processes, $n \geq 1$

Despite the advances in understanding the nature of the Pomeron in the last decades, we still need to learn how to fully compute the contributions from multiple-Pomeron exchange processes with $n \geq 3$

On the other hand, it is well-established that some unitarization schemes sum appropriately rescattering diagrams representing the exchange of several particular multiparticle states

$\Rightarrow$ these schemes are primarily based on phenomenological arguments

$\Rightarrow$ they are effective procedures for taking into account many of the properties of unitarity in the s-channel or,

at the very least,

for preventing the Froissart-Martin bound for σ_{tot} from being violated

■ We focus on two key unitarization schemes: the eikonal and the U-matrix approaches

Unitarization Schemes

■ Certain distinctive features of the high-energy $\mathcal{A}(s,t)$ are better illuminated when examined in the impact parameter b-representation:

$$2\,\mathrm{Im}\,H(s,b) = |H(s,b)|^2 + G_{in}(s,b) \tag{1}$$

$\Rightarrow$ $G_{in}(s,b)$ is the inelastic overlap function

□ After the integration over two-dimensional impact parameter space:

$$\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{in}(s)$$

where

$$\sigma_{tot}(s) = \frac{4\pi}{s}\mathrm{Im}\,\mathcal{A}(s,t=0) = 2\pi\int_0^\infty b\,db\,2\,\mathrm{Im}H(s,b)$$

$$\sigma_{el}(s) = \frac{\pi}{s^2} \int_{-\infty}^{0} dt\ |\mathcal{A}(s,t)|^2 = 2\pi \int_0^\infty b\,db\ |H(s,b)|^2$$

$$\sigma_{in}(s) = 2\pi \int_0^\infty b\,db\ \left(2\,\mathrm{Im}H(s,b) - |H(s,b)|^2\right)$$

□ After defining a function

$$\rho(s,b) = \frac{\mathrm{Re}\,H(s,b)}{\mathrm{Im}\,H(s,b)}$$

and solving the quadratic equation for Im $H(s,b)$ resulting from (1):

$$\mathrm{Im}\,H(s,b) = \frac{1 \pm \sqrt{1 - (1+\rho^2)\,G_{in}(s,b)}}{1+\rho^2} \tag{2}$$

$\Rightarrow$ We see that $G_{in}(s, b)$ must fit onto the interval

$$0 \leq G_{in}(s, b) \leq (1 + \rho^2)^{-1}$$

where we have required $\text{Im}\, H(s, b)$ be real.

■ The construction of unitarized scattering amplitudes relies on two formal steps:

Step 1. The choice of a Born term $\mathcal{F}(s, t)$ with the crossing-even and crossing-odd parts defined as

$$\mathcal{F}^{\pm}(s, t) = \frac{1}{2}\left[\mathcal{F}^{pp}(s, t) \pm \mathcal{F}^{\bar{p}p}(s, t)\right]$$

□ The correspondent crossing-even and crossing-odd Born amplitudes in b-space are given by

$$\chi^{\pm}(s,b) = \frac{1}{s}\int_0^{\infty} q\,dq\,J_0(bq)\mathcal{F}^{\pm}(s,-q^2)$$

Step 2. Consists of writing the scattering amplitude $H_{\bar{p}p}^{pp}(s,b)$ in terms of the Born amplitudes $\chi_{\bar{p}p}^{pp}(s,b)$

$\Rightarrow$ Once this is done, $\mathcal{A}_{\bar{p}p}^{pp}(s,t)$ is finally obtained from the inverse Fourier-Bessel transform of $H_{\bar{p}p}^{pp}(s,b)$:

$$\mathcal{A}_{\bar{p}p}^{pp}(s,t) = s\int_0^{\infty} b\,db\,J_0(bq)\,H_{\bar{p}p}^{pp}(s,b)$$

Eikonal Unitarization

■ The eikonal unitarization corresponds to the solution of equation (2) with the minus sign

⇒ The eikonal scheme (Es) leads us to the relation

$$H(s,b) = i\left[1 - e^{i\chi(s,b)}\right]$$

so that

$$\mathcal{A}_{[Es]}(s,t) = is\int_0^\infty b\,db\,J_0(bq)\left[1 - e^{i\chi(s,b)}\right]$$

