# Speed of sound from ultracentral nucleus-nucleus collisions using the mean transverse momentum

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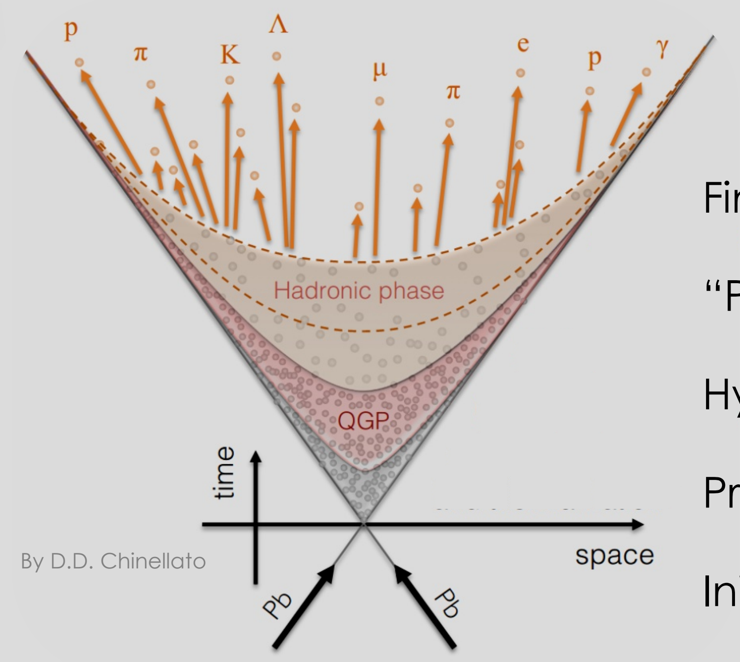
# Ultra-relativistic heavy-ion collisions

Observed particles
Final state observables

Nucleus-nucleus collisions @ modern colliders:
extract properties from the produced system

Currently best understood via **multi-stage hybrid hydrodynamic simulations**



By D.D. Chinellato

Final state dynamics [transport equations – UrQMD, SMASH]

“Particlization” [out-of-equilibrium corrections]

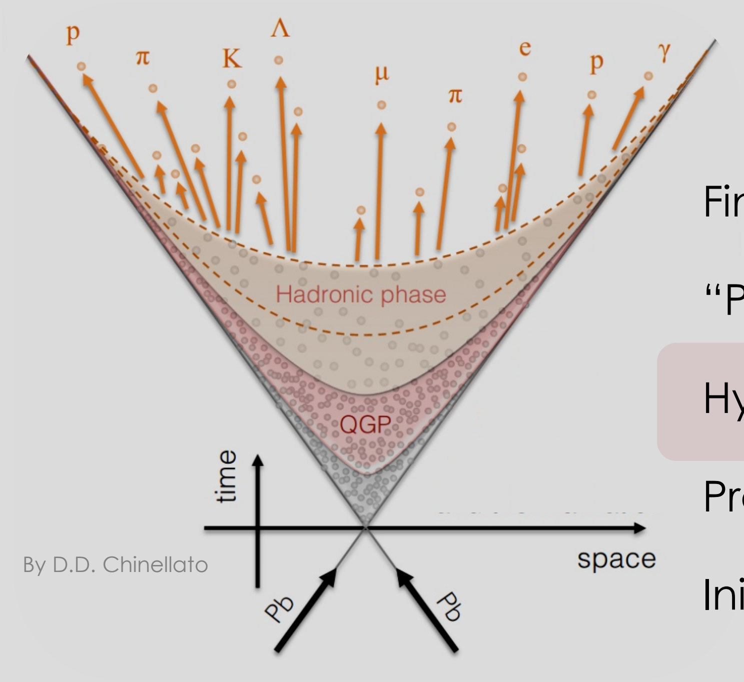
Hydrodynamical evolution [$\partial_\mu T^{\mu\nu} = 0$ + transport coefficients + EOS]

Pre-equilibrium phase [free-streaming, effective kinetic theory]

Initial conditions [MC-Glauber, MC-KLN, IP-Glasma, TRENTo, …]

# Ultra-relativistic heavy-ion collisions

Observed particles
Final state observables



By D.D. Chinellato

Information needed for such simulations currently determined via Bayesian analysis

**Focus on the hydrodynamic phase and the sound velocity of the QGP**

Final state dynamics [transport equations – UrQMD, SMASH]

“Particlization” [out-of-equilibrium corrections]

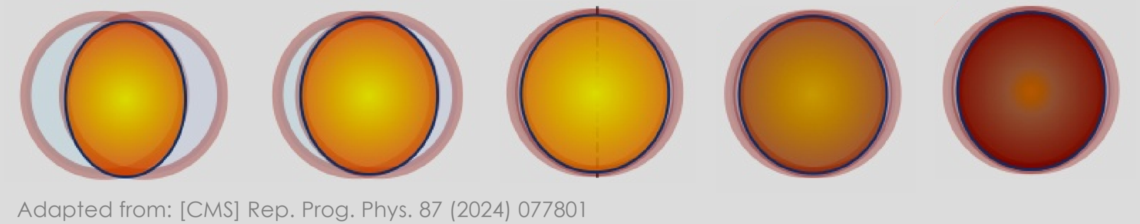
Hydrodynamical evolution $[\partial_\mu T^{\mu\nu} = 0$ + transport coefficients + EOS]

Pre-equilibrium phase [free-streaming, effective kinetic theory]

Initial conditions [MC-Glauber, MC-KLN, IP-Glasma, TRENTo, …]

# Ultra-relativistic heavy-ion collisions

Will make use of “ultracentral” collisions



Adapted from: [CMS] Rep. Prog. Phys. 87 (2024) 077801

Nearly vanishing impact parameter
Events with highest multiplicities

On average: fixed collision geometry, but multiplicity can vary up to ~15% along with $\langle p_T \rangle$

Excellent laboratory to study changes in system density at fixed geometry

speed of sound in the fluid: change in temperature due to system density

$$c_s^2 = \frac{dP}{d\varepsilon} = \frac{s}{T}\frac{dT}{ds} = \frac{d\ln T}{d\ln s}$$

[Vanishing net-baryon density]

$$\varepsilon = -P + Ts + \mu n \qquad dP = s dT + n d\mu$$

A. V. Giannini

# Relation to measurable quantities

Temperature: space-time dependent, decreasing with time

Relation involving $\langle p_T \rangle$ of produced particles and an effective temperature has been identified recently:

Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. 16, no.6, 615 (2020)

$$E = \int_{\text{frzout}} T^{0\mu} d\sigma_\mu = \epsilon(T_{\text{eff}})V_{\text{eff}}$$

$$S = \int_{\text{frzout}} s u^\mu d\sigma_\mu = s(T_{\text{eff}})V_{\text{eff}}$$

→ Total energy, including kinetic motion

The **total E and S of the medium at freeze-out** is put into a uniform, static fluid with **effective temperature $T_{\text{eff}}$** and **effective volume $V_{\text{eff}}$**

$\varepsilon\, ,\, s\, ,\, T_{\text{eff}}$ : related by EOS

**$T_{\text{eff}}$ : initial QGP temperature if total energy is conserved all the way to the freeze-out and measured**

# Relation to measurable quantities

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Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. 16, no.6, 615 (2020)

$$\langle p_T \rangle = 3\, T_{\mathrm{eff}}$$

$$T_{\mathrm{freeze-out}} < T_{\mathrm{eff}} < T_{\mathrm{initial}}$$

Shown to be independent of the transport coefficients and collision centrality

Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. 16, no.6, 615 (2020)

