Signals from the early universe

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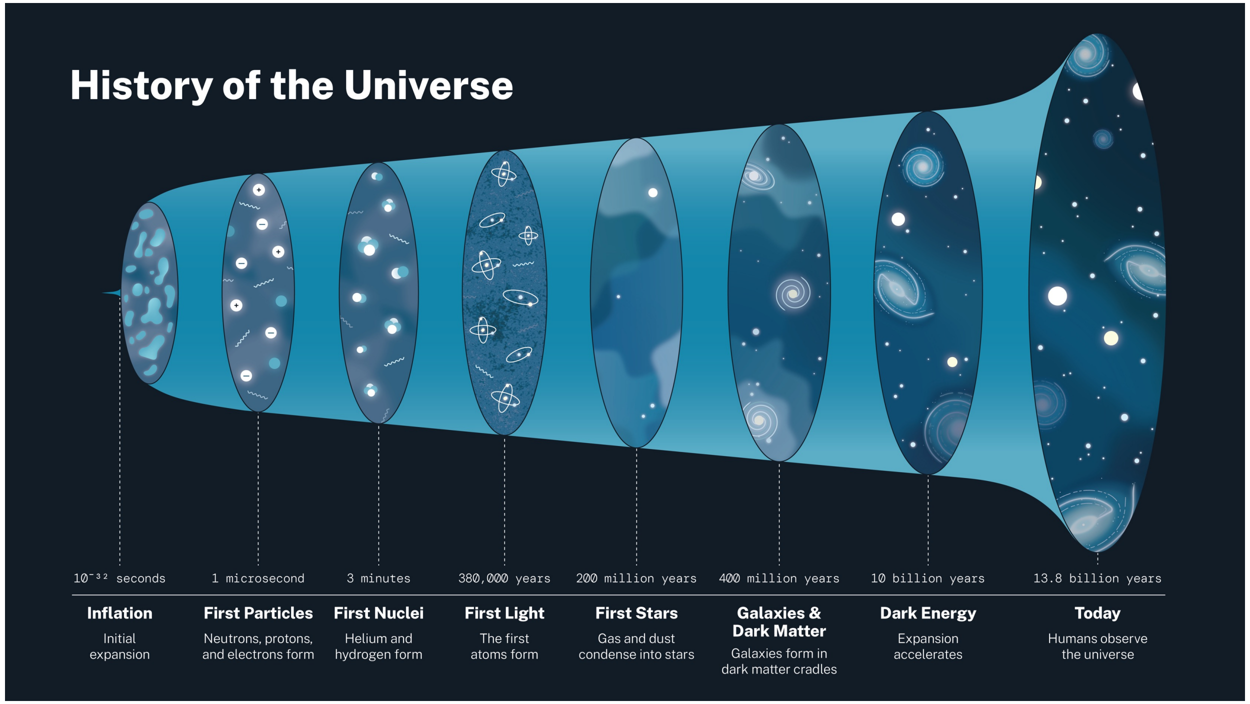
1. Introduction



- In the history of the universe, we concentrate on the epoch of recombination at redshifts 500<z<1100, temperatures 1350 K <T< 3000 K when helium and hydrogen atoms formed, and the universe became transparent to radiation

- In the cosmic microwave background (CMB) measured today at redshift z=0 and temperature T=2.725 K, can remnants of the spectral lines emitted at recombination be found as frequency fluctuations in the CMB? Specifically, remnants of the Lyα line emitted when hydrogen was formed?

Vladimir Kurt 1966: “But where are all the redshifted Lyman-α photons that were released during recombination?“

Recombination:
Lyα emission &
other spectral lines

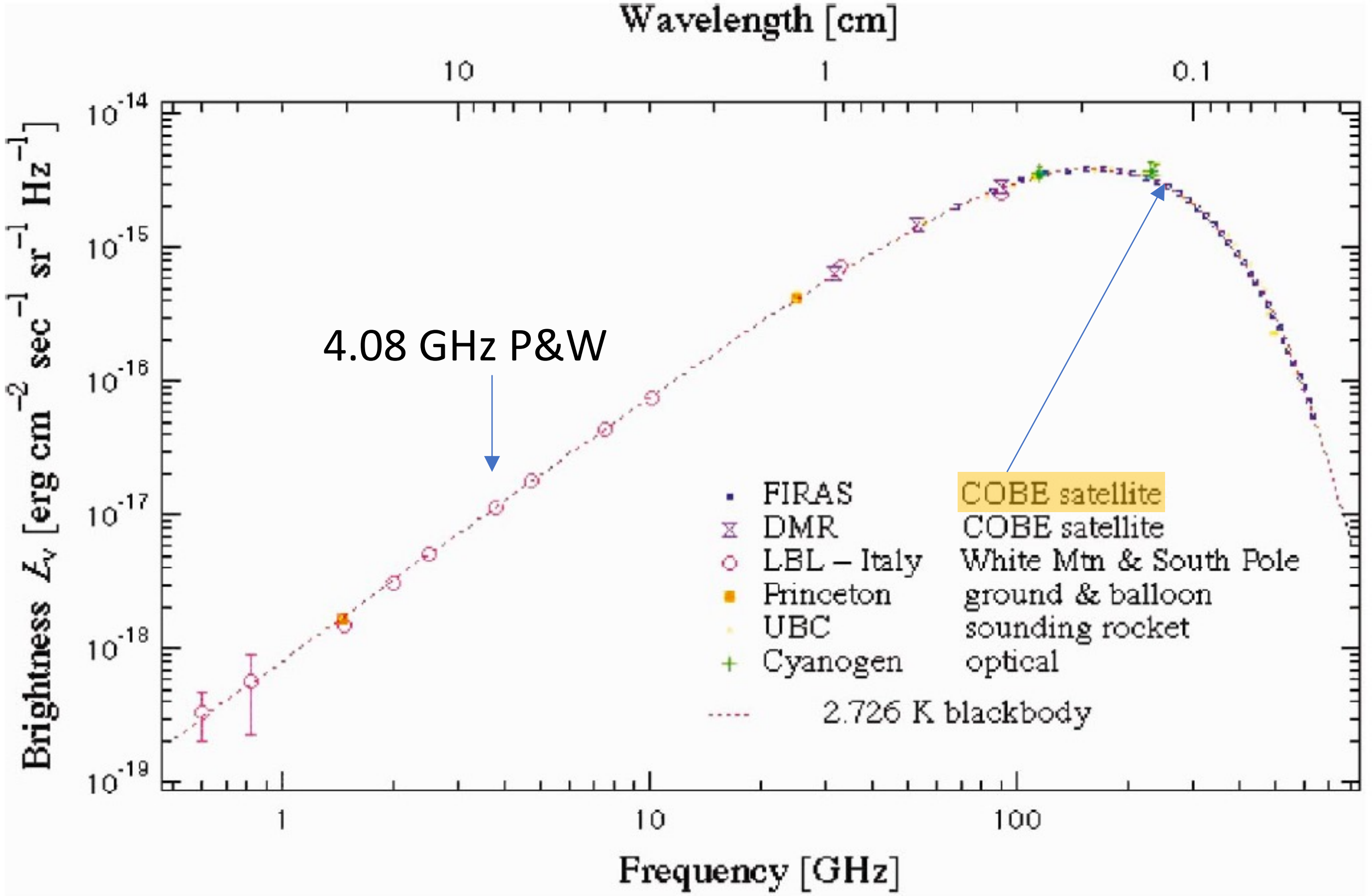
CMB detection;
Lyα remnants from
recombination, frequency fluctuations?
Signals from the early universe, beyond the CMB?