□ In the Es there is an upper limit on the imaginary part of $H(s,b)$,

$$0 \leq \mathrm{Im}H(s,b) \leq (1+\rho^2)^{-1}$$

Eikonal Unitarization

□ Solving the Unitarity Eq. (1) for $G_{in}(s,b)$ in terms of $\chi(s,b)$ yields

$$G_{in}(s,b) = 1 - e^{-2\,\mathrm{Im}\,\chi(s,b)}$$

$\Rightarrow$ The positivity condition on $G_{in}(s,b)$ and the upper limit on $\mathrm{Im}H(s,b)$ restrict the imaginary part of $\chi(s,b)$ over

$$0 \leq \mathrm{Im}\chi(s,b) \leq -\frac{1}{2}\ln\left(\frac{\rho^2}{1+\rho^2}\right)$$

$\Rightarrow$ In the limit of a perfectly absorbing profile $H(s,b)$ and $\chi(s,b)$ are purely imaginary

$\Rightarrow$ In this limit we have the asymptotic result $\sigma_{el}/\sigma_{tot} = 1/2$

U-matrix Unitarization

■ The *U*-matrix unitarization corresponds to the solution of the unitarity equation (2) with the plus sign

$\Rightarrow$ The *U*-matrix scheme (Us) leads us to the relation

$$H(s,b) = \frac{\chi(s,b)}{1 - i\chi(s,b)/2}$$

so that

$$\mathcal{A}_{[Us]}(s,t) = is \int_0^\infty b\, db\, J_0(bq) \left[\frac{2\chi(s,b)}{\chi(s,b) + 2i} \right]$$

□ In the Us the $\mathrm{Im}H(s,b)$ is constrained to lie in the interval

$$(1+\rho^2)^{-1} \leq \mathrm{Im}H(s,b) \leq 2(1+\rho^2)^{-1}$$

U-matrix Unitarization

☐ In the black disc and $\rho \to 0$ limits we have

$$\mathrm{Im}H(s,b) = 2$$

and

$$|H(s,b)|^2 = 4$$

⇒ These results lead us to the asymptotic behavior $\sigma_{el}/\sigma_{tot} = 1$

⇒ Thus $H(s,b)$ may exceed the black disc limit in this approach

Born Input Amplitudes

- The input Born amplitudes are associated with Reggeon exchange amplitudes

$\Rightarrow$ The corresponding amplitudes in the b-space are given by

$$\chi_i(s,b) = \frac{1}{s} \int \frac{d^2 q}{2\pi}\, e^{i\mathbf{q}\cdot\mathbf{b}}\, \mathcal{F}_i(s,t)$$

where $i = -, +, \mathbb{P}$, and $\mathbb{O}$.

$\Rightarrow$ The physical amplitudes in b-space are obtained by summing of all possible exchanges:

$$\chi^{pp}_{\bar{p}p}(s,b) = \chi_{\mathbb{P}}(s,b) + \chi_+(s,b) \pm \chi_-(s,b) \pm \xi_{\mathbb{O}}\chi_{\mathbb{O}}(s,b)$$

$\Rightarrow$ Here $\chi_{+}(s, b)$ ($\chi_{-}(s, b)$) is the $C = +1$ ($C = -1$) Reggeon contribution

$\Rightarrow$ $\chi_{\mathbb{P}}(s, b)$ ($\chi_{\mathbb{O}}(s, b)$) is the Pomeron (Odderon) contribution

$\Rightarrow$ $\xi_{\mathbb{O}}$ is the Odderon phase factor

☐ $\xi_{\mathbb{O}}$ is associated with the positivity property

☐ However, unlike Pomeron, the Odderon is not constrained by positivity requirements

From a theoretical standpoint, this implies that it is not possible to determine the phase of the Odderon mathematically

Specifically, the Born amplitude for each single exchange is

$$\mathcal{F}_i(s,t) = \beta_i^2(t)\eta_i(t)\left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

$\Rightarrow$ $\beta_i^2(t)$ is the elastic proton-Reggeon vertex

$\Rightarrow$ $\alpha_i(t)$ is the Regge trajectory

$\Rightarrow$ $\eta_i(t) = -ie^{-i\frac{\pi}{2}\alpha_i(t)}$ is the odd-signature factor

$\Rightarrow$ $\eta_i(t) = -e^{-i\frac{\pi}{2}\alpha_i(t)}$ is the even-signature factor

$\Rightarrow$ $s_0 \equiv 1\ \mathrm{GeV}^2$ is an energy scale

For Reggeons with positive charge-conjugation:

$$\beta_+(t) = \beta_+(0)\exp(r_+ t/2)$$

and

$$\alpha_+(t) = 1 - \eta_+ + \alpha'_+ t$$

Similarly, the Reggeons with negative charge-conjugation are described by the parameters $\beta_-(0)$, r_-, η_-, and α'_-