Equality with lower limit only if system is at rest, which is never achieved due to transverse flow

# Relation to measurable quantities

$$\langle p_T \rangle = 3\,T_{\mathrm{eff}}$$

$$T_{\mathrm{freeze-out}} < T_{\mathrm{eff}} < T_{\mathrm{initial}}$$

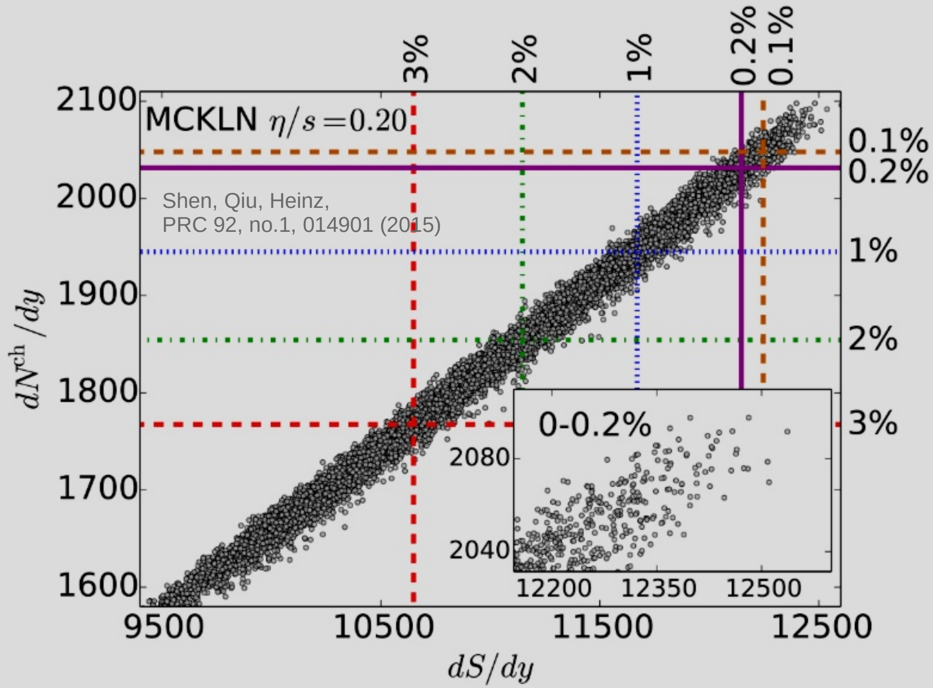
Entropy is well correlated to the number of produced particles, $\mathrm{S} \propto N_{\mathrm{ch}}$

$$c_s^2 = \frac{d\ln T}{d\ln s} = \frac{d\ln\langle p_T\rangle}{d\ln N_{\mathrm{ch}}}$$

$$\ln\langle p_T\rangle = const + c_s^2(T_{\mathrm{eff}})\ln N_{\mathrm{ch}}$$

For the most part, the above relation for $c_s^2$ has been tested at initial condition level

Gardim, Giacalone, Ollitrault, PLB 809 (2020) 135749



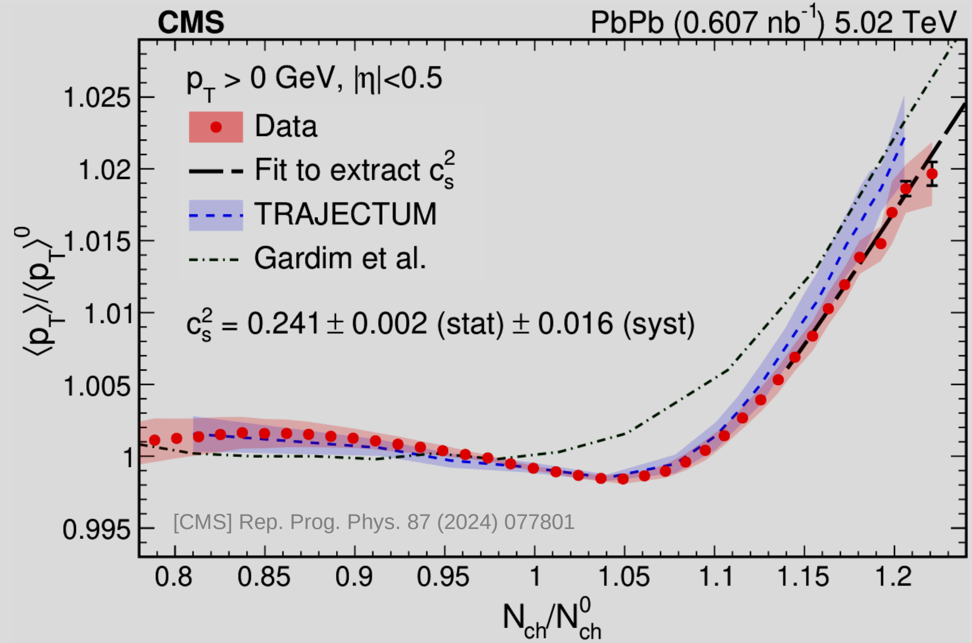
Shen, Qiu, Heinz, PRC 92, no.1, 014901 (2015)

# Recent CMS measurement

$$\langle p_T \rangle = 3\,T_{\text{eff}}$$

$$T_{\text{freeze-out}} < T_{\text{eff}} < T_{\text{initial}}$$

$$c_s^2 = \frac{d \ln \langle p_T \rangle}{d \ln N_{\text{ch}}}$$



[CMS] Rep. Prog. Phys. 87 (2024) 077801

"(...) *ultracentral* $\langle p_T \rangle$ *may not be a direct measurement of the speed of sound*"

Nijs, van der Schee, PLB 853 (2024) 138636

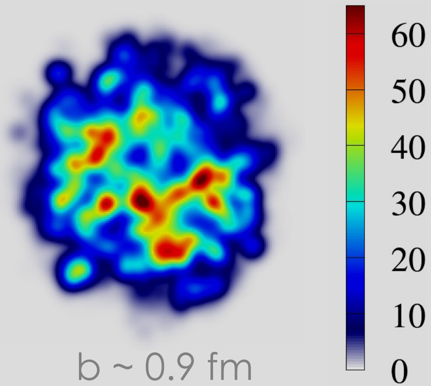
**Goal: assess more precisely the validity of the above relations by means of systematic hydrodynamic calculations**

# Setup of our simulations: initial conditions

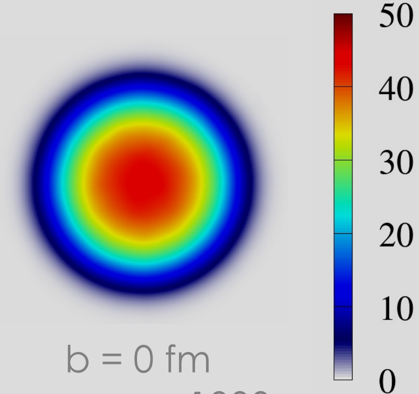
Duke's simulation chain (2020)