2. Cosmic microwave background (CMB)

- The cosmic background radiation (CBR) is thermalized essentially through Compton scattering and bremsstrahlung at very early times corresponding to redshifts $z>10^7$, producing Planck's equilibrium spectrum for the specific intensity (spectral radiance) of frequency ν at temperature T

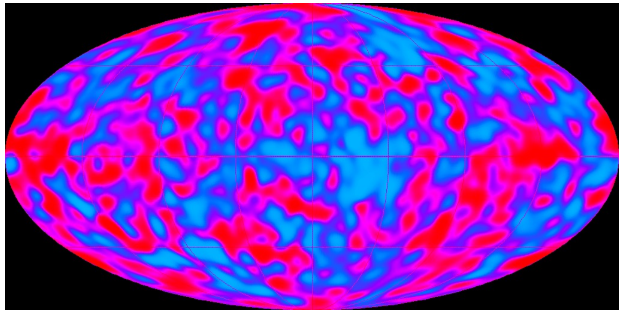
$$L(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

- Expansion retains the thermal spectrum, because both temperature and frequency are reduced with redshift as $(1+z)$, such that ν/T is unchanged. Hence, the CBR is visible today ($z=0$) as Cosmic microwave background (CMB) with temperature T= (2.725±0.001) K at ν~0.1-10^3 GHz

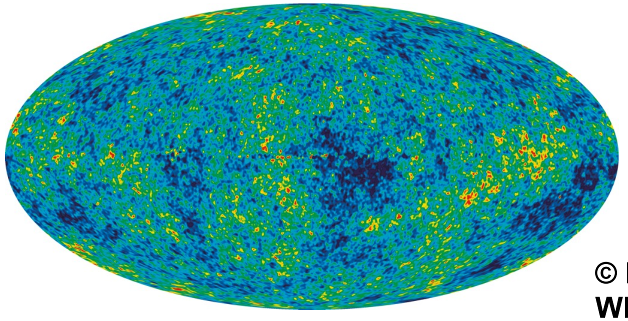


2.1 History of the CMB

- The cosmic microwave background radiation was predicted by Regener in 1933, and Dicke, Peebles et al. 1964
- First (accidental) detection by Penzias &Wilson 1964, with a horn antenna at a single frequency of 4.08 GHz
- COBE satellite results: Precise measurement of the blackbody spectrum (mean value) across the peak (FIRAS), and spatial structure (DMR): $\Delta T/T \sim 6 \times 10^{-6}$ (1992)
- WMAP space mission: More precise resolution of the spatial temperature fluctuations (2008)
- Planck mission: High-resolution measurements of the spatial fluctuations (2013, 2015)



© NASA and the COBE Collaboration



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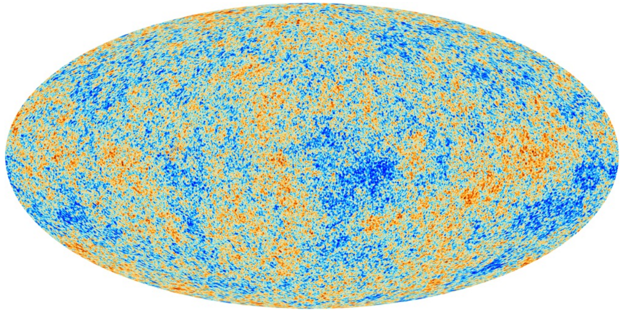

