Constraints on the neutrino extension of the Standard Model and baryon asymmetry of the Universe

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together with
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The Standard Model and Beyond

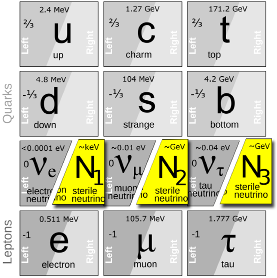
- In the past, the structure of the Standard Model was predicting where to expect new particles
- Higgs boson was the last predicted and discovered particle
- Standard Model is consistent up to the Planck scale

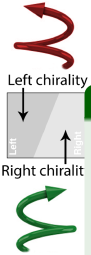
There are well-established phenomena that are not explained by the Standard Model:

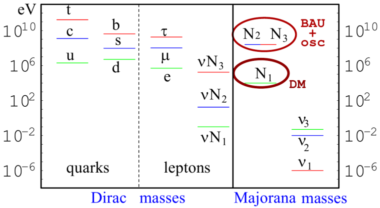
- Neutrino masses and active neutrino oscillations
- Dark matter
- Baryon asymmetry of the Universe

How can we explain them?

Why are Heavy Neutral Leptons interesting?







HNL can explain . . .

- . . . **neutrino oscillations**

  Bilenky & Pontecorvo'76; Minkowski'77; Yanagida'79; Gell-Mann et al.'79; Mohapatra & Senjanovic'80; Schechter & Valle'80

- . . . **Baryon asymmetry**

  Fukugita & Yanagida'86; Akhmedov, Smirnov & Rubakov'98; Pilaftsis & Underwood'04-05; Shaposhnikov+'05–

- . . . **Dark matter**

  Dodelson & Widrow'93; Shi & Fuller'99; Dolgov & Hansen'00; Abazajian+; Asaka, Shaposhnikov, Laine'06 –

Heavy Neutral Leptons

The simplest renormalizable extension of the Standard Model, consistent with neutrino experiments, contains 3 right-handed SU(2)×U(1) singlet neutrinos $N_I (I = 1, 2, 3)$ (Heavy Neutral Leptons) with gauge invariant Lagrangian is Neutrino Minimal Standard Model (νMSM):

$$\delta\mathcal{L} = i\bar{N}_I \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \Phi - \frac{M_I}{2} \bar{N}_I^c N_I + h.c.,$$

This approach can potentially explain all previously mentioned phenomena.

The lightest of the three sterile neutrinos is a dark matter particle candidate and should be unable to be detected in accelerators.
Therefore, in this work, we focused on 2 heavy right-handed neutrinos, whose parameters in the Lagrangian are constrained by:

- Failure to detect Sterile neutrinos in collider experiments (upper bound)
- The generation of baryon asymmetry of the early Universe (lower bound)

Constraints on parameters of the neutrino extension from collider experiments

Strategy

Problem statement

HNL are heavy and cannot be probed directly.
However, they can lead to charged lepton flavour violation (cLFV) processes, non-observation of which imposes restrictions on the model parameters.

Framework of the Effective Field Theory

We will work in the framework of the Effective Field Theory, where we can use a set of operators of mass dimension ≥ 5. It can be understood as an effective interaction generated by loop diagrams with HNL inside.

We will consider operators of mass dimension 6 that generate cLFV processes.
We want to strengthen model parameters from the existing cLFV constraints.

Starting Point

The observable experimental effects induced by heavy sterile neutrinos are expressed through the following Lagrangian parameters:

$$S_{\alpha\beta} = \sum_I S^I_{\alpha\beta} \equiv (FM^{-1*}M^{-1}F^\dagger)_{\alpha\beta}, \qquad R_{\alpha\beta} = \sum_I S^I_{\alpha\beta} \ln \frac{M_I}{M_W}$$

These parameters of the extended Lagrangian are constrained by:

- “forbidden processes” in SM;
- errors in measurements of known processes in SM.