For Pomeron exchange we adopt

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}} t + \frac{m_\pi^2}{32\pi^3}\, h(\tau)$$

where $\alpha_{\mathbb{P}}(0) = 1 + \epsilon$ and

$$h(\tau) = -\frac{4}{\tau} F_\pi^2(t) \left[2\tau - (1+\tau)^{3/2} \ln\left(\frac{\sqrt{1+\tau}+1}{\sqrt{1+\tau}-1} \right) + \ln\left(\frac{m^2}{m_\pi^2} \right) \right] \quad (3)$$

with $\epsilon > 0$, $\tau = 4m_\pi^2/|t|$, $m = 1$ GeV, and $m_\pi = 139.6$ MeV

$\Rightarrow$ $F_\pi(t)$ is the form factor of the pion-Pomeron vertex:

$$F_\pi(t) = \beta_\pi/(1 - t/a_1)$$

$\Rightarrow$ β_π specifies the value of the pion-Pomeron coupling

$\Rightarrow$ we take the additive quark model relation $\beta_\pi/\beta_{\boldsymbol{P}}(0) = 2/3$

The third term on the right-hand side of (3) corresponds to pion-loop insertions and is generated by t-channel unitarity

■ We investigated two different forms for the proton-Pomeron vertex

□ The first vertex, specifying our "Model I", is given by

$$\beta_{\mathbb{P}}(t) = \beta_{\mathbb{P}}(0)\exp\left(\frac{r_{\mathbb{P}}t}{2}\right)$$

□ The second proton-Pomeron vertex, referred to as "Model II", has the power-like form

$$\beta_{\mathbb{P}}(t) = \frac{\beta_{\mathbb{P}}(0)}{(1 - t/a_1)(1 - t/a_{\mathbb{P}})} \tag{4}$$

⇒ Note that the parameter a_1 in (4) is the same as the one in the expression for $F_{\pi}(t)$

⇒ we fix this parameter at $a_1 = m_{\rho}^2 = (0.776\,\mathrm{GeV})^2$

The total cross section, the elastic differential cross section, and the ρ parameter are expressed in terms of the physical amplitude $\mathcal{A}_{\bar{p}p}^{pp}(s,t)$,

$$\sigma_{tot}^{pp,\bar{p}p}(s) = \frac{4\pi}{s} \operatorname{Im} \mathcal{A}_{\bar{p}p}^{pp}(s, t=0)$$

$$\frac{d\sigma^{pp,\bar{p}p}}{dt}(s,t) = \frac{\pi}{s^2} \left| \mathcal{A}_{\bar{p}p}^{pp}(s,t) \right|^2$$

$$\rho^{pp,\bar{p}p}(s) = \frac{\operatorname{Re} \mathcal{A}_{\bar{p}p}^{pp}(s, t=0)}{\operatorname{Im} \mathcal{A}_{\bar{p}p}^{pp}(s, t=0)}$$

together with the replacements $\mathcal{A}_{\bar{p}p}^{pp}(s,t) = \mathcal{A}_{[Es]}^{pp,\bar{p}p}(s,t)$ or $\mathcal{A}_{[Us]}^{pp,\bar{p}p}(s,t)$, where

$$\mathcal{A}^{pp,\bar{p}p}_{[Es]}(s,t) = is\int_0^\infty b\,db\,J_0(bq)\left[1 - e^{i\chi^{pp}_{\bar{p}p}(s,b)}\right]$$

and

$$\mathcal{A}^{pp,\bar{p}p}_{[Us]}(s,t) = is\int_0^\infty b\,db\,J_0(bq)\left[\frac{2\chi^{pp}_{\bar{p}p}(s,b)}{\chi^{pp}_{\bar{p}p}(s,b) + 2i}\right]$$

The Odderon input

■ The Born amplitude for the Odderon contribution is represented as

$$\mathcal{F}_{\mathbb{O}}(s,t) = \beta_{\mathbb{O}}^2(t)\,\eta_{\mathbb{O}}(t)\left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{O}}(t)}$$

where $\eta_{\mathbb{O}}(t) = -ie^{-i\frac{\pi}{2}\alpha_{\mathbb{O}}(t)}$

□ In the formulation of "Model III", we employ an exponential form factor for the proton-Odderon vertex:

$$\beta_{\mathbb{O}}(t) = \beta_{\mathbb{O}}(0)\exp\left(\frac{r_{\mathbb{O}}t}{2}\right)$$

with $r_{\mathbb{O}} = r_{\mathbb{P}}/2$

The Odderon input

□ In the formulation of "Model IV", we adopt the power-like form for the proton-Odderon vertex:

$$\beta_{\mathbb{O}}(t) = \frac{\beta_{\mathbb{O}}(0)}{(1 - t/m_\rho^2)(1 - t/a_{\mathbb{O}})}$$

with $a_{\mathbb{O}} = 2a_{\mathbb{P}}$

$\Rightarrow$ The relationship between $a_{\mathbb{O}}$ and $a_{\mathbb{P}}$ that must satisfy the constraint $a_{\mathbb{O}} \geq a_{\mathbb{P}}$ to avoid non-physical amplitudes when using a power-like form factor

