

Recap of MAP 3 TeV lattice and IR design studies

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Lattice Challenges

Assuming we are able to accelerate enough muons to collider energy, the design of the collider ring itself is *not trivial* either.

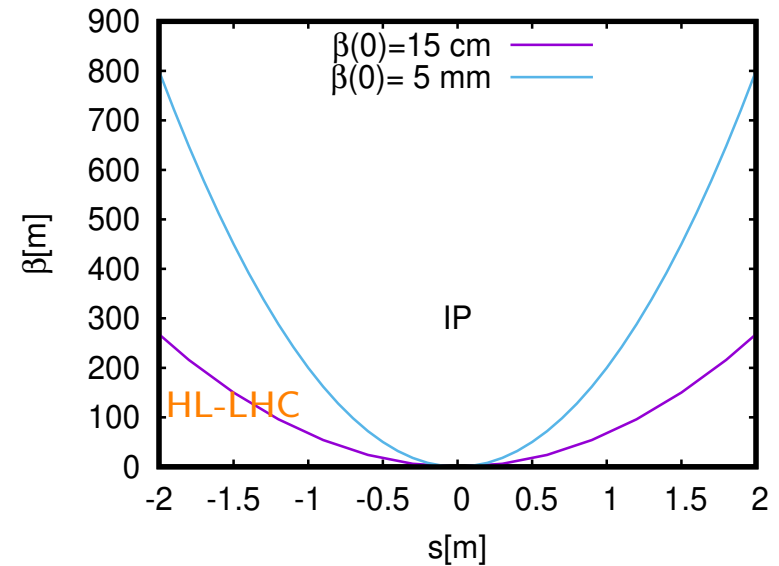
Beam size

$$\sigma = \sqrt{\beta\epsilon}$$

For a magnet free region

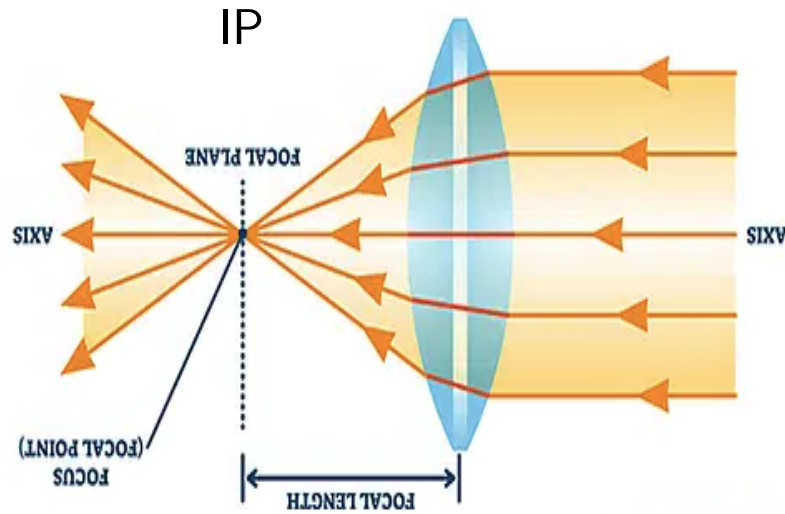
$$\beta(s) = \underbrace{\beta^*}_{\substack{\beta \text{ at IP} \\ \swarrow}} + \frac{s^2}{\beta^*}$$

β^* must be small for maximizing the luminosity!



What limits the free space available, for the experiments?

First quadrupole lens at $s = s^*$:



focal length $s^* \approx \frac{1}{K_q}$

integrated strength

$$\hat{\beta} = \beta(s^*) = \beta^* + \frac{s^{*2}}{\beta^*} \simeq \frac{s^{*2}}{\beta^*} \rightarrow s^* = \sqrt{\beta^* \hat{\beta}}$$

$$\Delta\xi \propto \hat{\beta} \times K_q = \frac{s^*}{\beta^*}$$

linear chromaticity

For a given β^*

- s^* must be small;
- K_q becomes large.

In this design we fixed ± 6 m space for the experiment.

