

# Revisit Lepton Flavor Violating Deep Inelastic Scattering

Masato Yamanaka (Hosei Univ., Japan)

M. Takeuchi, Y. Uesaka and MY, PLB772 (2017)

Y. Kiyoyama, M. Takeuchi, Y. Uesaka and MY, JHEP 04 (2022) 044

Y. Kiyoyama, M. Takeuchi, Y. Uesaka and MY, arXiv:2410.XXXXX

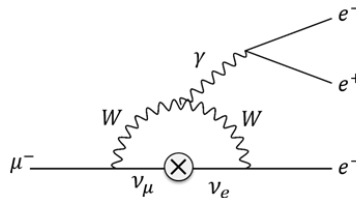
# Promising probe -- Lepton Flavor Violation (LFV) --

## In the SM

Lepton number is **always** conserved

## In the SM with $\nu$ oscillation

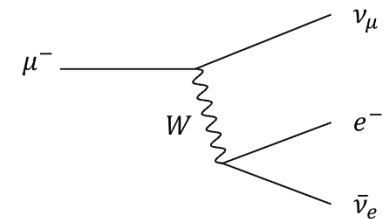
Will be discovered with  $10^{55} \mu$  😊



$$\text{BR} \approx 7.4 \times 10^{-55}$$

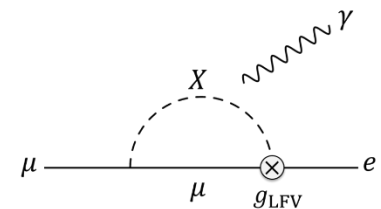
S. Petcov, Sov. J. Nucl. Phys. (1977)  
G. Hernandez-Tome, et al, EPJC (2019)

### Example: muon decay in SM



$$\begin{aligned} \mu \text{ number } +1 &\rightarrow +1 \\ e \text{ number } 0 &\rightarrow (+1) + (-1) \end{aligned}$$

### Ex.: muon decay in physics beyond SM



$$\begin{aligned} \mu \text{ number } +1 &\rightarrow 0 \\ e \text{ number } 0 &\rightarrow +1 \end{aligned}$$

# Promising probe -- Lepton Flavor Violation (LFV) --

## In the SM

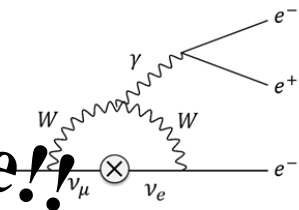
Lepton number is **always** conserved

## In the SM with $\nu$ oscillation

Will be discovered with  $10^{-15}$   $\mu$



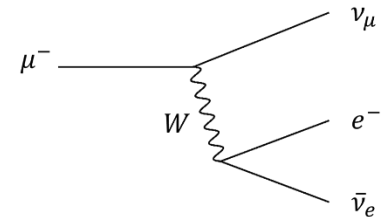
**Impossible!!**



$$BR \approx 7.4 \times 10^{-55}$$

S. Petcov, Sov. J. Nucl. Phys. (1977)  
G. Hernandez-Tome, et al, EPJC (2019)

### Example: muon decay in SM

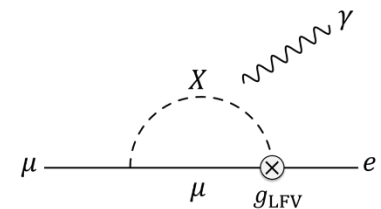


$$\begin{aligned} \mu \text{ number } +1 &\rightarrow +1 \\ e \text{ number } 0 &\rightarrow (+1) + (-1) \end{aligned}$$

## In the physics beyond the SM

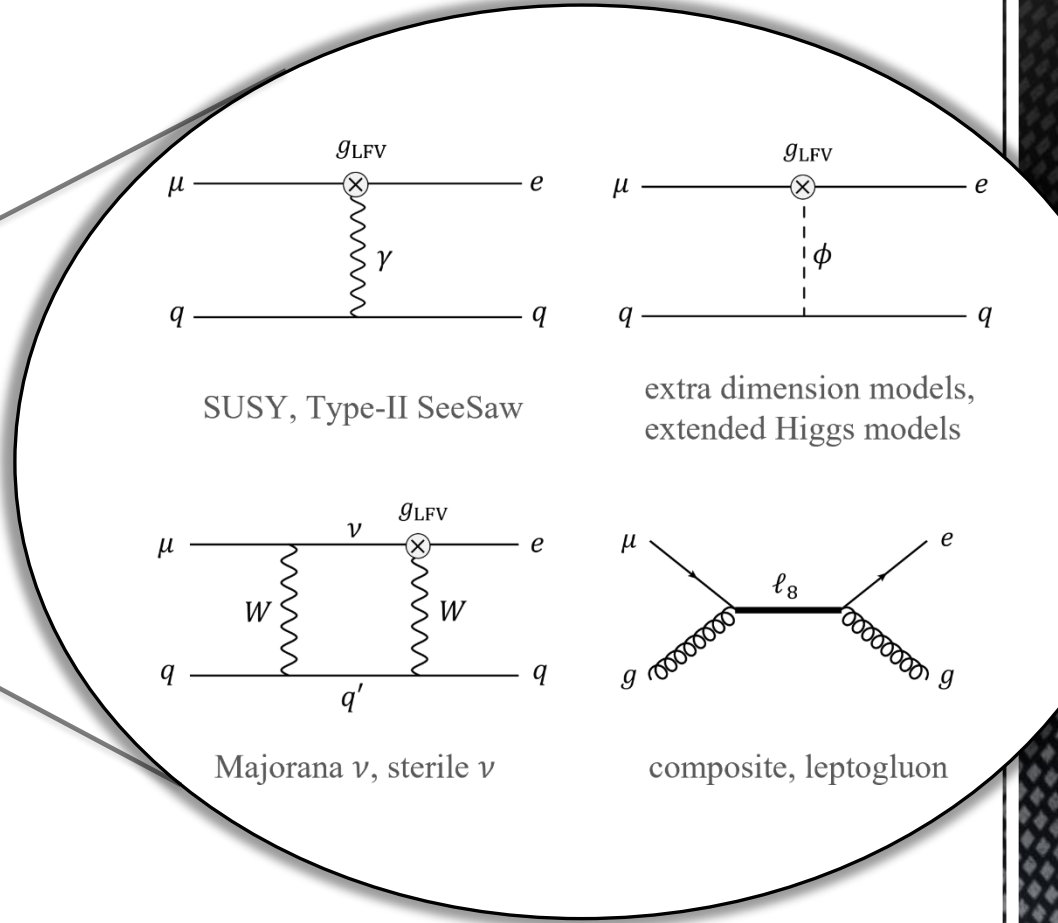
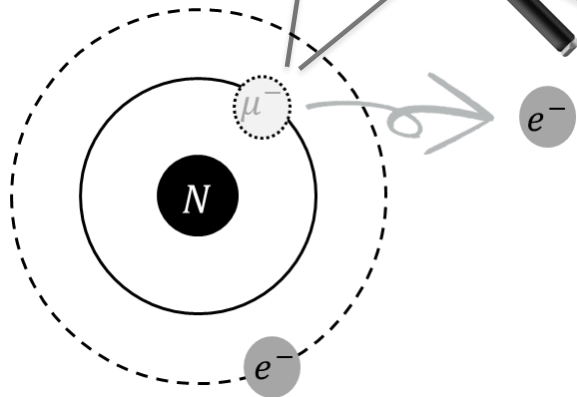
- accessible LFV rate by extra degrees of freedom  
(new particles, additional space dimension, ...)
- not only evidence, but also sensitive to the extra degrees

### Ex.: muon decay in physics beyond SM



$$\begin{aligned} \mu \text{ number } +1 &\rightarrow 0 \\ e \text{ number } 0 &\rightarrow +1 \end{aligned}$$

# How to unravel the physics behind the LFV



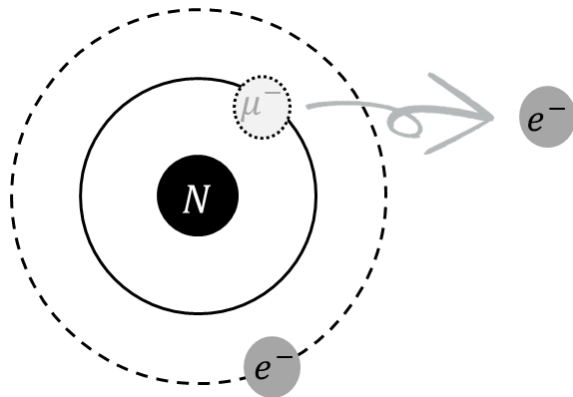
Unknown particle **indirectly** appears in LFV reactions