Constraints on observables

We have experimental constraints for our observables.

quantity	observable	upper limit (conservative)	upper limit (expected)
$\hat{S}_{ee} + \hat{S}_{\mu\mu}$	$\Gamma(Z \to ee^{+})$	$0.53 \cdot 10^{-3}$	–
$\hat{S}_{\tau\tau}$	$G_F^{\mu\tau} / G_F$	$0.64 \cdot 10^{-3}$	–
$\lvert\hat{S}_{e\mu}\rvert$	$\mathrm{BR}(\mu \to e\gamma)$	$6.8 \cdot 10^{-6}$	$2.6 \cdot 10^{-6}$
$\lvert\hat{S}_{e\tau}\rvert$	$\mathrm{BR}(\tau \to e\gamma)$	$4.5 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$
$\lvert\hat{S}_{\mu\tau}\rvert$	$\mathrm{BR}(\tau \to \mu\gamma)$	$5.2 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
$\lvert\hat{R}_{e\mu}\rvert$	$\mathrm{BR}(\mu Al \to e Al)$	$2.4 \cdot 10^{-7}$	$1.7 \cdot 10^{-8}$
$\lvert\hat{R}_{e\tau}\rvert$	$\mathrm{BR}(\tau \to eee)$	0.022	$3.0 \cdot 10^{-3}$
$\lvert\hat{R}_{\mu\tau}\rvert$	$\mathrm{BR}(\tau \to \mu\mu\mu)$	0.019	$4.2 \cdot 10^{-3}$

Rupert Coy's and Michele Frigerio's, *Effective approach to lepton observables: the seesaw case*, Phys. Rev. D **99**, 095040 (2019)

where $\hat{S}_{\alpha\beta} = M_W^2 S_{\alpha\beta}$, $\hat{R}_{\alpha\beta} = M_W^2 R_{\alpha\beta}$

Connection with observable quantities

We want to express observable parameters $\hat{S}_{\alpha\beta}$, $\hat{R}_{\alpha\beta}$ via νMSM parameters

- masses of active neutrinos m_i
- active neutrino oscillations parameters (mixing angles)
- mass of heavy sterile neutrinos M_1 and M_2
- difference of heavy sterile neutrino masses $\Delta M/M$
- $U^2_{tot} = \sum_{\alpha,I} |\Theta_{\alpha I}|^2 = \frac{v^2}{M^2}\mathrm{tr}(FF^\dagger) = \frac{\sum_i m_i}{M}\cosh(2\Im\mathfrak{m}\omega)$, where $\Theta_{\alpha I}$ is the mixing angle between active and sterile neutrino.

We will consider an interesting region for the current experimental search of HNL above the sea-saw line. i.e. under assumption:
$\cosh(2\Im\mathfrak{m}\omega) \approx \sinh(2\Im\mathfrak{m}\omega) = \frac{\exp(2\Im\mathfrak{m}\omega)}{2} \gg 1$

Simplified parametrization

Using Casas-Ibarra parametrisation $F = X\mathcal{R}\sqrt{M^{\mathrm{diag}}}$, $\quad X = \frac{i}{v} U_\nu \sqrt{m_\nu^{\mathrm{diag}}}$
we obtain for normal hierarchy:

$$S_{ee} = \frac{e^{2\mathfrak{Im}\omega}(M_1 + M_2)(X_{12} - iX_{13})\,(X_{12}^* + iX_{13}^*)}{4M_1M_2}$$

$$S_{\mu\mu} = \frac{e^{2\mathfrak{Im}\omega}(M_1 + M_2)(X_{22} - iX_{23})\,(X_{22}^* + iX_{23}^*)}{4M_1M_2}$$

$$S_{\tau\tau} = \frac{e^{2\mathfrak{Im}\omega}(M_1 + M_2)(X_{32} - iX_{33})\,(X_{32}^* + iX_{33}^*)}{4M_1M_2}$$

$$S_{e\mu} = \frac{e^{2\mathfrak{Im}\omega}(M_1 + M_2)(X_{12} - iX_{13})\,(X_{22}^* + iX_{23}^*)}{4M_1M_2}$$

$$S_{e\tau} = \frac{e^{2\mathfrak{Im}\omega}(M_1 + M_2)(X_{12} - iX_{13})\,(X_{32}^* + iX_{33}^*)}{4M_1M_2}$$

$$S_{\mu\tau} = \frac{e^{2\mathfrak{Im}\omega}(M_1 + M_2)(X_{22} - iX_{23})\,(X_{32}^* + iX_{33}^*)}{4M_1M_2}$$

General constraints on S_{ab} and R_{ab}

General constraints on S_{ab} and R_{ab} in the form of saturated Schwarz inequality

$$|S_{ab}|^2 = S_{aa} \cdot S_{bb} \qquad S_{\alpha\beta}\left(M_1 \ln \frac{M_2}{M_w} + M_2 \ln \frac{M_1}{M_w}\right) = R_{\alpha\beta}(M_1 + M_2)$$

These relations are valid in the case of different mass values of two RH neutrinos and massive active neutrinos

Previously similar restrictions in the form of saturated Schwarz inequality have been obtained in literature only for the partial case of massless active neutrinos or degenerate masses of heavy sterile neutrinos.