- Large transverse emittance ($\epsilon^n \approx 25 \mu\text{m}$).
- Low β^* (few mm):
 - Strong IR quadrupoles at large β :
 - * large chromaticity;
 - * large sensitivity to their misalignments and field errors.
- Small circumference, particularly important for short living particles!
- High density: $N \approx 2 \times 10^{12}$ per bunch.
 - Protection of magnets and detectors.
 - Neutrinos hotspots limit to ≈ 0.5 m field-free regions at beam energy ≈ 1.5 TeV
- $\sigma_\ell \leq \beta^*$ to avoid *hour-glass* effect, detrimental for the luminosity.
- Expected large momentum spread ($dp/p \approx 0.1\%$) requires
 - small $|\alpha_p|$ ($\approx 1 \times 10^{-5}$) over the momentum range to achieve short bunches with reasonable RF voltage;
 - sufficient Dynamic Aperture ($\gtrsim 3\sigma$) in presence of strong sextupoles and large dp/p .

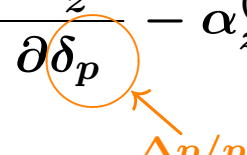
IR chromaticity correction

The “usual” global chromaticity correction using sextupoles in the arcs was unsatisfactory:
IR chromaticity must be corrected locally.

Montague chromatic functions

$$W_z \equiv \sqrt{A_z^2 + B_z^2}$$

$$A_z \equiv \frac{\partial \alpha_z^{(0)}}{\partial \delta_p} - \alpha_z^{(0)} B_z \quad B_z \equiv \frac{1}{\beta_z^{(0)}} \frac{\partial \beta_z}{\partial \delta_p} \quad (z = x/y)$$



 $\Delta p/p$

$$\frac{dA_z}{ds} = 2B_z \frac{d\mu_z^{(0)}}{ds} - \beta_z^{(0)} k \quad \text{and} \quad \frac{dB_z}{ds} = -2A_z \frac{d\mu_z^{(0)}}{ds}$$

$$k \equiv \begin{cases} +(K_1 - D_x K_2) & (\text{hor.}) \\ -(K_1 - D_x K_2) & (\text{vert.}) \end{cases} \quad \begin{array}{l} K_1 \equiv \text{quad. strength} \\ K_2 \equiv \text{sext. strength} \end{array}$$


- $A_z(s)$ becomes non-zero when going from the IP ($A_z=B_z=0$) through the IR quads.
- $B_z(s)=0$ as long as $d\mu_z^{(0)}/ds=0$.

A sextupole close to the FF quads (large $\beta_z \rightarrow d\mu_z^{(0)}/ds=0$) corrects A_z and keeps $B_z=0$.

- horizontal dispersion must be generated in the IR

Second order chromaticity

$$\xi_z^{(2)} = \frac{1}{8\pi} \int_0^C ds \left(-k B_z \pm 2K_2 \frac{dD_x^{(0)}}{d\delta_p} \right) \beta_z^{(0)} - \xi_z^{(1)}$$

lin. chrom. 