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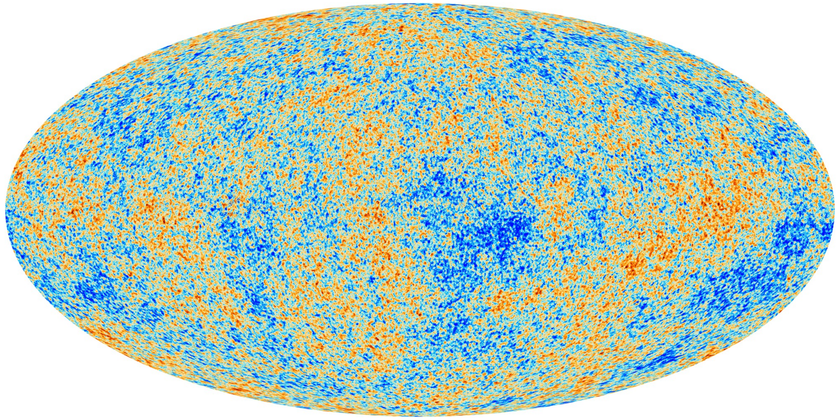
2.2 Spatial and frequency CMB anisotropies

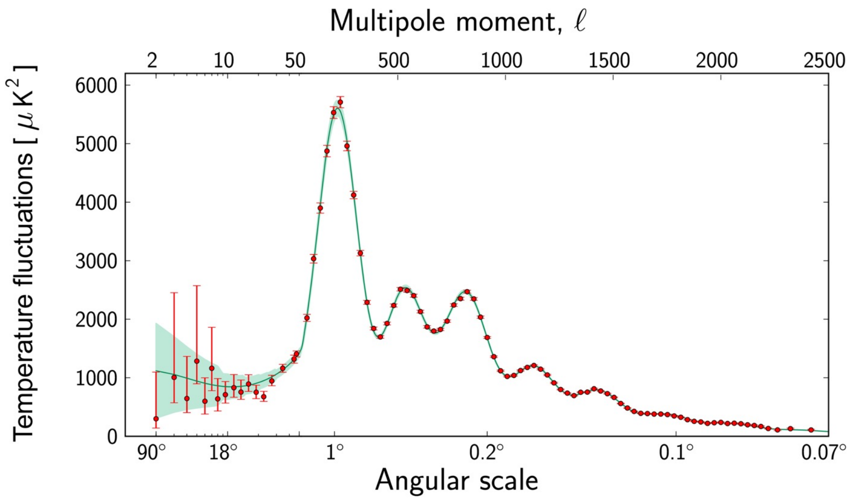
- Temperature fluctuations in the Cosmic Microwave Background detected by Planck at different angular scales on the sky. The sky map can be decomposed in spherical harmonics (multipole analysis), to obtain a CMB power spectrum.

- Since their discovery in 1992 by COBE, *T*-anisotropies have been the best way to study cosmology and the physics of the early universe

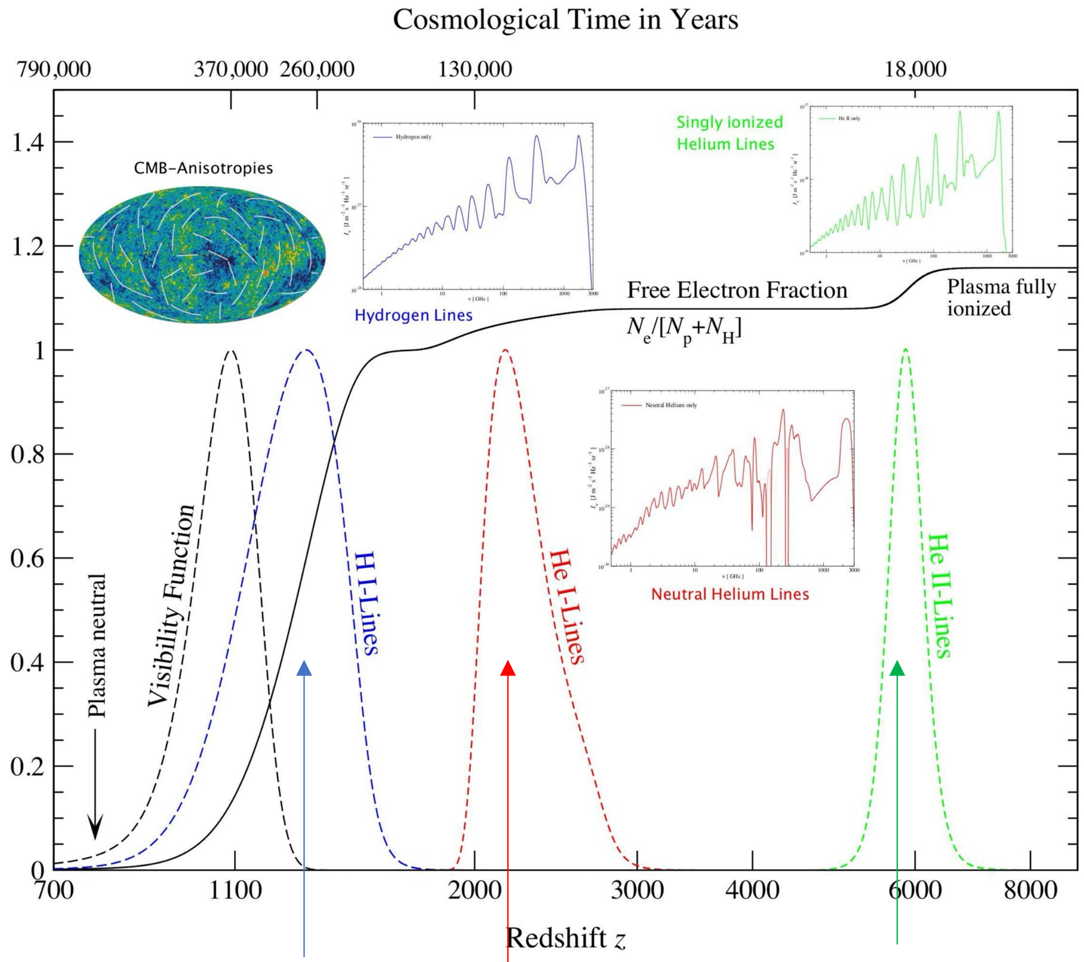
- The red dots (with error bars) in the power spectrum are Planck measurements, the green curve is the best fit within the ΛCDM standard model of cosmology, with a baryonic-matter fraction of 4.6%, dark matter 26.2%, dark energy 69.2 %. Age of the universe 13.799 Gy, H_0 = (67.8±0.038) km/s/Mpc (Planck 2015)