The Odderon input

■ From the standpoint of QCD (at the lowest order) the $C = +1$ amplitude arises from the exchange of two gluons and the $C = -1$ amplitude from the exchange of three gluons

■ Extensive theoretical studies have been directed towards uncovering corrections to these results, particularly in higher orders

□ In this scenario, the leading-log approximation allows for the summation of certain higher-order contributions to physical observables in high-energy particle scattering processes

$\Rightarrow$ This approach was widely used in the study of the QCD-Pomeron through the BFKL equation

The Odderon input

$\Rightarrow$ In BFKL equation terms of the order $(\alpha_s \ln(s))^n$ are systematically summed at high energy (large s) and small strong coupling α_s

$\Rightarrow$ The simplistic notion of bare two-gluon exchange gives way to the BFKL Pomeron, which, in an alternative representation, can be seen as the interaction of two reggeized gluons with one another

■ Beyond the BFKL Pomeron, the most elementary entity within perturbative QCD is the exchange involving three interacting reggeized gluons

□ The evolution of the three-gluon Odderon exchange as energy increases is governed by the BKP equation

$\Rightarrow$ A bound state solution of this Odderon equation was obtained with the intercept $\alpha_{\mathbb{O}}(0) = 1$

The Odderon input

- Based on these QCD findings, we adopt in this work the simplest conceivable form for the Odderon trajectory:

$$\alpha_{\mathbb{O}}(t) = 1$$

Results

■ The LHC has released exceptionally precise measurements of diffractive processes

□ These measurements, particularly the total and differential cross sections obtained from ATLAS and TOTEM Collaborations, enable us to determine the Pomeron and Odderon parameters accurately

$\Rightarrow$ However, these experimental results unveil a noteworthy tension between the TOTEM and ATLAS measurements

$\Rightarrow$ For instance, when comparing the TOTEM and the ATLAS result for σ_{tot}^{pp} at $\sqrt{s} = 8$ TeV, the discrepancy between the values corresponds to 2.6 σ

Results

■ In order to systematically explore the tension between TOTEM and ATLAS results, we perform global fits to pp and $\bar{p}p$ forward scattering data and to pp differential cross-section data while considering two distinct datasets, one with TOTEM measurements and the other with ATLAS measurements

□ The two data ensembles can be defined as follows:
Ensemble A: $\sigma_{tot}^{pp,\bar{p}p}$ data + $\rho^{pp,\bar{p}p}$ data + ATLAS data on $\frac{d\sigma}{dt}$ at 7, 8, and 13 TeV;
Ensemble T: $\sigma_{tot}^{pp,\bar{p}p}$ data + $\rho^{pp,\bar{p}p}$ data + TOTEM data on $\frac{d\sigma}{dt}$ at 7, 8, and 13 TeV

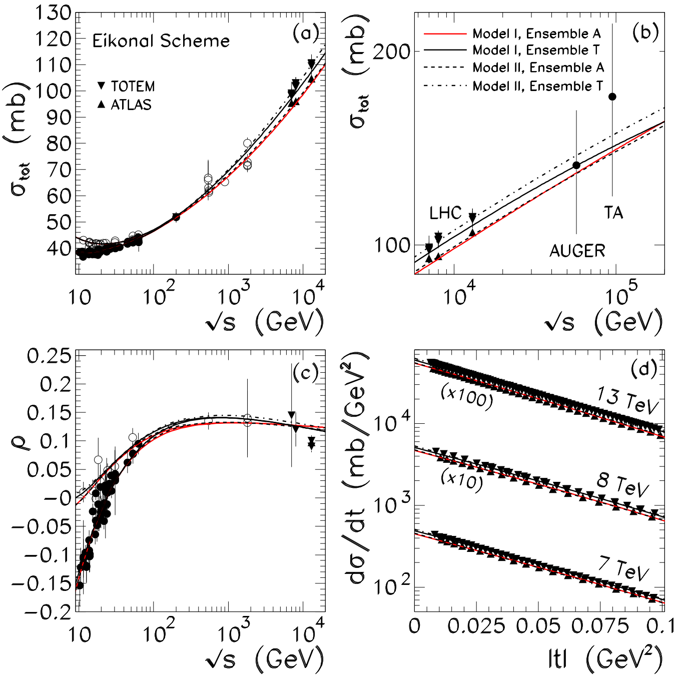
$\Rightarrow$ We carry out global fits to the two distinct ensembles using a χ^2 fitting procedure, where χ^2_{min} follows a χ^2 distribution with ν DoF

$\Rightarrow$ We adopt an interval $\chi^2 - \chi^2_{min}$ corresponding to a 90% confidence level (CL).