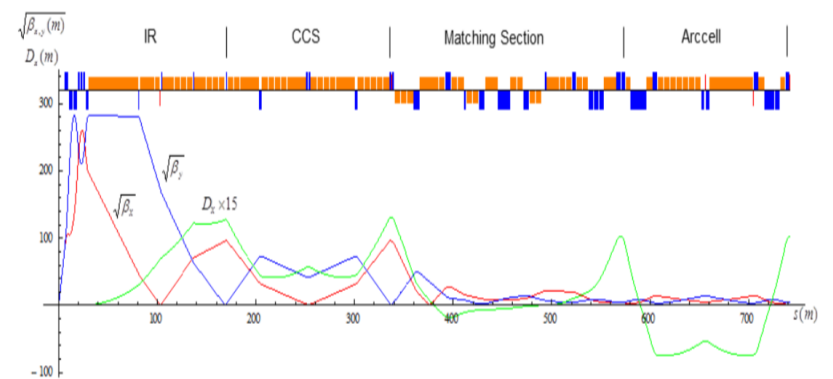
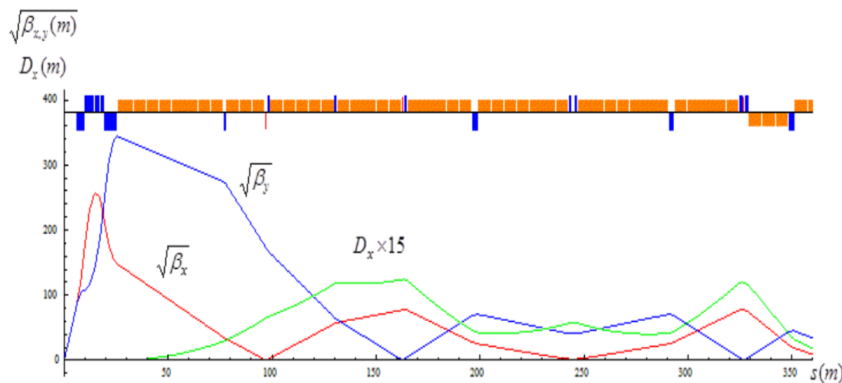
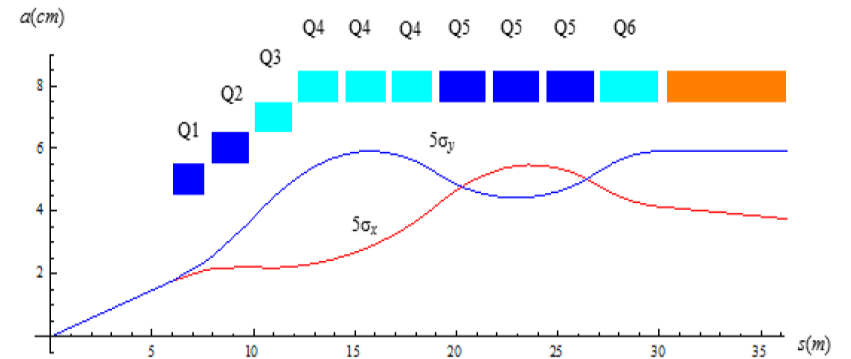
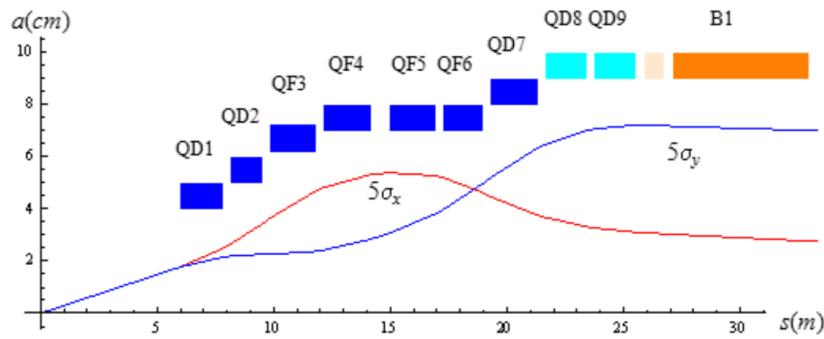
→ It may be necessary to correct $dD_x^{(0)}/d\delta_p$, in addition to $B_{x,y}$.

- It is convenient to focus first in the horizontal plane ($\hat{\beta}_y \gg \hat{\beta}_x$).
 - W_y is first corrected by a **single** sextupole at $\Delta\mu_y \approx 0$ from IP and very small β_x (for normal sextupole it ensures that the effect on detuning with amplitude and resonance driving terms are small, a consequence of $H = ax^3 - 3axy^2$).
 - W_x is corrected with one sextupole at $\Delta\mu_x = m\pi/2$ from IP and $\beta_x \gg \beta_y$;
 - * a **“twin”** sextupole at $-I$ reinforces the correction and cancels the aberrations.
- 1st order dispersion can be corrected by sextupoles at a low $\beta_{x,y}$ locations.
- D_x at all sextupoles should be as large as possible.

3 TeV c.o.m. case

2 IR designs, D-F-D triplet and F-D-F-D quadruplet, with $\beta^* = 5$ mm and $s^* = 6$ m.

- Quads in cyan include a dipole component.
- Space between quads for tungsten masks. Aperture: $\pm 5\sigma \pm 1.5$ cm for absorbers.
- IR chromaticity correction “à la Montague”.