Moreland, Bernhard, Bass, PRC 101, no.2, 024911(2020)

p+Pb @ 5.02 TeV
Pb+Pb @ 5.02 TeV
**Run using MAP values** (Table IV)
**Only b = 0 fm, smooth ic**



b ~ 0.9 fm
TRENTo



b = 0 fm
avg. over 1000
TRENTo ics

Consider a **smooth** initial condition for fixed impact parameter, b = 0 fm

Assess **multiplicity variations** by **rescaling** the initial condition

initial state fluctuations,
**energy variation**

# Setup of our simulations: recovering the “smoothness”

Duke’s simulation chain (2020)

Moreland, Bernhard, Bass, PRC 101, no.2, 024911(2020)

p+Pb @ 5.02 TeV

Pb+Pb @ 5.02 TeV

**Run using MAP values** (Table IV)

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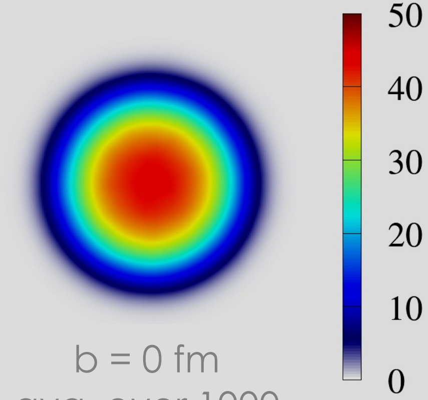


b = 0 fm
avg. over 1000
TRENTo ics

IC → Free-streaming → Hydrodynamics → **Particlization** → Final-state dynamics

Conversion from fluid to particles destroys the smoothness imposed at the initial state

Reduce discretization effects by sampling and running transport calculations (UrQMD) 1000 times for each hydro event

# Setup of our simulations: EOS

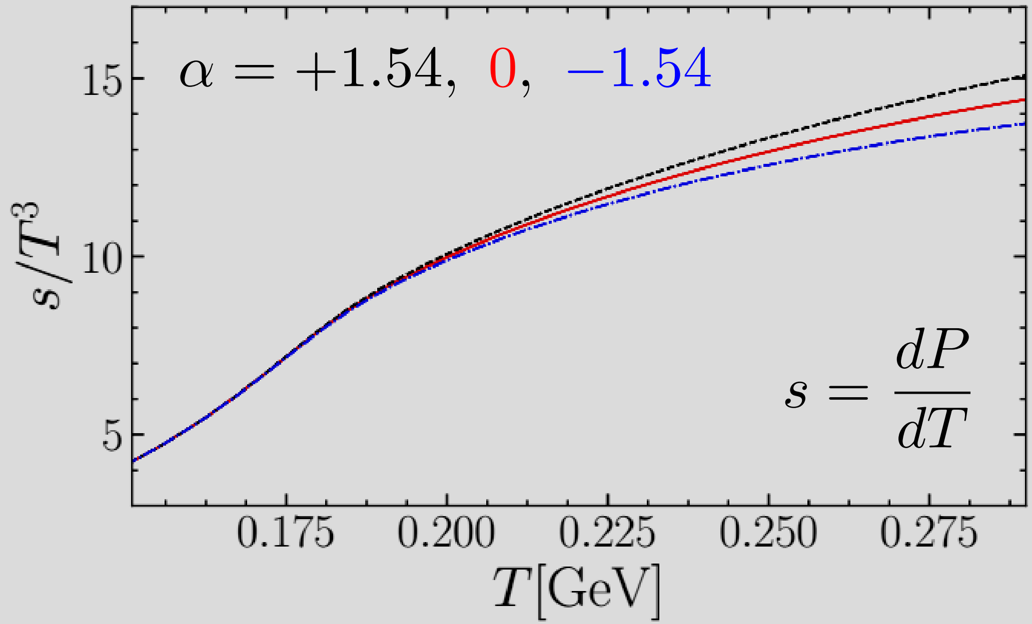
Duke's simulation chain (2020)

Moreland, Bernhard, Bass, PRC 101, no.2, 024911(2020)

EOS from **HotQCD**, matching a HRG at low temperatures, $T_{\text{frzout}} \equiv T_F = 151\,\text{MeV}$

A. Bazavov et al. [HotQCD], PRD 90, 094503 (2014)

$$P(T) \to P(T) + \alpha(T - T_F)^4, \quad T \geq T_F$$



**Small fluctuations around Lattice EOS**

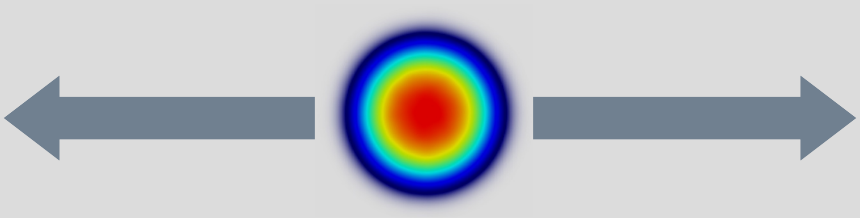
$T_{\text{eff}}$ averaged over the whole system (misses sharp EOS variations)

**Test whether it is possible to capture small changes in $c_s^2$ through experimental data**

# Results

25 values of charged particle multiplicities: 5 sets of 5 values

[0.5, 0.75, **1**, 1.25, 1.5]



Lower than average

Matches $\langle dN_{ch}/d\eta \rangle$ at 5.02 TeV

Larger than average

Simulate multiplicity fluctuations at fixed energy

Will be used to infer $c_s^2$ by a linear fit

**From produced particles:** $\langle p_T \rangle$ **&** $dN_{ch}/d\eta$
[after UrQMD]

**$T_{\text{eff}}$** from EOS @ hydro phase $$\frac{E}{S} = \frac{\epsilon(T_{\text{eff}})}{s(T_{\text{eff}})}$$

# Results

$\langle p_T \rangle$ **and** $dN_{ch}/d\eta$ **increase with collision energy**

Softer EOS (Harder EOS) → smaller (larger) $\langle p_T \rangle$

$\langle p_T \rangle \approx 3T_{\text{eff}}$ **to a good approximation for all cases!**

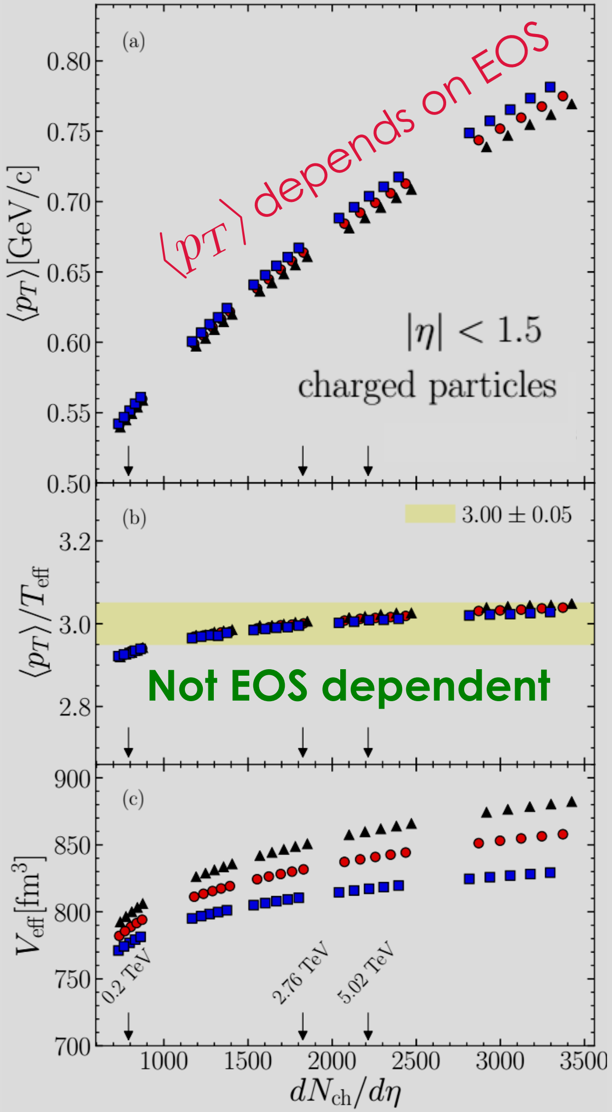
Very mild increase with collision energy

$$\langle p_T \rangle / T_{\text{eff}} = 2.93 \pm 0.05 \quad @ \quad 200\ \text{GeV}$$