Pomeron Analysis

Table: The Pomeron and secondary Reggeons parameters values obtained in global fits to Ensembles A and T after the eikonal unitarization.

	Ensemble A		Ensemble T	
	Model I	Model II	Model I	Model II
ϵ	0.1014±0.0033	0.1112±0.0013	0.1248±0.0027	0.1336±0.0023
$\alpha'_{\boldsymbol{P}}$ (GeV^{-2})	0.2938±0.0022	0.1148±0.0076	0.56×10^{-9} ±0.11	0.009±0.040
$\beta_{\mathbb{P}}(0)$	2.154±0.063	1.999±0.023	1.814±0.043	1.742±0.028
$r_{\mathbb{P}}$ (GeV^{-2})	2.375±0.019	—	7.448±0.087	—
$a_{\mathbb{P}}$ (GeV^{-2})	—	0.829±0.081	—	0.499±0.084
η_+	0.360±0.048	0.344±0.030	0.286±0.025	0.262±0.015
$\beta_+(0)$	4.56±0.47	4.37±0.34	4.02±0.21	3.93±0.14
η_-	0.556±0.010	0.550±0.089	0.536±0.067	0.530±0.064
$\beta_-(0)$	3.68±0.16	3.55±0.67	3.41±0.49	3.39±0.46
ν	226	226	350	350
χ^2/ν	0.86	0.83	0.74	0.65

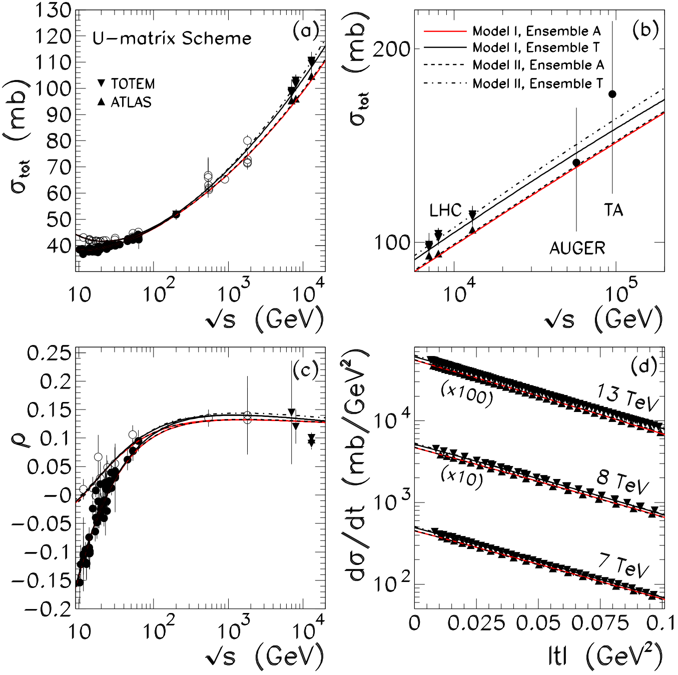


Eikonal Scheme
(a)
TOTEM
ATLAS
σ_{tot} (mb)
$\sqrt{s}$ (GeV)
(b)
Model I, Ensemble A
Model I, Ensemble T
Model II, Ensemble A
Model II, Ensemble T
LHC
TA
AUGER
(c)
ρ
(d)
(x100)
13 TeV
(x10)
8 TeV
7 TeV
$d\sigma/dt$ (mb/GeV^2)
|t| (GeV^2)

Pomeron Analysis

Table: The Pomeron and secondary Reggeons parameters values obtained in global fits to Ensembles A and T after the *U*-matrix unitarization.