Magnet data for the FF triplet

	QD1	QD2	QF3	QF4-6	QD7	QD8-9	B1
Aperture [mm]	80	100	124	140	160	180	180
Gradient [T/m]	-250	-200	161	144	125	-90	0
B_{dip} [T]	0	0	0	0	0	2	8
Length [m]	1.85	1.4	2.0	1.7	2.0	1.75	5.8

Magnet data for the FF quadruplet

	Q1	Q2	Q3	Q4	Q5	Q6	B
Aperture [mm]	90	110	130	150	150	150	150
Gradient [T/m]	267	218	-154	-133	129	-128	0
B_{dip} [T]	0	0	2	2	0	2	6.9
Length [m]	1.6	1.85	1.8	1.96	2.3	2.85	5.9

Nb₃Sn technology
 @4.5 K (1.9 K).
 Design optimized
 by ROXIE.

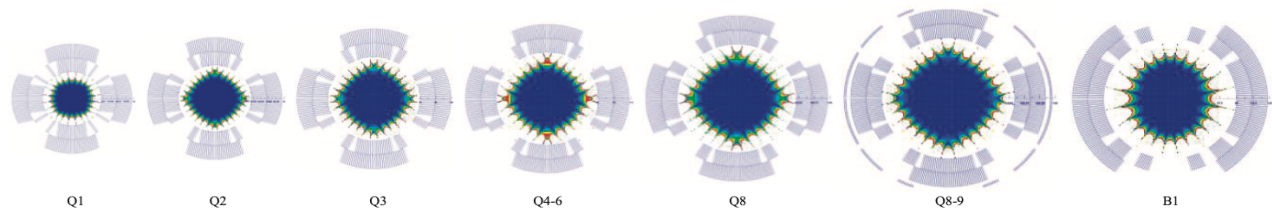
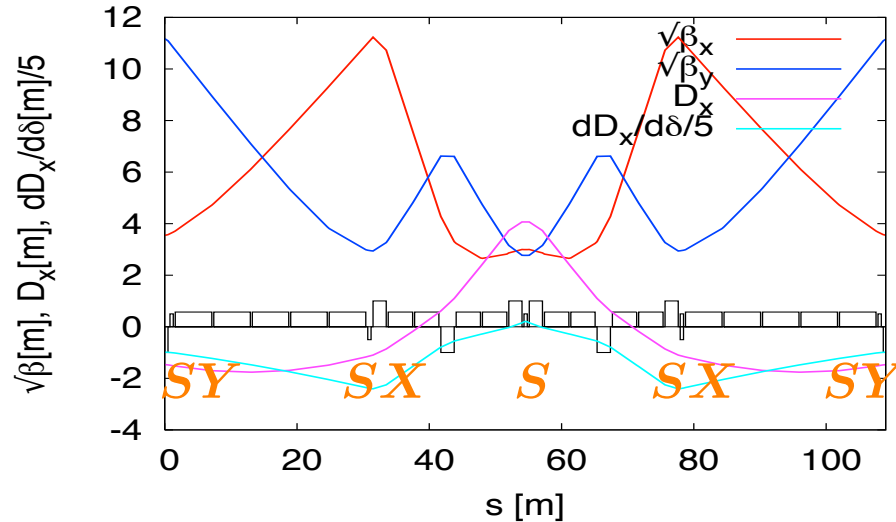


Figure 2: Coil cross-sections.

(V. V. Kashikhin, A. V. Zlobin)

Flexible momentum compaction arc cell

- Neutrinos hot spots limit length of straight sections to about 0.5 m
→ long arc quadrupoles replaced by **combined function** magnets.
- Large (positive) IR contribution to α_p must be compensated in the arcs.
- α_p must be small over the momentum range.



- Orthogonal chromaticity correction.
 - Phase advance and number of cells adjusted for canceling 3rd order resonance driving terms.
- Quads and sextupole in the middle control α_p and $d\alpha_p/d\delta_p$

Data for the arc magnets

	QDA1	QDA3	QFA2	QFA4	BEA1	BEA2	BEA3
Gradient [T/m]	-31	-35	85	85	10.2	10.2	10.2
B_{dip} [T]	8.9	8.9	7.9	7.9	10.4	10.4	10.4
Length [m]	3.34	5	4	4	6	6	5