Agrees with previous studies

Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. 16, no.6, 615 (2020)
Gardim, Krupczak, Nunes da Silva, PRC 109, no.1, 014904 (2024)

$V_{\text{eff}}$ shows some dependence on the EOS, significant uncertainty



$\langle p_T \rangle$ depends on EOS

**Not EOS dependent**

# Results 2: speed of sound

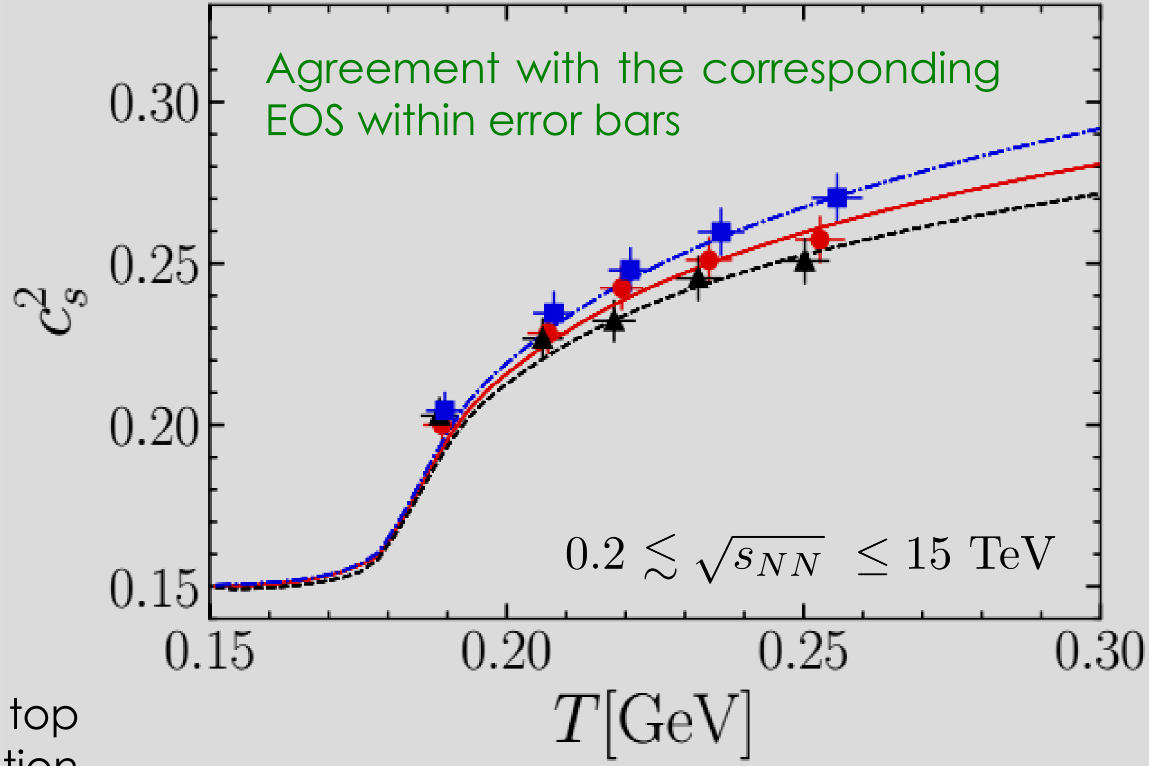
$$c_s^2(T_{\text{eff}}) = \frac{d\ln\langle p_T\rangle}{d\ln N_{\text{ch}}}$$

$$\ln\langle p_T\rangle = const + c_s^2(T_{\text{eff}})\ln N_{\text{ch}}$$

$$T_{\text{eff}} \propto \langle p_T\rangle$$

For each set of 5 points: fit the resulting $\langle p_T\rangle$ and extract $c_s^2$

$c_s^2$ overestimated by ~ 0.01 for RHIC's top energy; likely due to sharp EOS variation not built in $T_{\text{eff}}$



Errobars:
vertical → jackknife
Horizontal → ± 0.05 from $\langle p_T\rangle/T_{\text{eff}}$

# Results 2: speed of sound

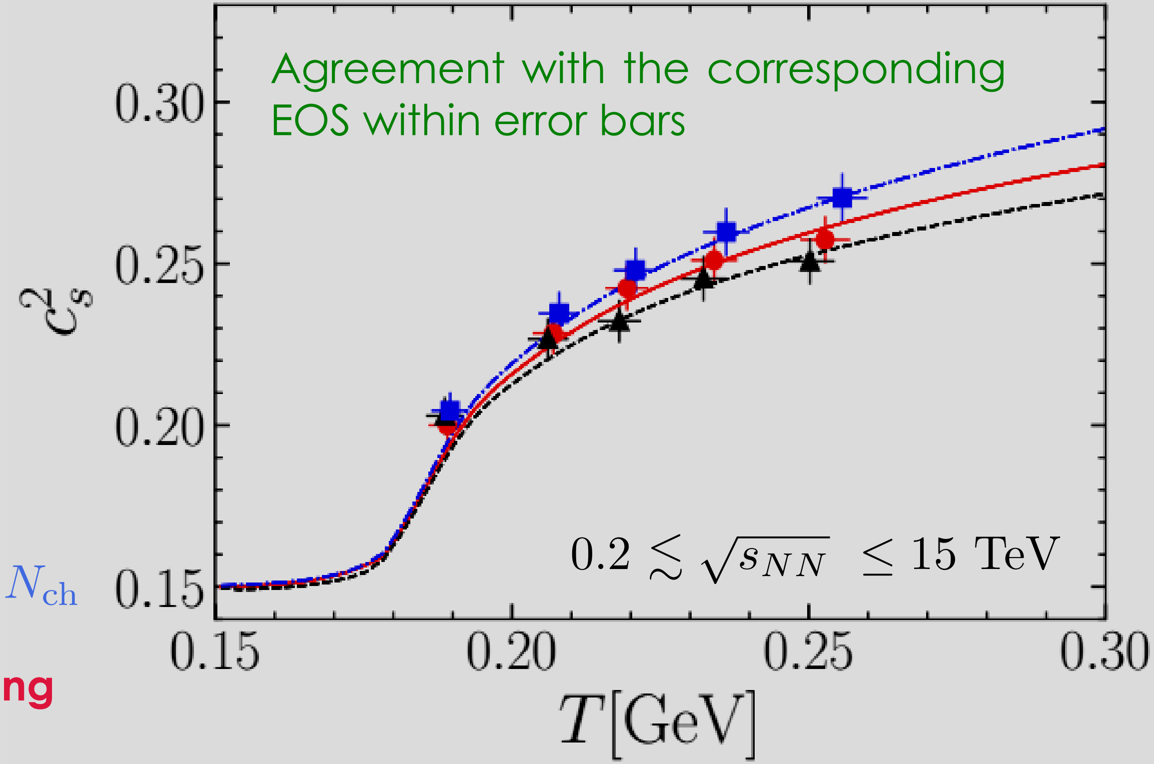
$$c_s^2(T_{\rm eff}) = \frac{d\ln\langle p_T\rangle}{d\ln N_{\rm ch}}$$