- In the following, we emphasize possible frequency distortions of the CMB that are due to partially thermalized recombination lines





3.1 Recombination transitions in helium and hydrogen



Hydrogen recombination (~76%)
$E_i(H)$=13.598 eV

Helium recombination (~ 24%)
$E_i(He)$=24.587 eV

Singly-ionized helium recombination
$E_i(He^+)$=54.418 eV

When recombination starts in the cooling early universe, first singly-ionized helium (He II) is formed, next neutral helium (He I), and then neutral hydrogen (H I, ~76%).

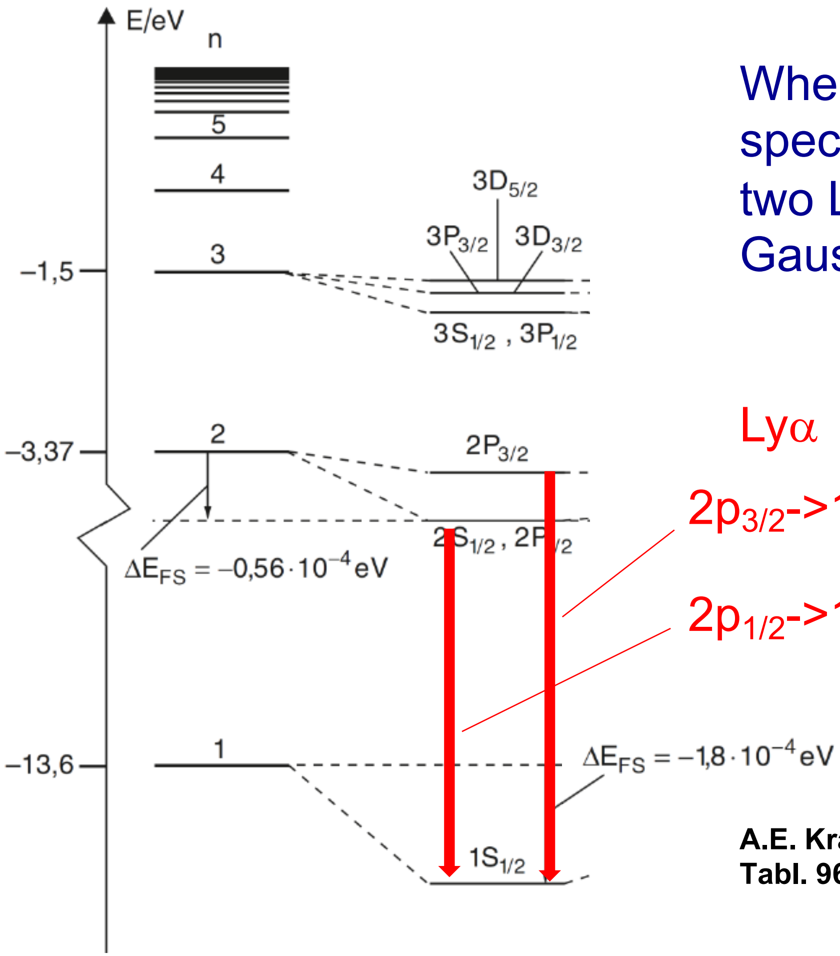
Among the various hydrogen lines, the 2p->1s Lyman-alpha transition is the most prominent (68% of all H I transitions).

The visibility function indicates the transparency of the universe for radiation.

In the present model calculation, the recombination era is taken to end at z=500 when the free electron fraction becomes negligible ($<10^{-3}$).

R.A. Sunyaev, J. Chluba, Astr. Nachr. 330, 657 (2009)

3.2 Lyman-α transition in hydrogen



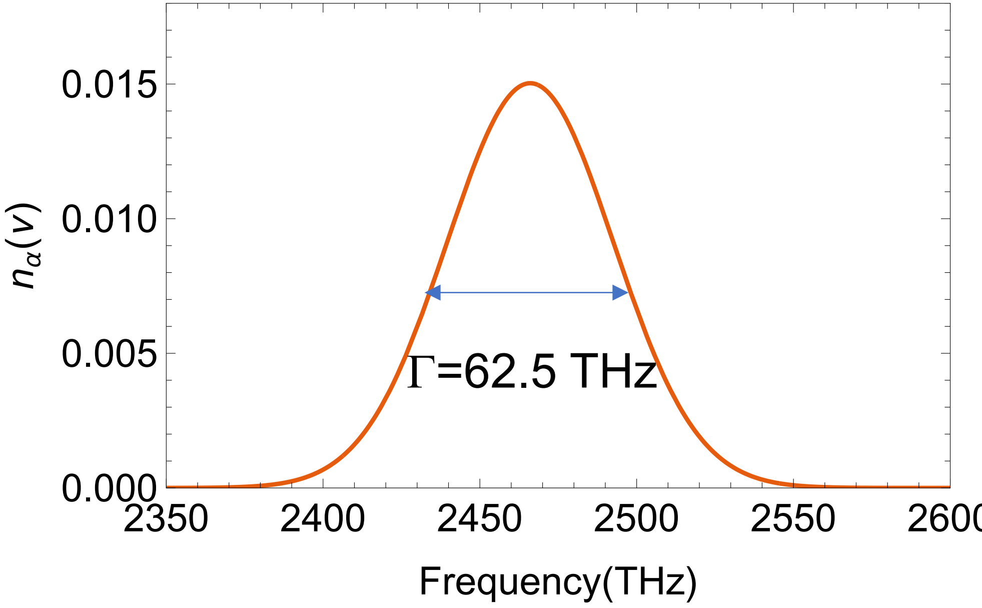
When emitted during recombination at T_{rec}~3000 K, spectral lines acquire a thermal width, and the two Lorentzian HF Lyα lines merge into a single Gaussian of width $\Gamma_\alpha \sim 1.8 T_{rec} \sim$ 62.5 THz