	Ensemble A		Ensemble T	
	Model I	Model II	Model I	Model II
ϵ	0.0911±0.0037	0.0981±0.0029	0.1129±0.0048	0.1150±0.0070
$\alpha'_{\mathbb{P}}$ (GeV^{-2})	0.4425±0.0085	0.2728±0.0089	0.05±0.14	0.10±0.12
$\beta_{\mathbb{P}}(0)$	2.271±0.075	2.140±0.056	1.926±0.085	1.92±0.11
$r_{\mathbb{P}}$ (GeV^{-2})	0.1051±0.0061	—	7.2±2.8	—
$a_{\mathbb{P}}$ (GeV^{-2})	—	40±20	—	0.62±0.49
η_+	0.356±0.057	0.369±0.049	0.325±0.050	0.314±0.053
$\beta_+(0)$	4.71±0.65	4.51±0.48	4.18±0.43	4.14±0.44
η_-	0.551±0.098	0.551±0.043	0.545±0.074	0.542±0.075
$\beta_-(0)$	3.59±0.74	3.54±0.34	3.43±0.54	3.43±0.54
ν	226	226	350	350
χ^2/ν	0.85	0.86	0.71	0.64


U−matrix Scheme
(a)
TOTEM
ATLAS
σ_tot (mb)
√s (GeV)
(b)
Model I, Ensemble A
Model I, Ensemble T
Model II, Ensemble A
Model II, Ensemble T
LHC
AUGER
TA
(c)
ρ
(d)
(x100)
13 TeV
(x10)
8 TeV
7 TeV
dσ/dt (mb/GeV²)
|t| (GeV²)

Pomeron $\oplus$ Odderon Analysis

Table: The Pomeron, Odderon and secondary Reggeons parameters values obtained in global fits to Ensembles A and T after the eikonal unitarization. We show the results with $\xi_{\mathbb{O}} = -1$.

	Ensemble A		Ensemble T	
	Model III	Model IV	Model III	Model IV
ϵ	0.1017±0.0043	0.1043±0.0026	0.1247±0.0048	0.1335±0.0041
α'_P (GeV^{-2})	0.283±0.036	0.242±0.012	0.94×10^{-4} ±0.059	0.01±0.11
$\beta_{\mathbb{P}}(0)$	2.146±0.083	2.116±0.011	1.815±0.080	1.744±0.035
$r_{\mathbb{P}}$ (GeV^{-2})	2.58±0.68	—	7.45±0.13	—
$a_{\mathbb{P}}$ (GeV^{-2})	—	31±11	—	0.50±0.16
$\beta_{\mathbb{O}}(0)$	0.47±0.24	0.40±0.17	0.31±0.24	0.27±0.20
η_+	0.359±0.055	0.353±0.020	0.285±0.051	0.261±0.013
$\beta_+(0)$	4.52±0.54	4.47±0.29	4.00±0.38	3.91±0.16
η_-	0.4823±0.0019	0.482±0.077	0.490±0.030	0.489±0.077
$\beta_-(0)$	3.20±0.13	3.19±0.50	3.14±0.22	3.15±0.50
ν	225	225	349	349
χ^2/ν	0.84	0.80	0.73	0.65

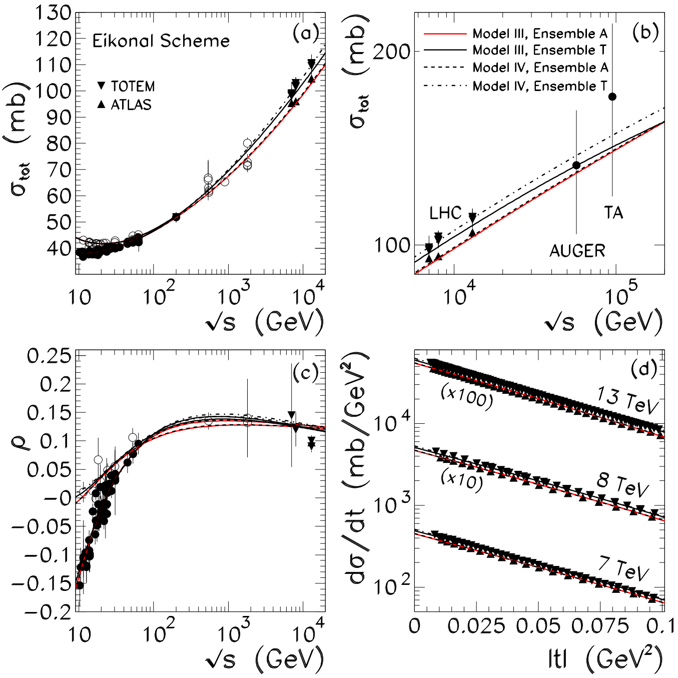
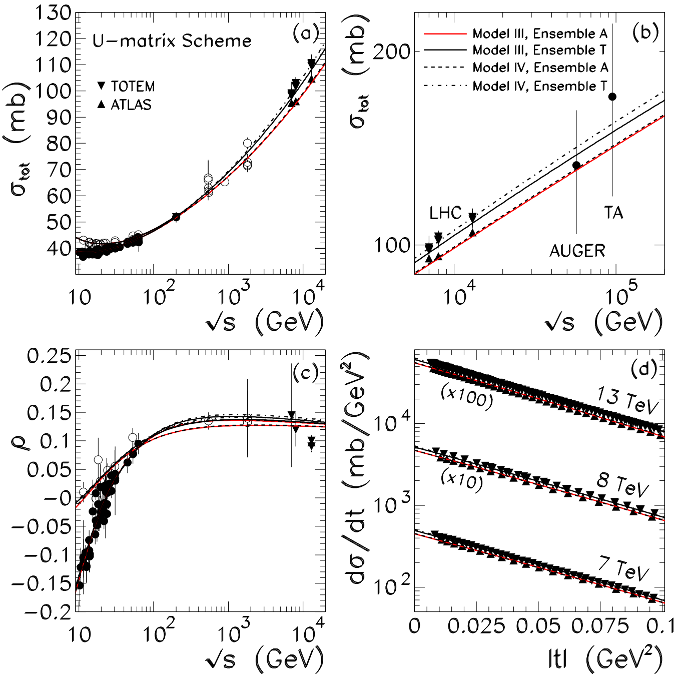