$$\ln\langle p_T\rangle = const + c_s^2(T_{\rm eff})\ln N_{\rm ch}$$

$$T_{\rm eff} \propto \langle p_T\rangle$$

b = 0 + smooth initial condition: $\langle p_T\rangle$ promoted to a single value function of $N_{\rm ch}$

**Not true anymore if b = 0 + fluctuating initial conditions!**



Errobars:
vertical → jackknife
Horizontal → ± 0.05 from $\langle p_T\rangle / T_{\rm eff}$

# Discussion 1: kinematic cuts, $p_T$ & $\eta$

$$c_s^2(T_{\rm eff}) = \frac{d\ln\langle p_T\rangle}{d\ln N_{\rm ch}}$$

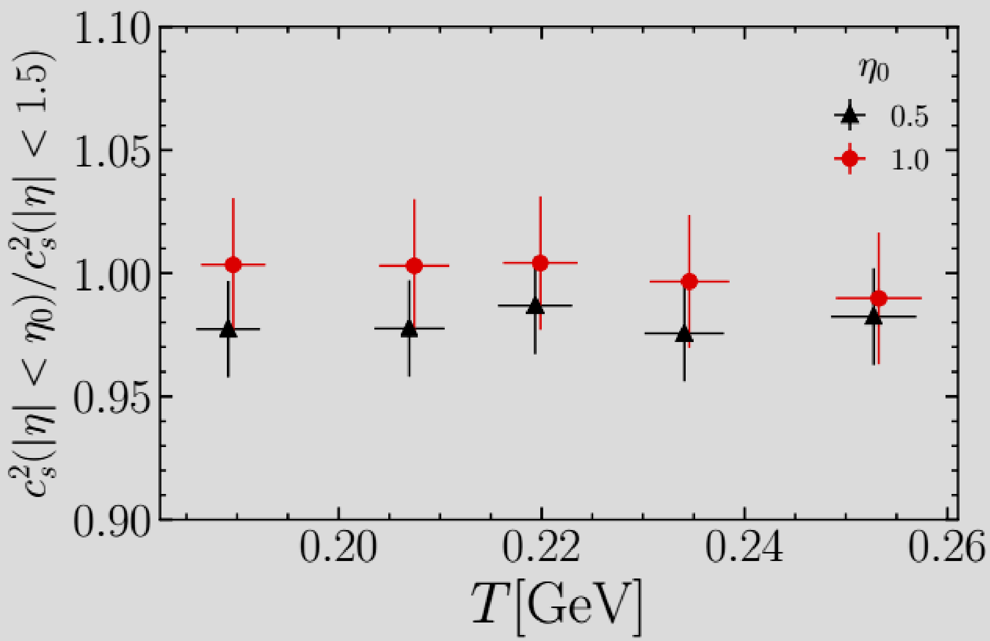
Requires $\langle p_T\rangle$ without any cut

Feasible by extrapolating the measured spectra, as done by CMS

Narrower $\eta$ interval miss particles with low $p_T$ , thus $\langle p_T\rangle$ increases

Smaller $c_s^2$ but effect is smaller than error bar reported by CMS!

**Result is still robust!!**



# Discussion 2: centrality determination and self-correlations

CMS: different detector for centrality and ( $N_{\rm ch}, \ \langle p_T \rangle$ ) determination, eliminating self-correlations

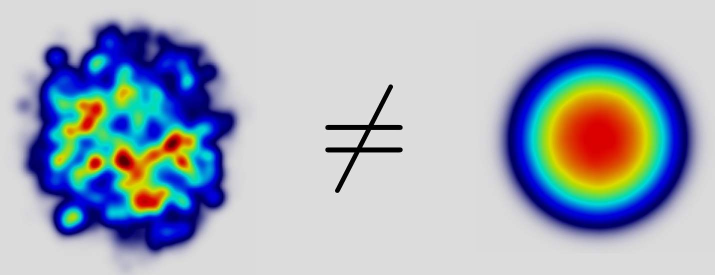
Analysis still feasible if centrality detector and the analysis detector overlap: assuming everything was done with the detector that measured $N_{\rm ch}$

$$\begin{aligned}\sigma^2_{N_{\rm ch}} &= stat. + dynamical \\ &= \langle N_{\rm ch} \rangle + \sigma^2_{\rm dyn}\end{aligned}$$

$$c_s^2(T_{\rm eff}) \ \rightarrow \ \frac{\sigma_{dyn}}{\sigma_{N_{\rm ch}}} c_s^2(T_{\rm eff}) = \frac{d \ln \langle p_T \rangle}{d \ln N_{\rm ch}}$$

$$c_s^2(T_{\rm eff}) = \left(1 - \frac{\langle N_{\rm ch} \rangle}{\sigma^2_{N_{\rm ch}}}\right)^{-1/2} \frac{d \ln \langle p_T \rangle}{d \ln N_{\rm ch}}$$

# Discussion 3: local energy density fluctuations



$\mathrm{V_{eff}}$ is tricky! Depends on how one models initial density fluctuations!

Following previous study: assumed conservative assumption that the increase of the multiplicity in ultracentral collisions results from an homogeneous increase of the density

**In out setup: increase in energy is equivalent to increase in multiplicity**

Not true anymore in presence of fluctuations! Experiment can shed light on this!

Increase of $\langle p_T \rangle$ measured by CMS matches the one from increasing collision energy

Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. 16, no.6, 615 (2020)

(2.76 TeV → 5.02 TeV)