Improved Constraints

Using these new relations between observables, we can improve previous experimental constraints **by an order of magnitude**

quantity	upper limit (calculated)	upper limit (old table)
$\lvert\hat{S}_{e\tau}\rvert$	$0.58 \cdot 10^{-3}$	$4.5 \cdot 10^{-3}(1.8 \cdot 10^{-3})$
$\lvert\hat{S}_{\mu\tau}\rvert$	$0.58 \cdot 10^{-3}$	$5.2 \cdot 10^{-3}(1.4 \cdot 10^{-3})$

Constraints for U^2_{tot}, M

Using best values of known parameters of active neutrino oscillation and an approximation $M_1 \approx M_2 \approx M$ we can rewrite our observable parameters as:

For Normal Ordering:

$$|S_{ab}| = U^2_{tot}\, TrigNO_{ab}(a_2), \qquad |R_{ab}| = U^2_{tot}\, TrigNO_{ab}(a_2) \ln \frac{M}{Mw}$$

$$U^2_{tot} \leq 6.85 \cdot 10^{-4} (2.62 \cdot 10^{-4}) \qquad U^2_{tot} \ln \frac{M}{M_w} \leq 9.78 \cdot 10^{-4} (1.71 \cdot 10^{-6})$$

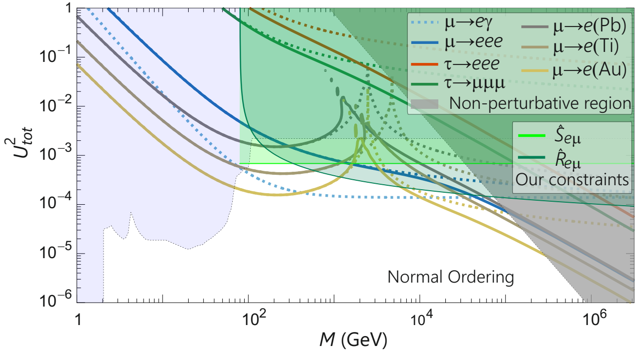
For Inverted Ordering:

$$|S_{ab}| = U^2_{tot}\, TrigIO_{ab}(a_1 - a_2), \qquad |R_{ab}| = U^2_{tot}\, TrigIO_{ab}(a_1 - a_2) \ln \frac{M}{Mw}$$

$$U^2_{tot} \leq 4.91 \cdot 10^{-4} (1.88 \cdot 10^{-4}) \qquad U^2_{tot} \ln \frac{M}{M_w} \leq 7.00 \cdot 10^{-4} (1.23 \cdot 10^{-6})$$

$Trig_{ab}$ - trigonometrical functions of Majorana phases of active neutrinos

Constraints on U^2_{tot}, M (present data)



Our constraints from present experiments.
Pink line - from our constraint on $\hat{S}_{e\mu}$. Purple line - from our constraint on $\hat{R}_{e\mu}$

We compare with a more complicated model in

Urquía-Calderón, K.A., Timiryasov, I. & Ruchayskiy, O. *Heavy neutral leptons — Advancing into the PeV domain.* J. High Energ. Phys. 2023, **167** (2023)

Constraints on parameters of the neutrino extension from baryon asymmetry of the Universe

Baryon asymmetry

Baryon asymmetry in the neutrino modified SM

$$\Delta B = \frac{28}{79}\mathrm{Tr}\Delta_N|_{T_W} = \frac{28}{79}\frac{\pi^{\frac{3}{2}}\sin^3\phi}{384\cdot 3^{\frac{1}{3}}\Gamma(\frac{5}{6})}\frac{m_{sol}^{\frac{1}{2}}m_{atm}^{\frac{3}{2}}M_1^{\frac{1}{2}}M_2^{\frac{3}{2}}M_0^{\frac{7}{3}}}{v^4 T_W(\Delta M_{21}^2)^{\frac{2}{3}}}\delta_{CP}$$

$$\frac{n_B}{s} = 7\cdot 10^{-4}\mathrm{Tr}\Delta_N|_{T_W}$$

T. Asaka and M. Shaposhnikov, *The νMSM, dark matter and baryon asymmetry of the Universe*, Phys. Lett. B, **620** 17 (2005)