(a)
Eikonal Scheme
TOTEM
ATLAS
σ_{tot} (mb)
$\sqrt{s}$ (GeV)
(b)
Model III, Ensemble A
Model III, Ensemble T
Model IV, Ensemble A
Model IV, Ensemble T
LHC
AUGER
TA
(c)
ρ
(d)
(x100)
13 TeV
(x10)
8 TeV
7 TeV
$d\sigma/dt$ (mb/GeV^2)
|t| (GeV^2)

Table: The Pomeron, Odderon and secondary Reggeons parameters values obtained in global fits to Ensembles A and T after the *U*-matrix unitarization. We show the results with $\xi_{\mathbb{O}} = -1$.

	Ensemble A		Ensemble T	
	Model III	Model IV	Model III	Model IV
ϵ	0.0938±0.0045	0.0978±0.0047	0.1115±0.0035	0.1148±0.0060
$\alpha'_{\mathbb{P}}$ (GeV^{-2})	0.364±0.029	0.273±0.031	0.10±0.15	0.106±0.098
$\beta_{\mathbb{P}}(0)$	2.215±0.075	2.146±0.066	1.951±0.063	1.919±0.093
$r_{\mathbb{P}}$ (GeV^{-2})	1.57±0.58	—	6.2±3.0	—
$a_{\mathbb{P}}$ (GeV^{-2})	—	40±24	—	0.63±0.41
$\beta_{\mathbb{O}}(0)$	0.44±0.20	0.23±0.15	0.32±0.18	0.27±0.18
η_+	0.374±0.031	0.369±0.026	0.327±0.071	0.313±0.046
$\beta_+(0)$	4.62±0.50	4.49±0.64	4.18±0.72	4.12±0.38
η_-	0.490±0.047	0.48±0.33	0.49±0.21	0.50±0.12
$\beta_-(0)$	3.18±0.18	3.08±0.79	3.11±0.42	3.17±0.71
ν	225	225	349	349
χ^2/ν	0.83	0.84	0.71	0.64


U−matrix Scheme
(a)
TOTEM
ATLAS
σ_{tot} (mb)
$\sqrt{s}$ (GeV)
(b)
Model III, Ensemble A
Model III, Ensemble T
Model IV, Ensemble A
Model IV, Ensemble T
LHC
TA
AUGER
(c)
ρ
(d)
(x100)
13 TeV
(x10)
8 TeV
7 TeV
$d\sigma/dt$ (mb/GeV^2)
|t| (GeV^2)

Conclusions and Perspectives

■ The presence of the Odderon immediately impacts the behavior of total cross sections, particularly generating different growth patterns for $\sigma_{tot}^{pp}(s)$ and $\sigma_{tot}^{\bar{p}p}(s)$ at high energies

$\Rightarrow$ With an asymptotic non-zero crossing-odd term $\mathcal{A}^{-}(s,t)$ in the scattering amplitude, it is possible to demonstrate that $|\Delta\sigma|$ can be at most $|\Delta\sigma| = k \ln s$ in the limit $s \to \infty$, where k is a constant