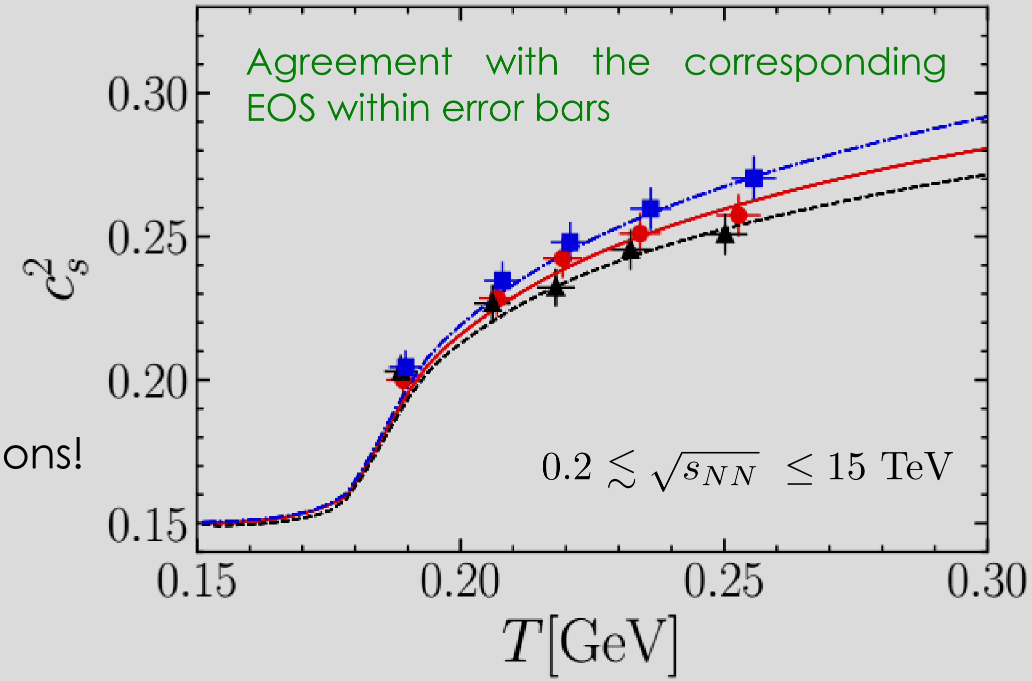
# Final remarks

$c_s^2$ from data: robust and perhaps surprisingly precise within a hydrodynamic description

Might be a consequence of our setup!
Need to check!

Important caveat: modeling of initial fluctuations!

Nijs, van der Schee, PLB 853 (2024) 138636



Agreement with the corresponding EOS within error bars

Important to assess uncertainties from the non-hydrodynamic production at high transverse momentum

# Backup slides

# Setup of our simulations: EOS and $\alpha$

Duke's simulation chain (2020)

Moreland, Bernhard, Bass, PRC 101, no.2, 024911(2020)

EOS from **HotQCD**, matching a HRG at low temperatures, $T_{\mathrm{frzout}} \equiv T_F = 151\mathrm{MeV}$

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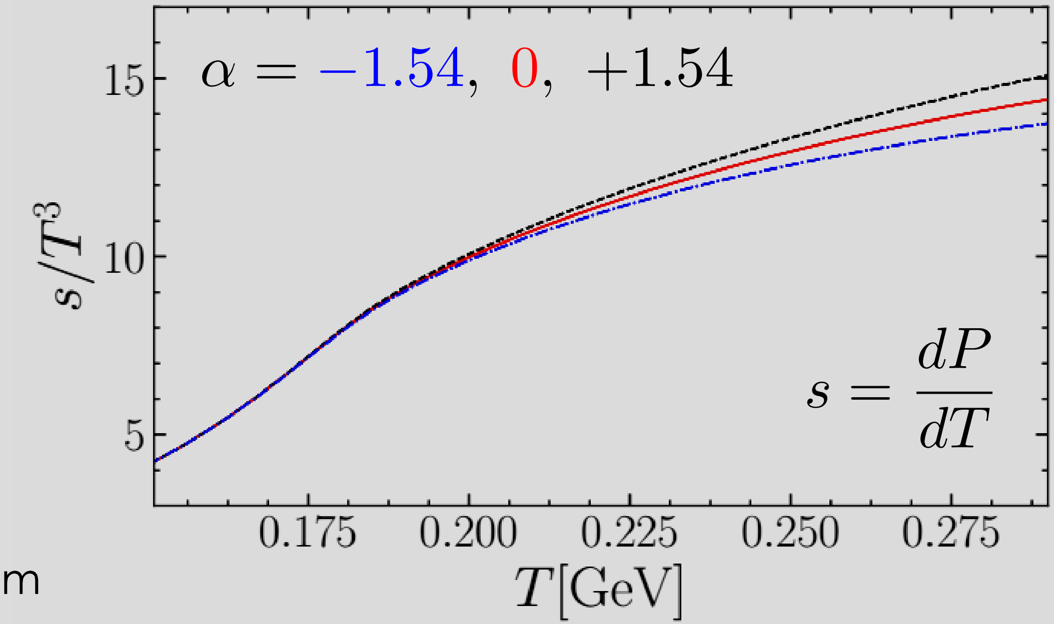
$$P(T) \rightarrow P(T) + \alpha(T - T_F)^4, \quad T \geq T_F$$



Weakly coupled regime, $T \rightarrow \infty$

$P/T^4 \propto$ degrees of freedom

Nijs, van der Schee, arXiv:2312.04623

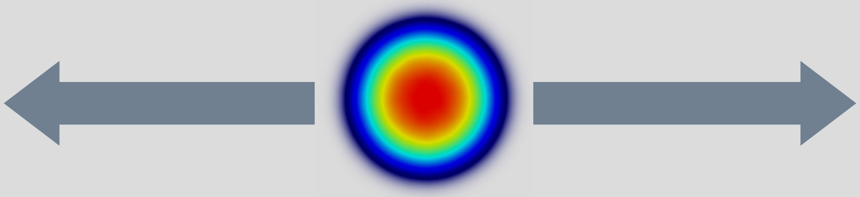
$\alpha$ reflect a change in degrees of freedom

**Test whether it is possible to capture small changes in $c_s^2$ through experimental data**

# Results 1: $\langle p_T \rangle$ & $dN_{ch}/d\eta$ from produced particles

25 values of ch. particle multiplicities: 5 sets of 5 values   [used to extract $c_s^2$ later]

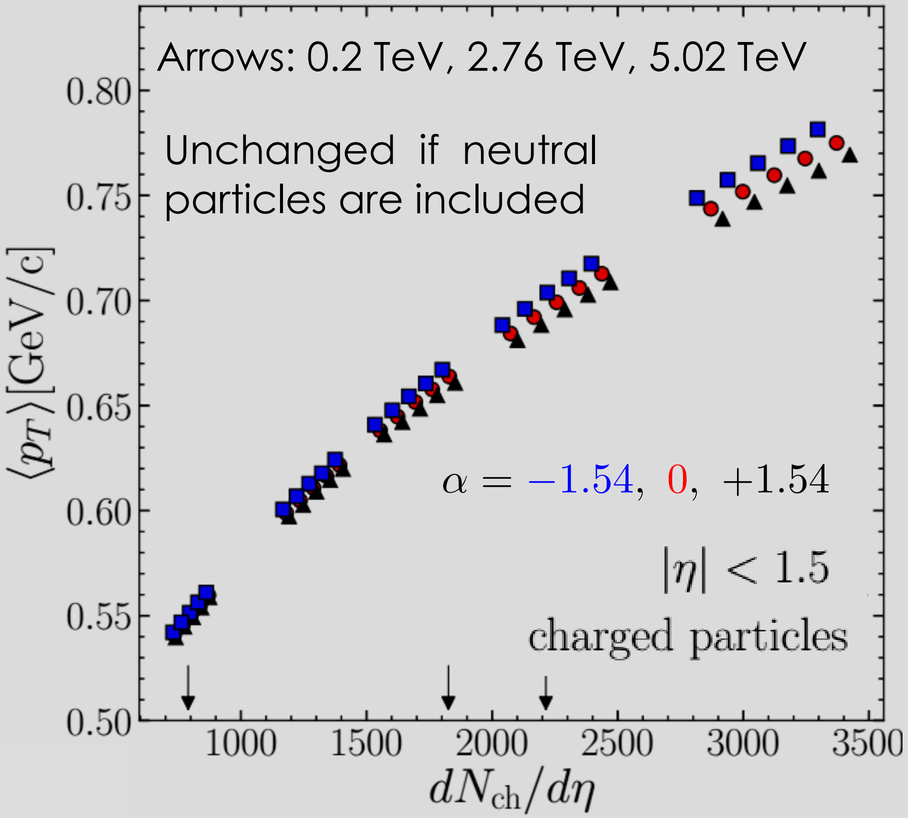
[0.5, 0.75, **1**, 1.25, 1.5]