Not appeared in direct observables

# How to unravel the physics behind the LFV

## Important and necessary

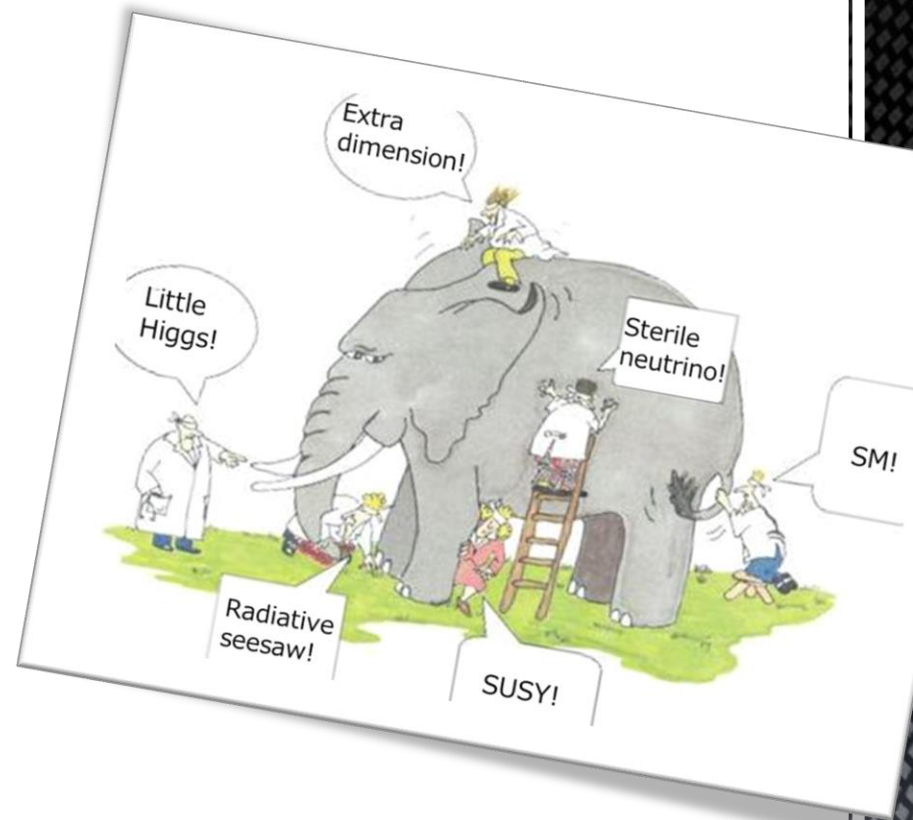
1. **Many LFV observables as possible**  
to draw “unknown” from various angles
2. **Accurate connection** between LFV parameters and observables



Unknown particle **indirectly** appears in LFV reactions

Not appeared in direct observables

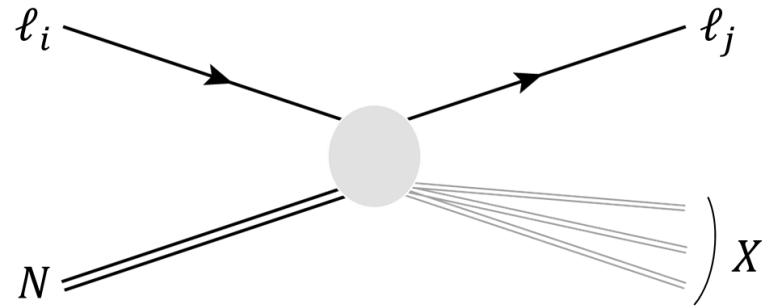
*The more LFV processes,  
the “elephant” is more clearly illustrated!*



# Lepton Flavor Violating Deep-Inelastic Scattering (LFV-DIS)

**LFV-DIS: Promising process to search for LFV**

$$\ell_i + N \rightarrow \ell_j + X \quad (N: \text{nucleon}, X: \text{hadron})$$

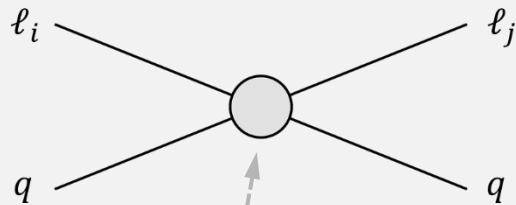


- ❑ Many experiments for cross check (LHeC,  $\nu$ -factory, ILC, ...)
- ❑ Complementary with  $\tau$  hadronic LFV ( $\tau \rightarrow e\pi\pi$ , etc) and LFV at LHC ( $pp \rightarrow \mu\tau$ , etc)
- ❑ Probe to the chirality of LFV ope. using polarized beam
- ❑ Large number of event  $\propto$  (beam intensity  $N_{\ell_i}$ )  $\times$  (nucleon density  $\sim$  mole number)  
@fixed target exp.

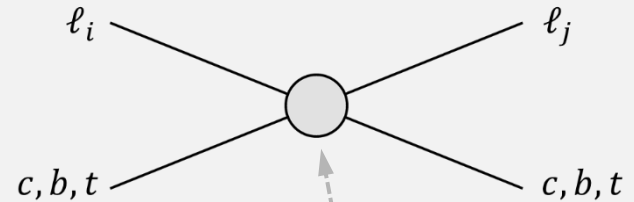
# Types of interaction between LFV mediator and quarks

## Universal

$$q \in \{u, d, c, s, t, b\}$$

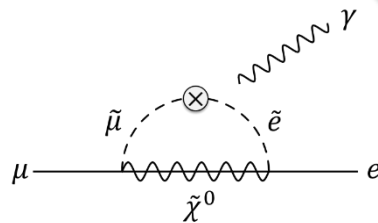


## Non-Universal



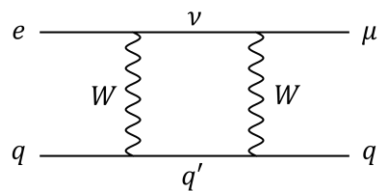
### Photonic dipole type

(SUSY, type-II SeeSaw models, etc)



### Box type

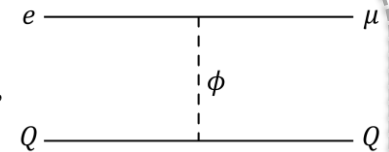
(sterile neutrino models, etc)



etc

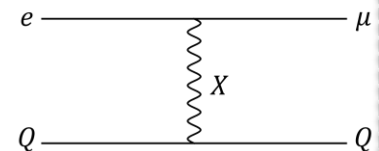
### Higgs LFV

(Extended Higgs models, etc)



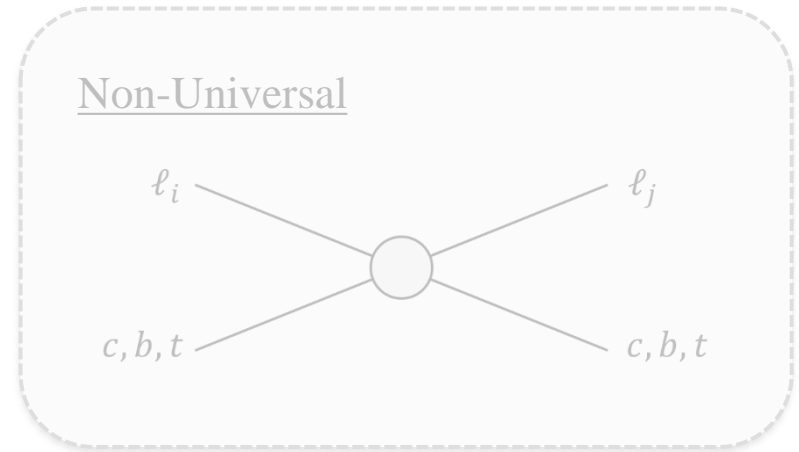
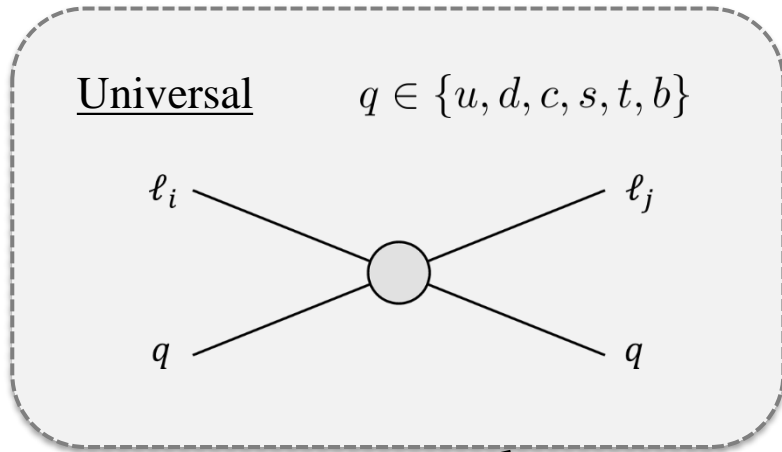
### Massive gauge boson