■ After introducing the Odderon, the eikonal scheme demonstrates a slight advantage over the U-matrix scheme, mirroring the scenario where the Pomeron is the sole asymptotically dominant entity

■ We observe that for an Odderon with a phase factor $\xi_{\mathbb{O}} = +1$, all eight $\beta_{\mathbb{O}}(0)$ values obtained are consistent with zero (errors significantly surpassing central values)

Conclusions and Perspectives

$\Rightarrow$ Consequently, the remaining parameters assume values very closely resembling the scenario where the Pomeron dominates the scattering amplitude

- The Odderon phase is well-defined and is equal to $\xi_{\mathbb{O}} = -1$

- An ongoing analysis focusing solely on high-energy data, considering exclusively the contributions from Pomeron and Odderon, is imperative to ascertain the stability of the Odderon phase factor

- Ongoing investigations involving a two-channel model are underway, focusing on the study of eikonal and *U*-matrix unitarization schemes within the context of our analysis

THANK YOU

Resummations in QCD

■ Every physical observable can be written, in pQCD, as a power series in α_s

$\Longrightarrow$ in these series the coupling constant is accompanied by large logarithms, which need to be resummed

$\Longrightarrow$ according to the type and to the powers of logarithms that are effectively resummed one gets different evolution equations

■ The solution of the DGLAP equation sums over all orders in α_s the contributions from leading, single, collinear logarithms of the form $\alpha_s \ln\left(Q^2/Q_0^2\right)$

$\Longrightarrow$ it does not include leading, single, soft singularities of the form $\alpha_s \ln\left(1/x\right)$, which are treated instead by the BFKL equation

■ The BFKL equation describes the x-evolution of PDFs at fixed Q^2

Resummations in QCD

■ The phase space regions which contribute these logarithms enhancements are associated with configurations in which successive partons have strongly ordered transverse, k_T, or longitudinal, $k_L \equiv x$, momenta:

$\Rightarrow \alpha_s L_Q \sim 1$, $\alpha_s L_x \ll 1$: $Q^2 \gg k^2_{T,n} \gg \cdots \gg k^2_{T,1} \gg Q^2_0$

$\Rightarrow \alpha_s L_x \sim 1$, $\alpha_s L_Q \ll 1$: $x \ll x_n \ll \cdots \ll x_1 \ll x_0$

■ At small-x and slow Q^2 (where gluons are dominant) we do not have strongly ordered k_T

$\Rightarrow$ we have to integrate over the full range of k_T

$\Rightarrow$ this leads us to work with the *unintegrated* gluon PDF $\tilde{g}(x, k_T^2)$:

$$xg(x, Q^2) = \int^{Q^2} \frac{dk_T^2}{k_T^2} \tilde{g}(x, k_T^2)$$

Positivity

■ The phase factor is associated with the positivity property

$\Rightarrow$ However, unlike Pomeron, the Odderon is not constrained by positivity requirements

$\Rightarrow$ From a theoretical standpoint, this implies that it is not possible to determine the phase of the Odderon mathematically

□ This issue can be succinctly grasped: in the forward direction the physical amplitudes $\mathcal{F}^{pp}_{\bar{p}p}(s)$ can be written as $\mathcal{F}^{pp}_{\bar{p}p}(s) = F^{+}(s) \pm F^{-}(s)$

□ Considering that the only relevant contributions are those arising from the Pomeron and the Odderon exchanges, we can write the symmetric and antisymmetric amplitudes as $F^{+}(s) = R_{\mathbb{P}}(s) + iI_{\mathbb{P}}(s)$ and $F^{-}(s) = R_{\mathbb{O}}(s) + iI_{\mathbb{O}}(s)$

□ From the optical theorem, we have $s\sigma_{tot}^{pp,\bar{p}p}(s) = 4\pi \operatorname{Im} \mathcal{F}_{\bar{p}p}^{pp}(s) > 0$, which implies that

$$\operatorname{Im} \mathcal{F}_{\bar{p}p}^{pp}(s) = I_{\mathbb{P}}(s) \pm I_{\mathbb{O}}(s) > 0$$

and, in turn,

$$I_{\mathbb{P}}(s) > |I_{\mathbb{O}}(s)|$$

As a consequence,

$$I_{\mathbb{P}}(s) = \frac{s}{2}\left[\sigma_{tot}^{pp}(s) + \sigma_{tot}^{\bar{p}p}(s)\right] > 0$$

while

$$I_{\mathbb{O}}(s) = \frac{s}{2}\left[\sigma_{tot}^{pp}(s) - \sigma_{tot}^{\bar{p}p}(s)\right]$$

is not bound by the same positivity requirements