Lower than average

Matches $\langle dN_{ch}/d\eta \rangle$

Larger than average



**From produced particles:** $\langle p_T \rangle$ **&** $dN_{ch}/d\eta$
[after UrQMD]

$\langle p_T \rangle$ increase with multiplicity → increase in $T_{\mathrm{eff}}$

Larger $\langle p_T \rangle$ at fixed multiplicity → smaller $V_{\mathrm{eff}}$

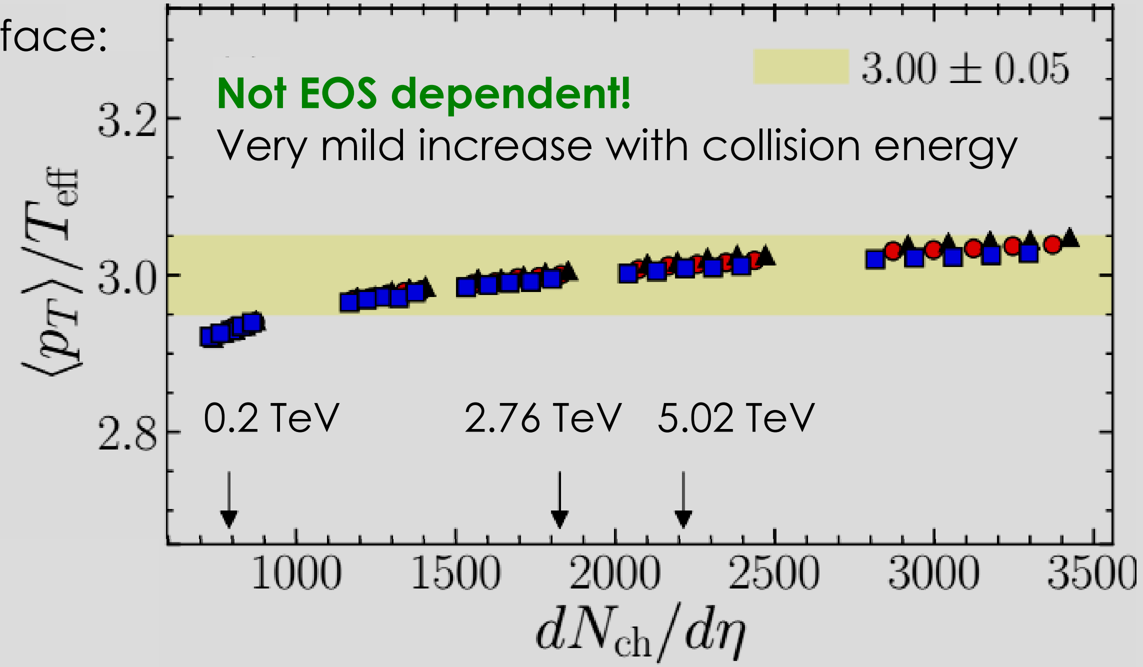
# Results 2: $T_{\text{eff}}$

For each hydrodynamic hypersurface: evaluate $E$ and $S$ at $T_{\text{F}}$

$$\begin{cases} E = \displaystyle\int_{\text{frzout}} T^{0\mu} d\sigma_\mu = \epsilon(T_{\text{eff}}) V_{\text{eff}} \\ S = \displaystyle\int_{\text{frzout}} s u^\mu d\sigma_\mu = s(T_{\text{eff}}) V_{\text{eff}} \end{cases}$$

From same EOS @ hydro phase

$$\frac{E}{S} = \frac{\epsilon(T_{\text{eff}})}{s(T_{\text{eff}})}$$



**Not EOS dependent!**

Very mild increase with collision energy

$$\langle p_T \rangle / T_{\text{eff}} = 3,00 \pm 0.05$$

$$\langle p_T \rangle / T_{\text{eff}} = 2,93 \pm 0.05 \quad @ \quad 200 \text{ GeV}$$

Agrees with previous studies

Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. 16, no.6, 615 (2020)
Gardim, Krupczak, Nunes da Silva, PRC 109, no.1, 014904 (2024)

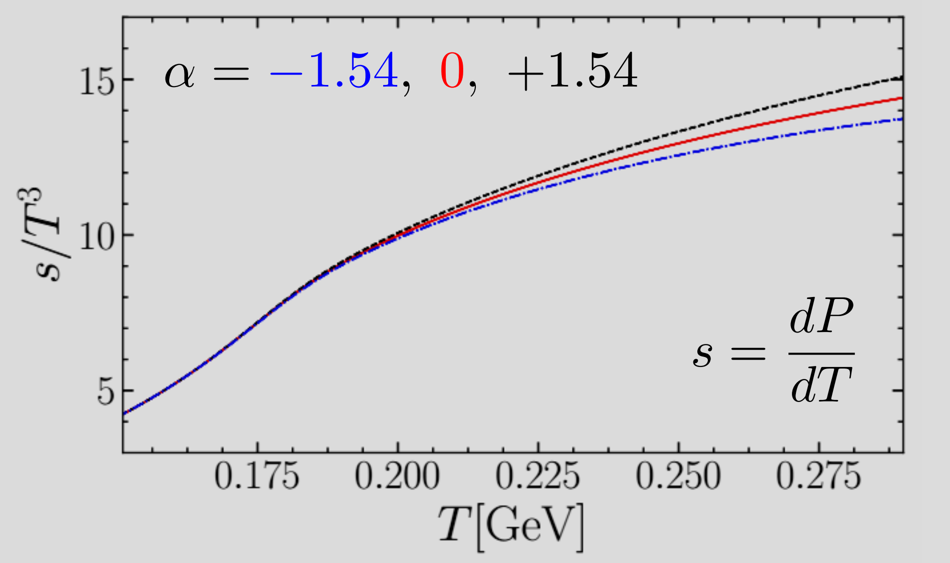
$T_{\text{eff}}$ $[V_{\text{eff}}]$ : temperature [volume] of of a fluid at rest that would have the same energy and entropy as at the end of the hydrodynamic evolution