Lyα

$2p_{3/2}$->$1s_{1/2}$: 2466.071 THz

$2p_{1/2}$->$1s_{1/2}$: 2466.060 THz

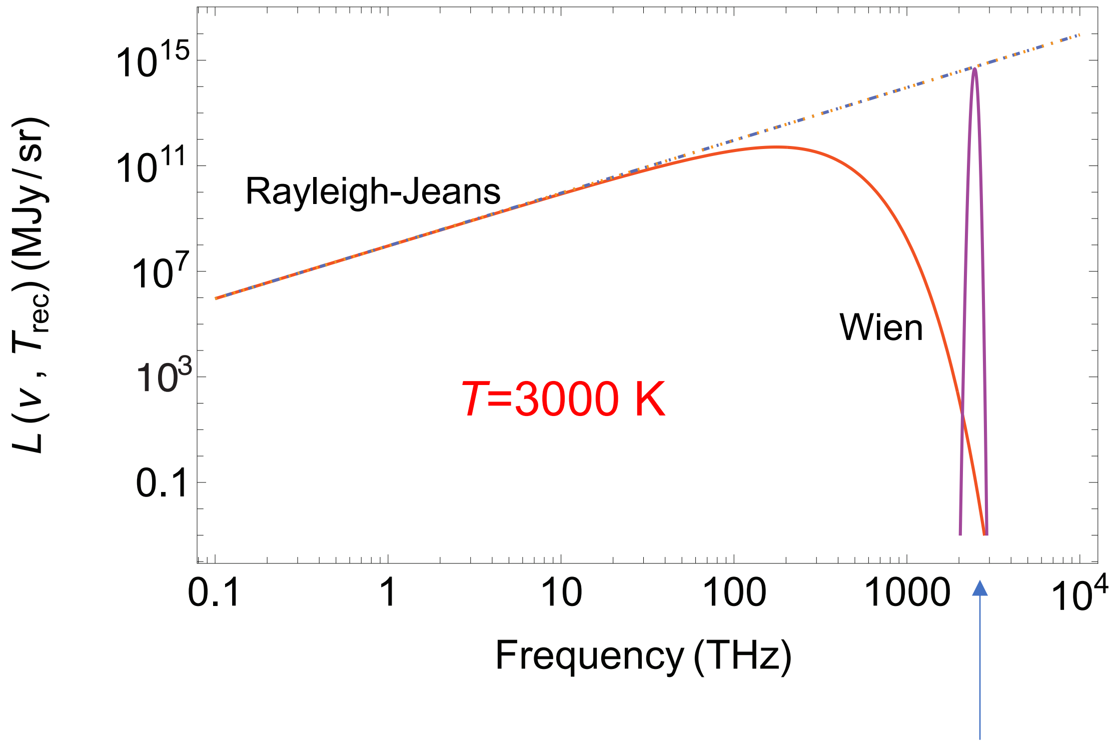
A.E. Kramida, At. Data Nucl. Data Tabl. 96, 586-644 (2010)



(natural line width of 2p state: 100 MHz)

About 68% of the hydrogen lines emitted during recombination are Lyα

Lyman-α transition and the blackbody cosmic background radiation (CBR) spectrum at t = 380 ky



Lyα @ 2466 THz,
Doppler-broadened

Goal: Account for the partial thermalization of the Lyα line in the recombination era

G. Wolschin, Sci. Rep. (SpringerNature) 14, 4935 (2024)

3.3 Partial thermalization processes of Lyα

Frequency transfer of the Lyα line occurs via

- true emission and absorption,
- Hubble expansion
- resonant scattering
- inelastic scattering

Photons diffuse in frequency through resonant scattering due to random kicks from the thermal velocities of hydrogen atoms, and drift toward lower frequencies due to energy loss via atomic recoil.

Raman scattering converts incident ultraviolet (UV) photons around the Lyman resonance lines into optical-infrared (IR) photons.

e.g. C.M. Hirata, J. Forbes, Lyman-alpha transfer in primordial hydrogen recombination, Phys. Rev. D 80, 023001 (2009)
M. Kokubo, Rayleigh and Raman scattering cross-sections and phase matrices of the ground-state hydrogen atom, and their astrophysical implications, MNRAS 529, 2131–2149 (2024)

In the present mesoscopic model, time-dependent drift and diffusion coefficients plus expansion encompass the various frequency-transfer processes – without going into microscopic detail

4. Partial thermalization of Lyα, and CMB frequency anisotropies

4.1 Nonlinear boson diffusion equation (NBDE) and analytical solution

The nonlinear boson diffusion equation (NBDE) for the single-particle occupation-number distribution n(ν,t) has been derived from the collision term in the quantum Boltzmann equation as

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial \nu}\left[J\, n(1+n) + n\frac{\partial D}{\partial \nu}\right] + \frac{\partial^2}{\partial \nu^2}\left[D\, n\right]$$

with the diffusion coefficient *D*(ν,*t*) as second moment of the transition probabilities, and the drift coefficient *J*(ν,*t*) as first moment. The equation has the Bose-Einstein equilibrium distribution as its stationary limit,

$$n_\infty(\nu) = n_{\mathrm{eq}}(\nu) = \frac{1}{e^{(2\pi\nu-\mu)/T} - 1}$$

where $T \equiv \lim_{t\to\infty}[-D(\nu,t)/J(\nu,t)],\; D(\nu,t)/J(\nu,t) = \mathrm{const}\,\forall t,$ and the chemical potential μ is a parameter.