(Extra dimension models, Flavor symmetry models, etc)



etc

# Flavor universal interaction type



$$\sigma_{\ell_i N \rightarrow \tau X} = \sum_{q=u,d} \int dx dy \frac{d^2 \hat{\sigma}_{\ell_i q \rightarrow \tau q}}{dx dy} f_q(x, Q^2)$$

Parton distribution function (PDF)

Familiar scheme using subprocess  
with massless valence quarks

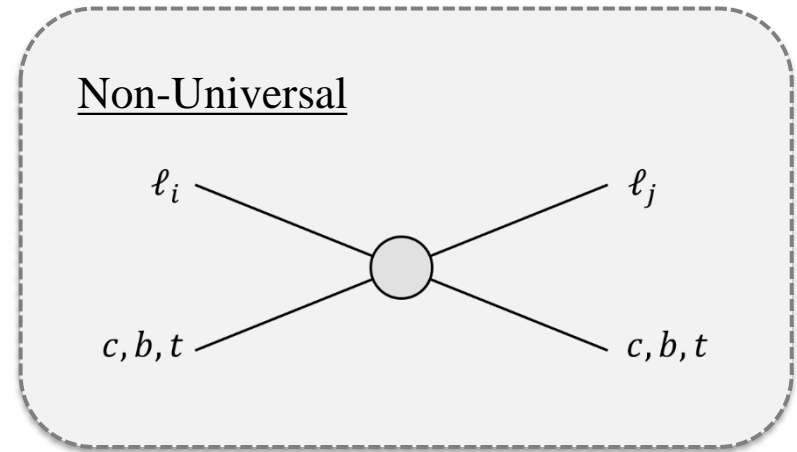
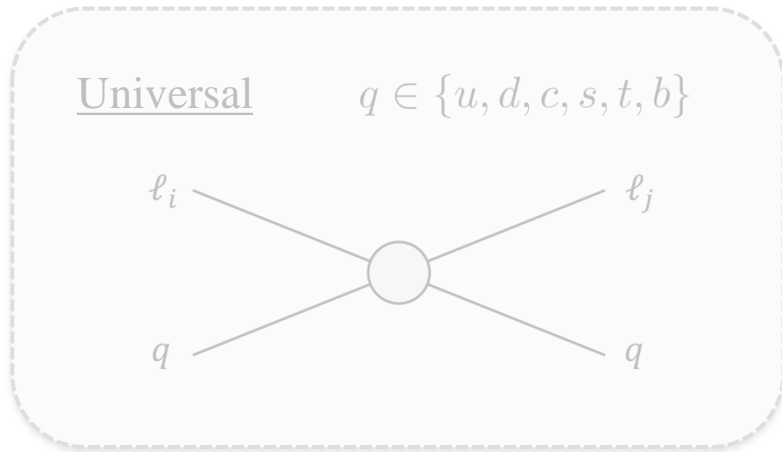
S. Gninenko, M. Kirsanov, et al, MPLA (2002),  
M. Sher, I. Turan, PRD (2004),  
A. Abada, V. Romeri, A. Teixeira, JHEP (2016),  
and so on

$x$  : Bjorken variable

$y$  : scattering inelasticity



# Flavor non-universal interaction type



Aim **Precisely connect LFV parameters and DIS observables**

No heavy quarks in nucleon  $\implies$  **What is subprocess for  $eN \rightarrow \tau X$ ?**

Inaccessible  $t$ -threshold due to too heavy  $\implies$  **How to identify the interaction and mediator mass?**

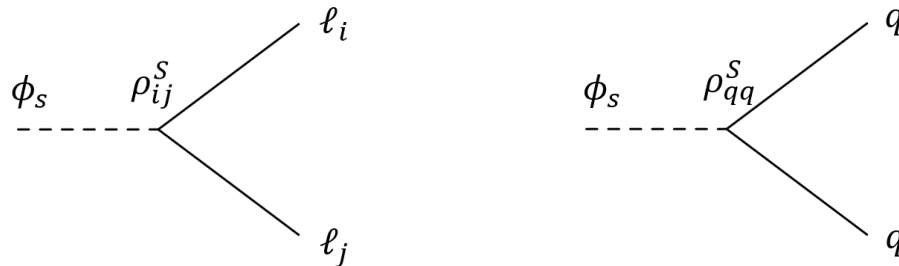
# **LFV interaction and cross section**

# LFV interaction (toy model)

Applicable to a variety of models  
(extended) Higgs model, leptoquark,  
R-parity violating SUSY, flavor sym.,  
extra dimension model, etc.

- LFV mediator : scalar
- Scenario : mediator dominantly couples with heavy fermion

$$\mathcal{L}_S = - \sum_{i,j} (\rho_{ij}^S \bar{\ell}_j P_L \ell_i \phi_S + h.c.) - \sum_q \rho_{qq}^S \bar{q} q \phi_S$$



Coupling with quarks (flavor diagonal only)

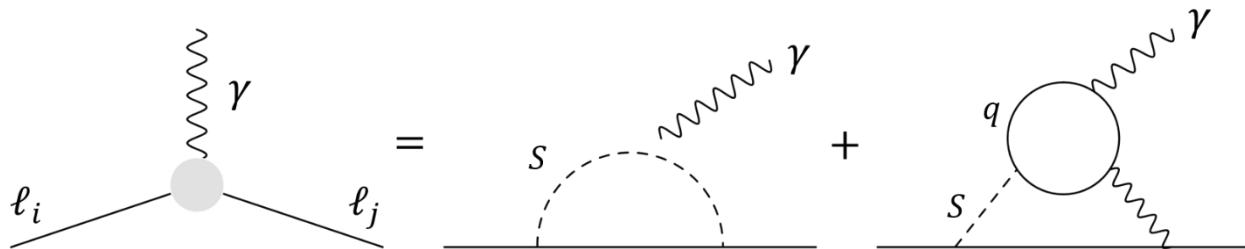
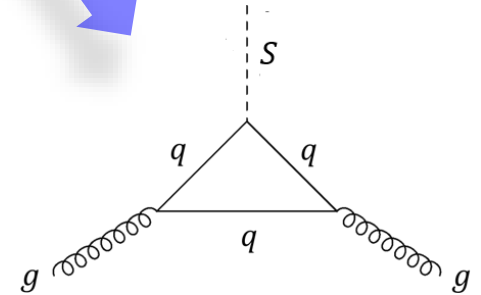
A) Proportional to quark mass  $\rho_{cc}^S : \rho_{bb}^S : \rho_{tt}^S = m_c : m_b : m_t$

B) Couple with 1 flavor only example :  $\rho_{bb}^S \neq 0, \rho_{cc}^S = \rho_{tt}^S = 0$

# LFV interaction (toy model)

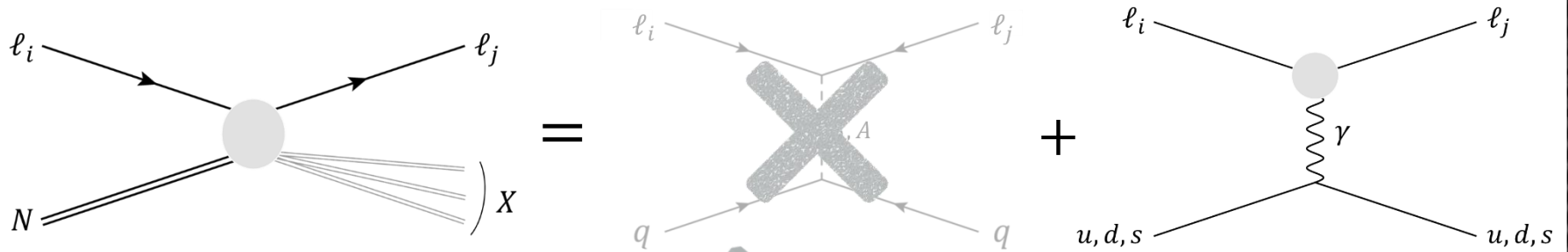
$$\mathcal{L}_S = - \sum_{i,j} (\rho_{ij}^S \bar{\ell}_j P_L \ell_i \phi_S + h.c.) - \sum_q \rho_{qq}^S \bar{q} q \phi_S$$