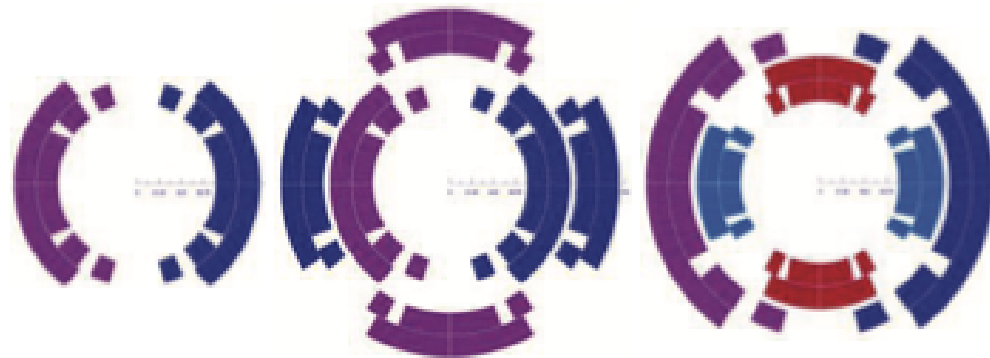


Figure 4: Bending dipole (left) and combined-function quadrupoles with the dipole coil inside (center) and outside (right) of the main quadrupole coil. The color shades represent the current directions in the coils.

(V. V. Kashikhin, A. V. Zlobin)

Matching Section

The matching section used for

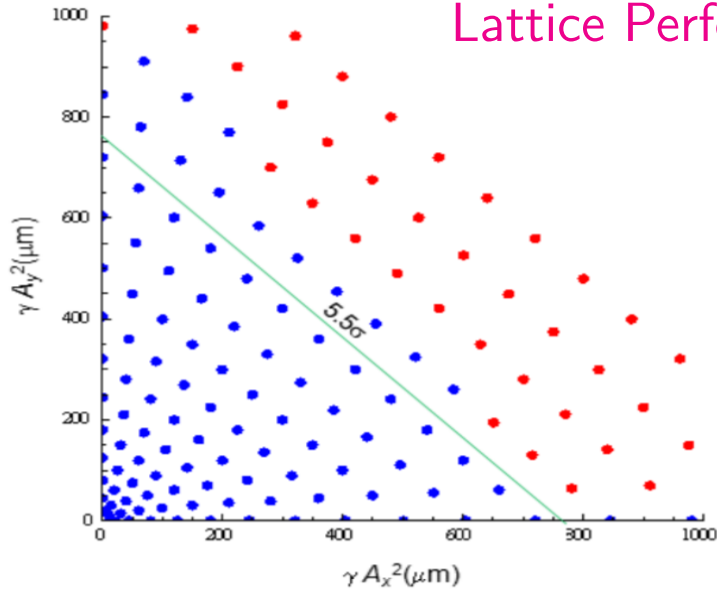
- matching the IR to the arc cell;
- adjusting β^* (≈ 3 mm to 3 cm), w/o changing the IR and the arc, for
 - coping with possible larger than expected emittance
 - commissioning purposes.

It may host also

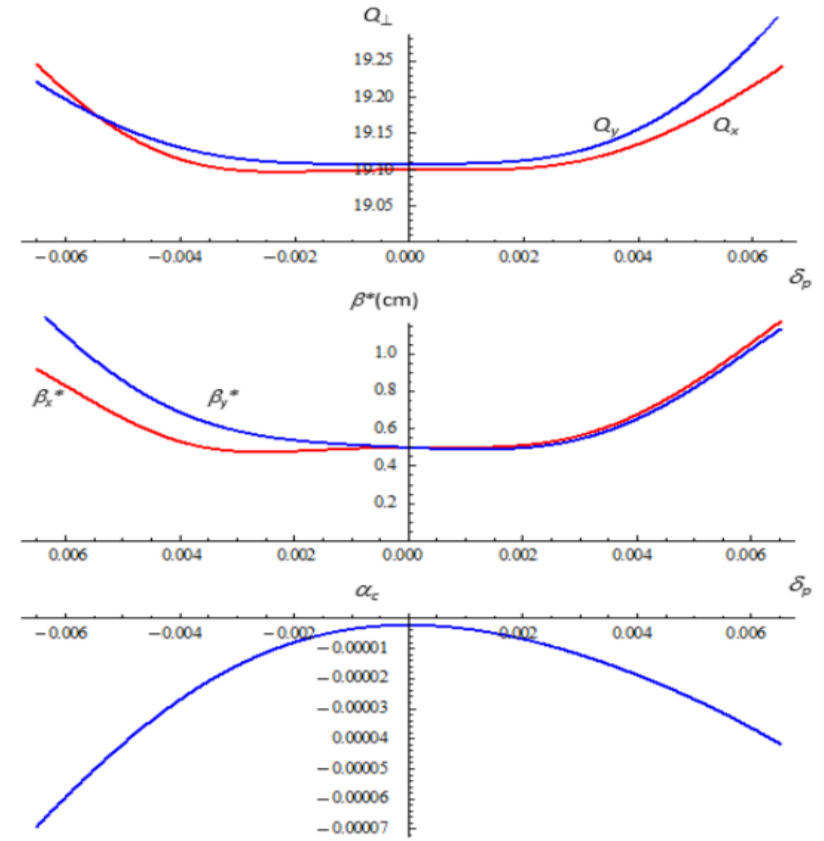
- RF cavities
- Injection elements
- halo removal, if studies show it is needed