G. Wolschin, Physica A 499, 1-10 (2018)

In the limit of frequency-independent transport coefficients, the NBDE takes the form

$$\frac{\partial n}{\partial t} = -J\,\frac{\partial}{\partial \nu}\left[n\,(1+n)\right] + D\,\frac{\partial^2 n}{\partial \nu^2}\ .$$

For any given initial nonequilibrium distribution $n_0(\nu)$, it can be solved through the nonlinear transformation

$$n\,(\nu, t) = T\,\partial_\nu \ln \mathcal{Z}(\nu, t) - \frac{1}{2} = \frac{T}{\mathcal{Z}}\,\partial_\nu \mathcal{Z} - \frac{1}{2}\,,$$

where the generalized (time-dependent) partition function $\mathcal{Z}(\nu, t)$ obeys a linear diffusion equation

$$\frac{\partial}{\partial t}\mathcal{Z}(\nu, t) = D\frac{\partial^2}{\partial \nu^2}\mathcal{Z}(\nu, t)$$

and can be written as an integral over the corresponding Green's function and an exponential function $F(x)$ that depends on the initial condition $n_0(\nu)$,

$$\mathcal{Z}_{\text{bound}}(\nu, t) = \int_0^{+\infty} G_{\text{bound}}(\nu, x, t)\,F(x)\,dx$$

With the infrared boundary condition of the occupation-number distribution $\lim_{\nu\downarrow 0} n(\nu, t) = \infty \; \forall \, t$, Green's function becomes

$$G_{\text{bound}}(\nu, x, t) = G_{\text{free}}(\nu, x, t) - G_{\text{free}}(\nu, -x, t)$$

with a Gaussian in (ν-x) as the free Green's function, F(x) contains the initial condition $n_0(\nu)$

$$G_{\text{free}}(\nu, x, t) = \frac{1}{\sqrt{4\pi D t}} \exp\left[-\frac{(\nu - x)^2}{4Dt}\right].$$

$$F(x) = \exp\left[-\frac{1}{2D}\left(Jx + 2J\int^x n_0(y)\,dy\right)\right].$$

The occupation-number distribution is then obtained through the above nonlinear transformation. For sufficiently simple initial distributions, all integrals may be calculated analytically:

- Example for a nonlinear partial differential equation that can be solved exactly.

In the present case of the Lyα initial condition (thermally broadened Gaussian),

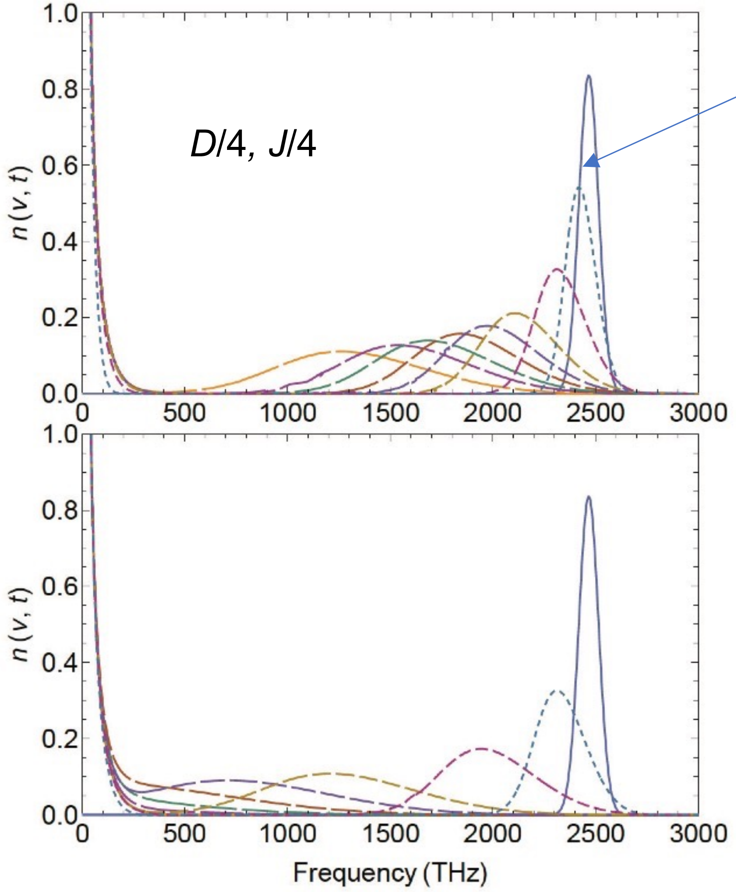
$$\int^x n_0(y)\,dy = \int^x \frac{N_\alpha}{\sqrt{2\pi}\,\sigma_\alpha} \exp\left[\frac{(y - \nu_\alpha)^2}{2\sigma_\alpha^2}\right] dy$$

$$= \frac{N_\alpha}{2\sigma_\alpha} \operatorname{erf}\left[\frac{x - \nu_\alpha}{\sqrt{2}\sigma_\alpha}\right],$$

the integral for $\mathcal{Z}_{\text{bound}}$ is carried out numerically, and $n(\nu,t)$ is obtained from the nonlinear transformation.