We have expressed it in terms of observables

$$\Delta B = \frac{7\pi^{\frac{3}{2}}\sin^3\phi}{912\cdot 3^{\frac{1}{3}}\Gamma(\frac{5}{6})}\frac{M_0^{\frac{7}{3}}M_1^8}{M_W^6 T_W(\Delta M_{21}^2)^{\frac{5}{3}}}\times\Big(\hat{S}_{ee}\mathrm{Im}\left[\hat{S}_{e\mu}^*\hat{R}_{e\mu}+\hat{S}_{e\tau}^*\hat{R}_{e\tau}\right]+ + \hat{S}_{\mu\mu}\mathrm{Im}\left[\hat{S}_{\mu e}^*\hat{R}_{\mu e}+\hat{S}_{\mu\tau}^*\hat{R}_{\mu\tau}\right]+\hat{S}_{\tau\tau}\mathrm{Im}\left[\hat{S}_{\tau e}^*\hat{R}_{\tau e}\hat{S}_{\tau\mu}^*\hat{R}_{\tau\mu}\right]\Big)$$

Careful assumptions

Under approximation $\cosh(2\mathfrak{Im}\omega) = \sinh(2\mathfrak{Im}\omega) = \frac{\exp(2\mathfrak{Im}\omega)}{2}$

$$S_{\alpha\beta}\left(M_1 \ln\frac{M_2}{M_w} + M_2 \ln\frac{M_1}{M_w}\right) = R_{\alpha\beta}(M_1 + M_2)$$

Consequently

$$\mathrm{Im}\left[S^*_{\alpha\beta}R_{\alpha\beta}\right] = 0, \qquad \Delta B = 0$$

Only taking into account a difference $\cosh(2\mathfrak{Im}\omega) \neq \sinh(2\mathfrak{Im}\omega)$ we get

For the case of normal ordering the top estimate is:

$$\mathrm{Im}\left[S^*_{\alpha\beta}R_{\alpha\beta}\right] \leq \frac{\ln\frac{M2}{M1}}{M1 + M2}|S_{\alpha\beta}|\frac{\sqrt{m_3^2 + 4m_2^2 + 8m_3m_2}}{2v^2}$$

Baryon asymmetry in terms of $S_{\alpha\beta}$, $R_{\alpha\beta}$

For the case of normal ordering we get:

$$\frac{n_B}{s} \leq \frac{7 \cdot 10^{-4} \pi^{\frac{3}{2}} \sin^3 \phi}{384 \cdot 12^{\frac{1}{3}} \Gamma(\frac{5}{6})} \frac{M_0^{\frac{7}{3}} M^{\frac{11}{3}} \sqrt{m_3^2 + 4m_2^2 + 8m_3 m_2}}{T_W v^2 M_W^4} \left(\frac{M}{\Delta M_{21}}\right)^{\frac{2}{3}} \sum_{\alpha, \beta \neq \alpha} \hat{S}_{\alpha\alpha} |\hat{S}_{\alpha\beta}|$$

And using constraints on the observables, we get:
normal ordering

$$\frac{n_B}{s} \leq 4.6 \left(\frac{M}{\Delta M_{21}}\right)^{\frac{2}{3}} (M/1\mathrm{GeV})^{\frac{11}{3}}$$

inverted ordering

$$\frac{n_B}{s} \leq 10.2 \left(\frac{M}{\Delta M_{21}}\right)^{\frac{2}{3}} (M/1\mathrm{GeV})^{\frac{11}{3}}$$

On the other hand, we know $n_B/s = (8.8 - 9.8) \cdot 10^{-11}$

Therefore, the upper bound on the observable parameters $S_{\alpha\beta}$ and $R_{\alpha\beta}$ from collider experiments and the lower bound from baryon asymmetry differ by many orders of magnitude.