Two types of effective LFV interaction from the coupling with fermions



# Subprocess of LFV-DIS $\ell_i N \rightarrow \ell_j X$

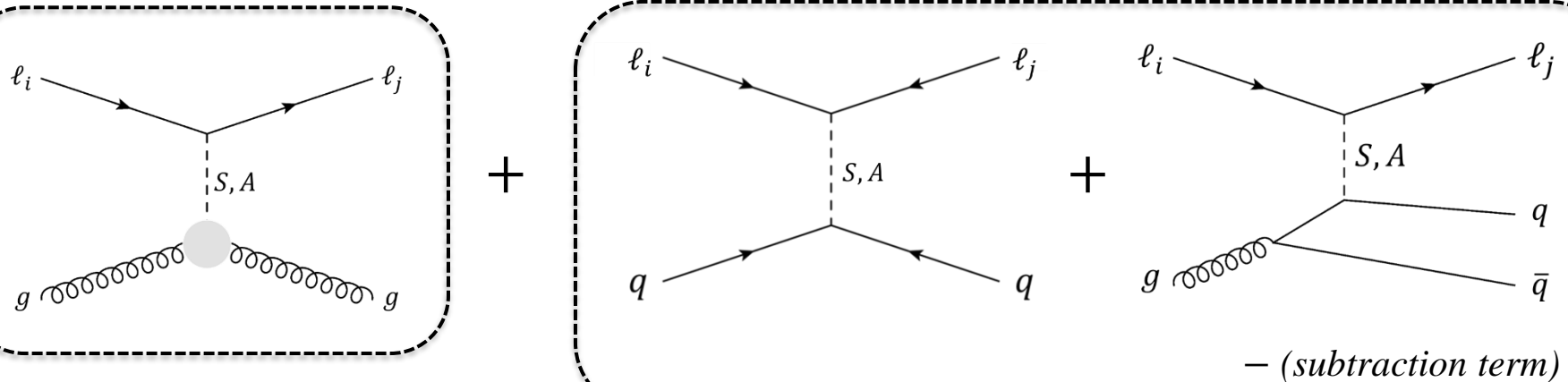
Take into account the (1)  $\phi gg$  coupling (2) quark-number conservation



Improved handling

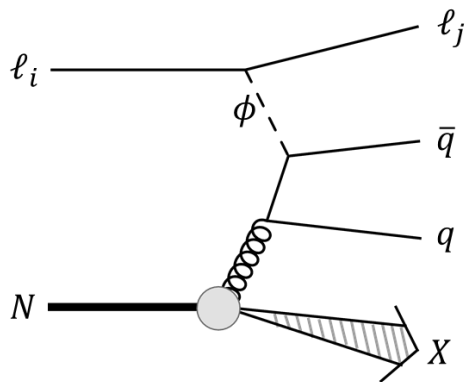
M. Takeuchi, Y. Uesaka, MY, PLB (2017)

ACOT scheme [PRD50 (1994)]



# ACOT scheme

M. Aivazis, J. Collins, F. Olness, W. Tung, PRD50 (1994)



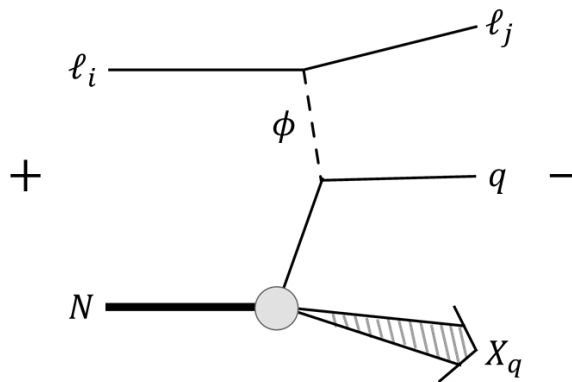
contribution with  $m_q \neq 0$



Subprocess associated with  $c, b, t$   
@low  $Q^2$

Lowest-order subprocess due  
to  $Q$ -number conservation

amplitude  $\propto \ln(Q^2/m_q^2)$



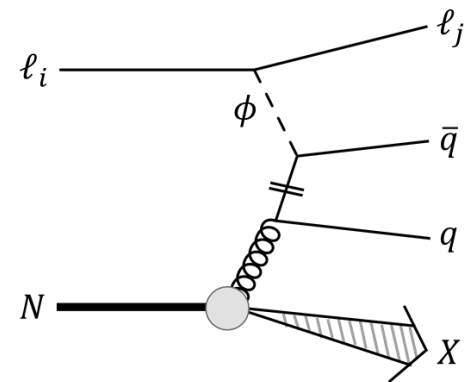
contribution with  $m_q = 0$



Subprocess associated with  $c, b, t$   
@high  $Q^2$

Subprocess described by  $c, b, t$  PDF

$c, b, t$  PDF : constructed by  
renormalizing divergent part for  
 $Q^2 \gg m_q^2$  with QCD correction



Subtraction contribution  
to cancel double counting

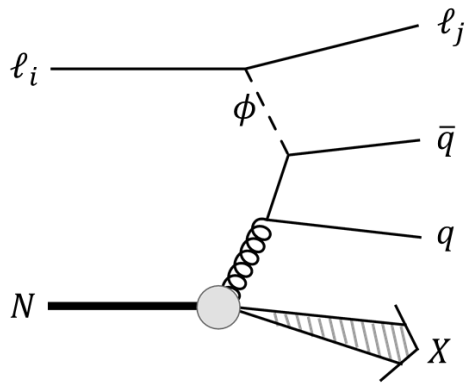
**Scheme for DIS XS with  
 $Q$ -number conservation  
and  $Q$ -mass**

Scalar structure function  
involving ACOT

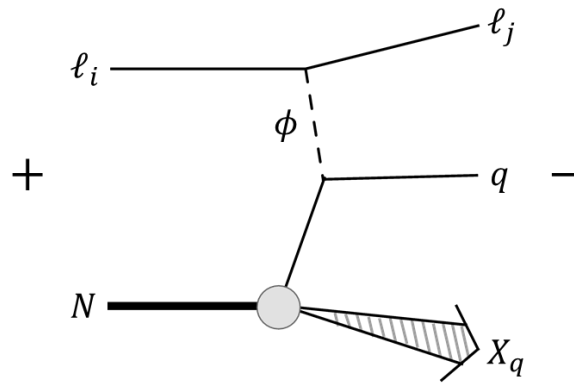
arXiv:2107.10840

# ACOT scheme

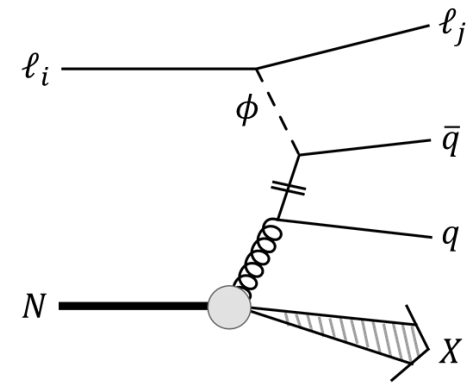
M. Aivazis, J. Collins, F. Olness, W. Tung, PRD50 (1994)



contribution with  $m_q \neq 0$



contribution with  $m_q = 0$



Subtraction contribution to cancel double counting

**Scheme formula interpolating two different energy scales ( $Q^2 \sim 4m_q^2$  and  $Q^2 \gg m_q^2$ )**

$$\frac{d^2\sigma}{dxdy} = \int_0^1 d\xi \left\{ \frac{d^2\hat{\sigma}_{l_i g \rightarrow l_j q \bar{q}}}{dxdy} f_g(\xi, Q^2) + \frac{d^2\hat{\sigma}_{l_i q \rightarrow l_j q}}{dxdy} f_q(\xi, Q^2) + \frac{d^2\hat{\sigma}_{l_i \bar{q} \rightarrow l_j \bar{q}}}{dxdy} f_q(\xi, Q^2) \right. \\ \left. - \left( \frac{d^2\hat{\sigma}_{l_i q \rightarrow l_j q}}{dxdy} + \frac{d^2\hat{\sigma}_{l_i \bar{q} \rightarrow l_j \bar{q}}}{dxdy} \right) \frac{\alpha_s}{2\pi} \int_\xi^1 \frac{dz}{z} f_g(z, Q^2) P_{gq}(\xi/z) \ln\left(\frac{Q^2}{m_q^2}\right) \right\}$$

Subtraction part

# Cross section

See e.g. T. Stavreva, F. I. Olness, et al, PRD85 (2012)  
M. Takeuchi, Y. Uesaka, M.Y., PLB772 (2017)

$x$  : Bjorken variable  
 $y$  : measure of inelasticity

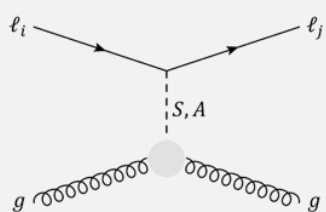
■ Momentum fraction :  $\xi = \frac{Q^2 + w^2}{Q^2} x$

■ Invariant mass of  $\hat{X}$  :  $w^2 = (p_q + p_{q'})^2$

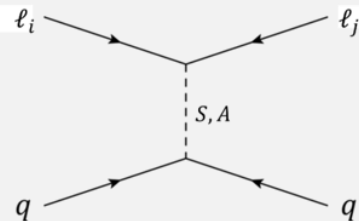
■ Momentum transfer :  $Q^2 = -(p_i - p_f)^2$

$$\sigma_{\ell_i N \rightarrow \ell_j X} = \sum_{\hat{p}=g,q} \int dx dy \int_0^1 d\xi \frac{d^2 \hat{\sigma}_{\ell_i \hat{p} \rightarrow \ell_j \hat{X}}}{dx dy} f_{\hat{p}}(\xi \hat{p}, Q^2)$$

PDF (function of  $\xi$ , not  $x$ !)