- The transport coefficients are related to the equilibrium temperature through the fluctuation-dissipation relation $T = -D/J$, and to the equilibration timescale according to $\tau_{eq}=a_\tau D/J^2$, with a proportionality constant a_τ ⟹ determine τ_{eq} and a_τ in the cosmological context, or use the drift J as a parameter, and compute the diffusion coefficient from the dissipation-fluctuation relation at the equilibrium temperature T.

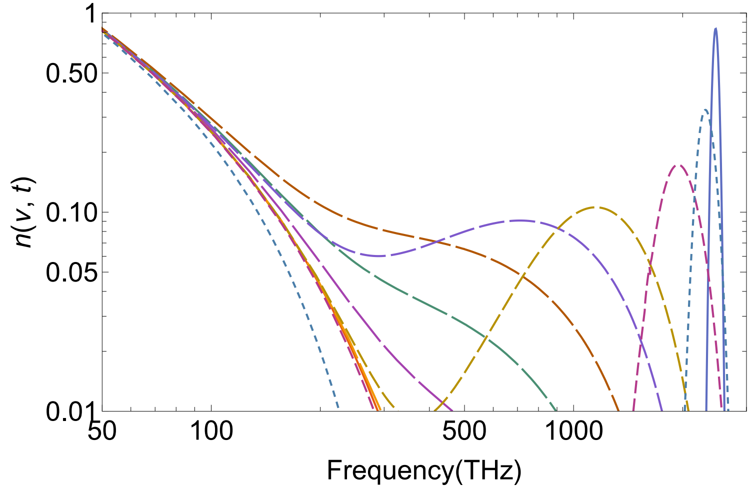
- Test the properties of the NBDE assuming complete thermalization



Thermally broadened Lyα line, 2466 THz
$T=T_{rec}$; no expansion

$$D = 63\,\mathrm{THz}^2/\mathrm{ky}$$
$$J = -1\,\mathrm{THz/ky}$$
$$t/10^3\,\mathrm{ky} = 0.1;\ 0.4;\ 1.1;\ 1.5;\ 2;\ 2.5;\ 3;\ 4$$

Drift is mainly caused by atomic recoil, diffusion by Doppler broadening and boosting of Lyα. In the NBDE, each of these processes is included implicitly, but not detailed explicitly.



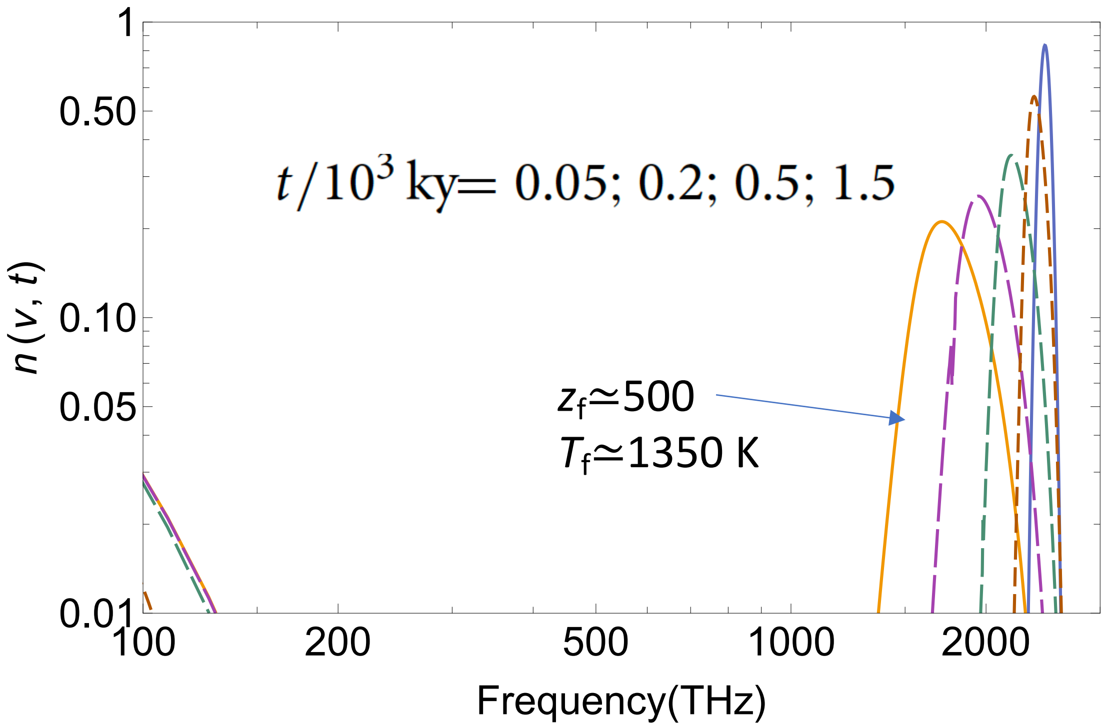
4.2 Partial thermalization of the recombination Lyα-line

Partial thermalization during the recombination era occurs between redshifts $z_i \simeq 1100$ at temperature $T_i \simeq 3000$ K, and $z_f \simeq 500$ at temperature $T_f \simeq 1350$ K.
In this period, time-dependent transport coefficients are required, keeping their ratio $T(t) = - D(t)/J(t)$ = const at each time, $T(t) = T_{cmb}(1+z)$, and an exponential relation $z(T)$

$$D(t) = D_0 \exp(-t/\tau_{eq}) \, ,$$
$$J(t) = J_0 \exp(-t/\tau_{eq}) \, ,$$

with $J_0 = - 1$ THz/ky, $D_0 = - 0.45\ T_{rec}/J_0 = 28$ THz^2/ky, $\tau_{eq} = 1.5\text{x}10^3$ ky.