It must not include long straights: it is not easy to change the quads without affecting the dispersion... A dipole chicane was introduced, but this solution requires moving the chicane dipoles when β^* is changed.

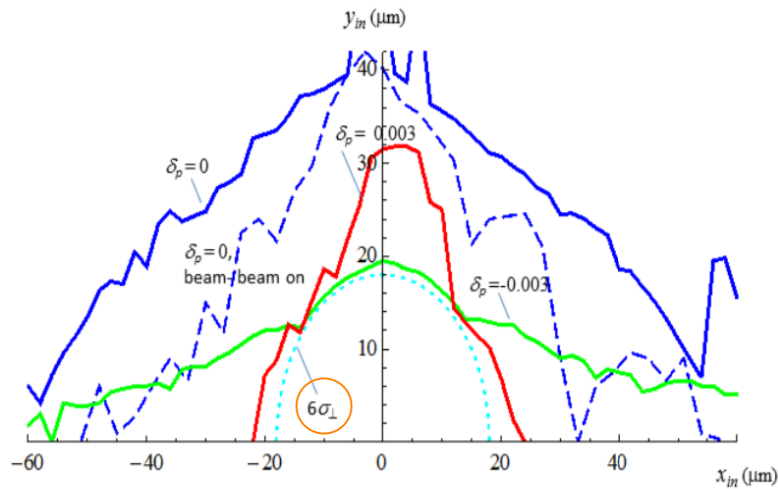
Lattice Performance for the 3 TeV MC



$$\#\sigma = \sqrt{\gamma A^2 / \epsilon^N}$$



$$\sigma_p / p = 0.1\%$$



$$\sigma^* = 3\mu\text{m}$$

Design parameters

	Higgs Factory		High Energy Collider	
Beam energy [TeV]	0.063	0.75	1.5	3
\mathcal{C} [Km]	0.3	2.5	4.3	6.3
IP's #	1	2	2	2
β^* [cm]	1.7	1	0.5	1
σ_ℓ [cm]	6.3	1	0.5	1
α_p	0.079	-1.3×10^{-5}	-0.5×10^{-5}	-1.2×10^{-3}
ϵ_\perp^N [μm]	300	25	25	25
σ_p/p [%]	0.004	0.1	0.1	0.1
n_b	1	1	1	1
N_μ	4×10^{12}	2×10^{12}	2×10^{12}	2×10^{12}
f_{rf} [GHz]	0.2	1.3	1.3	-
V_{rf} [MV]	0.1	12	50	-
Repetition rate [Hz]	15	15	12	15
Average \mathcal{L} [$\text{cm}^{-2}\text{sec}^{-1}$]	8×10^{31}	1.25×10^{34}	4.6×10^{34}	7.1×10^{34}

Summary

Some of the challenges related to the design of a Muon Collider and possible approaches for overcoming them have been shown.

- The 3 TeV collider conceptual designs is relatively mature. The related studies on magnets, energy deposition and beam-beam effects haven't pointed out to showstoppers.
 - The design assumed fields compatible with already available technology.
- The effect of misalignments, field errors and fringe fields has been not studied for the 3 TeV case.
 - IR quads fringe field studies by V. Kapin have shown a reduction of the DA in the 1.5 TeV collider Fermilab design.