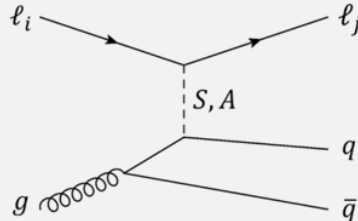


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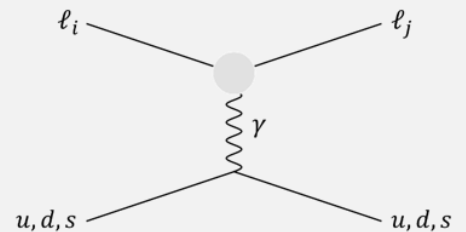
ACOT scheme, PRD50 (1994)

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- (subtraction term)

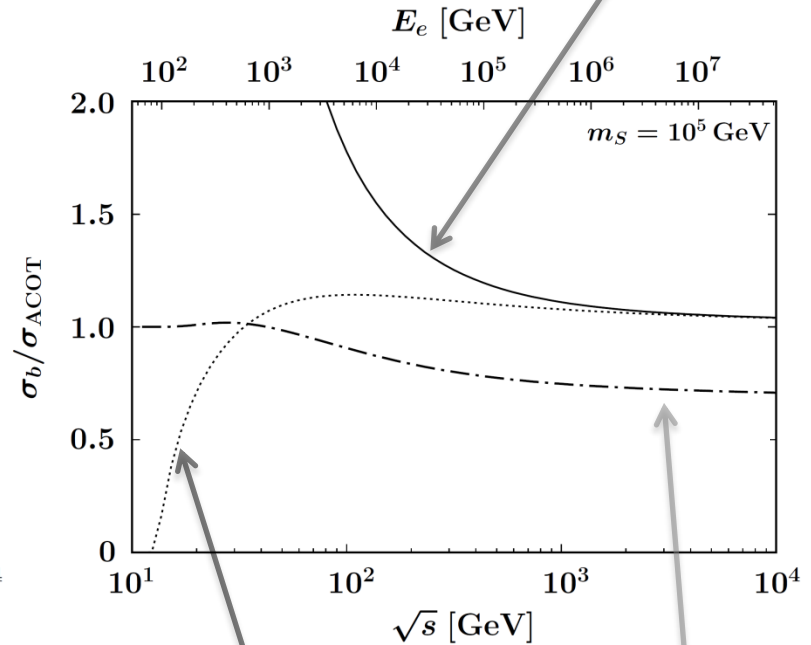
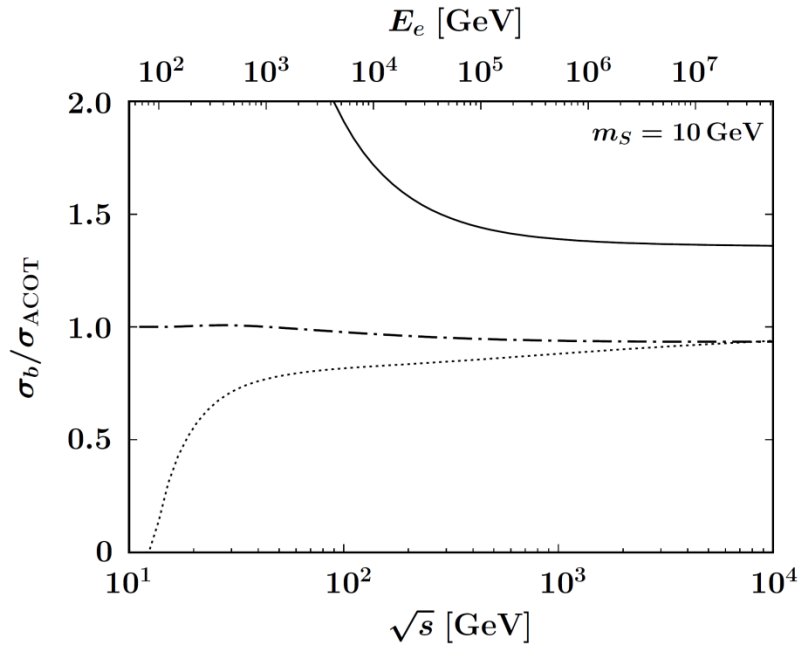
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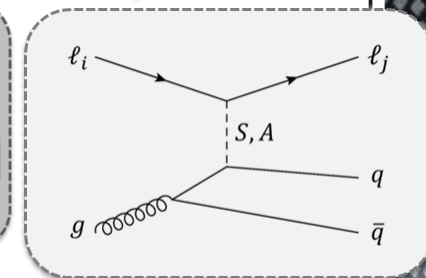
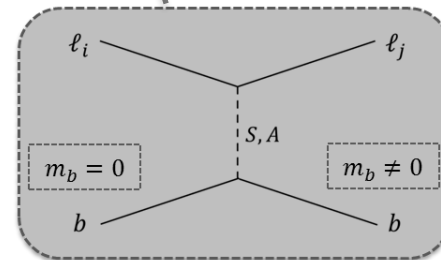


# Numerical analysis

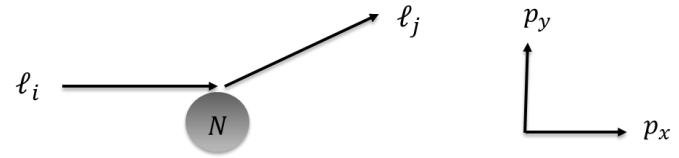
# Scheme dependence on LFV-DIS



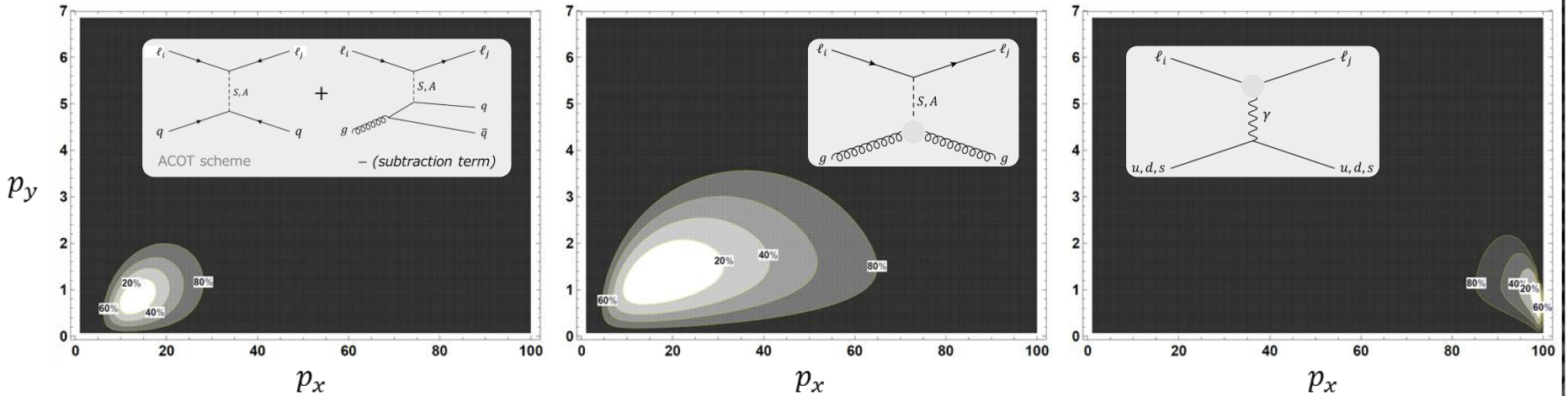
- ◆ Finite  $m_b$  effect even far above the  $b\bar{b}$  threshold
- ◆ **ACOT scheme: Indispensable for LFV-DIS mediated by scalar, especially for small  $m_S$**
- ◆  $\ell_i g \rightarrow \ell_j b\bar{b}$ : Acceptable alternative at the scale of next(-next) generation experiments