⟹ solve the NBDE:



The Lyα line is shifted with time to lower frequencies, reduced in height, and broadened. A low-frequency Bose enhancement emerges, creating a Rayleigh-Jeans equilibrium part, but the total distribution remains far from equilibrium.

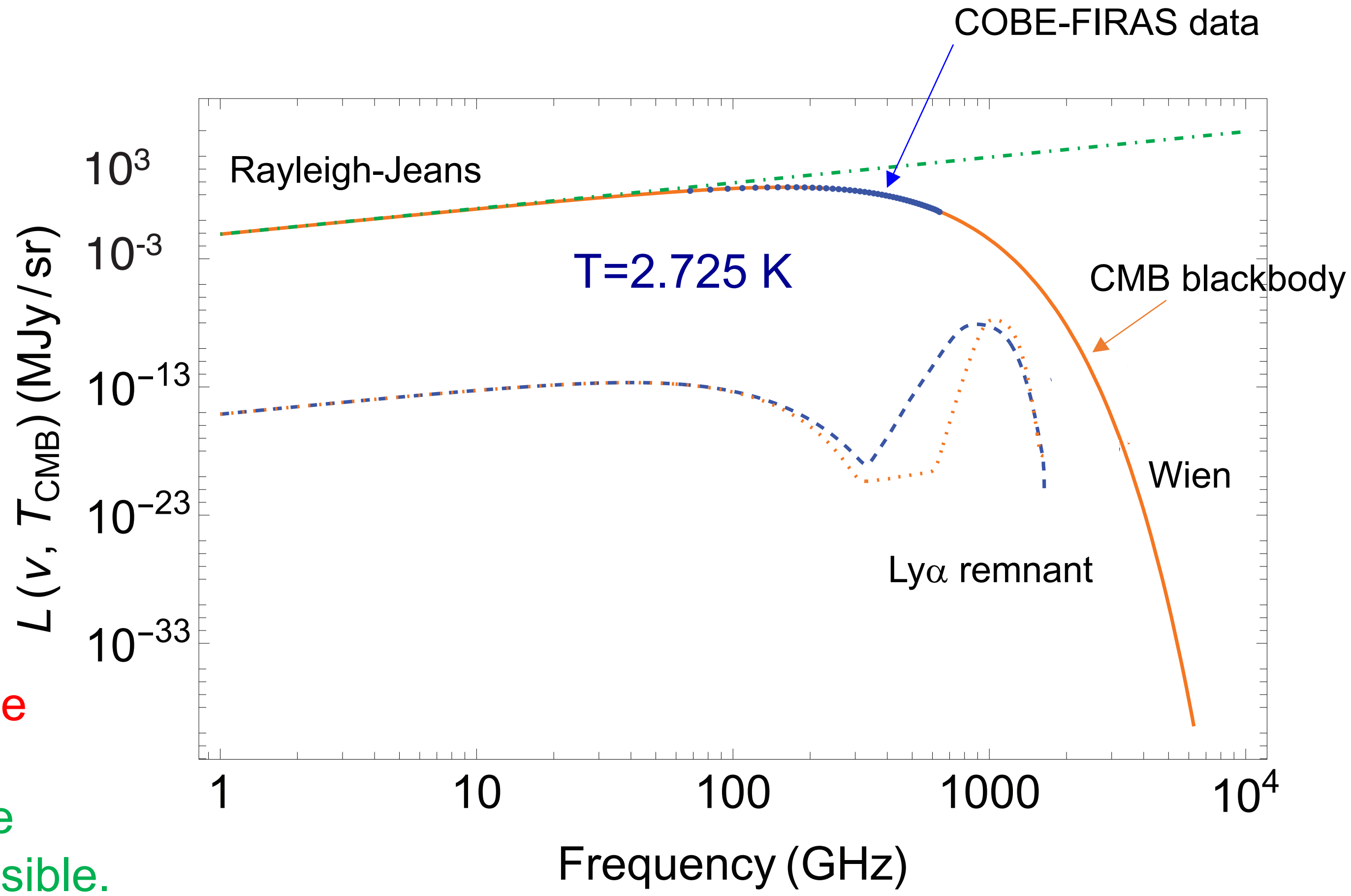
4.3 Possible CMB frequency anisotropies

Convert the occupation-number density of photons to the corresponding specific-intensity spectrum, and propagate the distribution function from z = 500 to z = 0, dashed curve:

The normalization is according to the ratio of photons to baryons ~1.6×10^9, yielding an upper limit for the distortion.

The result is robust when reducing the amplitudes of drift J_0 and diffusion D_0 by factors of two (dotted curve):

- ❑ The frequency distortion of the CMB remains about 7 orders of magnitude below the signal – too low to be observable with present instrumentation.
- ❑ With improved spectrometer sensitivity, the detection of the line remnants may be possible.



G. Wolschin, Sci. Rep. (SpringerNature) 14, 4935 (2024)

5. Summary and Conclusion

- A nonlinear boson diffusion equation (NBDE) is presented as a kinetic model for thermalization problems in many areas of physics. For frequency-independent coefficients, it can be solved analytically through a nonlinear transformation.

- The model is applied to the partial thermalization of the Lyα line emitted during recombination in the early universe. Possible remnants of the line in today‘s CMB are calculated using the NBDE.

- The frequency distortion of the CMB remains about 7 orders of magnitude below the signal – too low to be observable with present instrumentation.

- With improved spectrometer sensitivity, it appears possible in the future to detect remnants of the recombination lines – specifically, the Lyα line – as messengers from the early universe through frequency distortions of the CMB blackbody spectrum.

Thank you for your attention !