Conclusions

- We obtained new constraints in the form of saturated Schwarz inequality between observable parameters of neutrino extension of SM for the case of heavy right-handed neutrinos of different masses and massive active neutrinos:
  $|S_{ab}|^2 = S_{aa} \cdot S_{bb} \qquad S_{\alpha\beta}\left(M_1 \ln \frac{M_2}{M_w} + M_2 \ln \frac{M_1}{M_w}\right) = R_{\alpha\beta}(M_1 + M_2)$
- Using this expression we were able to improve experimental constraints for observables S_{ab}
- We obtained a new expression for baryon asymmetry via observable parameters.
- We have shown that the upper bound on the observable parameters $S_{\alpha\beta}$ and $R_{\alpha\beta}$ (from the non-observation of forbidden processes in the SM) and the lower bound (from baryon asymmetry) differ by many orders of magnitude.

These results are presented in detail in

V. Gorkavenko, O. Khasai, O. Ruchayskiy, M. Tsarenkova, *Constraints on the parameters of the neutrino extension of the Standard Model.* **arXiv:2408.02107** (2024)

Thank you for attention!

Experimental bounds for non-diagonal observables

Observable	Experimental value	Constraint
$\mathrm{BR}(Z \to \nu\nu)$	$N_\nu = 2.9840 \pm 0.0082$ [52]	$1.05(\hat{S}_{ee} + \hat{S}_{\mu\mu}) + \hat{S}_{\tau\tau} \lesssim 3.5 \times 10^{-3}$
m_W	80.379 ± 0.012 GeV [53]	$\hat{S}_{ee} + \hat{S}_{\mu\mu} \lesssim 1.3 \times 10^{-3}$
$\Gamma(Z \to e^+e^-)$	83.92 ± 0.12 MeV [52]	$\hat{S}_{ee} + \hat{S}_{\mu\mu} \lesssim 0.53 \times 10^{-3}$
$a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}}$	$(-8.7 \pm 3.6) \times 10^{-13}$ [54]	$\hat{S}_{ee} \lesssim 9.3$
$\Gamma(Z \to \mu^+\mu^-)$	83.99 ± 0.18 MeV [52]	$\hat{S}_{ee} + \hat{S}_{\mu\mu} \lesssim 1.4 \times 10^{-3}$
$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	$(2.74 \pm 0.73) \times 10^{-9}$ [55]	$\hat{S}_{\mu\mu} \lesssim 0.2$
$\Gamma(Z \to \tau^+\tau^-)$	84.08 ± 0.22 MeV [52]	$\hat{S}_{ee} + \hat{S}_{\mu\mu} \lesssim 2.9 \times 10^{-3}$
$G_F^{\mu\tau}/G_F^{e\tau}$	1.0018 ± 0.0014 [56]	$\hat{S}_{ee} \lesssim 2.6 \times 10^{-3}$
$G_F^{e\tau}/G_F$	1.0011 ± 0.0015 [56]	$\hat{S}_{\mu\mu} \lesssim 1.0 \times 10^{-3}$
$G_F^{\mu\tau}/G_F$	1.0030 ± 0.0015 [56]	$\hat{S}_{\tau\tau} \lesssim 0.64 \times 10^{-3}$

Rupert Coy and Michele Frigerio. *Effective approach to lepton observables: the seesaw case.* Phys. Rev. D, 99(9):095040, 2019