# Momentum distribution



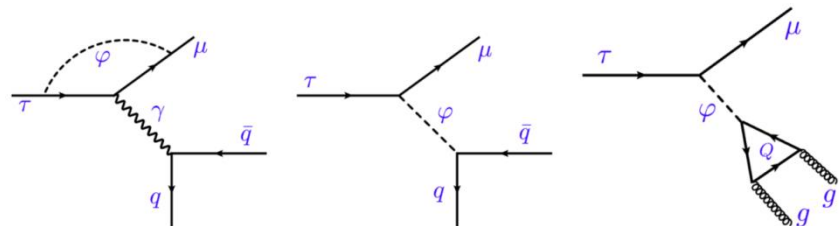
Beam energy = 100 GeV,  $m_S = 10$  GeV



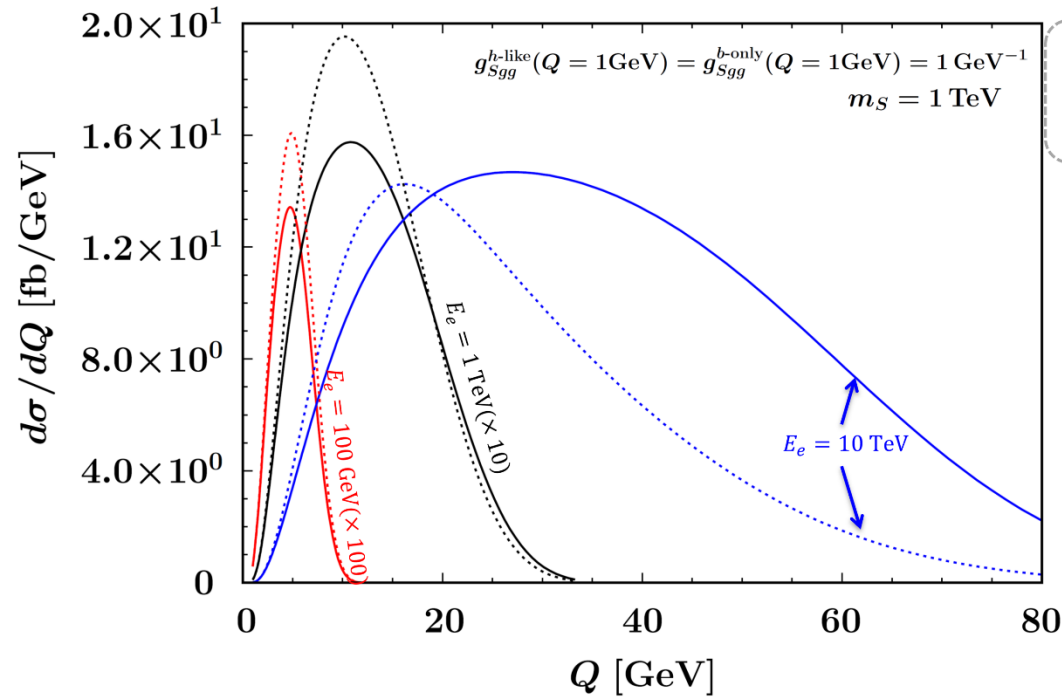
- ◆ Completely different distribution for each subprocess
- ◆ Sensitive probe to the interactions of mediator and quark/gluon/photon
- ◆ Synergy with  $\tau$  LFV decay

A. Celis, V. Cirigliano, E. Passemar, PRD89 (2014)

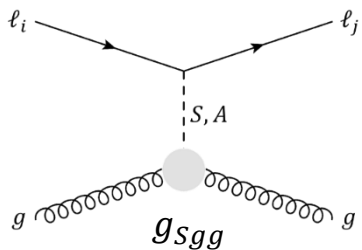
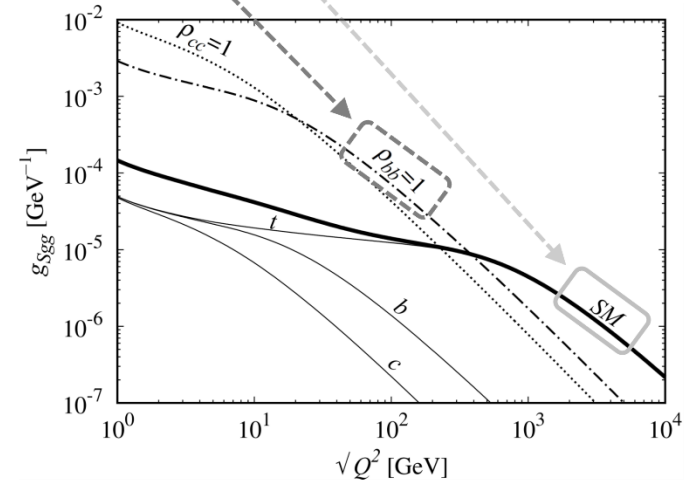
T. Husek, K. Monsalvez-Pozo, J. Portoles, JHEP01 (2021)



# Interaction between mediator and quarks



— Proportional to  $m_q$   $\rho_{cc}^S : \rho_{bb}^S : \rho_{tt}^S = m_c : m_b : m_t$   
 ..... Couple with  $b$  only  $\rho_{bb}^S \neq 0, \rho_{cc}^S = \rho_{tt}^S = 0$

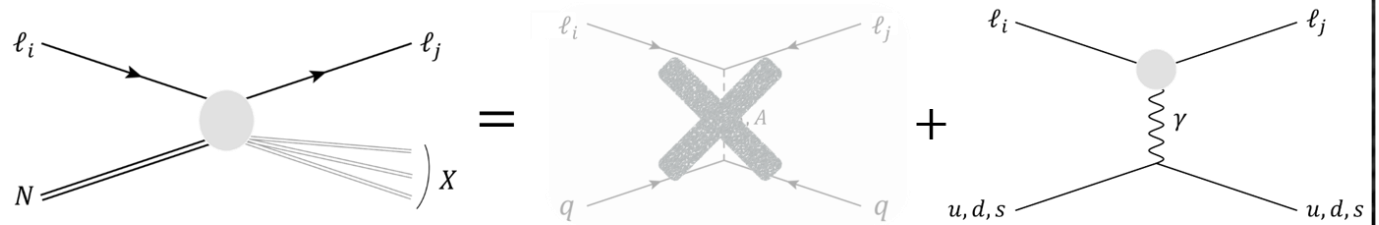


Mediator-gluon-gluon coupling  $g_{Sgg}$  strongly depends on

- (i) momentum transfer  $\sqrt{Q^2}$
- (ii) internal quark mass  $m_q$

**Differential cross section identifies interaction between mediator and quarks**

# Summary



❑ LFV-DIS: Promising process to search for LFV

❑ **Analysis on LFV-DIS taking into account important ingredients**

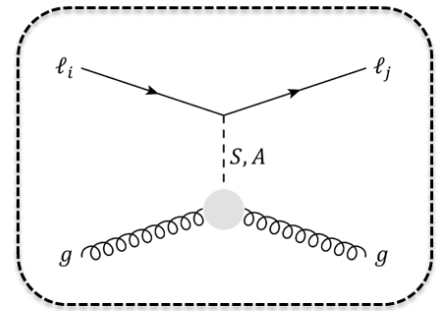
(1) **gluon contribution  $\ell_i g \rightarrow \ell_j g$**

(2) **ACOT scheme**

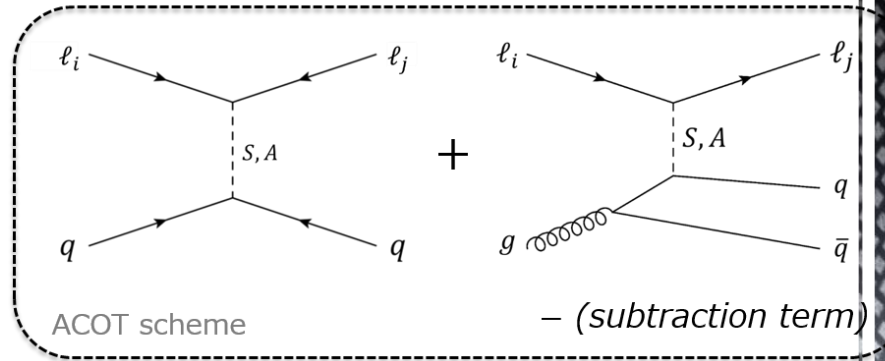
❑ Heavy quark effects ( $q$ -number conservation and finite mass) and the gluon contribution could change the cross section by an order

❑ Different distributions in each subprocess enable to identify the LFV operator

Improved handling



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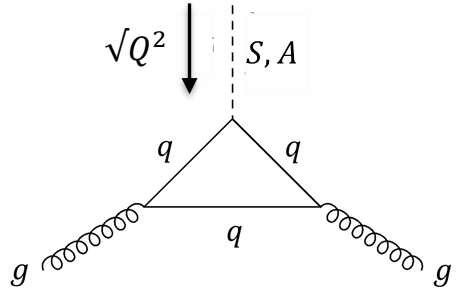


ACOT scheme

– (subtraction term)