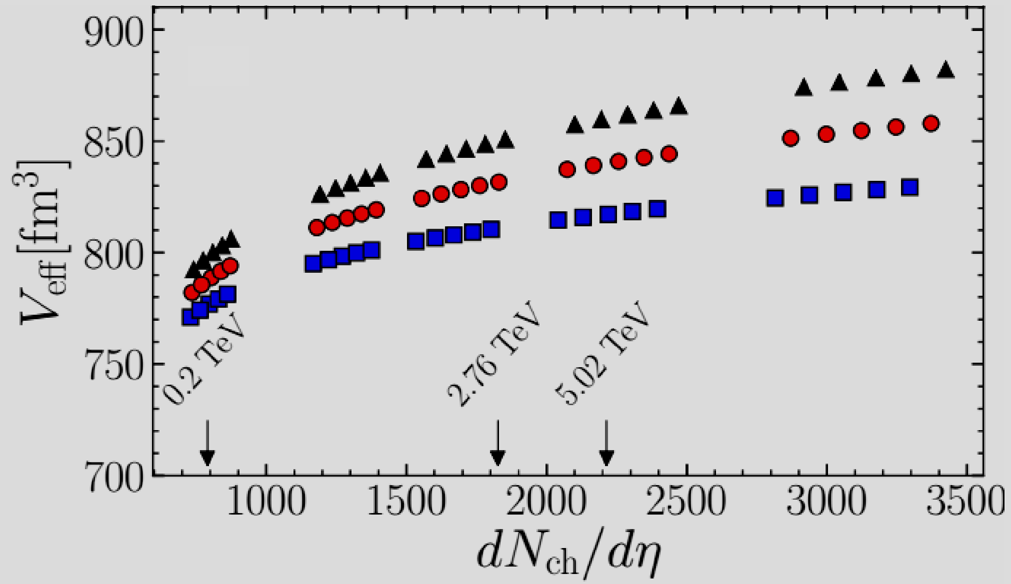
# Results 3: $V_{\rm eff}$

$V_{\rm eff}$ shows some dependence with EOS

Soft EOS: larger system size, smaller $\langle p_T \rangle$

Hard EOS: smaller system size, larger $\langle p_T \rangle$





Mild increase with charged particle multiplicity for all cases

For $\alpha = 1.54$ , Increase of ~11% from

$0.2 \leq \sqrt{s_{NN}} \leq 15$ TeV

# Extrapolation of particle multiplicities

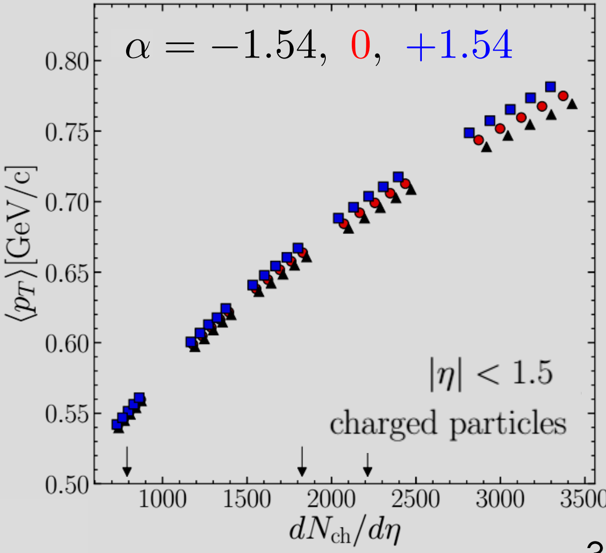
1. For fixed collision energy: extrapolate linearly, using the first two centrality bins, down to "0% centrality" (b ~ 0 fm)

2. Take into account the different η intervals and rescale by same factor as in our hydrodynamical calculation (increase < 5%, in practice)

3. Extrapolate 0.2 TeV value for Au+Au to Pb+Pb assuming $dN_{\mathrm{ch}}/d\eta$ is proportional to mass number: 208 / 197

4. Values for energies larger than LHC regime, $dN/d\eta \propto s_{NN}^{0.155}$

|η| < 1 for 0.2 TeV
|η| < 0.5 for 2.76 TeV & 5.02 TeV
|η| < 1.5, for our calculation

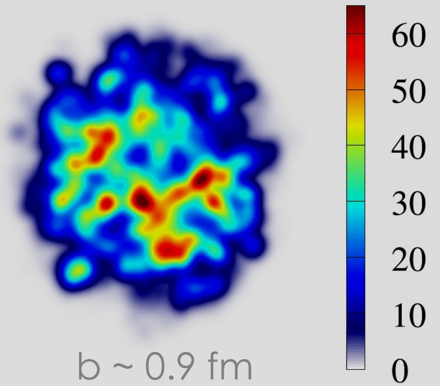


# Initial conditions and initial state fluctuations

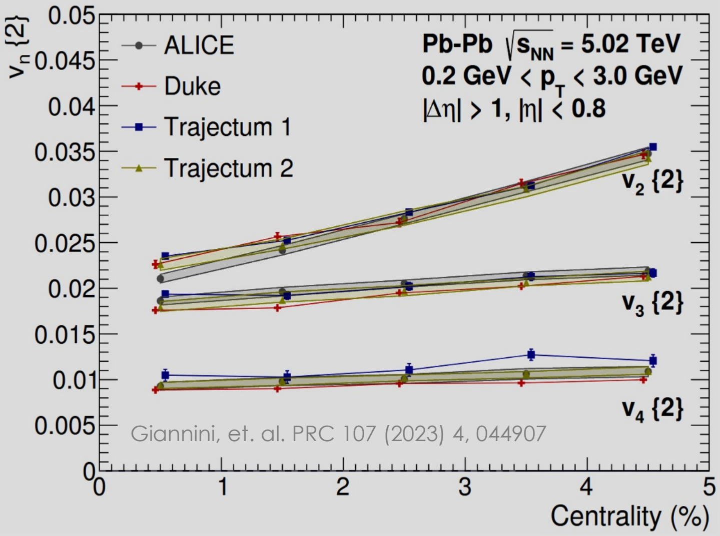
Duke’s simulation chain (2020)

Moreland, Bernhard, Bass, PRC 101, no.2, 024911(2020)

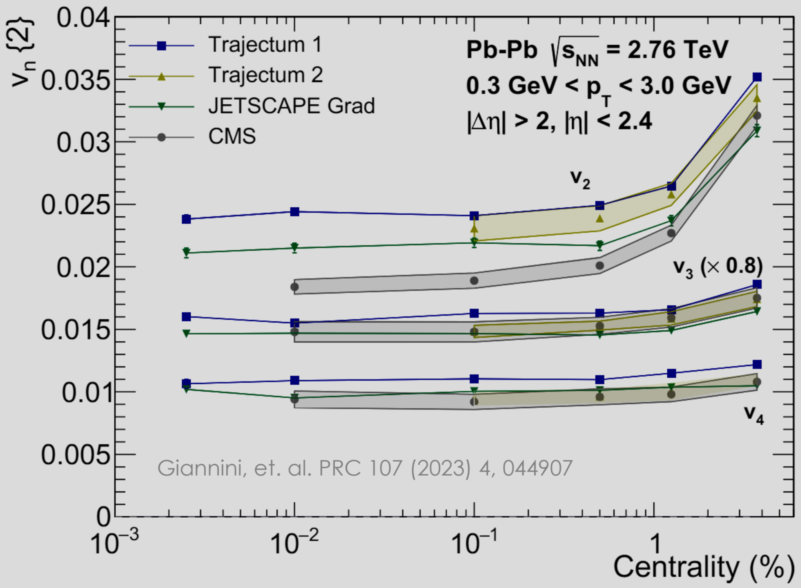
p+Pb @ 5.02 TeV
Pb+Pb @ 5.02 TeV
**Run using MAP values** (Table IV)



b ~ 0.9 fm
TRENTo



Giannini, et. al. PRC 107 (2023) 4, 044907



Giannini, et. al. PRC 107 (2023) 4, 044907

Same disagreement seen in other Bayesian constrained models

Modeling of initial state fluctuations in high-energy nuclear collisions still an open question