Experimental bounds for diagonal observables

Observable	Experimental upper limit	Constraint
$\mathrm{BR}(h \to e\mu)$	$3.5(0.3) \times 10^{-4}$ (95% C.L.) [57,58]	$\lvert\hat{R}_{e\mu}\rvert \lesssim 81(24)$
$\mathrm{BR}(Z \to e\mu)$	7.5×10^{-7} (95% C.L.) [59]	$\lvert\hat{R}_{e\mu}\rvert \lesssim 0.065$
$\mathrm{BR}(\mu \to e\gamma)$	$4.2(0.6) \times 10^{-13}$ (90% C.L.) [60,61]	$\lvert\hat{S}_{e\mu}\rvert \lesssim 6.8(2.6) \times 10^{-6}$
$\mathrm{BR}(\mu \to eee)$	$1.0 \times 10^{-12}(10^{-16})$ (90% C.L.) [62,63]	$\lvert\hat{R}_{e\mu}\rvert \lesssim 5.6 \times 10^{-5}(5.6 \times 10^{-7})$
$\mathrm{BR}(\mu Au \to eAu)$	7×10^{-13} (90% C.L.) [64]	$\lvert\hat{R}_{e\mu}\rvert \lesssim 9.7 \times 10^{-6}$
$\mathrm{BR}(\mu Ti \to eTi)$	$4.3 \times 10^{-12}(10^{-18})$ (90% C.L.) [65–67]	$\lvert\hat{R}_{e\mu}\rvert \lesssim 3.5 \times 10^{-5}(1.7 \times 10^{-8})$
$\mathrm{BR}(\mu Al \to eAl)$	10^{-16} (90% C.L.) [68]	$\lvert\hat{R}_{e\mu}\rvert \lesssim 2.4 \times 10^{-7}$
$\mathrm{BR}(h \to e\tau)$	$6.9(0.3) \times 10^{-3}$ (95% C.L.) [57,58]	$\lvert\hat{R}_{e\tau}\rvert \lesssim 22(4.5)$
$\mathrm{BR}(Z \to e\tau)$	9.8×10^{-6} (95% C.L.) [69]	$\lvert\hat{R}_{e\tau}\rvert \lesssim 0.24$
$\mathrm{BR}(\tau \to e\gamma)$	$3.3(0.5) \times 10^{-8}$ (90% C.L.) [70,71]	$\lvert\hat{S}_{e\tau}\rvert \lesssim 4.5(1.8) \times 10^{-3}$
$\mathrm{BR}(\tau \to eee)$	$2.7(0.05) \times 10^{-8}$ (90% C.L.) [71,72]	$\lvert\hat{R}_{e\tau}\rvert \lesssim 0.022\ (3.0 \times 10^{-3})$
$\mathrm{BR}(h \to \mu\tau)$	$0.014(3 \times 10^{-3})$ (95% C.L.) [58,73]	$\lvert\hat{R}_{\mu\tau}\rvert \lesssim 31(4.5)$
$\mathrm{BR}(Z \to \mu\tau)$	1.2×10^{-5} (95% C.L.) [74]	$\lvert\hat{R}_{\mu\tau}\rvert \lesssim 0.26$
$\mathrm{BR}(\tau \to \mu\gamma)$	$4.4(0.3) \times 10^{-8}$ (90% C.L.) [70,75]	$\lvert\hat{S}_{e\tau}\rvert \lesssim 5.2(1.4) \times 10^{-3}$
$\mathrm{BR}(\tau \to \mu\mu\mu)$	$2.1(0.1) \times 10^{-8}$ (90% C.L.) [72,75]	$\lvert\hat{R}_{\mu\tau}\rvert \lesssim 0.019\ (4.2 \times 10^{-3})$
$\lvert d_e\rvert$	1.1×10^{-29} e cm (90% C.L.) [76]	$\lvert\mathrm{Im}(\hat{S}_{e\mu}\hat{S}_{e\tau}\hat{S}_{\mu e})\rvert \lesssim 0.02$

Rupert Coy and Michele Frigerio. *Effective approach to lepton observables: the seesaw case.* Phys. Rev. D, 99(9):095040, 2019

Casas-Ibarra matrix parametrisation

$$F = \frac{i}{v} U_\nu \sqrt{m_\nu^{\mathrm{diag}}} \mathcal{R} \sqrt{M^{\mathrm{diag}}}$$

$(m_\nu^{\mathrm{diag}})_{ij} = \delta_{ij} m_i$, m_i – light neutrino masses.

$$U_\nu = V^{(23)} U_\delta V^{(13)} U_{-\delta} V^{(12)} \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)\,, \quad U_{\pm\delta} = \mathrm{diag}(e^{\mp i\delta/2}, 1, e^{\pm i\delta/2})$$

The non vanishing $V^{(ij)}$ are given by:

$$V^{(ij)}_{ii} = V^{(ij)}_{jj} = \cos\theta_{ij}\ ,\ \ V^{(ij)}_{ij} = -V^{(ij)}_{ji} = \sin\theta_{ij}\ ,\ \ V^{(ij)}_{kk} = 1 \quad \text{for } k \neq i, j.$$

θ_{ij} – light neutrino mixing angles, δ – Dirac phase, $\alpha_{1,2}$ – Majorana phases.

One complex angle ω and elements of M^{diag} are unknown parameters

$$\mathcal{R}^{\mathrm{NO}} = \begin{pmatrix} 0 & 0 \\ \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix}, \qquad \mathcal{R}^{\mathrm{IO}} = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \\ 0 & 0 \end{pmatrix}$$