**Backup slides**

# $\phi_{S(A)}gg$ effective coupling



$$\mathcal{L}_G = g_{Sgg} \phi_S G_{\mu\nu}^a G^{a\mu\nu} + g_{Agg} \phi_A G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

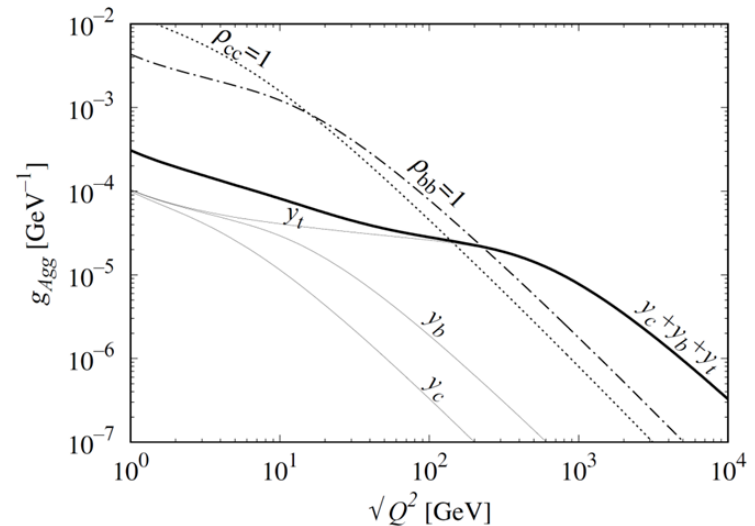
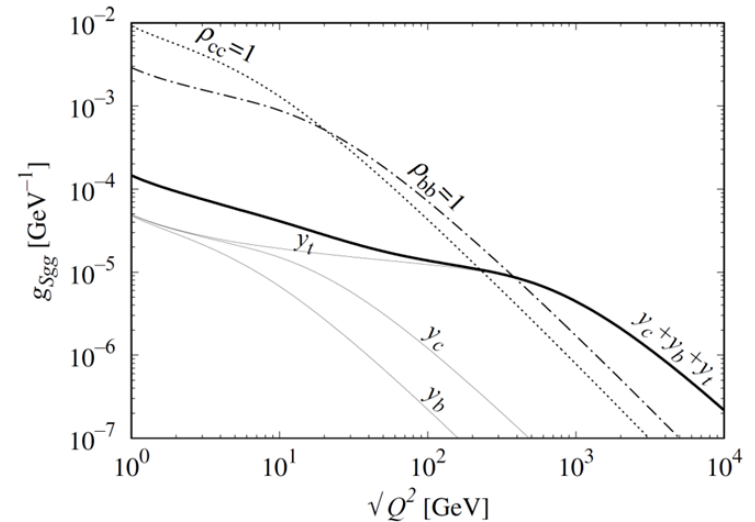
**Carefully handle following points to determine LFV ope.**

- Strong dependence of momentum transfer
- Pattern of mediator-quark interaction
- Sizable contributions of  $c$ - and  $b$ -quarks in addition to  $t$ -quark

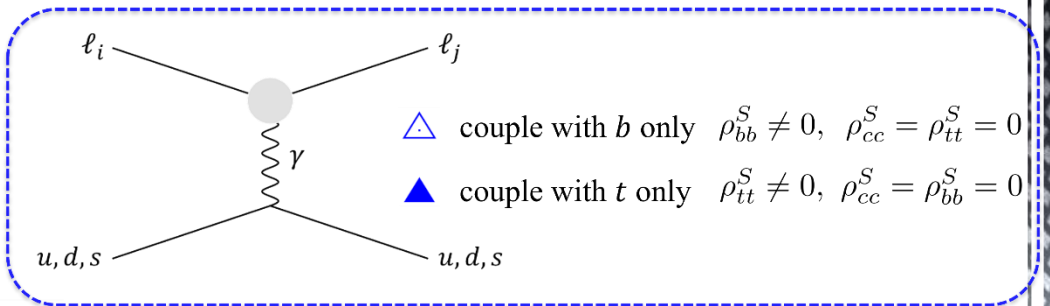
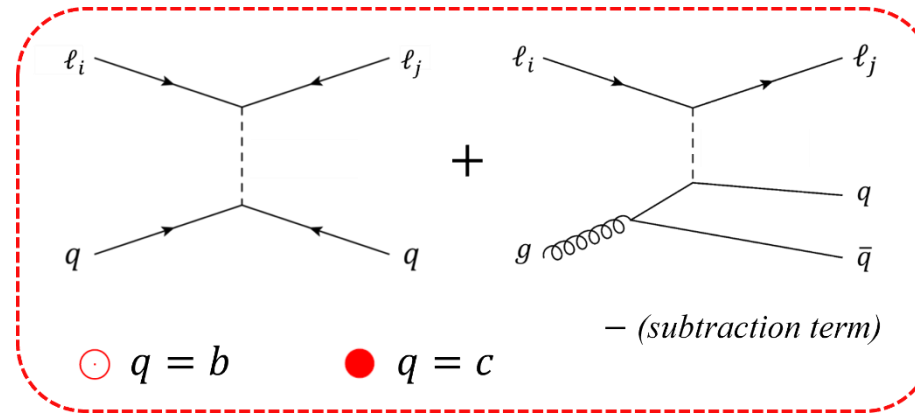
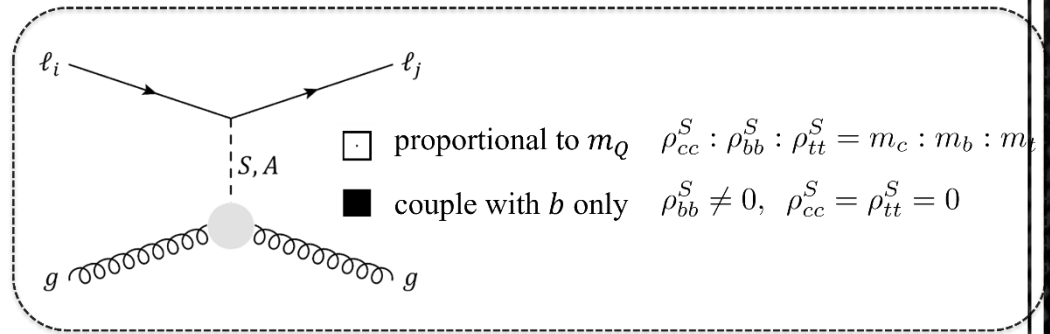
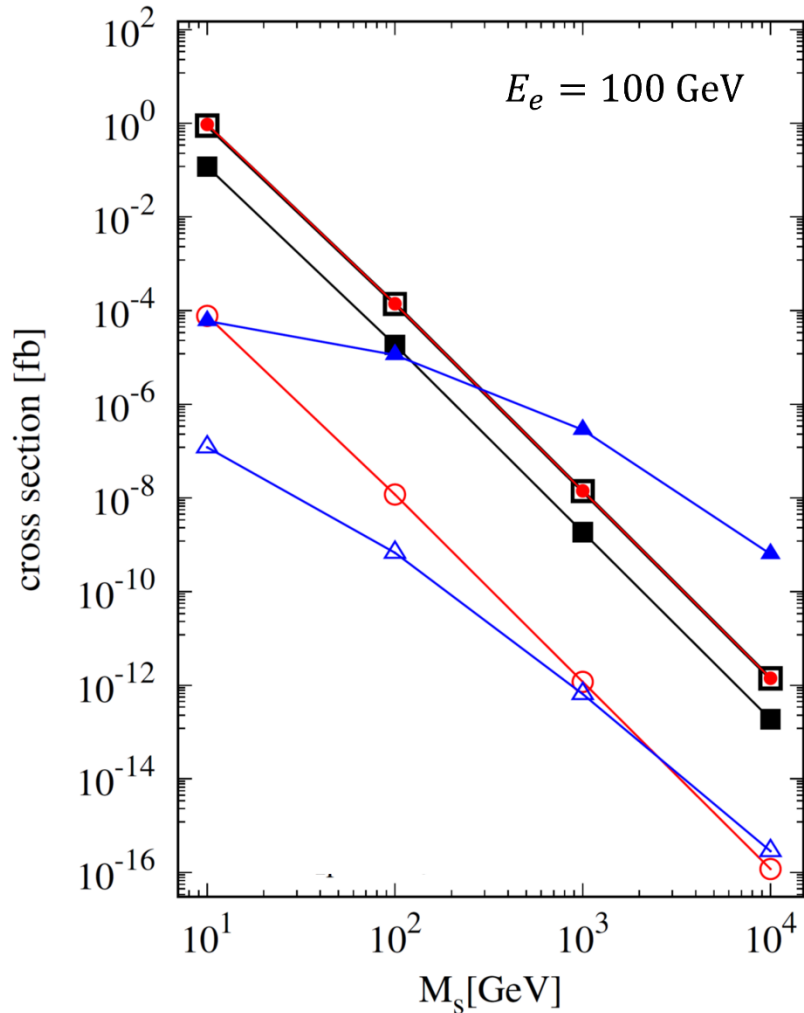
(a)  $\rho_{cc}^{S(A)} = 1, \rho_{bb}^{S(A)} = \rho_{tt}^{S(A)} = 0$

(b)  $\rho_{bb}^{S(A)} = 1, \rho_{cc}^{S(A)} = \rho_{tt}^{S(A)} = 0$

(c)  $\rho_{cc}^{S(A)} = y_c, \rho_{bb}^{S(A)} = y_b, \rho_{tt}^{S(A)} = y_t$



# Mediator mass dependence

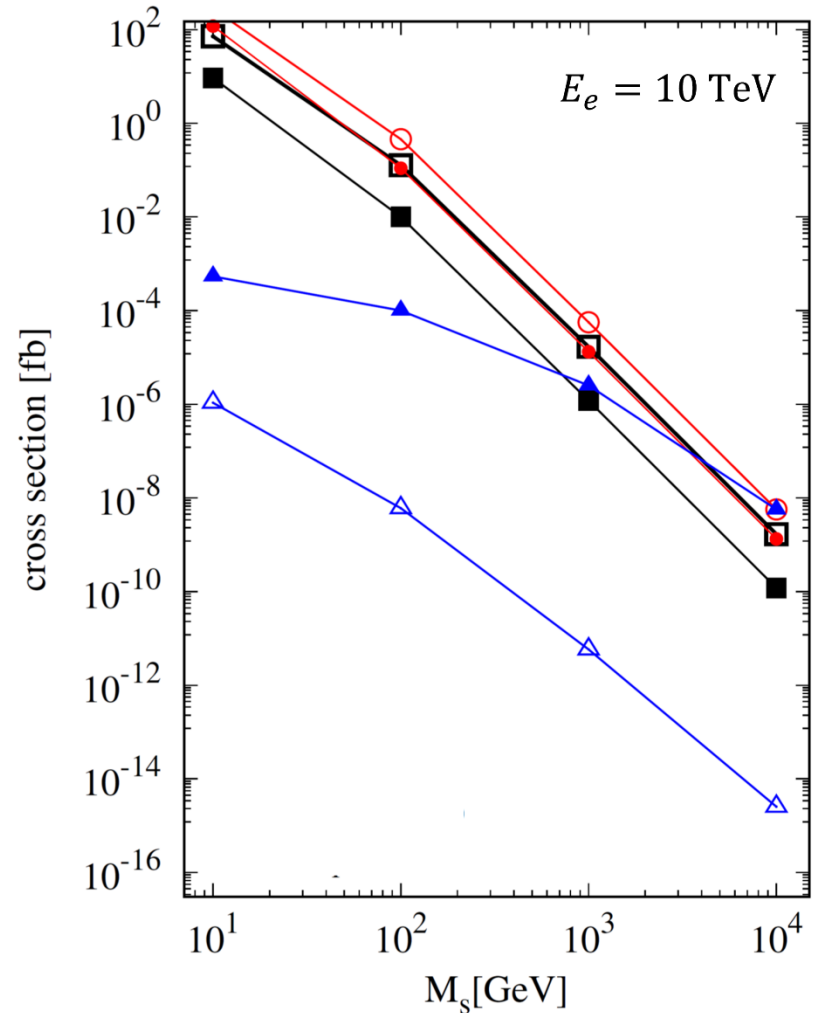
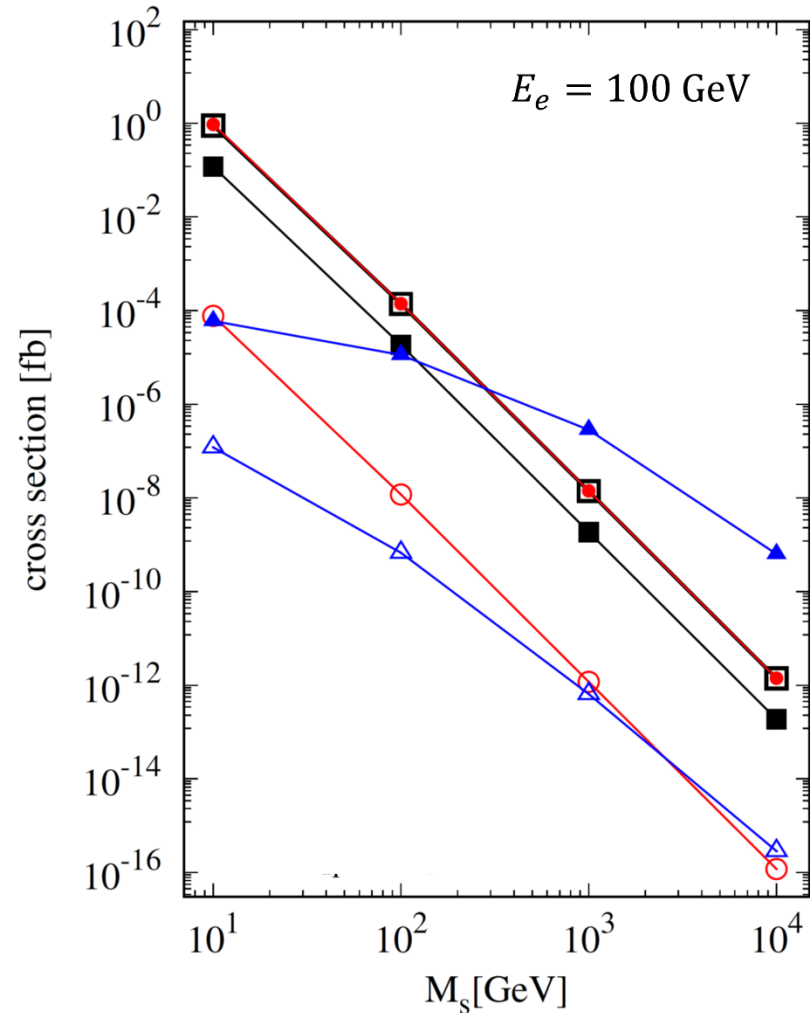




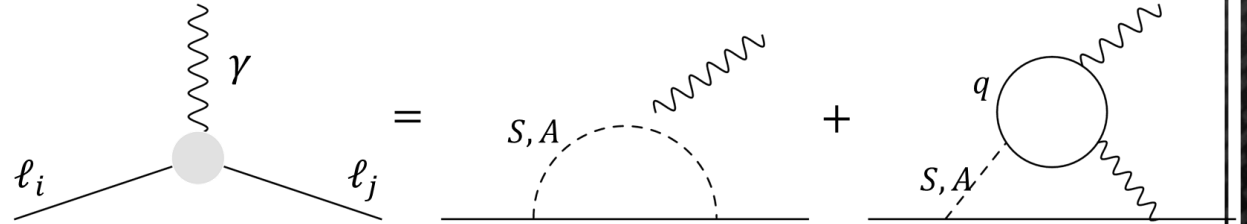
# Mediator mass dependence

$M_s$  : measure  $\sigma$  for each subprocess with different  $E_e$

Not sensitive to interaction between Med. and quark ...



# Photonic dipole



$$\mathcal{L}_{\text{dipole}} = -\frac{e}{2}m_j \sum_{X=S,A} \left( A_{ij}^X \bar{\ell}_j \sigma^{\mu\nu} P_L \ell_i F_{\mu\nu} + A_{ji}^X \bar{\ell}_j \sigma^{\mu\nu} P_R \ell_i F_{\mu\nu} \right)$$

$$A_{ij} = \frac{1}{16\pi^2 v^2} \left( A_1 + A_2^{t,b} + A_2^W \right)$$

e.g. coefficients in 2HDM as a function of scalar mass

Sensitive to models and mediator mass

$m_\phi$ [GeV]	125	200	300	400	500
$10^3 \times \tilde{A}_1^f(r_{\tau/\phi})$	2.0025	0.8872	0.4345	0.2605	0.1747
$10^3 \times \tilde{A}_2^{t,H}(r_{t/\phi})$	6.2431	4.6631	3.4720	2.7435	2.2504
$10^3 \times \tilde{A}_2^{t,A}(r_{t/\phi})$	8.9039	6.5746	4.8361	3.7840	3.0785
$10^3 \times \tilde{A}_2^{b,H}(r_{b/\phi})$	0.0407	0.0208	0.0114	0.0073	0.0052
$10^3 \times \tilde{A}_2^{b,A}(r_{b/\phi})$	0.0508	0.0255	0.0138	0.0088	0.0062
$10^3 \times \tilde{A}_{2,\phi}^W(r_{W/\phi})$	-14.0380	-8.8698	-5.1773	-2.9841	-1.5079

**Event rate via the dipole operator is useful for model discrimination**